2025年2月28日 星期五 15:17

The problem of Sorting

Input: Sequence $\langle a_1, a_2, \cdots, a_n \rangle$ of numbers.

Output: permutation $\langle a_1', a_2', \cdots a_n' \rangle$ such that. $a_1' \le a_2' \le \cdots \le a_n'$

Ex. 8 2 4 9 3 6 ✓

Output: 2 3 4 6 8 9 ✓

Insertion Sort

Insertion_Sort (A, n) ▷ A [1, … n]

```
for j ← 2 to n
    key ← A[j]
    i ← j - 1
    while i > 0 and A[i] > key
        A[i+1] ← A[i]
        i ← i - 1
    A[i+1] ← key
```

8 → 2 4 9 3 6
i j

0 8 8 4 9 3 6
← i

2 8 4 9 3 6
i j

2 4 8 9 3 6
i j

2 4 8 9 3 6
i j

2 3 4 8 9 6
i j

2 3 4 6 8 9

Insertion_Sort (A, n)

	cost	times
for j ← 2 to n	c_1	n
key ← A[j]	c_2	$n-1$
i ← j - 1	c_3	$n-1$
while i > 0 and A[i] > key	c_4	$\sum_{j=2}^{n} t_j$
A[i+1] ← A[i]	c_5	$\sum_{j=2}^{n} (t_j - 1)$
i ← i - 1	c_6	$\sum_{j=2}^{n} (t_j - 1)$
A[i+1] ← key	+ c_7	$n-1$

$$T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

Best Case: already sorted. $A[i] \le key$

$t_j = 1$

$$T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4(n-1) + c_7(n-1)$$

$$= \underbrace{(c_1 + c_2 + c_3 + c_4 + c_7)}_{a} n - \underbrace{(c_2 + c_3 + c_4 + c_7)}_{b}$$

$$= an + b = \Theta(n)$$

Worse Case: reverse sorted order.

$t_j = j$

$$T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} j + c_5 \sum_{j=2}^{n} (j-1) + c_6 \sum_{j=2}^{n} (j-1) + c_7(n-1)$$

$$\sum_{j=2}^{n} j = (2 + \cdots n) = \frac{(2+n)(n-1)}{2}$$

$$\sum_{j=2}^{n} (j-1) = (1 + 2 + \cdots n-1) = \frac{(n-1+1)(n-1)}{2}$$

$$T(n) = \underbrace{\left(\frac{c_4}{2} + \frac{c_5}{2} + \frac{c_6}{2}\right)}_{a} n^2 + \underbrace{\left(c_1 + c_2 + c_3 + c_7 - \frac{c_4}{2} - \frac{c_5}{2} - \frac{c_6}{2}\right)}_{b} n - \underbrace{(c_2 + c_3 + c_4 + c_7)}_{c}$$

$$= an^2 + bn + c = \Theta(n^2)$$