Spatial Confidence Sets

For Standardized Effect Size Images

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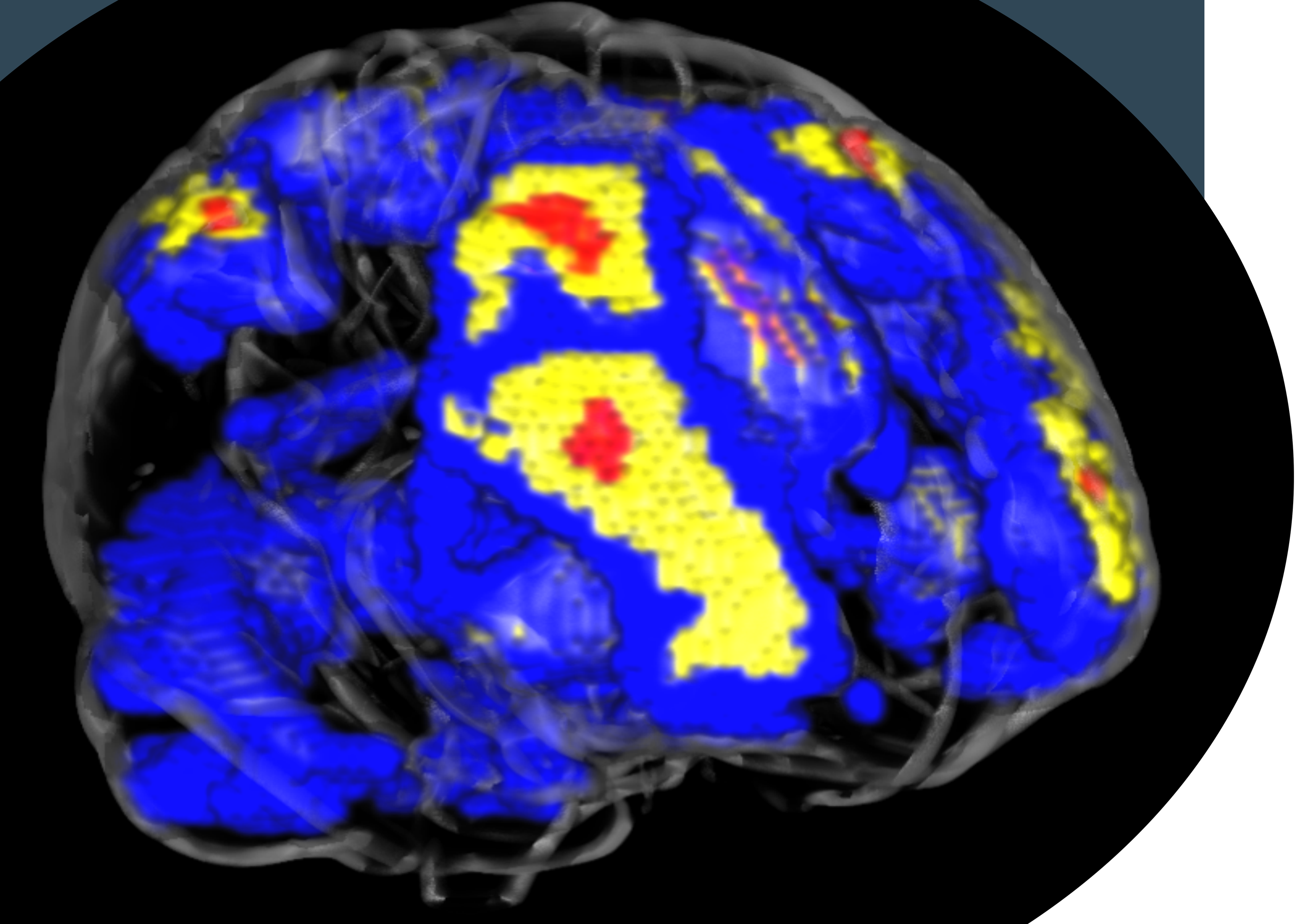
Want to know more about the Confidence Sets?

See our bioRxiv preprint for full details on the methods presented here, and our NeuroImage article, where we developed the CSs for raw %BOLD change effect size images!

What are Confidence Sets (CSs)?

Traditionally, statistical inference is used in task-fMRI to identify brain regions where the null hypothesis, i.e. an effect size of zero, can be rejected. Confidence Sets (CSs), on the other hand, are 3D analogs of confidence intervals. For Cohen's *d* effect size maps, the CSs combine information about the magnitude and reliability of effects to identify areas of the brain where we can assert standardized effect sizes have exceeded, and fallen short of, a *non-zero* threshold *c*. This approach facilitates a more direct style of inference on the true biological effects of interest in a task-fMRI study.



Interpreting the CSs for a c = 0.5 Cohen's *d* threshold

For a 95% confidence level, the CSs comprise of two sets of voxels:

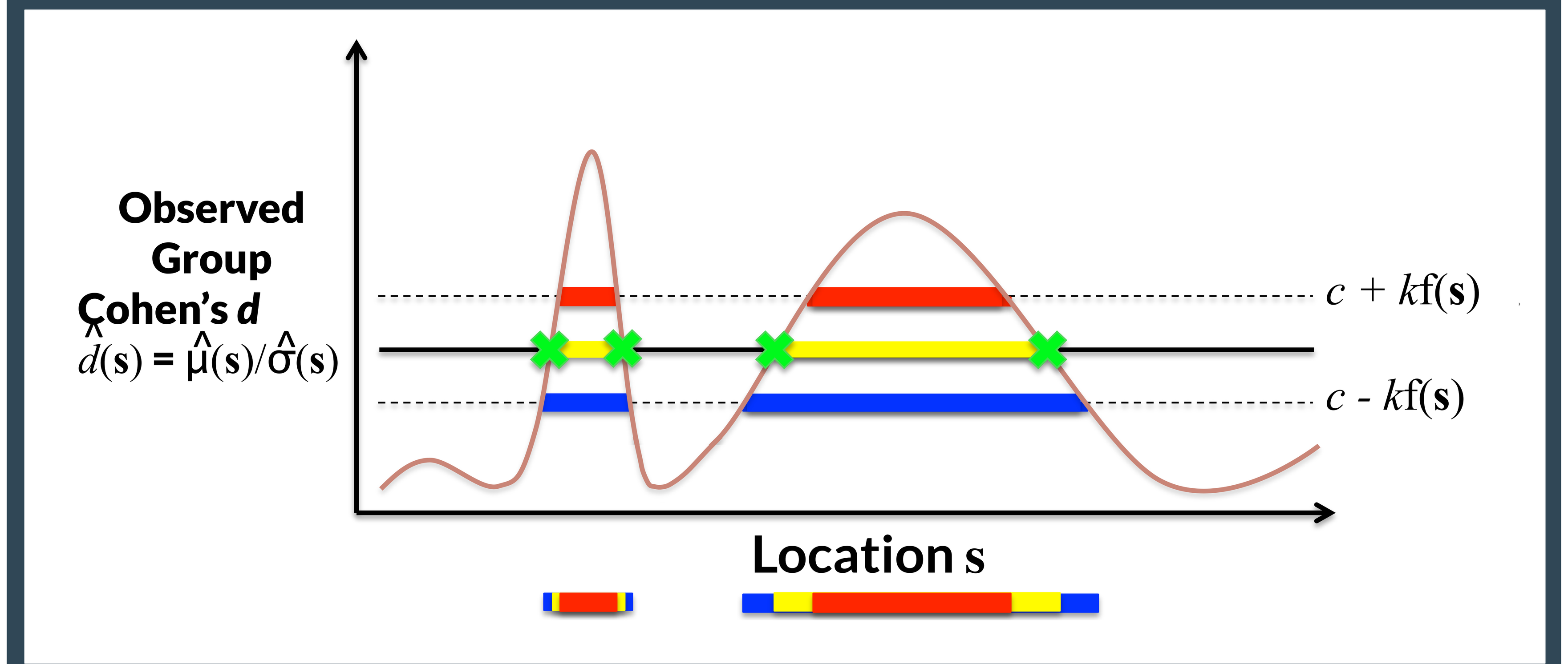
All RED voxels lie in the upper CS. We have 95% confidence over the whole brain that all red voxels have a true Cohen's *d* effect size of at least 0.5.

All BLUE voxels (overlapped by yellow and red) lie in the lower CS. We have 95% confidence over the whole brain that all voxels outside the blue (i.e. background voxels) have a true Cohen's *d* effect size of less than 0.5.

The confidence statement holds simultaneously for both CSs.

All YELLOW voxels (overlapped by red) lie in the point estimate set, obtained by thresholding the group-level Cohen's *d* effect size map at 0.5. This is the 'best guess' from the data of all voxels that have a Cohen's *d* effect size of at least 0.5.

Methods



A 1D Intuition of how to compute the CSs

The image above provides a demonstration of how the CSs are computed for a realization of the signal-plus-noise model:

$$Y_i(s) = \mu(s) + \varepsilon_i(s) \qquad i = 1, \ldots, N$$

across a one-dimensional spatial domain. For each location s, the $Y_i(s)$ are the subject-level observations, $\mu(s)$ is the true underlying mean %BOLD signal change across the observations, and the $\varepsilon_i(s)$ are i.i.d. mean-zero errors with common variance $\sigma^2(s)$ and some unspecified spatial correlation. In this setting, the *true* Cohen's *d* effect size for the group is given by $d(s) = \mu(s)/\sigma(s)$.

The yellow voxels are obtained by thresholding the observed group Cohen's *d* map at the threshold *c*; this is the best guess from the data of the set of voxels whose true Cohen's *d* effect size is greater than c. The red upper CS and blue lower CS are computed by thresholding the Cohen's *d* map at $c + k\hat{f}(s)$ and $c - k\hat{f}(s)$ respectively, where $\hat{f}(s)$ is a function of the standard deviation $\hat{\sigma}(s)$, and k is a critical value determined as the 95th percentile of the maximum distribution of the absolute process of specialized Cohen's *d* residuals over the estimated boundary of where $d(s) = c$ (green X's in the figure), obtained using a Wild *t*-Bootstrap schema.

From this procedure, we have 95% confidence that all red voxels have a true Cohen's *d* effect size greater than *c*, and simultaneously, all voxels outside the blue have a true Cohen's *d* effect size less than c.

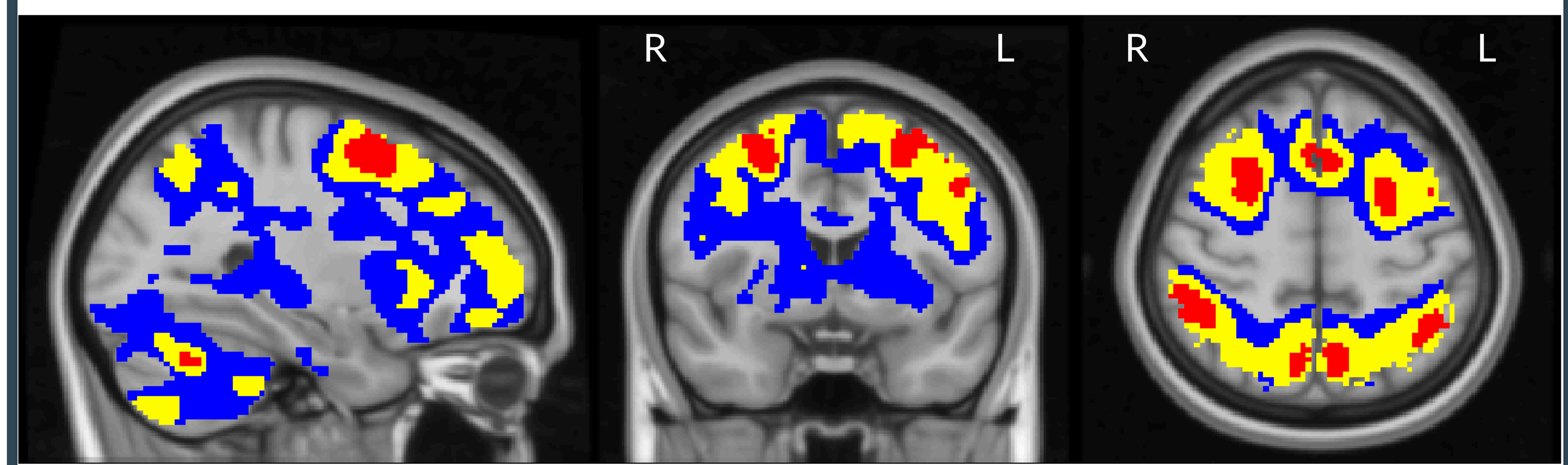
Conclusion

We have presented a method of Confidence Sets to obtain simultaneous confidence statements on brain areas where standardized effect sizes exceed (and fall short of) a non-zero value. As the availability of population neuroimaging studies with arbitrarily large power increases, tools like this for interpreting all significant effects are becoming more important.

Results

1.

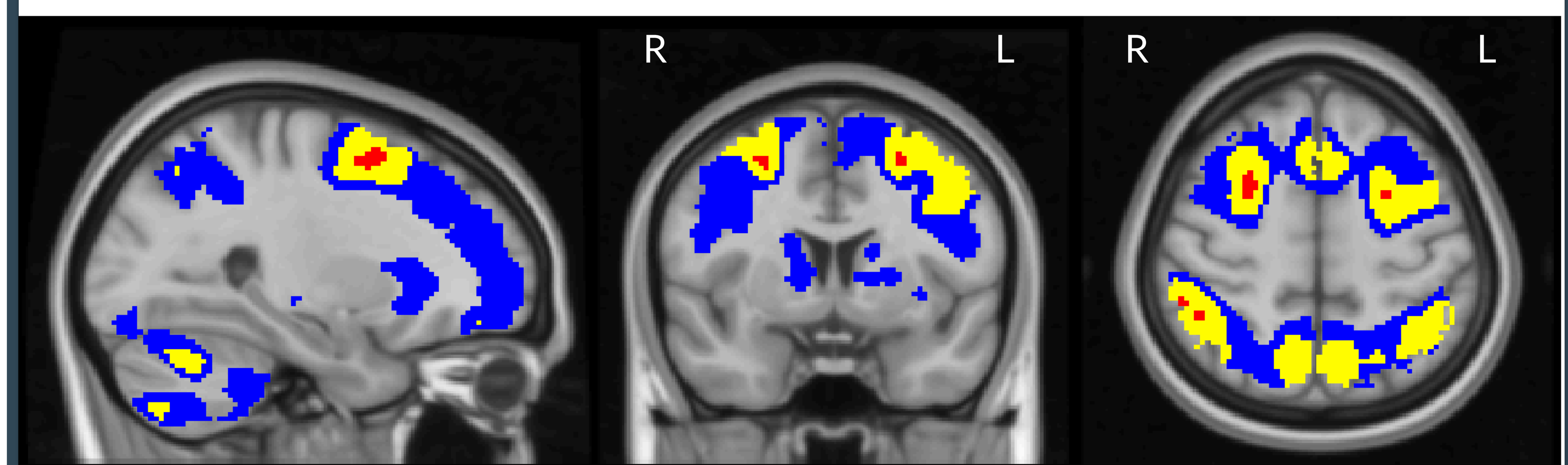
HCP: 2-back vs 0-back contrast
N = 80, *c* = 0.5 Cohen's *d*



CSs computed on the '2-back vs 0-back' subject-level contrast maps taken from the HCP Unrelated 80 working memory task dataset, using a *c* = 0.5 Cohen's *d* threshold. We are 95% confident that all voxels in the red upper CS have a Cohen's *d* effect size of at least 0.5, and simultaneously, all voxels outside the blue lower CS (i.e. grey, background voxels) have a Cohen's *d* effect size less than 0.5. The yellow region (overlapped by red) is the point estimate set, the best guess from the data of where the Cohen's *d* effect size is larger than 0.5. The red upper CS has localized regions in the frontal gyrus, paracingulate gyrus, cerebellum and precuneus, which we can assert with 95% confidence have attained at least a 0.5 Cohen's *d* effect size.

2.

HCP: 2-back vs 0-back contrast
N = 80, *c* = 0.8 Cohen's *d*



To demonstrate the role of the threshold, we computed CSs on the same 80 subject's HCP data as in 1. , but this time using a larger threshold of *c* = 0.8 Cohen's *d*. By increasing the threshold, the red upper CSs and blue lower CSs have both shrunk: there are less voxels we can confidently declare have an effect size greater than 0.8 (compared to 0.5), and at them same time, there are more voxels we can confidently assert have an effect size less then 0.8. Comparing to 1., it is notable that the red upper CS has vanished in the right cerebellar and insular cortex here; although we are 95% confident some voxels in these two regions have an effect size greater than 0.5, there are no voxels we can assert have an effect size greater than 0.8.