The future of gravitational theories in the era of the gravitational wave astronomy

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Abstract

We discuss the future of gravitational theories in the framework of gravitational wave (GW) astronomy after the recent GW detections (the events GW150914, GW151226, GW170104, GW170814, GW170817 and GW170608). In particular, a calculation of the frequency and angular dependent response function that a GW detector would see if massive modes from $f(R)$ theories or scalar tensor gravity (STG) were present, allowing for sources incident from any direction on the sky, is shown. In addition, through separate theoretical results which do not involve the recent GW detections, we show that $f(R)$ theories of gravity having a third massless mode are ultimately ruled out while there is still room for STG having a third (massive or massless) mode and for $f(R)$ theories of gravity having a third massive mode.

Keywords: Gravitational theories; gravitational waves; gravitational wave astronomy; interferometer response function.

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To the memory of Ron Drever and Adalberto Giazotto

1 Introduction

The discovery of GW emissions from the compact binary system with two neutron stars (NS) PSR1913+16 [1] excited interest in GWs although the first efforts at direct detection started before that, by involving the design, implementation, and advancement of extremely sophisticated GW detection technology (see the recent review [30] for the history of GW research). Physicists working in this field of research need this technology to conduct thorough investigations of GWs in order to advance science. The main motivation for searching for GWs is to use them as a probe of the systems that produce them. The first observation of GWs from a binary black hole (BH) merger (event GW150914) [2], which occurred in the 100th anniversary of Albert Einstein's prediction of GWs [3], has recently shown that this ambitious challenge has been won. The event GW150914 represented a cornerstone for science and for gravitational physics in particular. In fact, this remarkable event equipped scientists with the means to give definitive proof of the existence of GWs, the existence of BHs having mass greater than 25 solar masses and the existence of binary systems of BHs which coalesce in a time less than the age of the Universe [2]. As a consequence of the event GW150914, the Nobel Prize in Physics 2017 has been awarded to Rainer Weiss, Barry Barish and Kip Thorne.

A subsequent analysis of GW150914 constrained the graviton Compton wavelength of those alternative gravity theories (AGTs) in which the graviton is massive and placed a lower bound of 10^{13} km, corresponding to a graviton mass $m_g \leq 1.2 \times 10^{-22} \frac{eV}{c^2}$ [4]. Within their statistical uncertainties, the LIGO Scientific Collaboration and the Virgo Collaboration have not found evidence of violations of the general theory of relativity (GTR) in the genuinely strong-field regime of gravity [4]. After the event GW150914, the LIGO Scientific Collaboration and the Virgo Collaboration announced other five new GW detections, the events GW151226 [33], GW170104 [34], GW170814 [35], GW170817 [36] and GW170608 [37]. All the cited events again arise from binary BH coalescences with the sole exception ofthe event GW170817, which represent the first GW detection from a NS merger [36]. After such events, the bound on the graviton mass is even more compelling: $m_g \leq 7.7 \times 10^{-23} \frac{eV}{c^2}$ [34]. Notice that this does not mean that the analyses in [4, 34] show that the true theory of gravity is massless. In fact, the analyses in [4, 34] have not shown that the graviton mass is zero, just that it is small. One expects that LIGO and the other GW interferometers will never show that the mass is exactly 0, they can only place increasingly precise bounds on its value. On the other hand, the possibility that AGTs are still alive after the event GW150914 has been emphasized in [5]. In fact, in [6] two important questions have been raised, verbatim: "*Does gravity really behave as predicted by Einstein in the vicinity of black holes, where the fields are very strong? Can dark energy and the acceleration of the Universe be explained if we modify Einstein's gravity?*" The current situation is that "*We are only just beginning to answer these questions*" [6].

Among the various kinds of AGTs, $f(R)$ theories and STG seem to be the most popular among gravitational physicists because they could be, in principle, important in order to solve some problem of standard cosmology like the Dark Matter and Dark Energy problems [7, 8, 9, 12]. These theories attempt to extend the framework of the GTR by modifying the Lagrangian, with respect to the standard Einstein-Hilbert gravitational Lagrangian, through the addition of high-order terms in the curvature invariants (terms like R^2, $R^{ab}R_{ab}$, $R^{abcd}R_{abcd}$, $R\Box R$, $R\Box^k R$) and/or terms with scalar fields non-minimally coupled to geometry (terms like $\phi^2 R$) [7, 8, 9, 12, 18]. In this paper we will focus on these two classes of AGTs. Criticisms on such theories arises from the fact that lots of them can be excluded by requirements of cosmology and solar system tests [9, 11, 15, 24]. Thus, one needs the additional assumption that the variation from the standard GTR must be weak [12].

For the sake of completeness, as the number of predictions of AGTs which are highlighted in this paper cannot be tests using the recent GW detections, it could be useful for the reader to know some ways in which those predictions might be, in principle, tested in the future. Following [9], one sees that strong gravity tests are considered, together with GWs and stellar system tests, the fundamental gravitational tests for the 21st Century. Those systems for which the simple first order post-Newtonian approximation (PNA) is no longer appropriate are called *strong-field systems* (SFSs) [9]. Usually, SFSs contain strongly relativistic objects, such as NS or BHs, where the first order PNA breaks down [9]. The key point is that in AGTs the strong-field internal gravity of the bodies should leave imprints on the orbital motion of the objects [9]. SFSs are also connected with GWs, because GWs can affect the evolution of the SFS. In fact, as the first order PNA does not contain the effects of the GW back-reaction, we need a solution of the equations substantially beyond the first order PNA [9]. Concerning stellar system tests, the discovery of the binary pulsar B1913+16 [1] had importance in the gravitational physics also beyond GWs. Another key point is indeed the effects of strong relativistic internal gravitational fields on orbital dynamics [9]. Gravitational theory is today tested with pulsars, including binary and millisecond pulsars [9]. Assuming that both members of the system are NS, the formulas for the periastron shift, the gravitational redshift/second-order Doppler shift parameter, the Shapiro delay coefficients, and the rate of change of orbital period can be obtained [9]. On one hand, the near equality of NS masses in typical double NS binary pulsars makes bounds obtained not competitive with the Cassini bound [24] because dipole radiation is somewhat suppressed, see [9] and Section 2 of this paper. On the other hand, more promising tests of dipole radiation arise from a binary pulsar system having dissimilar objects, such as a white dwarf (WD) or BH companion [9]. An important example is the NS–WD system J1738+0333, which yields much more stringent bounds, surpassing the Cassini bound [9]. In years to come, the experiments that have been cited will be further improved and perfected in order to search for new physics beyond Einstein's GTR.

The main new results of this paper will be shown in next Section and are the following:

- We perform a calculation of the frequency and angular dependent response function that a GW detector would see if massive modes were present, allowing for sources incident from any direction on the sky. This will permit, in principle, to discriminate between massless and massive modes in $f(R)$ theories and STG, while such a discrimination was not possible in previous GW literature.

- Through separate theoretical results which do not involve the recent GW detections, we show that $f(R)$ theories of gravity having a third massless mode are ultimately ruled out while there is still room for STG having a third (massive or massless) mode and for $f(R)$ theories of gravity having a third massive mode. This issue has an important consequence on the debate on the equivalence or non-equivalence between $f(R)$ theories an STG [7, 9, 12, 31].

2 Viability of $f(R)$ theories and scalar tensor gravity through gravitational waves

We emphasize that in discussing dipole and monopole radiation we closely follow the papers [14, 19, 20].

In the framework of GWs, the more important difference between the GTR and the cited two classes of AGTs ($f(R)$ theories and STG) is the existence, in the latter, of dipole and monopole radiation [14, 19]. In the GTR, for slowly moving systems, the leading multipole contribution to gravitational radiation is the quadrupole one, with the result that the dominant radiation-reaction effects are at order $(\frac{v}{c})^5$, where v is the orbital velocity. The rate, due to quadrupole radiation in the GTR, at which a binary system loses energy is given by (we work with $16\pi G = 1$, $c = 1$ and $\hbar = 1$ in the following) [14, 19]

$$\left(\frac{dE}{dt}\right)_{quadrupole} = -\frac{8}{15}\eta^2\frac{m^4}{r^4}(12v^2 - 11\dot{r}^2). \tag{1}$$

η and m are the reduced mass parameter and total mass, respectively, given by $\eta = \frac{m_1 m_2}{(m_1+m_2)^2}$, and $m = m_1 + m_2$. r, v, and $\dot{r}$ represent the orbital separation, relative orbital velocity, and radial velocity, respectively.

In $f(R)$ theories and STG, eq. (1) is modified by PN corrections to monopole and dipole radiation, and even a cross-term between dipole and octupole radiation as [9]

$$\frac{dE}{dt} = -\frac{8}{15}\alpha^3\eta^2\left(\frac{m}{r}\right)^4(k_1 v^2 - k_2\dot{r}^2), \tag{2}$$

where α is a two-body gravitational interaction parameter [9], and the parameters $k_1$and k_2 have been calculated in [25].

The important modification in $f(R)$ theories and STG is the additional energy loss caused by dipole modes. By analogy with electrodynamics, dipole radiation is a $(v/c)^3$ effect, potentially much stronger than quadrupole radiation. However, in $f(R)$ theories and STG, the gravitational "*dipole moment*" is governed by the difference $S \equiv s_1 - s_2$ between the bodies, where s_i is a measure of the self-gravitational binding energy per unit rest mass of each body [14, 19]. s_i represents the "*sensitivity*" of the total mass of the body to variations in the background value of the Newton constant, which, in this theory, is a function of the scalar field (an "effective"scalar field in the case of $f(R)$ theories [11]) [14, 19]:

$$s_i = \left(\frac{\partial(\ln m_i)}{\partial(\ln G)} \right)_N . \tag{3}$$

G is the effective Newtonian constant at the star and the subscript N denotes holding baryon number fixed.

To first order in $\frac{1}{w}$, where w is the coupling parameter of the scalar field [19], the energy loss caused by dipole radiation is given by [14, 19]

$$\left(\frac{dE}{dt} \right)_{dipole} = -\frac{2}{3}\eta^2 \frac{m^4}{r^4} S^2. \tag{4}$$

In $f(R)$ theories and STG, the sensitivity of a BH is always $s_{BH} = 0.5$ [14, 19], while the sensitivity of a NS varies with the equation of state and mass. For example, $s_{NS} \approx 0.12$ for a NS of mass order $1.4M_{\odot}$, being $M_{\odot}$ the solar mass [14, 19].

Binary BH systems are not at all promising for studying dipole modes because $s_{BH1} - s_{BH2} = 0$, a consequence of the no-hair theorems for BHs [14, 19]. BHs indeed radiate away any scalar field, so that a binary BH system in $f(R)$ theories and STG behaves as in the GTR. Similarly, binary NS systems are also not effective testing grounds for dipole radiation [14, 19]. This is because NS masses tend to cluster around the Chandrasekhar limit of $1.4M_{\odot}$, and the sensitivity of NSs is not a strong function of mass for a given equation of state. Thus, in systems like the binary pulsar, dipole radiation is naturally suppressed by symmetry, and the bound achievable cannot compete with those from the solar system [14, 19]. Hence the most promising systems are mixed: BH-NS, BH-WD or NS-WD.

The emission of monopole radiation in $f(R)$ theories and STG is very important in the collapse of quasi-spherical astrophysical objects because in this case the energy emitted by quadrupole modes can be neglected [14, 20]. The authors of [20] have shown that, in the formation of a NS, monopole waves interact with the detectors as well as quadrupole ones. In that case, the field-dependent coupling strength between matter and the scalar field has been assumed to be a linear function of the scalar field φ. In the notation of this paper such a coupling strength is given by $k^2 = \frac{16\pi}{|2\omega+3|}$ in eq. (2) of [14]. Then [20]

$$k^2 = \alpha_0 + \beta_0(\varphi - \varphi_0) \tag{5}$$

and the amplitude of the scalar polarization results [20]

$$\Phi \propto \frac{\alpha_0}{d} \tag{6}$$

where d is the distance of the collapsing NS expressed in meters.

For the following discussion the key point is that STG and $f(R)$ theories have an additional GW polarization which, in general, is massive with respect to the two standard polarizations of the GTR; see [10, 11, 12, 13, 14]. As GW detection is performed in a laboratory environment on Earth, one typically uses the coordinate system in which space-time is locally flat and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics. This is the so-called gauge of the local observer [10, 13, 14, 16]. In such a gauge the GWs manifest themselves by exerting tidal forces on the masses (the mirror and the beam-splitter in the case of an interferometer) [10, 13, 14, 16]. By putting the beam-splitter in the origin of the coordinate system, the components of the separation vector are the coordinates of the mirror. The effect of the GW is to drive the mirror to have oscillations [10, 13, 14, 16]. Let us consider a mirror that has the initial (unperturbed) coordinates x_{M0}, y_{M0} and z_{M0} , where there is a GW propagating in the z direction.

In the GTR the GW admits only the standard $+$ and $\times$ polarizations [10, 16]. We label the respective metric perturbations as h_+ and $h_\times$. To the first order approximation of h_+ and $h_\times$ the motion of the mirror due to the GW is [10, 16]

$$\begin{aligned} x_M(t) &= x_{M0} + \tfrac{1}{2}[x_{M0}h_+(t) - y_{M0}h_\times(t)] \\ y_M(t) &= y_{M0} - \tfrac{1}{2}[y_{M0}h_+(t) + x_{M0}h_\times(t)] \\ z_M(t) &= z_{M0}. \end{aligned} \tag{7}$$

STG can be have a third additional mode that is massless [10, 12, 14]. In this case, calling h_Φ the metric perturbation due to the additional GW polarization, to the first order approximation of h_+, $h_\times$ and h_Φ the motion of the mirror due to the GW is [10, 14]

$$\begin{aligned} x_M(t) &= x_{M0} + \tfrac{1}{2}[x_{M0}h_+(t) - y_{M0}h_\times(t)] + \tfrac{1}{2}x_{M0}h_\Phi(t) \\ y_M(t) &= y_{M0} - \tfrac{1}{2}[y_{M0}h_+(t) + x_{M0}h_\times(t)] + \tfrac{1}{2}y_{M0}h_\Phi(t) \\ z_M(t) &= z_{M0}. \end{aligned} \tag{8}$$

$f(R)$ theories have a third additional mode which is generally massive [11, 12, 13, 14]. The cases of STG and $f(R)$ theories having a third massive additional mode are totally equivalent [11, 12, 13, 14]. This is not surprising because it is well known that there is a more general conformal equivalence between $f(R)$ theories and STG [7, 9, 12, 31]. Again, we call h_Φ the metric perturbation due to the additional GW polarization. To the first order approximation of h_+, $h_\times$ and h_Φ the motion of the mirror due to the GW in STG and $f(R)$ theories having a third massive additional mode is [12, 13, 14]

$$\begin{aligned} x_M(t) &= x_{M0} + \tfrac{1}{2}[x_{M0}h_+(t) - y_{M0}h_\times(t)] + \tfrac{1}{2}x_{M0}h_\Phi(t) \\ y_M(t) &= y_{M0} - \tfrac{1}{2}[y_{M0}h_+(t) + x_{M0}h_\times(t)] + \tfrac{1}{2}y_{M0}h_\Phi(t) \\ z_M(t) &= z_{M0} + \tfrac{1}{2}z_{M0}\tfrac{m^2}{\omega^2}h_\Phi(t), \end{aligned} \tag{9}$$

where m and ω are the mass and the frequency of the GW's third massive mode, which is interpreted in terms of a wave packet [12, 13, 14]. We also recall that the relation between the mass and the frequency of the wave packet is given by [11, 13, 14]

$$m=\sqrt{(1-v_G^2)}\omega, \tag{10}$$

where v_G is the group-velocity of the wave-packet. Inserting eq. (10) in the third of eqs. (9) one gets

$$\begin{array}{c}
x_M(t)=x_{M0}+\frac{1}{2}[x_{M0}h_+(t)-y_{M0}h_\times(t)]+\frac{1}{2}x_{M0}h_\Phi(t)\\
\\
y_M(t)=y_{M0}-\frac{1}{2}[y_{M0}h_+(t)+x_{M0}h_\times(t)]+\frac{1}{2}y_{M0}h_\Phi(t)\\
\\
z_M(t)=z_{M0}+\frac{(1-v_G^2)}{2}z_{M0}h_\Phi(t).
\end{array} \tag{11}$$

The presence of the little mass m implies that the speed of the third massive mode is less than the speed of light; this generates the longitudinal component and drives the mirror oscillations of the z direction [11, 13, 14], which is shown by the third of eqs. (9).

Now, we perform a calculation of the signal that a GW detector would see if massive modes were present, allowing for sources incident from any direction on the sky. For the third massive additional mode the equation for geodesic deviation gives [13]

$$\begin{array}{c}
\widetilde{R}^1_{010}=-\frac{1}{2}\ddot{h}_\Phi\\
\\
\widetilde{R}^2_{020}=-\frac{1}{2}\ddot{h}_\Phi\\
\\
\widetilde{R}^3_{030}=\frac{1}{2}m^2h_\Phi,
\end{array} \tag{12}$$

where the $\widetilde{R}^i_{0i0}$ are the non zero components of the linearized Riemann tensor [13]. Now, let us consider a GW propagating in an arbitrary direction $\widehat{n}$ with the arm of the interferometer in the $\hat{u}$ and $\hat{v}$ directions, see Figure 1. Eq. (12) can be rewritten in compact form as

$$\begin{array}{c}
\widetilde{R}^i_{0j0}=\frac{1}{2}\left(\begin{array}{ccc}
-\ddot{h}_\Phi & 0 & 0\\
0 & -\ddot{h}_\Phi & 0\\
0 & 0 & m^2h_\Phi
\end{array}\right)=\\
\\
=\frac{1}{2}\ddot{h}_\Phi\left(\delta_{ij}-\widehat{n}_i\widehat{n}_j\right)x_j-\frac{1}{2}m^2h_\Phi\left(\widehat{n}_i\widehat{n}_j\right)x_j.
\end{array} \tag{13}$$

It is possible to associate to the interferometer a *polarization tensor* defined by [28]

$$d^{ij}\equiv\frac{1}{2}(\hat{v}^i\hat{v}^j-\hat{u}^i\hat{u}^j). \tag{14}$$

In that case, the signal induced by a generic GW polarization is the phase shift which is given by [28, 29]

$$s(t)\sim d^{ij}\widetilde{R}_{i0j0}. \tag{15}$$

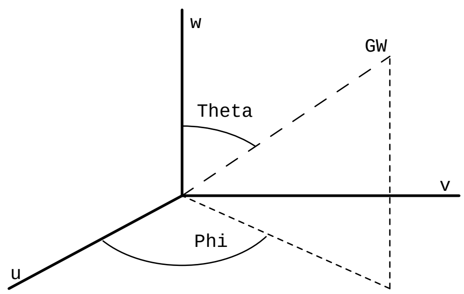

Figure 1: a GW propagating in an arbitrary direction, adapted from ref. [10]

Using eqs. (13) and (14), one gets

$$s(t) \sim -\sin^2\theta\cos 2\phi. \tag{16}$$

The angular dependence (16) is different from the two well known ones arising from the standard tensor modes of the GTR which are $(1+\cos^2\theta)\cos 2\phi$ for the $+$ polarization and $-\cos\theta\sin 2\vartheta$ for the $\times$ polarization respectively [29]. Now, let us see what happens for the third additional massless mode in STG. In that case, eq. (12) reduces to

$$\begin{array}{c}
\widetilde{R}^1_{010} = -\frac{1}{2}\ddot{h}_\Phi \\
\\
\widetilde{R}^2_{020} = -\frac{1}{2}\ddot{h}_\Phi,
\end{array} \tag{17}$$

which can be rewritten in compact form as

$$\begin{array}{c}
\widetilde{R}^i_{0j0} = \frac{1}{2} = \left(\begin{array}{cc} -\ddot{h}_\Phi & 0 \\ 0 & -\ddot{h}_\Phi \end{array}\right) \\
\\
= \frac{1}{2}\ddot{h}_\Phi\,(\delta_{ij} - \widehat{n}_i\widehat{n}_j)\,x_j.
\end{array} \tag{18}$$

Then, using eqs. (18) and (14), one gets again

$$s(t) \sim -\sin^2\theta\cos 2\phi, \tag{19}$$

which is the same result of eq. (16). Thus, by using this approach one cannot discriminate between massless and massive modes. On the other hand, the results of eqs. (16) and (19) are well known, [9, 29]. Now, in order to discriminate between massless and massive modes we will compute the frequency and angular dependent response function of a GW interferometric detector for massive modes. In fact, the angular dependences (16) and (19) have been computed with the implicit, standard assumption that the GW-wavelength is much larger than the distance between the test masses, which are the two mirrors and the beam-splitter for interferometers like LIGO [10, 29]. This low frequency approximation does not permit to discriminate between massless and massive modes because the angular dependence is the same in both of the cases. We will go beyond the low frequency approximation in the next analysis. To compute the response function of the interferometer to a massive mode from arbitrary propagating directions we must perform a spatial rotation of the coordinate system as

$$\begin{aligned} u &= -x\cos\theta\cos\phi + y\sin\phi + z\sin\theta\cos\phi \\ v &= -x\cos\theta\sin\phi - y\cos\phi + z\sin\theta\sin\phi \\ w &= x\sin\theta + z\cos\theta, \end{aligned} \tag{20}$$

or, in terms of the x, y, z frame:

$$\begin{aligned} x &= -u\cos\theta\cos\phi - v\cos\theta\sin\phi + w\sin\theta \\ y &= u\sin\phi - v\cos\phi \\ z &= u\sin\theta\cos\phi + v\sin\theta\sin\phi + w\cos\theta. \end{aligned} \tag{21}$$

The test masses are the beam splitter and the mirror of the interferometer, and we will suppose that the beam splitter is located in the origin of the coordinate system. Hence, Eqs. (11) represent the motion of the mirror like it is due to the massive mode of the GW. The mirror of Eqs. (11) is situated in the u direction. Thus, using Eqs. (11), (20) and (21) the u coordinate of the mirror is given by

$$\begin{aligned} u_M = &-\left(x_{M0} + \tfrac{1}{2}x_{M0}h_\Phi(t)\right)(\cos\theta\cos\phi) \\ &+\left(y_{M0} + \tfrac{1}{2}y_{M0}h_\Phi(t)\right)\sin\phi \\ &+\left(z_{M0} + \tfrac{(1-v_G^2)}{2}z_{M0}h_\Phi(t)\right)(z\sin\theta\cos\phi). \end{aligned} \tag{22}$$

In the same way, the v coordinate of the mirror is given by

$$\begin{aligned} v_M = &-\left(x_{M0} + \tfrac{1}{2}x_{M0}h_\Phi(t)\right)(\cos\theta\sin\phi) \\ &-\left(y_{M0} + \tfrac{1}{2}y_{M0}h_\Phi(t)\right)\cos\phi \\ &+\left(z_{M0} + \tfrac{(1-v_G^2)}{2}z_{M0}h_\Phi(t)\right)(z\sin\theta\sin\phi). \end{aligned} \tag{23}$$

Following [10, 12, 13, 32], a good way to analyse variations in the proper distance (time) is by means of “bouncing photons”. A photon can be launched from the interferometer’s beam-splitter to be bounced back by the mirror. The “bouncing photons analysis” was created in [32]. Actually, it has strongly generalized to angular dependences and scalar waves in [10, 12, 13] but this is the first time that such an analysis is performed in order to compute the frequency and angular dependent response function for massive modes. We will consider a photon propagating in the u axis. The analysis is similar for a photon propagating in the v axis. By using eq. (22), the unperturbed coordinates for the beam-splitter and the mirror are $u_b = 0$ and $u_m = L$, where $L = \sqrt{x_{M0}^2 + y_{M0}^2 + z_{M0}^2}$ is the length of the interferometer arms. Then, the unperturbed propagation time between the two masses is

$$T = L. \tag{24}$$

From eq. (22), one gets the displacements of the two masses under the influence of the massive mode of the GW as

$$\delta u_{BS}(t) = 0 \tag{25}$$

and

$$\delta u_M(t) = \frac{1}{2} A h_\Phi(t + L \sin\theta \cos\phi), \tag{26}$$

where

$$A \equiv -x_{M0} \cos\theta \cos\phi + y_{M0} \sin\phi + z_{M0} \sin\theta \cos\phi. \tag{27}$$

Therefore, the relative displacement in the u direction, which is defined by

$$\delta L(t) = \delta u_M(t) - \delta u_{BS}(t) \tag{28}$$

gives a "signal" in the u direction

$$\frac{\delta T(t)}{T}|_u = \frac{\delta L(t)}{L} = \frac{1}{2}\frac{A}{L} h_\Phi(t + L \sin\theta \cos\phi). \tag{29}$$

But one sees that for a large separation between the test masses (in the case of LIGO the distance between the beam-splitter and the mirror is four kilometres), the definition (28) for relative displacements becomes unphysical because the two test masses are taken at the same time and therefore cannot be in a casual connection [10, 13, 32]. Thus, the correct definitions for the bouncing photon are

$$\delta L_1(t) = \delta u_M(t) - \delta u_{BS}(t - T_1) \tag{30}$$

and

$$\delta L_2(t) = \delta u_M(t - T_2) - \delta u_{BS}(t), \tag{31}$$

where T_1 and T_2 are the photon propagation times for the forward and return trip correspondingly. Through the new definitions, the displacement of one test mass is compared with the displacement of the other at a later time to allow for finite delay from the light propagation [10, 13, 32]. The propagation times T_1 and T_2 in Eqs. (30) and (31) can be replaced with the nominal value T because the test mass displacements are already first order in h_Φ [10, 13, 32]. In this way, the total change in the distance between the beam splitter and the mirror in one round-trip of the photon is

$$\delta L_{r.t.}(t) = \delta L_1(t-T) + \delta L_2(t) = 2\delta u_m(t-T) - \delta u_{BS}(t) - \delta u_{BS}(t-2T), \tag{32}$$

and in terms of the amplitude of the massive GW mode:

$$\delta L_{r.t.}(t) = Ah_{\Phi}(t + L\sin\theta\cos\phi - L). \tag{33}$$

The change in distance (33) leads to changes in the round-trip time for photons propagating between the beam-splitter and the mirror in the u direction:

$$\frac{\delta_1 T(t)}{T}|_u = \frac{A}{L} h_{\Phi}(t + L\sin\theta\cos\phi - L). \tag{34}$$

One observes that in the last calculation, which concerns the variations in the photon round-trip time which come from the motion of the test masses inducted by the massive GW mode, it has been implicitly assumed that the propagation of the photon between the beam-splitter and the mirror of the interferometer is uniform as if it were moving in a flat space-time. But the presence of the tidal forces indicates that the space-time is curved instead. As a result, one must analyse one more effect after the first discussed, that requires spacial separation [10, 13, 32]. From equation (26) the tidal acceleration of a test mass caused by the massive GW mode in the u direction is

$$\ddot{u}(t + u\sin\theta\cos\phi) = \frac{1}{2}A\ddot{h}_{\Phi}(t + u\sin\theta\cos\phi). \tag{35}$$

This is equivalent to the presence of a gravitational potential [10, 13, 32]:

$$V(u,t) = -\frac{1}{2}A\int_0^u \ddot{h}_{\Phi}(t + l\sin\theta\cos\phi)dl, \tag{36}$$

generating the tidal forces. Thus, and the motion of the test mass is governed by the Newtonian equation [10, 13, 32]

$$\ddot{\vec{r}} = -\bigtriangledown V. \tag{37}$$

Now, we can discuss the second effect. Let us consider the interval for photons propagating along the u -axis

$$ds^2 = g_{00}dt^2 + du^2. \tag{38}$$

The condition for a null trajectory ($ds = 0$) gives the coordinate velocity of the photons [10, 13, 32]

$$v_p^2 \equiv (\frac{du}{dt})^2 = 1 + 2V(t,u), \tag{39}$$

which to first order in h_{Φ} is approximated by

$$v_p \approx \pm[1 + V(t,u)], \tag{40}$$

with $+$ and $-$ for the forward and return trip respectively. By knowing the coordinate velocity of the photon, the propagation time for its travelling between the beam-splitter and the mirror can be defined as [10, 13, 32]

$$T_1(t) = \int_{u_{BS}(t-T_1)}^{u_M(t)} \frac{du}{v_p} \tag{41}$$

and

$$T_2(t) = \int_{u_M(t-T_2)}^{u_{BS}(t)} \frac{(-du)}{v_p}. \tag{42}$$

The calculations of these integrals would be complicated because the u_M boundaries of them are changing with time [10, 13, 32]

$$u_{BS}(t) = 0 \tag{43}$$

and

$$u_M(t) = L + \delta u_M(t). \tag{44}$$

But, to first order in h_Φ, these contributions can be approximated by $\delta L_1(t)$ and $\delta L_2(t)$ (see Eqs. (30) and (31)) [10, 13, 32]. Hence, the combined effect of the varying boundaries is given by $\delta_1 T(t)$ in eq. (34). Therefore, one needs to compute only the times for photon propagation between the fixed boundaries, i.e 0 and L. Such propagation times are denoted with $\Delta T_{1,2}$ to distinguish from $T_{1,2}$. In the forward trip, the propagation time between the fixed limits is

$$\Delta T_1(t) = \int_0^L \frac{du}{v(t', u)} \approx L - \int_0^L V(t', u) du, \tag{45}$$

where t' is the delay time (i.e. t is the time at which the photon arrives in the position L, so $L - u = t - t'$ [10, 13, 32]) which corresponds to the unperturbed photon trajectory: $t' = t - (L - u)$. Similarly, the propagation time in the return trip is

$$\Delta T_2(t) = L - \int_L^0 V(t', u) du, \tag{46}$$

where now the delay time is given by $t' = t - u$. The sum of $\Delta T_1(t - T)$ and $\Delta T_2(t)$ gives the round-trip time for photons travelling between the fixed boundaries. Then, the deviation of this round-trip time (distance) from its unperturbed value $2T$ is

$$\begin{aligned} \delta_2 T(t) = &-\textstyle\int_0^L [V(t - 2L + u, u) du \\ &-\textstyle\int_L^0 V(t - u, u)] du, \end{aligned} \tag{47}$$

and, using Eq. (36), it is

$$\begin{aligned} \delta_2 T(t) = &\tfrac{1}{2} A \textstyle\int_0^L [\int_0^u \ddot{h}_\Phi(t - 2T + l(1 + \sin\theta\cos\phi)) dl \\ &-\textstyle\int_0^u \ddot{h}_\Phi(t - l(1 - \sin\theta\cos\phi) dl] du. \end{aligned} \tag{48}$$

Thus, the total round-trip proper distance in presence of the massive GW mode is:

$$T_t = 2T + \delta_1 T + \delta_2 T, \tag{49}$$

and

$$\delta T_u = T_t - 2T = \delta_1 T + \delta_2 T \tag{50}$$

is the total variation of the proper time (distance) for the round-trip of the photon in presence of the massive GW mode in the u direction. By using Eqs. (34), (48) and the Fourier transform of h_Φ defined by

$$\tilde{h}_\Phi(\omega) = \int_{-\infty}^{\infty} dt \, h_\Phi \exp(i\omega t), \tag{51}$$

the quantity (50) can be computed in the frequency domain by using the derivation and translation theorems of the Fourier transform as

$$\tilde{\delta} T_u(\omega) = \tilde{\delta}_1 T(\omega) + \tilde{\delta}_2 T(\omega) \tag{52}$$

where

$$\tilde{\delta}_1 T(\omega) = -i\omega \exp[i\omega T(1 - \sin\theta\cos\phi)] A \, \tilde{h}_\Phi(\omega) \tag{53}$$

and

$$\begin{array}{c}
\tilde{\delta}_2 T(\omega) = -\frac{A\omega^2}{2} \exp\left(2i\omega T\right) \left[\frac{T}{i\omega(1+\sin\theta\cos\phi)} + \frac{\exp[i\omega T(1+\sin\theta\cos\phi)]-1}{\omega^2(1+\sin\theta\cos\phi)^2}\right] \tilde{h}_\Phi(\omega) \\
\\
+\frac{A\omega^2}{2} \left[\frac{T}{i\omega(1-\sin\theta\cos\phi)} + \frac{\exp[i\omega T(1-\sin\theta\cos\phi)]-1}{\omega^2(1-\sin\theta\cos\phi)^2}\right] \tilde{h}_\Phi(\omega).
\end{array} \tag{54}$$

In this way, one finds the response function of the u arm of the interferometer to the massive GW mode as

$$\begin{array}{c}
H_u^{massive}(\omega) \equiv \frac{\tilde{\delta} T_u(\omega)}{T \, \tilde{h}_\Phi(\omega)} = \\
\\
= -i\omega \exp[i\omega T(1 - \sin\theta\cos\phi)] \frac{A}{T} \\
\\
-\frac{A\omega^2}{2T} \exp\left(2i\omega T\right) \left[\frac{T}{i\omega(1+\sin\theta\cos\phi)} + \frac{\exp[i\omega T(1+\sin\theta\cos\phi)]-1}{\omega^2(1+\sin\theta\cos\phi)^2}\right] \\
\\
+\frac{A\omega^2}{2T} \left[\frac{T}{i\omega(1-\sin\theta\cos\phi)} + \frac{\exp[i\omega T(1-\sin\theta\cos\phi)]-1}{\omega^2(1-\sin\theta\cos\phi)^2}\right].
\end{array} \tag{55}$$

The computation for the v arm is parallel to the one above. With the same way of thinking of previous analysis, defining

$$B \equiv -x_{M0} \cos\theta \sin\phi - y_{M0} \cos\phi + z_{M0} \sin\theta \sin\phi, \tag{56}$$

a straightforward similar computation permits to find the response function of the v arm of the interferometer to the massive GW mode as

$$
\begin{array}{c}
H_v^{massive}(\omega) \equiv \frac{\tilde{\delta} T_v(\omega)}{T\,\tilde{h}_\Phi(\omega)} = \\
\\
= -i\omega \exp[i\omega T(1-\sin\theta\sin\phi)]\frac{B}{T} \\
\\
-\frac{B\omega^2}{2T}\exp\left(2i\omega T\right)\left[\frac{T}{i\omega(1+\sin\theta\sin\phi)}+\frac{\exp[i\omega T(1+\sin\theta\sin\phi)]-1}{\omega^2(1+\sin\theta\sin\phi)^2}\right] \\
\\
+\frac{B\omega^2}{2T}\left[\frac{T}{i\omega(1-\sin\theta\sin\phi)}+\frac{\exp[i\omega T(1-\sin\theta\sin\phi)]-1}{\omega^2(1-\sin\theta\sin\phi)^2}\right].
\end{array}
\tag{57}
$$

The total response function to the massive GW mode is given by the difference of the two response function of the two arms:

$$
H_{tot}^{massive}(\omega) \equiv H_u^m(\omega) - H_v^m(\omega), \tag{58}
$$

and using Eqs. (55) and (57) one gets

$$
\begin{array}{c}
H_{tot}^{massive}(\omega) = \frac{\tilde{\delta} T_{tot}(\omega)}{T\tilde{h}_\Phi(\omega)} = \\
\\
= -i\omega \exp[i\omega T(1-\sin\theta\cos\phi)]\frac{A}{T} - i\omega\exp[i\omega T(1-\sin\theta\sin\phi)]\frac{B}{T} \\
\\
-\frac{A\omega^2}{2T}\exp\left(2i\omega T\right)\left[\frac{T}{i\omega(1+\sin\theta\cos\phi)}+\frac{\exp[i\omega T(1+\sin\theta\cos\phi)]-1}{\omega^2(1+\sin\theta\cos\phi)^2}\right] \\
\\
-\frac{B\omega^2}{2T}\exp\left(2i\omega T\right)\left[\frac{T}{i\omega(1+\sin\theta\sin\phi)}+\frac{\exp[i\omega T(1+\sin\theta\sin\phi)]-1}{\omega^2(1+\sin\theta\sin\phi)^2}\right] \\
\\
+\frac{A\omega^2}{2T}\left[\frac{T}{i\omega(1-\sin\theta\cos\phi)}+\frac{\exp[i\omega T(1-\sin\theta\cos\phi)]-1}{\omega^2(1-\sin\theta\cos\phi)^2}\right] \\
\\
+\frac{B\omega^2}{2T}\left[\frac{T}{i\omega(1-\sin\theta\sin\phi)}+\frac{\exp[i\omega T(1-\sin\theta\sin\phi)]-1}{\omega^2(1-\sin\theta\sin\phi)^2}\right].
\end{array}
\tag{59}
$$

On the other hand, the frequency and angular dependent response function of a GW interferometric detector for the third massless mode in STG is well known (see for example [12]) and is given by [12]

$$
\begin{array}{c}
H_{tot}^{massless}(\omega) = \frac{\sin\theta}{2i\omega L}\{\cos\phi[1+\exp(2i\omega L) - 2\exp i\omega L(1+\sin\theta\cos\phi)]+ \\
\\
-\sin\phi[1+\exp(2i\omega L)-2\exp i\omega L(1+\sin\theta\sin\phi)]\},
\end{array}
\tag{60}
$$

which is different from eq. (59). Thus, in principle, the frequency and angular dependent response functions (59) and (60) can be used to discriminate between massless and massive modes in STG and massive $f(R)$ gravity. Now, one notes that in the low frequency approximation, that is when $\omega \to 0$, one gets

$$
H_{tot}^{massless}(\omega) \approx H_{tot}^{massive}(\omega) \approx -\sin^2\theta\cos 2\phi. \tag{61}
$$

Then, one finds again the angular dependences (16) and (19). Thus, the angular dependences (16) and (19) are sufficient to discriminate between the GTR on one hand and STG and $f(R)$ gravity on the other hand, but they are not sufficient to discriminate between massless and massive modes. In order to discriminate between massless and massive modes one must look at higher frequencies by using the frequency and angular dependent response functions (59) and (60). This is, in principle, possible, because the frequency-range for earth based gravitational antennas is the interval $10Hz \leq f \leq 10KHz$ [10]. Thus, <u>eq. (59) is very important and represents the main result of this paper</u>. It is indeed a completely new and original result which can, in principle, be used to find massive modes arising from STG and $f(R)$ gravity in the motion of the interferometer's test masses. Instead, such a discrimination between massive and massless modes was <u>not</u> possible in previous GW literature.

The recent detections imply that the graviton mass must be $m_g \leq 7.7 \times 10^{-23} \frac{eV}{c^2}$ (in standard units) [34]. An important point is that the graviton mass m_g has not to be confused with the quantity m in eq. (9). In fact, the LIGO constraint in [34] is not on the extra polarization mode, which vanishes in the GTR limit, but on the tensor modes. A further clarification is needed in order to avoid confusion. Eqs. (8) and (9) could give the reader an incorrect impression of how the works [4, 34] placed constraints on the graviton mass using the recent GW detections. In fact, eqs. (8) and (9) show a difference in the $z-$motion of the mirrors when the AGT has a third propagation mode. However, LIGO has no sensitivity to motion in the $z-$direction. Of course, if the GW does not come from overhead the motion will be in the sensitive direction. In any case, even in the massless case there is a change in the motion of the mirrors due to the h_Φ term. To detect that in the data, it is necessary to measure the polarization of the GWs, but LIGO is not very well set up to do that, since the two detectors are almost aligned. The addition of Virgo [22, 23] will make that measurement, in principle, possible, see also the final discussion of this paper on the realization of a network of interferometers. Moreover, a key point is that, at the present time, the constraint on the mass is made indirectly [4, 34]. It comes from the lack of observed dispersion in the GW signal - an inspiraling binary radiates at different frequencies as the orbit decays. These different frequencies propagate at different speeds in massive gravity theories. Thus, the observed signal suffers dispersion [4, 34]. As we stressed above, this is in the tensor part of the signal. But the tensor part of the signal is the same in the GTR as well as in STG and $f(R)$ theories. This can be immediately understood by writing down explicitly, the corresponding line-elements. In the standard GTR the line element for a GW propagating in the $z-$direction can be written down as [16]

$$ds^2 = dt^2 - dz^2 - (1 + h_+)dx^2 - (1 - h_+)dy^2 - 2h_\times dxdy, \qquad (62)$$

where h_+ and $h_\times$ are expressed in terms of synchronous coordinates in the transverse-traceless (TT) gauge [16]. If the third mode of STG is massless the line-element in the TT gauge can be extended with the one more polarization

h_Φ as [10, 12, 14]

$$ds^2 = dt^2 - dz^2 - (1 + h_+ + h_\Phi)dx^2 - (1 - h_+ + h_\Phi)dy^2 - 2h_\times dxdy. \qquad (63)$$

On the other hand, as previously stressed, STG and $f(R)$ theories can have the third mode being massive. In that case, it is impossible to extend the TT gauge to the third mode because of the presence of the small mass m which generates a GW longitudinal component [12, 13, 14]. Then, gauge transformations permit to find the line-element as [12, 13, 14]

$$\begin{aligned} ds^2 = dt^2 - dz^2 - (1 + h_+)dx^2 - (1 - h_+)dy^2 - 2h_\times dxdy+ \\ +(1 + h_\Phi)(dt^2 - dz^2 - dx^2 - dy^2). \end{aligned} \qquad (64)$$

Then, one sees immediately that the tensor modes in $f(R)$ theories and STG are the same as in the GTR independently on the issue that the third additional mode is massless or massive. In fact, setting $h_\Phi = 0$ in eqs. (63) and (64), one sees immediately that both eqs. (63) and (64) reduce to eq. (62). Thus, as the tensor modes in $f(R)$ theories and STG are massless and the same as in the GTR, the analysis in [4] does not work for these two classes of theories. In fact, there are other constraints on massive theories of gravity, from weak-lensing, which are stronger than those from the recent GW detections. Similarly, there are laboratory experiments that constraint Yukawa-deviations from Newtonian gravity, that place much stricter bounds on $f(R)$ gravity. A common argument is to invoke a chameleon mechanism [21] that screens the deviations on certain scales. If screening is invoked then one could argue that any constraints obtained apply only to this system or only to BH binaries. In any case, the constraint here is coming from the propagation of the GWs over cosmological distances rather than from processes occurring on the scale of the binary, so it is not a local constraint. Cosmological GWs can also put constrains on the inflaton field [26, 27]. A further clarification is needed. Following eqs. (7) - (9) one argues that in GTR, $f(R)$ gravity and STG theories the tensor modes are the same and hence LIGO can not distinguish between them. While it is true that the detector responds in the same way to these modes in all theories, the evolution of the modes themselves may not be the same. In STG there may be additional channels into which energy is radiated as GWs. In the above discussion we identified the third massless scalar mode. While it is right that the detector cannot distinguish a monochromatic scalar mode signal from one in the tensor modes, if such additional modes exist they will cause the binary system to inspiral more quickly. This more rapid inspiral will be visible in the phase evolution of the tensor modes and so LIGO can still place constraints on the existence of such modes even if they are not directly observed.

We know that STG can be massless [10, 12, 14]. Thus, let us see what happens in the case of massless $f(R)$ theories, that, to our knowledge, has not been analysed in the literature. In order to linearize the $f(R)$ theories one uses the identifications [11]

$$\Phi \rightarrow \frac{df(R)}{dR} \qquad \text{and} \qquad \frac{dV}{d\Phi} \rightarrow \frac{2f(R) - R\frac{df(R)}{dR}}{3}. \qquad (65)$$

The mass of the extra polarisation mode is given by [11]

$$\frac{dV}{d\Phi} \simeq m^2 \delta\Phi, \tag{66}$$

where $\delta\Phi$ is the variation of the effective scalar field Φ near a minimum for the effective potential V, see [11] for details. Thus, for $m = 0$ one gets

$$2f(R) = R\frac{df(R)}{dR}. \tag{67}$$

By separating the variables eq. (67) is easily solved as

$$f(R) = R^2. \tag{68}$$

In the general case of $f(R)$ theories, to first order in $h_{\mu\nu}$ and $\delta\Phi$, calling, $\widetilde{R}_{\mu\nu}$ and $\widetilde{R}$ the linearized quantity which correspond to $R_{\mu\nu}$ and R, (where $R_{\mu\nu}$ and R are the Ricci tensor and the Ricci scalar respectively) the linearized field equations are [11]

$$\begin{array}{c} \widetilde{R}_{\mu\nu} - \frac{\widetilde{R}}{2}\eta_{\mu\nu} = (\partial_\mu \partial_\nu h_\Phi - \eta_{\mu\nu} \Box h_\Phi) \\ \\ \Box h_\Phi = m^2 h_\Phi. \end{array} \tag{69}$$

For the particular case of $f(R) = R^2$, eqs. (69) become

$$\begin{array}{c} \widetilde{R}_{\mu\nu} - \frac{\widetilde{R}}{2}\eta_{\mu\nu} = (\partial_\mu \partial_\nu h_\Phi - \eta_{\mu\nu} \Box h_\Phi) \\ \\ \Box h_\Phi = 0. \end{array} \tag{70}$$

This case is the exact analogous of STG having a third massless mode that has been discussed in detail in [10], see eqs. (23) of [10]. Thus, following the analysis in [10] step by step one arrives to the line element for the third component of the GW [10]

$$ds^2 = dt^2 - dz^2 - (1 + h_\Phi)\left(dx^2 + dy^2\right), \tag{71}$$

that is the part of eq. (63) arising from the additional GW mode h_Φ. Thus, from the mathematical point of view, also $f(R)$ theories admit a third massless GW polarization. But we recall that the class of αR^n theories (where n is not restricted to be an integer and $\alpha > 0$ has the dimensions of a mass squared [17]), is viable only for $n = 1 + \varepsilon$ with $0 \leq \varepsilon \ll 1$ [15, 16, 17]. Consequently, since the R^2 theory is not viable, we've discovered <u>a second, interesting new result</u>: the only $f(R)$ theory having a third massless GW mode is ruled out by our previous analysis. Thus, the extra polarization mode in $f(R)$ theories must be *always massive*. This also means that the only massless $f(R)$ theory which results viable is the GTR, for which it is $f(R) = R$. This result has an important consequence on the debate on the equivalence or non-equivalence between $f(R)$ theories an STG [7, 12]. In fact, despite it is well known that there is a general conformal equivalence between STG and$f(R)$ theories, there is a big debate on the possibility that such a conformal equivalence should be a *physical equivalence* too, see [7, 9, 12, 31]. Clearly, our result implies indeed that these two classes of theories have only a conformal equivalence because, differently from $f(R)$ theories, STG admit a third massless polarization mode.

3 Conclusion remarks

The GTR is not yet ultimately confirmed by the results of the LIGO Scientific Collaboration and the Virgo Collaborations in [2, 4], [33 - 37]. In fact, on one hand, in principle, there is still room for AGTs having massive tensor polarizations with a graviton mass $m_g \leq 7.7 \times 10^{-23} \frac{eV}{c^2}$. On the other hand, there is still room for $f(R)$ theories and STG. In fact, the recent GW detections did not put constraints on the these two classes of theories. Thus, we understand which is the key point here. Only a perfect knowledge of the motion of the interferometer's mirror will permit one to determine if the GTR is the definitive theory of gravity. In order to ultimately conclude that the GTR is the definitive theory of gravity, one must prove that the oscillations of the interferometer's mirror are in fact governed by eqs. (7). Otherwise, if one proves that the oscillations of the interferometer's mirror are in fact governed by eqs. (8) or eqs. (9), then the GTR must be extended. In this framework also the results of this paper on the frequency and angular dependent response functions (59) and (60) could be useful, because they can permit to discriminate between massless and massive modes in STG and $f(R)$ theories.

On the other hand, at the present time, the sensitivity of the current ground based GW interferometers is not sufficiently high to determine if the oscillations of the interferometer's mirror are governed by eqs. (7), or if they are governed by eqs. (8) or eqs. (9). That sensitivity is also not sufficiently high to determine the frequency and angular dependent response functions (59) and (60). A network including interferometers with different orientations is indeed required and we're hoping that future advancements in ground-based projects and space-based projects will have a sufficiently high sensitivity. Such advancements would enable physicists to determine, with absolute precision, the direction of GW propagation and the motion of the various involved mirrors. In other words, in the nascent GW astronomy we hope not only to obtain new, precious astrophysical information, but we also hope to be able to discriminate between eqs. (7), eqs. (8), and eqs. (9) and also to discriminate between the frequency and angular dependent response functions (59) and (60). Such advances in GW technology would equip us with the means and results to ultimately confirm the GTR or, alternatively, to ultimately clarify that the GTR must be extended.

Summarizing, in this paper we have discussed the future of gravitational theories in the framework of GW astronomy. In particular, we performed a calculation of the frequency and angular dependent response function that a GW detector would see if massive modes were present, allowing for sources incident from any direction on the sky. In addition, we have shown that massive (in terms of the third additional polarization) $f(R)$ theories of gravity and STG are still alive, while there is no room for $f(R)$ theories of gravity having a massless extra polarization mode.

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