

# CONTENTS

CHAPTER	MULTIPLE SYSTEMS	PAGE
1.1	Classical states	2
1.2	Probabilistic states	3

# Chapter 1

## Multiple Systems

### 1.1 Classical states

Suppose we have just two systems:

- $X$  is a system with classical state set  $\Sigma$
- $Y$  is a system with classical state set  $\Gamma$

we can place them side-by-side to obtain a single classical system.

#### Definition 1.1.1: Compound classical system (pair)

The system created by joining two classical states  $X, Y$ :

$$XY$$

#### Question 1

What are the classical states of  $(X, Y)$ ?

The classical state set of  $(X, Y)$  is the *Cartesian product* of the two sets:

$$\Sigma \times \Gamma = \{(a, b) : a \in \Sigma \text{ and } b \in \Gamma\}$$

#### Note:

We're making the assumption that we know which system is which, and there's no confusion between them.

This generalizes well for  $n$  systems:

#### Definition 1.1.2: Compound classical system

Given  $n$  classical systems  $X_1, \dots, X_n$ , the **compound system** formed by them is written as an  $n$ -tuple:

$$(X_1, \dots, X_n)$$

or as a string:

$$X_1 \dots X_n$$

#### Proposition 1.1.1

The classical state set of  $X_1 \dots X_n$  with states  $\Sigma_1, \dots, \Sigma_n$  is the set:

$$\Sigma_1 \times \dots \times \Sigma_n$$

**Note:**

As per convention, cartesian products of states are ordered *lexicographically*, and significance decreases from left to right.

## 1.2 Probabilistic states

A probability is associated to each Cartesian product of the classical state sets of the individual systems.

**Definition 1.2.1: Statistical independence**

Given a probabilistic state of  $(X, Y)$ , we say that  $X$  and  $Y$  are independent if,  $\forall a \in \Sigma, \forall b \in \Gamma$ :

$$Pr((X, Y) = (a, b)) = Pr(X = a)Pr(Y = b)$$

By expressing the state of  $(X, Y)$  as a vector:

$$|\pi\rangle = \sum_{(a,b) \in \Sigma \times \Gamma} p_{ab} |ab\rangle$$