Name:_____

No calculator

You're in the right place! This pretest is just to understand what the math club coaches need to teach, so don't worry about your scores. Just have fun!

Directions:

- You have 30 minutes to complete this pretest.
- You do not need to provide units in your answers.
- Try to provide exact answers (no rounding).
- Please **box or circle** your answers and make sure they are **legible**.
- **Please show your work.** This pretest is used to determine what topics we need to cover, not to evaluate you. (We want to see what you can do, not if you can take 20 minutes to solve one question!)

Remember, you are not expected to finish this pretest or know all the material on it. Also, feel free to skip around! If you do not know how to answer a question or it is taking too long, leave it for later and work on a different question.

1. Evaluate $5x + 2x + 3 \bullet 2$ when $x = 3$.

The expression becomes 7x+6 = 7*3 + 6 = **27**.

2. If $4x + 3y = 34$ and $2x + y = 14$, find $\frac{x}{y}$.

Multiply the second equation by 2 to get 4x + 2y = 28. Subtract that from the first equation to get y = 34-28 = 6. Then 2x + 6 = 14, so 2x = 8 and x = 4. x/y = 4/6 = **2/3**.

3. How many positive multiples of 3 are between 1 and 50?

It goes from 3 to 48, i.e. 3*1 to 3*16. Then the answer is 16-1+1 = **16**.

4. A cube has a volume of 8 cubic centimeters. A second cube has an edge length that is two times that of the first. What is the volume of the second cube in cubic centimeters?

The first cube has side length equal to the cube root of 8, or 2 centimeters. Then the second cube has side length 4, so the volume of the second cube is 4^3 = **64.**

5. Aline graphs a line through the points (2,5) and (8, 8). If (4, A) is a point on Aline's line, what is A?

The slope is (8-5)/(8-2) = 3/6 = 1/2. Using point-slope form, we get that the line is (y-5) = ½(x-2), so A-5 = ½(4-2) = 1, and A = **6**.

6. Let a 🙄 b = a^2+b. What is 3 🙄 (5 🙄 2)?

We work first inside parentheses: 5 🙄 2 = 5^2+2 = 27. Then we need 3 🙄 27. This is 3^2 + 27 = **36**.

7. Equilateral triangle ABC has $AB = BC = AC = 12$. Square $DEFG$ has ***side length** equal to one fourth of the perimeter of triangle ABC. What is the area of square $DEFG$?

The perimeter of ABC is 12+12+12 = 36. Then square DEFG has ***side length** ¼ * 36 = 9, and the area of the square is 9^2 = **81**. **NOTE: there was a typo where it said "perimeter" instead of side length. This would lead to the answer of (9/4)^2 = 81/16 = 5.063.**

8. Compute the number of positive divisors of 54.

We prime factorize 54 as 2*3*3*3 = 2^1 * 3^3. Then the number of divisors is (1+1)(3+1) = **8**.

9. How many ways are there to rearrange the letters in the word "MUSES"?

Pretending all the letters are distinct, we get 5! = 120 permutations. Then we divide by 2! = 2 because there are 2 S's. Thus there are 120/2 = **60** permutations.

10. If $x^2 - y^2 = 10$ and $x - y = 2$, what is $x + y$?

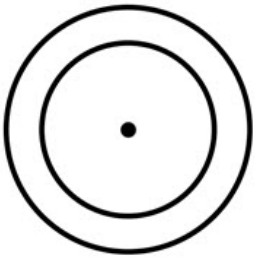
Using the difference of squares identity, we derive (x-y)(x+y) = 10. Then plugging in, we have that 2(x+y) = 10, so x+y = **5**.

11. If Dr. Meek rolls 2 dice, what is the probability that the product of the two numbers is even?

It's easier to use complementary counting in this case. The product of any two numbers is even as long as at least one of the two is even. Then P(odd product) = P(1st roll odd) * P(2nd roll odd) = ½ * ½ = ¼. Thus P(even product) = 1 - P(odd product) = 1-¼ = **3/4**.

12. Triangle ABC has $AB = 10$, $BC = 24$, $AC = 26$. What is the length of the altitude from B to AC?

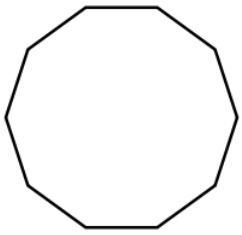
This is a 5-12-13 triangle and is thus right. Then the area of ABC = ½ * 10 * 24 = 120. Let h be the altitude from B. Then the area is also equal to ½ * h * AC = ½ * h * 26 = 13h. Thus 13h = 120, so h = **120/13**. (*You can also use similar triangles to solve this.)



13. Circle A has a radius of 4. It is inside and concentric with Circle B, which has a radius of 6. What is the area of the region inside Circle B but outside Circle A?

Circle A has area 4^2 * pi = 16pi. Circle B has area 6^2 * pi = 36pi. The difference between these two areas is 36pi - 16pi = **20pi**.

14. How many degrees are in an interior angle of a regular decagon? (A decagon is a polygon with 10 sides.)



Solution 1. The sum of the interior angles is (10-2)*180 = 1440 degrees, because you can draw 10-2 = 8 triangles inside of the decagon. Then each interior angle is 1440/10 = **144** degrees, as all interior angles are equal for regular polygons.
Solution 2. The sum of the exterior angles is always 360 degrees. Because it's a regular polygon, each exterior angle is equal, so an exterior angle measures 360/10 = 36 degrees. Then an interior angle measures 180-36 = **144** degrees.

15. Simplify $(\sqrt{63} + 2\sqrt{7})\sqrt{3}$.

Note that 63 = 9 * 7 = 3^2 * 7. Then $\sqrt{63} = \sqrt{9} * \sqrt{7} = 3\sqrt{7}$, so $(\sqrt{63} + 2\sqrt{7})\sqrt{3} = (3\sqrt{7} + 2\sqrt{7})\sqrt{3}$. This equals $5\sqrt{7} \cdot \sqrt{3} = 5\sqrt{21}$. The answer is **5sqrt(21).**

16. Andy says, "We have A ways to distribute 4 identical baseballs to 8 *different* people." Bob says, "Yes, and we have B ways to distribute the 4 baseballs among 8 *identical* boxes". Compute $A - B$.

We use stars and bars to find A. There are 4 stars and 7 bars, so it's $\binom{7+4}{4}$. Thus, $A = \binom{11}{4} = \frac{11!}{4!7!} = \frac{11*10*9*8}{4*3*2*1} = 11 * 10 * 3 = 330$. Computing B is not too difficult: the only ways to distribute 4 identical objects among 8 identical boxes is (1,1,1,1,0,0,0,0), (2,1,1,0,0,0,0,0), (2,2,0,0,0,0,0,0), (3,1,0,0,0,0,0,0), and (4,0,0,0,0,0,0,0). Then B = 5. Therefore, A - B = 330 - 5 = **325**.

17. Joseph has 7 marbles in a bag—there are 2 red marbles, 1 blue marble, and 4 purple marbles. His best friend Nate decides to be sneaky, so he flips a coin to decide whether or not he should add an extra red marble before Joseph finds out. After Nate's actions, what is the probability that a randomly chosen marble is purple?

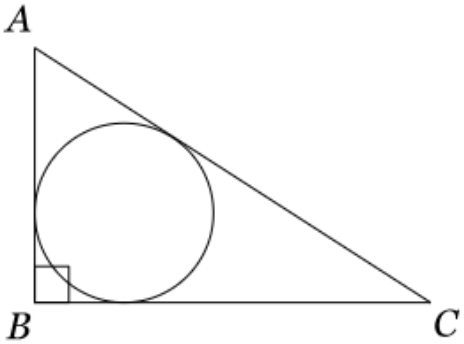
There are originally 7 total marbles and 4 purple marbles, so if Nate does not tamper with the marbles, P(choose a purple marble) = 4/7. If Nate does add an extra red marble, then there are 8 total marbles and 4 purple marbles, so P(choose a purple marble) = 4/8 = ½. There is a ½ probabilities for both events to occur, so the total probability is (½ * 4/7) + (½ * ½) = 4/14 + ¼ = 8/28 + 7/28 = **15/28**.

18. Yonda takes 6 hours to tile the floor. Zachary takes 8 hours to tile the floor. Working together, how long will it take (in hours) for both of them to tile the floor?

Yonda tiles ⅙ floor per hour. Zachary tiles ⅛ floor per hour. Thus, together they tile ⅙ + ⅛ = 7/24 floor per hour. (Why couldn't we just add the numbers 6 and 8?) Then, it will take them **24/7** hours to tile the floor.

19. To attend the play *Our Town*, an adult ticket costs x^2 dollars and a child ticket costs $2x$ dollars. One adult ticket and 2 child tickets together cost \$16.25. How much does 1 child ticket cost, in dollars?

We have that $x^2 + 4x = 16.25 \Rightarrow 4x^2 + 16x - 65 = 0$. Factoring, we get that $(2x - 5)(2x+13) = 0$. Thus $x = 5/2$ or $x = -13/2$. But clearly x cannot be negative, because then the child ticket would cost negative dollars, which is not allowed. Thus $x = 5/2$ and a child ticket costs **5** dollars.



20. Right triangle ABC has circle I inscribed in it. If $\angle B = 90°$, $\angle C = 30°$, and $AB = 3$ inches, what is the radius of circle I, in inches?

This is a 30-60-90 triangle, so since AB = 3 inches, $BC = 3\sqrt{3}$ inches and AC = 6 inches. Thus $s = \frac{a+b+c}{2} = \frac{9+3\sqrt{3}}{2}$ and the radius of circle I = s - b = $\frac{9+3\sqrt{3}}{2} - \frac{12}{2} = \frac{3\sqrt{3}-3}{2}$. The answer is **(3sqrt(3)-3)/2**.