1. 采用Recursion-Tree方法求解递推式 $T(n) = T(n/4) + T(n/2) + n^2$.

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***T*(*n*)**

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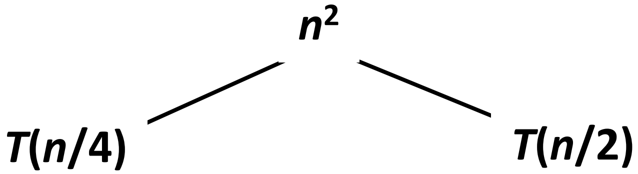

n^2

$T(n/4)$

$T(n/2)$

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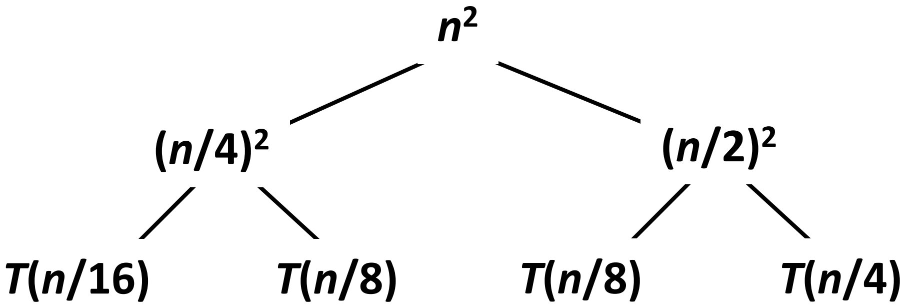

n^2

$(n/4)^2$ $(n/2)^2$

$T(n/16)$ $T(n/8)$ $T(n/8)$ $T(n/4)$

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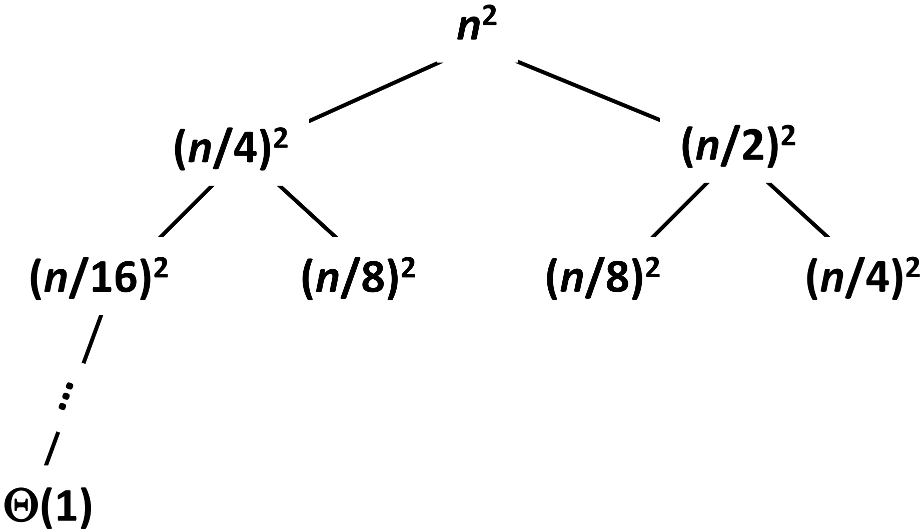

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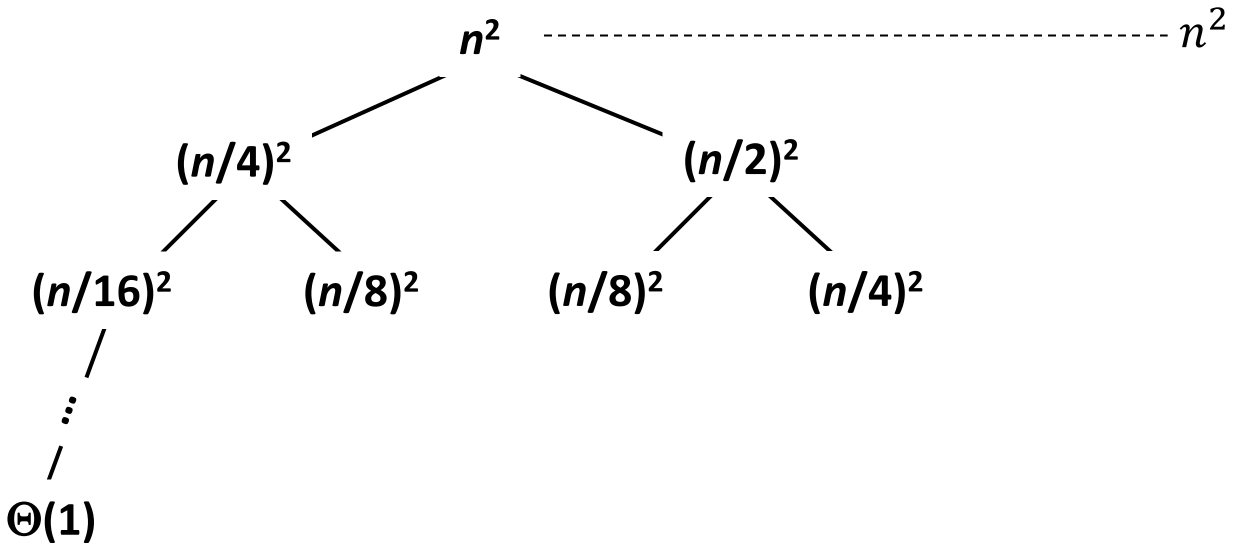

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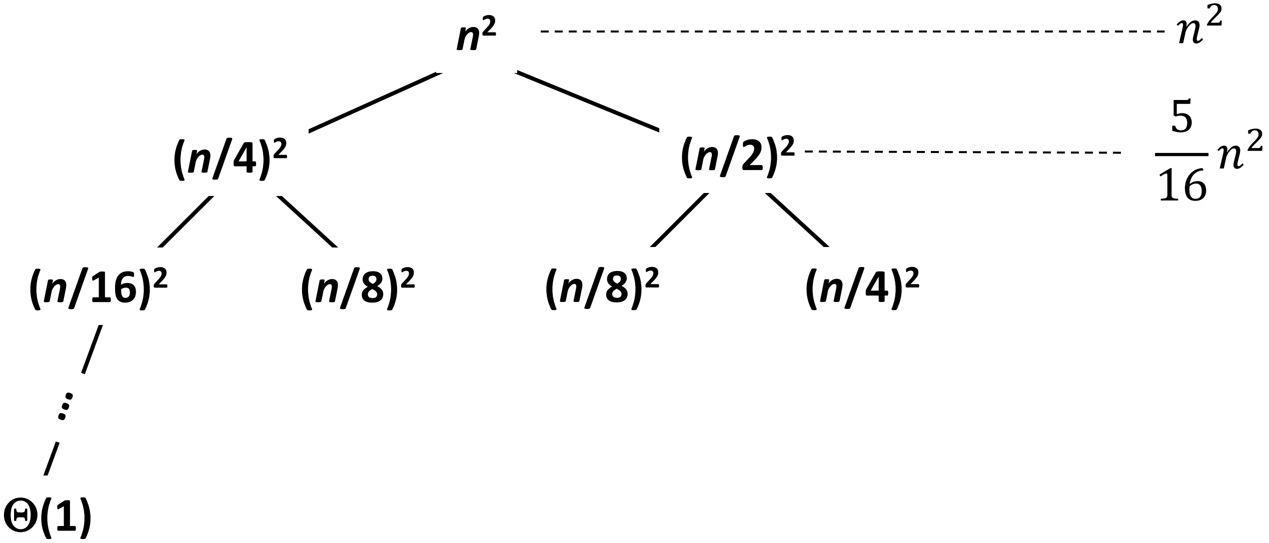

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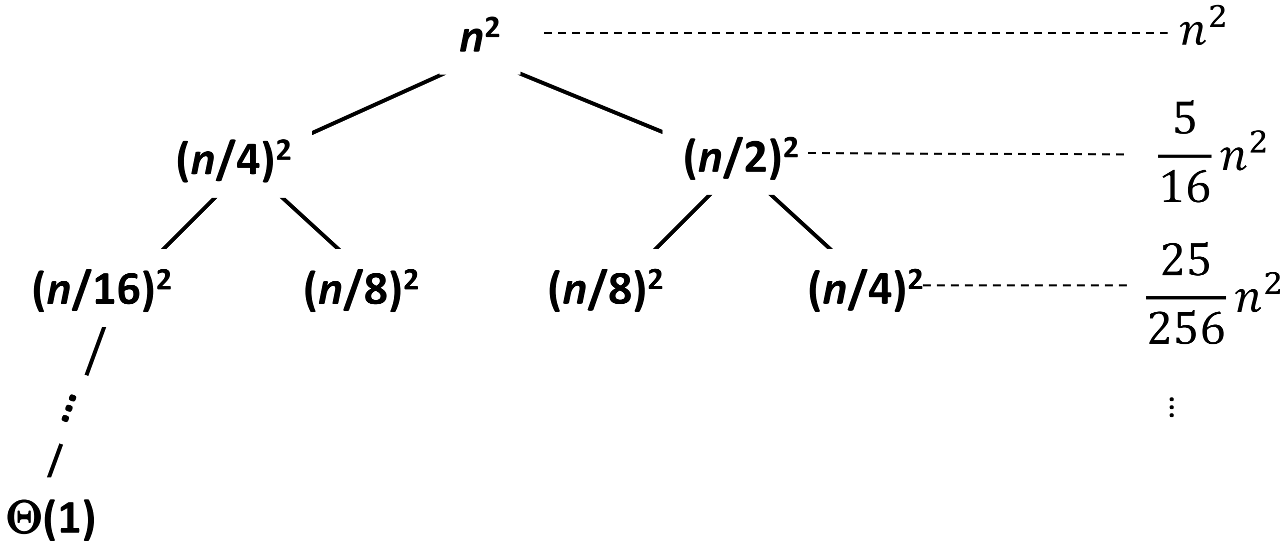

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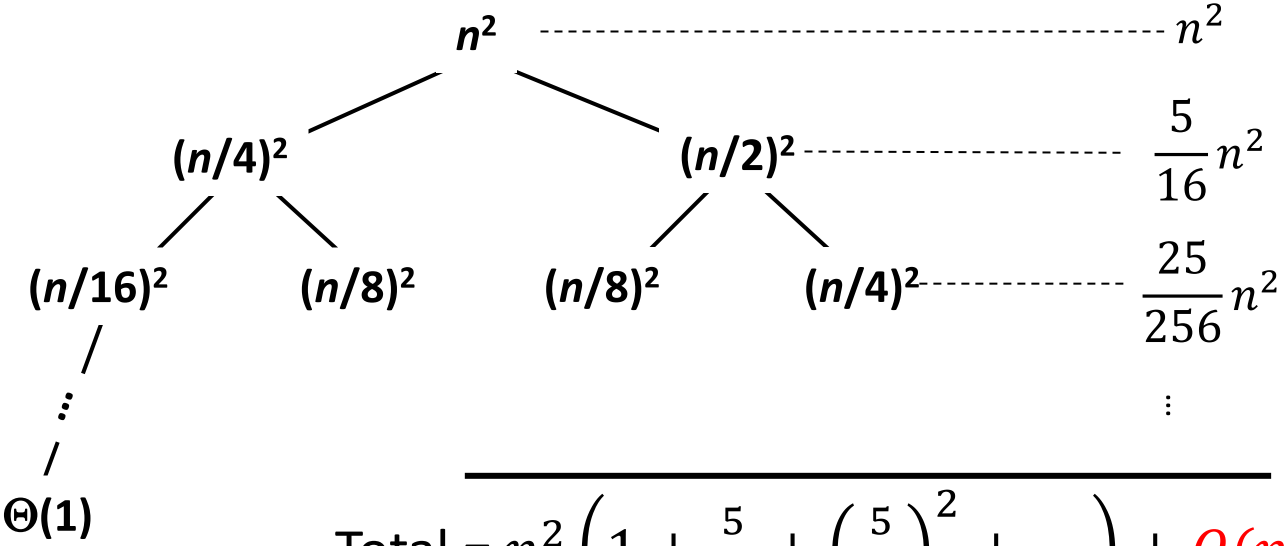

Total = $n^2\left(1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \cdots\right) + O(n)$

= $O(n^2)$ *geometric series*

2. 采用Master Theorem求解下列递推式.

a) $T(n) = 4T(n/2) + n$

b) $T(n) = 4T(n/2) + n^2$

c) $T(n) = 4T(n/2) + n^3$

d) $T(n)=3T(n/4)+n\log n$

Master Theorem. Suppose that $T(n)$ is a function on the non-negative integers that satisfies the recurrence:

$$T(n) = aT(n/b) + f(n)$$

with $T(0) = 0 \; and \; T(1) = \Theta(1)$, where n/b means either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Case 1. If $f(n) = O(n^k)$ for some constant $k < \log_b a$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.

Case 2. If $f(n) = \Theta(n^k log^p n)$ for $p \geq 0$ and $k = \log_b a$, then $T(n) = \Theta\left(n^k log^{p+1} n\right)$.

Case 3. If $f(n) = \Omega(n^k)$ for some constant $k > \log_b a$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta\left(f(n)\right)$.

2. 采用Master Theorem求解下列递推式.

a) T(n) = 4T(n/2) + n

a = 4, b = 2,

$n^{log_b a} = n^2$; and

$f(n) = n = O(n^1)$.

Case 1: $k < \log_b a$,

then $T(n) = \Theta(n^2)$.

Master Theorem. Suppose that $T(n)$ is a function on the non-negative integers that satisfies the recurrence:

$$T(n) = aT(n/b) + f(n)$$

with $T(0) = 0 \; and \; T(1) = \Theta(1)$, where n/b means either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Case 1. If $f(n) = O(n^k)$ for some constant $k < \log_b a$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.

Case 2. If $f(n) = \Theta(n^k log^p n)$ for $p \geq 0$ and $k = \log_b a$, then $T(n) = \Theta\left(n^k log^{p+1} n\right)$.

Case 3. If $f(n) = \Omega(n^k)$ for some constant $k > \log_b a$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta\left(f(n)\right)$.

2. 采用Master Theorem求解下列递推式.

b) $T(n) = 4T(n/2) + n^2$

$a = 4$, $b = 2$,
$n^{\log_b a} = n^2$; and
$f(n) = n^2$.
Case 2: $f(n) = (n^2 \log^0 n)$,
and $k = 2$,
then $T(n) = \Theta(n^2 \log n)$.

Master Theorem. Suppose that $T(n)$ is a function on the non-negative integers that satisfies the recurrence:

$$T(n) = aT(n/b) + f(n)$$

with $T(0) = 0$ and $T(1) = \Theta(1)$, where n/b means either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Case 1. If $f(n) = O(n^k)$ for some constant $k < \log_b a$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.

Case 2. If $f(n) = \Theta(n^k log^p n)$ for $p \geq 0$ and $k = \log_b a$, then $T(n) = \Theta\left(n^k log^{p+1} n\right)$.

Case 3. If $f(n) = \Omega(n^k)$ for some constant $k > \log_b a$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta\left(f(n)\right)$.

2. 采用Master Theorem求解下列递推式.

c) $T(n) = 4T(n/2) + n^3$

$a = 4$, $b = 2$,
$n^{\log_b a} = n^2$; *and*
$f(n) = n^3$.
Case 3: $f(n) = \Omega(n^3)$,
and $4(n/2)^3 \leq cn^3$ *(reg. cond.) for* $c = ½$,
then $T(n) = \Theta(n^3)$.

Master Theorem. Suppose that $T(n)$ is a function on the non-negative integers that satisfies the recurrence:

$$T(n) = aT(n/b) + f(n)$$

with $T(0) = 0 \; and \; T(1) = \Theta(1)$, where n/b means either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Case 1. If $f(n) = O(n^k)$ for some constant $k < \log_b a$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.

Case 2. If $f(n) = \Theta(n^k log^p n)$ for $p \geq 0$ and $k = \log_b a$, then $T(n) = \Theta\left(n^k log^{p+1} n\right)$.

Case 3. If $f(n) = \Omega(n^k)$ for some constant $k > \log_b a$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta\left(f(n)\right)$.

2. 采用Master Theorem求解下列递推式.

d) $T(n)=3T(n/4)+n\log n$

$a = 3$, $b = 4$,
$n^{\log_b a} = n^{0.793}$; and
$f(n) = n\log n$.
Case 3: $f(n) = \Omega(n^1)$, and $3(n/4)\log(n/4) \leq cn\log n$ (reg. cond.) for $c = 3/4$,
then $T(n) = \Theta(n\log n)$.

Master Theorem. Suppose that $T(n)$ is a function on the non-negative integers that satisfies the recurrence:

$$T(n) = aT(n/b) + f(n)$$

with $T(0) = 0 \; and \; T(1) = \Theta(1)$, where n/b means either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Case 1. If $f(n) = O(n^k)$ for some constant $k < \log_b a$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.

Case 2. If $f(n) = \Theta(n^k log^p n)$ for $p \geq 0$ and $k = \log_b a$, then $T(n) = \Theta\left(n^k log^{p+1} n\right)$.

Case 3. If $f(n) = \Omega(n^k)$ for some constant $k > \log_b a$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta\left(f(n)\right)$.