Von Bertalanffy Growth Model Fitting - No Sex Term

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```
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```

```
LMBL <- read.csv("Data/Clean-Data/2016_largemouth-bass_long-format.csv") %>%
  select(FID:BI.len) %>%
  arrange(FID,Agei)
### Making factors factors
LMBL$FID <- factor(LMBL$FID)</pre>
LMBL$Site <- factor(LMBL$Site)</pre>
LMBL$SEXCON <- factor(LMBL$SEXCON)</pre>
LMBL$Sex <- factor(LMBL$Sex)</pre>
str(LMBL)
## 'data.frame':
                   337 obs. of 11 variables:
## $ FID : Factor w/ 126 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 ...
## $ Site : Factor w/ 11 levels "2","4","6","8",..: 6 6 6 6 6 6 6 6 6 6 ...
## $ AgeCap: int 4 4 4 4 4 4 4 4 4 ...
## $ RadCap: num 0.94 0.94 0.94 0.94 0.988 ...
## $ LenCap: int 347 347 347 347 292 292 292 348 348 ...
          : int 658 658 658 658 415 415 415 557 557 ...
## $ SEXCON: Factor w/ 5 levels "0","1","3","6",..: 5 5 5 5 3 3 3 3 3 3 ...
          : Factor w/ 3 levels "0", "1", "2": 3 3 3 3 2 2 2 2 2 2 ...
## $ Agei : int 1 2 3 4 1 2 3 4 1 2 ...
## $ Radi : num 0.433 0.69 0.803 0.927 0.567 ...
## $ BI.len: num 155 252 295 342 165 ...
headtail(LMBL)
##
      FID Site AgeCap RadCap LenCap WTg SEXCON Sex Agei
                                                           Radi
                     4 0.9402
                                347 658
## 1
        1
             11
                                            8 2 1 0.4328 154.6790
                                             8 2 2 0.6898 252.0903
## 2
        1
             11
                     4 0.9402
                                 347 658
                                 347 658
                                            8 2 3 0.8028 294.9210
## 3
        1
             11
                     4 0.9402
## 335 132 15972
                     7 1.0474
                                 395 971
                                            3 1 5 0.9567 359.9617
## 336 132 15972
                     7 1.0474
                                 395 971
                                             3 1 6 1.0119 381.2860
## 337 132 15972
                     7 1.0474
                                 395 971
                                                       7 1.0365 390.7892
datgr = groupedData(BI.len ~ Agei|FID, data = LMBL,
                 labels = list(x = "Age", y = "Size"),
                 units = list(x = (Years), y = (mm))
    Creating the von Bertalanffy function.
LVB <- function(x, Linf, K, t0){
  y = Linf * (1 - exp(-K * (x - t0)))
 у
}
LVB <- vbFuns()
LVB(5, 422.8, 0.39, -0.40)
## [1] 371.3351
```

```
LVB(5, Linf = c(422.8, 0.39, -0.40)) ### Should be the same output
## [1] 371.3351
VBGM Fit via NLME to Both Sexes, {nlme.mod}
nlme.mod <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,</pre>
                            fixed = list(Linf ~ 1, K ~ 1, t0 ~ 1),
                            random = Linf+K+t0 ~ 1,
                            start = list(fixed =
                                      c(Linf = 389.3647,
                                        K = 0.4359,
                                        t0 = -0.3127)),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=100,
                                        pnlsMaxIter=100,
                                        msMaxIter=100,
                                        niterEM=100))
#save(nlme.mod,
# file = "model-output/nlme.mod.rda")
## [1] "Iterations = 4"
## Nonlinear mixed-effects model fit by REML
    Model: BI.len ~ LVB(Agei, Linf, K, t0)
##
   Data: datgr
##
          AIC
                   BIC
                          logLik
##
     2833.928 2872.039 -1406.964
##
## Random effects:
## Formula: list(Linf ~ 1, K ~ 1, t0 ~ 1)
## Level: FID
## Structure: General positive-definite, Log-Cholesky parametrization
           StdDev
                       Corr
           60.8327092 Linf
## Linf
## K
            0.1289488 -0.873
            0.4352377 -0.670 0.856
## Residual 4.7627527
##
## Fixed effects: list(Linf ~ 1, K ~ 1, t0 ~ 1)
           Value Std.Error DF t-value p-value
## Linf 425.6965 7.665872 209 55.53139
                                              0
         0.4009 0.015719 209 25.50485
                                              0
## t0
        -0.3666 0.043430 209 -8.44185
                                              0
## Correlation:
     Linf
##
## K -0.884
## t0 -0.609 0.804
```

QЗ

Max

Standardized Within-Group Residuals:

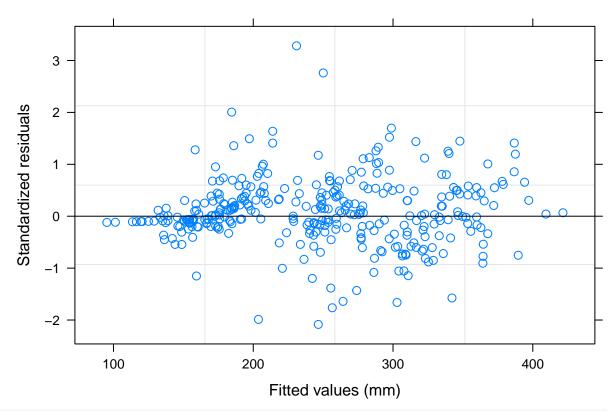
Q1

Med

-2.0847753 -0.1981814 0.0278886 0.4229808 3.2823985

Min

```
##
## Number of Observations: 337
## Number of Groups: 126
## Approximate 95% confidence intervals
##
## Fixed effects:
            lower
                                   upper
                         est.
## Linf 410.5841783 425.6965204 440.8088624
        0.3699297 0.4009183 0.4319070
## t0
        -0.4522482 -0.3666309 -0.2810135
## attr(,"label")
## [1] "Fixed effects:"
##
## Random Effects:
   Level: FID
##
                    lower est.
                                       upper
## sd(Linf) 47.75866119 60.8327092 77.4858093
## sd(K)
              0.09216863 0.1289488 0.1804062
## sd(t0)
              0.36935346 0.4352377 0.5128741
## cor(Linf,K) -0.93279735 -0.8731908 -0.7670862
## cor(Linf,t0) -0.81409867 -0.6699900 -0.4481523
## cor(K,t0)
            0.75059662 0.8559247 0.9188331
##
## Within-group standard error:
## lower
            est.
                      upper
## 4.015377 4.762753 5.649237
       numDF denDF F-value p-value
## Linf
        1 209 39477.28 <.0001
## K
          1
              209 2945.99 <.0001
          1 209 71.26 <.0001
## t0
```



```
fixef(nlme.mod)
```

Linf K t0 ## 425.6965204 0.4009183 -0.3666309

ranef(nlme.mod)

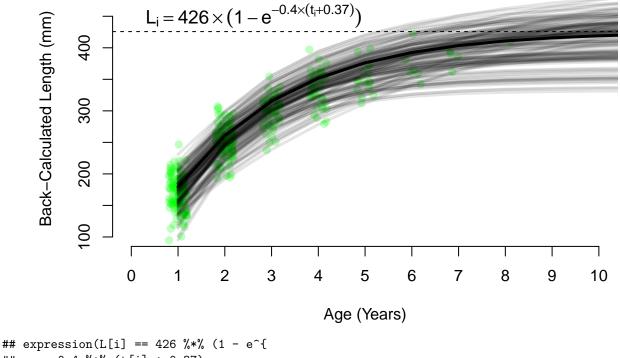
```
##
                               K
                                            t0
              Linf
                     0.254102333
## 88
       -93.7655080
                                  0.850327251
##
  87
       -89.5309586
                     0.241018178
                                  0.807633915
                     0.213040893
                                  0.717194451
## 85
       -80.2756842
##
  86
       -78.0302853
                    0.206370369
                                  0.695787193
##
   83
       -75.3219004
                    0.198380479
                                  0.670217061
##
  27
       -74.4837813
                    0.195920011
                                  0.662357689
##
   89
       -70.5055240
                     0.184315577
                                   0.625378025
   69
       -66.7437139
                     0.173450991
                                  0.590875294
##
##
  84
       -60.7334347
                     0.156297618
                                  0.536593022
## 82
       -56.7359272
                     0.145020539
                                  0.501000964
##
   110 -43.7085356
                     0.108951631
                                  0.387272004
  76
       -40.9992916
##
                    0.101578141
                                  0.363955618
##
   80
       -40.2353417
                     0.099507050
                                  0.357396318
##
   74
       -39.0369131
                     0.096265119
                                  0.347117958
       -38.2839127
##
   78
                     0.094232701
                                  0.340666810
   77
       -37.9900873
                    0.093440611
                                  0.338150867
##
##
  75
       -35.4334599
                     0.086571503
                                  0.316286181
## 72
       -34.1825761
                     0.083226102
                                  0.305602753
##
  121 -34.1367211
                     0.083103665
                                  0.305211262
  73
       -10.2531257
##
                     0.022104939
                                  0.098367680
##
  7
        -5.1373475
                    0.010435941
                                  0.050915992
## 70
         5.4358424 -0.009054611 -0.058044561
```

```
5.4592382 -0.009087252 -0.058306379
        6.9699866 -0.011073280 -0.075396289
## 122
        7.5302443 -0.011747867 -0.081819863
## 96
## 16
        12.3531788 -0.016174613 -0.138395152
## 39
        14.1472228 -0.017237878 -0.159646049
       83.1805110 -0.226345819 -0.421180510
## 104
## 29
        16.5716994 -0.018262076 -0.188133723
## 71
        76.1181072 -0.217421642 -0.502318020
## 15
        20.4980390 -0.019143259 -0.233057683
## 34
        22.2430684 -0.019305381 -0.252387533
## 119
       68.5556673 -0.189287798 -0.261228152
       -87.3945906 0.159934847 0.434778395
## 65
## 91
       -71.8797729 0.164307295 0.529623403
      -72.5208369 0.157019130 0.493945783
## 107 50.9380516 -0.170610018 -0.559899453
## 64
       -40.6743335 0.042804181 0.161924862
## 67
       -44.5197633 0.061143030 0.206853864
        40.5098677 -0.145836216 -0.526376796
      38.8853475 -0.135024683 -0.420431485
## 120
       13.3701851 -0.070680594 -0.082612646
## 66
        35.2282493 -0.123895270 -0.368067142
## 131 -39.9027149 0.107761773 0.392898942
        17.5988554 -0.078215182 -0.138379185
## 68
## 106 -32.8342277 0.098353224 0.369698476
## 130 -30.9233230  0.087232442  0.328753142
      -31.5441222 0.122549806 0.471819735
      -30.4175534 0.098318308 0.373911453
## 56
       16.2260378 -0.070960912 -0.180191484
## 112 -12.1176351 0.009225910 0.074871498
## 126
       3.7573486 -0.034119839 -0.066668531
## 53
        -4.9322721 -0.008237626 0.016178226
## 115 -19.5897005 0.055708011 0.221511133
## 117
       13.7330580 -0.053081267 -0.205915325
        6.0197569 -0.026408005 -0.085735797
## 33
       -15.1967909 0.090824135 0.362590935
## 108 -11.5832215 0.066657109 0.265891961
## 127 -13.0763895 0.090891072 0.364520334
## 95
        6.4447668 -0.014588334 -0.063531173
        17.9159489 -0.060167949 -0.331951918
## 57
        1.7147396 0.020904280 0.075744444
## 102
## 100
        8.9650067 -0.014620009 -0.080450062
## 8
        15.7849615 -0.035082721 -0.212953075
## 125
        4.6771434 0.067692615 0.268132168
## 123
       15.9741933 -0.018372610 -0.144805850
## 14
        20.4862748 -0.045252794 -0.338809870
        21.9593209 -0.047771744 -0.393442937
## 23
## 54
        16.3288729 0.049478834 0.174557258
## 52
      -73.3878437 0.167216683 0.497155460
## 2
        46.2684987 -0.225471248 -0.964255731
## 97
        18.8247175
                   0.081900056 0.325276646
## 18
       -60.6897244 0.065822138 0.075977175
## 92
       24.2833476 0.061999373 0.225010550
## 19
       30.5406512 -0.017638826 -0.245465882
## 114 30.2783334 0.013437375 -0.045038926
```

```
30.5900672 0.054245365 0.177186538
       32.0130709 0.004888832 -0.102958124
## 25
## 45
       32.7407168
                   0.055482821 0.180262579
      -24.1980353
                   0.001771404 -0.070703355
## 43
##
  36
       -41.2370782
                   0.071076175 0.069316779
##
  90
       41.4174993 0.022939601 -0.022754211
## 46
       32.1206158 -0.045481794 -0.608836678
## 61
       12.0809816 -0.081178442 -0.435853646
## 17
      -17.1189292 0.009467309 -0.098306636
## 118
      24.1400040 -0.070149278 -0.174038710
      -37.5590692 0.010448970 0.085143563
## 113 -57.4836597 0.105911110 0.232544310
## 47
        36.6778559 -0.194094834 -0.512475080
## 103
       85.0945465 -0.184806818 -0.253603681
## 30
       21.6241329 -0.113606416 -0.274434821
## 94
        13.0463836 -0.109045286 -0.510903418
## 50
        7.6772732 -0.160758874 -0.754503398
##
  32
       18.8423544 -0.011500207 -0.219451161
  129 -31.3895085 0.048286533 0.141988495
      -26.2563435 -0.085581390 -0.408891614
## 31
       48.4141916 -0.129249249 -0.266371420
## 3
       -28.8049055
                  0.042189431 -0.064866856
      -28.2235852 0.066161338 0.294133556
## 1
## 109 -22.1428517 -0.033826426 0.184543733
      -16.9999997 0.027025016 -0.021977793
      -30.1476525 -0.035440764 -0.279231852
## 116 -34.0509764 0.016645187 0.222360867
  48
        6.3788969 -0.022631486 0.048965519
##
  38
      -12.9872176 -0.017337489 -0.537493698
## 93
      -10.8413781 0.046596875 0.142831978
## 41
       75.8494011 -0.150791340 -0.361141014
## 20
        47.9671915 -0.123178187 -0.572165599
## 105
        7.5355958 -0.062207231 -0.643223351
## 5
       49.5248474 -0.081452492 -0.063056461
## 4
        36.8841703 -0.173255202 -0.442855846
## 128
       78.1186269 -0.174222121 -0.143621610
## 42
        ## 49
       71.1358837 -0.233586456 -0.542669407
## 44
        32.5580286 -0.067297659 -0.243573735
## 22
        8.2148233 0.031849559 0.106664425
       70.3178106 -0.178991963 -0.730004858
## 13
        0.8241870 -0.018052706 -0.006550835
## 12
## 9
        39.5732351 -0.046386743 -0.183247825
## 40
       68.1604558 -0.076820382 -0.130041373
## 26
        45.1782200 -0.162970172 -0.015381762
## 132 -24.6260301 0.069678281 0.290659452
## 37
       89.1641423 -0.163747006 -0.523779348
## 11
       52.8137176 -0.025319517 -0.252902255
## 10
      113.7101496 -0.240192906 -0.486463371
## 24
       56.8967254 -0.063958567 0.229931178
coef(nlme.mod)[1,3]
```

[1] 0.4836964

```
Axes <- seq(100,450,by=50)
Years <- seq(0,10,by=1)
## plot individual fish data
## plot the fixed parameter model
## plot individual fish models
plot(jitter(LMBL$Agei),LMBL$BI.len,
     col=rgb(0,1,0,0.25, maxColorValue=1),
     pch=19,
     ylim=c(100,475),
     xlim=c(0,10),
     xlab = "Age (Years)",
     ylab = "Back-Calculated Length (mm)",
     bty="n",
     yaxt="n",
     xaxt="n")
axis(2,at = Axes)
axis(1,at = Years)
abline(h=425.6965204,lty=2)
x \leftarrow seq(1,11,by=1)
lines(x, fixef(nlme.mod)[1] * (1 - exp(-fixef(nlme.mod)[2] * (x - (fixef(nlme.mod)[3])))),
      lwd=3,
      col="black")
for(i in 1:126){
lines(x, coef(nlme.mod)[i,1] * (1 - exp(- coef(nlme.mod)[i,2]
* (x - (coef(nlme.mod)[i,3]))), lwd=3, col=gray(0,0.1)) }
legend("topleft",
       legend = print(expression(L[i] == 426 \% \% (1 - e **{-0.40 \% \% (t[i] + 0.37)}))),
       bty="n",
       cex=1.15)
```



-0.4 %*% (t[i] + 0.37) ## }))

Modeling Fixed Effects for Sex

1 89

8

Lets look at the von Bertalanffy growth model fits with a fixed sex term on the parameter estimates.

In order to fit the sex model I will have to remove the individual with no sex. In order to compare the sex model to the no sex model I will have to re fit the no sex model without the individual with unknown sex. this means making new df excluding this individual. making a new grouped data object. and rerunning the nlme function.

Removing the individual with no sex (FID=89??)

1 0.4328

136 38

```
### Just looking at data
head(LMBL)
     FID Site AgeCap RadCap LenCap WTg SEXCON Sex Agei
##
                                                             Radi
                                                                     BI.len
                    4 0.9402
## 1
                                 347 658
                                               8
                                                    2
                                                         1 0.4328 154.6790
       1
           11
## 2
       1
           11
                    4 0.9402
                                 347 658
                                               8
                                                   2
                                                         2 0.6898 252.0903
                                                   2
## 3
           11
                    4 0.9402
                                 347 658
                                               8
                                                         3 0.8028 294.9210
       1
                    4 0.9402
                                               8
                                                    2
                                                         4 0.9269 341.9589
       1
           11
                                 347 658
## 5
       2
           11
                    4 0.9884
                                 292 415
                                               3
                                                    1
                                                         1 0.5665 164.5096
       2
                                               3
                                                         2 0.7181 210.3203
## 6
           11
                    4 0.9884
                                 292 415
                                                    1
### Finding fish with unknown sex
(unknown.sex <- filterD(LMBL,Sex==0))</pre>
     FID Site AgeCap RadCap LenCap WTg SEXCON Sex Agei
##
```

0

0

1 0.3983 124.5434

```
### Getting row number for fish with the unknown sex
(FID89 <- as.numeric(row.names(LMBL[LMBL$Sex==0,])))</pre>
## [1] 230
### removing the fish with unknown sex from the data set
length(LMBL$FID) ### just seeing the number of rows in the data set
## [1] 337
length(unique(LMBL$FID)) ### just seeing the number of fish
## [1] 126
LMBL <- LMBL[-c(FID89),]
length(LMBL$FID)
## [1] 336
length(unique(LMBL$FID)) ### Good! looks like only FID 89 was removed
## [1] 125
### Lets make sure there is no empty row in my data
LMBL <- filterD(LMBL,!is.na(FID))</pre>
### and lets just take a quick look at the data
str(LMBL)
## 'data.frame':
                   336 obs. of 11 variables:
## $ FID : Factor w/ 125 levels "1","2","3","4",...: 1 1 1 1 2 2 2 2 3 3 ...
## $ Site : Factor w/ 11 levels "2","4","6","8",..: 6 6 6 6 6 6 6 6 6 ...
## $ AgeCap: int 4 4 4 4 4 4 4 4 4 ...
## $ RadCap: num 0.94 0.94 0.94 0.94 0.988 ...
## $ LenCap: int 347 347 347 347 292 292 292 292 348 348 ...
          : int 658 658 658 658 415 415 415 557 557 ...
## $ SEXCON: Factor w/ 4 levels "1", "3", "6", "8": 4 4 4 4 2 2 2 2 2 2 ...
          : Factor w/ 2 levels "1", "2": 2 2 2 2 1 1 1 1 1 1 ...
## $ Sex
## $ Agei : int 1 2 3 4 1 2 3 4 1 2 ...
## $ Radi : num 0.433 0.69 0.803 0.927 0.567 ...
## $ BI.len: num 155 252 295 342 165 ...
headtail(LMBL)
      FID Site AgeCap RadCap LenCap WTg SEXCON Sex Agei Radi BI.len
##
## 1
        1
            11
                 4 0.9402
                                347 658
                                         8 2 1 0.4328 154.6790
## 2
                    4 0.9402
                                347 658
                                           8 2 2 0.6898 252.0903
        1
             11
                                           8 2 3 0.8028 294.9210
## 3
             11
                    4 0.9402 347 658
## 334 132 15972
                     7 1.0474
                                395 971
                                           3 1 5 0.9567 359.9617
                                           3 1 6 1.0119 381.2860
## 335 132 15972
                     7 1.0474
                                395 971
395 971
                                395 971
                                           3 1 7 1.0365 390.7892
## 336 132 15972
                     7 1.0474
Remake Grouped Data Object
datgr = groupedData(BI.len ~ Agei | FID, data = LMBL,
                 labels = list(x = "Age", y = "Size"),
                 units = list(x = "(Years)", y = "(mm)"))
```

Re-Fitting VBGM Without Sex Terms After Removing Unkown Sex Fish

```
nlme.mod2 <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,</pre>
                            fixed = list(Linf \sim 1, K \sim 1, t0 \sim 1),
                            random = Linf+K+t0 ~ 1,
                            start = list(fixed =
                                       c(Linf = 389.3647,
                                        K = 0.4359
                                         t0 = -0.3127)),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=100,
                                        pnlsMaxIter=100,
                                        msMaxIter=100,
                                        niterEM=100))
#save(nlme.mod2,
     file = "model-output/nlme.mod2.rda")
## [1] "Iterations = 8"
## Nonlinear mixed-effects model fit by REML
     Model: BI.len ~ LVB(Agei, Linf, K, t0)
##
   Data: datgr
##
          AIC
                  BIC
                         logLik
##
     2825.199 2863.28 -1402.599
##
## Random effects:
## Formula: list(Linf ~ 1, K ~ 1, t0 ~ 1)
## Level: FID
  Structure: General positive-definite, Log-Cholesky parametrization
            StdDev
##
                       Corr
            60.2706899 Linf
## Linf
## K
             0.1636979 -0.896
             0.4051268 -0.743 0.880
## Residual 5.1861779
##
## Fixed effects: list(Linf ~ 1, K ~ 1, t0 ~ 1)
           Value Std.Error DF t-value p-value
## Linf 422.1732 7.758468 209 54.41450
                                               0
         0.4092 0.019320 209 21.18083
                                               0
## K
        -0.3564 0.042235 209 -8.43892
## t0
## Correlation:
##
     Linf
## K -0.897
## t0 -0.680 0.849
##
## Standardized Within-Group Residuals:
           Min
                        Q1
                                   Med
                                                            Max
## -1.94148193 -0.17864102 0.01619288 0.39153583 3.07200909
##
## Number of Observations: 336
## Number of Groups: 125
## Approximate 95% confidence intervals
##
##
  Fixed effects:
##
              lower
                           est.
                                      upper
```

```
## Linf 406.8783142 422.1731988 437.4680835
## K
           0.3711197
                        0.4092061
                                     0.4472924
                      -0.3564181
##
         -0.4396794
                                   -0.2731568
   attr(,"label")
##
   [1] "Fixed effects:"
##
##
##
    Random Effects:
     Level: FID
##
##
                        lower
                                     est.
                                               upper
## sd(Linf)
                 46.20689210 60.2706899 78.6150269
## sd(K)
                  0.07580475
                               0.1636979
                                           0.3535001
## sd(t0)
                  0.30276138
                               0.4051268
                                           0.5421027
   cor(Linf,K)
                 -0.96608505 -0.8959355 -0.7026344
##
   cor(Linf,t0) -0.90420164 -0.7425586 -0.3948487
   cor(K,t0)
                  0.74950984
                               0.8803736
                                           0.9450186
##
##
    Within-group standard error:
##
                 est.
                          upper
##
   3.325330 5.186178 8.088353
##
        numDF denDF
                      F-value p-value
## Linf
             1
                 209 38273.06 <.0001
## K
                 209
                      2870.19 <.0001
             1
## t0
             1
                 209
                         71.22
                               <.0001
                                                0
           3
                                                    0
           2
     Standardized residuals
           1
           0
                                                                                   0
          -1
                                 0
                                                                         0
                                          0
          -2
                   100
                                         200
                                                              300
                                                                                    400
                                             Fitted values (mm)
```

Not too different from nlme.mod as expected.

Full Sex Model, {sexmod.lkt}

Sex terms on all model parameters

$\{L_{\infty}, K\}$ Sex Model, $\{\text{sexmod.lk}\}$

$\{L_{\infty}, t_0\}$ Sex Model, {sexmod.lt}

```
file = "model-output/sexmod.lt.rda")
```

$\{K, t_0\}$, Sex Model, $\{sexmod.kt\}$

Comparing Models

Summary

The von Bertlanffy growth model was fit to 337 observations of length at age from 126 individuals. Five additional individuals with out age estimates were removed from the growth analysis. The maximum asymptotic length (L_{∞}) fit to both male and females was estimated to be 426 mm (SE=7.7). The brody growth rate coefficient (K) was estimated as 0.40 (SE=0.02). The estimated value of t_0 is -0.37 (SE=0.04). Differences in the estimated growth parameters for different sexes were explored and found to be not significant (Need to report statistics).