Test 2 – Take-Home

MATH 377-01
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Name

Instructions: This is the take-home part of Test 2. This take-home is due at 9:00 am CST on Tuesday, November 5. The only resources you may use for this test are your class notes, your book, the notes on Canvas, a calculator, R, and your brain. You may not consult any other reference. After you have received a copy of this test, you may not convey any information about this test to anyone else. All questions should be directed to Dr. Hendricks.

You are to write/type relevant work and answers/solutions on $8\frac{1}{2}$ in $\times$ 11 in sheets of paper. Direct output from R is not appropriate for your answers. You should take your numerical values from R and use them in an appropriate context. All graphs/plots are to be generated in R and included in your file that contains your work/answers. Hand drawn graphs or graphs generated outside of R will receive no credit.

Whenever you use R for a problem, you are to save your commands and output and then submit an electronic copy of this file. Writing only the answer is not sufficient.

By the due date, you are to submit two PDF files:

- "Test 2 - Take-Home" assignment in Canvas. Upload (exactly) one PDF file that contains a copy of the test and the sheets of paper that have your answers, any work that was done by hand, and graphs/plots generated in R. You should consider your answers as being part of a report and should be formatted as such. **Only the answers included on the PDF that is submitted to the "Test 2 - Take-Home" assignment will be graded.** Submitting your work in multiple PDF files will result in a grade of 0 for the test.
- "Test 2 - R Work" assignment in Canvas. Upload (exactly) one PDF file that contains your R commands and output from the R Console showing your work and values in R for the test. The first entry in your R file should be a comment line that has your name listed. A comment is entered by preceding the comment with the "#" character. For example:

  ```
  > # David Hendricks / Test 2
  ```

  Your R work will only be checked for appropriate work, completeness, and academic integrity.

Rounding: Round numerical answers to four decimal places.

Failure to sign and date in the spaces provided or failure to follow directions may result in your receiving a zero on this test.

If you are unable to print your test, then you should write the appropriate statements about abiding by the rules for the test and sign your name on a sheet of paper and include it with a scan of your work.

By signing below, I have read the rules for this test, and I agree to abide by these rules.

Signature A Burgos　　　　Date 10/31/24

Hypothesis Test Steps

0. Give the name of the test you are using and explain why you are using it.

1. Given a claim, identify the null hypothesis and the alternative hypothesis, and express them both in symbolic form.

2. Given a claim and sample data, calculate the value of the test statistic.

3. Given a significance level, identify the critical value(s) and the rejection region.

4. Given a value of the test statistic, identify the P-value.

5. Make a decision: reject H_0 or fail to reject H_0.

6. State the conclusion of a hypothesis test in simple, non-technical terms.

How to Word Your Conclusion to a Hypothesis Test

	Decision: Reject H_0	Decision: Fail to Reject H_0
Claim: H_0	"There is sufficient evidence to warrant rejection of the claim that ... (original claim)."	"There is not sufficient evidence to warrant rejection of the claim that ... (original claim)."
Claim: H_a	"The sample data support the claim that ... (original claim)."	"There is not sufficient sample evidence to support the claim that ... (original claim)."

Hypothesis Test Template

Template

Test used: ____________________

H_0: __________
H_a: __________

Test statistic: __________

Critical value(s): _____________
Rejection region: _____________

p-value: __________

Decision: Reject H_0 or Fail to reject H_0
Reason for decision:

Conclusion:

Example

Test used: One mean t-test

H_0: $\mu = 12$ $\alpha = 0.05$
H_a: $\mu \neq 12$ (claim) $n = 42$

Test statistic: $t = 2.3211$

Critical value: 2.019541

```
> qt(1-0.05/2,41)=2.019541
```

Rejection region: $t \geq 2.0195$ or $t \leq -2.0195$

p-value: 1.9747

```
> 2*pt(2.3211,41)=1.974668
```

Reject H_0. The test statistic lies in the rejection region and the p-value is less than α.

There is sufficient sample evidence to conclude that the true population mean is different from 12.

1. (15 points) A study is designed to test the hypothesis H_0: $\mu = 500$ versus H_a: $\mu \neq 500$. A random sample of 32 was selected from a specified population that has a standard deviation of $\sigma = 60$, and the measurements were summarized to $\overline{y} = 520$.

(a) With $\alpha = 0.01$, is there substantial evidence that the population mean is different from 500?

(b) Calculate the probability of making a Type II error if $\mu_a = 525$.

(c) Find the values of the power curve for rejecting H_0: $\mu = 500$ for the following values of μ: 450, 460, 470, 480, 490, 510, 520, 530, 540, and 550.

(d) If the sample size is increased to 64, what is the probability of making a Type II error if the actual value of the population mean is 525.

(e) If $\alpha = 0.01$, what sample size is needed to have a probability of Type II error of at most 0.04 if the actual mean is 520 (and $\mu_0 = 500$)?

2. (10 points) Sunspots have been observed for many centuries. Records of sunspots from ancient Persian and Chinese astronomers go back thousands of years. Some archaeologists think sunspot activity may somehow be related to prolonged periods of drought in the southwestern United States. Let X be a random variable representing the average number of sunspots observed in a four-week period. A random sample of 40 such periods from the Spanish colonial times gave a sample mean of $\overline{x} = 47.0$. Previous studies of sunspot activity during this period indicate that $\sigma = 35$. It is thought that for thousands of years, the mean number of sunspots per four-week period was about $\mu = 41$. Sunspot activity above this level may or may not be linked to gradual climate change.

(a) Construct and interpret a 95% confidence interval for the mean sunspot activity during Spanish colonial period.

(b) Test the claim that the mean sunspot activity during Spanish colonial period was higher than 41. Use $\alpha = 0.05$.

3. (10 points) The file `377-coli.txt` contains the cost of living index for housing and the cost of living index for groceries. The data are given for 36 randomly selected metropolitan areas in the United States.

(a) Use the data to perform an appropriate t-test to determine if there is sufficient evidence to conclude that the mean cost of living index for housing is higher than that for groceries in these areas. Use $\alpha = 0.05$.

(b) Construct and interpret a 95% confidence interval for the difference in the cost of living index for housing and for groceries.

4. (10 points) The pathogen *Phytophithora capsici* causes bell peppers to wilt and die. Because bell peppers are an important commercial crop, this disease has undergone a great deal of agricultural research. It is thought that too much water aids the spread of the pathogen. Two fields are under study. The first step in the research project is to compare the mean soil water content for the two fields. The units are percent water by volume of soil.

Field A Samples							
8.1	8.5	8.4	7.3	8.0	7.1	13.9	12.2
13.4	11.3	12.6	12.6	12.7	12.4	11.3	12.5

Field B Samples						
10.2	10.7	15.5	10.4	9.9	10.0	16.6
15.1	15.2	13.8	14.1	11.4	11.5	11.0

(a) Assume that the distribution of soil water content in each field is mound-shaped and symmetric, use a 5% level of significance to test the claim that field A has, on average, a lower soil water content than field B.

(b) Construct and interpret a 95% confidence interval for the difference in the mean water content in each field.

5. (10 points) Several neurosurgeons wanted to determine whether a dynamic system (Z-plate) reduced the operative time relative to a static system (ALPS plate). The neurosurgeons obtained the data displayed below on operative times, in minutes, for the two systems.

Dynamic							**Static**		
370	360	510	445	295	315	490	430	445	455
345	450	505	335	280	325	500	455	490	535

(a) Is there sufficient evidence to conclude that the mean operative time is less with the dynamic system that with the static system. Use $\alpha = 0.01$.

(b) Construct and interpret a 99% confidence interval for the difference in the mean time between dynamic and static systems.

1.

Test Used: Two-tailed Z-test

$$H_0 : \mu = 500$$

$$H_a : \mu \neq 500$$

Test statistic: z = 1.885618

Critical Value: 2.575829

Rejection Region: (z < -2.575829, z > 2.575829)

p-value: 0.5865993

Decision: Fail to reject H0 because it is within the rejection region and p value is greater than alpha.

Conclusion: There is not enough evidence to conclude that the sample mean is different than 500.

a.

Test statistic: 1.885618

Critical Value: 2.575829

Not enough evidence to reject because z is not greater than 2.5758.

b.

Probability of Type II Error: 0.5866

mu	Power

c.

mu	Power
450	0.98375039
460	0.88403595
470	0.59971053
480	0.24503475
490	0.05144925
510	0.05144925
520	0.24503475
530	0.59971053
540	0.88403595
550	0.98375039

d.

Probability of Type II Error: 0.224374

e.

Required Sample Size to achieve Beta <= 0.04: 169

2.

Test Used: Right tail Z-test

$$H_0 : \mu = 41$$

$$H_a : \mu > 41$$

Test statistic: z = 1.084209

Critical Value: 1.644854

Rejection Region: (z <= -1.644854, z => 1.644854) p-value: 0.139136

Decision: Fail to reject H0 due to the p-value being greater than alpha and z being within the rejection region.

Conclusion: There is not enough evidence at the 0.05 significance level to conclude that mean sunspot activity was greater than 41.

a.

95% Confidence Interval: (36.15359 , 57.84641)

b.

z = 1.084209

Critical Value = 1.644854

p-value = 0.139136

3.

Test Used: Paired T-test

$$H_0 : \mu_0 - \mu_1 \leq 0$$

$$H_a : \mu_0 - \mu_1 > 0$$

Test statistic: t = 1.8824

Critical Value: 1.689572

Rejection Region: t >= 1.689572

p-value: 0.03406

Decision: Reject H0 because the value of 't' is greater than the critical value and alpha is greater than the p-value.

Conclusion: There is sufficient evidence to conclude that the mean cost of living index for housing is higher than that for groceries in these metropolitan areas.

a.

There is sufficient evidence to conclude that the mean cost of living index for housing is higher than that for groceries in these metropolitan areas.

b.

The mean lies within the (0.4381481, 8.117408) interval.

4.

a.

Test Used: Two-sample T-test

$$H_0 : \mu_a \geq \mu_b$$

$$H_a : \mu_a < \mu_b$$

Test statistic: t = -2.0059

Critical Value: -1.701131

Rejection Region: t < -1.701131

p-value: 0.0274

Decision: Fail to reject H0, the p-value is greater than alpha, and the test statistic is not in the rejection region.

Conclusion: At the 0.05 significance level, there is not sufficient evidence to conclude that field A has, on average, a lower soil water content than field B.

b.

$(-0.334, 4.178)$. We can be 95% confident that the true difference in mean water content is between -0.334% and 4.178%. Since this interval contains zero, this agrees with our hypothesis test conclusion that we cannot conclude there is a significant difference in mean water content between the fields.

5.

a.

Test Used: Two-sample T-test

$$H_0 : \mu_d \geq \mu_s$$

$$H_a : \mu_d < \mu_s$$

Test statistic: t = -2.6804

Critical Value: -2.554701

Rejection Region: t < -2.554701

p-value: 0.007679

Decision: Reject H0 due to alpha being greater than p-value and test statistic is in the rejection region.

Conclusion: There is enough evidence to reject the claim that the mean operative time of the dynamic system is lass than with the static.

b.

(-166.9086, 19.5276)

Since the mean difference includes 0, we cannot conclude that there is a significant difference between the operative times of the systems at the 99% confidence level.