# Homework 1

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## 1 Hypothesis testing, the t-distribution

### 1.1

$$s^2_{\text{pooled}} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and if $n_1 = n_2$ and $s_1 = s_2$:

$$s_{\text{pooled}} = \frac{1}{2}\left(\frac{1}{n-1}\right)\left(\sum_{i=1}^{n}(x_i - \bar{x})^2 + (y_i - \bar{y})^2\right)$$

### 1.2

$$\begin{aligned}
\mathbb{E}[s^2_{\text{pooled}}] &= \frac{1}{2}\left(\frac{1}{n-1}\right)\mathbb{E}\left[\left(\sum_{i=1}^{n}(x_i - \bar{x})^2 + (y_i - \bar{y})^2\right)\right] \\
&= \frac{1}{2}\left(\frac{1}{n-1}\right)\left[\sum_{i=1}^{n}\left(\mathbb{E}\left[(x_i - \bar{x})^2\right] + \mathbb{E}\left[(y_i - \bar{y})^2\right]\right)\right] \\
&= \frac{\mathbb{V}[X] + \mathbb{V}[Y]}{2(n-1)} \\
&= \frac{(n-1)\sigma^2 + (n-1)\sigma^2}{2(n-1)} \\
\mathbb{E}[s^2_{\text{pooled}}] &= \sigma^2
\end{aligned}$$

## 2 Questions from Freedman

## 3 Questions from Broad Street