

# **Enrichment of ontological taxonomies using a neural network approach**

## **Bachelorarbeit**

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vorgelegt von  
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# 1 Introduction

**Knowledge bases (KB)** like DBpedia, YAGO and Wikidata store information about every significant real-world entity and the relations between those entities. Using different approaches like human collaboration in Wikidata or information from Wikipedia data boxes in YAGO, these KBs are populated [17]. The issue of incompleteness arises in this context, as it is not realistic to assume that every bit of knowledge can or should be captured in the KBs. **Ontologies** are necessary for modeling, sharing, and exchanging knowledge [20], and are therefore integral in the functioning of a KB [43]. They provide data schemas, concept vocabularies and rules [29] to which a KB has to adhere. Therefore, it should be a priority to ensure the completeness of the ontology in a KB. Types of ontologies range from lightweight ontologies like glossaries and taxonomies (concept hierarchies) to heavyweight ontologies like data models and description logics [43].

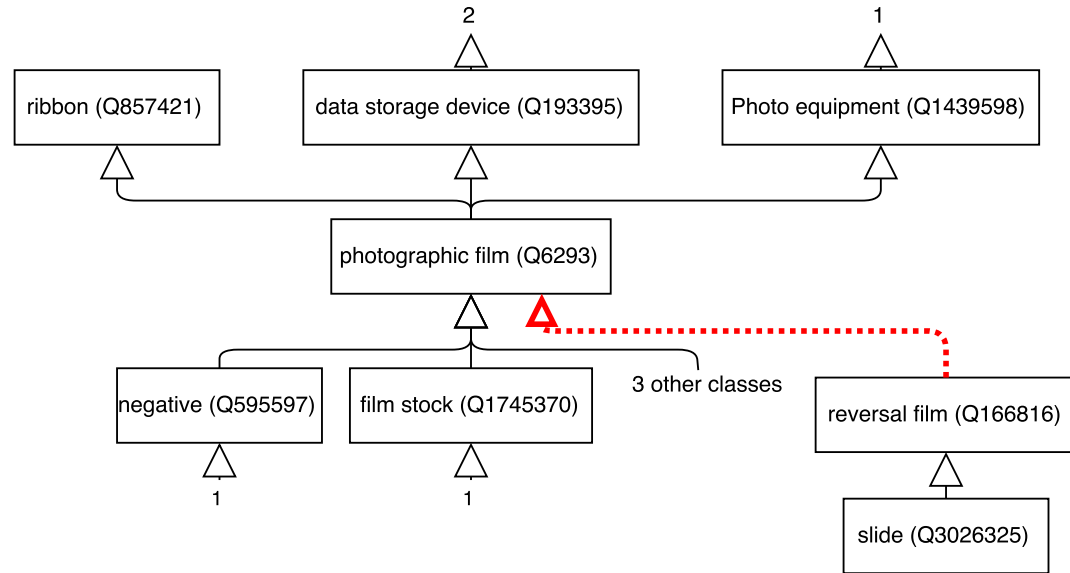


Figure 1: Classification of orphan class in Wikidata’s taxonomy

The thesis’ goal is the development and evaluation of approaches to increase the completeness of ontological taxonomies. Specifically, the problem of classifying **orphan classes** is solved. Orphan classes are classes in a taxonomy, which have no superclasses (parents) and are not the root of the taxonomy. An example, for such a classification problem is given in Figure 1. An extract of the Wikidata taxonomy is shown. The class *reversal film* is an orphan class in the taxonomy, since it has no superclass. A possible solution is represented by the dotted arrow, which places *photographic film* as superclass of *reversal film*. It has to be noted that additionally all superclasses of *photographic film* are also valid solutions to the problem for the orphan class *reversal film*, since the subclass-of relationship is transitive. A solution

to the problem should find the most specific superclass for a given orphan class [13]. An algorithm is developed with the task of classifying such orphan classes into a given taxonomy. The algorithm will use neural word embeddings, as developed by Mikolov et al. [30]. This is motivated by the effectiveness of neural networks in other context-sensitive tasks like language processing [30] [22] [5], image generation [19] etc. Neural word embeddings are especially interesting, since they have been used to great effect in the construction of taxonomies [16], graph representation [6] and semantic similarity tasks [31] [25]. Different approaches in using neural word embeddings for classification of orphan classes will be considered and evaluated in this work.

In this work, the taxonomy of Wikidata is considered as case example, as it provides a high number of entities (24, 507, 102 in 2016/11/07 [3]), and a big taxonomy consisting of 1, 299, 501 classes [3]. At this time, Wikidata’s taxonomy is mostly maintained by human editors. The solution developed by the thesis should be able to support the editors in improving the completeness of the taxonomy by reducing the number of orphan classes and refining the existing taxonomic relations between classes.

The thesis is structured as follows. In Chapter 2 the problem will be formally defined (2.1, 2.2, 2.3, 2.4) and related work in the fields of neural networks (2.6) and ontology learning (2.7) discussed for the use in this work. Section 2.6 introduces the notion of neural networks and discusses work dealing with neural word and graph embeddings. Section 2.7 compares the problem, solved by the thesis, to related work in the field of ontology learning. Subsequently, the problem is classified, solutions to similar problems analyzed and the novelty of the work justified. Chapter 3 describes the Wikidata data set used as case example for the implementation and evaluation of the proposed hybrid algorithm. Chapter 4 describes the components of the developed baseline algorithm and variations, which potentially improve the effectiveness of the method. Chapter 5 describes the evaluation methodology and presents the results. The baseline algorithm and proposed improvements are evaluated.

Formatting of text obeys the following rules. *Italic text* indicates Wikidata entities like *entity (Q35120)* or *instance of (P31)*. **Bold text** indicates the first occurrence of a relevant concept, such as **orphan class**. Such concepts are described in the following sentences and, if necessary, formally defined. Figures with long descriptions are framed to clearly separate continuous text and description of the figure. In-line quotes are indicated by quotation marks. The corresponding citation is placed after the quote. Multi-line quotes are indented. The quote is introduced by a short sentence, which contains the citation.

## 2 Background and related work

### 2.1 Wikidata

Wikidata is an open, free, multilingual and collaborative KB. It is a structured knowledge source for other Wikimedia projects. It tries to model the real world, meaning every concept, object, animal, person, etc. Wikidata is mostly edited and extended by humans, which in general improves the quality of entries compared to fully-automated systems, because different editors can validate and correct occurring errors. However, Wikidata, like most knowledge bases, is incomplete in terms of missing facts about different entities [11] and therefore has to be operated under the **Open World Assumption (OWA)**. OWA states that if a statement is not contained in a knowledge base, it is not necessarily false but rather unknown [17].

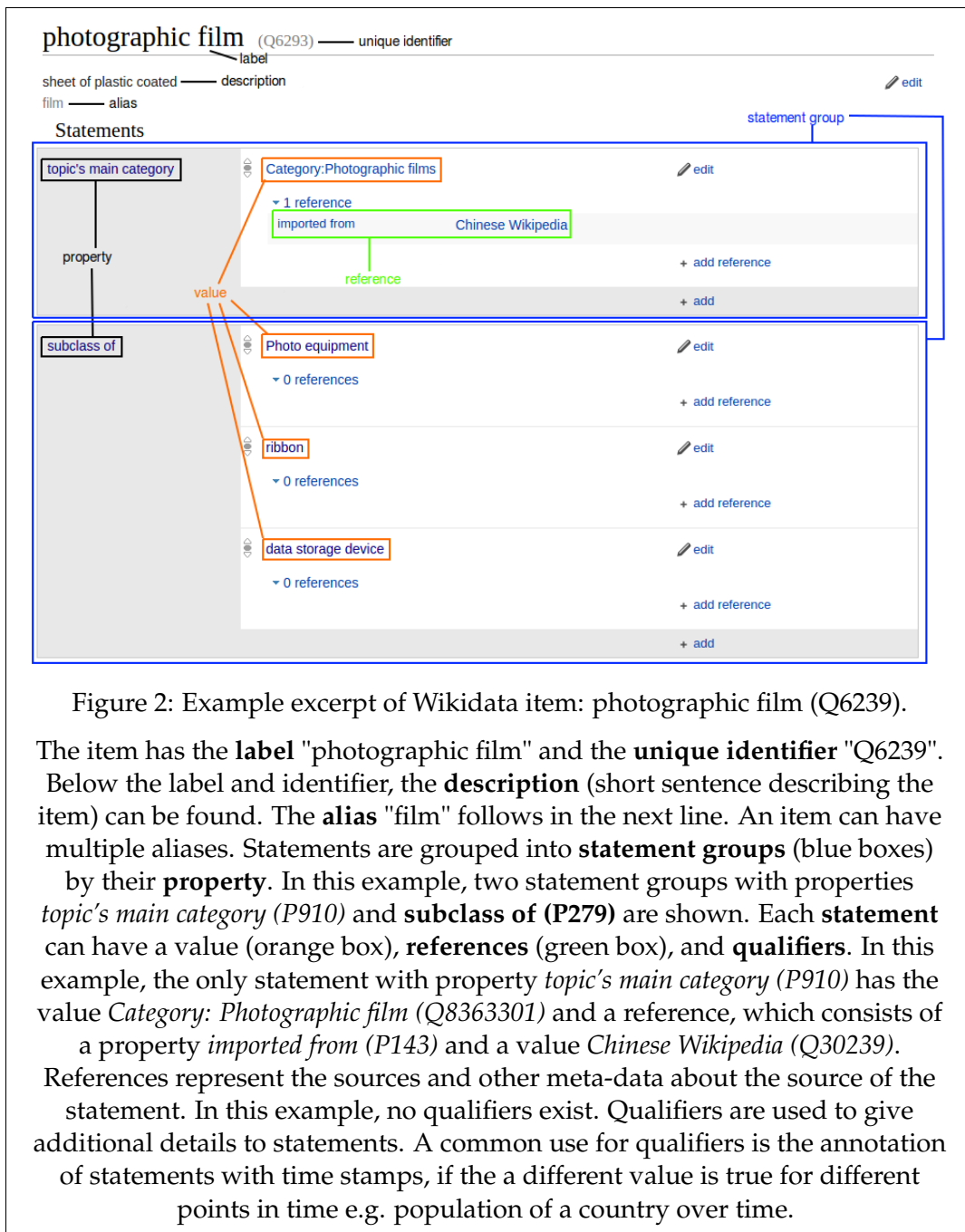
In Wikidata **items** and **properties** exist. Items are the aforementioned concepts, objects, etc. Properties are used to make claims about items, e.g. *photographic film* (Q6293) is a *subclass of* (P279) *data storage device* (Q193395) (see Figure 2). Each item and property has a unique identifier, which starts with the letter Q for items and the letter P for properties and is followed by a numeric code. The identifiers in Wikidata are essential to avoid ambiguity and make items and properties multilingual.

Items consist of labels, aliases and descriptions in different languages. Sitelinks connect items to their corresponding pages of Wikimedia projects like Wikipedia articles. Most importantly, an item is described by **statements**. Statements are in their simplest form a pair of property and value, assigned to a specific item. A value is either a literal value or another item. It should be noted that an item can have multiple statements with the same property. The set of statements with the same property is called statement group. Statements can be annotated with qualifiers, which specify the context of the statement, e.g. population at a certain point of time. Additionally, references can be used for statements to include its source. See Figure 2 for an example of a Wikidata item.

Following, the concepts item and statement are formally defined:

**Definition 1 (Item).** An **item** is a tuple  $(id, label, aliases, description, sitelinks)$ :

- $id \in \mathbb{N}$  is the numerical item ID;
- $label \in String$  is the English label of the item;
- $aliases \in \mathcal{P}(String)$  is the set of English synonyms for the label;
- $description \in String$  is a short sentence describing the item;
- $sitelinks \in String \times String$  is a set of tuples  $(site, title)$ , where *site* refers to a specific site of the Wikimedia projects, e.g. enwiki, and *title* is the corresponding article title of the item on this site.



For example, the item *photographic film* (Q6239) (Figure 2) can be formally represented, as follows:

```
(
  6239,
  "photographic film",
  {"film"},
  "sheet of plastic coated with light-sensitive chemicals",
  {"("enwiki", "Photographic film"), ("dewiki", "Fotografischer Film"), ... }
)
```

**Definition 2 (Statement).** A **statement** is a tuple (*itemid*, *pid*, *value*, *refs*, *qualifiers*):

- *itemid*  $\in \mathbb{N}$  is a numerical item ID, to which the statement belongs;
- *pid*  $\in \mathbb{N}$  is a numerical property ID;
- *value* is either a constant value like string, int, etc., or an item ID;
- *refs* is a set of references, containing the source of information for the statement;
- *qualifiers* is a set of qualifiers, which further specifies the statement.

For example, *photographic film* (Q6239) (Figure 2) can be formally represented, as follows:

$$(6239, 910, Q30239, \{\dots\}, \emptyset)$$

In Wikidata, there is no strict distinction between classes and instances. Both groups are represented as items. This leads to the issue, that recognizing, whether an item is a class or instance is not trivial. Based on which statements connect two items, a distinction can be made. A class is any item, which has instances, subclasses or is the subclass of another class. In Wikidata, the properties *instance of* (P31) and *subclass of* (P279) exist, which describe these relations between items. Therefore to identify whether an item is a class, it needs to be checked, whether the items fulfills any of the three mentioned criteria.

**Definition 3 (Class).** Given a set of items *I* and a set of statements *R*.  $c = (classid, \_, \_, \_, \_) \in I$  is a **class**, if at least one of the following assertions are true:

$$\exists i = (instanceid, \_, \_, \_, \_) \in I \exists s = (itemid, pid, value, \_, \_) \in R :$$

$$instanceid = itemid \wedge pid = 31 \wedge value = classid \text{ (has instance)}$$

$$\exists s = (itemid, pid, \_, \_, \_) \in I : itemid = classid \wedge pid = 279 \text{ (is subclass)}$$

$$\exists i = (subclassid, \_, \_, \_, \_) \in I \exists s = (itemid, pid, value, \_, \_) \in R :$$

$$itemid = subclassid \wedge pid = 279 \wedge value = classid \text{ (has subclass)}$$

\_ is used as an anonymous placeholder, for the purpose of not naming unused elements in tuples. For example, *photographic film* (Q6293) (Figure 2) is a class, because it is the subclass of three other classes.

## 2.2 Taxonomy

“Ontologies are (meta)data schemas, providing a controlled vocabulary of concepts, each with an explicitly defined and machine processable semantics” [29]. Additionally it is possible for ontologies to contain axioms used for validation and constraint enforcement. Ontologies enable the modeling and sharing of knowledge in a specific domain and support the knowledge exchange via web by extending syntactic to semantic interoperability [20]. In comparison, a KB like Wikidata can be seen as an instantiation of such an ontology, since every KB has to be conceptualized by an ontology [43]. Different types of ontologies can be grouped by their level of formality and expressiveness. Wong et al. [43] differentiates ontologies as lightweight and heavyweight ontologies (see Figure 3). **Taxonomies** are concept or class hierarchies. They typically represent a parent-child structure, which can be formalized with a single relationship called for example *subclass-of* in the case of Wikidata. The observed taxonomy in Wikidata belongs to the category of lightweight ontologies, specifically principled, informal hierarchies, as the only enforced rule for the subclass-of relation is that it should connect two entities [1].

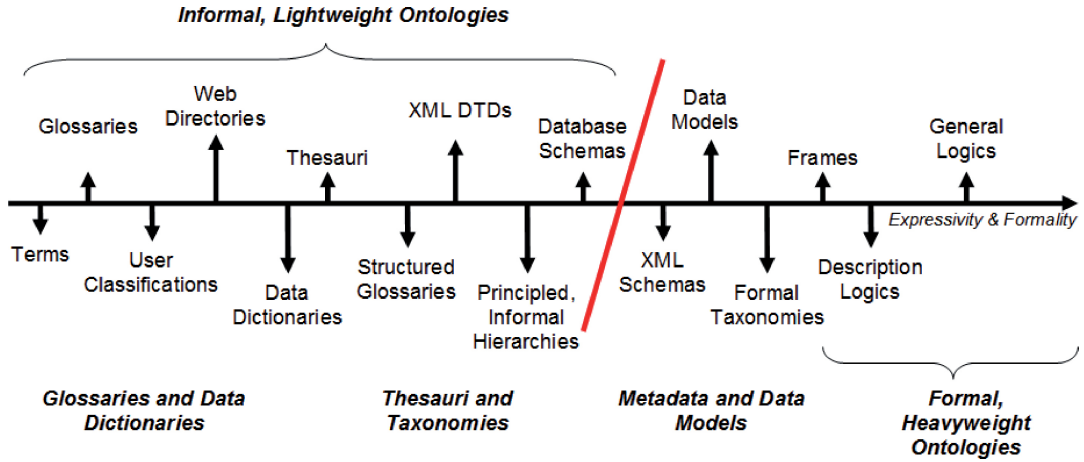


Figure 3: The spectrum of ontology kinds [43]

For the purpose of developing a formal definition of the thesis’ problem statement the notion of taxonomy needs to be formalized. Cimiano defines a heavyweight ontology, which includes a taxonomy, as follows [9]:

**Definition (Ontology).** An **ontology** is a structure

$$\mathcal{O} := (C, \leq_C, R, \sigma_R, \leq_R, \mathcal{A}, \sigma_A, \mathcal{T})$$

consisting of

- four disjoint sets  $C$ ,  $R$ ,  $\mathcal{A}$ , and  $\mathcal{T}$  whose elements are called concept identifiers, relation identifiers, attribute identifiers and data types, respectively,
- a semi-upper lattice  $\leq_C$  on  $C$  with top element  $root_C$ , called concept hierarchy or taxonomy,
- a function  $\sigma_R : R \rightarrow C^+$  called relation signature,
- a partial order  $\leq_R$  on  $R$ , called relation hierarchy, where  $r_1 \leq_R r_2$  implies  $|\sigma_R(r_1)| = |\sigma_R(r_2)|$  and  $\pi_i(\sigma_R(r_1)) \leq_C \pi_i(\sigma_R(r_2))$ , for each  $1 \leq i \leq |\sigma_R(r_1)|$ , and
- a function  $\sigma_A : \mathcal{A} \rightarrow C \times \mathcal{T}$ , called attribute signature,
- a set  $\mathcal{T}$  of datatypes such as strings, integers, etc.

Hereby,  $\pi_i(t)$  is the  $i$ -th component of tuple  $t$ . [...] Further, a semi-upper lattice  $\leq$  fulfills the following conditions:

$\forall x (x \leq x)$  (reflexive)

$\forall x \forall y (x \leq y \wedge y \leq x \implies x = y)$  (anti-symmetric)

$\forall x \forall y \forall z (x \leq y \wedge y \leq z \implies x \leq z)$  (transitive)

$\forall x x \leq top$  (top element)

$\forall x \forall y \exists z (z \geq x \wedge z \geq y \wedge \forall w (w \geq x \wedge w \geq y \implies w \geq z))$  (supremum)

So every two elements have a unique most specific supremum.

This definition by Cimiano [9] can be mapped to Wikidata. In the Wikidata context,  $C$  is the set of classes (see Definition 3 (Class)),  $R$  is the set of properties, which target other items, and  $\mathcal{A}$  is the set of properties, which target literals like strings.

As shown in the preceding definition by Cimiano [9], a taxonomy can be modeled as a semi-upper lattice  $\leq_C$ . This induces two important assumptions about the structure and to some degree completeness of the observed taxonomies. First, there is only one *root class*, top element of the lattice, of which every other class is (transitively) a subclass. Second, because of the supremum property, the taxonomy is fully connected, which implies that every class has at least one root class, excluding the root class  $root_C$ . Wikidata's taxonomy does therefore not fulfill the definition, as it is not fully connected.

In the following, definitions will be presented, which attempt to model an incomplete taxonomy based on the already presented data model and structure of Wikidata. Refer to Appendix A for the necessary definitions on graphs.

In Wikidata, a class can have multiple superclasses, therefore a tree structure is not sufficient to model the taxonomy. However a **directed acyclic graph (DAG)**, can model the taxonomy. The acyclic constraint is necessary to ensure that no class is transitively a subclass of itself.

**Definition 4** (Taxonomy). A **taxonomy**  $T = (C, S)$  is a DAG, where  $C$  is a set of class identifiers, and  $S$  is the set of edges, which describe the subclass-of relation between two classes. such that  $c_1$  is the subclass of  $c_2$ , if  $(c_1, c_2) \in S$ .

**Definition 5** (Subclass-of relation). The transitive binary relation  $\triangleleft_T$  on the taxonomy  $T = (C, S)$  represents the transitive subclass relationship of two classes in  $T$ . Given  $c_1, c_2 \in C$ ,  $c_1 \triangleleft_T c_2$  applies, if there is a walk  $W = (c_1, \dots, c_2)$  with length  $n \geq 1$ , which connects  $c_1$  and  $c_2$ .  $\triangleleft_T$  is transitive,  $\forall c_1, c_2, c_3 \in C : c_1 \triangleleft_T c_2 \wedge c_2 \triangleleft_T c_3 \implies c_1 \triangleleft_T c_3$ .

If the taxonomy defined by Cimiano [9] is mapped on this graph-based taxonomy model, the following assumption is true, for  $T = (C, S)$ :

$$|\{c \in C \mid \neg \exists s \in C : c \triangleleft_T s\}| = 1 \quad (1)$$

Only one class in this taxonomy has no superclasses. This class is called **root class**. However in the case of Wikidata, this assumption does not hold true. The following state is the case:

$$|\{c \in C \mid \neg \exists s \in C : c \triangleleft_T s\}| \geq 1 \quad (2)$$

There are classes other than the root class, which also have no superclasses. These classes will be called orphan classes.

**Definition 6** (Root class). Given a taxonomy  $T = (C, S)$ , the **root class**  $root_T$  is a specific, predefined class with no superclasses in  $T$ . For  $root_T$ ,  $|succ_T(root_T)| = 0$  applies.

**Definition 7** (Orphan class). Given a taxonomy  $T = (C, S)$  with a root class  $root_T$ , a class  $u \in C$  is called **orphan class**, if  $u \neq root_T \wedge |succ_T(u)| = 0$ .

In Wikidata, the root class is *entity* (Q35120) [2]. All other classes, which are not subclasses of *entity* (Q35120), are therefore either orphan classes, or subclasses of orphan classes. In Chapter 3, it is shown that 97% of all classes are subclasses of the root class *entity* (Q35120). This set  $R = \{c \in C \mid c \triangleleft_T root_T \vee c = root_T\}$  will be referred to as **root taxonomy** in later sections.



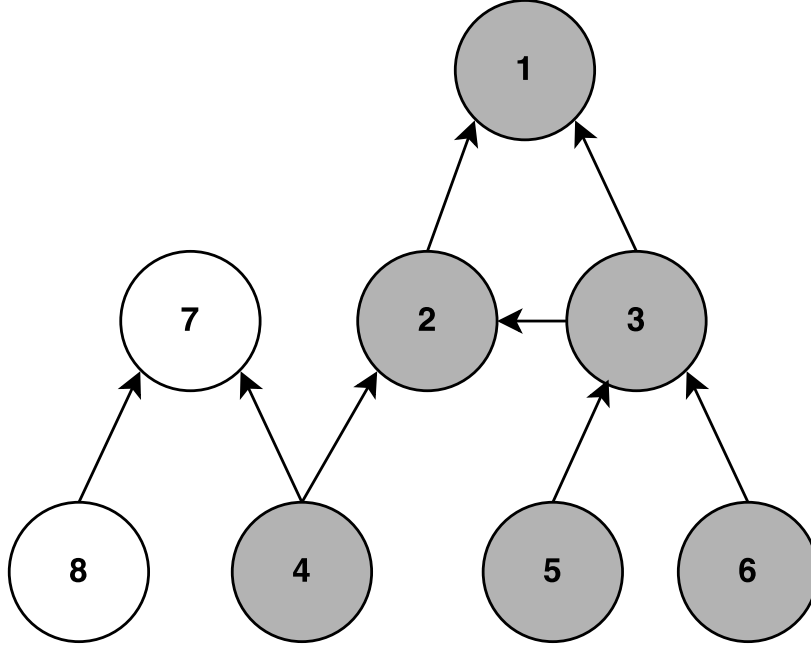


Figure 4: Example: Taxonomy DAG

A **taxonomy**  $T = (C, S)$  with **root class**  $root_T = 1$  can be defined as DAG (see Definition 4). Each node represents a class with its class identifier. The directed edges represent the **subclass-of** relation between the classes. Node 1 is the root of the taxonomy. All gray-colored nodes are connected to  $root_T$  and therefore part of the **root taxonomy**. It can be seen that the white-colored classes 7 and 8 are not connected to  $root_T = 1$ , therefore they do not belong to the root taxonomy. Class 7 is an **orphan class**, since it is not the root class and has no outgoing edges.

### 2.3 Similarity

For the task of ontology learning [20] as well as classification, e.g. k-nearest-neighbors and SVM [7], the concept of **similarity** is of importance. A **similarity measure** computes the similarity between two objects [42]:

**Definition 8** (Similarity measure).  $sim : \Omega \times \Omega \mapsto [0, 1]$  is called **similarity measure** for a set of objects  $\Omega$ . Given two objects  $A, B \in \Omega$ ,  $sim(A, B) = 1$  if  $A = B$  and  $sim(A, B) < 1$  if  $A \neq B$ .

Lin [28] states three intuitions, which a similarity measure between two objects  $A$  and  $B$  should fulfill:

- "The similarity between  $A$  and  $B$  is related to their commonality. The more commonality they share, the more similar they are." [28]

- "The similarity between  $A$  and  $B$  is related to the differences between them. The more differences they have, the less similar they are." [28]
- "The maximum similarity between  $A$  and  $B$  is reached when  $A$  and  $B$  are identical, no matter how much commonality they share." [28]

Therefore a similarity measure  $\text{sim}(A, B)$  should return 1 only if  $A = B$  and 0 if the  $A$  and  $B$  share no commonalities. A similarity measure should factor in both commonalities and differences to compute the similarity between objects [28].

Many types of information can be encoded as feature vectors in  $\mathbb{R}^d$ , e.g. words [27] [30], graphs [38] [6], etc., where  $d \in \mathbb{N}$  is the dimension of the vector space. The similarity between feature vectors can be measured using a distance measure between two vectors  $\vec{p}, \vec{q} \in \mathbb{R}^d$  [42]. Distance measures  $\delta$  have the signature:  $\delta : \mathbb{R}^d \times \mathbb{R}^d \mapsto \mathbb{R}_0^+$ .  $\delta(\vec{p}, \vec{q}) = 0$  implies  $\text{sim}(\vec{p}, \vec{q}) = 1$ , while  $\delta(\vec{p}, \vec{q}) \rightarrow \infty$  implies  $\text{sim}(\vec{p}, \vec{q}) = 0$ . Therefore it is necessary to find a mapping of distance to similarity, if a distance measure should be used as similarity measure. A typical distance measure is the  $L_S$ -Norm [42]:

$$L_S : \delta_S(\vec{p}, \vec{q}) = \sqrt[S]{\sum_{i=0}^{n-1} |\vec{p}_i - \vec{q}_i|^S} \quad (3)$$

with  $S \in \mathbb{R}^+$ . For example, the  $L_2$ -Norm is the euclidean distance:

$$L_2 : \delta_2(\vec{p}, \vec{q}) = \sqrt{\sum_{i=0}^{n-1} |\vec{p}_i - \vec{q}_i|^2} \quad (4)$$

Similarity between vectors can now be defined by mapping the distances to similarity values using a correspondence function [42]:

**Definition 9** (Correspondence function).  $h : \mathbb{R}^+ \mapsto [0, 1]$  is a **correspondence function**, if it fulfills the following properties:

1.  $h(0), h(\infty) = 0$ ,
2.  $\forall x, y : x > y \implies h(x) \leq h(y)$

Aggarwal et al. [4] show that  $L_S$  distance measures especially suffer the curse of dimensionality for increasing values of  $S$ . The curse of dimensionality implies that "the ratio of the distances of the nearest and farthest neighbors to a given target in high dimensional space is almost 1 for a wide variety of data distributions and distance functions" [4]. Aggarwal et al. [4] also show that for increasing dimensions, the distance between the nearest and farthest neighbors converges against  $\infty$  for  $L_1$ , against some constant  $c \in \mathbb{R}$  for  $L_2$ , and for higher  $L_S$ -norms the distance converges against 0. This implies that  $L_S$  norms with higher  $S$  are bad at contrasting between vectors in high-dimensional space [4]. Therefore the distance-based kNN used in

the thesis should use either the  $L_1$ - or the  $L_2$ -norm as distance measure to avoid the low contrast associated with the other  $L_S$  measures.

In ontology learning, **semantic similarity** is used to great effect, e.g for clustering objects to create hierarchies[20] [43] or mapping between different ontologies [13] [39]. Semantic similarity compares the semantic content of objects or documents. This can be achieved by comparing which features can be found in both objects (commonalities) and which features are unique to the compared objects (differences). Rodríguez and Egenhofer [39] develops a semantic similarity measure for comparing entity classes in ontologies. Given two objects  $a, b \in \Omega$ ,  $A$  and  $B$  are their corresponding descriptions sets, e.g. for Wikidata the aliases and statements.  $\alpha$  is a function, which defines the importance of differences between  $a$  and  $b$ .  $A \cap B$  is the set of commonalities, and  $A/B$  the set of differences between the  $a$  and  $b$ . The defined similarity function is not symmetric,  $\text{sim}(a, b) \neq \text{sim}(b, a)$ .

$$\text{sim}(a, b) = \frac{|A \cap B|}{|A \cap B| + \alpha(a, b)|A/B| + (1 - \alpha(a, b))|B/A|} \quad (5)$$

for  $0 \leq \alpha \leq 1$  [39].

Calculating the similarity between vectors is advantageous to calculating the similarity between objects in an ontology, because vectors abstract from the specific objects and their relations to other objects in the ontology. Using neural word or graph embeddings, which are presented in Sections 2.6.2 and 2.6.3, enables the representation of classes and instances in ontologies as feature vectors. The mentioned curse of dimensionality is a non-issue as it applies, if the the number of irrelevant of features is high. This is typically solved by reducing the dimension of the feature vectors to include only relevant features [14]. Neural embeddings however seem to implicitly capture only relevant features [30], therefore no dimensionality reduction is necessary, if neural embeddings are used.

## 2.4 Problem statement

The task of this thesis is the classification of orphan classes in Wikidata. In other words a function is needed, which given an orphan class  $u$  of a taxonomy  $T = (C, S)$  with a root class  $\text{root}_T$ , find an appropriate superclass for  $u$ .

An optimal similarity function  $\text{sim}_{\text{opt}}$  is assumed for the definition of the problem statement. Let  $s \in C$  be the most specific, appropriate superclass of  $u$ . The following property should be fulfilled by  $\text{sim}_{\text{opt}}$ :

$$\forall c \in C \setminus \{s\} 1 = \text{sim}_{\text{opt}}(u, u) > \text{sim}_{\text{opt}}(u, s) > \text{sim}_{\text{opt}}(u, c) \quad (6)$$

This definition implies that the most similar class to  $u$  in the taxonomy  $T$ , other than  $u$  itself, is the most appropriate superclass  $s$  of  $u$ .

The thesis' problem is defined as follows:

**Definition 10** (Problem definition). Given a taxonomy  $T = (C, S)$  with root class  $root_T$  and a similarity function  $sim$  over  $T$ , find a function  $f : C \mapsto C$ , which, given an orphan class  $u \in C$ , returns a class  $s = f(u)$ , fulfilling the following criteria:

$$\forall p \in P : \neg(p \triangleleft_T u) \text{ no children} \quad (7)$$

$$s = \max_{s \in C} (sim_{msp}(u, c)) \text{ most specific parent} \quad (8)$$

The stated problem induces several challenges, which will be listed here, but addressed in later sections.

1. **High number of labels.** An orphan class has to be assigned to a class in the root taxonomy. The only restricting condition is that the chosen class cannot be a subclass of the orphan class. As shown in Section 3, 97% of 12999501 classes are part of the root taxonomy. Classification methods, like SVM or neural networks, usually classify with a small number of labels. A classification method, which is able to handle over a million labels, is required.
2. **Representation of items.** Items in Wikidata are structured information, similar to nodes in RDF graphs and Wikidata can be represented as such [38] [15]. As motivated in Section 1, the solution should exploit the apparent power of neural networks. Neural networks, which are introduced in Section 2.6, require input to be represented as vectors. Therefore, it will be necessary to map items to vectors.

## 2.5 k-nearest-neighbors classification

Based on the characteristics of the classification problem, described by the problem statement, and the challenges attached to it, the **k-nearest-neighbor algorithm (kNN)** seems like an appropriate tool for solving the task. Nearest-neighbors classification is a lazy method, as it does not require training before testing. This is useful for applications with high amounts of data, large numbers of classes, and changing data [44] [7]. For the considered use case of classification in Wikidata, these are very important strengths, as the number of classes in the taxonomy is very high and Wikidata is being constantly edited.

kNNs can be defined as a similarity-based classification method. kNN uses a pairwise similarity or distance measure (see Section 2.3). Access to the features of the classified objects is therefore not required [7].

Given a set of classified objects  $\Phi \subset \mathbb{R}^n$  with the set of classes  $C$ , where the function  $h : \Phi \mapsto C$  returns the class for a given object. Given an unclassified object  $\vec{u} \in \mathbb{R}^n$ , the parameter  $k$  defines the number of nearest neighbors  $\{\vec{n}_i \in \Phi | k = 1, \dots, k\}$  of  $\vec{u}$ , which are considered for the classification decision. The unknown is classified by a vote. Each nearest neighbor  $n_i$  votes for its own class  $c_i = h(n_i)$ . The class, which has the majority vote, is then chosen as the class of  $\vec{u}$ .

The votes of each neighbor can be weighted based on different criteria. Uniform weights give each vote the same weight, therefore the distance between the neighbors to the unknown is not relevant for this weighting scheme. Figure 5 describes a classification using kNN with uniform weights.

Results produced by uniform weighted kNN are not always desirable. Assume the case, in which the parameter  $k$  was chosen too high for a given unknown, which leads to the problem that very dissimilar objects could be part of the  $k$  nearest neighbors. Using uniform weighting would be problematic, since the dissimilar objects would negatively influence the majority vote. Votes of distant objects are not as relevant to the classification as close objects [7]. This leads to the criteria of **affinity**, which states that similar neighbor should be given larger weights [7]. A typical weighting approach to achieve affinity is distance-based weighting. The distance between each neighbor and the unknown is computed. For example in  $\mathbb{R}^n$ , the euclidean norm  $\delta_2$  could be used. Each neighbor  $\vec{n}_i$  of  $\vec{u}$  is assigned the inverse distance as weight  $w_i = \delta_2(\vec{u}, \vec{n}_i)^{-1}$ . Therefore the weight of each neighbor is directly proportional to its similarity to  $\vec{u}$ .

## 2.6 Neural networks

The notion of neural networks is introduced with the example for feedforward networks with backpropagation. The task of representing semantic information encoded in text and graphs as feature vectors using neural embeddings is then discussed. Solutions for generating word and graph embeddings by Mikolov et al. [30], Ristoski and Paulheim [38], Cao et al. [6] are presented.

### 2.6.1 Introduction to neural networks

A **neural network** is a triple  $(N, C, W)$ , where  $N$  are **neurons**,  $C = \{(i, j) | i, j \in N\}$  is a set of **connections** between neurons, and  $W$  are the weights corresponding to the connections, where  $W_{ij}$  is the weight for the connection  $(i, j) \in C$  [24]. Each neuron has an activation function  $f_{act}$ , which commonly is the Fermi function

$$f_{act} = \frac{1}{1 + e^{-x}} \quad (9)$$

mapping values to the interval of  $(0, 1)$ . This function is non-linear and therefore allows the neural network to solve non-linear problems. Input neurons typically use the identity function  $id$  as activation function [24]. The output of a neuron is computed by summing up the products of weights and inputs of the neuron, which is called net input, and applying the activation function on this value:

$$o_i = f_{act}\left(\sum_{k=1}^n w_{ki} o_k\right) \quad (10)$$

The input for input neurons (neurons on the first layer) are the inputs of the network.

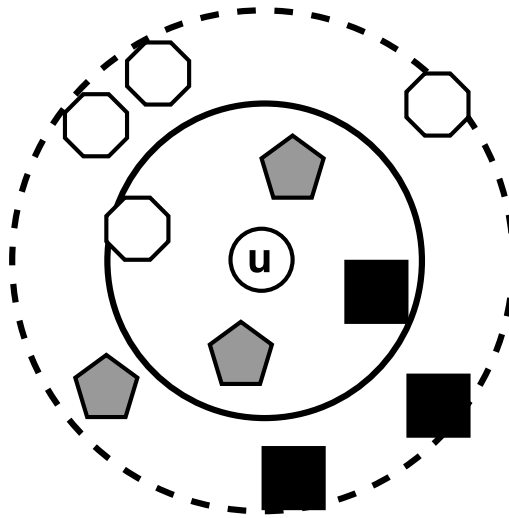


Figure 5: Example for kNN with uniform weights.

kNN considers only the closest  $k$  neighbors in its classification decision. In this example, the circle with name  $u$  is the unknown class, which has to be classified. 10 other objects, where each belongs to one of the 3 classes (square, pentagon, octagon), are present. Different weighting schemes are used by kNN. Each neighbor votes on the class of  $u$ . The weight controls to what degree a single neighbor can influence the class of  $u$ . The simplest approach is uniform weighting, which is used in this example. Uniform weighting assigns each object the same weight. Therefore, the distance of the neighbors to  $u$  does not matter in the classification decision.

If  $k = 4$  then all objects in the inner circle are considered. 2 objects have the class pentagon, while the classes square and octagon are represented with 1 object each in the 4 neighbors. Using uniform weighting, the class pentagon gets 2 votes and therefore  $u$  would be classified as pentagon.

Changing the parameter  $k$  has influence on the results produced by kNN. If  $k$  would be set as 10 instead of 4, all object in the outer dashed circle would be considered. The classes square and pentagon have each 3 objects in this set of neighbors, while the class octagon has 4 objects in the set. Using uniform weights would therefore classify  $u$  as octagon.

It could be argued that the classification result using  $k = 10$  may be inaccurate, since all objects of class octagon have a high distance to  $u$  in comparison to the objects in class pentagon. Therefore the decision made by  $k = 4$  may be more accurate. This problem can be solved by applying other weighting schemes than uniform weights to kNN. A typical approach, described in Section 2.5, is distance-based weights. This approach assigns higher weights to closer neighbors and could therefore solve the previously mentioned inaccuracy.



Figure 6: Example for feedforward neural network

A **feedforward neural network** consists of an input layer with  $s_i$  input neurons, an output layer with  $s_o$  output neurons and  $d$  hidden layers with  $h$  neurons per layer. A neural network with many hidden layers is called deep neural network. The connections in a feedforward network only connect the neurons of a layer  $i$  with the next layer  $i + 1$ . This means that a neuron can never influence itself. Networks, which allow this, are called recurrent neural networks. See in Figure 6 a feedforward neural network with  $s_i = 2$ ,  $s_o = 1$ ,  $d = 1$ ,  $h = 2$  with already initialized weights.

**Forward propagation** pushes an input through all layers of the network in a step-wise manner (layer per layer). In the first step, the input vector  $\vec{x}$  is pushed to the hidden layer and outputs of the hidden layer are computed, which results in a vector  $\vec{h}$  in the hidden layer:

$$\vec{h} = f_{act}\left(\begin{bmatrix} w_{1,3} & w_{2,3} \\ w_{1,4} & w_{2,4} \end{bmatrix} \vec{x}\right) \quad (11)$$

In the second step, the output of the hidden layer is pushed to the output layer resulting in the output  $y^*$ :

$$y^* = f_{act}\left(\begin{bmatrix} w_{3,5} & w_{4,5} \end{bmatrix} \vec{h}\right) \quad (12)$$

Assuming the expected output for an input  $\vec{x}$  is  $y$ , the error function is defined as

$$E = \frac{1}{2}(y - y^*)^2 \quad (13)$$

Training objective is to minimize the error function  $E$ . This can be accomplished by adjusting the weights  $w_{ij} \in W$ . Rather than brute-forcing the optimal weights for a network, **backpropagation** using **gradient descent** is applied in training. The idea of gradient descent is to look at the slope of the error function in respect to the current weights and adjust the weights into the descending direction. To do so the partial gradient in respect to each weight  $w_{ij}$  has to be derived from the error

function [32]:

$$\frac{\partial E}{\partial w_{ij}} = \delta_j o_i \text{ with} \quad (14)$$

$$\delta_j = \frac{\partial E}{\partial o_j} \frac{\partial o_j}{\partial \text{net}_j} \quad (15)$$

Subsequently, the weights can be updated by adding  $\Delta w_{ij} = -\alpha \frac{\partial E}{\partial w_{ij}}$  to each corresponding weight [32]. Repeating the backpropagation for a set of training samples will train the neural network to approximate a specific function.

In the following subsections, the concept of embeddings will be used to generate word and graph vector representations. By learning a certain task like predicting the probability for a certain word to appear in a context, a neural network implicitly learns hidden embeddings for the inputs. Embeddings are, in the case of Word2Vec [30], the connection weights between the input and hidden layer. More specifically, if the weights are represented as a matrix (e.g. see Equation 11), each row is an embedding for the corresponding input neuron.

### 2.6.2 Word embeddings

Mikolov et al. [30] introduce two neural network language models, **Continuous Bag-of-Words (CBOW)** and **Skip-gram (SG)**, which have proven to be very effective at creating word embeddings. The generated word embeddings of both models encode the semantics and linguistic regularities of words. Words, which are semantically close, are also close in the word embedding vector space. Calculating the offset between words makes it possible to answer more complex semantic questions. It is possible to answer more complex questions than similarity about the relationship between words. For example, the question "What is the word that is similar to *small* in the same sense as *biggest* is similar to *big*?" can be answered by calculating the offset between *big* and *biggest* and adding the vector of *small* to it:  $\text{vector}(\text{"biggest"}) - \text{vector}(\text{"big"}) + \text{vector}(\text{"small"})$  [30].

Further research in this topic has shown that **SG with negative sampling (SGNS)** is generally more effective in generating high-quality word embeddings than CBOW [30] [31] [27], as it is better in representing rare words. Therefore only SGNS will be introduced in this thesis. The following description is based on work by Mikolov et al. [30] [31], Levy and Goldberg [25] [26] [18], Rong [40], and Levy et al. [27].

SGNS uses a word corpus  $w \in V_W$  and the corresponding context corpus  $c \in V_C$ .  $V_W$  and  $V_C$  are vocabularies. Given a sequence of words  $w_1, \dots, w_n$ , e.g. a text corpus like a Wikipedia text dump, the context to a word  $w_i$  are the  $2 * L$  words around it:  $w_{i-L}, \dots, w_{i-1}, w_{i+1}, \dots, w_{i+L}$ .  $L \in \mathbb{N}$  is the size of the context window. The network (see Figure 7) is trained for the task of predicting the context of an input word. Each word  $w \in V_W$  is mapped to an embedding vector  $\vec{w} \in \mathbb{R}^d$ , and each context  $c \in V_C$  is mapped to an embedding vector  $\vec{c} \in \mathbb{R}^d$ , where  $d$  is the embedding size. Both embeddings are parameters, which are learned by the network. The



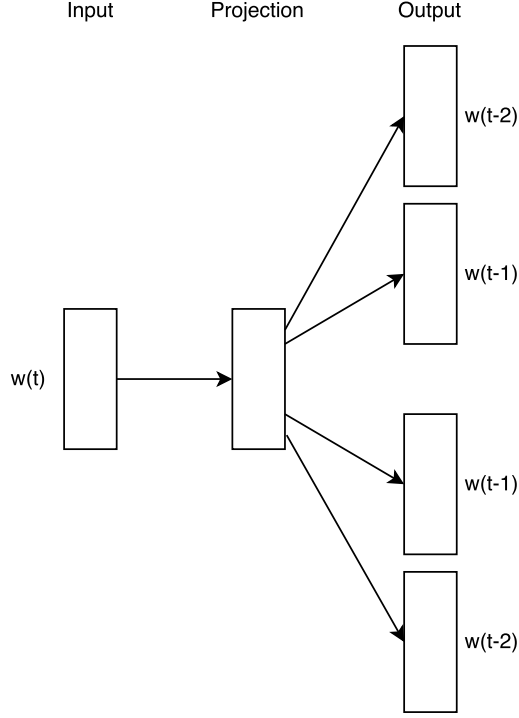


Figure 7: Skip-gram model

embeddings can be represented as matrices  $W \in \mathbb{R}^{|V_W| \times d}$  and  $C \in \mathbb{R}^{|V_C| \times d}$ , where  $W$  maps the word input to the projection layer and  $C$  maps the projection to the output layer, and therefore the context of the entered word [40].

Given a word  $w$  and a context  $c$  the model wants to maximize the probability  $p(D = 1|w, c)$ , that  $(w, c)$  is in the data  $D$ . The probability distribution is modeled as [25]:

$$p(D = 1|w, c) = \frac{1}{1 + e^{-\vec{w} \cdot \vec{c}}} \quad (16)$$

, which leads to the maximization objective [26]:

$$\max_{\vec{w}, \vec{c}} \sum_{(w, c) \in D} \log \frac{1}{1 + e^{-\vec{w} \cdot \vec{c}}} \quad (17)$$

This problem has a trivial solution with  $\vec{c} = \vec{w}$  and  $\vec{c} \cdot \vec{w} = K$ , for a large enough  $K$  [26] [25]. Using negative sampling solves the problem of having a trivial solution and also benefits the quality of word embeddings, as it increases the distance between word-context pairs, which do not occur in the data. Negative sampling is represented with the probability  $p(D = 0|w, c) = 1 - p(D = 1|w, c)$ , that a pair  $(w, c)$  does not occur in the data  $D$ . The negative sampling training objective can be

written as [26]:

$$\max_{\vec{w}, \vec{c}} \left( \sum_{(w,c) \in D} \log \sigma(\vec{c} \cdot \vec{w}) + \sum_{(w,c) \in D'} \log \sigma(-\vec{c} \cdot \vec{w}) \right) \quad (18)$$

, where  $D'$  is a set of negative training samples, which are not in  $D$ , and  $\sigma(x) = \frac{1}{1+e^x}$ . For this objective  $p(D = 1|w, c)$  needs to produce small values for  $(w, c) \in D'$  and high values for  $(w, c) \in D$ , which counteracts the trivial solution possible for objective 17.

Additionally Mikolov et al. [30] introduces parameters to further influence the quality of word embeddings. Rare words that occur less than a certain threshold are not considered as words or context. Additionally very frequent words are down-sampled (occur less often). This is done before context generation and increases the effective size of context windows [25], which improves the quality of word embeddings [31].

Research has shown that the effectiveness of word embeddings is highly dependent on the choice of hyperparameters [27] [31]. Experiments by Levy et al. [27] indicate that SGNS prefers a high number of negative samples. Additionally sub-sampling of very frequent words (e.g. "in", "a") benefits the embeddings, since these words provide less information than less frequent words [31].

SGNS has a log-linear computational complexity  $O$  in respect to the vocabulary size  $V$

$$O = E \times T \times C \times (D + D \times \log_2(V)) \quad (19)$$

where  $E$  is the number of training epochs,  $T$  is the number of words in the training set,  $C$  is the context window, and  $D$  is the word embedding size [30].

### 2.6.3 Graph embeddings

Similar to generating vector embeddings for words, it is beneficial to create graph embeddings. As it allows the extraction of useful information about vertices in its graph context [6].

Where sentences are directly representable as linear sequences and can therefore be directly used in SGNS, this is not the case for graph structures like RDF graphs or protein networks. Ristoski and Paulheim [38] proposes the use of **graph walks** to generate linear sequence samples. Given an directed, weighted graph  $G = (V, E)$ , graph walk will generate all graph walks  $P_v$  of depth  $d$  starting in vertex  $v$ , for all vertices  $v \in V$ . Breadth-first search is used for generating the graph walks. This results in a set of sequences of the format  $v_i \rightarrow e_{ij} \rightarrow v_j \rightarrow \dots$ , where  $v_i, v_j \in V$  and  $e_{ij} \in E$ . The number of generated walks increases exponentially with depth  $d$ . Therefore, instead of generating all graph walks for each vertice, a random walk approach as developed by Perozzi et al. [35] is used, where the number of walks per vertice is limited. A **random walk**  $W_v$  rooted at vertice  $v$  consists of random variables  $W_v^1, W_v^2, \dots, W_v^k$ .  $W_v^{i+1}$  is a vertice, which is chosen randomly from the

neighbors of  $W_v^i$ . Random walking allows an easier parallelization, since multiple workers can simultaneously generate walks in different parts of the graph [35]. Evaluation results show that this approach outperforms standard feature generation approaches [38].

Cao et al. [6] develop a deep neural network for graph representation (DNCR). Instead of generating graph walks, a **random surfing** model similar to Google’s PageRank is used to generate a probabilistic co-occurrence matrix, which indicates the probability of reaching a vertice  $j$  after a number of steps  $k$  from a starting vertice  $v_i$ . Similar to PageRank a teleportation probability  $\alpha$  is used, which indicates the chance whether the random surfing continues or is reset to the starting vertice. A row  $p_k$  of the co-occurrence matrix is therefore defined as follows [6]:

$$p_k = \alpha \cdot p_{k-1}A + (1 - \alpha)p_0 \quad (20)$$

with  $p_{0_i} = 1, p_{0_j} = 0, j \neq i$ . Based on the co-occurrence relation, a vector representation  $r$  can be defined. It can be assumed that vertices, which are close to the original vertex should have higher weight than distant vertices. This leads to the vector representation  $r$  for the starting vertex  $v_i$  [6]:

$$r = \sum_{k=1}^K w(k) \cdot p_k^* \quad (21)$$

with  $p_k^* = p_{k-1}^*A = p_0A^k$  being the probabilities of arriving in exactly  $k$  steps, if no random restart occurs, and  $w$  being a decreasing weight function. Finally a stacked denoising autoencoder is used to produce a non-linear mapping from the representations to low dimensional vectors. Stacked implies that the autoencoder has multiple hidden layers (deep neural network), which allows the learning better embeddings with each layer [6]. The neural network encodes and decodes the inputted vectors, which performs a meaningful dimension reduction by removing redundant information and noise [6]. Evaluation with comparison to other word embedding approaches like SGNS [30], DeepWalk [35], etc. show that the combination of random surfing and deep neural networks is effective, as the approach performs better than the other baselines. However, the approach has a linear complexity in regards to the number of vertices in the graph, while SGNS has a log-linear computational complexity [30]. This is problematic in the thesis’ use case, since Wikidata contains over 20 million vertices.

Random surfing has multiple advantages to sampling approaches like graph walking. Linear sequences have finite lengths and can therefore fail to capture relevant contextual information. Using random surfing overcomes this problem as it is able to consider walks of every length. Additionally, a desired property of embedding approaches is the ability to weight context based on its distance to original word or vertice [30] [6]. Random surfing allows, similarly to Word2Vec (see Section 2.6.2, the weighting of words based on its distance, which is important to create good word representations [6].

## 2.7 Ontology learning

Manually building ontologies is an expensive, tedious and error-prone process [20]. Maedche and Staab [29] recognize that the manual construction of ontologies results in a **knowledge acquisition bottleneck**, which motivates the research into the field of **ontology learning (OL)**. The field of OL supports ontology engineers in the construction and maintenance of ontologies by providing OL techniques in the form of tools like *OntoEdit* [29]. The process of OL can be divided into different subtasks. OL tools consist of different components, which automatically or semi-automatically support this process. The field of ontology learning exploits different research fields like natural language processing, machine learning, ontology engineering, etc. [8].

The process of OL and the components of a generic OL architecture are summarized and the task of the thesis is categorized. Following, basic algorithms for learning taxonomic relations are summarized. Both subsections are based on work by Cimiano et al. [8], Maedche and Staab [29], and Hazman et al. [20]. Finally, related work, which exploits neural networks, is analyzed and compared to the thesis' task. The novelty and additional benefits of this work are justified.

### 2.7.1 Process and architecture for ontology learning

The process of OLg can be divided into subtasks. Maedche and Staab [29] define a ontology learning cycle, consisting of the following steps:

1. **Import/Reuse** is the merging of existing structures and mapping between the structures and the target ontology.
2. **Extraction** defines the modeling of the target ontology by feeding from web documents.
3. **Prune** takes the generated model and adjusts the ontology to its purpose.
4. **Refine** completes the ontology at a fine granularity.
5. **Apply** applies the resulting ontology on its target application. This serves as a measure of validation.

These 5 steps can be repeated as often as necessary to include additional domains and updating it with new content. The thesis' task can be categorized to the *Refine* step, as its goal is to improve the completeness of an existing ontology.

Cimiano et al. [8] introduces a generic OL architecture and its major components. The described tool is semi-automatic, meaning that it support an ontology engineer rather than fully automatizing the process of ontology construction and maintenance. The architecture consists of the following components:

1. **Ontology management component** provides an interface between the ontology and learning algorithms. Learned concepts, relations and axioms should

be added to the ontology using this component. It is also used for manipulating the ontology. More specifically, for the importing, browsing, modification, versioning and evolution of ontologies.

2. **Coordination component** is the user interface, which should allow the ontology engineer to choose input data, learning and resource processing methods.
3. **Resource processing component** allows the discovery, import, analysis and transformation of unstructured, semi-structured and structured input. For this purpose the component needs different natural language processing components to parse and analyze input on different levels, word to sentence-level.
4. **Algorithm library component** contains the algorithms, which are applied for the purpose of ontology learning. These algorithms are generic standard machine learning methods, as well as specialized OL methods. Additionally, different similarity and collocation measures should be available.

In the context of Wikidata, there exists no single comprehensive OL tool. However, in the Wikimedia technology space different tools exist, which mainly support the task of refining and maintaining the current ontology. For example, Stratan [41] develops a taxonomy browser for Wikidata, which is able to evaluate the quality of the taxonomy by detecting different types of cycles, redundancies, errors and unlinked classes. This tool is an ontology learning component, as it provides the ability to browse and evaluate the ontology of Wikidata.

### 2.7.2 Approaches for learning taxonomic relations

A subgroup of algorithms for OL is concerned with learning taxonomic relations. The following approaches, categorized by Cimiano et al. [8], use text as input.

**Lexico-syntactic patterns** are word patterns in patterns, which are used to identify hypernym-hyponym pairs (superclass-subclass pairs) in natural text. For example, such a pattern is

$$NP_{hyper} \text{ such as } \{NP_{hyponym},\}^* \{(and \mid or)\} NP_{hyponym}$$

, where  $NP$  stands for noun phrase, and  $NP_{hyper}$  is a hypernym or superclass, while  $NP_{hyponym}$  are hyponyms or subclasses. These patterns provide reasonable results, but the manual creation of patterns is involve high cost and time investments [43].

**Clustering** uses some measure of similarity to organize objects into groups. This can be achieved by representing the words or terms as vectors [10], on which different distance or similarity measures can be applied. Clustering methods can be categorized to three different types. Agglomerative clustering initializes each term as its own cluster and merges in each step the most similar terms into one cluster. Divisive clustering approaches the problem the opposite way by starting with a single cluster, containing all words, and then dividing them into smaller groups. Both

approaches generate hierarchies. Agglomerative clustering is doing so bottom-up and divisive clustering top-down.

**Phrase analysis** analyses noun phrases directly. It is assumed that nouns, which have additional modifiers are subclasses of the noun without modifiers. For example, this could be applied on the labeled unlinked classes of Wikidata. For example the classes *Men's Junior European Volleyball Championship* (Q169359) and *Women's Junior European Volleyball Championship* (Q169956) could be subclasses of *European Volleyball Championship* (Q6834). In this case, phrase analysis interprets *Men's Junior* and *Women's Junior* as modifiers, which denote these classes as specialization to Q6834.

**Classification**-based approaches can be used, when a taxonomy is already present. In this case, classification can be used to add unclassified concepts to the existing hierarchy. Challenging with this task is that a taxonomy typically contains a large amount of classes and therefore classification methods like SVM are not suited for the task. Specific algorithms, which only consider, a subset of relevant classes are necessary to carry out an efficient classification. For example, Pekar and Staab [34] solves this problem by exploiting the taxonomy's tree structure using tree-ascending or tree-descending algorithms.

The algorithm developed by this thesis uses a classification-based approach, since a large taxonomy is already given. Instead of exploiting the hierarchic structure of the taxonomy, a variant of k-nearest-neighbors is used to address the "high number of labels" problem (mentioned in Section 2.4).

### 2.7.3 Neural networks in ontology learning

Fu et al. [16] develop a method to construct a semantic hierarchy (taxonomy) given a hyponym (subclass) and a list of corresponding hypernyms (superclasses), as shown in Figure 8. In this example, the input is the subclass *computer* (Q68) and its superclasses *tool* (Q39546) etc. Fu et al. [16]'s solution uses neural word embeddings, as discussed in Section 2.6.2, to construct a more specific and precise concept hierarchy. For this example, the direct superclasses of *computer* (Q68) are *tool* (Q39546) and *electrical apparatus* (Q2425052), while the other superclasses are transitive superclasses to *computer* (Q68).

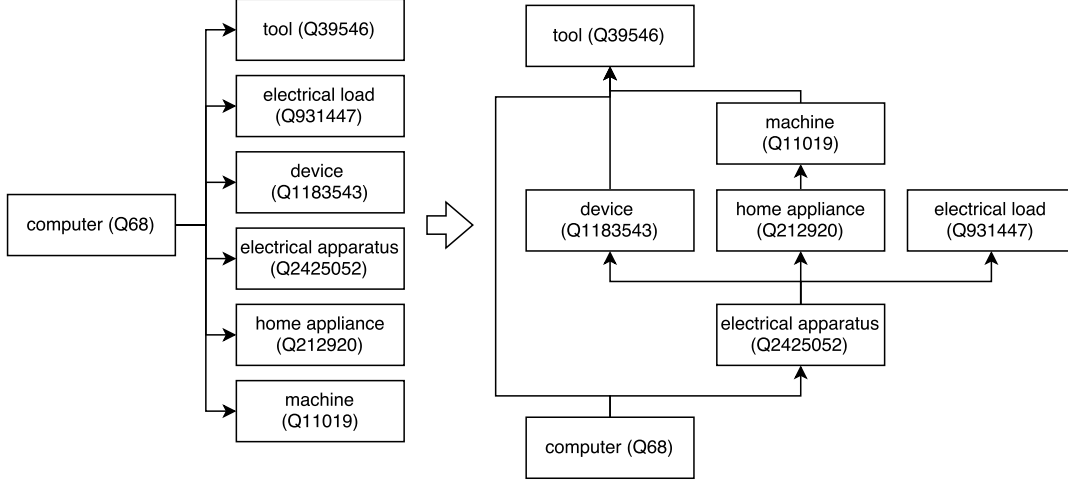


Figure 8: Construction of semantic hierarchy in Fu et al. [16].

The approach proposed by Fu et al. [16] exploits the linguistic regularities of neural word embeddings. It is assumed that there is a subclass vector offset between the subclass and its superclass embeddings, which behaves similar to the prominent example by Mikolov et al. [30]: Queen = King – Man + Woman. Computing such subclass offsets for random subclass-superclass pairs for Fu et al.’s dataset as well as the Wikidata dataset [3] analyzed in this thesis (see Section 4.3.2) shows that clusters of very similar subclass offsets exist. Analysis by Fu et al. reveals that the offsets in one cluster refer to similar topic, e.g. single cluster concerned about people’s occupations [16]. The subclass offsets in the Wikidata dataset reveal similar characteristic (see Section 4.3.2).

Fu et al. [16] conclude that the subclass-of relation is too complex to be described by a translation. Instead a linear projection is trained by minimizing the mean squared error between projection of the subclass  $\vec{x} \in \mathbb{R}^d$  and the actual superclass  $\vec{y} \in \mathbb{R}^d$ , where  $d$  is the embedding size. The following minimization problem is thereby solved:

$$\Phi^* = \min_{\Phi} \frac{1}{N} \sum_{(\vec{x}, \vec{y})} \|\Phi \vec{x} - \vec{y}\|^2 \quad (22)$$

where  $N$  is the number of  $(\vec{x}, \vec{y})$  subclass-superclass pairs in the training data [16]. The problem described by this minimization problem is a multivariate linear regression problem [16].

Fu et al. argues that because of the varied types of subclass-of relations, a single matrix is not adequate at fitting all subclass-superclass pairs. Therefore piecewise linear projection is implemented. The subclass-superclass pairs are clustered by their subclass-of offsets and a projection for each cluster is trained. Piecewise linear projection learns a projection matrix  $\Phi_k$  for each identified cluster in the data set [16]:

$$\Phi^* = \min_{\Phi_k} \frac{1}{N_k} \sum_{(\vec{x}, \vec{y}) \in C_k} \|\Phi_k \vec{x} - \vec{y}\|^2 \quad (23)$$

where  $N_k$  is the amount of word pairs in the  $k^{th}$  cluster  $C_k$  [16]. The number of clusters is fine-tuned based on the data set [16].

To determine whether a given pair of concepts  $\vec{x}, \vec{y} \in \mathbb{R}^d$  have a subclass-superclass relation, the cluster  $k$ , which has the closest cluster center to the subclass offset  $\vec{y} - \vec{x}$ , is chosen and the projection  $\Phi_k \vec{x}$  computed [16]. The pair  $(\vec{x}, \vec{y})$  is a subclass-superclass pair, if either the distance between the projection  $\Phi_k \vec{x}$  and  $\vec{y}$  falls below a certain threshold  $\delta$  or if there exist another concept  $\vec{z}$ , which is superclass of  $\vec{x}$  and subclass of  $\vec{y}$ , and therefore allows the use of the transitivity of the subclass-of relation [16]. This approach allows one subclass to have multiple superclasses and is also able to identify the case, in which no valid subclass-of relation exists.

Fu et al. evaluated the piecewise linear projection on a test set of 418 subclasses and their superclasses. The embeddings are trained on a 30 million Chinese sentence corpus and the projection is trained on 15247 hypernym-hyponym word pairs. The approach achieves an F-score of 73.74% on a manually labeled dataset, and outperforms existing state-of-the-art methods.

As argued in Chapter 2.6, the use of neural word embeddings seems promising, which is supported by the good results of Fu et al.'s work. A similar approach, which is presented in Chapter ??, can be applied in the given task. The main difference in the problems is the expected input, which is one of the reasons why Fu et al. [16]'s approach works so well. Fu et al. assumes that a subclass and its superclasses are given as input. This is advantageous for multiple reasons. Only a small set of classes has to be considered for the classification decision, which are all related to subclass and are therefore related to similar topics. Additionally, the input already provides a set of correct subclass-of relations. In the thesis' case, the only given input is the subclass. Therefore every class in the taxonomy has to be assumed as a possible superclass rather than a small subset. The problem statement also assumes that for each class a superclass has to be found. Consequently the identification of subclass-of relations by Fu et al. is not applicable anymore and another approach has to be implemented. Although piecewise linear projection is still a promising approach to use neural word embeddings for the subclass classification, the difficulty of the given problem implies worse results than in the presented work by Fu et al. [16].



### 3 Analysis of the Wikidata taxonomy

For the task of developing an algorithm, which takes orphan classes as input, it is necessary to know, what information the classes carry and if there are common characteristics among classes.

The taxonomy contained in the Wikidata dump of 2016/11/07 was analyzed. It contains a total of 24,507,102 items, of which 1,299,501 are classes. Classes were recognized as defined in Section 2.1.

#### 3.1 Root taxonomy

The state of the taxonomy was captured in regards to the root class *entity* (Q35120) (see Figure 9). 1,260,842 classes are currently subclasses of *entity* (Q35120). 97% of all classes are therefore nodes in the root taxonomy. This implies a high agreement in the Wikidata community on which class is considered root, and thereby also supports the modeling decision made in Section 2.2, which assumes that a taxonomy only has one root, and this root is *entity* (Q35120) in Wikidata.



Figure 9: Distance of subclasses to root class *entity* (Q35120). Wikidata (2016/11/07)

In Figure 9, the levels of the root taxonomy tree are represented. Most classes have a shortest distance between 6 and 10 to the root. While the highest distance to the root is 20. The most distant classes generally belong to concepts of biological taxonomies. The most distant classes are subclasses of the genus *Homo* (Q171283).

The developed algorithm will likely place orphan classes into the same levels of the taxonomy, in which most classes already reside. If the algorithm places classes in levels close to the root, it would indicate that the classification method is not specific enough. On the opposite, placing classes into distant levels would indicate incorrectness of the algorithm, since only a small subset of classes occurs in these levels.

### 3.2 All classes

All 1,299,501 classes were analyzed. 98.2% of all classes have an English label. In Figure 10, the percentage of classes with a specific amount of instances is shown. It can be seen that a majority of classes have no instances. The median of instances per class is 0, while the average is  $\approx 15$ . The high average in comparison to the low median implies that there are some classes with a very high amount of instances. Figure 11 shows the percentage of classes with a specific amount of subclasses. The median of subclasses per class is 0, while the average is  $\approx 1.3$ . Most classes do not have subclasses or instances. Subsequently, this implies that almost all classes should have the *subclass of* (P279), since the other two criteria used to identify classes do not apply. In Figure 12, the percentage of classes with a certain amount of properties is shown. The median of properties is 7, while the average is  $\approx 6.65$ . The similar median and average implies that few outliers exist. Figure 13 supports this conclusion that the majority of classes have the *subclass of* (P279) property. Other very common properties like *found in taxon* (P703), *Entrez Gene ID* and *UniProt ID* (P637) indicate that at least  $\approx 80\%$  of all classes are related to genes and proteins. The set of classes identified with these properties is mostly entered by bots, which map information from other knowledge bases like NCBI Entrez to Wikidata [36]. The characteristics of these classes is dissimilar to the human curated part of Wikidata, which is shown in the following Section 3.3. Additionally, the method developed by this thesis should find practical application by supporting humans in improving Wikidata's taxonomy. Therefore, the set of classes related to proteins and genes is not of relevance to the problem, and should not be considered in the development of an solution. Following, the set of relevant classes, which are of interest to human editors, is analyzed.



Figure 10: Percentage of all classes with a specific amount of instances. Wikidata (2016/11/07)

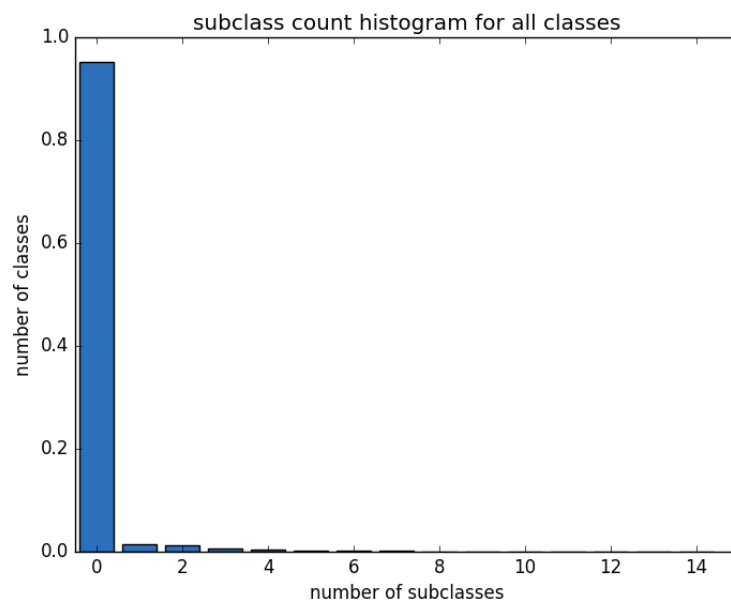


Figure 11: Percentage of all classes with a specific amount of subclasses. Wikidata (2016/11/07)



Figure 12: Percentage of all classes with a specific amount of unique properties. Wikidata (2016/11/07)



Figure 13: Frequency of properties in all classes. Wikidata (2016/11/07)

### 3.3 Relevant classes

Following the observations in the previous section, the set of relevant classes was analyzed. A class is considered relevant, if it does not have an irrelevant property. The properties considered irrelevant identify classes representing genes and proteins. The following properties are irrelevant:

- *Entrez Gene ID* (P351)
- *genomic start* (P644)
- *genomic end* (P645)
- *strand orientation* (P2548)
- *UniProt ID* (P352)
- *NCBI Locus tag* (P2393)
- *encoded by* (P702)
- *encodes* (P688)
- *molecular function* (P680)
- *RefSeq RNA ID* (P639)
- *Mouse Genome Informatics ID* (P671)
- *Ensembl Transcript ID* (P704)
- *HGNC gene symbol* (P353)
- *HomoloGene ID* (P593)
- *Gene Ontology ID* (P686)
- *InterPro ID* (P2926)

These properties were handpicked mostly based on the property frequency in Figure 13.

A total of 195,124 classes are relevant classes, which  $\approx 15.01\%$  of all classes. Of all relevant classes 87.92% have an English label, which is  $\approx 10\%$  percent less than the ratio of all classes. The distribution of instances and subclasses is similar to the set of all classes (Figures 14 and 15). Both the percentage of classes with 0 instances, as well as the classes with 0 subclasses, are about 15% lower than all classes. Accordingly the instance average increases from  $\approx 15$  to  $\approx 99.5$ , and the subclass average increases from  $\approx 1.3$  to  $\approx 7.02$ . The median remains unchanged at 0 for both instances and subclasses. Comparing the distribution of properties of all classes (Figure 12) to relevant classes (Figure 16) shows a shift to the left, which implies a decrease of properties per class in the set of relevant classes. The average decreased from  $\approx 6.65$  to 4.82 and the median decreased from 7 to 4. Genes have simultaneously occurring properties, e.g. *genomic end* (P645), *genomic start* (P644), *strand orientation* (P2548), and *Entrez Gene ID* (P351), which results in a high percentage of classes having a similar number of properties, which is 7 for the set of all classes. Removing these classes consequently lowers the property median. The property frequency (see Figure 17) shows a range of different properties, which are not concerned with the same subject, as it was the case for the set of all classes. The property frequency shows that classes concerned with countries (*country* (P17)) and sports (*sport* (P641)) are relatively common. But even the most common topic related properties occur with a frequency of less than 30%, therefore it is concluded that the data is not biased to a certain topic, as it was for the set of all classes. As a result,

the hybrid algorithm will be developed and evaluated in regards to relevant classes rather than all classes.

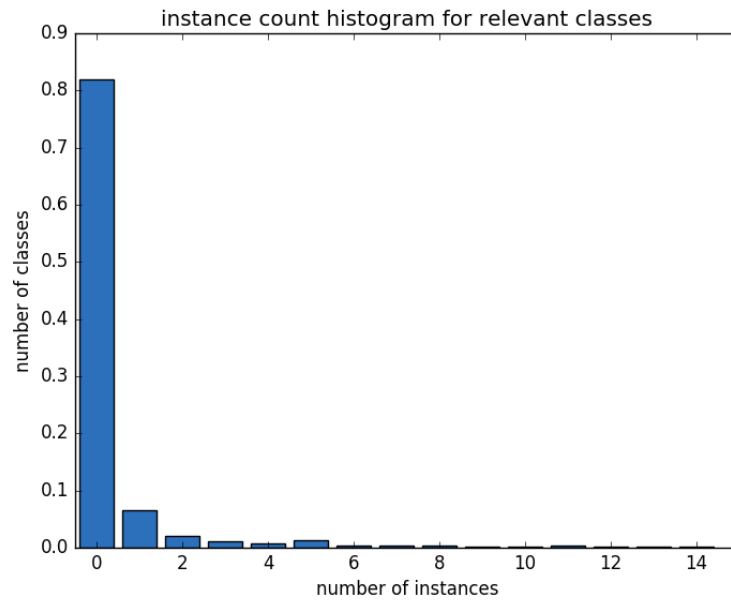


Figure 14: Percentage of relevant classes with a specific amount of instances. Wiki-data (2016/11/07)

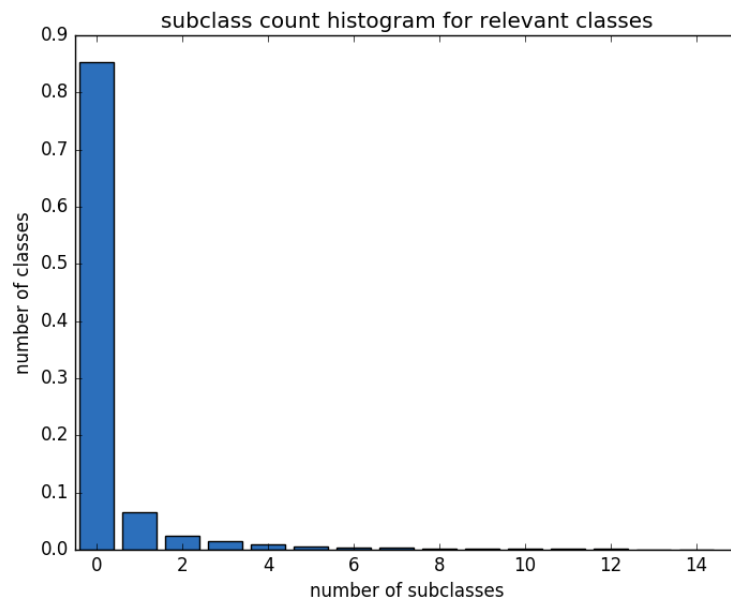


Figure 15: Percentage of relevant classes with a specific amount of subclasses. Wiki-data (2016/11/07)

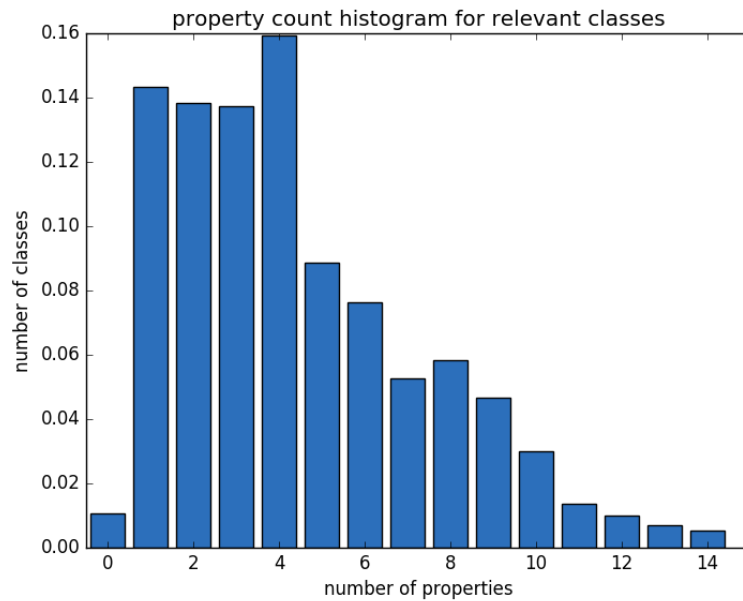


Figure 16: Percentage of relevant classes with a specific amount of unique properties. Wikidata (2016/11/07)

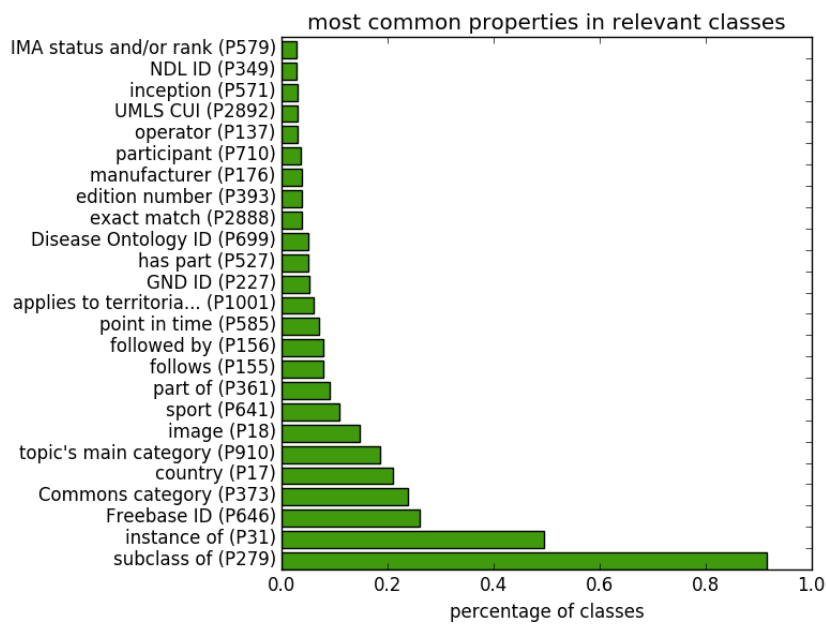


Figure 17: Frequency of properties in relevant classes. Wikidata (2016/11/07)



### 3.4 Orphan classes

Orphan classes were identified by checking whether a class does not have the *subclass of* (P279) property. The final goal of the thesis is to develop a method, which is able to classify orphan classes with a good accuracy. However, orphan classes can not be used for testing the method, because a gold standard for orphan classes is missing. Instead a gold standard can be extracted from the set of relevant classes. To ensure that the gold standard evaluation is relevant to the given problem, the set of orphan classes has to be analyzed and compared to the set of relevant classes. If the characteristics of both sets are similar, the gold standard evaluation is directly relevant to the performance in classifying orphan classes.

16,373 classes were identified as orphan classes, which is  $\approx 1.26\%$  of all classes.  $\approx 84.33\%$  of all orphan classes have an English label, which is only  $\approx 5\%$  lower than the relevant classes. The distribution of instances for orphan classes (Figure 18) is very different to the relevant classes (Figure ??). The majority of orphan classes have at least 1 instance rather than 0 instances. Consequentially, the median of instances per class is 1 for orphan classes and the average is  $\approx 4.66$ , which is much lower than the average  $\approx 86.34$  of relevant classes. This, in combination with the higher median for orphan classes, implies that orphan classes have none to very few outliers in regards to many instances. Additionally, it can be hypothesized that orphan classes are often created for the purpose of classifying single instances by human editors, as a mean to provide a better description for the instance, but the orphan class itself is not further described. The subclass distribution (Figure 19) shows that  $\approx 25\%$  of orphan classes have at least 1 subclass, which is  $\approx 20\%$  higher than the relevant classes. However, the median of subclasses per class remains 0, while the average is  $\approx 0.852$  and therefore lower than the average 6.5 of relevant classes, which indicates very low or not existing amount of outliers in the set of orphan classes. The same behavior was already observed for the instance distribution. The distribution of properties per class (Figure 20) is very similar to the relevant classes (Figure 16). The only difference is the percentage of classes with 0 properties, which is  $\approx 12.5\%$  for orphan classes and lower than 1% for relevant classes. This observation supports the previously made hypothesis, which claims that orphan classes are mainly created for the purpose of classifying instances, since 12.5% of orphan classes is not described at all. The corresponding property median remains at 3 and the average is  $\approx 4.78$ , which is almost identical to the relevant classes. The most frequent properties for orphan classes, as shown in Figure 21, shows very similar properties to the relevant classes (Figure 17). This, in addition to the similar property median, is a good indicator that orphan and relevant classes have common characteristics, since their descriptions are similar. Therefore, it can be concluded that the set of relevant classes should be adequate at representing the set of orphan classes in the implementation and evaluation of the thesis' hybrid algorithm.

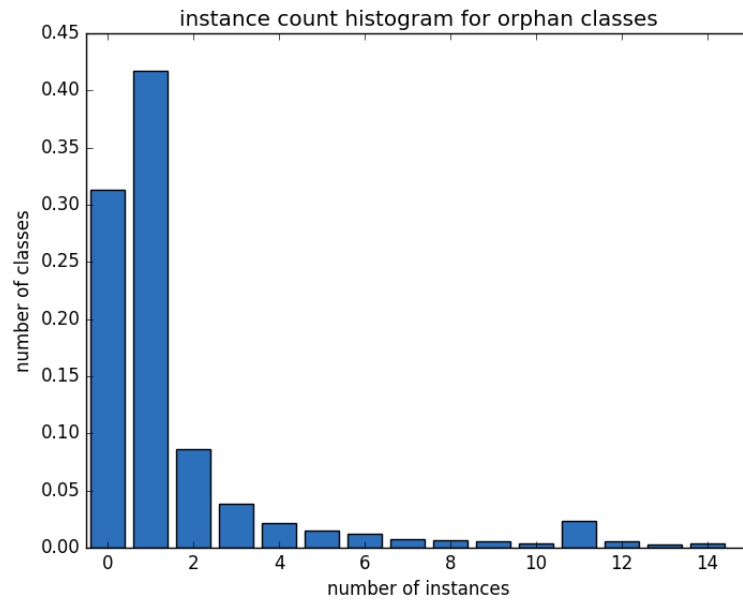


Figure 18: Percentage of orphan classes with a specific amount of instances. Wiki-data (2016/11/07)

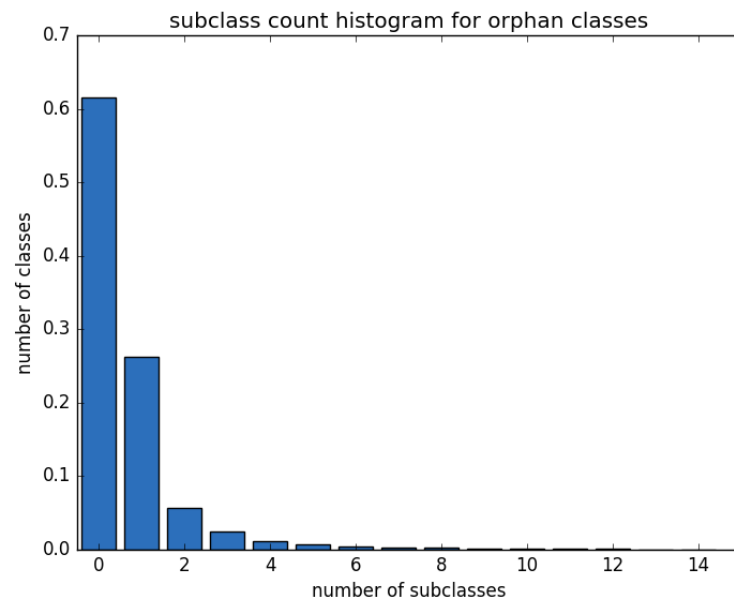


Figure 19: Percentage of orphan classes with a specific amount of subclasses. Wiki-data (2016/11/07)

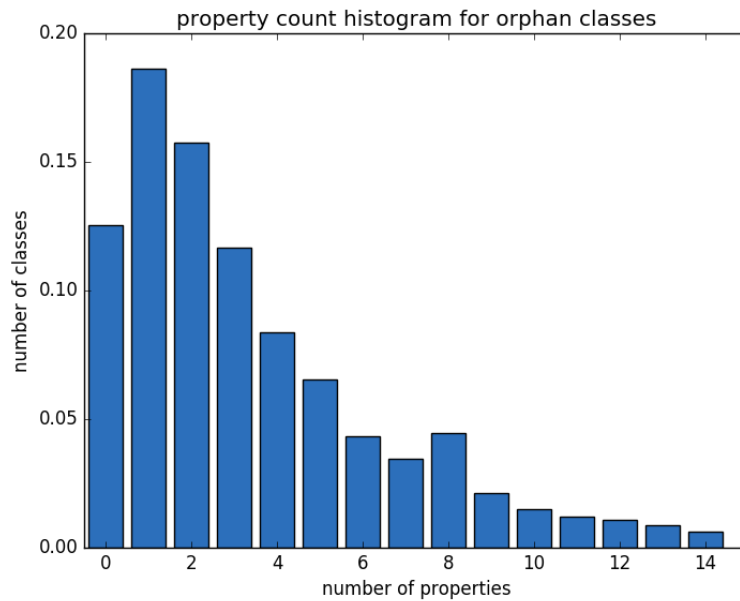


Figure 20: Percentage of orphan classes with a specific amount of unique properties. Wikidata (2016/11/07)

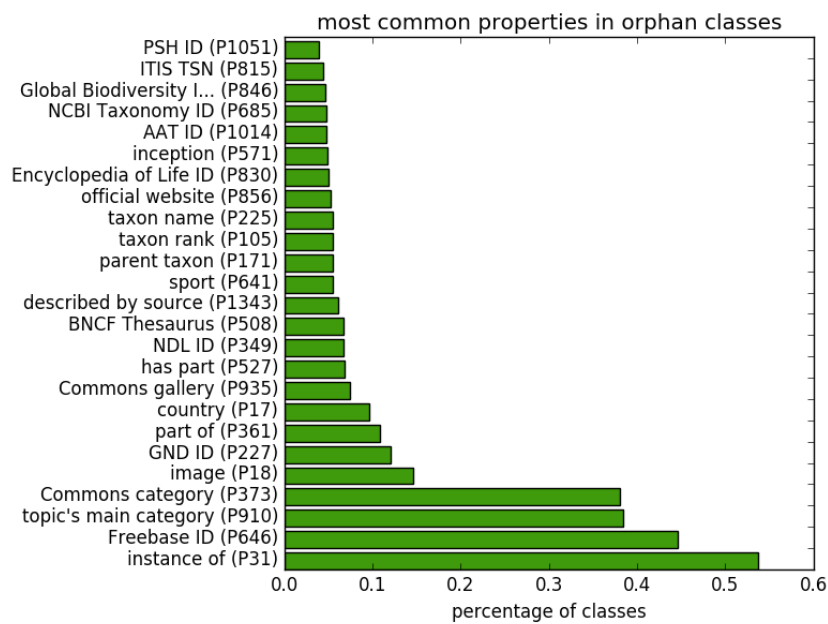


Figure 21: Frequency of properties in orphan classes. Wikidata (2016/11/07)

## 4 Hybrid algorithm

As motivated in the previous chapters, a hybrid algorithm using neural word embeddings is implemented to solve the defined problem. In this chapter, the required components for the algorithm are presented. For each component, possible implementations are proposed and implemented. Additionally, only relevant classes, as described in Section 3.3, are considered in this chapter. Therefore training of the algorithm uses only of relevant classes and their corresponding instances.

### 4.1 Components

The hybrid algorithm exploits neural word embeddings to solve the classification task, which is motivated by the power of word embeddings in similarity tasks [30] and in taxonomy construction [16].

For the task of computing word embeddings, the SGNS model (Section 2.6.2) will be used, which has shown to generate effective word embeddings relatively fast [30] [27]. It has been proven through theoretic analysis as well as experiments that SGNS creates very effective word embeddings, if its hyperparameters are chosen well [30] [27].

Based on Levy et al. [27], Mikolov et al. [30] and Mikolov et al. [31] the following hyperparameters are used:

- Embedding size: 300
- Subsampling frequency:  $10^{-5}$
- Context window: 2
- Negative samples: 15

The original paper by Mikolov et al. [30] uses context windows of size 2. Bigger context windows show slight improvements in the quality of word embeddings [27]. However triple sentences (see Section 4.2.1) only have a maximum context of 2, e.g.  $w_1 w_2 w_3$  the maximum distance between words in the same sentence is 2. Therefore triple sentences would not benefit from bigger context sizes. To preserve comparability between variations of the algorithm, the same context size is also used for all types of SequenceGen components. The implementation of SGNS in the gensim library by Řehůřek and Sojka [37] is used.

Two additional components can be identified for the hybrid classification algorithm using SGNS. The data flow and IO of the components is visualized in Figure 22.

1. **SequenceGen.** SGNS requires a linear sequence of words as input (see Section 2.6.2). The SequenceGen component transforms any possible kind of data into a set of sentences, which can be used for training the SGNS. Possible data sets include Wikidata and Wikipedia. Wikidata’s highly interlinked structure

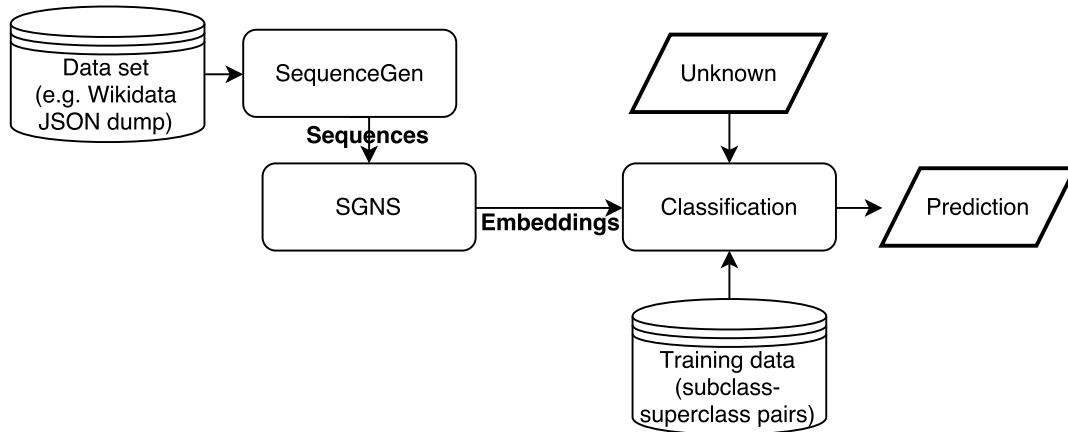


Figure 22: Data flow between components

is more alike to an RDF graph [15]. Using Wikidata therefore requires an approach, which maps the graph to linear sequences. If the encyclopedic text provided by Wikipedia is used, a different challenge has to be solved. Since the gold standard consists of pairs of subclass-superclass pairs, which were acquired from Wikidata [3], a mapping from words to Wikidata IDs would have to be implemented.

2. **Classification.** The Classification component is trained on a set of gold standard subclass-superclass pairs. It uses the word embeddings generated by SGNS to make a classification decision. The word embeddings produced by SGNS group similar words close to each other and preserve linguistic regularities. These properties can be exploited by different classifiers. kNN can exploit the first characteristic using a similarity or distance measure. The second characteristic can be exploited by computing the vector offset or learning a linear projection representing the subclass-of relationship between two classes [16]. The classification component should implement a single-label multiclass classification.

## 4.2 SequenceGen

### 4.2.1 Triple sentences

Triple sentences uses Wikidata as input data set. Statements in Wikidata can be represented as triples of  $(source, property, target)$ , where *source* is a Wikidata item ID, *property* is a Wikidata property ID, and *target* is either a Wikidata item ID or a literal. These triples represents simple 3-word sentences, which are sufficient as training input for Word2Vec. Appereances of literals are removed from the generated sentences to reduce the amount of noise in the data. In comparison to natural text, the triple sentence contain a very low amount of noise, which may improve the

quality of embeddings in comparison to using for example Wikipedia as data set.

Figure 23 shows an example mapping of Wikidata to triple sentences. Edges between entities, e.g.  $(Q42, P31, Q5)$ , are expressed as triple sentences, e.g. "Q42 P31 Q5", while edges between an entity and a literal, e.g.  $(Q42, P1559, \text{"Douglas Adams"})$ , ignore the literal and are therefore double sentences, e.g. "Q42 P1559".

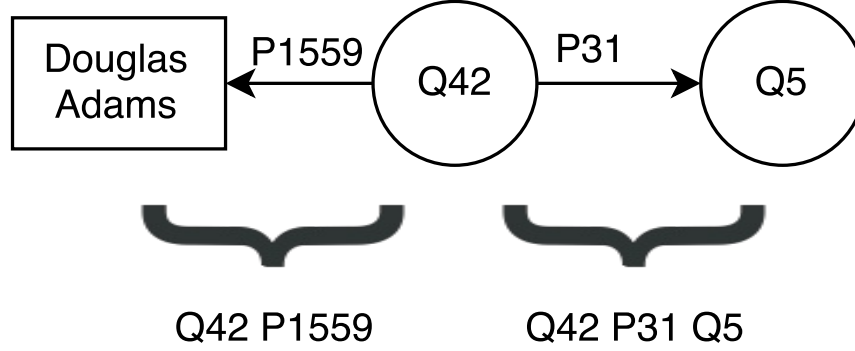


Figure 23: Generation of triple sentences from Wikidata

#### 4.2.2 Graph walk sentences

A graph walk, developed by Ristoski and Paulheim [38] and explained in Section 2.6.3, can be used for generating longer sentences instead of triple sentences. The longer sequences are able to better capture contextual information than triples. This is achieved by creating walks of a depth  $d$ , starting from each class. Evaluation of RDF2Vec shows that Skip-gram and CBOW both produce better word embeddings using graph walk [38]. Therefore it is expected that graph walk will also improve the performance of the hybrid algorithm. Computing all walks for each vertex is however not feasible, since the number of walks for each vertex is potentially exponential [38]. The exponential runtime can be counteracted by computing a limited number of random walks instead of computing all walks [35]. Additionally, the starting vertices for each walk will be sampled from the set of classes rather than the set of all items. The quality of word embeddings in regards to classes is a major concern, while the word embeddings for other items are only optional for the task of classification. The proposed measure will ensure that each class occurs at least once in the graph walk sequences and the frequency of classes in the generated set will be higher than by random sampling from all items.

The set of random walk sequences will be combined with the set of triple sentences, which are used in the baseline, to constitute a new training data set for the SGNS. The following parameters are used for the graph walk :

- Depth of walks: 4
- Maximum number of walks per vertice: 10

Ristoski and Paulheim [38] applies 200 maximum walks per vertice for the Wikidata dataset. Because of time and hardware constraints only 10 walks per vertice are computed, which should allow a "good" coverage of classes in the generated sequences.

Figure 24 shows an excerpt of the Wikidata graph. 2 random walks of depth 4 are generated beginning at the node Q42. The walks by the Arabic and Roman numerals. The walk indicated by Arabic numerals has a length of 3 and ends at the node Q223557. If no further successors can be found from a given node, the walk ends prematurely. The walks indicated by roman numerals has a length of 4 and ends at node Q215627. The Wikidata graph contains cycles, as is shown between nodes Q5 and Q8205328. The random walks can therefore also display cycles and therefore visit nodes multiple times. The output of graph walk sentences with 2 maximum walks and depth 4 with source node Q42 (based on Figure 24 is the following:

- "Q42 P31 Q5 P1542 Q8205328 P279 Q223557" (1, 2, 3, 4)
- ""Q42 P31 Q5 P1542 Q8205328 P170 Q5 P279 Q215627" (I, II, III, IV, V)

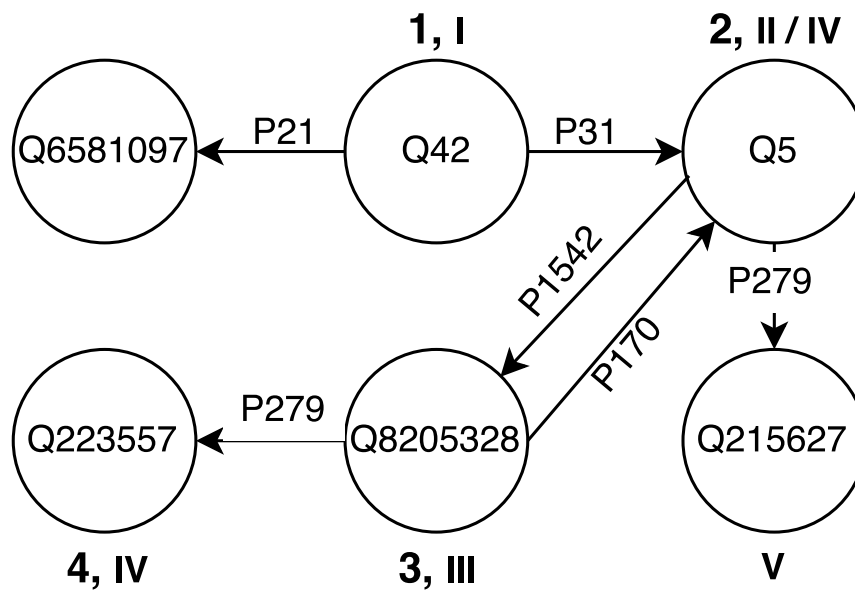


Figure 24: Two random graph walks in Wikidata with depth 4.

### 4.3 Classification

#### 4.3.1 k-nearest-neighbors

Similar classes are grouped close to each other in the embedding vector space. Intuitively a kNN classifier (see Section 2.5) should be able to use this property to great effect.

Distance-based kNN using the euclidean norm is implemented as classification component. The kNN implementation provided by the scikit-learn library [33] is used.

#### 4.3.2 Vector offset

A possible approach for classification is to exploit the linear regularities encoded in the word embeddings. As shown in Section 2.6.2, it is possible to answer semantic questions by applying algebraic operations on word embeddings [30]. Given a subclass-superclass pair with word embeddings  $\vec{x}, \vec{y} \in \mathbb{R}^d$  and the word embedding  $\vec{u} \in \mathbb{R}^d$  of an unknown class, the superclass with word embedding  $\vec{p} \in \mathbb{R}^d$  of  $\vec{u}$  could be calculated, as follows

$$\vec{p} = \vec{y} - \vec{x} + \vec{u}$$

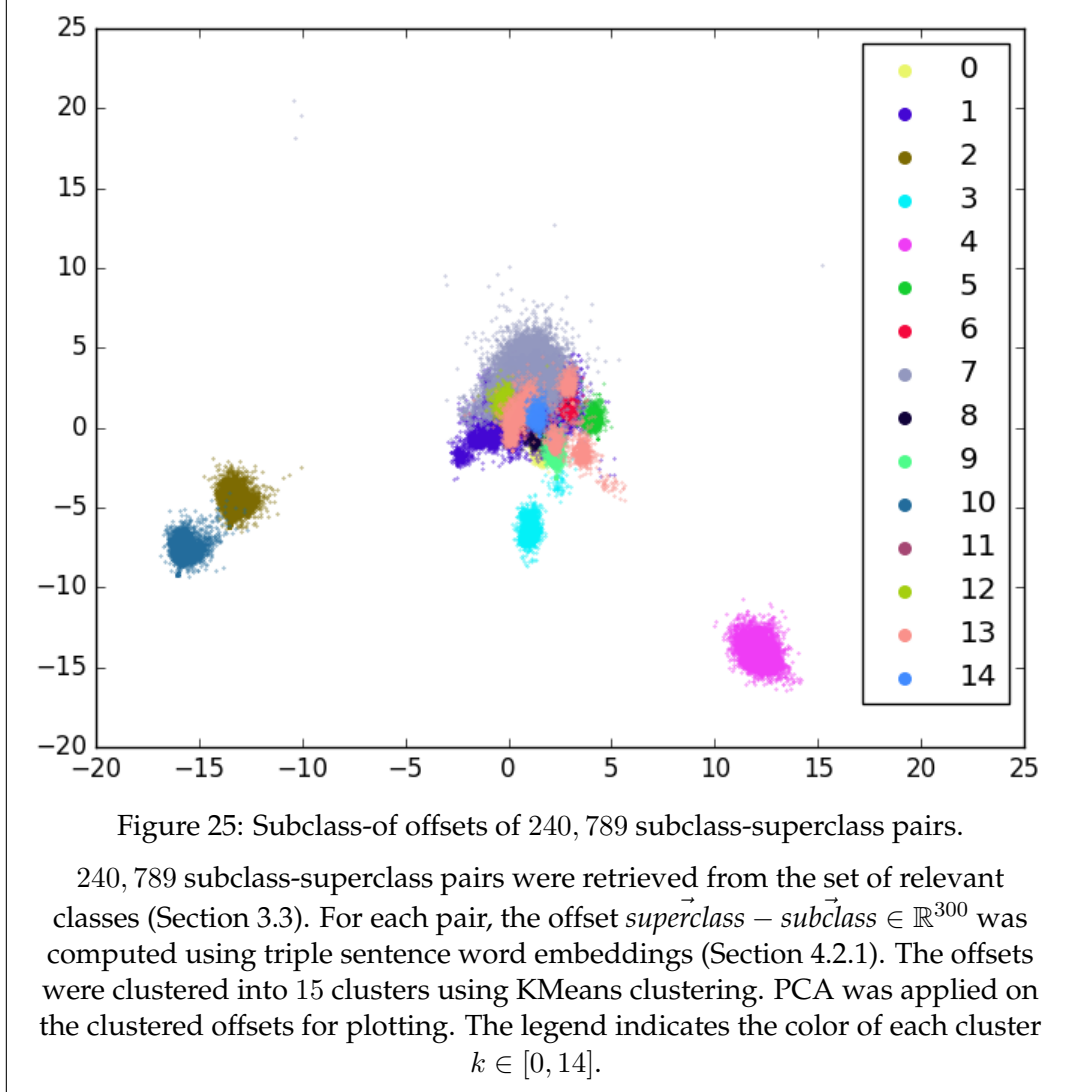
where  $d \in \mathbb{N}$  is the embedding size. The subclass-of relationship is represented by the offset  $\vec{y} - \vec{x}$ . Using the subclass offsets would simplify the difficult task of classifying an unknown class in a big taxonomy to a vector addition, which represents a translation. This is however only possible, if the subclass-of offset is similar for all existing subclass-superclass pairs [16].

For the use case of Wikidata, it can be shown that the offsets do not fulfill this criteria. It can rather be seen that different clusters of similar subclass-of offsets exist. Using the word embeddings created by SGNS trained on triple sentences, the subclass-of offsets for 240789 subclass-superclass pairs were computed. The subclass-superclass pairs were extracted from the set of relevant classes (Section 3.3). Using Principal Component Analysis (PCA), the 300-dimensional vectors were reduced to 2-dimensional vectors, which are shown in Figure 25.

The subclass offsets with dimension  $d = 300$  were clustered into  $K = 15$  clusters using KMeans. The clusters 1, 7 and 13, which attribute to 71.5% of all offsets contain a variation subclass-of relation types, which is most likely the case because 15 clusters are not sufficient to separate all different types of subclass-of relations. Therefore algorithms like piecewise linear projection [16], which exploit the similarity of specific types of subclass-of relations, need to use more clusters  $K$ . The other clusters show very specific types of offsets. Table 1 shows the list of 15 clusters with their percentage of all offsets, their topic and a subclass-superclass pair in the cluster.

Clusters, which describe similar topics are close together, e.g. clusters 2 and 10 are related to proteins, clusters 6, 9 and 11 are related to vehicles. An error in filtering for relevant classes occurred (Section 3.3), since two clusters in the set are related to proteins, which is 11.9% of all relevant subclass-superclass relations. Because the percentage of protein-related classes is relatively low, further filtering of the relevant dataset is not pursued. It can be seen, that the clusters 2, 4 and 10 have very different orientations in respect to the origin  $(0, 0)$  than the other clusters. Therefore directly using vector offsets would not be applicable, as offsets can not represent the complex





$k$	perc.	topic	example
0	1%	wine (mostly Italian wines)	<i>Greek wine</i> $\triangleleft$ <i>wine</i> , <i>Alghero frizzante rosato</i> $\triangleleft$ <i>wine</i>
2	8.3%	protein families	<i>G kinase-anchoring protein 1</i> $\triangleleft$ <i>protein family</i>
3	3.3%	Spanish mayors (Alcalde)	<i>mayor of Granollers</i> $\triangleleft$ <i>Alcalde</i>
4	6.5%	badminton championships	<i>Jamaican International Badminton Championships</i> $\triangleleft$ <i>badminton championship</i>
5	0.6%	food	<i>Sour cream doughnut</i> $\triangleleft$ <i>food</i>
6	1.6%	locomotives	<i>GWR 103 President</i> $\triangleleft$ <i>tender locomotive</i> , <i>GE C31-8</i> $\triangleleft$ <i>diesel-electric locomotive</i>
8	0.8%	tennis tournaments	<i>ECC Antwerp</i> $\triangleleft$ <i>tennis tournament</i>
9	1.5%	aircrafts	<i>Martin KF-1</i> $\triangleleft$ <i>aircraft</i>
10	3.6%	protein domains	<i>ISX02-like transposase domain</i> $\triangleleft$ <i>protein domain</i>
11	0.4%	military aircrafts	<i>Heinkel He 50</i> $\triangleleft$ <i>military aircraft</i>
12	1%	diseases	<i>cervical squamotransitional carcinoma</i> $\triangleleft$ <i>disease</i>
14	0.5%	ambassadors	<i>Ambassador of the Republic of China to the United States</i> $\triangleleft$ <i>ambassador</i>

Table 1: Analysis of 15 subclass-of clusters, which are plotted in Figure 25

subclass-of relation. Fu et al. [16] reached a similar conclusion and proposes the use of linear projections as means to represent the subclass-of relation. Additionally, it can be concluded that 15 clusters is too low for usage in piecewise linear projection, since 3 clusters, representing 71.5% of all offsets, exist, which do not have definite topics. Therefore cluster sizes  $k$  of at least 30 should be considered as parameter for piecewise linear projection. In the following section, the approach by Fu et al., which is described in Section 2.7.3, is adjusted for usage in the hybrid algorithm.

#### 4.3.3 Linear projection

Following the idea of using vector offsets to find the correct superclass  $\vec{p} \in \mathbb{R}^d$  for a given unknown  $\vec{u} \in \mathbb{R}^d$ , the translation represented by the subclass offset can be extended by adding a linear projection matrix  $\Phi \in \mathbb{R}^{d \times d}$ . This follows the idea of Fu et al. [16], who uses linear projection without translation to find the superclasses for a given unknown. The superclass  $\vec{p}$  for an unknown  $\vec{u}$  is accordingly computed:

$$\vec{p} = \Phi \vec{u} + \vec{v} = \begin{bmatrix} \Phi & \vec{v} \end{bmatrix} \begin{bmatrix} \vec{u} \\ 1 \end{bmatrix} \quad (24)$$

where  $\vec{v} \in \mathbb{R}^d$  is a translation vector. Combining the translation and projection into a single matrix  $T = \begin{bmatrix} \Phi & \vec{v} \end{bmatrix}$  enables the use of Fu et al. [16]’s approach for finding

$T^* \in \mathbb{R}^{d \times d}$ , which is the best fit for the given subclass-superclass pairs.  $T^*$  can be computed by minimizing the following mean squared error [16]:

$$T^* = \arg \min_T \frac{1}{N} \sum_{(\vec{x}, \vec{y})} \|T\vec{x} - \vec{y}\|^2 \quad (25)$$

where  $N$  is the number of  $(\vec{x}, \vec{y})$  subclass-superclass pairs in the training data [16].

Fu et al. [16] proposes an improvement to the linear projection. As shown in Figure 25, the subclass offsets appear in clusters, therefore it is intuitive to assume that training a matrix  $T_k \in \mathbb{R}^{d \times d}$  for different clusters  $k$  increases the accuracy of the method [16]. However, the approach by Fu et al. [16] is not applicable for the given problem. In Fu et al.'s stated problem, a an unknown class can have zero, one, or multiple superclasses, but in the thesis' problem an unknown class is always assigned one superclass. Additionally, the input for the thesis' classification is a single unknown class, therefore it cannot be decided in which subclass-of offset cluster the unknown class belongs. Instead of using subclass-of clusters to train the piecewise projections, the subclass-superclass pairs  $(\vec{x}, \vec{y})$  are clustered by their input vector  $\vec{x}$ . This leads to the following training objective for piecewise linear projections [16]:

$$T_k^* = \arg \min_{T_k} \frac{1}{N_k} \sum_{(\vec{x}, \vec{y}) \in C_k} \|T_k \vec{x} - \vec{y}\| \quad (26)$$

where  $N_k \in \mathbb{N}$  is the amount of word pairs in the  $k^{th}$  cluster  $C_k = \{(\vec{x}_i, \vec{y}_i) | \vec{x}_i \text{ in cluster } k, i \in [1, N]\}$  [16]. Both training objectives describe multivariate linear regression tasks. Fu et al. [16] uses stochastic gradient descent (SGD) to solve the objective. SciPy's SGD regressor [21] is used to train the linear projection matrices.

The projection for a given unknown  $\vec{u}$  using piecewise linear projection can be computed as follows:

$$\vec{p} = T_k^* \begin{bmatrix} \vec{u} \\ 1 \end{bmatrix} \quad (27)$$

where  $\vec{u}$  in cluster  $k$ . The appropriate superclass is subsequently the closest class embedding to the computed projection.

The thesis' evaluation will show whether linear projection is superior to the distance-based kNN in the given task, and to what degree it benefits from word embeddings using graph walk sentences instead of triple sentences.

## 5 Evaluation

### 5.1 Method

The hybrid algorithms are evaluated using a gold standard. The gold standard is fetched from the set of relevant classes (Section 3.3) in the 2016/11/07 Wikidata dump [3]. 208,502 classes and their corresponding superclasses were retrieved.

The gold standard set  $G$  consists of sample tuples  $(c_i, S_i)$ , where  $c_i \in R$  is a subclass and  $S_i = succ(c_i)$  is the set of direct superclasses of  $c_i$ , where  $R$  is the set of relevant classes and  $i \in [1, |G|]$ . Because it is assumed that every class, except the root class, should have a superclass,  $|S_i| > 0$  applies for all gold samples  $(c_i, S_i)$  with  $i \in [1, |G|]$ . Therefore the computation of the  $F_1$ -score as an evaluation measure is applicable, because only correct and false classification can occur. Instead the first applied evaluation measure is the accuracy  $acc_{alg}$ , which is calculated as follows for an algorithm  $alg$ :

$$acc_{alg} = \frac{1}{|G|} \sum_{i=1}^{|G|} \begin{cases} 1, & \text{for } p_{alg}(c_i) \in S_i \\ 0, & \text{for } p_{alg}(c_i) \notin S_i \end{cases} \quad (28)$$

where  $p_{alg} : R \mapsto R$  is the prediction function of  $alg$ , which returns the predicted superclass of  $alg$  given an unknown class.

However, accuracy alone is not sufficient in evaluating a classification in a hierarchical context like taxonomies [23]. Accuracy ignores the taxonomy and other possible dimensions, as it only measures how often the algorithm is able to correctly guess the superclass. Dellschaft and Staab states that a good measure should "allow[s] to evaluate an ontology along multiple dimensions" [12]. Another dimension, next to the accuracy of the algorithm, which can be observed is the taxonomic relation between the predictions and their corresponding gold standards. Given two algorithms  $algA$  and  $algB$  with equal accuracies  $acc_{algA} = acc_{algB}$ , assume  $algB$  predicts either correctly or predicts the root class, while  $algA$  predicts either correctly or overspecializes by predicting the subclass of the gold standard. Which one of the two algorithms performs better? If only the accuracy is considered, both algorithms are equally good and it would not matter, which algorithm would be chosen for practical application. But in practice  $algA$  would be preferred, since even false predictions are useful to some degree, while  $algB$ 's false predictions are not usable. Therefore the taxonomy dimension should be included in the evaluation. For this the **taxonomic overlap** measure, defined by Dellschaft and Staab [12], is also used in the evaluation. The taxonomic overlap counts how many superclasses are shared by the prediction and the gold standard for each sample. This is called the **semantic upwards cotopy**  $sc$  defined as follows [12]:

$$sc(c, T) = \{c_i | c_i, c \in C \wedge c \triangleleft_T c_i\} \quad (29)$$

where  $T = (C, \_)$  is a taxonomy and  $c \in C$  is a class in the taxonomy  $T$ . Based on the semantic upwards cotopy, the taxonomic overlap  $to_{sc}$  between classes  $c_1$  and  $c_2$

in  $C$  is defined as follows [12]:

$$to_{sc}(c_1, c_2, T) = \frac{|sc(c_1, T) \cap sc(c_2, T)|}{|sc(c_1, T) \cup sc(c_2, T)|} \quad (30)$$

The taxonomic overlap between two classes ranges between 0, if the classes have no common superclasses, and 1 if the classes are identical or all direct superclasses are identical. To apply the taxonomic overlap in the evaluation the overlaps for all prediction-gold pairs have to be summed up, which leads to the **average taxonomic overlap**  $ato_{sc,alg}$  for an algorithm  $alg$ :

$$ato_{sc,alg} = \frac{1}{|G|} \sum_{i=1}^{|G|} \max_{s \in S_i} (to_{sc}(p_{alg}(c_i), s, T)) \quad (31)$$

where  $G$  is gold standard, which samples  $(c_i, S_i)$  with  $i \in [1, |G|]$  and  $p_{alg}$  the prediction function of an algorithm  $alg$ . The maximum overlap between prediction and gold is chosen, if there are multiple possible superclass for a sample, since the taxonomic overlap is a function, which should be maximized by the algorithm [23].

Each possible combination of components is evaluated as a separate hybrid algorithm. For each classification component different hyperparameters are tested to find a good configuration. A naming scheme is used to uniquely identify each hybrid algorithm with their corresponding hyperparameters. Because the SGNS configuration is constant over all hybrid algorithms, an algorithm combination can be identified by their SequenceGen and Classification component, as well as their classification parameters. Consequently, hybrid algorithms have the following name format:

$$\langle \text{SequenceGen} \rangle + \langle \text{Classification} \rangle (\langle \text{hyperparameters} \rangle)$$

To shorten the identifiers, the following abbreviations for components are used:

component	abbreviation
triple sentences	ts
graph walk sentences	gw
distance-based kNN	distknn
piecewise linear projection	linproj

The following hybrid algorithms are evaluated:

- ts+distknn( $k=?$ ) with  $k = 5, 10, 15, 20$
- ts+linproj( $c=?$ ) with  $c = 1, 25, 50$
- gw+distknn( $k=?$ ) with  $k = 5, 10, 15, 20$
- gw+linproj( $c=?$ ) with  $c = 1, 25, 50$

where  $k$  is the number of neighbors in kNN and  $c$  is the number of clusters in piecewise linear projection.

## **5.2 About the dataset**

## **5.3 Results**

## **6 Conclusion and future work**

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## Appendices

### A Appendix: Graphs

**Definition 11** (Directed graph). A **directed graph**  $G$  is an ordered pair  $G = (V, E)$ , where  $V$  is a set of vertices, and  $E = \{(v_1, v_2) \mid v_1, v_2 \in V\}$  is a set of ordered pairs called directed edges, connecting the the vertices.

**Definition 12** (Subgraph). Let  $G = (V, E)$  and  $H = (W, F)$  be directed graphs.  $H$  is called **subgraph** of  $G$ , if  $W \subseteq V$  and  $F \subseteq E$ .

**Definition 13** (Predecessor). Let  $G = (V, E)$  be a directed graph.  $v_1 \in V$  is a **predecessor** of  $v_2 \in V$ , if there exists an edge so that  $(v_1, v_2) \in E$ . Let  $v \in V$  be a vertice of  $G$ , then  $pred_G(v) = \{w \mid (w, v) \in E\}$  is the set of predecessors of  $v$ .

**Definition 14** (Successor).  $v_1 \in V$  is a **successor** of  $v_2 \in V$ , if there exists an edge so that  $(v_2, v_1) \in E$ . Let  $v \in V$  be a vertice of  $G$ , then  $succ_G(v) = \{w \mid (v, w) \in E\}$  is the set of successors of  $v$ .

**Definition 15** (Walk). Let  $G = (V, E)$  be a directed graph. A **walk**  $W$  of length  $n \in \mathbb{N}$  is a sequence of vertices  $W = (v_1, \dots, v_n)$  with  $v_1, \dots, v_n \in V$ , so that  $(v_i, v_{i+1}) \in E \forall i = 1, \dots, n - 1$ .

**Definition 16** (Connected (Vertice)). Let  $G = (V, E)$  be a directed graph. Vertices  $u, v \in V$  are **connected**, if  $u = v$ , or  $u \neq v$  and there is walk between  $u$  and  $v$  or  $v$  and  $u$ .

**Definition 17** (Connected (Directed graph)). Let  $G$  be a directed graph.  $G$  is **connected**, if every pair of vertices in  $G$  are connected.

**Definition 18** (Cycle). A walk  $W = (v_1, \dots, v_n)$  of length  $n$  is called a **cycle**, if  $v_1 = v_n$ .

**Definition 19** (Directed acyclic graph). A directed graph  $G$  is called **directed acyclic graph**, if there are no cycles in  $G$ .

**Definition 20** (Directed weighted graph). A **directed weighted graph**  $G$  is an ordered pair  $G = (V, E)$ , where  $V$  is a set of vertices, and  $E = \{(v_1, v_2, e_{v_1 v_2}) \mid v_1, v_2 \in V\}$ , where  $e_{v_1 v_2}$  is the edge weight between the vertices  $v_1$  and  $v_2$ .