

Enrichment of ontological taxonomies using a neural network approach

Bachelorarbeit

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1 Introduction

TODO: Motivation. Related work. Justify novelty. Solution. Evaluation.

The thesis is structured as follows. Section 2 defines the concepts related to Wikidata, taxonomies, similarity measures and k-nearest-neighbors classification. Subsequently, the problem statement is formalized and its corresponding challenges listed. Section 3 explains the notion of neural networks at the example of a simple feedforward network with backpropagation. Subsequently, neural networks for graph and word representation are compared in regards to suitability in the problem's context. Section 4 compares the problem, solved by the thesis, to related work in the field of ontology learning. The problem is classified and solutions to similar problems analyzed. The novelty of the work is justified. Section 5 presents the results of the analysis of Wikidata's taxonomy. Characteristics of unlinked classes are identified, and a specific subset of unlinked classes chosen, which fulfills necessary requirements as input for an algorithm. Section 6 describes the developed baseline algorithm, which combines Word2Vec by Mikolov et al. [19] and k-nearest-neighbors. Possible variations of the algorithm are presented and compared in regards to possible gains and problems. One of these variations is also implemented for evaluation purposes. Section 7 describes the evaluation methodology and presents the results. The baseline algorithm, using multiple different sets of different hyperparameters, and a variation of it will be compared. Section 8 summarizes the results of the thesis and lists possible future work, which could extend upon it.

2 Foundations

2.1 Wikidata

Wikidata is an open, free, multilingual and collaborative knowledge base. It is a structured knowledge source for other Wikimedia projects. It tries to model the real world, meaning every concept, object, animal, person, etc. Wikidata is mostly edited and extended by humans, which in general improves the quality of entries compared to fully-automated systems, because different editors can validate and correct occurring errors. However, Wikidata, like most knowledge bases, is incomplete and therefore has to be operated under the *Open World Assumption* (OWA). OWA states that if a statement is not contained in a knowledge base, it is not necessarily false but rather unknown [13].

In Wikidata items and properties exist. Items are the aforementioned concepts, objects, etc. While properties are used to make claims about items, e.g. *photographic film* (Q6293) is a *subclass of* (P279) *data storage device* (Q193395) (see Figure 1). Each item and property has a unique identifier, which starts with the letter Q for items and the letter P for properties and is followed by a numeric code. The identifiers in Wikidata are essential to avoid ambiguity and to make items and properties multi-

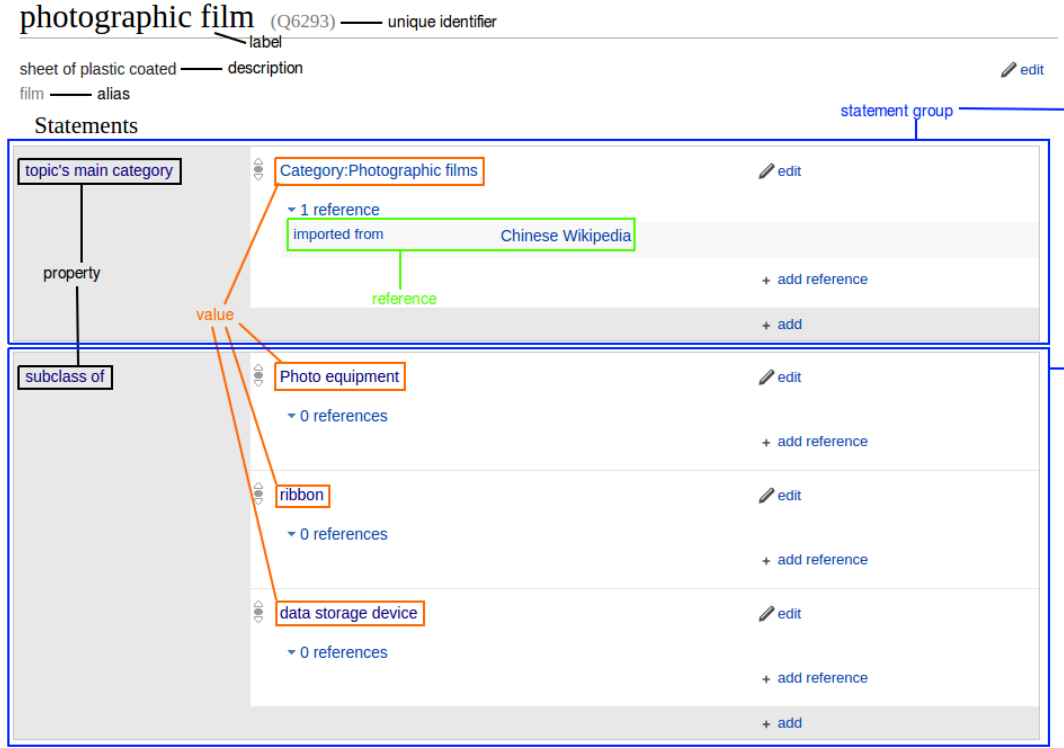


Figure 1: Example of Wikidata class: photographic film (Q6239)

lingual.

Items consist of labels, aliases and descriptions in different languages. Sitelinks connect items to their corresponding pages of Wikimedia projects like Wikipedia articles. Most importantly item are described by statements. Statements are in their simplest form a pair of property and value, assigned to a specific item. A value is either a literal value or another item. It should be noted that an item can have multiple statements with the same property. The set of statements with the same property is called statement group. Statements can be annotated with qualifiers, which specify the context of the statement, e.g. population at a certain point of time. Additionally, references can be used for statements to include its source. See Figure 1 for an example of a Wikidata item.

Following, the terms of item and statement are defined in the context of Wikidata.

Definition 1 (Item). An item is a tuple $(id, label, aliases, description, sitelinks)$:

- $id \in \mathbb{N}$ is the numerical item ID;
- $label \in String$ is the English label of the item;
- $aliases \in \mathcal{P}(String)$ is the set of English synonyms for the label;

- $description \in String$ is a short sentence describing the item;
- $sitelinks \in String \times String$ is a set of tuples $(site, title)$, where $site$ refers to a specific site of the Wikimedia projects, e.g. enwiki, and $title$ is the corresponding article title of the item on this site.

Definition 2 (Statement). A statement is a tuple $(itemid, pid, value, refs, qualifiers)$:

- $itemid \in \mathbb{N}$ is a numerical item ID, to which the statement belongs;
- $pid \in \mathbb{N}$ is a numerical property ID;
- $value$ is either a constant value like string, int, etc., or an item ID;
- $refs$ is a set of references, containing the source of information for the statement;
- $qualifiers$ is a set of qualifiers, which further specifies the statement.

In Wikidata, there is no strict distinction between classes and instances. Both groups are represented as items. This leads to the issue, that recognizing, whether an item is a class or instance is not trivial. Based on which statements connect two items, a distinction can be made. A class is any item, which has instances, subclasses or is the subclass of another class. In Wikidata, the properties *instance of* (P31) and *subclass of* (P279) exist, which describe this relation between items. Therefore to identify whether an item is a class, it needs to be checked, whether the items fulfills any of the three above criteria.

Definition 3 (Class). Given a set of items I and a set of statements R . $c = (classid, _, _, _, _) \in I$ is a class, if at least one of the following assertions are true:

$$\begin{aligned}
&\exists i = (instanceid, _, _, _, _) \in I \exists s = (itemid, pid, value, _, _) \in R : \\
&\quad instanceid = itemid \wedge pid = 31 \wedge value = classid \text{ (has instance)} \\
&\exists s = (itemid, pid, _, _, _) \in I : itemid = classid \wedge pid = 279 \text{ (is subclass)} \\
&\exists i = (subclassid, _, _, _, _) \in I \exists s = (itemid, pid, value, _, _) \in R : \\
&\quad itemid = subclassid \wedge pid = 279 \wedge value = classid \text{ (has subclass)}
\end{aligned}$$

$_$ is used as an anonymous placeholder, for the purpose of not naming unused elements in tuples. For example, *photographic film* (Q6293) (Figure 1) is a class, because it is the subclass of three other classes.

2.2 Taxonomy

“Ontologies are (meta)data schemas, providing a controlled vocabulary of concepts, each with an explicitly defined and machine processable semantics” [18]. Additionally it is possible for ontologies to contain axioms used for validation and constraint

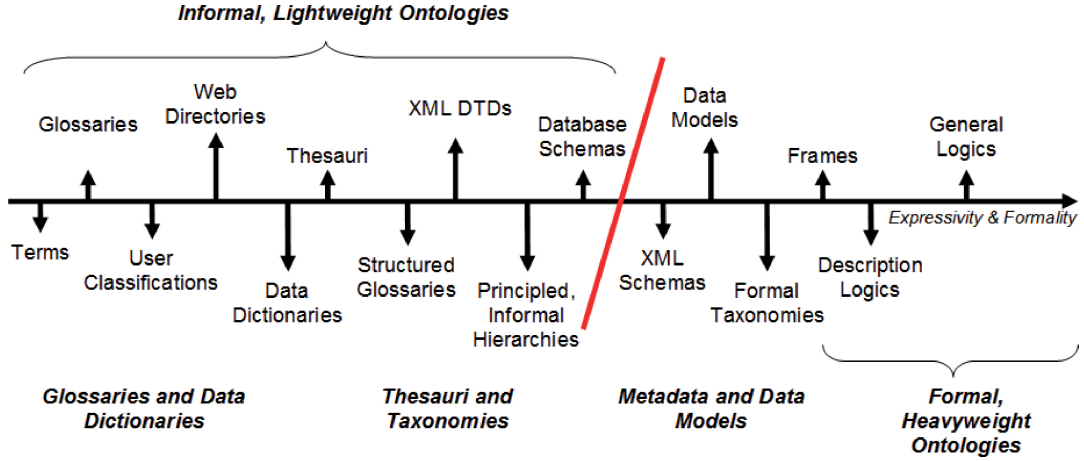


Figure 2: The spectrum of ontology kinds. [27]

enforcement. Ontologies enable the modeling and sharing of knowledge in a specific domain and support the knowledge exchange via web by extending syntactic to semantic interoperability [15]. In comparison, a knowledge base like Wikidata can be seen as an instantiation of such an ontology, since every knowledge base has to be conceptualized by an ontology [27]. Different types of ontologies can be grouped by their level of formality and expressiveness. Wong et al. [27] differentiates ontologies as lightweight and heavyweight ontologies (see Figure 2). *Taxonomies* are concept or class hierarchies. They typically represent a parent-child structure, which can be formalized with a single relationship called for example *subclass-of* in the case of Wikidata. The observed taxonomy in Wikidata belongs to the category of lightweight ontologies, specifically *principled, informal hierarchies*, as the only enforced rule for the subclass-of relation is that it should connect two entities [2].

For the purpose of developing a formal definition of the thesis' problem statement the notion of taxonomy needs to be formalized. Cimiano [7] defines a heavyweight ontology, which includes a taxonomy, as follows:

Definition (Ontology). An ontology is a structure

$$\mathcal{O} := (C, \leq_C, R, \sigma_R, \leq_R, \mathcal{A}, \sigma_A, \mathcal{T})$$

consisting of

- four disjoint sets C , R , \mathcal{A} , and \mathcal{T} whose elements are called concept identifiers, relation identifiers, attribute identifiers and data types, respectively,
- a semi-upper lattice \leq_C on C with top element $root_C$, called concept hierarchy or taxonomy,
- a function $\sigma_R : R \rightarrow C^+$ called relation signature,

- a partial order \leq_R on R , called relation hierarchy, where $r_1 \leq_R r_2$ implies $|\sigma_R(r_1)| = |\sigma_R(r_2)|$ and $\pi_i(\sigma_R(r_1)) \leq_C \pi_i(\sigma_R(r_2))$, for each $1 \leq i \leq |\sigma_R(r_1)|$, and
- a function $\sigma_A : \mathcal{A} \rightarrow C \times \mathcal{T}$, called attribute signature,
- a set \mathcal{T} of datatypes such as strings, integers, etc.

Hereby, $\pi_i(t)$ is the i -th component of tuple t . [...] Further, a semi-upper lattice \leq fulfills the following conditions:

- $\forall x(x \leq x)$ (reflexive)
- $\forall x \forall y(x \leq y \wedge y \leq x \implies x = y)$ (anti-symmetric)
- $\forall x \forall y \forall z(x \leq y \wedge y \leq z \implies x \leq z)$ (transitive)
- $\forall x x \leq \text{top}$ (top element)
- $\forall x \forall y \exists z(z \geq x \wedge z \geq y \wedge \forall w(w \geq x \wedge w \geq y \implies w \geq z))$ (supremum)

So every two elements have a unique most specific supremum. "

A taxonomy can be modeled as a semi-upper lattice. This induces two important assumptions about the structure and to some degree completeness of the observed taxonomies. First, there is only one *root class*, top element of the lattice, of which every other class is (transitively) a subclass. Second, because of the supremum property, the taxonomy is fully connected, which means each class, but the root class, has a superclass. Wikidata's taxonomy does therefore not fulfill the definition by Cimiano [7], as it is not fully connected.

In the following, definitions will be presented, which attempt to model an incomplete taxonomy based on the already presented data model and structure of Wikidata. Refer to Appendix A for the necessary definitions on graphs.

In Wikidata, a class can have multiple superclasses, therefore a tree structure is not sufficient to model the taxonomy. However a directed acyclic graph, can model the taxonomy. The acyclic constraint is necessary to ensure that no class is transitively a subclass of itself.

Definition 4 (Taxonomy). A taxonomy $T = (C, S)$ is a directed acyclic graph, where C is a set of class identifiers, and S is the set of edges, which describe the subclass-of relation between two classes. such that c_1 is the subclass of c_2 , if $(c_1, c_2) \in S$.

Definition 5 (Subclass-of relation). The transitive binary relation \triangleleft_T on the taxonomy $T = (C, S)$ represents the subclass relationship of two classes in T . Given $c_1, c_2 \in C$, $c_1 \triangleleft_T c_2$, if there is a walk $W = (c_1, \dots, c_2)$ with length $n \geq 1$, which connects c_1 and c_2 . \triangleleft_T is transitive, $\forall c_1, c_2, c_3 \in C : c_1 \triangleleft_T c_2 \wedge c_2 \triangleleft_T c_3 \implies c_1 \triangleleft_T c_3$.

If the taxonomy defined by Cimiano [7] is mapped on this graph-based taxonomy model, the following assumption is true, for $T = (C, S)$:

$$|\{c \in C \mid \neg \exists s \in C : c \triangleleft_T s\}| = 1 \quad (1)$$

Only one class in this taxonomy has no superclasses. This class is called *root class*. However in the case of Wikidata, this assumption does not hold true. The following state is the case:

$$|\{c \in C \mid \neg \exists s \in C : c \triangleleft_T s\}| > \quad (2)$$

There are classes other than the root class, which also have no superclasses. These classes will be called *unlinked classes*.

Definition 6 (Root class). Given a taxonomy $T = (C, S)$, the root class $root_T$ is a specific, predefined class with no superclasses in T . For $root_T$, $|succ_T(root_T)| = 0$ applies.

Definition 7 (Unlinked class). Given a taxonomy $T = (C, S)$ with a root class $root_T$, a class $u \in C$ is called unlinked class, if $u \neq root_T \wedge |succ_T(u)| = 0$.

In Wikidata, the root class is *entity* (Q35120) [3]. All other classes, which are not subclasses of *entity* (Q35120), are therefore either unlinked classes, or subclasses of unlinked classes. In Section 5, it will be shown that the Wikidata taxonomy graph is not fully connected. But the component, which contains the root class *entity* (Q35120), contains 97% of all classes. This component will be referred to as *root taxonomy* in later sections.

2.3 Similarity

- semantic similarity e.g. distributional similarity
Lin [17]
Rodríguez and Egenhofer [24]
- geometrical similarity e.g. distance based-similarity, cosine similarity

For the task of ontology learning [15] as well as classification, e.g. k-nearest-neighbors, the concept of similarity is of importance. A basic intuition of similarity is for example given by Lin [17]. Similarity is related to the commonalities and differences between two objects. More commonalities implies higher similarity. Vice versa, more differences implies lower similarity. Two identical objects should have the maximum similarity. In addition, only identical objects should be able to achieve maximum similarity. A similarity measure computes the similarity between two objects [26]:

Definition 8 (Similarity measure). $sim : O \times O \mapsto [0, 1]$ is called similarity measure for a set of objects O .

A similarity of 1 implies identical objects and a similarity of 0 implies that there are no commonalities between the input objects.

TODO: Whole paragraph needs citations In ontology learning, semantic similarity is used to great effect . Semantic similarity compares the semantic content of

objects or documents . This can be achieved by comparing which features can be found in both objects (commonalities) and which features are unique to the compared objects (differences). Rodríguez and Egenhofer [24] develops a semantic similarity measure for comparing entity classes in ontologies. **Write**

Similarity between vectors can be computed by calculating their distance [26]. **Write**

2.4 Problem statement

The task of this thesis is the classification of unlinked classes in Wikidata. In other words a function is needed, which given an unlinked class u of a taxonomy $T = (C, S)$ with a root class $root_T$, find an appropriate superclass for T . Doan et al. [10] suggests that for the task of placing a class into an appropriate position in T , either finding the most similar class, most specific superclass, or most general subclasses of u , are sensible approaches. This induces that the appropriate superclass for an unlinked class u is the superclass of the most similar class to u . Therefore we can define the problem, as follows:

Definition 9 (Problem definition). Given a taxonomy $T = (C, S)$ with root class $root_T$ and a similarity function sim over T , find a function $f : \mathbb{N} \mapsto \mathcal{P}(\mathbb{N})$, which, given an unlinked class $u \in C$, returns a set of classes $P = f(u)$, fulfilling the following criteria:

$$\neg(\forall p \in S : p \triangleleft_T u \text{ no children}) \quad (3)$$

$$= succ(\max_{c \in C}(sim(u, s))) \text{ superclasses of most similar classes} \quad (4)$$

Because it is possible for a class to have multiple superclasses, it is necessary to define f in such a way, that it returns a set of classes. Therefore the described problem is a multi-label classification problem.

The stated problem induces several challenges, which will be listed here, but addressed in later sections.

1. **Multi-label classification.** Algorithms for classification typically map entered objects to one label **TODO:Reference**. It has to be decided whether the solution will be simplified to assign one superclass per input, or attempt multi-label classification.
2. **High number of labels.** An entered, unlinked class has to be assigned to a class in the root taxonomy. The only restricting condition is that the chosen class cannot be a subclass of the unlinked class. As shown in Section 5 97% of 12999501 classes are part of the root taxonomy. Classification like SVM or neural networks are usually used for classification with a small number of labels. SVM is for example a binary classification method. A classification method, which is able to handle over a million of labels, has to be found.

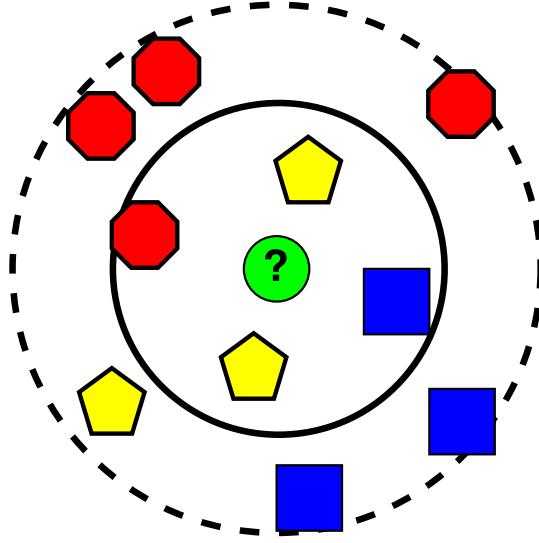


Figure 3: Example for k-nearest neighbors for 3 classes with $k=4$ and $k=10$.

3. **Representation of items.** Items in Wikidata are structured information, similar to nodes in RDF graphs. Items are nodes and statements can be simplified to triples of $(itemid, pid, value)$. As motivated in Section 1, the solution should exploit the apparent power of neural networks. Neural networks, which are introduced in Section 3, require input to be represented as vectors. Therefore it will be necessary to map items to vectors.

2.5 k-nearest-neighbors classification

Chen et al. [5]

Zhang and Zhou [28]

Based on the characteristics of the classification problem, described by the problem statement, and the challenges attached to it, the k-nearest-neighbor algorithm (kNN) seems like an appropriate tool for solving the task. Nearest-neighbors classification is a lazy method, as it does not require training before testing. This is useful for applications with high amounts of data, large numbers of classes, and changing data [28]. For the considered use case of classification in Wikidata, these are very important strengths, as the number of classes in the taxonomy is very high and Wikidata is being constantly edited.

k-nearest-neighbors is a distance- or similarity-based classification method, which maps its input to a single label. This means, that kNN, given an input u only requires the distance or similarity between u and all training samples. Additional knowledge about the structure or content of the data is not required.

Algorithm 1 (weighted kNN). Let O be a specific set of objects, e.g. \mathbb{R}^n , with a similarity measure sim . Let $n \in \mathbb{N}$ be the number of labels. Let $T = \{(o_i, l_i)\}$ with

$i = 0, \dots, m - 1$ and be the set of training samples with m samples, where $o_i \in O$ is an object and $l_i \in [0, n - 1]$ is its corresponding class. Let $u \in O$ be an unclassified object. **TODO: insert missing definitions**

1. Find the set K of k nearest neighbors to u in T . $K = \{\}$
2. Each item $k \in K$ a weight is assigned based on its similarity to u . $w_k = \text{sim}(k, u) / \text{sum}(\text{sim}(o_j, u))$.
3. Each item votes for its label using its weight. The label which has the highest vote score is assigned as label to u . l

The task defined in this thesis can be considered a multi-label classification problem. Zhang and Zhou [28] developed a multi-label nearest neighbor algorithm, which is based on kNN. **TODO: explain it**

3 Neural networks

Word embeddings are vector representations of words.

3.1 Introduction to neural networks

Notion of neural networks will be introduced.

3.2 Word2Vec

Mikolov et al. [19]

Levy et al. [16]

Goldberg and Levy [14]

suitable for capturing linear structures [4]

Mikolov et al. [19] introduce two neural network language models, Continuous Bag-of-Words and Skip-gram, which have proven to be very effective at creating word embeddings. The generated word embeddings of both models encode the semantics and linguistic regularities of words. Words, which are semantically close, are also close in the word embedding vector space. Calculating the offset between words makes it possible to answer more complex semantic questions. It is possible to answer more complex questions than similarity about the relationship between words. For example, the question "What is the word that is similar to *small* in the same sense as *biggest* is similar to *big*?" can be answered by calculating the offset between *big* and *biggest* and adding the vector of *small* to it: $\text{vector}(\text{"biggest"}) - \text{vector}(\text{"big"}) + \text{vector}(\text{"small"})$ [19].

Continuous Bag-of-Words (CBOW) is a feedforward network with one hidden layer. The model is trained for the task of predicting a given word w based on its context c .

A k -skip- n -gram for a sentence $w_1 \dots w_n$ is defined as follows **cite Guthrie: TODO: add indices to sum**

$$\{w_{i_1}, w_{i_2}, \dots, w_{i_n} \mid \sum i_j - i_{j-1} < k\} \quad (5)$$

The skip-gram model solves the opposite task to CBOW. The target word is now the input and the model is trained to predict its context.

3.3 RDF2Vec

combination of Word2Vec and graph walking, which can be found in dnn for gr Ristoski and Paulheim [23] propose two approaches to create embeddings for entities in RDF graphs, graph walks and Weisfeiler-Lehman subtree RDF graph kernels. Both approaches create linear sequences from the RDF graph, which then are applied on the CBOW and Skip-gram models with negative sampling (see Section 3.2).

Given an undirected, weighted graph $G = (V, E)$, graph walk will generate all graph walks P_v of depth d starting in vertex v , for all vertices $v \in V$. Breadth-first search is used for generating the graph walks. This results in a set of sequences of the format $v_i \rightarrow e_{ij} \rightarrow v_j \rightarrow \dots$, where $v_i, v_j \in V$ and $e_{ij} \in E$. **TODO: finish this explanation**

it will be fun to explain the second approach

3.4 Deep neural networks for graph representation

Cao et al. [4] deep neural networks for graph representation

Raghu et al. [22] expressivity of deep neural networks

The idea of representing semantic objects like words or vertices, which was already explored in the previous section, can be applied similarly to graphs. Cao et al. [4] develop a deep neural network for graph representation. Based on the idea of generating linear sequences of vertices using a graph walk approach like DeepWalk **cite** or similar to the previously presented RDF2Vec **cite**. Cao et al. develop a random surfing model similar to Google's PageRank using a teleportation probability α , which indicates the chance to return to the starting vertex and restarting the process. The random surfing generates a probabilistic co-occurrence matrix, which contains the probabilities to reach a certain vertex after a determined amount of steps. The advantage of random surfing instead of random walking is the fact, that linear sequences have finite lengths and can therefore fail to capture relevant contextual information.

3.5 Comparison

4 Ontology learning

General concepts. Classification of considered problem in the task of ontology learning. Related work.

Petrucci et al. [21] neural network for ontology learning
Fu et al. [12] word embeddings for word hierarchies
Hazman et al. [15] Survey on ontology learning
Cimiano et al. [6] Ontology learning intro
Maedche and Staab [18] OntoEdit
Wong et al. [27] Ontology learning from text

Manually building ontologies is an expensive, tedious and error-prone process [15]. Maedche and Staab [18] recognize that the manual construction of ontologies results in a *knowledge acquisition bottleneck*, which motivates the research into the field of *ontology learning (OL)*. The field of ontology learning supports ontology engineers in the construction and maintenance of ontologies by providing OL techniques in the form of tools like *OntoEdit* [18]. The process of OL can be divided into different subtasks. OL tools consist of different components, which automatically or semi-automatically support this process. The field of ontology learning exploits different research fields like natural language processing, machine learning, ontology engineering, etc. [6].

The process of ontology learning and the components of a generic OL architecture are summarized and the task of the thesis is categorized. Following, basic algorithms for learning taxonomic relations are summarized. Both subsections are based on work by Cimiano et al. [6], Maedche and Staab [18], and Hazman et al. [15]. Finally, related work, which exploits neural networks, is analyzed and compared to the thesis' task. The novelty and additional benefits of this work are justified.

4.1 Process and architecture for ontology learning

The process of ontology learning can be divided into subtasks. Maedche and Staab [18] define a ontology learning cycle, consisting of the following steps:

1. *Import/Reuse* is the merging of existing structures and mapping between the structures and the target ontology.
2. *Extraction* defines the modeling of the target ontology by feeding from web documents.
3. *Prune* takes the generated model and adjusts the ontology to its purpose.
4. *Refine* completes the ontology at a fine granularity.
5. *Apply* applies the resulting ontology on its target application. This serves as a measure of validation.

These 5 steps can be repeated as often as necessary to include additional domains and updating it with new content. The thesis' task can be categorized to the *Refine* step, as its goal is to improve the completeness of an existing ontology.

Cimiano et al. [6] introduces a generic ontology learning architecture and its major components. The described tool is semi-automatic, meaning that it support an ontology engineer rather than fully automatizing the process of ontology construction and maintenance. The architecture consists of the following components:

1. *Ontology management component* provides an interface between the ontology and learning algorithms. Learned concepts, relations and axioms should be added to the ontology using this component. It is also used for manipulating the ontology. More specifically, for the importing, browsing, modification, versioning and evolution of ontologies.
2. *Coordination component* is the user interface, which should allow the ontology engineer to choose input data, learning and resource processing methods.
3. *Resource processing component* allows the discovery, import, analysis and transformation of unstructured, semi-structured and structured input. For this purpose the component needs different natural language processing components to parse and analyze input on different levels, word to sentence-level.
4. *Algorithm library component* contains the algorithms, which are applied for the purpose of ontology learning. These algorithms are generic standard machine learning methods, as well as specialized OL methods. Additionally, different similarity and collocation measures should be available.

In the context of Wikidata, there exists no single comprehensive OL tool. However, in the **Wikidata technology space**??? different tools exist, which mainly support the task of refining and maintaining the current ontology. For example, Stratan [25] develops a taxonomy browser for Wikidata, which is able to evaluate the quality of the taxonomy by detecting different types of cycles, redundancies, errors and unlinked classes. This tool is an ontology learning component, as it provides the ability to browse and evaluate the ontology of Wikidata.

4.2 Approaches for learning taxonomic relations

A subgroup of algorithms for ontology learning is concerned with learning taxonomic relations. The following approaches, categorized by Cimiano et al. [6], use text as input.

Lexico-syntactic patterns are word patterns in patterns, which are used to identify hypernym-hyponym pairs (superclass-subclass pairs) in natural text. For example, such a pattern is

$$NP_{hyper} \text{ such as } \{NP_{hypo},\}^* \{(and \mid or)\} NP_{hypo}$$

, where NP stands for noun phrase, and NP_{hyper} is a hypernym or superclass, while NP_{hypo} are hyponyms or subclasses. These patterns provide reasonable results, but

the manual creation of patterns is involve "high"**not really high, but it could be automatized** cost and time investments [27].

Clustering uses some measure of similarity to organize objects into groups. This can be achieved by representing the words or terms as vectors [8], on which different distance or similarity measures can be applied. Clustering methods can be categorized to three different types. Agglomerative clustering initializes each term as its own cluster and merges in each step the most similar terms into one cluster. Divisive clustering approaches the problem the opposite way by starting with a single cluster, containing all words, and then dividing them into smaller groups. Both of these approaches generate hierarchies, agglomerative bottom-up and divisive top-down **maybe not say that or formulate it another way**.

Phrase analysis analyses noun phrases directly. It is assumed that nouns, which have additional modifiers are subclasses of the noun without modifiers. For example, this could be applied on the labeled unlinked classes of Wikidata. For example the classes *Men's Junior European Volleyball Championship* (Q169359) and *Women's Junior European Volleyball Championship* (Q169956) could be subclasses of *European Volleyball Championship* (Q6834). In this case, phrase analysis interprets *Men's Junior* and *Women's Junior* as modifiers, which denote these classes as specialization to Q6834.

Classification-based approaches can be used, when a taxonomy is already present. In this case, classification can be used to add unclassified concepts to the existing hierarchy. Challenging with this task is that a taxonomy typically contains a large amount of classes and therefore classification methods like SVM are not suited for the task. Specific algorithms, which only consider, a subset of relevant classes are necessary to carry out an efficient classification. For example, Pekar and Staab [20] solves this problem by exploiting the taxonomy's tree structure using tree-ascending or tree-descending algorithms.

The algorithm developed by this thesis uses a classification-based approach, since a large taxonomy is already given. Instead of exploiting the hierarchic structure of the taxonomy, a variant of k-nearest-neighbors is used to address the "high number of labels" problem (mentioned in Section 2.4).

4.3 Related work

Fu et al. [12] develop a method to construct semantic hierarchies given a hyponym (subclass) and a list of corresponding superclasses. The solution uses neural word embeddings generated with the Skip-gram model [19], which was presented in Section 3.2. Using the linguistic regularities of these word embeddings, Fu et al. train a linear projection on existing sets of hypernym-hyponym pairs. A linear projection is used instead of an offset, because it was observed that the subclass-of relation is too complex to be represented as a simple offset. The embeddings are trained on a 30 million Chinese sentence corpus. And the projection is trained on 15247 hypernym-hyponym word pairs. The handpicked testing data consists of 418 entities and their hypernyms. The method achieves an F-score of 73.74% on a manually

labeled dataset, and outperforms existing state-of-the-art methods. **A bit skeptical about these really good evaluation results. The test set is not very big, maybe it was just chosen right?**

Petrucci et al. [21] develop a recurrent neural network for ontology learning. The purpose of the work is a first step to verify whether neural networks are able to support the ontology learning process. The task solved by the specific network is the translation of encyclopedic text to logical formulas. This task is divided into the task of sentence transduction, identify logical structure and generate corresponding formula template, and sentence tagging, identifying the roles of words in the input sentence. The tagged sentence can then be mapped **unto (is this a real word?)** the generated formula template. This is achieved by the use of a recurrent neural network for sentence tagging and a recurrent encoder-decoder for sentence transduction. Both are supported by gated recursive units, which provide a short-term-memory effect. The developed network is, at this time, under evaluation. Therefore, no statement about the effectiveness of recursive neural networks for the task of ontology learning can be made based on the paper.

TODO: Interpret related work in context of this thesis.

5 Analysis of the Wikidata taxonomy

For the task of developing an algorithm, which takes unlinked classes as input, it is necessary to know, what information the classes carry and if there are certain patterns among the classes. For this purpose an analysis of the taxonomy needs to be carried out, which may answer questions.

The taxonomy contained in the Wikidata dump of 2016/11/07 was analyzed. Classes were recognized as defined in Section 2.1. Unlinked classes were identified by checking whether a class does not have the *subclass of* (*P279*) property. The taxonomy contained a total of 1299501 classes at this time.

5.1 Root taxonomy

The state of the taxonomy was captured in regards to the root class *entity* (*Q35120*) (see Figure 4). 1260842 classes are currently subclasses of *entity* (*Q35120*). 97% of all classes are therefore nodes in the root taxonomy. This implies a high agreement in the Wikidata community on which class is considered root, and thereby also supports the modeling decision made in Section 2.2, which assumes that a taxonomy only has one root, and this root is *entity* (*Q35120*) in Wikidata. **Last sentences of this paragraph are mostly interesting trivia, but have currently no further use.** The longest shortest distance between the root and a leaf class is 20. The classes *Q639064*, *Q15978631* and *Q151055* fulfill this characteristics. Each of them is subclass of *Homo* (*Q171283*).

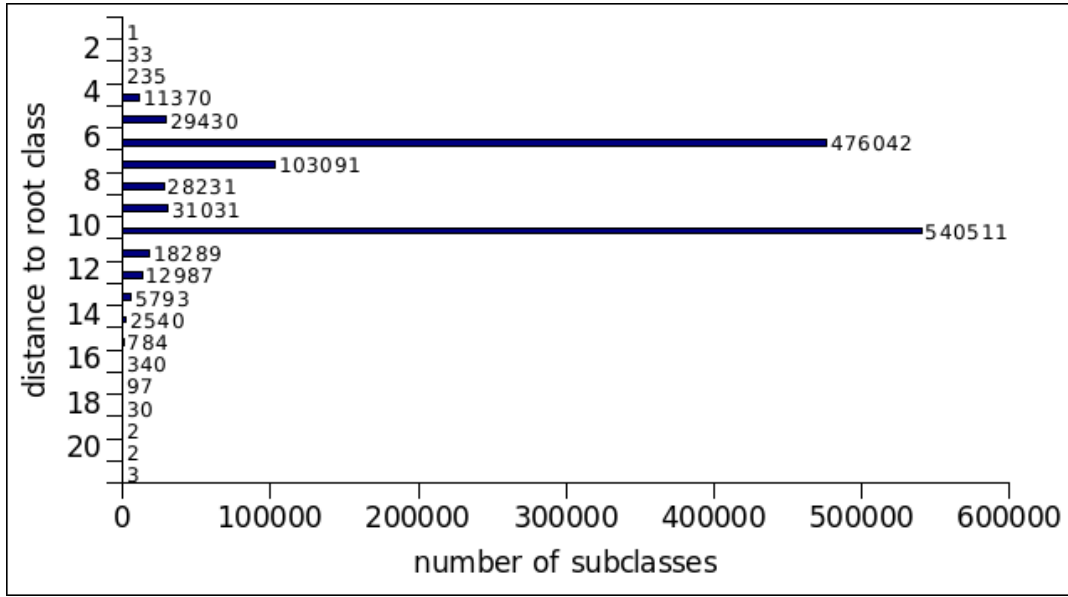


Figure 4: Distance of subclasses to root class *entity* (Q35120). Wikidata (2016/11/07)

5.2 Unlinked classes

The characteristics of all unlinked classes and the root class were analyzed. This set contains a total of 16373 classes. 13807 classes have an English label and 11534 classes have a corresponding English Wikipedia page.

Regarding the number of unique properties (or statement groups) per class (see Figure 5) the median is 3 and the average is ≈ 4.8 . Combined with the analyzed unique property frequency (see Figure 6), it can be seen that many unlinked classes are linked to other knowledge bases and taxonomies, e.g. Freebase or GND, using an identifier. Another observation is that 901 of the unlinked classes have properties related to taxons, which is a, to some degree, separate taxonomy in Wikidata, created with properties like *taxon name* (P225), *parent taxon* (P171), etc.

Most unlinked classes have only 0 to 1 instances, the median is 1 and the average is ≈ 4.65 instances per class (see Figure 7). There are however outliers with big numbers of instances, which skew the average number of instances per class. This may for example imply, that classes are created mainly for the purpose of grouping newly created instances.

The median and average for subclasses per class are 0 and ≈ 0.85 respectively (see Figure 8). This implies that the graph components of the unlinked classes are very small, and in most cases contain only the respective unlinked class.

The question has to be asked, whether an algorithm should try to handle the complete set of unlinked classes, or only a specific subset, which fulfills certain requirements. The answer depends on what information such an algorithm requires as input. The full set of unlinked classes is problematic for use in an algorithm as

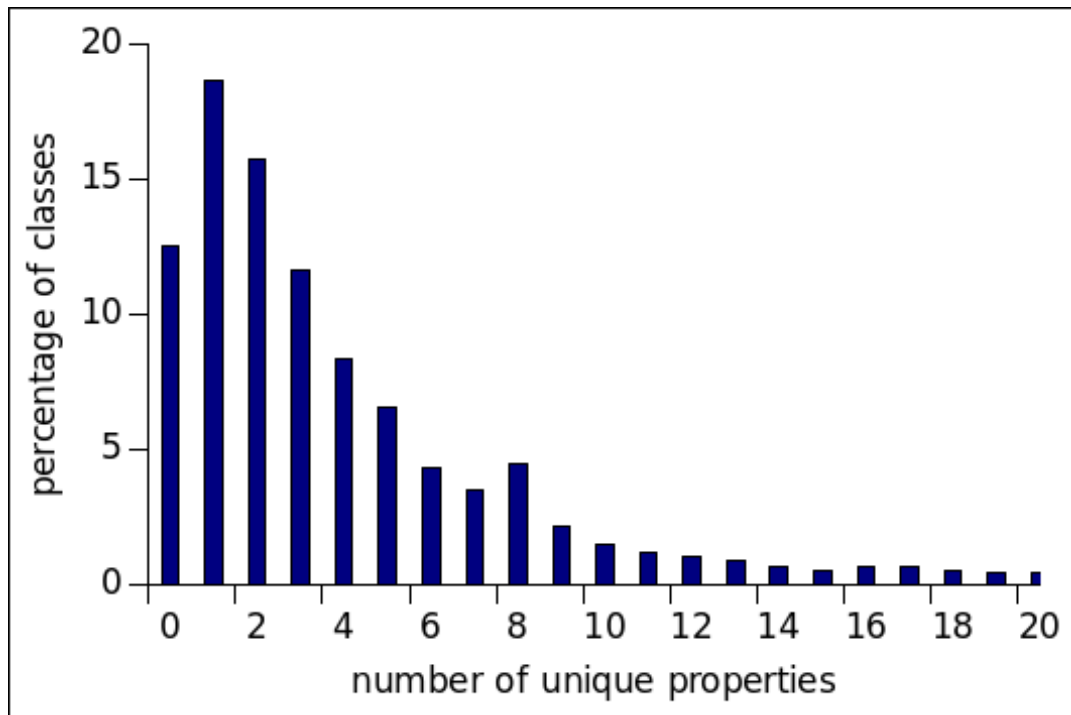


Figure 5: Percentage of unlinked classes with a specific amount of unique properties. Wikidata (2016/11/07)

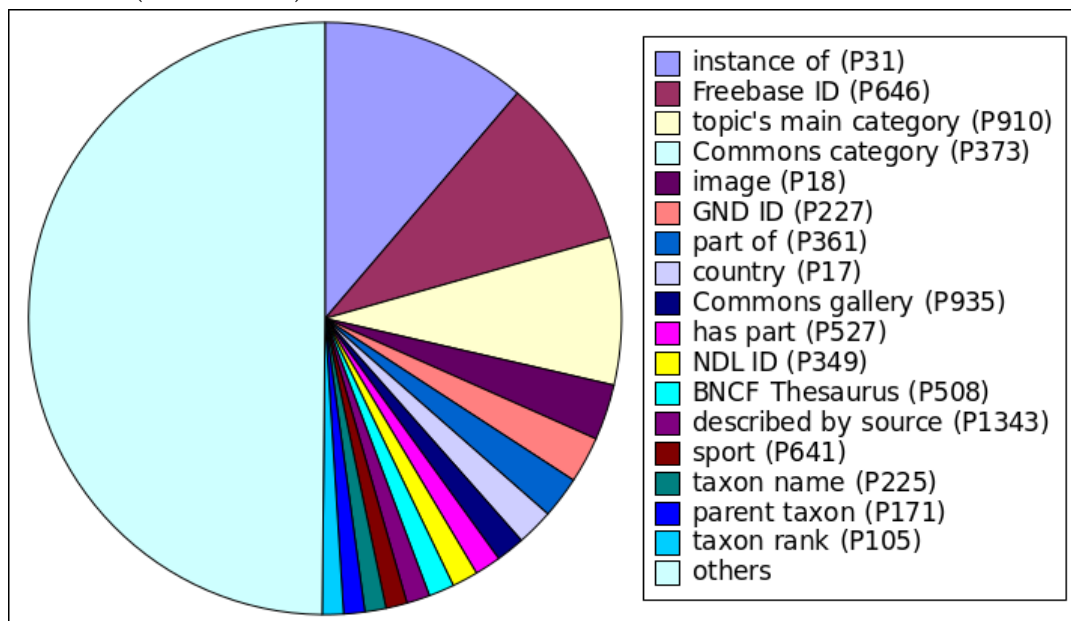


Figure 6: Frequency of properties in unlinked classes. Wikidata (2016/11/07)

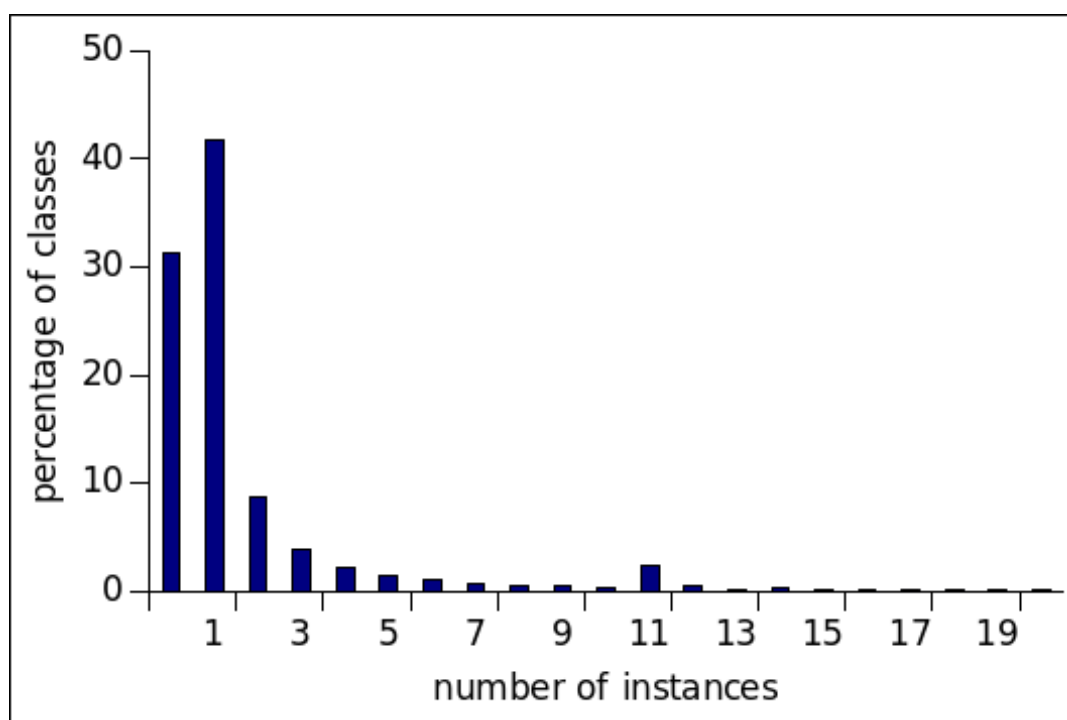


Figure 7: Percentage of unlinked classes with a specific amount of instances. Wiki-data (2016/11/07)

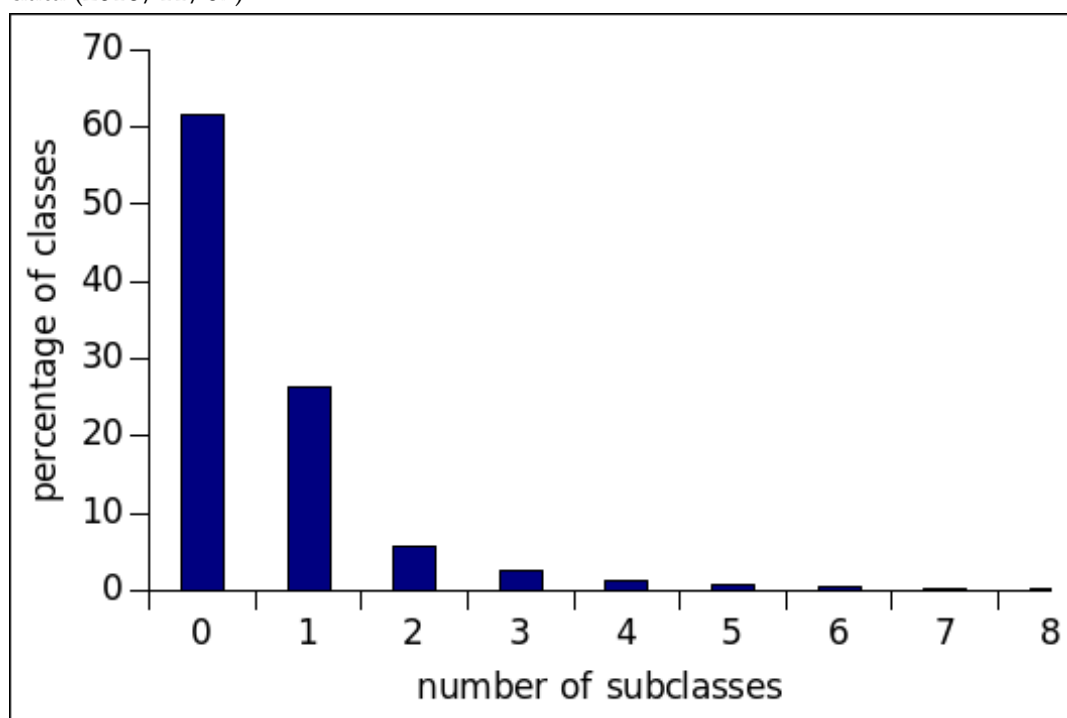


Figure 8: Percentage of unlinked classes with a specific amount of subclasses. Wiki-data (2016/11/07)

there are few shared characteristics over all classes. $\approx 16\%$ of classes are not labeled and $\approx 31.4\%$ of classes have no instances. Both labels and instances are useful characteristics for a class to have. The label allows the recognition of the class in natural text and ensures that the class fulfills a basic quality criteria, since the label is the first characteristic of a Wikidata item, which should be created by a user [1]. Instances represent classes and thereby describe them. For example, Rodríguez and Egenhofer [24] use instances to compute semantic similarity between classes. Therefore, I propose to only consider the set of classes, which have a label in the English language and at least 1 instance, as input for the algorithm.

5.3 Labeled, instantiated, unlinked classes

Following this argument, the subset of labeled, instantiated and unlinked classes was analyzed. This set contains a total of 9157 classes. 7557 classes have an English Wikipedia page, which is $\approx 82.5\%$ of all analyzed classes in the subset, and an increase of $\approx 12\%$ in comparison to the full set of unlinked classes.

The average of unique properties per class increases from 4.8 to 5.5, while the median remains at 3 (see Figure 9). The frequency of different properties appearing in classes however is more distributed (see Figure 10). This implies that the classes of this subset are related to different topics, and share only few commonalities.

As expected the average of instances per class increased to ≈ 7.37 from ≈ 4.65 . However the median is still 1 (see Figure 11). $\approx 59\%$ of classes have only 1 instance.

At the same time the average of subclasses per class halved to ≈ 0.4 (see Figure 12). **can i just do such an assumption? the data supports it a bit, but i am not searching for counterclaims. and the assumption is not directly relevant to the thesis.** Combined with the unchanged median of 1 instance per class, supports the assumption, that classes are mainly created for the purpose of grouping instances. While adding the newly created class to the main taxonomy is being neglected.

The classes of the subset have, on average, a better level of description, more unique properties and instances, than the full set of unlinked classes. Between these classes the number of commonalities is lower, because the same properties occur not as frequently for different classes. The lack of commonalities between unlinked classes is not very problematic, as the algorithm would not compare unlinked classes with each other but with classes in the root taxonomy.

5.4 Discussion (TODO: new subsection title)

After completing the analysis, it can be seen that *entity* (Q35120) is the root class of the taxonomy, and Wikidata's taxonomy is in a good state, since 97% of classes are part of the root taxonomy. **TODO: not sure if i can just say that it is in a good state** Unlinked classes share few commonalities, and as shown above, a percentage of classes lack even basic information like a label. For developing an algorithm it is necessary to set a baseline, of what can be expected from the input. In this case, input is required to be labeled and have at least one instance. The benefit of this specific

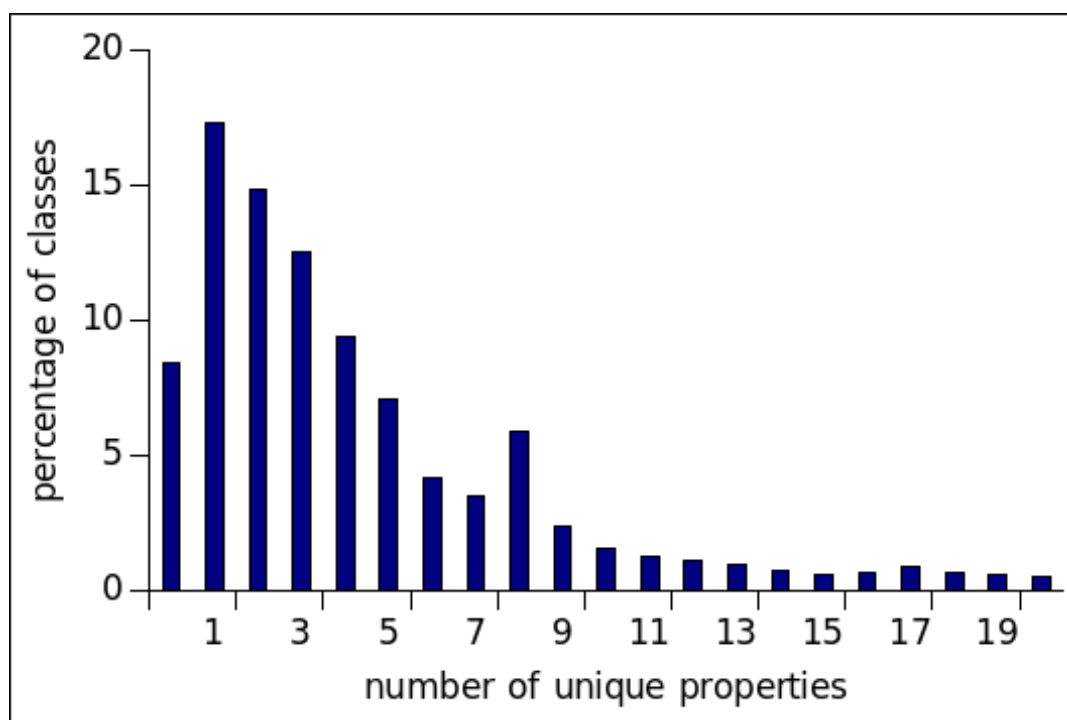


Figure 9: Percentage of unlinked, labeled, instantiated classes with a specific amount of unique properties. Wikidata (2016/11/07)

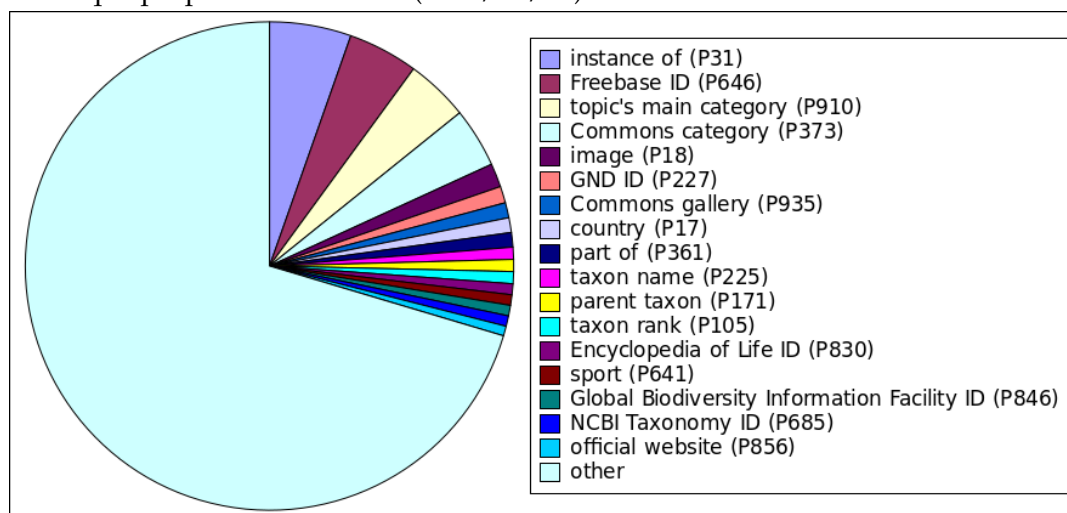


Figure 10: Frequency of properties in unlinked, labeled, instantiated classes. Wikidata (2016/11/07)

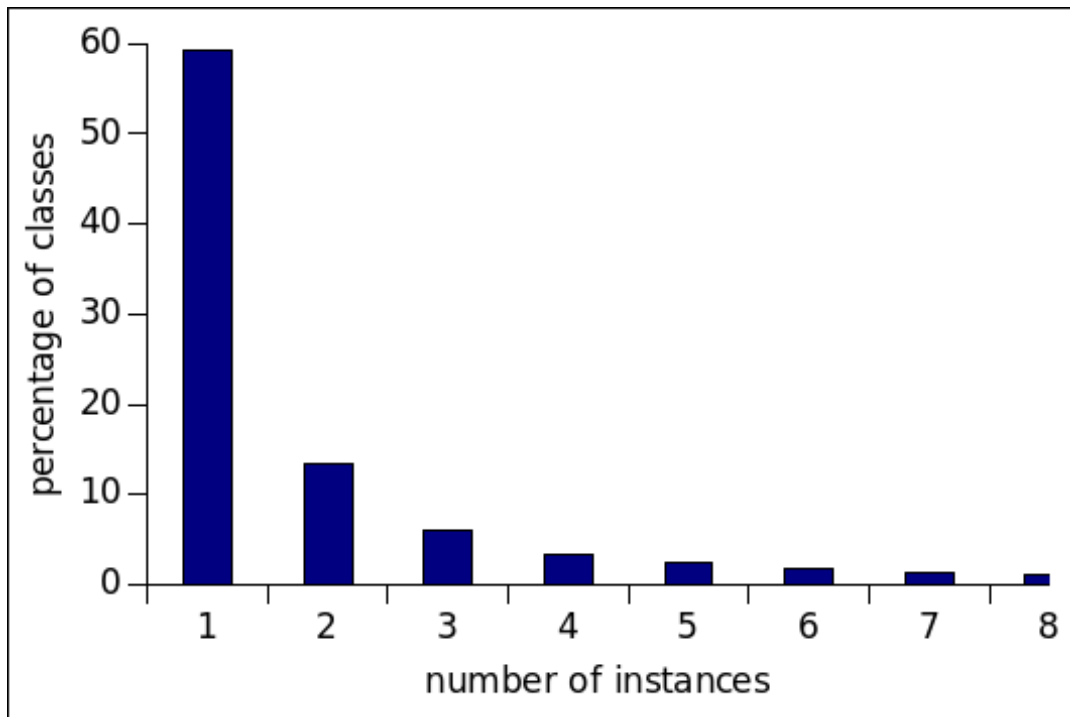


Figure 11: Percentage of unlinked, labeled, instantiated classes with a specific amount of instances. Wikidata (2016/11/07)

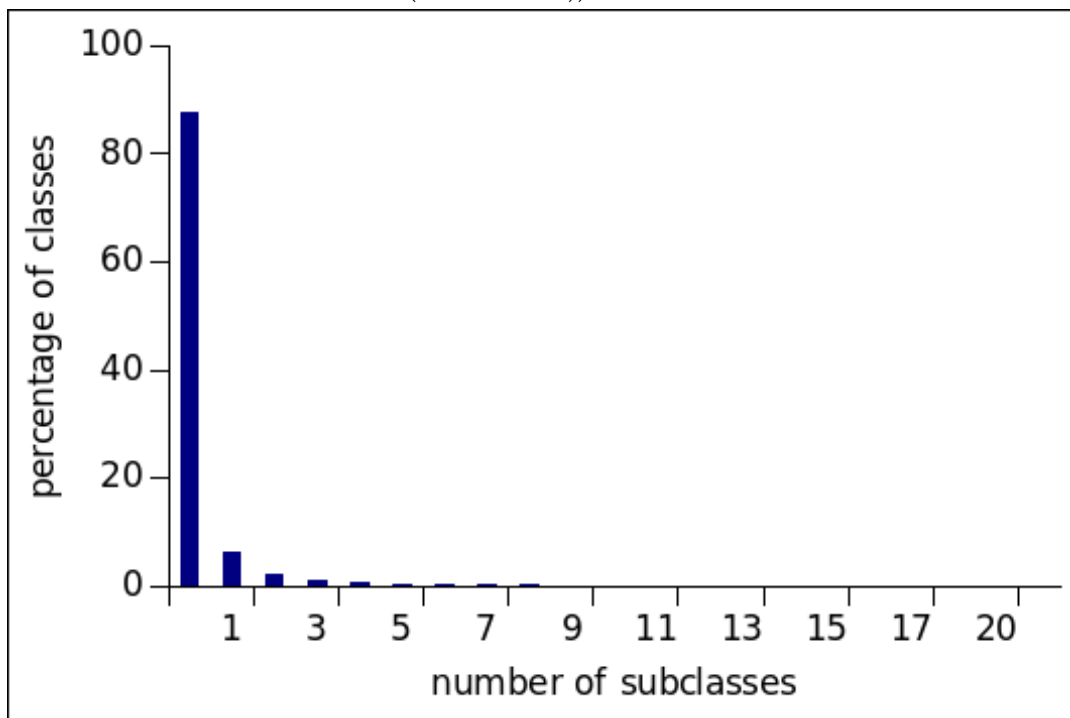


Figure 12: Percentage of unlinked, labeled, instantiated classes with a specific amount of subclasses. Wikidata (2016/11/07)

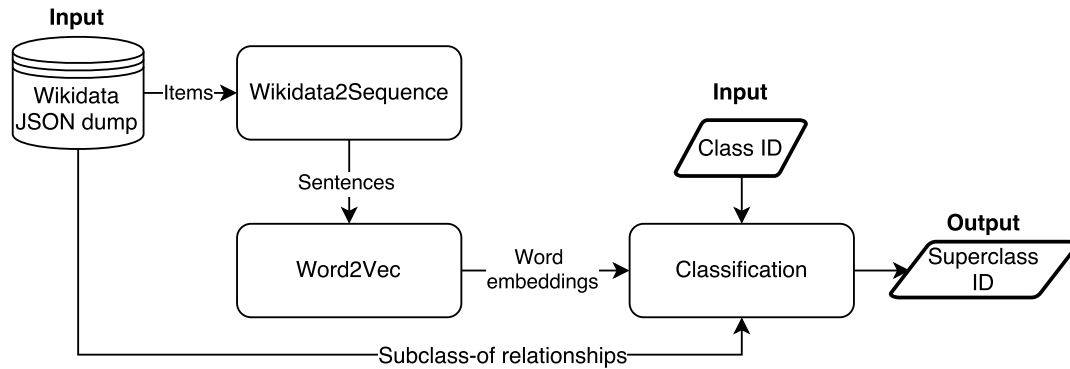


Figure 13: Data flow between components

restriction is, that a basic level of descriptiveness can be assumed for every class. A label additionally allows supplementing the information, provided by Wikidata, to be supplemented with natural text sources, like Wikipedia, since it is now possible to recognize occurrences of the class in text.

6 Combined algorithm

6.1 Components

The combined algorithm will use a Word2Vec model to generate word embeddings, which then can be used in a classification task. This leads to the identification of three main components. These main components will be implemented by a baseline algorithm and for each variation an improvement is proposed, which in general replaces one of the components. The data flow and input/output between the components is visualized in Figure 13

1. **Wikidata2Sequence.** For the task of generating word embeddings the Skip-gram model with negative sampling will be used. As mentioned in Section 3.2, both Word2Vec models require a linear sequence of words as input. Wikidata's highly interlinked structure is more alike to an RDF graph [11]. Therefore the first required component for the algorithm is a mapping from Wikidata to word sequences. The output of this method is required to be at least one sequence of words, which is called sentence. Therefore multiple sentences represent a text. Each word in the sequence has to be either a Wikidata item or property ID, or the data type for a literal value (e.g. replace all strings with *string*).
2. **Word2Vec.** The Skip-gram model with negative sampling (SGNS) will be used in every variation of the algorithm. It has been proven through theoretic analysis as well as experiments that SGNS creates very effective word embeddings, if its hyperparameters are chosen well. **TODO: citations**

3. **Classification.** The final component of the algorithm is a classification method. The word embeddings produced by SGNS group similar words close to each other and preserve linguistic regularities. kNN can exploit the first characteristic using a similarity or distance measure. The second characteristic can be exploited by computing the vector offset or learning a linear projection representing the subclass-of relationship between two classes [12]. For the purpose of evaluation each classification component should have the same output, this will allow a comparison between the different variations of the algorithm. Therefore each classification method will output the Wikidata ID (e.g. Q35120) of the most likely superclass.

Wikidata2Sequence and Word2Vec are executed once to compute the word embeddings of the corresponding Wikidata dump. The classification method will then be trained on the word embeddings and can be applied repeatedly to unlinked classes.

6.2 Baseline

The baseline algorithm uses a very basic Wikidata2Sequence component and simple weighted kNN for classification. If not otherwise mentioned, the variations will use the same hyperparameters and components as the baseline algorithm.

1. **Wikidata2Sequence.** Trivial sentences will be generated from statements. Each statement can be transformed to a triple $(itemid, pid, value)$, where value is either another item ID or a data type. **Insert example.**
2. **Word2Vec.** As mentioned above a Skip-gram model with negative sampling will be used. Based on **cite levy?** and **cite mikolov** the following hyperparameters have been proven to be effective in the task of creating word embeddings and are therefore used:
 - **TODO: add hyperparameters**
3. **Classification.** A weighted kNN will be used for the classification task. Each neighbor votes for all of its superclasses with the same weighted vote. **Which distance or similarity measure will be used to identify the k nearest neighbors.** The weight of each vote is based on its similarity to the input class. **TODO: What is k?**

6.3 Variation: Graph walk

A graph walk, developed by Ristoski and Paulheim [23] and explained in Section 3.3 can be used for generating longer sentences instead of triple sentences. The longer sequences are able to better capture contextual information than three word sentences. This is achieved by creating paths of a depth d , starting from each class. Evaluation of RDF2Vec shows that Skip-gram and CBOW both produce better word

embeddings this way [23].. Therefore it is expected that graph walk will also improve the performance of this algorithm. **TODO: choice of hyperparameters**

6.4 Variation: Vector offset

6.5 Variation: Linear projection

6.6 Variation: Wikipedia

6.7 Implementation details

Is this section necessary?

7 Evaluation

7.1 Method

Dellschaft and Staab [9]

7.2 Generation of gold standard

7.3 Results

8 Conclusion and future work

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Appendices

A Appendix: Graphs

Definition 10 (Directed graph). A directed graph G is an ordered pair $G = (V, E)$, where V is a set of vertices, and $E = \{(v_1, v_2) \mid v_1, v_2 \in V\}$ is a set of ordered pairs called directed edges, connecting the the vertices.

Definition 11. Subgraph Let $G = (V, E)$ and $H = (W, F)$ be directed graphs. H is called subgraph of G , if $W \subseteq V$ and $F \subseteq E$.

Definition 12 (Predecessor). Let $G = (V, E)$ be a directed graph. $v_1 \in V$ is a predecessor of $v_2 \in V$, if there exists an edge so that $(v_1, v_2) \in E$. Let $v \in V$ be a vertex of G , then $pred_G(v) = \{w \mid (w, v) \in E\}$ is the set of predecessors of v .

Definition 13 (Successor). $v_1 \in V$ is a successor of $v_2 \in V$, if there exists an edge so that $(v_2, v_1) \in E$. Let $v \in V$ be a vertex of G , then $succ_G(v) = \{w \mid (v, w) \in E\}$ is the set of successors of v .

Definition 14 (Walk). Let $G = (V, E)$ be a directed graph. A walk W of length $n \in \mathbb{N}$ is a sequence of vertices $W = (v_1, \dots, v_n)$ with $v_1, \dots, v_n \in V$, so that $(v_i, v_{i+1}) \in E \forall i = 1, \dots, n - 1$.

Definition 15 (Connected (Vertice)). Let $G = (V, E)$ be a directed graph. Vertices $u, v \in V$ are connected, if $u = v$, or $u \neq v$ and there is walk between u and v or v and u .

Definition 16 (Connected (Directed graph)). Let G be a directed graph. G is connected, if every pair of vertices in G are connected.

TODO: the current definition of component is useless.

Definition 17 (Component). Let G and H be directed graphs. H is a component of G , if H is a connected subgraph of G and H is not subgraph of another connected subgraph of G , which has more vertices or edges than H .

Definition 18 (Cycle). A walk $W = (v_1, \dots, v_n)$ of length n is called a cycle, if $v_1 = v_n$.

Definition 19 (Directed acyclic graph). A directed graph G is called directed acyclic graph, if there are no cycles in G .

TODO: definitions

Definition 20 (Weighted graph).

Definition 21 (Graph walk).