Contents

Chapter 1

Multiple Systems

1.1 Classical states

Suppose we have just two systems:

- X is a system with classical state set Σ
- Y is a system with classical state set Γ

we can place them side-by-side to obtain a single classical system.

Definition 1.1.1: Compound classical system (pair)

The system created by joining two classical states X, Y:

$$XY$$

Question 1

What are the classical states of (X, Y)?

The classical state set of (X, Y) is the *Cartesian product* of the two sets:

$$\Sigma \times \Gamma = \{(a, b) : a \in \Sigma and b \in \Gamma\}$$

Note:

We're making the assumption that we know witch system is witch, and there's no confusion between them.

This generalizes well for n systems:

Definition 1.1.2: Compound classical system

Given n classical systems $X_1, ..., X_n$, the **compound system** formed by them is written as an n-tuple:

$$(X_1, ..., X_n)$$

or as a string:

$$X_1 ... X_n$$

Proposition 1.1.1

The classical state set of $X_1 ... X_n$ with states $\Sigma_1, ..., \Sigma_n$ is the set:

$$\Sigma_1 \times ... \times \Sigma_n$$

Note:

As per convention, cartesian products of states are ordered *lexicographically*, and significance decreases from left to right.

1.2 Probabilistic states

A probability is associated to each Cartesian product of the classical state sets of the individual systems.

Definition 1.2.1: Statistical independence

Given a probabilistic state of (X, Y), we say that X and Y are independent if, $\forall a \in \Sigma, \forall b \in \Gamma$:

$$Pr((X, Y) = (a, b)) = Pr(X = a)Pr(Y = b)$$

By expressing the state of (X, Y) as a vector:

$$|\pi\rangle = \sum_{(a,b)\in\Sigma\times\Gamma} p_{ab}|ab\rangle$$