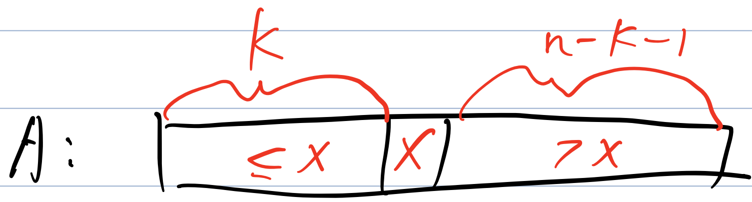


$$T(n) = T(k) + T(n-k-1) + \overset{cn}{\Theta}(n)$$

$$T(n) = T(k) + T(n-k-1) + cn$$

$k: 0, 1, 2, \cdots n-1$

$$E\quad T(n) = \begin{cases} T(0) + T(n-1), & k=0 \\ T(1) + T(n-2), & k=1 \\ T(2) + T(n-3), & k=2 \\ \vdots & \vdots \\ T(n-1) + T(0), & k=n-1 \end{cases}$$

$$E[T(n)] = \frac{1}{n} E\left[\sum_{k=0}^{n-1} \left(T(k) + T(n-k-1) + cn\right)\right]$$

$$= \frac{2}{n} E\left[\sum_{k=0}^{n-1} \left(T(k)\right)\right] + E[cn]$$

$$\frac{1}{n}\sum_{0}^{n-1} T(k) + \frac{1}{n}\sum_{0}^{n-1} \underline{T(n-k-1)} + cn$$

$T(0)+\cdots T(n-1)$ $\qquad$ $T(n-1)+\cdots T(0)$

$$E[T(n)] = \frac{2}{n}\sum_{k=0}^{n-1} E[T(k)] + cn \leftarrow T(0), T(1)$$

$$= \frac{2}{n}\sum_{k=2}^{n-1} E[T(k)] + cn \le an\lg n$$

$$E[T(n)] = O(n\lg n)$$

Proof: $E[T(n)] = \frac{2}{n}\sum_{k=2}^{n-1} E[T(k)] + cn$

$$\le an\lg n$$

1. $k<n$, $E[T(k)] \le ak\lg k$

2. $E[T(n)] = \frac{2}{n}\sum_{k=2}^{n-1} E[T(k)] + cn$

$$\le \frac{2}{n}\sum_{k=2}^{n-1} ak\lg k + cn$$

$$\boxed{\text{Fact: } \sum_{k=1}^{n-1} k \lg k \le \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2 \text{ (Exercise)}}$$

$$\le \cancel{\frac{2a}{n}}\left(\cancel{\frac{1}{2}} n^2 \lg n - \frac{1}{8} n^2\right) + cn$$

$$= an\lg n - \frac{an}{4} + cn$$

$$= an\lg n - \underbrace{\left(\frac{an}{4} - cn\right)}_{\text{residual } \ge 0}$$

$$E[T(n)] \le an\lg n$$

$$\frac{an}{4} - cn \ge 0 \Rightarrow a \ge 4c$$

D and Q.

Recurrency

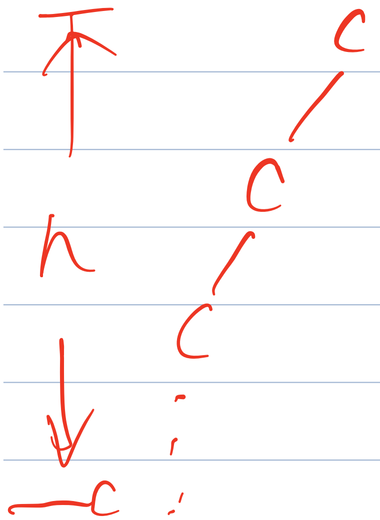
$n!$ $\Theta(n)$

Prime(n)

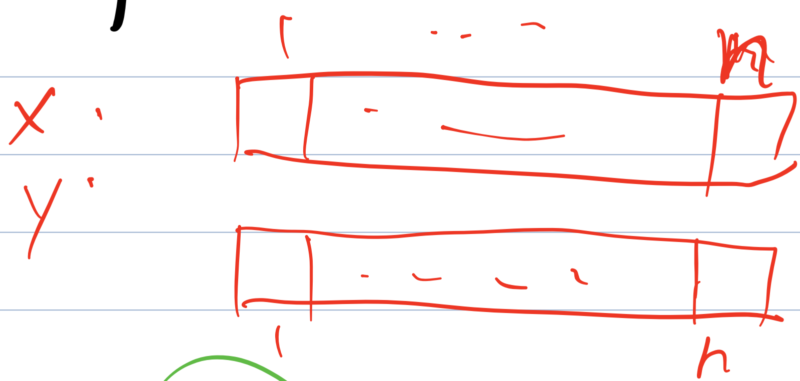
$n \le 1$; return 1; C.

return $n *$ Prime$(n-1)$

$$T(n) = c + T(n-1) = \Theta(n)$$



Big number Mult.



Binary

$\Theta(n^2)$

$$\begin{array}{r} 1101 \\ \underline{0110} \\ 01 \end{array}$$

$n = 1$

$s = 0$

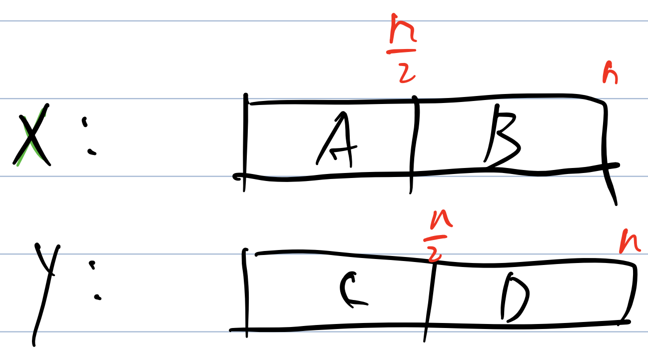
~~while (x & 1)~~

~~s += y~~

~~y ← y << 1~~

~~x ← x >> 1~~

return s.



$$x * y = (A * 2^{\frac{n}{2}} + B)(C * 2^{\frac{n}{2}} + D)$$

$$= AC \cdot 2^{n} + AD \cdot 2^{\frac{n}{2}} + BC * 2^{\frac{n}{2}} + BD$$

$$T(n) = 4T\left(\frac{n}{2}\right) + cn$$

$a = 4,\ b = 2,$

$$= \Theta(n^2)$$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$$X \cdot Y = AC \cdot 2^{n} + AD \cdot 2^{\frac{n}{2}} + BC \cdot 2^{\frac{n}{2}} + BD$$

$$= AC \cdot 2^{n} + \left((A-B)(D-C) + AC + BD\right) \cdot 2^{\frac{n}{2}} + BD$$

$$T(n) = 3T\left(\frac{n}{2}\right) + cn$$

$$\Rightarrow = \Theta(n^{\log_2 3}) = \Theta(n^{1.59\cdots})$$