Fitting von Bertalanffy Growth Models with Fixed Sex Effects

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Load Required Packages and Data

```
LMBL <- read.csv("Data/Clean-Data/2016_largemouth-bass_long-format.csv") %>%
  select(FID:BI.len) %>%
  arrange(FID,Agei)

### Making factors factors
LMBL$FID <- factor(LMBL$FID)
LMBL$Site <- factor(LMBL$Site)
LMBL$SEXCON <- factor(LMBL$SEXCON)
LMBL$Sex <- factor(LMBL$Sex)
```

```
str(LMBL)
```

```
## 'data.frame':    337 obs. of  11 variables:
##  $ FID   : Factor w/ 126 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 ...
##  $ Site  : Factor w/ 11 levels "2","4","6","8",..: 6 6 6 6 6 6 6 6 6 6 ...
##  $ AgeCap: int  4 4 4 4 4 4 4 4 4 4 ...
##  $ RadCap: num  0.94 0.94 0.94 0.94 0.988 ...
##  $ LenCap: int  347 347 347 347 292 292 292 292 348 348 ...
##  $ WTg   : int  658 658 658 658 415 415 415 415 557 557 ...
##  $ SEXCON: Factor w/ 5 levels "0","1","3","6",..: 5 5 5 5 3 3 3 3 3 3 ...
##  $ Sex   : Factor w/ 3 levels "0","1","2": 3 3 3 3 2 2 2 2 2 2 ...
##  $ Agei  : int  1 2 3 4 1 2 3 4 1 2 ...
##  $ Radi  : num  0.433 0.69 0.803 0.927 0.567 ...
##  $ BI.len: num  155 252 295 342 165 ...
```

```
headtail(LMBL)
```

```
##     FID  Site AgeCap RadCap LenCap WTg SEXCON Sex Agei   Radi   BI.len
## 1     1    11      4 0.9402    347 658      8   2    1 0.4328 154.6790
## 2     1    11      4 0.9402    347 658      8   2    2 0.6898 252.0903
## 3     1    11      4 0.9402    347 658      8   2    3 0.8028 294.9210
## 335 132 15972      7 1.0474    395 971      3   1    5 0.9567 359.9617
## 336 132 15972      7 1.0474    395 971      3   1    6 1.0119 381.2860
## 337 132 15972      7 1.0474    395 971      3   1    7 1.0365 390.7892
```

Creating the von Bertalanffy function.

```
LVB <- function(x, Linf, K, t0){
  y = Linf * (1 - exp(-K * (x - t0)))
  y
}
LVB <- vbFuns()

LVB(5, 422.8, 0.39, -0.40)
```

```
## [1] 371.3351
```

```r
LVB(5, Linf = c(422.8, 0.39, -0.40)) ### Should be the same output
```

```
## [1] 371.3351
```

Modeling Fixed Effects for Sex

Lets look at the von Bertalanffy growth model fits with a fixed sex term on the parameter estimates.

In order to fit the sex model I will have to remove the individual with no sex. In order to compare the sex model to the no sex model I will have to re-fit the no sex model without the individual with unknown sex. This means making new df excluding this individual. Then, making a new grouped data object. Finnaly, I will need to rerun the nlme function with out this fish to get the nlme.mod2 output [*This was All done in the fit-von-bertalanffy-growth-model.rmd file*].

Removing the individual with no sex (FID=89??)

```r
### Just looking at data
head(LMBL)
```

```
##   FID Site AgeCap RadCap LenCap WTg SEXCON Sex Agei   Radi   BI.len
## 1   1   11      4 0.9402    347 658      8   2    1 0.4328 154.6790
## 2   1   11      4 0.9402    347 658      8   2    2 0.6898 252.0903
## 3   1   11      4 0.9402    347 658      8   2    3 0.8028 294.9210
## 4   1   11      4 0.9402    347 658      8   2    4 0.9269 341.9589
## 5   2   11      4 0.9884    292 415      3   1    1 0.5665 164.5096
## 6   2   11      4 0.9884    292 415      3   1    2 0.7181 210.3203
```

```r
### Finding fish with unknown sex
(unknown.sex <- filterD(LMBL,Sex==0))
```

```
##   FID Site AgeCap RadCap LenCap WTg SEXCON Sex Agei   Radi   BI.len
## 1  89    8      1 0.4328    136  38      0   0    1 0.3983 124.5434
```

```r
### Getting row number for fish with the unknown sex
(FID89 <- as.numeric(row.names(LMBL[LMBL$Sex==0,])))
```

```
## [1] 230
```

```r
### removing the fish with unknown sex from the data set
length(LMBL$FID) ### just seeing the number of rows in the data set
```

```
## [1] 337
```

```r
length(unique(LMBL$FID)) ### just seeing the number of fish
```

```
## [1] 126
```

```r
LMBL <- LMBL[-c(FID89),]
length(LMBL$FID)
```

```
## [1] 336
```

```r
length(unique(LMBL$FID)) ### Good! looks like only FID 89 was removed
```

```
## [1] 125
```

```r
### Lets make sure there is no empty row in my data
LMBL <- filterD(LMBL,!is.na(FID))
### and lets just take a quick look at the data
str(LMBL)
```

```
## 'data.frame':    336 obs. of  11 variables:
##  $ FID   : Factor w/ 125 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 ...
##  $ Site  : Factor w/ 11 levels "2","4","6","8",..: 6 6 6 6 6 6 6 6 6 6 ...
##  $ AgeCap: int  4 4 4 4 4 4 4 4 4 4 ...
##  $ RadCap: num  0.94 0.94 0.94 0.94 0.988 ...
##  $ LenCap: int  347 347 347 347 292 292 292 292 348 348 ...
##  $ WTg   : int  658 658 658 658 415 415 415 415 557 557 ...
##  $ SEXCON: Factor w/ 4 levels "1","3","6","8": 4 4 4 4 2 2 2 2 2 2 ...
##  $ Sex   : Factor w/ 2 levels "1","2": 2 2 2 2 1 1 1 1 1 1 ...
##  $ Agei  : int  1 2 3 4 1 2 3 4 1 2 ...
##  $ Radi  : num  0.433 0.69 0.803 0.927 0.567 ...
##  $ BI.len: num  155 252 295 342 165 ...
```

```r
headtail(LMBL)
```

```
##     FID  Site AgeCap RadCap LenCap WTg SEXCON Sex Agei   Radi   BI.len
## 1     1    11      4 0.9402    347 658      8   2    1 0.4328 154.6790
## 2     1    11      4 0.9402    347 658      8   2    2 0.6898 252.0903
## 3     1    11      4 0.9402    347 658      8   2    3 0.8028 294.9210
## 334 132 15972      7 1.0474    395 971      3   1    5 0.9567 359.9617
## 335 132 15972      7 1.0474    395 971      3   1    6 1.0119 381.2860
## 336 132 15972      7 1.0474    395 971      3   1    7 1.0365 390.7892
```

Remake Grouped Data Object

```r
datgr = groupedData(BI.len ~ Agei|FID, data = LMBL,
                    labels = list(x = "Age", y = "Size"),
                    units = list(x = "(Years)", y = "(mm)"))
```

Full Sex Model, {sexmod.lkt}

Sex terms on all model parameters

I'm going to skip this for now it seems to be taking forever. I have this model output already converged, however, I did so with ML estimation instead of REML so I cannot compare with other model fits. I dont think this model should be used anyways since I'm not aware of a biological reason male and female largemouth bass would have a different t_0.

```r
sexmod.lkt <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                            fixed = list(Linf ~ Sex-1, K ~ Sex-1, t0 ~ Sex-1),
                            random = Linf+K+t0 ~ 1,
                            start = list(fixed =
                                      c(Linf = c(389.3647,389.3647),
                                        K = c(0.4359,0.4359),
                                        t0 = c(-0.3127,-0.3127))),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=1000,
                                        pnlsMaxIter=100,
                                        msMaxIter=100,
```

```
                                        niterEM=100))
save(sexmod.lkt,
     file = "model-output/sexmod.lkt.rda")
```

{L_∞, K} Sex Model, {sexmod.lk}

I'm going to skip this for now it seems to be taking forever. I have this model output already converged, however, I did so with ML estimation instead of REML so I cannot compare with other model fits.

Unfortunaltly, I do think this is the sex model that would make the most sense biologically except for maybe the L_∞ of K sex models.

```
sexmod.lk <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                            fixed = list(Linf ~ Sex-1, K ~ Sex-1, t0 ~ 1),
                            random = Linf+K+t0 ~ 1,
                            start = list(fixed =
                                      c(Linf = c(389.3647,389.3647),
                                        K = c(0.4359,0.4359),
                                        t0 = -0.3127)),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=1000,
                                        pnlsMaxIter=100,
                                        msMaxIter=100,
                                        niterEM=100))
save(sexmod.lk,
     file = "model-output/sexmod.lk.rda")
```

{L_∞, t_0} Sex Model, {sexmod.lt}

```
sexmod.lt <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                            fixed = list(Linf ~ Sex-1, K ~ 1, t0 ~ Sex-1),
                            random = Linf+K+t0 ~ 1,
                            start = list(fixed =
                                      c(Linf = c(389.3647,389.3647),
                                        K = 0.4359,
                                        t0 = c(-0.3127,-0.3127))),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=1000,
                                        pnlsMaxIter=100,
                                        msMaxIter=100,
                                        niterEM=100))
#save(sexmod.lt,
#     file = "model-output/sexmod.lt.rda")
```

{K, t_0}, Sex Model, {sexmod.kt}

```
sexmod.kt <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                            fixed = list(Linf ~ 1, K ~ Sex-1, t0 ~ Sex-1),
```

```
                             random = Linf+K+t0 ~ 1,
                             start = list(fixed =
                                       c(Linf = 389.3647,
                                         K = c(0.4359,0.4359),
                                         t0 = c(-0.3127,-0.3127))),
                            method= "REML",
                            control=list(opt="nlminb",
                                         maxIter=1000,
                                         pnlsMaxIter=100,
                                         msMaxIter=100,
                                         niterEM=100))
#save(sexmod.kt,
#     file = "model-output/sexmod.kt.rda")
```

$\{L_\infty\}$, Sex Model, {sexmod.l}

```
sexmod.l <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                             fixed = list(Linf ~ Sex-1, K ~ 1, t0 ~ 1),
                             random = Linf+K+t0 ~ 1,
                             start = list(fixed =
                                       c(Linf = c(389.3647,389.3647),
                                         K = 0.4359,
                                         t0 = -0.3127)),
                            method= "REML",
                            control=list(opt="nlminb",
                                         maxIter=1000,
                                         pnlsMaxIter=100,
                                         msMaxIter=100,
                                         niterEM=100))
#save(sexmod.l,
#     file = "model-output/sexmod.l.rda")
```

{K}, Sex Model, {sexmod.k}

```
sexmod.k <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                             fixed = list(Linf ~ 1, K ~ Sex-1, t0 ~ 1),
                             random = Linf+K+t0 ~ 1,
                             start = list(fixed =
                                       c(Linf = 389.3647,
                                         K = c(0.4359,0.4359),
                                         t0 = -0.3127)),
                            method= "REML",
                            control=list(opt="nlminb",
                                         maxIter=1000,
                                         pnlsMaxIter=100,
                                         msMaxIter=100,
                                         niterEM=100))
#save(sexmod.k,
#     file = "model-output/sexmod.k.rda")
```

$\{t_0\}$, Sex Model, {sexmod.t}

```
sexmod.t <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                           fixed = list(Linf ~ 1, K ~ 1, t0 ~ Sex-1),
                           random = Linf+K+t0 ~ 1,
                           start = list(fixed =
                                     c(Linf = 389.3647,
                                       K = c(0.4359,0.4359),
                                       t0 = -0.3127)),
                          method= "REML",
                          control=list(opt="nlminb",
                                       maxIter=1000,
                                       pnlsMaxIter=100,
                                       msMaxIter=100,
                                       niterEM=100))
#save(sexmod.t,
#     file = "model-output/sexmod.t.rda")
```

zero intercept sex models ???

I will now try fitting the same sex models with a zero intercept (I think this is right might be slope). this is done by removing the -1 from VBPARAM ~ Sex-1 argument.

Rather than go through all of these right now which maybe I should I will just pick my favorits and go from there.

$\{L_\infty$, K$\}$ Intercept Sex Model {sexmod.lk.int}

```
sexmod.lk.int <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                           fixed = list(Linf ~ Sex, K ~ Sex, t0 ~ 1),
                           random = Linf+K+t0 ~ 1,
                           start = list(fixed =
                                     c(Linf = c(389.3647,389.3647),
                                       K = c(0.4359,0.4359),
                                       t0 = -0.3127)),
                          method= "REML",
                          control=list(opt="nlminb",
                                       maxIter=100,
                                       pnlsMaxIter=1000,
                                       msMaxIter=100,
                                       niterEM=100))
save(sexmod.lk.int,
     file = "model-output/sexmod.lk.int.rda")
```

$\{L_\infty\}$ Intercept Sex Model {sexmod.l.int}

```
sexmod.l.int <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                           fixed = list(Linf ~ Sex, K ~ 1, t0 ~ 1),
                           random = Linf+K+t0 ~ 1,
                           start = list(fixed =
```

```
                                      c(Linf = c(389.3647,389.3647),
                                        K = 0.4359,
                                        t0 = -0.3127)),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=100,
                                        pnlsMaxIter=100,
                                        msMaxIter=100,
                                        niterEM=100))
#save(sexmod.l.int,
#     file = "model-output/sexmod.l.int.rda")
```

{K} Intercept Sex Model {sexmod.k.int}

```
sexmod.k.int <- nlme::nlme(BI.len ~ LVB(Agei, Linf, K, t0), data = datgr,
                            fixed = list(Linf ~ 1, K ~ Sex, t0 ~ 1),
                            random = Linf+K+t0 ~ 1,
                            start = list(fixed =
                                      c(Linf = 389.3647,
                                        K = c(0.4359,0.4359),
                                        t0 = -0.3127)),
                           method= "REML",
                           control=list(opt="nlminb",
                                        maxIter=100,
                                        pnlsMaxIter=1000,
                                        msMaxIter=100,
                                        niterEM=100))
save(sexmod.k.int,
     file = "model-output/sexmod.k.int.rda")
```

Look at Model Fits

```
load("model-output/nlme.mod2.rda")

#load("model-output/sexmod.lkt.rda")
#load("model-output/sexmod.lk.rda")
load("model-output/sexmod.lt.rda")
load("model-output/sexmod.kt.rda")
load("model-output/sexmod.l.rda")
load("model-output/sexmod.k.rda")
load("model-output/sexmod.t.rda")

#load("model-output/sexmod.lkt.int.rda")
#load("model-output/sexmod.lk.int.rda")
#load("model-output/sexmod.lt.int.rda")
#load("model-output/sexmod.kt.int.rda")
load("model-output/sexmod.l.int.rda")
#load("model-output/sexmod.k.int.rda")
#load("model-output/sexmod.t.int.rda")
```

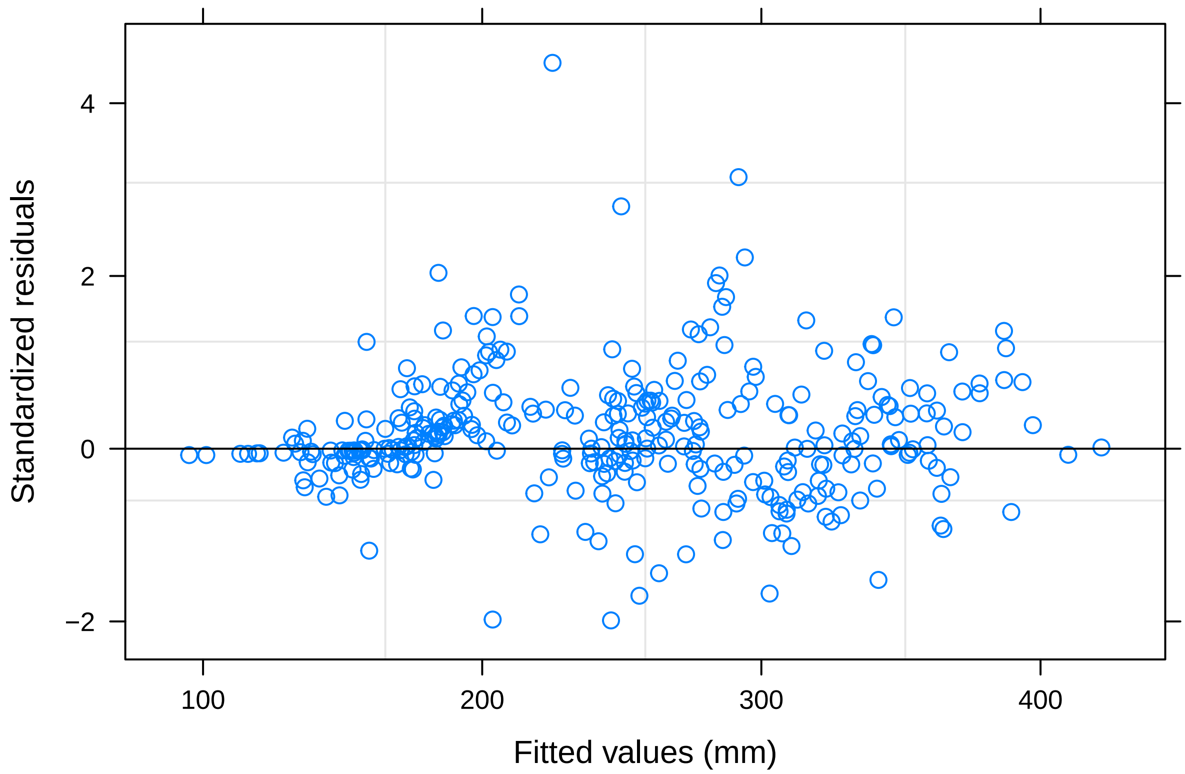
```
## [1] "Iterations = 11"
```

```
## Nonlinear mixed-effects model fit by REML
##   Model: BI.len ~ LVB(Agei, Linf, K, t0)
##  Data: datgr
##        AIC      BIC    logLik
##   2809.756 2851.613 -1393.878
##
## Random effects:
##  Formula: list(Linf ~ 1, K ~ 1, t0 ~ 1)
##  Level: FID
##  Structure: General positive-definite, Log-Cholesky parametrization
##                 StdDev     Corr
## Linf.(Intercept) 60.7965255 Ln.(I) K
## K                 0.1283636 -0.880
## t0                0.4334910 -0.675  0.857
## Residual          4.7920294
##
## Fixed effects: list(Linf ~ Sex - 1, K ~ 1, t0 ~ 1)
##             Value Std.Error  DF  t-value p-value
## Linf.Sex1 425.4479  8.432956 208 50.45062       0
## Linf.Sex2 439.0623  8.546468 208 51.37354       0
## K           0.3855  0.015426 208 24.98993       0
## t0         -0.3887  0.043652 208 -8.90441       0
##  Correlation:
##           Lnf.S1 Lnf.S2 K
## Linf.Sex2  0.695
## K         -0.824 -0.810
## t0        -0.582 -0.543  0.811
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -1.98806932 -0.16716238  0.08612533  0.49651811  4.46707175
##
## Number of Observations: 336
## Number of Groups: 125
```

```
## Approximate 95% confidence intervals
##
##  Fixed effects:
##                 lower        est.       upper
## Linf.Sex1 408.8228527 425.4478748 442.0728969
## Linf.Sex2 422.2135142 439.0623179 455.9111216
## K           0.3550875   0.3854992   0.4159109
## t0         -0.4747482  -0.3886919  -0.3026356
## attr(,"label")
## [1] "Fixed effects:"
##
##  Random Effects:
##   Level: FID
##                               lower       est.      upper
## sd(Linf.(Intercept))     47.29589478 60.7965255 78.1509163
## sd(K)                     0.08225568  0.1283636  0.2003170
## sd(t0)                    0.36854094  0.4334910  0.5098876
## cor(Linf.(Intercept),K)  -0.95392287 -0.8799812 -0.7052801
## cor(Linf.(Intercept),t0) -0.85453505 -0.6745742 -0.3500078
```

```
## cor(K,t0)                0.71194480  0.8565798  0.9314982
##
##  Within-group standard error:
##    lower     est.    upper
## 3.978896 4.792029 5.771337
```

```
##            numDF denDF   F-value p-value
## Linf.Sex       2   208 20992.203  <.0001
## K              1   208  3025.994  <.0001
## t0             1   208    79.289  <.0001
```




```r
fixef(sexmod.l)
```

```
##   Linf.Sex1   Linf.Sex2           K          t0
## 425.4478748 439.0623179   0.3854992  -0.3886919
```

```r
ranef(sexmod.l)
```

```
##     Linf.(Intercept)            K           t0
## 88      -96.09305881 0.2496292093 0.8778496208
## 87      -89.71634582 0.2299266925 0.8060792419
## 85      -82.95964757 0.2115588716 0.7468871005
## 86      -77.70709952 0.1958531873 0.6897164706
## 83      -78.14569333 0.1979820667 0.7007736752
## 27      -77.33178413 0.1957044398 0.6930647115
## 69      -69.82363331 0.1749195555 0.6230373032
## 84      -64.00480662 0.1590706823 0.5699787840
## 82      -55.36020645 0.1352840741 0.4867700302
## 110     -41.59308625 0.0994054508 0.3675613527
## 76      -44.97292140 0.1085899502 0.4020755279
## 80      -44.23785084 0.1066780798 0.3957214618
```

```
## 74      -43.08478201  0.1036844814  0.3857683022
## 78      -42.36027990  0.1018068294  0.3795228400
## 77      -42.07756079  0.1010748338  0.3770872465
## 75      -39.61713785  0.0947213152  0.3559277853
## 72      -31.44858559  0.0735853195  0.2812061103
## 121     -38.36861410  0.0915087775  0.3452123299
## 73      -14.78535586  0.0326456096  0.1416824757
## 7        -9.33227805  0.0199076831  0.0919711559
## 70        2.59650888 -0.0046519742 -0.0287300445
## 35        9.05285476 -0.0159100796 -0.0958419431
## 122       4.41004452 -0.0078158523 -0.0487268109
## 96        5.08015594 -0.0089454399 -0.0561899263
## 16       11.06966834 -0.0182679640 -0.1235655905
## 39       19.74085320 -0.0313431748 -0.2108471731
## 104      82.54502836 -0.2103654214 -0.3968855837
## 29       16.72887576 -0.0263503782 -0.1861070159
## 71       79.46948218 -0.2114759643 -0.5069152114
## 15       22.48102556 -0.0344777419 -0.2472160284
## 34       25.22455345 -0.0383900372 -0.2755990785
## 119      73.69401660 -0.1862658555 -0.2676293222
## 65     -100.02326809  0.1750895921  0.4623489442
## 91      -82.68369317  0.1732261340  0.5528137735
## 63      -83.77929709  0.1676353390  0.5183505487
## 107      54.52263239 -0.1694459383 -0.5759494216
## 64       25.12954217 -0.0885749956 -0.0467231656
## 67      -39.01050775  0.0592790369  0.2051042654
## 58       35.66599126 -0.1261669497 -0.4989858552
## 120      43.51544091 -0.1379908263 -0.4396728066
## 111      -0.09413534 -0.0322496580 -0.0196023412
## 66       28.70884997 -0.1007363986 -0.3300284050
## 131     -32.11550846  0.0974113483  0.3888868324
## 68       27.41415694 -0.0934513760 -0.1711742602
## 106     -24.81436000  0.0880314038  0.3646556978
## 130     -39.77728309  0.0934919005  0.3498516275
## 98      -22.80619481  0.1091107860  0.4681242005
## 56      -38.65531884  0.1024489672  0.3931534643
## 62       22.69659817 -0.0828540379 -0.2049449464
## 112      -8.65403808  0.0146259551  0.0785762766
## 126       2.07655839 -0.0172354832 -0.0458231645
## 53       -5.68305605 -0.0141200085  0.0152281508
## 115     -27.42134377  0.0608156280  0.2406847141
## 117      13.23178442 -0.0392944275 -0.1912866231
## 33        7.76717018 -0.0170549400 -0.0780114119
## 99      -21.27742723  0.0896608522  0.3764918093
## 108     -17.66314246  0.0667728402  0.2796688503
## 127     -18.84934672  0.0889246482  0.3776454724
## 95        3.90927248 -0.0172890406 -0.0617326941
## 57       18.72368927 -0.0665833840 -0.3478239749
## 102       8.94791098  0.0162173705  0.0663144160
## 100       6.46642748 -0.0176731244 -0.0796954323
## 8        21.00275953 -0.0341884834 -0.2220286045
## 125       0.94423040  0.0616232248  0.2750909498
## 123      14.48365925 -0.0240810860 -0.1504971351
## 14       28.30126764 -0.0507628296 -0.3640891219
```

```
## 23       23.57033465 -0.0587978372 -0.4209700874
## 54       26.91834003  0.0335802823  0.1577805812
## 52      -67.67100661  0.1566168121  0.4940275538
## 2        44.47104159 -0.2093412299 -0.9483670010
## 97       17.01145598  0.0695919096  0.3278387503
## 18      -55.19423844  0.0613897925  0.0655661812
## 92       23.10221450  0.0488304742  0.2241374009
## 19       33.70262975 -0.0353936479 -0.2760913646
## 114      43.00613158 -0.0080337664 -0.0770408973
## 101      30.33459374  0.0385482970  0.1727547366
## 25       45.54786275 -0.0181798087 -0.1408979644
## 45       32.77538137  0.0387162952  0.1750980276
## 43      -40.28095395  0.0260437090 -0.0280308221
## 36      -33.98666867  0.0608699393  0.0494886599
## 90       44.54638473 -0.0004402106 -0.0441008572
## 46       42.83192412 -0.0848684592 -0.7062088924
## 61       14.00717860 -0.0904596881 -0.4748269899
## 17      -11.76250713  0.0091272765 -0.1071188917
## 118      22.77491566 -0.0530521030 -0.1488778949
## 60      -56.58679297  0.0456054836  0.1471054576
## 113     -53.68880232  0.1037904471  0.2265014278
## 47       34.02658996 -0.1759375494 -0.4801123988
## 103      83.84105200 -0.1681926199 -0.2269882342
## 30       14.91679728 -0.0871042149 -0.2207281986
## 94       10.29562027 -0.0894073113 -0.4773824082
## 50        5.43907984 -0.1424575183 -0.7240886416
## 32       13.70577437 -0.0169527928 -0.2344326840
## 129     -27.91755348  0.0515999792  0.1417999253
## 51      -28.64853876 -0.0640683984 -0.3678230912
## 31       47.11179378 -0.1285115754 -0.2860008118
## 3       -24.57320923  0.0418710341 -0.0764804713
## 1       -42.03344810  0.0840974327  0.3223150096
## 109     -38.20615560 -0.0101696335  0.2267788759
## 21      -29.86623126  0.0415670775  0.0007227505
## 59      -29.73983420 -0.0209739806 -0.2607241769
## 116     -33.50097865  0.0308018993  0.2419064932
## 48       -5.75087978 -0.0092515444  0.0695525327
## 38      -23.61611474 -0.0130562339 -0.5448368463
## 93       -6.70133976  0.0482401767  0.1395892740
## 41       75.40440608 -0.1352248712 -0.3384178502
## 20       44.15527750 -0.1216279154 -0.5955794570
## 105      -0.05858524 -0.0627155281 -0.6704132068
## 5        49.66620144 -0.0664264716 -0.0413644047
## 4        31.54156751 -0.1671937643 -0.4652021951
## 128      76.73694528 -0.1571835898 -0.1150986143
## 42        4.52341585  0.0229754547 -0.1585003277
## 49       71.10607154 -0.2180382471 -0.5207899642
## 44       33.96846623 -0.0554912227 -0.2307003625
## 22       12.53298864  0.0340155131  0.1034585949
## 13       70.69343157 -0.1646342085 -0.7143871605
## 12        1.89267094 -0.0052933949  0.0090573986
## 9        42.72483895 -0.0391283441 -0.1805153722
## 40       70.96688093 -0.0674434095 -0.1218287269
## 26       44.95547912 -0.1469194627  0.0095931128
```

```
## 132     -23.69567555  0.0807245820  0.3034811306
## 37       83.52697465 -0.1569631880 -0.5355728353
## 11       44.88516203 -0.0249299264 -0.2639530728
## 10      106.70572989 -0.2289074646 -0.4929933572
## 24       44.28354951 -0.0497220001  0.2503322309
```

```r
coef(sexmod.l)[1,3]
```

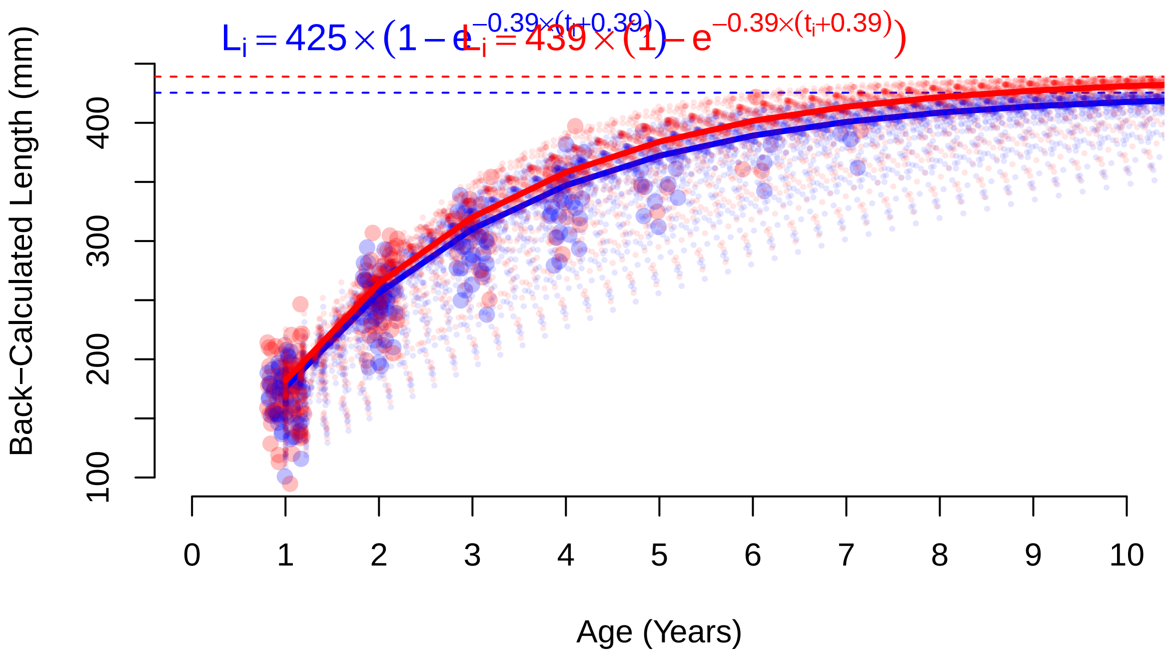
```
## [1] 0.6351284
```

```r
Axes <- seq(100,450,by=50)
Years <- seq(0,10,by=1)

## plot individual fish data
## plot the fixed parameter model
## plot individual fish models

### rgb(0,1,0,0.25, maxColorValue=1)

plot(jitter(LMBL$Agei),LMBL$BI.len,
     col=ifelse(LMBL$Sex==1,rgb(0,0,1,0.25, maxColorValue=1),rgb(1,0,0,0.25, maxColorValue=1)),
     pch=19,
     ylim=c(100,500),
     xlim=c(0,10),
     xlab = "Age (Years)",
     ylab = "Back-Calculated Length (mm)",
     bty="n",
     yaxt="n",
     xaxt="n")

axis(2,at = Axes)
axis(1,at = Years)

abline(h=425.4478748,lty=2,col="blue") ### Males
abline(h=439.0623179,lty=2,col="red") ### females

x <- seq(1,11,by=1)
lines(x, fixef(sexmod.l)[1] * (1 - exp(-fixef(sexmod.l)[3] * (x - (fixef(sexmod.l)[4])))),
      lwd=3,
      col="blue") ### Males

for(i in 1:125){
lines(x, coef(sexmod.l)[i,1] * (1 - exp(- coef(sexmod.l)[i,3]
* (x - ( coef(sexmod.l)[i,4] )))),lwd=3,col=rgb(0,0,1,0.1),lty=3) } ### Males

lines(x, fixef(sexmod.l)[2] * (1 - exp(-fixef(sexmod.l)[3] * (x - (fixef(sexmod.l)[4])))),
      lwd=3,
      col="red") ### Females

for(i in 1:125){
lines(x, coef(sexmod.l)[i,2] * (1 - exp(- coef(sexmod.l)[i,3]
* (x - ( coef(sexmod.l)[i,4] )))),lwd=3,col=rgb(1,0,0,0.1),lty=3) } ### Females

legend("topleft",
       legend = print(expression(L[i]==425 %*% (1 - e **{-0.39 %*% (t[i] + 0.39)}))),
```

```
       bty="n",
       cex=1.15,
       text.col = "blue") ### VB Equation Males
```

```
## expression(L[i] == 425 %*% (1 - e^{
##     -0.39 %*% (t[i] + 0.39)
## }))
```

```
legend("top",
       legend = print(expression(L[i]==439 %*% (1 - e **{-0.39 %*% (t[i] + 0.39)}))),
       bty="n",
       cex=1.15,
       text.col = "red") ### VB Equation Females
```




```
## expression(L[i] == 439 %*% (1 - e^{
##     -0.39 %*% (t[i] + 0.39)
## }))
```

{sexmod.l.int}

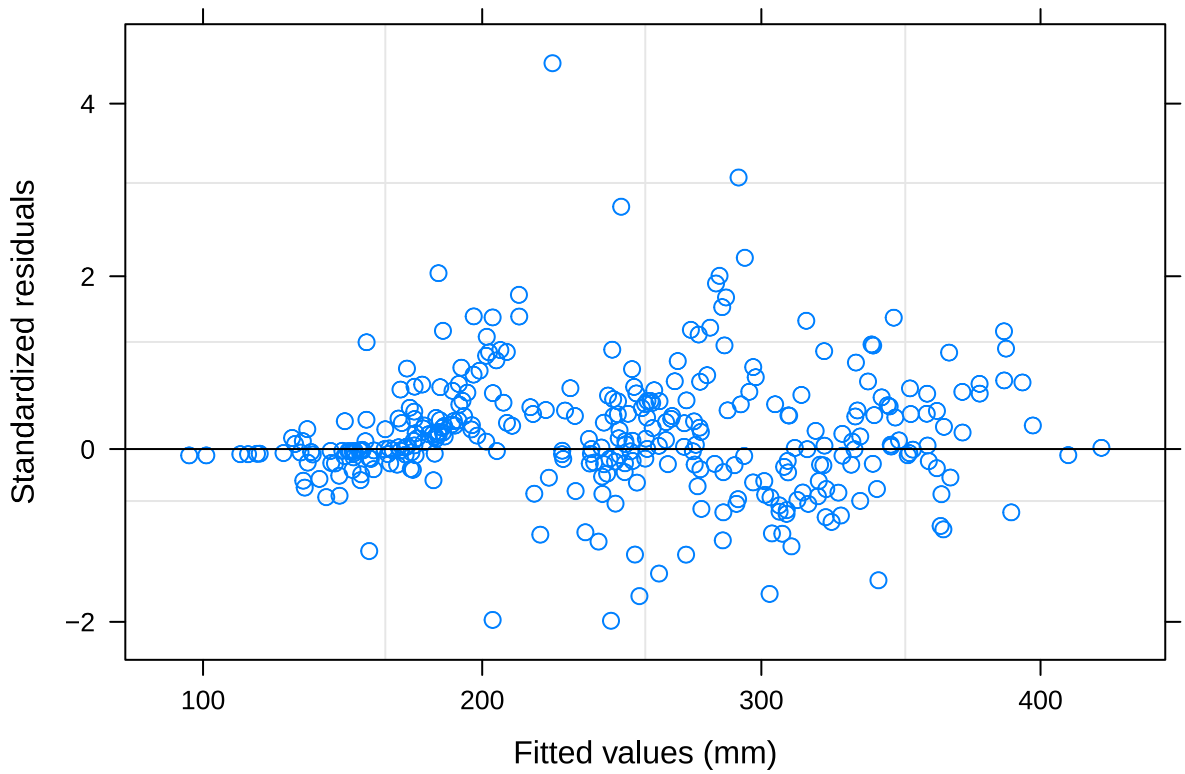
```
## [1] "Iterations = 8"
```

```
## Nonlinear mixed-effects model fit by REML
##   Model: BI.len ~ LVB(Agei, Linf, K, t0)
##  Data: datgr
##        AIC      BIC    logLik
##   2809.756 2851.613 -1393.878
##
## Random effects:
##  Formula: list(Linf ~ 1, K ~ 1, t0 ~ 1)
##  Level: FID
##  Structure: General positive-definite, Log-Cholesky parametrization
##                  StdDev     Corr
## Linf.(Intercept) 60.7967245 Ln.(I) K
```

```
## K                 0.1283638 -0.880
## t0                0.4334913 -0.675  0.857
## Residual          4.7920262
##
## Fixed effects: list(Linf ~ Sex, K ~ 1, t0 ~ 1)
##                     Value Std.Error  DF  t-value p-value
## Linf.(Intercept) 425.4479  8.432969 208 50.45055  0.0000
## Linf.Sex2         13.6145  6.626469 208  2.05456  0.0412
## K                  0.3855  0.015426 208 24.98990  0.0000
## t0                -0.3887  0.043652 208 -8.90441  0.0000
##  Correlation:
##           Ln.(I) Lnf.S2 K
## Linf.Sex2 -0.376
## K         -0.824  0.004
## t0        -0.582  0.040  0.811
##
## Standardized Within-Group Residuals:
##        Min         Q1        Med         Q3        Max
## -1.9880697 -0.1671674  0.0861218  0.4965188  4.4670194
##
## Number of Observations: 336
## Number of Groups: 125
```

```
## Approximate 95% confidence intervals
##
##  Fixed effects:
##                        lower        est.       upper
## Linf.(Intercept) 408.8228916 425.4479383 442.0729850
## Linf.Sex2          0.5508558  13.6145061  26.6781564
## K                  0.3550873   0.3854991   0.4159108
## t0                -0.4747486  -0.3886923  -0.3026359
## attr(,"label")
## [1] "Fixed effects:"
##
##  Random Effects:
##   Level: FID
##                               lower       est.      upper
## sd(Linf.(Intercept))     47.2998329 60.7967245 78.1449211
## sd(K)                     0.0822892  0.1283638  0.2002362
## sd(t0)                    0.3685501  0.4334913  0.5098757
## cor(Linf.(Intercept),K)  -0.9538721 -0.8799822 -0.7055677
## cor(Linf.(Intercept),t0) -0.8543408 -0.6745775 -0.3506498
## cor(K,t0)                 0.7122032  0.8565806  0.9314297
##
##  Within-group standard error:
##    lower     est.    upper
## 3.979148 4.792026 5.770963
```

```
##                  numDF denDF  F-value p-value
## Linf.(Intercept)     1   208 41958.70  <.0001
## Linf.Sex             1   208    25.59  <.0001
## K                    1   208  3026.00  <.0001
## t0                   1   208    79.29  <.0001
```




```
fixef(sexmod.l.int)
```

```
## Linf.(Intercept)        Linf.Sex2                K               t0
##      425.4479383       13.6145061        0.3854991       -0.3886923
```

```
ranef(sexmod.l.int)
```

```
##      Linf.(Intercept)            K            t0
## 88       -96.09321168  0.249629216  0.8778497795
## 87       -89.71646144  0.229926649  0.8060792709
## 85       -82.95977400  0.211558879  0.7468872657
## 86       -77.70719271  0.195853142  0.6897164842
## 83       -78.14581280  0.197982078  0.7007738464
## 27       -77.33190265  0.195704451  0.6930648794
## 69       -69.82374377  0.174919577  0.6230374893
## 84       -64.00491304  0.159070714  0.5699789812
## 82       -55.36027694  0.135284052  0.4867700371
## 110      -41.59316212  0.099405607  0.3675619827
## 76       -44.97302255  0.108590018  0.4020757671
## 80       -44.23795275  0.106678158  0.3957217311
## 74       -43.08489280  0.103684495  0.3857686046
## 78       -42.36037907  0.101806889  0.3795229856
## 77       -42.07766547  0.101074941  0.3770876183
## 75       -39.61724203  0.094721427  0.3559281313
## 72       -31.44864984  0.073585352  0.2812061590
## 121      -38.36871533  0.091508864  0.3452125665
## 73       -14.78547244  0.032645824  0.1416830699
## 7         -9.33238402  0.019907888  0.0919717676
## 70         2.59650714 -0.004651973 -0.0287298236
## 35         9.05294684 -0.015910237 -0.0958422729
```

```
## 122       4.41006677 -0.007815898 -0.0487266969
## 96        5.08018809 -0.008945504 -0.0561898641
## 16       11.06978320 -0.018268166 -0.1235659235
## 39       19.74105967 -0.031343472 -0.2108479542
## 104      82.54496875 -0.210365424 -0.3968868192
## 29       16.72905323 -0.026350657 -0.1861076140
## 71       79.46950468 -0.211476009 -0.5069162518
## 15       22.48124838 -0.034478045 -0.2472167359
## 34       25.22479098 -0.038390341 -0.2755997795
## 119      73.69391981 -0.186265869 -0.2676302733
## 65     -100.02319355  0.175089167  0.4623491570
## 91      -82.68368794  0.173225942  0.5528140347
## 63      -83.77927882  0.167635108  0.5183507967
## 107      54.52279514 -0.169446065 -0.5759499153
## 64       25.12919521 -0.088574597 -0.0467223168
## 67      -39.01039355  0.059278761  0.2051042226
## 58       35.66620325 -0.126167131 -0.4989862363
## 120      43.51558507 -0.137990994 -0.4396733347
## 111      -0.09425749 -0.032249427 -0.0196017741
## 66       28.70896021 -0.100736514 -0.3300286636
## 131     -32.11545849  0.097411401  0.3888871447
## 68       27.41406913 -0.093451318 -0.1711741220
## 106     -24.81437527  0.088031647  0.3646562165
## 130     -39.77736091  0.093491966  0.3498519420
## 98      -22.80627003  0.109110987  0.4681244493
## 56      -38.65539609  0.102449091  0.3931538749
## 62       22.69661412 -0.082854113 -0.2049450236
## 112      -8.65412241  0.014626145  0.0785767240
## 126       2.07651641 -0.017235379 -0.0458228154
## 53       -5.68314390 -0.014119918  0.0152285630
## 115     -27.42143945  0.060815746  0.2406850924
## 117      13.23188265 -0.039294551 -0.1912867296
## 33        7.76716359 -0.017054895 -0.0780112098
## 99      -21.27755277  0.089661157  0.3764923833
## 108     -17.66328388  0.066773074  0.2796692712
## 127     -18.84954620  0.088925050  0.3776460809
## 95        3.90924046 -0.017289038 -0.0617324750
## 57       18.72393853 -0.066583766 -0.3478245719
## 102       8.94783747  0.016217557  0.0663146882
## 100       6.46641009 -0.017673142 -0.0796952723
## 8        21.00286562 -0.034188583 -0.2220287689
## 125       0.94408297  0.061623499  0.2750912683
## 123      14.48369588 -0.024081171 -0.1504971693
## 14       28.30144864 -0.050762933 -0.3640893678
## 23       23.57054539 -0.058798029 -0.4209703548
## 54       26.91830145  0.033580509  0.1577806102
## 52      -67.67100387  0.156616739  0.4940278079
## 2        44.47128270 -0.209341310 -0.9483672770
## 97       17.01134313  0.069592204  0.3278389153
## 18      -55.19409047  0.061389300  0.0655657506
## 92       23.10212166  0.048830737  0.2241375045
## 19       33.70270698 -0.035393685 -0.2760915941
## 114      43.00613990 -0.008033666 -0.0770411406
## 101      30.33452595  0.038548498  0.1727546169
```

```
## 25        45.54789055 -0.018179758 -0.1408982465
## 45        32.77531900  0.038716509  0.1750978762
## 43       -40.28076273  0.026043041 -0.0280316648
## 36       -33.98660907  0.060869745  0.0494886303
## 90        44.54633812 -0.000440037 -0.0441009923
## 46        42.83279728 -0.084869076 -0.7062110233
## 61        14.00756590 -0.090460297 -0.4748284428
## 17       -11.76237532  0.009126989 -0.1071192445
## 118       22.77487750 -0.053052020 -0.1488776381
## 60       -56.58692157  0.045605649  0.1471060589
## 113      -53.68883477  0.103790475  0.2265017561
## 47        34.02670528 -0.175937613 -0.4801129254
## 103       83.84098477 -0.168192527 -0.2269882642
## 30        14.91681887 -0.087104238 -0.2207282512
## 94        10.29576932 -0.089407526 -0.4773829926
## 50         5.43918603 -0.142457607 -0.7240889652
## 32        13.70590016 -0.016953019 -0.2344331770
## 129      -27.91761112  0.051600105  0.1418003821
## 51       -28.64853320 -0.064068421 -0.3678230268
## 31        47.11181097 -0.128511634 -0.2860011019
## 3        -24.57322697  0.041871051 -0.0764802187
## 1        -42.03358631  0.084097628  0.3223155713
## 109      -38.20639930 -0.010169173  0.2267800448
## 21       -29.86633252  0.041567161  0.0007231562
## 59       -29.73986759 -0.020973920 -0.2607238433
## 116      -33.50107675  0.030802150  0.2419071911
## 48        -5.75106398 -0.009251272  0.0695532693
## 38       -23.61608258 -0.013056306 -0.5448368653
## 93        -6.70140794  0.048240332  0.1395897464
## 41        75.40449262 -0.135224941 -0.3384182792
## 20        44.15542456 -0.121628143 -0.5955802976
## 105       -0.05855515 -0.062715513 -0.6704130066
## 5         49.66613827 -0.066426352 -0.0413641175
## 4         31.54159520 -0.167193820 -0.4652027309
## 128       76.73685725 -0.157183462 -0.1150984113
## 42         4.52340652  0.022975477 -0.1585001555
## 49        71.10611590 -0.218038213 -0.5207903342
## 44        33.96849902 -0.055491259 -0.2307004581
## 22        12.53292218  0.034015664  0.1034590390
## 13        70.69356966 -0.164634298 -0.7143878331
## 12         1.89260155 -0.005293252  0.0090578443
## 9         42.72486870 -0.039128366 -0.1805154458
## 40        70.96693576 -0.067443468 -0.1218289491
## 26        44.95539192 -0.146919319  0.0095934612
## 132      -23.69573821  0.080724696  0.3034814946
## 37        83.52707432 -0.156963302 -0.5355736071
## 11        44.88516133 -0.024929947 -0.2639532188
## 10       106.70574808 -0.228907441 -0.4929939169
## 24        44.28339194 -0.049721817  0.2503327326
```

```
coef(sexmod.l.int)[1,3]
```

```
## [1] 0.6351283
```

Comparing Models

```r
AIC(nlme.mod2,sexmod.lt,sexmod.kt,sexmod.l,sexmod.k,sexmod.t,sexmod.l.int)
```

```
## Warning in AIC.default(nlme.mod2, sexmod.lt, sexmod.kt, sexmod.l,
## sexmod.k, : models are not all fitted to the same number of observations
```

```
##              df      AIC
## nlme.mod2    10 2825.199
## sexmod.lt    12 2827.788
## sexmod.kt    12 2837.204
## sexmod.l     11 2809.756
## sexmod.k     11 2824.301
## sexmod.t     11 2829.548
## sexmod.l.int 11 2809.756
```

```r
BIC(nlme.mod2,sexmod.lt,sexmod.kt,sexmod.l,sexmod.k,sexmod.t,sexmod.l.int)
```

```
## Warning in BIC.default(nlme.mod2, sexmod.lt, sexmod.kt, sexmod.l,
## sexmod.k, : models are not all fitted to the same number of observations
```

```
##              df      BIC
## nlme.mod2    10 2863.280
## sexmod.lt    12 2873.413
## sexmod.kt    12 2882.830
## sexmod.l     11 2851.613
## sexmod.k     11 2866.158
## sexmod.t     11 2871.404
## sexmod.l.int 11 2851.613
```

```r
anova(nlme.mod2,sexmod.lt,sexmod.kt,sexmod.l,sexmod.k,sexmod.t,sexmod.l.int)
```

```
##              Model df      AIC      BIC    logLik   Test   L.Ratio p-value
## nlme.mod2        1 10 2825.199 2863.280 -1402.599
## sexmod.lt        2 12 2827.788 2873.413 -1401.894 1 vs 2  1.410824  0.4939
## sexmod.kt        3 12 2837.204 2882.830 -1406.602
## sexmod.l         4 11 2809.756 2851.613 -1393.878 3 vs 4 25.448099  <.0001
## sexmod.k         5 11 2824.301 2866.157 -1401.150
## sexmod.t         6 11 2829.548 2871.404 -1403.774
## sexmod.l.int     7 11 2809.756 2851.613 -1393.878
```

```r
anova(nlme.mod2,sexmod.l,sexmod.l.int)
```

```
##              Model df      AIC      BIC    logLik   Test L.Ratio p-value
## nlme.mod2        1 10 2825.199 2863.280 -1402.599
## sexmod.l         2 11 2809.756 2851.613 -1393.878 1 vs 2 17.4423  <.0001
## sexmod.l.int     3 11 2809.756 2851.613 -1393.878
```

```r
anova(sexmod.l,nlme.mod2,sexmod.l.int)
```

```
##              Model df      AIC      BIC    logLik   Test  L.Ratio p-value
## sexmod.l         1 11 2809.756 2851.613 -1393.878
## nlme.mod2        2 10 2825.199 2863.280 -1402.599 1 vs 2 17.44230  <.0001
## sexmod.l.int     3 11 2809.756 2851.613 -1393.878 2 vs 3 17.44229  <.0001
```

```r
anova(sexmod.l,sexmod.k,nlme.mod2)
```

```
##          Model df      AIC      BIC    logLik   Test L.Ratio p-value
## sexmod.l     1 11 2809.756 2851.613 -1393.878
```

```
## sexmod.k      2 11 2824.301 2866.157 -1401.150
## nlme.mod2     3 10 2825.199 2863.280 -1402.599 2 vs 3 2.89759  0.0887
```

```r
anova(sexmod.l,nlme.mod2)
```

```
##           Model df      AIC      BIC    logLik   Test L.Ratio p-value
## sexmod.l      1 11 2809.756 2851.613 -1393.878
## nlme.mod2     2 10 2825.199 2863.280 -1402.599 1 vs 2 17.4423  <.0001
```

```r
anova(nlme.mod2,sexmod.l)
```

```
##           Model df      AIC      BIC    logLik   Test L.Ratio p-value
## nlme.mod2     1 10 2825.199 2863.280 -1402.599
## sexmod.l      2 11 2809.756 2851.613 -1393.878 1 vs 2 17.4423  <.0001
```

```r
anova(sexmod.l,sexmod.l.int)
```

```
##              Model df      AIC      BIC    logLik
## sexmod.l         1 11 2809.756 2851.613 -1393.878
## sexmod.l.int     2 11 2809.756 2851.613 -1393.878
```

```r
anova(sexmod.l.int,sexmod.l)
```

```
##              Model df      AIC      BIC    logLik
## sexmod.l.int     1 11 2809.756 2851.613 -1393.878
## sexmod.l         2 11 2809.756 2851.613 -1393.878
```

See sexmod.l

There is a significant difference between the nlme.mod2 and sexmod.l with sexmod.l being the model that best describes the data (loglikliehood,df=11, L Ratio = 17.4423, $p<0.0001$).