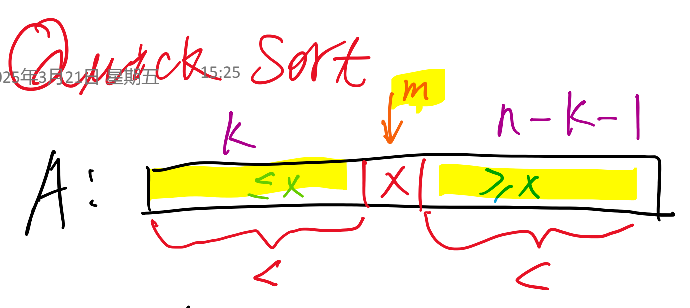
Quick Sort

2025年3月21日 星期五 15:25



Quick_Sort(A, p, q) $T(n)$

$x \leftarrow A[p]$ c

m ← Partition(A, p, q, x) $\Theta(n)$

Quick_Sort(A, p, m-1) $T(k)$

Quick-Sort(A, m+1, q) $T(n-k-1)$

Partition(A, p, q, x) $T(n) = \Theta(n)$

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i ← p
for j ← p+1 to q
    if A[j] ≤ x
        i ← i+1
        swap(A[i], A[j])
swap(A[p], A[i])
return i
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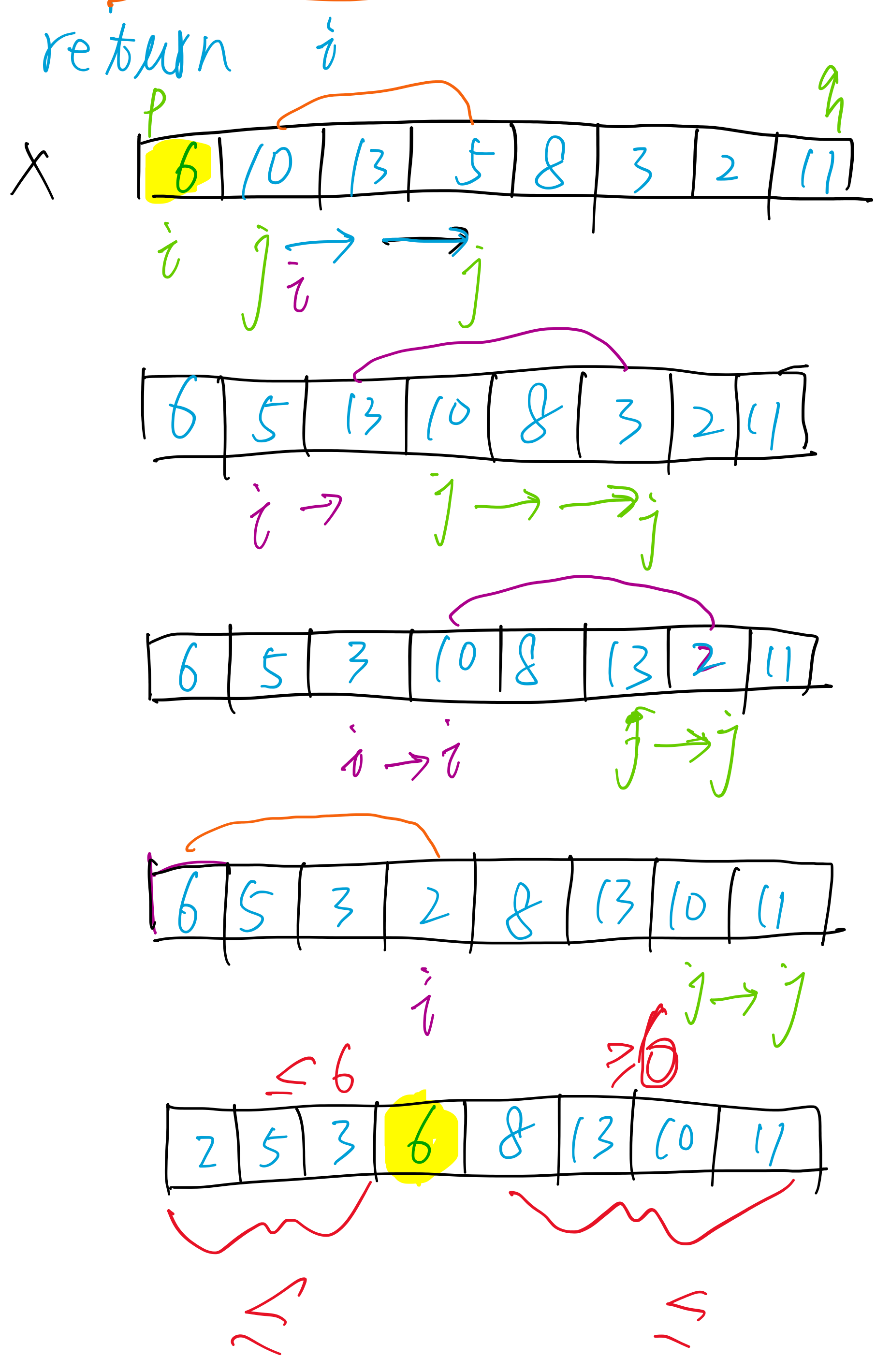
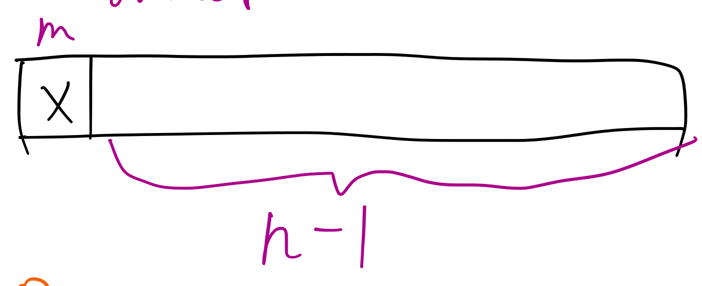


Quick sort(A, 1, n)

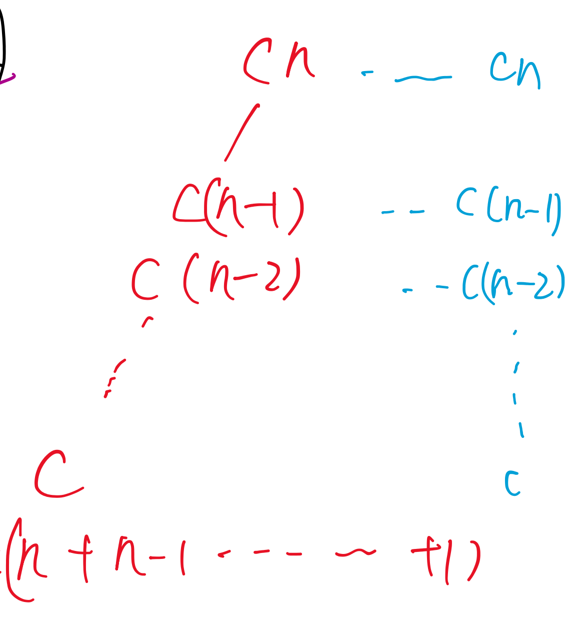
$T(n) = T(k) + T(n-k-1) + cn$

increase order



$T(n) = T(0) + T(n-1) + cn$

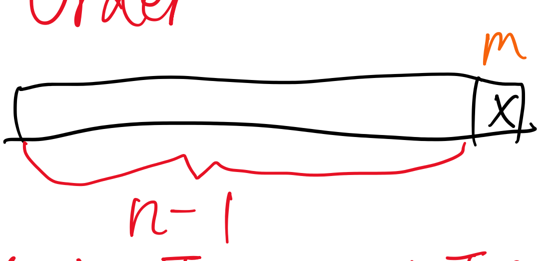
$= T(n-1) + cn$



$T(n) = c(n + n-1 \cdots\cdots +1)$

$= \frac{(n+1)n}{2} c = \Theta(n^2)$

Decreas Order



$T(n) = T(n-1) + T(0) + cn$

$= T(n-1) + cn$

$= \Theta(n^2)$

Best Case: $k = \frac{n}{2}$

$T(n) = T(k) + T(n-k-1) + cn$

$= T(\frac{n}{2}) + T(\frac{n}{2} - 1) + cn$

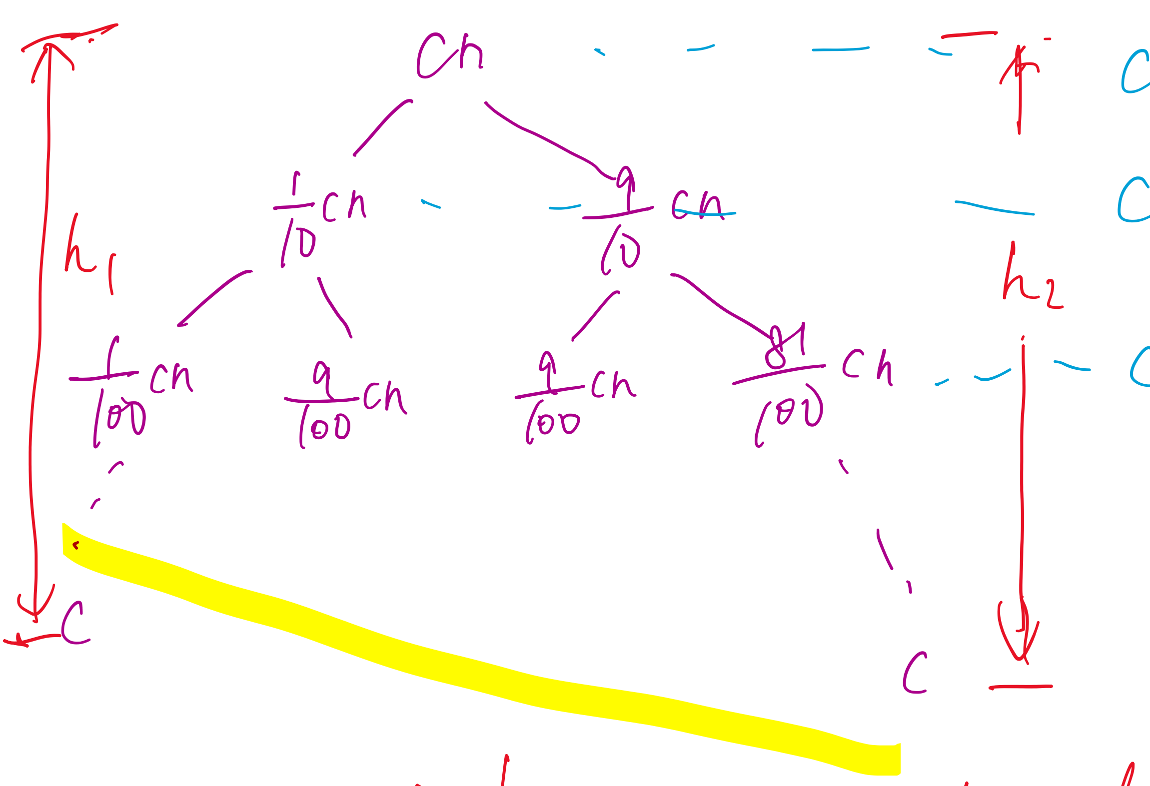
$\leq 2T(\frac{n}{2}) + cn$

$= O(n \lg n)$

$k = \frac{1}{10} n$

$T(n) = T(\frac{1}{10}n) + T(\frac{9}{10}n - 1) + cn$

$\leq T(\frac{1}{10}n) + T(\frac{9}{10}n) + cn$



$(\frac{1}{10})^{h_1} n = 1 \Rightarrow h_1 = \log_{10} n$

$(\frac{9}{10})^{h_2} n = 1 \Rightarrow h_2 = \log_{\frac{10}{9}} n$

$cn \cdot \log_{10} n = cn \cdot h_1 \leq T(n) \leq cn \cdot h = cn \log_{\frac{10}{9}} n$

$T(n) = \Theta(n \lg n)$

$L(n) = 2U(\frac{n}{2}) + cn$ lucky

$U(n) = L(n-1) + cn$ unlucky

$L(n) = 2U(\frac{n}{2}) + cn$

$= 2\left(L(\frac{n}{2} - 1) + \frac{c}{2}n\right) + cn$

$= 2L(\frac{n}{2} - 1) + cn + cn$

$= 2L(\frac{n}{2} - 1) + c'n$

$= \Theta(n \lg n)$ lucky!