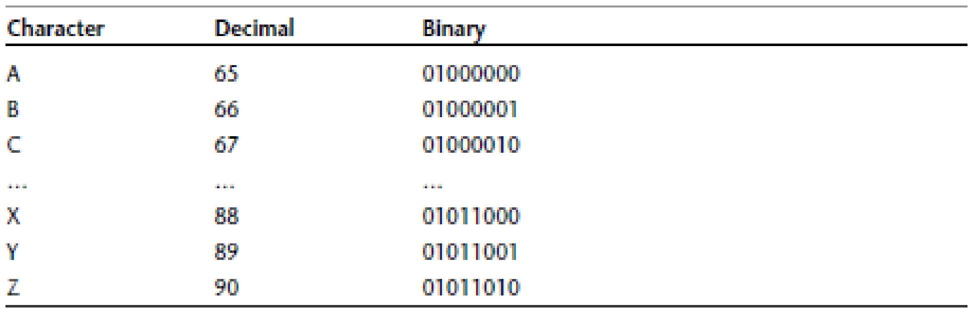
CSCI205 Computer Science III
Binary Tree and Heap Application: Data Compression

The Huffman Code

Here is an algorithm that uses a binary tree and tree based heap to losslessly compress data. It's called the Huffman code, after David Huffman who discovered it in 1952. Data compression is important in many situations. An example is sending data over the Internet, transmission can take a long time unless the data is compressed. An important aspect of compression is being able to inflate the data back to its original content. Your assignment will be to create an implementation of this algorithm.

Character Codes

Each character in a normal uncompressed text file is represented in the computer by one byte (for the venerable ASCII code) or by two bytes (for the newer Unicode, which is designed to work for all languages.) In these schemes, every character requires the same number of bits. The following table shows how some characters are represented in binary using the ASCII code. As you can see, every character takes 8 bits.



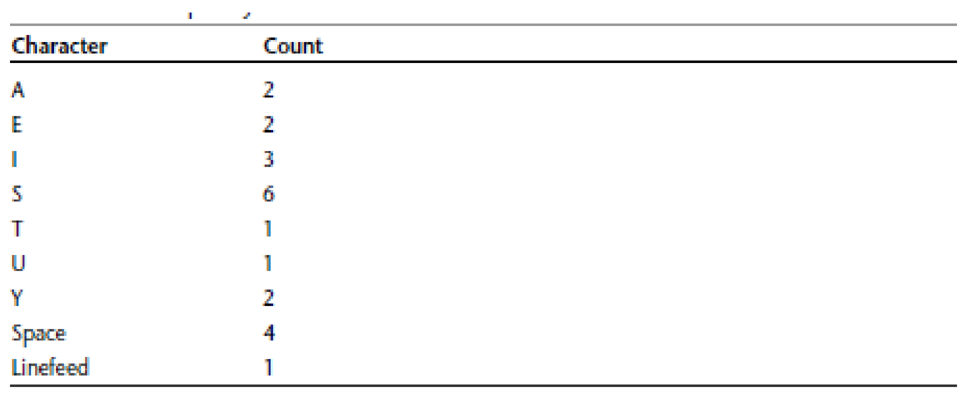
Character	Decimal	Binary
A	65	01000000
B	66	01000001
C	67	01000010
…	…	…
X	88	01011000
Y	89	01011001
Z	90	01011010

There are several approaches to compressing data. For text, the most common approach is to reduce the number of bits that represent the most-used characters. In English, E is often the most common letter, so it seems reasonable to use as few bits as possible to encode it. On the other hand, Z is seldom used, so using a large number of bits is not so bad. Suppose we use just two bits for E, say 01. We can't encode every letter of the alphabet in two bits because there are only four 2-bit combinations: 00, 01, 10, and 11. Can we use these four combinations for the four most-used characters? Unfortunately not.

We must be careful that no character is represented by the same bit combination that appears at the beginning of a longer code used for some other character. For example, if E is 01, and X is 01011000, then anyone decoding 01011000 wouldn't know if the initial 01 represented an E or the beginning of an X.

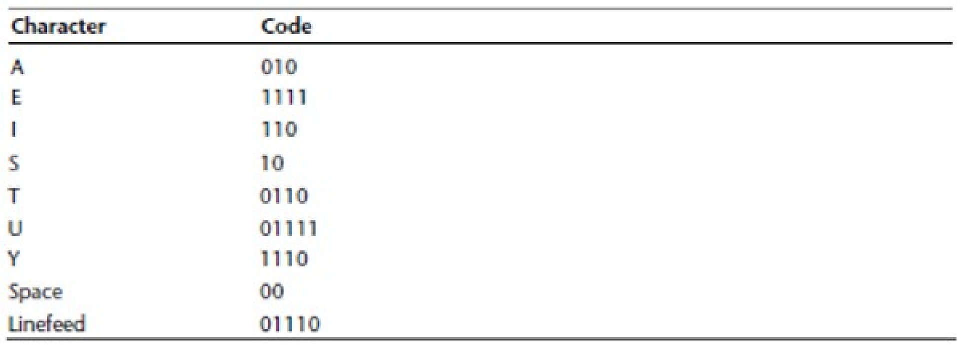
This leads to a rule: No code can be the prefix of any other code. Something else to consider is that in some messages E might not be the most-used character. If the text is a C++ source file, for example, the ; (semicolon) character might appear more often than E.

Solution: For each message, we make up a ***new code*** tailored to that particular message. Suppose we want to send the message SUSIE SAYS IT IS EASY. The letter S appears a lot, and so does the space character. We might want to make up a table showing how many times each letter appears. This is called a frequency table (histogram)



Character	Count
A	2
E	2
I	3
S	6
T	1
U	1
Y	2
Space	4
Linefeed	1

The characters with the highest counts should be coded with a small number of bits. The following table shows how we might encode the characters in the Susie message.



Character	Code
A	010
E	1111
I	110
S	10
T	0110
U	01111
Y	1110
Space	00
Linefeed	01110

We use 10 for S and 00 for the space. We can’t use 01 or 11 because they are prefixes for other characters. What about 3-bit combinations? There are eight possibilities: 000, 001, 010, 011, 100, 101, 110, and 111. A is 010 and I is 110. Why aren’t any other combinations used? We already know we can’t use anything starting with 10 or 00; that eliminates four possibilities. Also, 011 is used at the beginning of U and the linefeed, and 111 is used at the beginning of E and Y. Only two 3-bit codes remain, which we use for A and I. In a similar way we can see why only three 4-bit codes are available.

Thus, the entire message is coded as

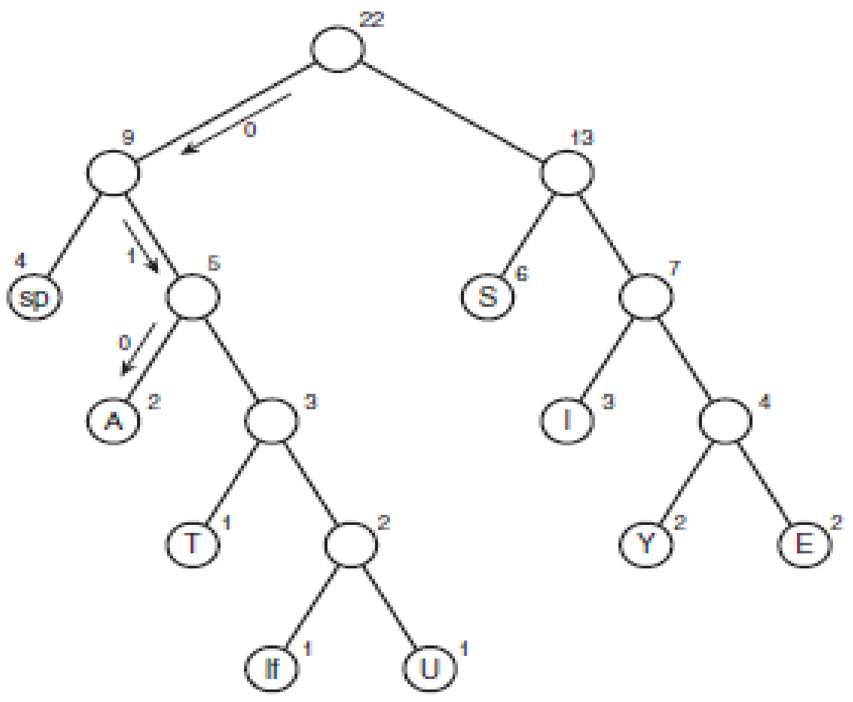
```
S  U     S  I   E   ‘ ‘ S  A   Y    S ‘ ‘ I   T   ‘ ‘ I   S ‘ ‘ E    A   S  Y   ‘\n’

10 01111 10 110 1111 00 10 010 1110 10 00 110 0110 00 110 10 00 1111 010 10 1110 01110
```

For sanity reasons we show this message broken into the codes for individual characters. Of course, in reality all the bits would run together; there is no space character in a binary message, only 0s and 1s.

Decoding with the Huffman Tree

We'll see later how to create Huffman codes. First, we'll examine the somewhat easier process of decoding. Suppose we received the string of bits shown in the preceding section. How would we transform it back into characters? We can use a kind of binary tree called a Huffman tree.

The characters in the message appear in the tree as leaf nodes. The higher their frequency in the message, the higher up they appear in the tree. The number outside each circle is the frequency. The numbers outside non-leaf nodes are the sums of the frequencies of their children. We'll see later why this is important. How do we use this tree to decode the message? For each character you start at the root. If you see a 0 bit, you go left to the next node, and if you see a 1 bit, you go right. Try it with the code for A, which is 010. You go left, then right, then left again and you find yourself on the A node. This is shown by the arrows in the above diagram. You'll see you can do the same with the other characters. If you have the patience, you can decode the entire bit string this way.



```
S  U     S  I   E   ‘ ‘S  A   Y   S‘ ‘I   T   ‘ ‘I  S‘ ‘E   A   S  Y   ‘\n’

10 01111 10 110 1111 00 10 010 1110 10 00 110 0110 00 110 10 00 1111 010 10 1110 01110
```

Creating the Huffman Tree

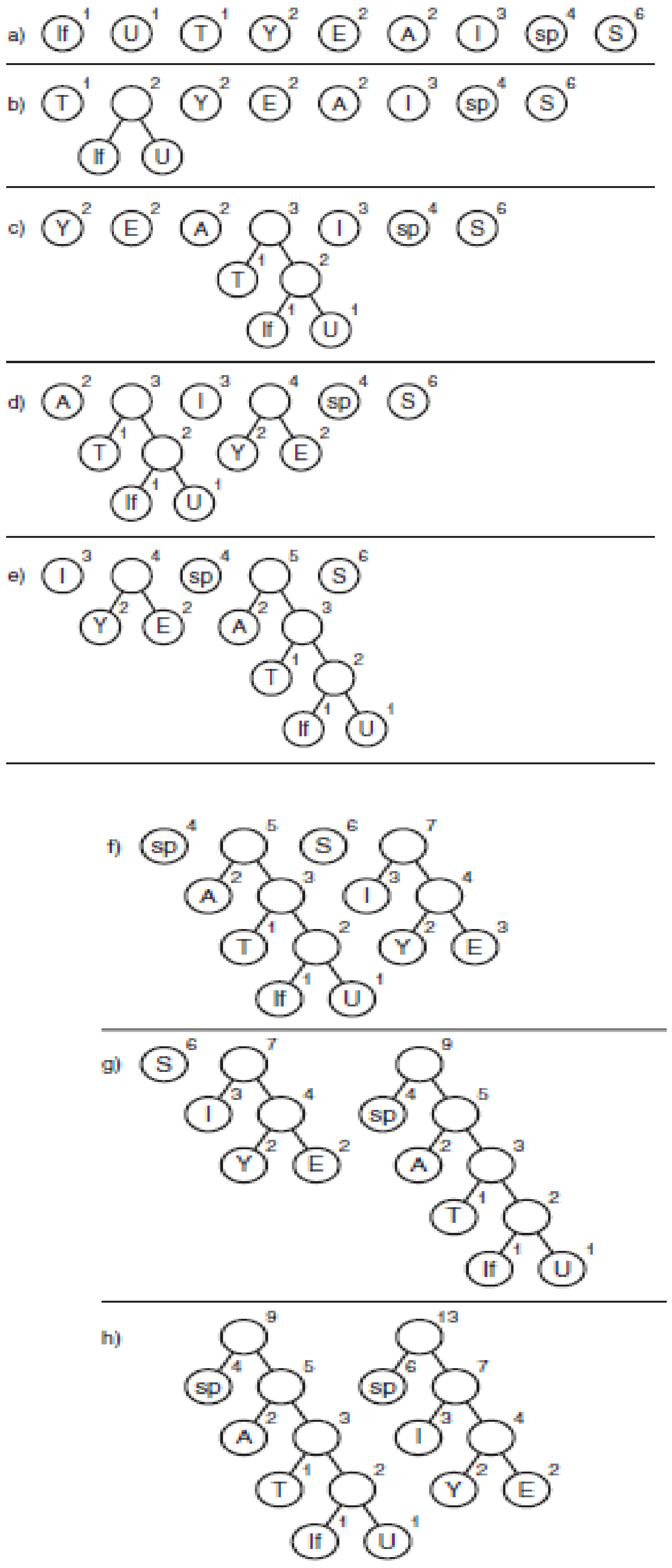
We've seen how to use the Huffman tree for decoding, but how do we create this tree? There are many ways to handle this problem. We'll base our approach on the Node and Tree classes (although routines that are specific to search trees, like find(), insert(), and delete() are no longer relevant).

Here is the algorithm for constructing the tree:

1. Make a Node object (or Tree object depending on the design) for each character used in the message. For our Susie example that would be nine nodes. Each node has two data items: the character and that character's frequency in the message.
2. Make a tree object for each of these nodes. The node becomes the root of the tree.
3. Insert these trees in a priority queue. They are ordered by ascending frequency, with the ***smallest frequency having the highest priority***. That is, when you remove a tree, it's always the one with the least-used character.

Now do the following:

1. Remove two trees from the priority queue, and make them into children of a new node. The new node has a frequency that is the sum of the children's frequencies; its character field can be left blank.
2. Insert this new three-node tree back into the priority queue.
3. Keep repeating steps 1 and 2. The trees will get larger and larger, and there will be fewer and fewer of them. When there is only one tree left in the queue, it is the Huffman tree and you're done.


a) lf 1 U 1 T 1 Y 2 E 2 A 2 I 3 sp 4 S 6
b) T 1 2 Y 2 E 2 A 2 I 3 sp 4 S 6 lf U
c) Y 2 E 2 A 2 3 I 3 sp 4 S 6 T 1 2 lf 1 U 1
d) A 2 3 I 3 4 sp 4 S 6 T 1 2 Y 2 E 2 lf 1 U 1
e) I 3 4 sp 4 5 S 6 Y 2 E 2 A 2 3 T 1 2 lf 1 U 1
f) sp 4 5 S 6 7 A 2 3 I 3 4 T 1 2 Y 2 E 3 lf 1 U 1
g) S 6 7 9 I 3 4 sp 4 5 Y 2 E 2 A 2 3 T 1 2 lf 1 U 1
h) 9 13 sp 4 5 sp 6 7 A 2 3 I 3 4 T 1 2 Y 2 E 2 lf 1 U 1

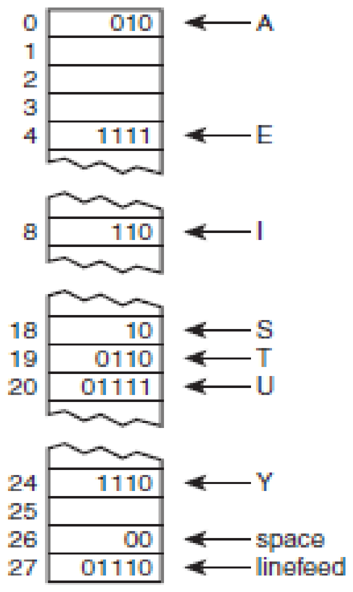
Coding the Message

Now that we have the Huffman tree, how do we code a message? We start by creating a code table, which lists the Huffman code alongside each character. To simplify the discussion, let's assume that, instead of the ASCII code, our computer uses a simplified alphabet that has only uppercase letters with 28 characters. A is 0, B is 1, and so on up to Z, which is 25. A space is 26, and a linefeed is 27. We number these characters so their numerical codes run from 0 to 27. (This is not a compressed code, just a simplification of the ASCII code, the normal way characters are stored in the computer.) Our code table would be an array of 28 cells. The index of each cell would be the numerical value of the character: 0 for A, 1 for B, and so on. The contents of the cell would be the Huffman code for the corresponding character. Not every cell contains a code; only those that appear in the message.

Such a code table makes it easy to generate the coded message: For each character in the original message, we use its code as an index into the code table. We then repeatedly append the Huffman codes to the end of the coded message until it's complete.

Creating the Huffman Code

How do we create the Huffman code to put into the code table? The process is like decoding a message. We start at the root of the Huffman tree and follow every possible path to a leaf node. As we go along the path, we remember the sequence of left and right choices, recording a 0 for a left edge and a 1 for a right edge. When we arrive at the leaf node for a character, the sequence of 0s and 1s is the Huffman code for that character. We put this code into the code table at the appropriate index number. This process can be handled by calling a method that starts at the root and then calls itself recursively for each child. Eventually, the paths to all the leaf nodes will be explored and the code table will be complete.



0
010
A
1
2
3
4
1111
E
8
110
I
18
10
S
19
0110
T
20
01111
U
24
1110
Y
25
26
00
space
27
01110
linefeed