Goal Programming Example

A company produces two products. Relevant information for each product is shown in the Table below.

	Product 1	Product 2
Labor required	4 hours	2 hours
Contribution to profit	$4	$2

The company has a goal of $48 in profits and incurs a $1 penalty for each dollar it falls short of this goal. A total of 32 hours of labor are available. A $2 penalty is incurred for each hour of overtime (labor over 32 hours) used, and a $1 penalty is incurred for each hour of available labor that is unused. Marketing considerations require that at least 7 units of product 1 be produced and at least 10 units of product 2 be produced. For each unit (of either product) by which production falls short of demand, a penalty of $5 is assessed.

Formulate an LP than can be used to minimize the total penalty incurred by the company.

Solution:

Goal 1: Have $48 in profits.

Goal 2: Not exceed 32 hours labor.

Goal 3: Meet demand for product 1.

Goal 4: Meet demand for product 2.

Let

x_i = Amount of product i produced, $i = 1, 2$

d_j^+ = Amount over jth goal, $j = 1, ..., 4$

d_j^- = Amount under jth goal, $j = 1, ..., 4$

$$\min \quad d_1^- + 2d_2^+ + d_2^- + 5d_3^- + 5d_4^-$$

$$
\begin{aligned}
\text{sub. to} \quad 4x_1 + 2x_2 + d_1^- - d_1^+ &= 48 \\
4x_1 + 2x_2 + d_2^- - d_2^+ &= 32 \\
x_1 + d_3^- - d_3^+ &= 7 \\
x_2 + d_4^- - d_4^+ &= 10 \\
x_i &\geq 0, \quad i = 1, 2 \\
d_i^+ &\geq 0 \\
d_i^- &\geq 0 \quad j = 1, 2, 3, 4.
\end{aligned}
$$