



PRE-ASSESSMENT: DISCRETE MATHEMATICS II (GFO1)
PGFO

Attempt #1
Status: Passed



1.

Consider the following pseudocode that merges two lists of numbers into one:

```
Merge0(List1, List2)
Set OUTlist to empty
While List1 is not empty OR List2 is not empty
  If one list is empty and the other is not,
    Remove the first number from the non-empty list and add it to OUTlist
  If both lists are non-empty,
    Remove the first number from List1 and add it to OUTlist
    Remove the first number from List2 and add it to OUTlist
Return OUTlist
```

If ListA is [1, 3, 5] and ListB is [2, 4, 6] then what is the result of Merge0 (ListA, Merge0 (ListB, ListA))?

	YOUR ANSWER	CORRECT ANSWER
[1, 2, 3, 4, 5, 6]		
[1, 1, 3, 2, 5, 3, 4, 5, 6]	✓	
[2, 1, 4, 3, 6, 5]		
[1, 2, 3, 1, 5, 4, 3, 6, 5]		✓



2.

Given this pseudocode:



```
S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
x = 2
```

```

While(x<11):
  For i in S:
    If  $0 \equiv i \pmod{x}$  and  $i \neq x$ :
      delete i from S
    end-If
  x = x + 1
end-For
end-While

```

What is *S* at the end of this code?

	YOUR ANSWER	CORRECT ANSWER
\emptyset		
{2, 3, 5, 7}		
{11, 13, 17, 19}		
{2, 3, 5, 7, 11, 13, 17, 19}		



3.

Given the pseudocode fragment:

```

x := 2
count := 4
while (count > 0)
  x := 2 × x
  count := count - 1
End-while

```

What is the final value for x?

	YOUR ANSWER	CORRECT ANSWER
4		
16		

	YOUR ANSWER	CORRECT ANSWER
32	✓	✓
64		



4.

Given this pseudocode that extracts a sample sequence from the data sequence of length N :

Function Sampler (Sequence Data)

Set Sample to an empty sequence

Set N to the length of Data

While $N \geq 1$

Append element N of Data to Sample

$N := N/2$

Return Sample

What is the worst-case run time for Function Sampler?

	YOUR ANSWER	CORRECT ANSWER
$O(\log_2 N)$		✓
$O(N/2)$		
$O(N)$	✓	
$O(\log_2 N)$		



5.

Given this algorithm:

Simple Sort

This algorithm sorts the elements of an array.

Input: numb, an array of n integers

Output: numb, in ascending order

```

for i = 1 to n
  for j = 1 to i - 1
    if numb(j) > numb(j + 1)
      temp = numb(j)
      numb(j) = numb(j + 1)
      numb(j + 1) = temp
    end for
  end for
end for

```

What is the asymptotic worst-case complexity?

	YOUR ANSWER	CORRECT ANSWER
$O(1)$		
$O(n)$		
$O(n^2)$	✓	✓
$O(n \log(n))$		



6.

Given the pseudocode:

```

procedure bubbleSort( A : list of sortable items )
  n = length(A)
  repeat
    newn = 0
    for i = 1 to n-1 inclusive do
      if A[i-1] > A[i] then
        swap(A[i-1], A[i])
        newn = i
      end if
    end for
    n = newn
  until n = 0
end procedure

```

What is the worst case performance?

	YOUR ANSWER	CORRECT ANSWER
$O(1)$		
$O(n)$		
$O(n^2)$	✓	✓
$O(n \log(n))$		



7.

Assume that the Sort(list L) function operates in $O(n \log n)$ time, where n is the length of the list (of numbers.)

Variables $L1$, $L2$, and $L3$ are lists of real numbers, all of length n .

Given the following pseudocode function:

```

Function Sort3(L1, L2, L3)
  K1 = Sort(L1)
  For each element, E1, of L1,
    Add E1 to each element of L2
  K2 = Sort(L2)
  For each element, E2, of L2
    Add E2 to each element of L3
  K3 = Sort(L3)
  End-For
End-For
L = Append lists K3, K1, and K2
return L

```

Which function dominates the run time of Sort3($L1$, $L2$, $L3$)?

	YOUR ANSWER	CORRECT ANSWER
$O(n \log n)$		
$O(n^2 \log n)$	✓	

	YOUR ANSWER	CORRECT ANSWER
$O(n^3 \log n)$		✓
$O(n^3)$		



8.

Which function is $O(n^3)$?

	YOUR ANSWER	CORRECT ANSWER
$4x + \sqrt{x}$		
$\sqrt[3]{x} + \ln x$		
$4x^3 + \sqrt{x-1}$	✓	✓
$3x^4 - \ln(x+2)$		



9.

Assume $n_0 = 1$.What is the big- O notation for the function $f(n) = n \times \log(n^2) + 7n^3 + 5n + 3$?

	YOUR ANSWER	CORRECT ANSWER
$O(n^3)$	✓	✓
$O(\log(n^2))$		
$O(n \times \log(n^2))$		

	YOUR ANSWER	CORRECT ANSWER
$O(n^2)$		



10.

What is the ones digit of the number 3^{902} ?

	YOUR ANSWER	CORRECT ANSWER
1		
3	✓	
7		
9		✓



11.

N is an integer between 0 and 9.

For how many values of N does $[123,3N2] = [0] \pmod{8}$, where 123,3N2 is a six-digit number?

	YOUR ANSWER	CORRECT ANSWER
2		
3	✓	✓
4		
5		



12.

Which set contains the multiplicative inverse of 13 mod 33?

	YOUR ANSWER	CORRECT ANSWER
{2, 10, 18, 26}		
{4, 12, 20, 28}		✓
{6, 14, 22, 30}		
{8, 16, 24, 32}	✓	



13.

What is the sequence of remainders obtained when using Euclid's algorithm to compute the greatest common divisor (GCD) of 178 and 20?

	YOUR ANSWER	CORRECT ANSWER
18 2 0		✓
8 2 0		
8 4 2 1 0	✓	
18 4 2 1 0		



14.

What is the decimal expansion of $(10011001)_2$?

	YOUR ANSWER	CORRECT ANSWER
29		

	YOUR ANSWER	CORRECT ANSWER
36		
153	✓	✓
306		



15.

What is the minimum number of bits required for the binary representation of a number greater than 32?

	YOUR ANSWER	CORRECT ANSWER
4		
5		
6	✓	✓
10		



16.

Let $N = 12 = 2^2 + 2^3$.

Given that $M^2 \equiv 51 \pmod{59}$, what is $M^{12} \pmod{59}$?

	YOUR ANSWER	CORRECT ANSWER
3	✓	
7		✓
30		

	YOUR ANSWER	CORRECT ANSWER
36		



17.

The binary representation of 21 is 10101.

What is $17^{21} \bmod n$?

	YOUR ANSWER	CORRECT ANSWER
$(17^2 \times 17^{10} + 17) \bmod n$		
$(17^4 \times 17^5 + 17) \bmod n$		
$(17^{12} \times 17^8 \times 17) \bmod n$		
$(17^{16} \times 17^4 \times 17) \bmod n$	✓	✓



18.

An encryption scheme uses the numerical position of a letter in the alphabet to encrypt characters, e.g., $A=1$, $D=4$, $Z=26$, etc., and spaces are ignored.

What is the encoding of "HAPPY BIRTHDAY" using this technique?

	YOUR ANSWER	CORRECT ANSWER
81662529182084125		
81662529082084125		
8116162529082084125		
8116162529182084125	✓	✓



19.

A preview is unavailable for this question.

YOUR ANSWER	CORRECT ANSWER
A preview is unavailable for this answer.	



20.

An individual has chosen a public key of $N = 391 = 17 \times 23$ and $e = 7$.

What is the private key using RSA encryption?

YOUR ANSWER	CORRECT ANSWER
56	✓
151	✓
352	
384	



21.

Given the recursive algorithm in this pseudocode:

RTC(n)

Input: A nonnegative integer, n

Output: A numerator or denominator (depending on parity of n) in an approximation of

If $n < 3$

Return (n + 1)

If $n \geq 3$

t = RTC(n - 1)

If n is odd

```

s:= RTC(n - 2)
Return (s + t)
If n is even
  r:= RTC(n - 3)
  Return (r + t)
If n is even
  print 'Your approximation is ' , RTC(n) , '/' , RTC(n - 1) , '

```

What is the output for the algorithm if the input n is 6?

	YOUR ANSWER	CORRECT ANSWER
Your approximation is 17/12.	✓	✓
Your approximation is 17/7.		
Your approximation is 12/7.		
Your approximation is 12/5.		



22.

The recurrence relation is given by $a_n = a_{n-1} + n$ with initial term $a_0 = 4$.

What is the value of a_5 ?

	YOUR ANSWER	CORRECT ANSWER
9		
14		
19	✓	✓
25		



23.

Which expression is equivalent to the given expression?

$$\sum_{i=-3}^6 (2i - 3)$$

YOUR
ANSWER

CORRECT
ANSWER

$$\sum_{i=-2}^5 (2i - 3)$$



$$9 + \sum_{i=-2}^5 (2i - 3)$$

$$9 + \sum_{i=-2}^6 (2i - 3)$$

$$-9 + \sum_{i=-3}^5 (2i - 3)$$



24.

In the proof for the sum of squares, the inductive hypothesis for any natural number is:

$$P(n): 0^2 + 1^2 + 2^2 + \dots + n^2 = n(n+1)(2n+1)/6, n \geq 0.$$

If $n = 0$, then the left-hand side of $P(0) = 0^2 = 0$, and right-hand side of $P(0) = 0 \times (0 + 1) \times (2 \times 0 + 1) / 6 = 0$.

What is the right-hand side of $P(n+1)$?

	YOUR ANSWER	CORRECT ANSWER
$n(n+1)(2n+1)/2$		
$(n+1)(2n^2+3n+6)/6$	✓	
$(n+1)(n+2)(2n+3)/6$		✓
$n(n+1)(2n+3)/6 + (n+1)^2$		



25.

In the inductive proof of:

$$4 + 9 + 14 + 19 + \cdots + (5n + 1) = \frac{n}{2}(3 + 5n)$$

The inductive hypothesis is that for any $n = k$,

$$4 + 9 + 14 + 19 + \cdots + (5k + 1) = \frac{k}{2}(3 + 5k)$$

What must be proven assuming the inductive hypothesis is true?

	YOUR ANSWER	CORRECT ANSWER
$4 + 9 + 14 + 19 + \cdots + (5k + 2) = \frac{k+1}{2}(3 + 5(k + 1))$		
$4 + 9 + 14 + 19 + \cdots + (5k + 1) + 5(k + 1) = \frac{k+1}{2}(3 + 5(k + 1))$		
$4 + 9 + 14 + 19 + \cdots + (5k + 1) + (5(k + 1) + 1) = \frac{k+1}{2}(3 + 5(k + 1))$	✓	✓
$4 + 9 + 14 + 19 + \cdots + (5k + 1) + (5(k + 1)) = \frac{k}{2}(3 + 5k)$		



26.

Given this algorithm written in pseudocode:

Algo(n)
 Input: A positive integer n
 Output: Answer, Algo(n)

If n = 1
 Answer = 3
 Else
 Answer = 3 × Algo(n-1)
 End if

What is Algo(4)?

	YOUR ANSWER	CORRECT ANSWER
7		
12		
64		
81	✓	✓



27.

Given this pseudocode:

input: sequence of numbers a_k
 k, length of sequence

 answer := a_1

 for i = 2 to k
 if ($a_i > \text{answer}$), then answer = a_i
 End-for

What is the value of answer for the sequence {-1, 4, -7, 10, 2} with k = 5?

	YOUR ANSWER	CORRECT ANSWER
-7		
-1		
4		
10	✓	✓



28.

Given the recurrence relation $a_{n+1} = 2 \times a_n - 1$, with $a_0 = 3$.

What is a_n when expressed as an explicit function of n ?

	YOUR ANSWER	CORRECT ANSWER
$2 + 2^n$	✓	
$3 + n \times 2^n$		
$1 + 2^{(n+1)}$		✓
$3 + n \times 2^{(n+1)}$		



29.

Suppose $a_{n+1} = 3a_n - 2a_{n-1}$ and $a_0 = 1$, $a_1 = 2$.

What is a_n expressed as a function of n ?

	YOUR ANSWER	CORRECT ANSWER
2^n	✓	✓
$n + 1$		

	YOUR ANSWER	CORRECT ANSWER
$(-1)^n + 3n$		
$(-2)^n + 4n$		



30.

A club of 12 people would like to choose a person for each office of president, a vice president, and a secretary.

How many different ways are there to select the officers so that only one person holds each office?

	YOUR ANSWER	CORRECT ANSWER
36		
1,320	✓	✓
1,728		
3,960		



31.

A fair coin is flipped n times and the results recorded.

How many different sequences of heads and tails are possible?

	YOUR ANSWER	CORRECT ANSWER
$2n$		
2^n	✓	✓
n^2		

	YOUR ANSWER	CORRECT ANSWER
$n!$		



32.

A mother, father, and their 3 children are having their picture taken. They will all be seated elbow-to-elbow on the living room couch, and the children will not be permitted to sit next to each other.

How many different arrangements are possible for the picture?

	YOUR ANSWER	CORRECT ANSWER
5		
6		
12		
120		



33.

How many ways can 7 soccer balls be divided among 3 coaches for practice?

	YOUR ANSWER	CORRECT ANSWER
21		
36		
210		
343		



34.

How many ways can a club select a president, a vice president, a secretary, and a treasurer if the club consist of 12 people?

	YOUR ANSWER	CORRECT ANSWER
48		
495		
11,880	✓	✓
20,736		



35.

A grocery store stocks 1-gallon cartons of skim milk, 1% milk, 2% milk, and whole milk. A customer is asked to buy 10 gallons of milk. The customer needs to buy at least one carton of each type of milk.

How many different ways can the kinds of milk to buy be selected?

	YOUR ANSWER	CORRECT ANSWER
84		✓
120		
210	✓	
286		



36.

Twenty people have volunteered for an experiment. Twelve of the volunteers are men and eight are women.

How many ways are there to select a group of five men and four women?

	YOUR ANSWER	CORRECT ANSWER
5!4!		
$C(20,9)$		
$C(12,5) \times C(8,4)$	✓	✓
$C(20,12) \times C(20,8)$		



37.

Which permutation of the set {1, 2, 3, 4, 5, 6} is first in lexicographic order?

	YOUR ANSWER	CORRECT ANSWER
(4, 6, 3, 2, 1, 5)		
(4, 5, 2, 3, 6, 1)		
(4, 5, 2, 3, 1, 6)	✓	✓
(4, 6, 5, 3, 2, 1)		



38.

Using lexicographic order, which 4-permutation of {1, 2, 3, 4, 5, 6} fits in the blank of the inequality chain $(3, 1, 4, 1) < (4, 2, 3, 2) < \underline{\hspace{2cm}} < (5, 3, 3, 2)$?

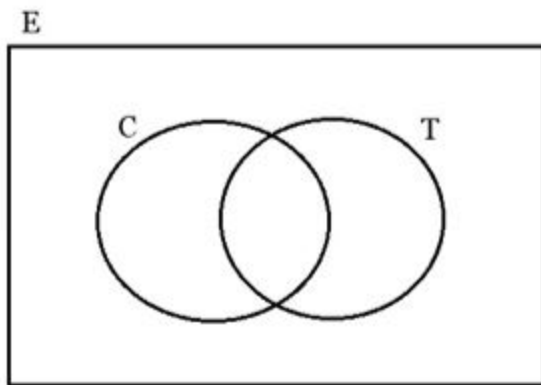
	YOUR ANSWER	CORRECT ANSWER
(3, 2, 3, 1)		
(3, 3, 4, 3)		

	YOUR ANSWER	CORRECT ANSWER
(4, 1, 3, 3)		
(4, 2, 4, 1)	✓	✓



39.

An 11-person musical ensemble is on a stage with a tuba and a cello. Only 1 of the 11 can play both instruments, and exactly 4 can play neither instrument. The number of ensemble members who play only the cello equals the number who play only the tuba.



How many members of the 11-person ensemble play only the cello?

	YOUR ANSWER	CORRECT ANSWER
1		
2		
3	✓	✓
4		



40.

How many people must be in a group to ensure that at least 2 individuals have the same first initial?

	YOUR ANSWER	CORRECT ANSWER
27		✓
52	✓	
53		
650		



41.

Set *A* has 8 elements, *B* has 10 elements, and their intersection has 6 elements.

How many elements are in the union of these two sets?

	YOUR ANSWER	CORRECT ANSWER
12	✓	✓
16		
18		
24		



42.

A company has three active projects: Domino, Falcon, and Giant. There are 19 employees charging time to Domino, 16 to Falcon, and 15 to Giant. Those totals include 8 employees charging both Domino and Falcon, 6 charging both Falcon and Giant, and 7 charging both Giant and Domino. Included in those subtotals is the 4-person IT team, all of whose members charge all three projects.

How many employees are working on the projects?

	YOUR ANSWER	CORRECT ANSWER
28		
33	✓	✓
37		
50		



43.

A random experiment consists of tossing a fair six-sided die repeatedly.

How many tosses are required to be certain that the probability that at least one '6' appears is greater than or equal to $1/2$?

	YOUR ANSWER	CORRECT ANSWER
3	✓	
4		✓
5		
6		



44.

Assume that a test for a disease gives a positive result for 1.5% of people who do not have the disease, but does not test negative if the person has the disease.

What is the probability that a person who tested positive has the disease, if 1% of people have the disease?

	YOUR ANSWER	CORRECT ANSWER
--	----------------	-------------------

	YOUR ANSWER	CORRECT ANSWER
.00		
.01		
.40	✓	✓
.60		



45.

There are two coins, one fair and one biased. The biased coin comes up heads with a probability 0.8 and tails with a probability 0.2.

One of the coins is selected at random and flipped ten times. The results of the coin flips are mutually independent.

The result of the 10 flips is H, T, T, H, H, T, H, H, T, H.

What is the probability that the coin flipped was the biased coin? (Round to the nearest tenth.)

	YOUR ANSWER	CORRECT ANSWER
0.2		
0.3	✓	✓
0.4		
0.5		



46.

A life insurance company issues standard or preferred policies.

Of the company's policyholders, 60% have standard policies and a probability of 0.01 of dying in the next year, and 40% have preferred policies and a probability of 0.08 of dying

in the next year.

A policyholder dies in the next year.

What is the conditional probability of the deceased having a preferred policy?

	YOUR ANSWER	CORRECT ANSWER
P(S) = 0.60, P(P) = 0.40, P(D P) = 0.08, P(D S) = 0.01, and the answer is 0.8421.	✓	✓
P(S) = 0.60, P(P) = 0.40, P(D P) = 0.01, P(D S) = 0.08, and the answer is 0.6154.		
P(S) = 0.40, P(P) = 0.60, P(D P) = 0.08, P(D S) = 0.01, and the answer is 0.9231.		
P(S) = 0.40, P(P) = 0.60, P(D P) = 0.01, P(D S) = 0.08, and the answer is 0.1579.		



47.

Which formula is the expected value of the random variable Y?

	YOUR ANSWER	CORRECT ANSWER
$\sum_{s=1}^n Y(s)p(s)$	✓	
$\sum_{s \in Y} Y(s)p(s)$		✓

YOUR
ANSWER

CORRECT
ANSWER

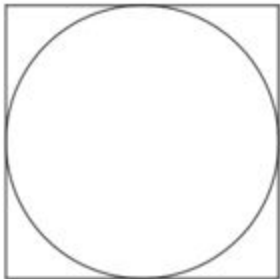
$$\sum_{s \in Y} |Y(s)|p(s)$$

$$\sum_{s \in Y} |Y(s)p(s)|$$



48.

A circle of radius 1 is inscribed in a square with side length 2 as shown in the diagram below:



If 1,000 points are selected randomly inside the square with all points equally likely to be selected, how many of those points are expected to lie inside the circle?

YOUR
ANSWER

CORRECT
ANSWER

632

657

720

785





49.

There are four different colored balls in a bag. There is equal probability of selecting the red, black, green, or blue ball.

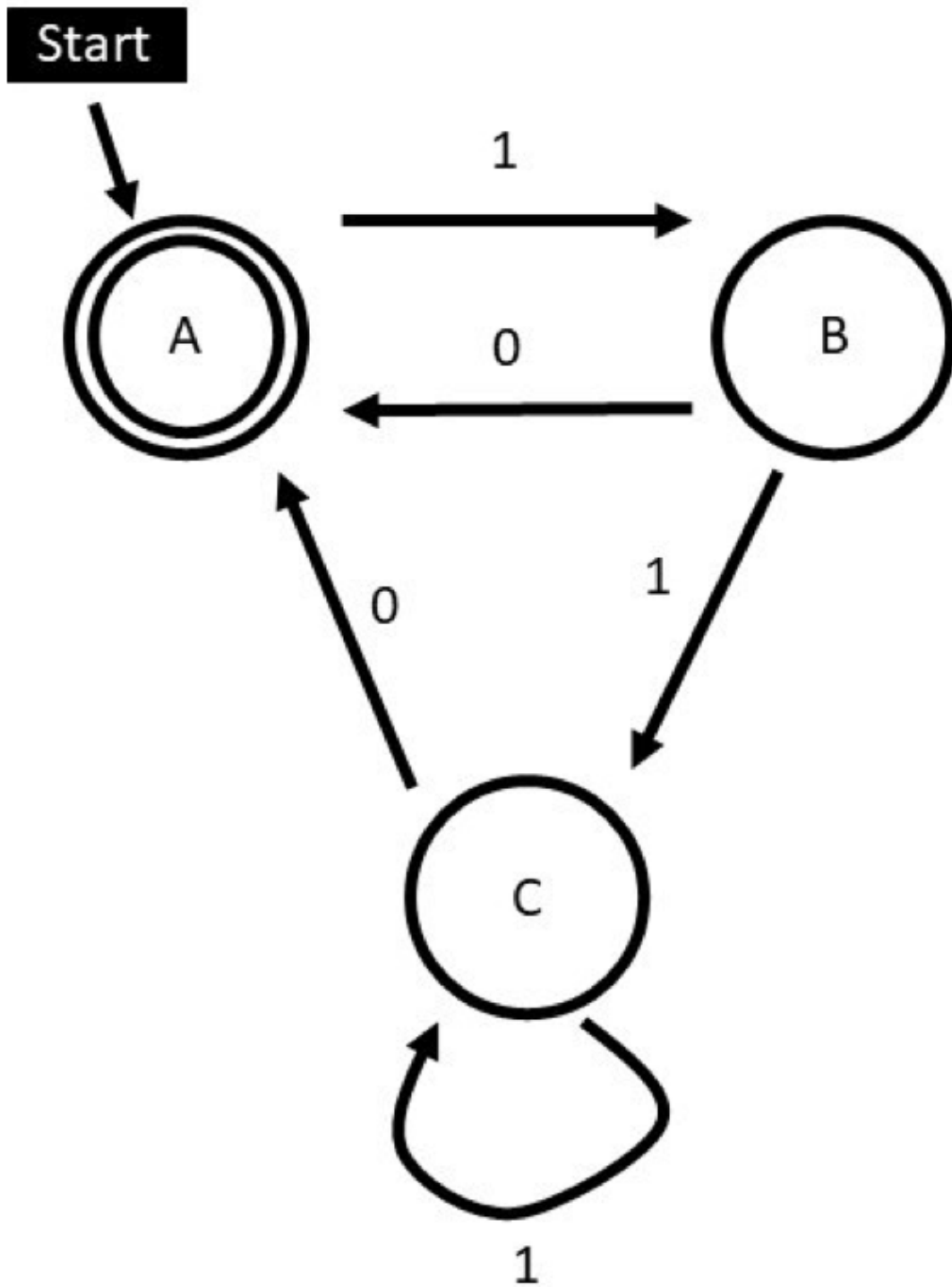
What is the expected value of getting a green ball out of 20 experiments with replacement?

	YOUR ANSWER	CORRECT ANSWER
4		
5	✓	✓
20		
80		



50.

Given the finite state machine (FSM):



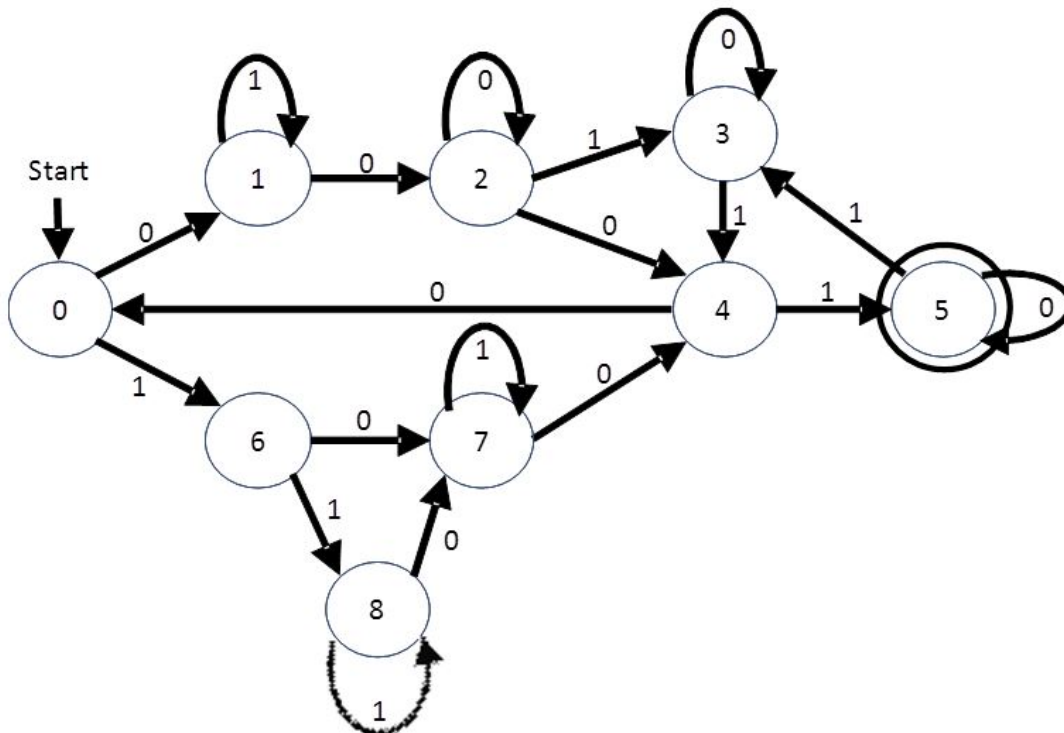
Which input is accepted by the FSM?

	YOUR ANSWER	CORRECT ANSWER
0000		
1101		
1110	✓	✓
1111		



51.

The diagram below shows a deterministic finite automaton with alphabet {0, 1}:



What state is the automaton in after processing the string "1011 0110 0011 1"?

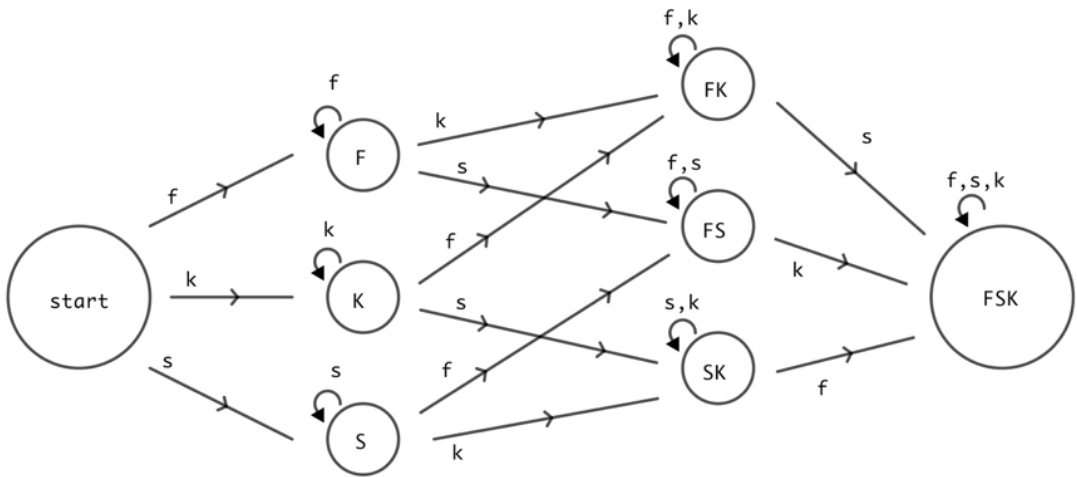
	YOUR ANSWER	CORRECT ANSWER
2		

	YOUR ANSWER	CORRECT ANSWER
3	✓	✓
4		
5		



52.

REMOVEThe finite state machine (FSM) describes setting a table with a knife, fork, and spoon. There only needs to be one of each, so once a state is reached, if a second addition of an existing utensil is attempted, the machine loops back to the same state.



Which statement is correct?

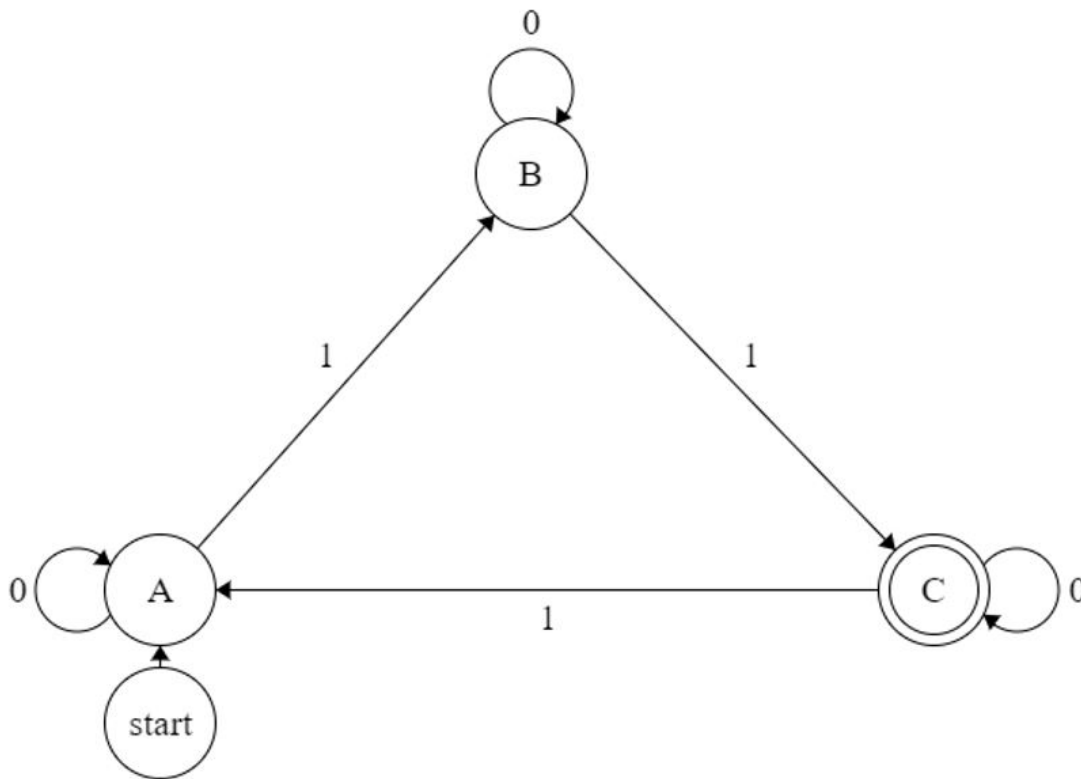
	YOUR ANSWER	CORRECT ANSWER
The only inputs that end at state FSK are the permutations of {f,s,k}.		
The only input that ends at state FSK is {f,s,k}.		
Only inputs that include the same number of f's, s's and k's will terminate at FSK.		

	YOUR ANSWER	CORRECT ANSWER
Any input that includes any number of f's, s's and k's will terminate at FSK.	✓	✓



53.

This finite state machine (FSM) has the input alphabet {0, 1} and the state set {A, B, C}.



Which input string will be accepted by the FSM?

	YOUR ANSWER	CORRECT ANSWER
000000		
001010	✓	✓
101010		



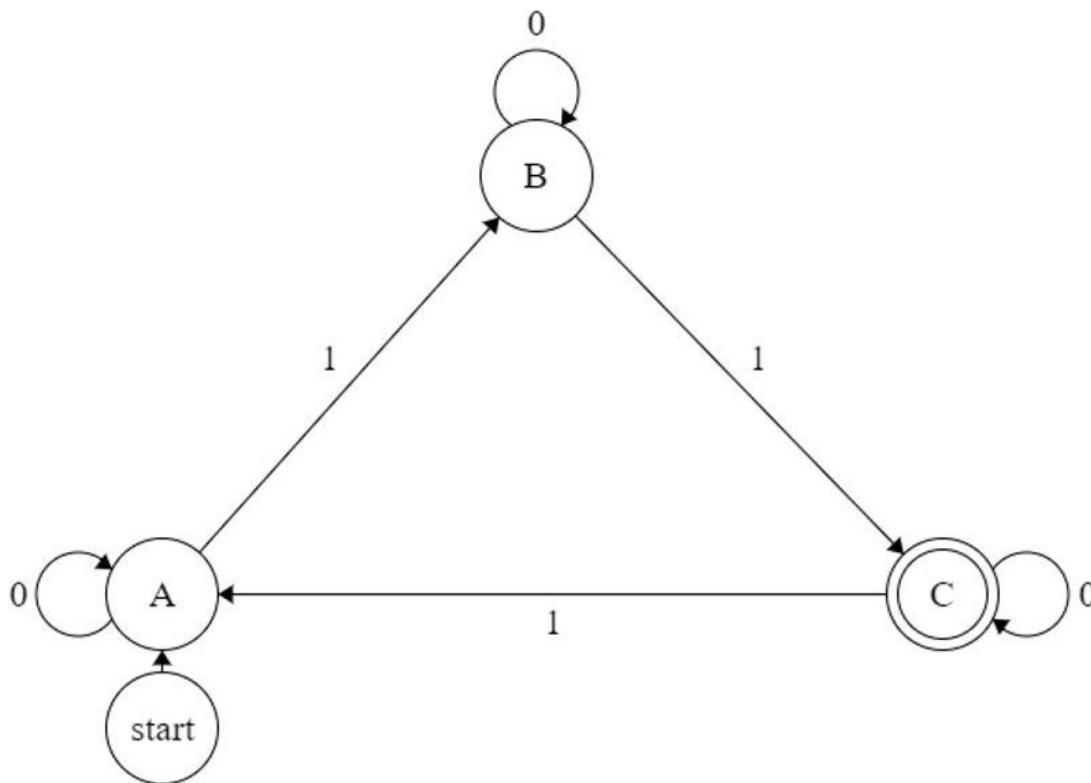
YOUR
ANSWERCORRECT
ANSWER

110101



54.

This finite state machine (FSM) has the input alphabet $\{0, 1\}$ and the state set $\{A, B, C\}$ with C being the only accepting state.



What is its current state if the input string 110110 has been fed to the FSM?

YOUR
ANSWERCORRECT
ANSWER

A

B



C

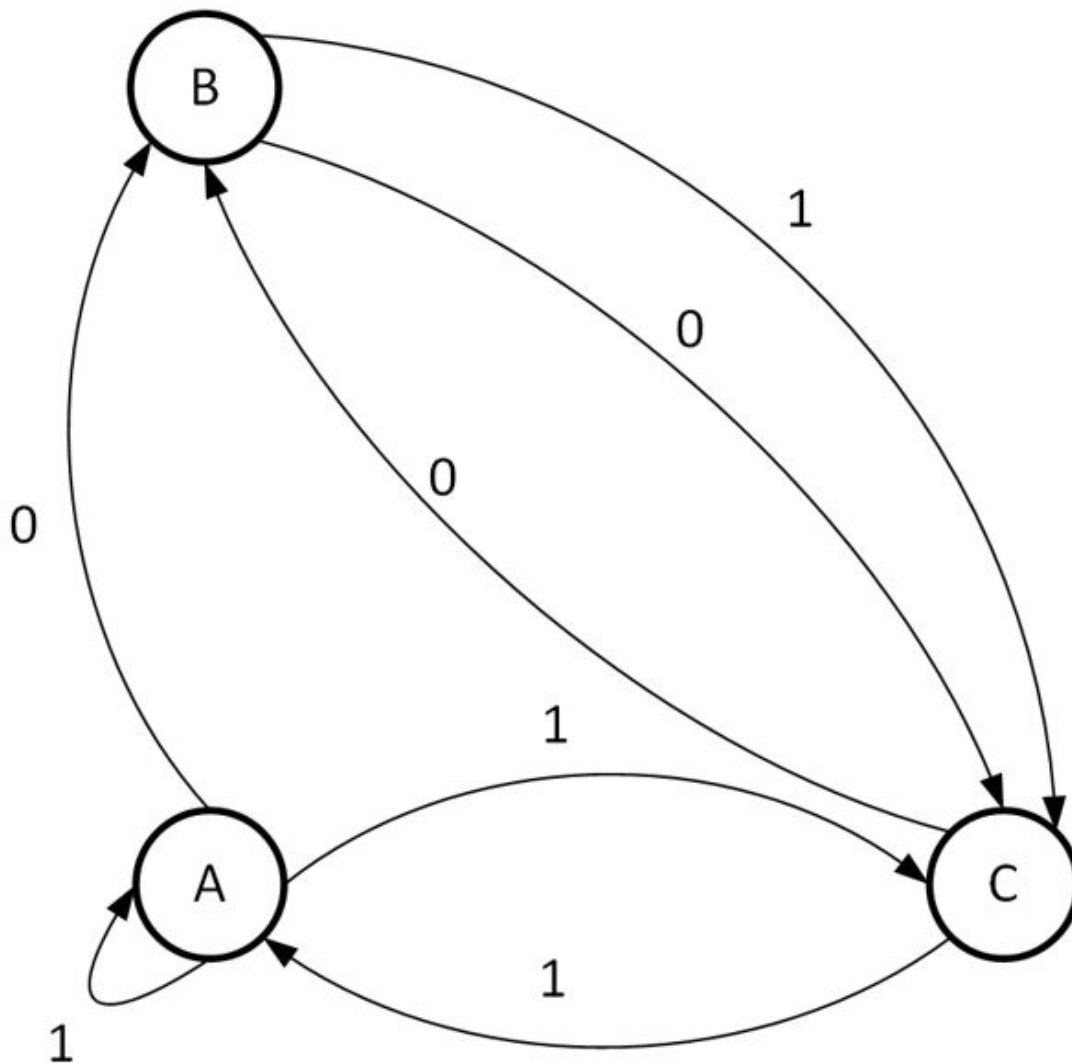
YOUR
ANSWERCORRECT
ANSWER

Input string is not valid.



55.

Given this finite state machine (FSM):



What is the current state after the FSM has processed the input sequence 11011 beginning in state B?

	YOUR ANSWER	CORRECT ANSWER
A	✓	✓
C		
{A, B}		
{A, C}		

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