order statistics

2025年3月26日 星期五 15:17

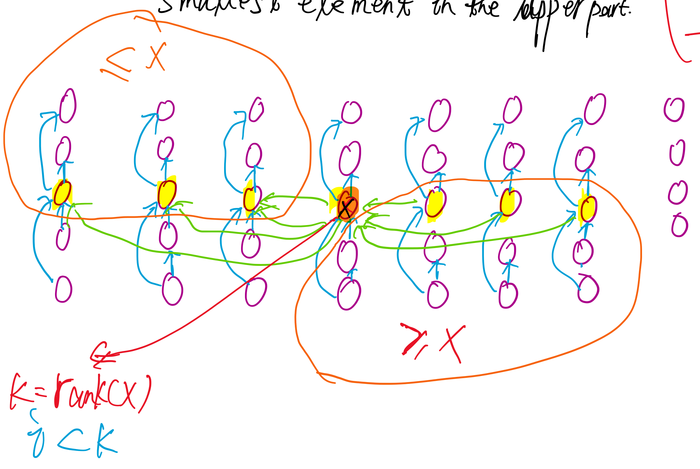
A: 

i-th smallest of n elements.

Rank(i)

Select(A, n, i) $T(n)$

1. Divide the n elements into group of 5. $c \cdot \lfloor \frac{n}{5} \rfloor$
   Find the median of each 5 element group. ~~$\Theta(n)$~~
2. Recursively Select the median x of the $\lfloor \frac{n}{5} \rfloor$ group medians to be the pivot. $T(\frac{n}{5})$
3. Partition around the pivot x. ~~$\Theta(n)$~~
   Let $k = \text{rank}(x)$.
4. if $i = k$ return x
   else if $i < k$
   then recursively Select the i-th smallest element in the lower part $T(\frac{3}{4}n)$
   else recursively Select the $(i-k)$-th smallest element in the upper part. $T(\frac{3}{4}n)$



$$T(n) = \Theta(n) + T(\tfrac{n}{5}) + T(\tfrac{3}{4}n)$$

$$T(n) = T(\tfrac{n}{5}) + T(\tfrac{3}{4}n) + cn \quad \le an.$$

$$\le \tfrac{1}{5}an + \tfrac{3}{4}an + cn \qquad \because \tfrac{1}{5}n < n,\ \tfrac{3}{4}n < n$$

$$= \tfrac{19}{20}an + cn$$

$$= an - \tfrac{1}{20}an + cn$$

$$= an - \underbrace{(\tfrac{a}{20} - c)n}_{\ge 0}$$

$\frac{a}{20} - c \ge 0 \Rightarrow a \ge 20c$

$T(n) \le an = O(n)$