

MONASH BUSINESS SCHOOL

ETC3550 Applied forecasting for business and economics

Ch11. Advanced forecasting methods OTexts.org/fpp3/



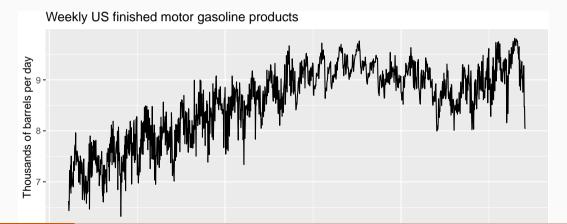
Outline

- 1 Complex seasonality
- 2 Vector autoregression
- 3 Neural network models
- 4 Bootstrapping and bagging

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- 1 Complex seasonality
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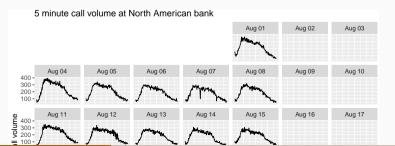
```
us_gasoline %>% autoplot(Barrels) +
labs(x = "Year", y = "Thousands of barrels per day",
    title = "Weekly US finished motor gasoline products")
```

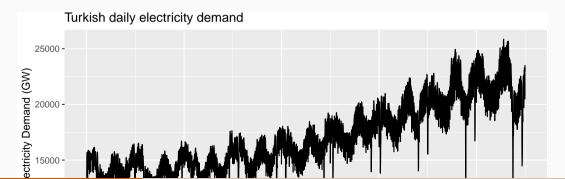


400 -

```
calls <- read_tsv("http://robjhyndman.com/data/callcenter.txt") %>%
  rename(time = `...1`) %>%
  pivot longer(-time, names to="date", values to="volume") %>%
 mutate(
    date = as.Date(date, format = "%d/%m/%Y"),
    datetime = as datetime(date) + time
  ) %>%
  as_tsibble(index = datetime)
calls %>%
  fill gaps() %>%
  autoplot(volume) +
  labs(x = "Weeks", y = "Call volume",
       title = "5 minute call volume at North American bank")
```

5 minute call volume at North American bank





TBATS

Trigonometric terms for seasonality

Box-Cox transformations for heterogeneity

ARMA errors for short-term dynamics

Trend (possibly damped)

Seasonal (including multiple and

non-integer periods)

$$y_{t} = \text{observation at time } t$$

$$y_{t}^{(\omega)} = \begin{cases} (y_{t}^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_{t} & \text{if } \omega = 0. \end{cases}$$

$$y_{t}^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_{i}}^{(i)} + d_{t}$$

$$y_{t}^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{N} s_{t-m_{i}}^{(i)} + d_{t}$$
$$\ell_{t} = \ell_{t-1} + \phi b_{t-1} + \alpha d_{t}$$
$$b_{t} = (1 - \phi)b + \phi b_{t-1} + \beta d_{t}$$

$$b_{t} = (1 - \phi)b + \phi b_{t-1} + \beta d_{t}$$

$$d_{t} = \sum_{i=1}^{p} \phi_{i} d_{t-i} + \sum_{i=1}^{q} \theta_{i} \varepsilon_{t-i} + \varepsilon_{t}$$

$$a_{t} = \sum_{i=1}^{k_{i}} \phi_{i} a_{t-i} + \sum_{j=1}^{k_{i}} \theta_{j} \varepsilon_{t-j} + \varepsilon_{t}$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_{j}^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_{j}^{(i)} + \gamma_{1}^{(i)} d_{t}$$

$$s_{i,t}^{(i)} = -s_{i,t-1}^{(i)} \sin \lambda_{j}^{(i)} + s_{i,t-1}^{*(i)} \cos \lambda_{j}^{(i)} + \gamma_{2}^{(i)} d_{t}$$

$$y_t$$
 = observation at time t

Box-Cox transformation

$$y_t^{(\omega)} = \begin{cases} (y_t^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^M s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^p \phi_i d_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-j} + \varepsilon_t$$

$$d_t + d_t$$

$$+ \beta d_t$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_{j}^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_{j}^{(i)} + \gamma_{1}^{(i)} d_{t}$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = -s_{i,t-1}^{(i)} \sin \lambda_{j}^{(i)} + s_{i,t-1}^{*(i)} \cos \lambda_{j}^{(i)} + \gamma_{2}^{(i)} d_{t}$$

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$
Box-Cox transformation
$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^M s_{t-m_i}^{(i)} + d_t \qquad M \text{ seasonal periods}$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

$$\begin{aligned} d_t &= \sum_{i=1}^p \phi_i d_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t \\ s_t^{(i)} &= \sum_{j=1}^{k_i} s_{j,t}^{(i)} & s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{i,t}^{(i)} &= -s_{i,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{i,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

$$y_t$$
 = observation at time t

$$y_t^{(\omega)} = \begin{cases} (y_t^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

Box-Cox transformation

$$y_{t}^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_{i}}^{(i)} + d_{t}$$

$$\ell_{t} = \ell_{t-1} + \phi b_{t-1} + \alpha d_{t}$$

$$b_{t} = (1 - \phi)b + \phi b_{t-1} + \beta d_{t}$$

M seasonal periods

$$b_{t} = (1 - \phi)b + \phi b_{t-1} + \beta d_{t}$$

$$d_{t} = \sum_{i=1}^{p} \phi_{i} d_{t-i} + \sum_{i=1}^{q} \theta_{i} \varepsilon_{t-j} + \varepsilon_{t}$$

global and local trend

$$d_{t} = \sum_{i=1}^{k} \phi_{i} d_{t-i} + \sum_{j=1}^{k} \theta_{j} \varepsilon_{t-j} + \varepsilon_{t}$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)} \qquad s_{j,t-1}^{(i)} \cos \lambda_{j}^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_{j}^{(i)} + \gamma_{1}^{(i)} d_{t}$$

$$s_{i,t}^{(i)} = -s_{i,t-1}^{(i)} \sin \lambda_{j}^{(i)} + s_{i,t-1}^{*(i)} \cos \lambda_{j}^{(i)} + \gamma_{2}^{(i)} d_{t}$$

$$y_t$$
 = observation at time t

$$y_t^{(\omega)} = \begin{cases} (y_t^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

M seasonal periods

Box-Cox transformation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^M s_{t-m_i}^{(i)} + d_t \quad \text{M seasonal periods}$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t \quad \text{global and local trend}$$

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^p \phi_i d_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_{j}^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_{j}^{(i)} + \gamma_{1}^{(i)} d_{t}$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = -s_{i,t-1}^{(i)} \sin \lambda_{j}^{(i)} + s_{i,t-1}^{*(i)} \cos \lambda_{j}^{(i)} + \gamma_{2}^{(i)} d_{t}$$

$$y_t$$
 = observation at time t $y_t^{(\omega)} = \begin{cases} (y_t^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$

Box-Cox transformation

$$y_{t}^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_{i}}^{(i)} + d_{t}$$

$$\ell_{t} = \ell_{t-1} + \phi b_{t-1} + \alpha d_{t}$$
g

global and local trend

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

$$\theta_{i\mathcal{E}_{t-i}} + \varepsilon_{t}$$
ARMA error

 $d_t = \sum_{i=1}^p \phi_i d_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-j} + \varepsilon_t$ $s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)}$ $s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \text{ cc Fourier-like seasonal terms}$ $s_{i,t}^{(i)} = -s_{i,t-1}^{(i)} \sin \lambda_{i}^{(i)} + s_{i,t-1}^{*(i)} \cos \lambda_{i}^{(i)} + \gamma_{2}^{(i)} d_{t}$

$$y_{t} = \text{observation at time } t$$

$$y_{t}^{(\omega)} = \begin{cases} (y_{t}^{\omega} - 1)/\omega & \text{if } \omega \neq 0; \\ \textbf{TBATS} \end{cases}$$

$$Trigonometric$$

$$y_{t}^{(\omega)} = \ell_{t} \text{Box-Cox}$$

$$\ell_{t} = \ell_{t} \text{ARMA}$$

$$b_{t} = (: t_{t} \text{Trend})$$

$$d_{t} = \sum_{i=1}^{k} \textbf{Seasonal}$$

$$s_{t}^{(i)} = \sum_{j=1}^{k_{i}} s_{j,t}^{(i)}$$

$$s_{j,t}^{(i)} = -s_{j,t-1}^{(i)} \cos \lambda_{j}^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_{j}^{(i)} + \gamma_{2}^{(i)} d_{t}$$

Complex seasonality

```
gasoline %>% tbats() %>% forecast() %>% autoplot()
```

Complex seasonality

```
calls %>% tbats() %>% forecast() %>% autoplot()
```

Complex seasonality

```
telec %>% tbats() %>% forecast() %>% autoplot()
```

TBATS

Trigonometric terms for seasonality

Box-Cox transformations for heterogeneity

ARMA errors for short-term dynamics

Trend (possibly damped)

Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

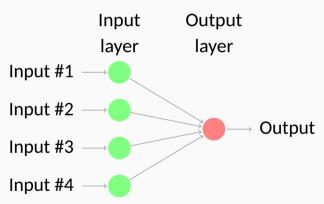
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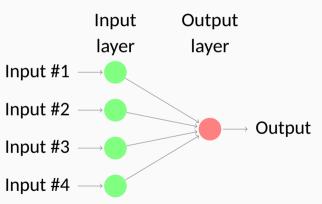
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Simplest version: linear regression

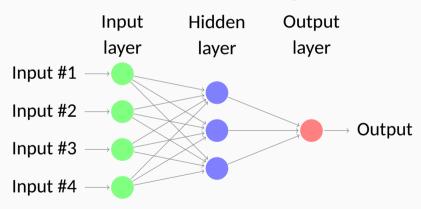


Simplest version: linear regression

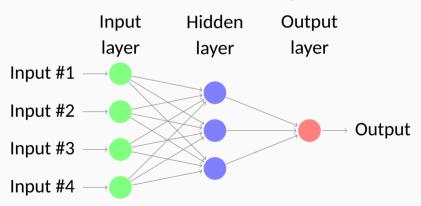


- Coefficients attached to predictors are called "weights".
- Forecasts are obtained by a linear combination of inputs.

Nonlinear model with one hidden layer



Nonlinear model with one hidden layer



A multilayer feed-forward network where each layer of nodes receives inputs from the previous layers.

Inputs to hidden neuron *j* linearly combined:

$$z_j = b_j + \sum_{i=1}^4 w_{i,j} x_i.$$

Modified using nonlinear function such as a sigmoid:

$$s(z)=\frac{1}{1+e^{-z}},$$

This tends to reduce the effect of extreme input values, thus making the network somewhat robust to outliers.

- Weights take random values to begin with, which are then updated using the observed data.
- There is an element of randomness in the predictions. So the network is usually trained several times using different random starting points, and the results are averaged.
- Number of hidden layers, and the number of nodes in each hidden layer, must be specified in advance.

NNAR models

- Lagged values of the time series can be used as inputs to a neural network.
- NNAR(p, k): p lagged inputs and k nodes in the single hidden layer.
- NNAR(p, 0) model is equivalent to an ARIMA(p, 0, 0) model but without stationarity restrictions.
- Seasonal NNAR(p, P, k): inputs $(y_{t-1}, y_{t-2}, \dots, y_{t-p}, y_{t-m}, y_{t-2m}, y_{t-Pm})$ and k neurons in the hidden layer.
- NNAR $(p, P, 0)_m$ model is equivalent to an ARIMA $(p, 0, 0)(P, 0, 0)_m$ model but without stationarity restrictions.

NNAR models in R

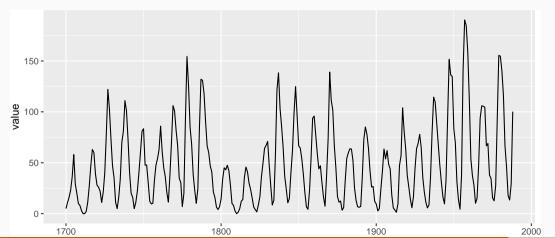
- The nnetar() function fits an NNAR(p, P, k)_m model.
- If *p* and *P* are not specified, they are automatically selected.
- For non-seasonal time series, default p = optimal number of lags (according to the AIC) for a linear AR(p) model.
- For seasonal time series, defaults are P = 1 and p is chosen from the optimal linear model fitted to the seasonally adjusted data.
- Default k = (p + P + 1)/2 (rounded to the nearest integer).

Sunspots

- Surface of the sun contains magnetic regions that appear as dark spots.
- These affect the propagation of radio waves and so telecommunication companies like to predict sunspot activity in order to plan for any future difficulties.
- Sunspots follow a cycle of length between 9 and 14 years.

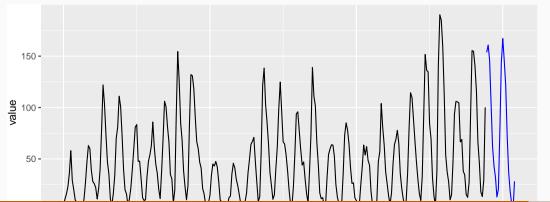
Sunspots

```
sunspots <- sunspot.year %>% as_tsibble()
sunspots %>% autoplot(value)
```

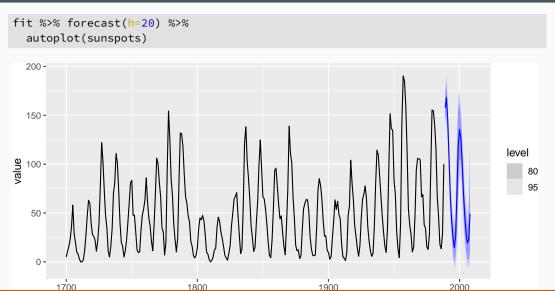


NNAR(9,5) model for sunspots

```
sunspots <- sunspot.year %>% as_tsibble()
fit <- sunspots %>% model(NNETAR(value))
fit %>% forecast(h=20, times = 1) %>%
  autoplot(sunspots, level = NULL)
```



Prediction intervals by simulation



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