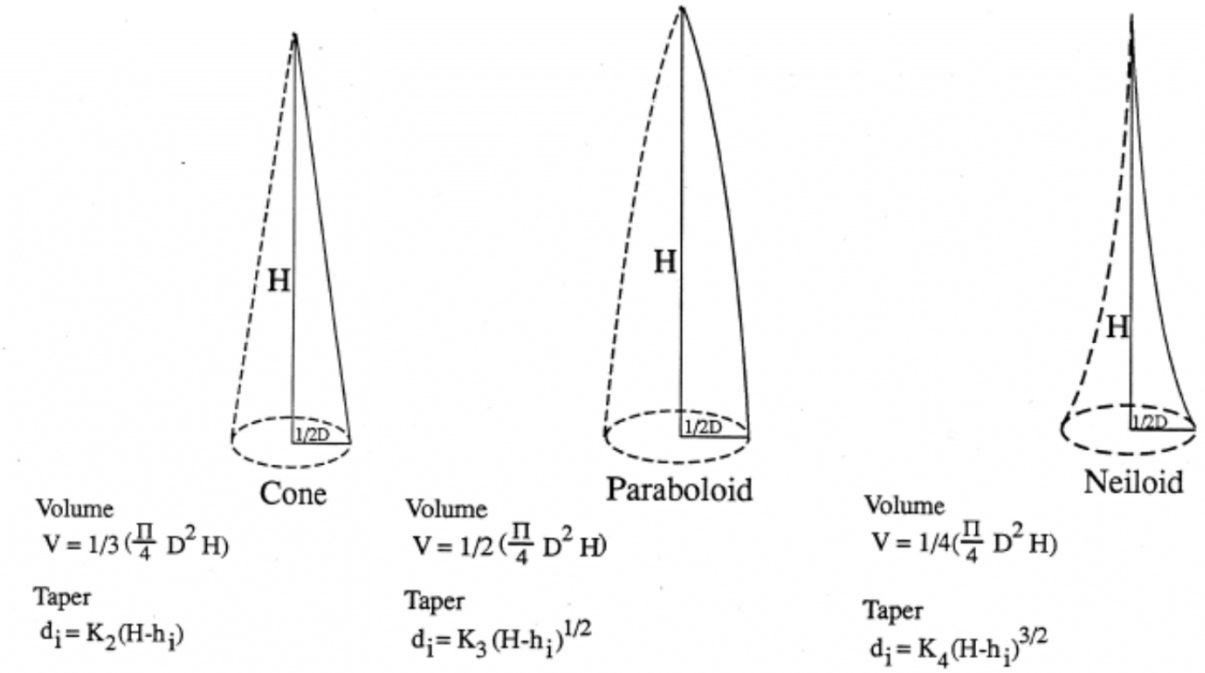
Note: Tree trunk described by 3 sections:

(1) top = cone

(2) middle = paraboloid

(3) base = neiloid

Examples of these shapes:

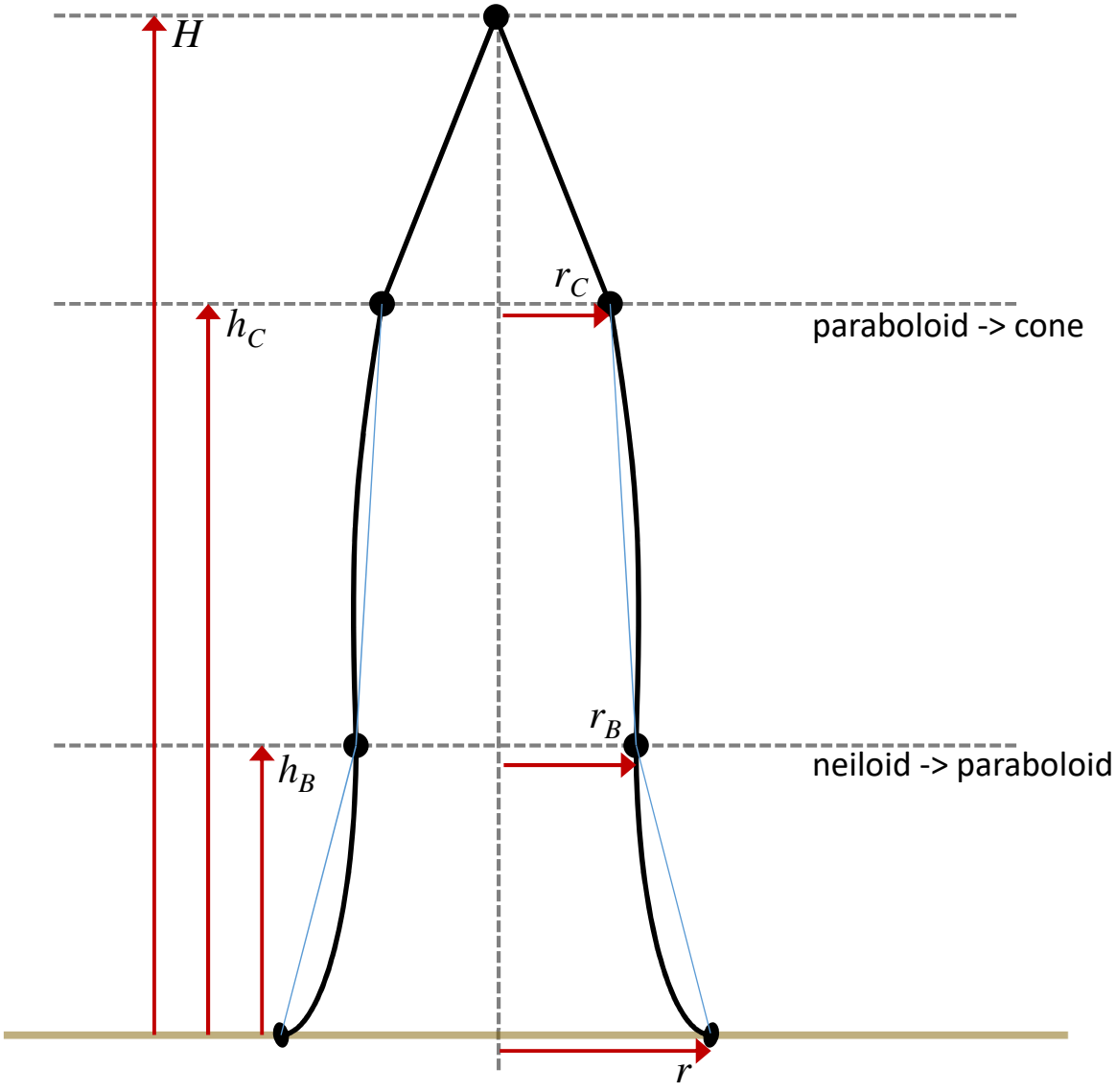


Geometry of trunk only

Quantities needed to "draw" trunk:

Variable names from original paper and supplemental materials:

H (tree height)
h_C (height that trunk transitions from paraboloid to cone)
h_B (height that trunk transitions from neiloid to paraboloid)
r (radius of trunk at base)
r_B (radius of trunk when transitions from neiloid to paraboloid)
r_C (radius of trunk when transitions from paraboloid to cone)



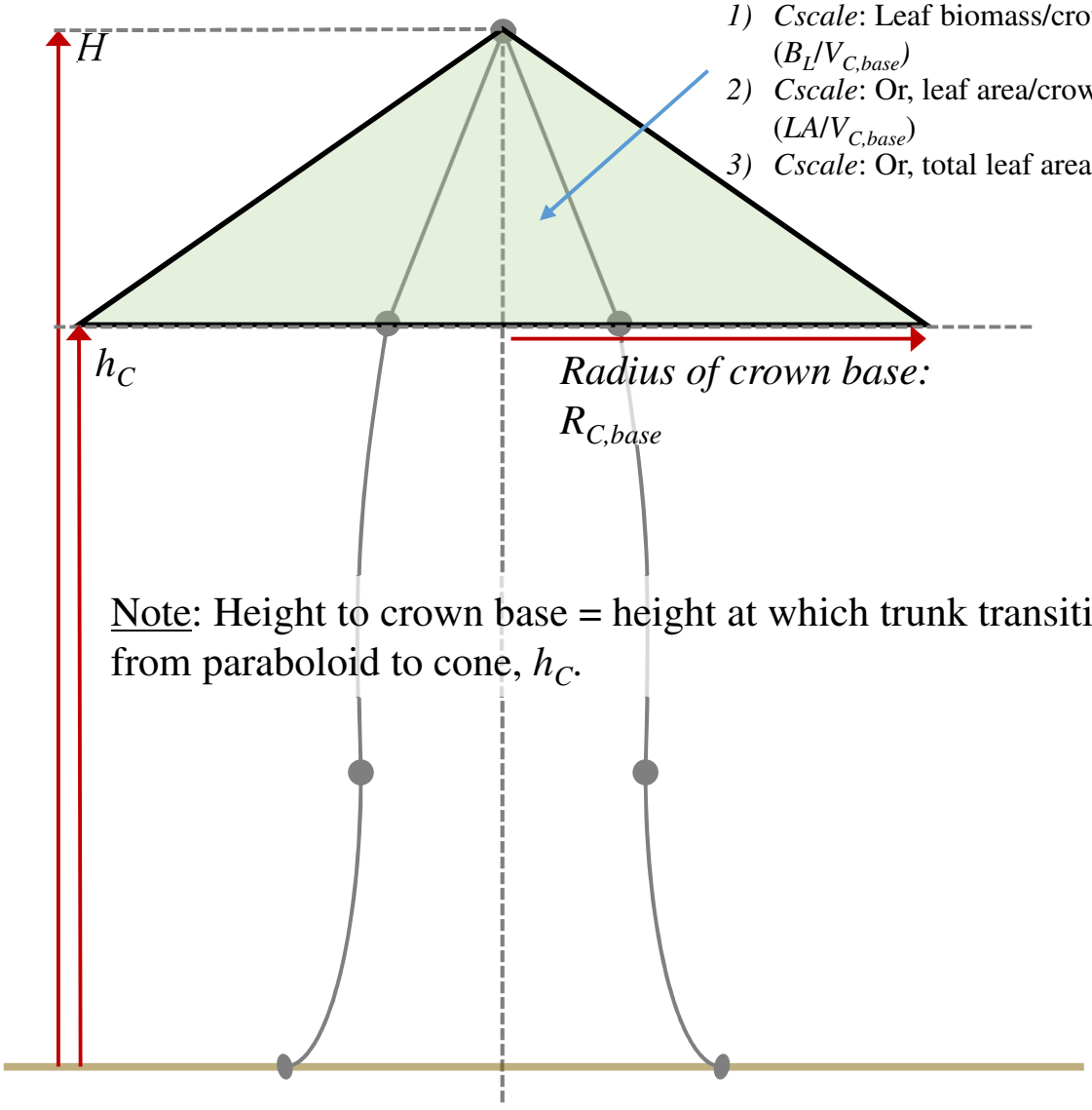
Crown overlaid on trunk

Quantities needed to "draw" crown:

Variable names from original paper and supplemental materials:

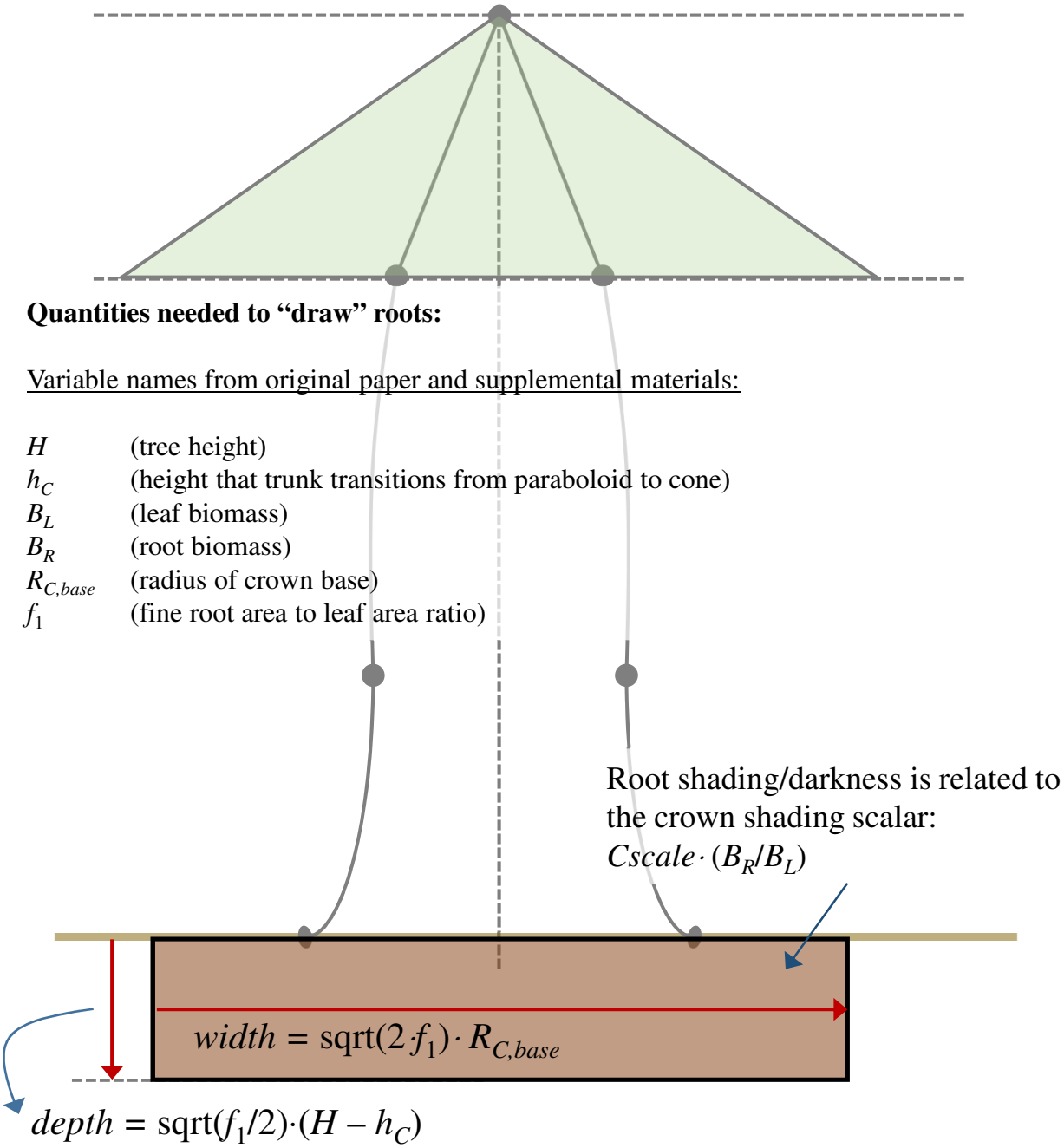
H (tree height)
h_C (height that trunk transitions from paraboloid to cone)
B_L (leaf biomass)
$V_{C,base}$ (total crown volume)
LA (total leaf area)
LAI_{tot} (total leaf area index, i.e., LAI of entire crown)
$R_{C,base}$ (radius of crown base)

Ideas for crown shading/darkness, scaled by:
1) *Cscale*: Leaf biomass/crown volume ($B_L/V_{C,base}$)
2) *Cscale*: Or, leaf area/crown volume ($LA/V_{C,base}$)
3) *Cscale*: Or, total leaf area index (LAI_{tot})



Note: Height to crown base = height at which trunk transition from paraboloid to cone, h_C.

Roots overlaid with trunk and crown



Quantities needed to "draw" roots:

Variable names from original paper and supplemental materials:

H	(tree height)
h_C	(height that trunk transitions from paraboloid to cone)
B_L	(leaf biomass)
B_R	(root biomass)
$R_{C,base}$	(radius of crown base)
f_1	(fine root area to leaf area ratio)

Simplifying assumptions for roots:

- Represent with a rectangle? If so,
- Solve for *width* and *depth* of rectangle by assuming:
  - root *width/depth* ratio = crown *diameter/depth* ratio
  - Area of root rectangle/area of crown triangle = f_1 (root area:crown area)

Where to find quantities?

Symbol	Definition	Input/output	Input/output name*
H	tree height	Output	h
h_C	height that trunk transitions from paraboloid to cone	Output	hC
h_B	height that trunk transitions from neiloid to paraboloid	Output	hB
r	radius of trunk at base	Output	r
r_B	radius of trunk when transitions from neiloid to paraboloid	Output	rB
r_C	radius of trunk when transitions from paraboloid to cone	Output	rC
B_L	leaf biomass	Output	bl
B_R	root biomass	Output	br
$V_{C,base}$	total crown volume	Output	not included**
LA	total leaf area	Output	la
LAI_{tot}	total leaf area index, i.e., LAI of entire crown	Output	LAI
$R_{C,base}$	radius of crown base	Output	not included**
f_1	fine root area to leaf area ratio	Input	f1 or p.f1
*when using fulloutput = TRUE in R			
**see next page for how to compute			

Info for computing $V_{C,base}$ and $R_{C,base}$

The equations below are taken from the supplement of Ogle and Pacala (2009). The following table indicates if the variables can be found in the input or output of the ACGCA model.

Symbol	Where to find	R	C
H_{max}	Input	p.hmax	p.hmax
ϕ_H	Input	p.phih	p.phih
η	Input	p.eta	p.eta
m	Input	p.M	p.M
α	Input	p.alpha	p.alpha
R_0	Input	p.R0	p.R0
R_{40}	Input	p.R40	p.R40
R_{BH}	Output	rBH	
H	Output	h	
BH	Constant (1.37 m)		

$R_{C,max}$ is the maximum potential radius at a crown ratio (crown depth/tree height) of m, and α describes the curvature of the crown. The actual crown ratio of the tree is given by $(1 - \eta)$; when $m > (1 - \eta)$, then the radius at the base of the crown is:

$$R_{C,base} = R_{C,max} \cdot \left(\frac{1-\eta}{m} \right)^{\alpha} \tag{22}$$

Modifying the model of Purves et al. (2007a), we describe $R_{C,max}$ as a function of trunk radius at breast height (r_{BH}), where BH is breast height (1.37 m):

$$R_{C,max} = \begin{cases} R_0 + (R_{40} - R_0) \cdot \dfrac{2 \cdot r_{BH} \cdot 100}{40} & H > BH \\ \dfrac{R_0 \cdot r}{\dfrac{H_{max}}{\phi} \cdot \ln\left(\dfrac{H_{max}}{H_{max} - BH} \right)} & H < BH \end{cases} \tag{23}$$

The crown volume between the top of the tree and distance z from the top is:

$$V_C(z) = \pi \cdot R_{C,max}{}^2 \cdot \frac{z}{1+2\cdot\alpha} \cdot \left(\frac{z}{H \cdot m} \right)^{2\alpha} \tag{25}$$

for $z \leq (1 - \eta) \cdot H$. The total volume of the crown ($V_{C,base}$) is given by evaluating Eqn 25 at $z = (1 - \eta) \cdot H$.