MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550 Applied forecasting

Ch10. Dynamic regression models

OTexts.org/fpp3/

Outline

1 Regression with ARIMA errors

2 Stochastic and deterministic trends

3 Dynamic harmonic regression

4 Lagged predictors

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1 Regression with ARIMA errors

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3 Dynamic harmonic regression

4 Lagged predictors

Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \ldots, x_{k,t}$.
- In regression, we assume that ε_t was WN.
- Now we want to allow ε_t to be autocorrelated.

Regression with ARIMA errors

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- In regression, we assume that ε_t was WN.
- Now we want to allow ε_t to be autocorrelated.

Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

where ε_t is white noise.

Residuals and errors

Example: η_t = ARIMA(1,1,1)

$$y_t = \beta_0 + \beta_1 x_{1,t} + \dots + \beta_k x_{k,t} + \eta_t,$$

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Residuals and errors

Example: η_t = ARIMA(1,1,1)

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$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

- Be careful in distinguishing η_t from ε_t.
- Only the errors ε_t are assumed to be white noise.
- In ordinary regression, η_t is assumed to be white noise and so $\eta_t = \varepsilon_t$.

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \dots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. *p*-values for coefficients usually too small ("spurious regression' ').
4. AIC of fitted models misleading.

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3. *p*-values for coefficients usually too small (“spurious regression’ ’).
4. AIC of fitted models misleading.

- Minimizing $\sum \varepsilon_t^2$ avoids these problems.
- Maximizing likelihood similar to minimizing $\sum \varepsilon_t^2$.

Regression with ARIMA errors

Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
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Regression with ARIMA errors

Model with ARIMA(1,1,1) errors

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Equivalent to model with ARIMA(1,0,1) errors

$$\begin{aligned} y'_t &= \beta_1 x'_{1,t} + \dots + \beta_k x'_{k,t} + \eta'_t, \\ (1 - \phi_1 B)\eta'_t &= (1 + \theta_1 B)\varepsilon_t, \end{aligned}$$

where $y'_t = y_t - y_{t-1}$, $x'_{t,i} = x_{t,i} - x_{t-1,i}$ and $\eta'_t = \eta_t - \eta_{t-1}$.

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

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Original data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t$$

$$\text{where} \quad \phi(B)(1 - B)^d \eta_t = \theta(B)\varepsilon_t$$

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$$\text{where} \quad \phi(B)(1-B)^d\eta_t = \theta(B)\varepsilon_t$$

After differencing all variables

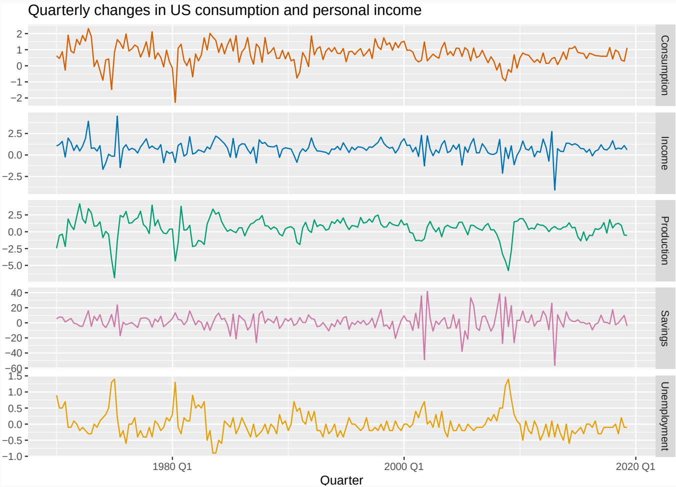
$$y'_t = \beta_1 x'_{1,t} + \dots + \beta_k x'_{k,t} + \eta'_t.$$

where $\phi(B)\eta'_t = \theta(B)\varepsilon_t$,
$y'_t = (1-B)^d y_t$, $x'_{i,t} = (1-B)^d x_{i,t}$, and $\eta'_t = (1-B)^d\eta_t$

Regression with ARIMA errors

- In R, we can specify an ARIMA(p, d, q) for the errors, and d levels of differencing will be applied to all variables ($y, x_{1,t}, \dots, x_{k,t}$).
- Check that ε_t series looks like white noise.
- AICc can be calculated for final model.
- Repeat procedure for all subsets of predictors to be considered, and select model with lowest AICc value.

US personal consumption and income



US personal consumption and income

```r
fit <- us_change %>% model(ARIMA(Consumption ~ Income))
report(fit)
```

```
## Series: Consumption
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##          ar1     ma1     ma2  Income  intercept
##        0.707  -0.617  0.2066  0.1976      0.595
## s.e.  0.107   0.122  0.0741  0.0462      0.085
##
## sigma^2 estimated as 0.3113:  log likelihood=-163
## AIC=338   AICc=339   BIC=358
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```
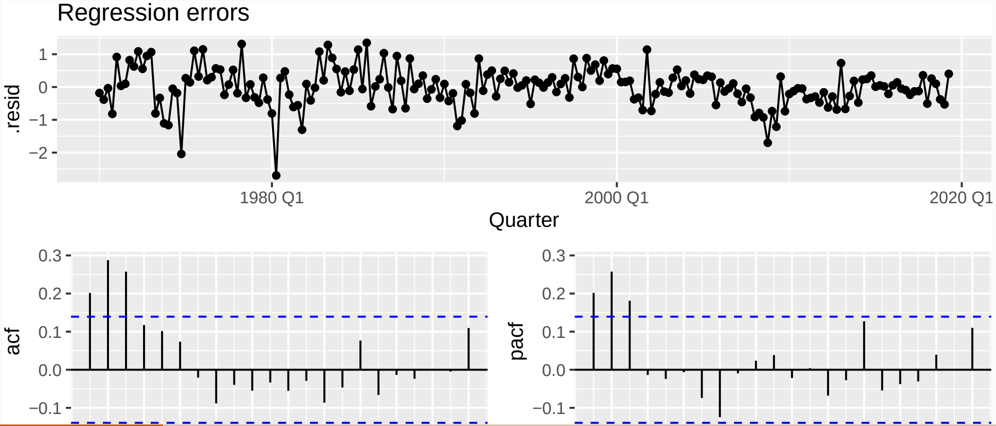

Write down the equations for the fitted model.

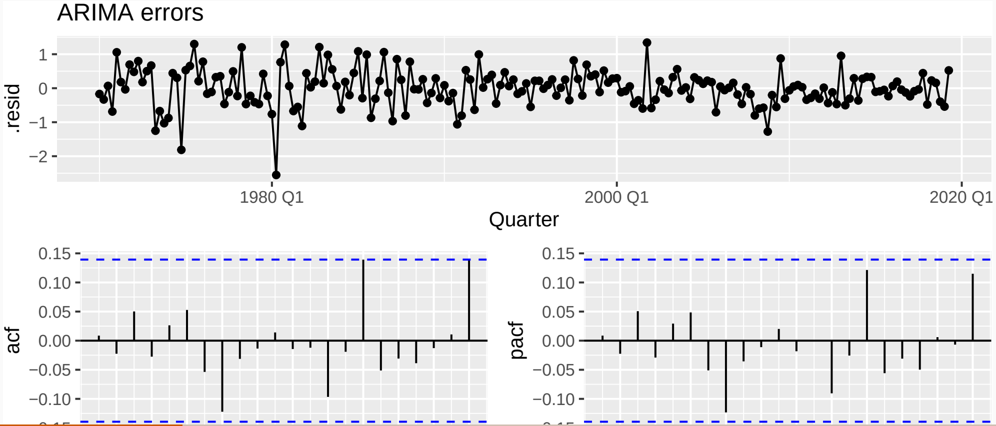
US personal consumption and income

```
residuals(fit, type='regression') %>%
  gg_tsdisplay(.resid, plot_type = 'partial') +
  labs(title = "Regression errors")
```



US personal consumption and income

```
residuals(fit, type='innovation') %>%
  gg_tsdisplay(.resid, plot_type = 'partial') +
  labs(title = "ARIMA errors")
```



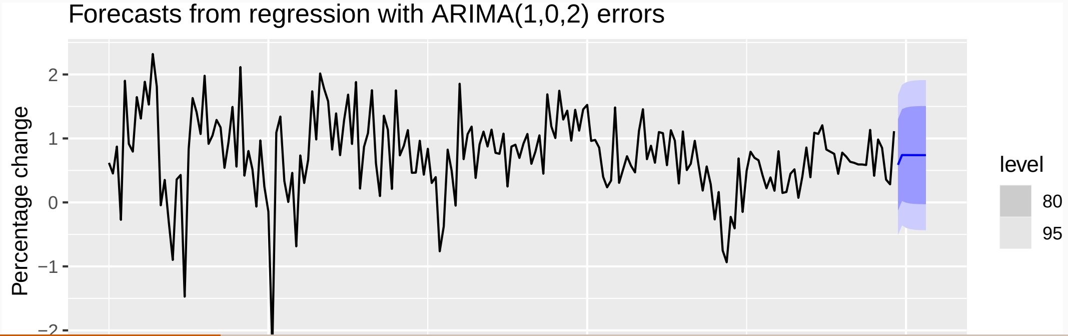
US personal consumption and income

```r
augment(fit) %>%
  features(.resid, ljung_box, dof = 5, lag = 12)
```

```
## # A tibble: 1 x 3
##   .model                          lb_stat lb_pvalue
##   <chr>                             <dbl>     <dbl>
## 1 ARIMA(Consumption ~ Income)        5.54     0.595
```

US personal consumption and income

```
us_change_future <- new_data(us_change, 8) %>%
  mutate(Income = mean(us_change$Income))
forecast(fit, new_data = us_change_future) %>%
  autoplot(us_change) +
  labs(x = "Year", y = "Percentage change",
       title = "Forecasts from regression with ARIMA(1,0,2) errors")
```



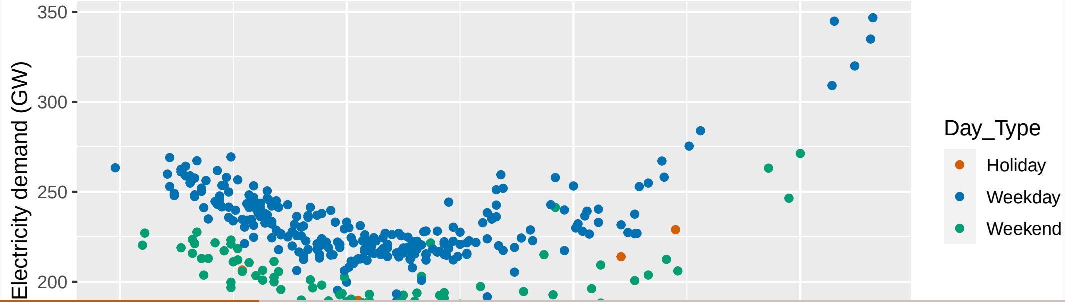
Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.

Daily electricity demand

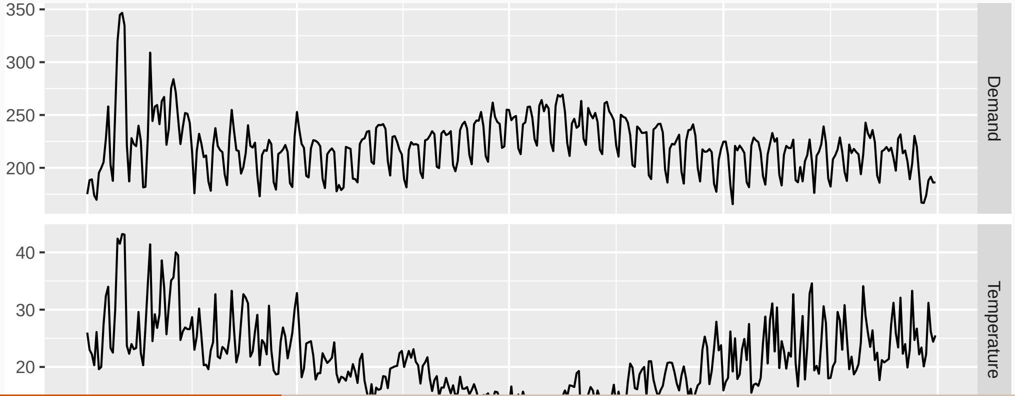
Model daily electricity demand as a function of temperature using quadratic regression with ARMA errors.

```
vic_elec_daily %>%
  ggplot(aes(x = Temperature, y = Demand, colour = Day_Type)) +
  geom_point() +
  labs(x = "Maximum temperature", y = "Electricity demand (GW)")
```



Daily electricity demand

```
vic_elec_daily %>%
  pivot_longer(c(Demand, Temperature)) %>%
  ggplot(aes(x = Date, y = value)) + geom_line() +
  facet_grid(name ~ ., scales = "free_y") + ylab("")
```



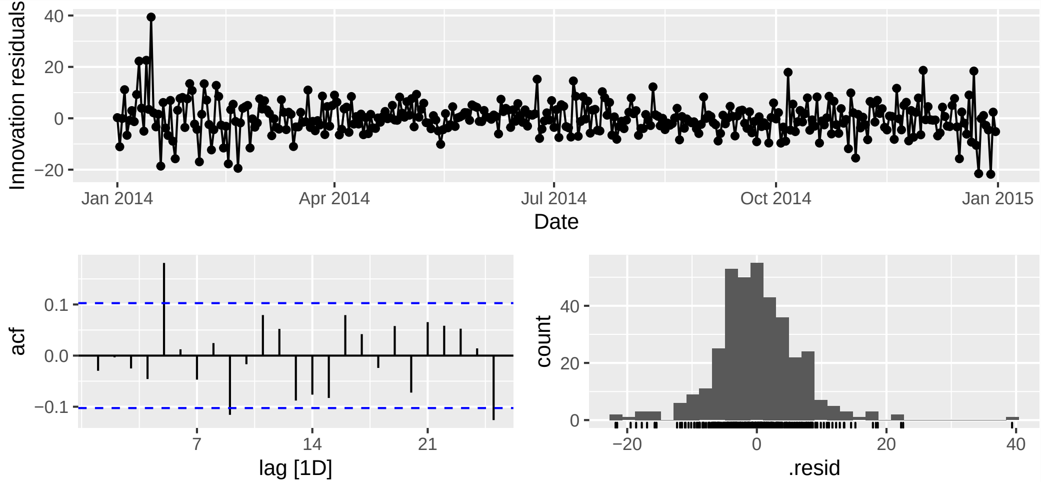
Daily electricity demand

```r
fit <- vic_elec_daily %>%
  model(ARIMA(Demand ~ Temperature + I(Temperature^2) +
                (Day_Type=="Weekday")))
report(fit)
```

```
## Series: Demand
## Model: LM w/ ARIMA(2,1,2)(2,0,0)[7] errors
##
## Coefficients:
##           ar1     ar2      ma1      ma2    sar1   sar2  Temperature
##       -0.1093  0.7226  -0.0182  -0.9381  0.1958  0.417       -7.614
## s.e.   0.0779  0.0739   0.0494   0.0493  0.0525  0.057        0.448
##       I(Temperature^2)  Day_Type == "Weekday"TRUE
##                 0.1810                      30.40
## s.e.            0.0085                       1.33
##
## sigma^2 estimated as 44.91:  log likelihood=-1206
## AIC=2432   AICc=2433   BIC=2471
```

Daily electricity demand

```
gg_tsresiduals(fit)
```



Daily electricity demand

```r
augment(fit) %>%
  features(.resid, ljung_box, dof = 9, lag = 14)
```

```
## # A tibble: 1 x 3
##   .model                                                    lb_stat lb_pvalue
##   <chr>                                                       <dbl>     <dbl>
## 1 "ARIMA(Demand ~ Temperature + I(Temperature^2) + (Day_~    28.4 0.0000304
```

Daily electricity demand

```r
# Forecast one day ahead
vic_next_day <- new_data(vic_elec_daily, 1) %>%
  mutate(Temperature = 26, Day_Type = "Holiday")
forecast(fit, vic_next_day)
```

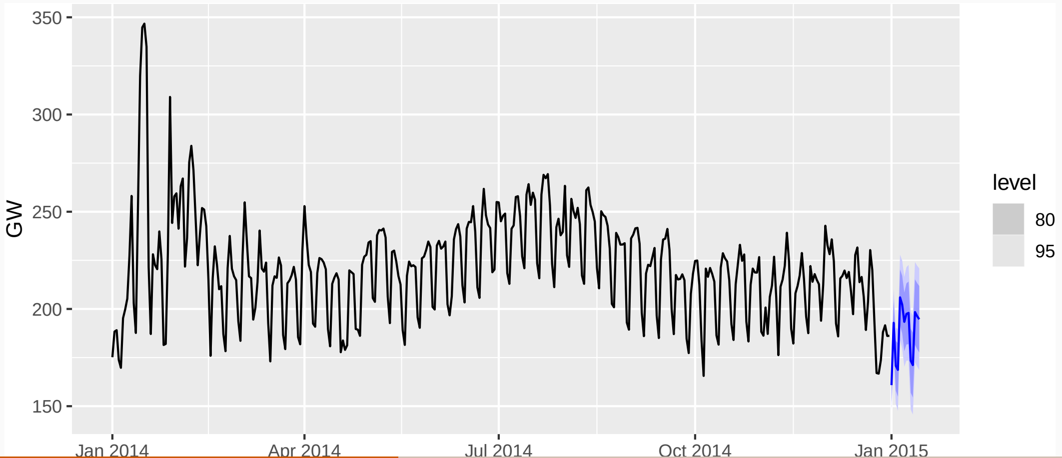
```
## # A fable: 1 x 6 [1D]
## # Key:     .model [1]
##   .model                    Date            Demand .mean Temperature Day_Type
##   <chr>                     <date>          <dist> <dbl>       <dbl> <chr>
## 1 "ARIMA(Demand ~ Tempera~ 2015-01-01 N(161, 45)  161.          26 Holiday
```

Daily electricity demand

```r
vic_elec_future <- new_data(vic_elec_daily, 14) %>%
  mutate(
    Temperature = 26,
    Holiday = c(TRUE, rep(FALSE, 13)),
    Day_Type = case_when(
      Holiday ~ "Holiday",
      wday(Date) %in% 2:6 ~ "Weekday",
      TRUE ~ "Weekend"
    )
  )
```

Daily electricity demand

```
forecast(fit, new_data = vic_elec_future) %>%
  autoplot(vic_elec_daily) + labs(y="GW")
```



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Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

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Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARIMA process with $d \geq 1$.

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARIMA process with $d \geq 1$.

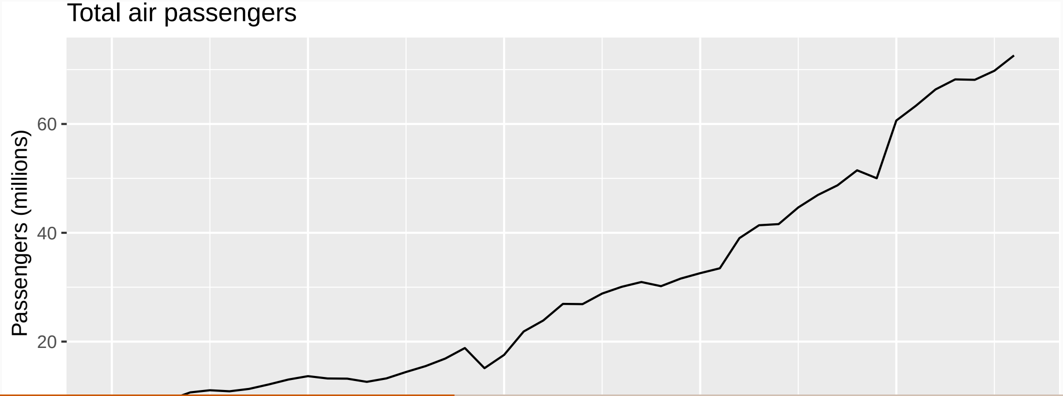
Difference both sides until η_t is stationary:

$$y'_t = \beta_1 + \eta'_t$$

where η'_t is ARMA process.

Air transport passengers Australia

```r
aus_airpassengers %>%
  autoplot(Passengers) +
  labs(y = "Passengers (millions)",
       title = "Total air passengers")
```



Air transport passengers Australia

Deterministic trend

```
fit_deterministic <- aus_airpassengers %>%
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
## Series: Passengers
## Model: LM w/ ARIMA(1,0,0) errors
##
## Coefficients:
##          ar1  trend()  intercept
##       0.9564    1.415      0.901
## s.e.  0.0362    0.197      7.075
##
## sigma^2 estimated as 4.343:  log likelihood=-101
## AIC=210   AICc=211   BIC=217
```

Air transport passengers Australia

Deterministic trend

```
fit_deterministic <- aus_airpassengers %>%
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
## Series: Passengers
## Model: LM w/ ARIMA(1,0,0) errors
##
## Coefficients:
##          ar1  trend()  intercept
##       0.9564    1.415      0.901
## s.e.  0.0362    0.197      7.075
##
## sigma^2 estimated as 4.343:  log likelihood=-101
## AIC=210   AICc=211   BIC=217
```

$$y_t = 0.901 + 1.415t + \eta_t$$
$$\eta_t = 0.956\eta_{t-1} + \varepsilon_t$$

Air transport passengers Australia

Stochastic trend

```
fit_stochastic <- aus_airpassengers %>%
  model(ARIMA(Passengers ~ pdq(d = 1)))
report(fit_stochastic)
```

```
## Series: Passengers
## Model: ARIMA(0,1,0) w/ drift
##
## Coefficients:
##       constant
##          1.419
## s.e.     0.301
##
## sigma^2 estimated as 4.271:  log likelihood=-98.2
## AIC=200   AICc=201   BIC=204
```

Air transport passengers Australia

Stochastic trend

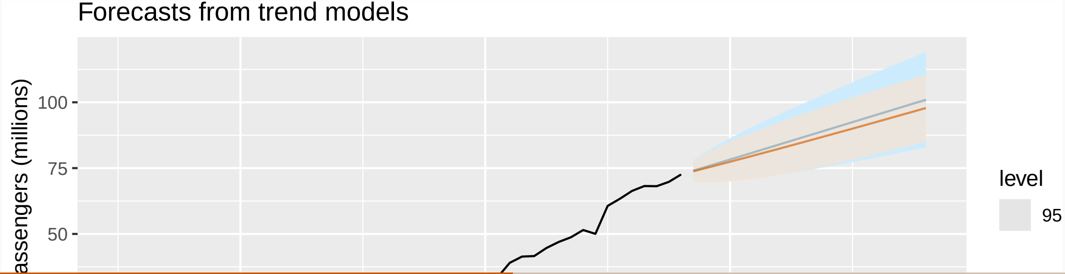
```
fit_stochastic <- aus_airpassengers %>%
  model(ARIMA(Passengers ~ pdq(d = 1)))
report(fit_stochastic)
```

```
## Series: Passengers
## Model: ARIMA(0,1,0) w/ drift
##
## Coefficients:
##       constant
##          1.419
## s.e.     0.301
##
## sigma^2 estimated as 4.271:  log likelihood=-98.2
## AIC=200   AICc=201   BIC=204
```

$$y_t - y_{t-1} = 1.419 + \varepsilon_t,$$
$$y_t = y_0 + 1.419t + \eta_t$$

Air transport passengers Australia

```
aus_airpassengers %>%
  autoplot(Passengers) +
  autolayer(fit_stochastic %>% forecast(h = 20),
    colour = "#0072B2", level = 95) +
  autolayer(fit_deterministic %>% forecast(h = 20),
    colour = "#D55E00", alpha = 0.65, level = 95) +
  labs(y = "Air passengers (millions)",
       title = "Forecasts from trend models")
```



Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.

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Dynamic harmonic regression

Combine Fourier terms with ARIMA errors

Advantages

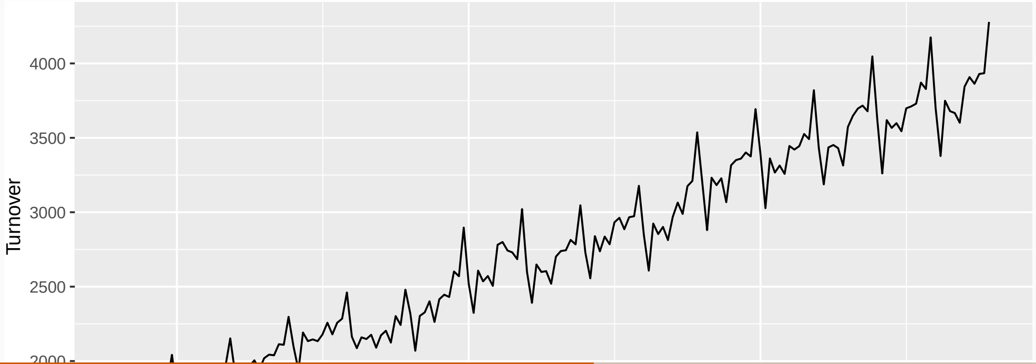
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```
aus_cafe <- aus_retail %>% filter(
    Industry == "Cafes, restaurants and takeaway food services",
    year(Month) %in% 2004:2018
  ) %>% summarise(Turnover = sum(Turnover))
aus_cafe %>% autoplot(Turnover)
```

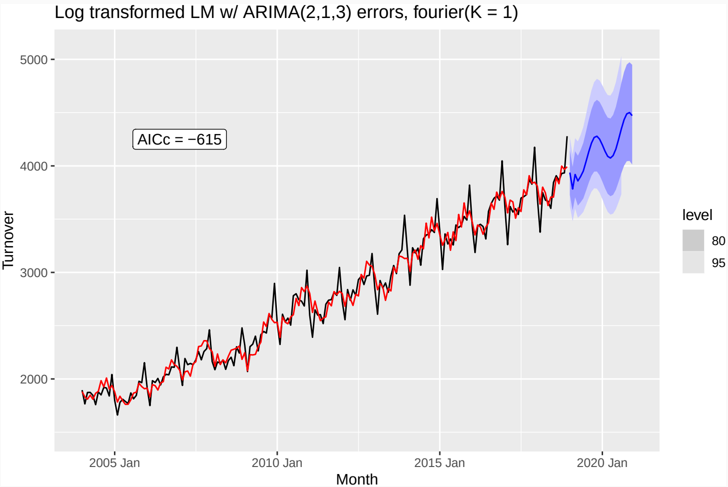


Eating-out expenditure

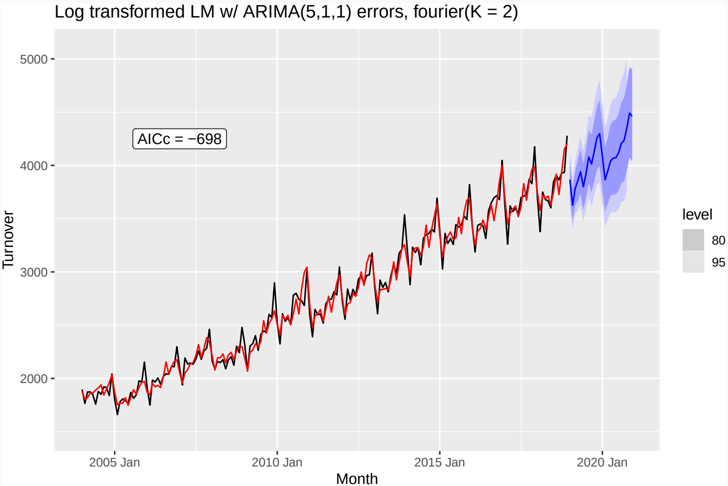
```
fit <- aus_cafe %>% model(
    `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0,0,0)),
    `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0,0,0)),
    `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0,0,0)),
    `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0,0,0)),
    `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0,0,0)),
    `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0,0,0)))
glance(fit)
```

.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	-918	-875

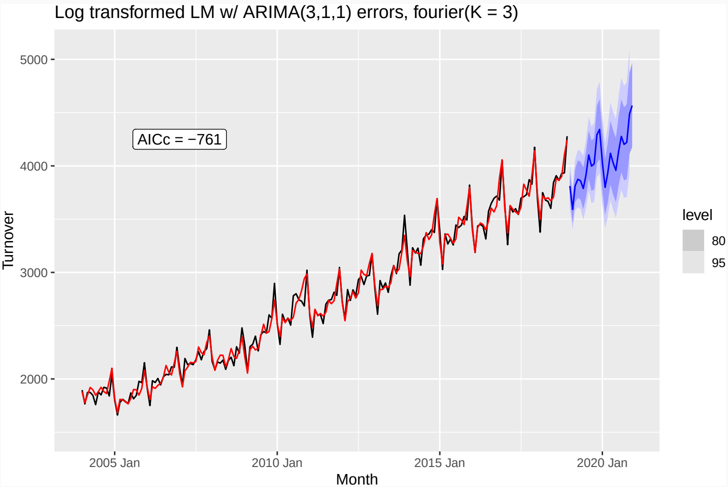
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



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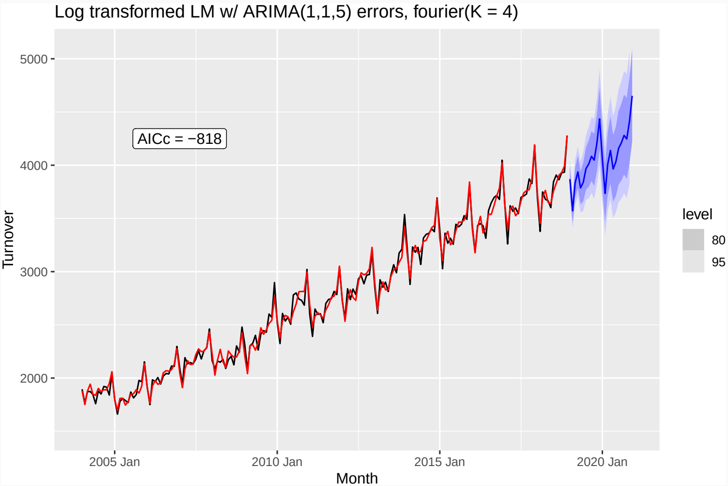

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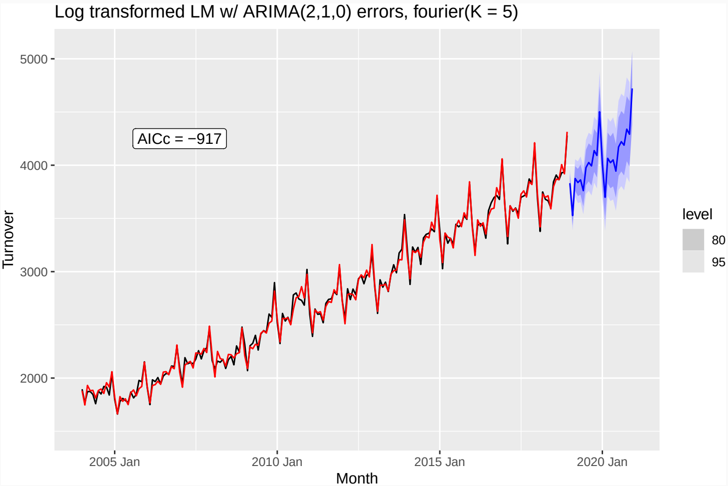

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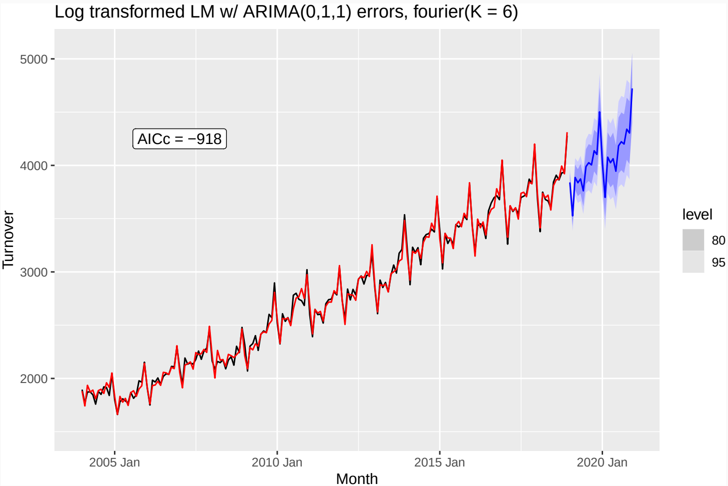


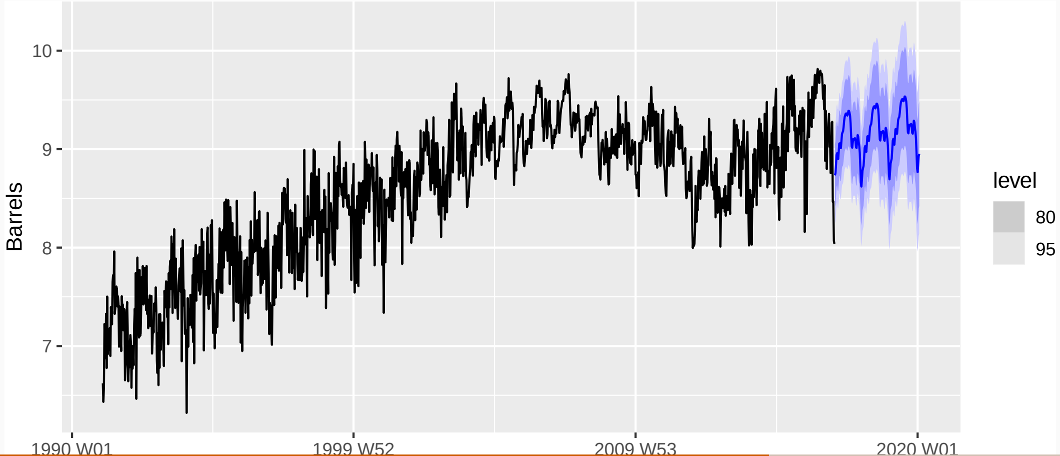
Example: weekly gasoline products

```r
fit <- us_gasoline %>%
  model(ARIMA(Barrels ~ fourier(K = 13) + PDQ(0,0,0)))
report(fit)
```

```
## Series: Barrels
## Model: LM w/ ARIMA(0,1,1) errors
##
## Coefficients:
##           ma1  fourier(K = 13)C1_52  fourier(K = 13)S1_52
##       -0.8934               -0.1121               -0.2300
## s.e.   0.0132                0.0123                0.0122
##       fourier(K = 13)C2_52  fourier(K = 13)S2_52
##                     0.0420                0.0317
## s.e.                0.0099                0.0099
##       fourier(K = 13)C3_52  fourier(K = 13)S3_52
##                     0.0832                0.0346
## s.e.                0.0094                0.0094
##       fourier(K = 13)C4_52  fourier(K = 13)S4_52
##                     0.0185                0.0398
## s.e.                0.0092                0.0092
##       fourier(K = 13)C5_52  fourier(K = 13)S5_52
```

Example: weekly gasoline products

```
forecast(fit, h = "3 years") %>%
  autoplot(us_gasoline)
```



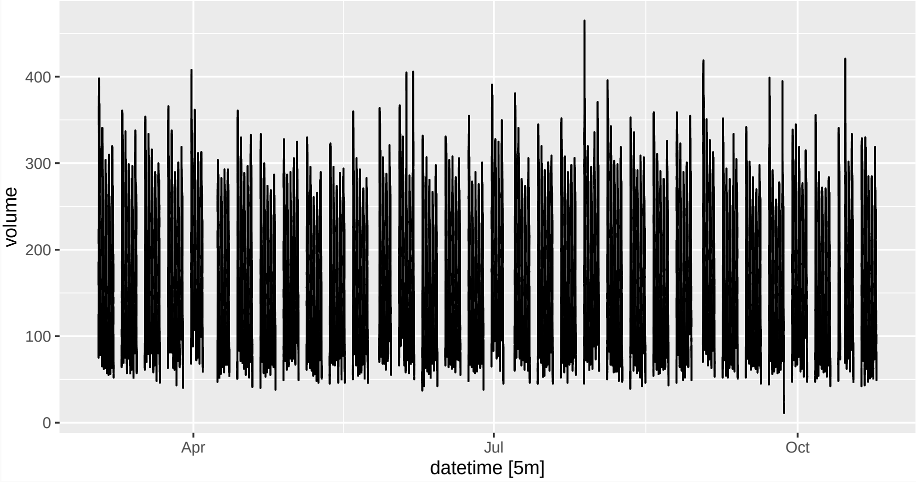
5-minute call centre volume

```
(calls <- readr::read_tsv("http://robjhyndman.com/data/callcenter.txt") %>%
  rename(time = `...1`) %>%
  pivot_longer(-time, names_to = "date", values_to = "volume") %>%
  mutate(
    date = as.Date(date, format = "%d/%m/%Y"),
    datetime = as_datetime(date) + time
  ) %>%
  as_tsibble(index = datetime))
```

```
## # A tsibble: 27,716 x 4 [5m] <UTC>
##    time   date       volume datetime
##    <time> <date>      <dbl> <dttm>
##  1 07:00  2003-03-03    111 2003-03-03 07:00:00
##  2 07:05  2003-03-03    113 2003-03-03 07:05:00
##  3 07:10  2003-03-03     76 2003-03-03 07:10:00
##  4 07:15  2003-03-03     82 2003-03-03 07:15:00
##  5 07:20  2003-03-03     91 2003-03-03 07:20:00
##  6 07:25  2003-03-03     87 2003-03-03 07:25:00
```

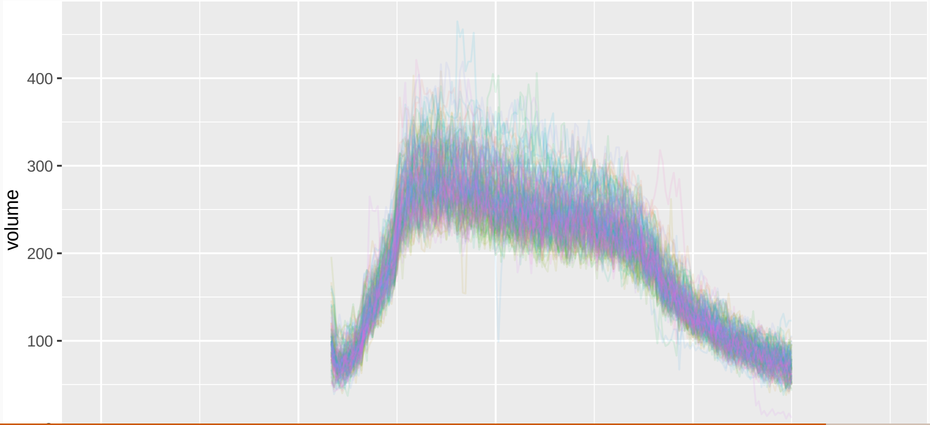
5-minute call centre volume

```
calls %>% fill_gaps() %>% autoplot(volume)
```



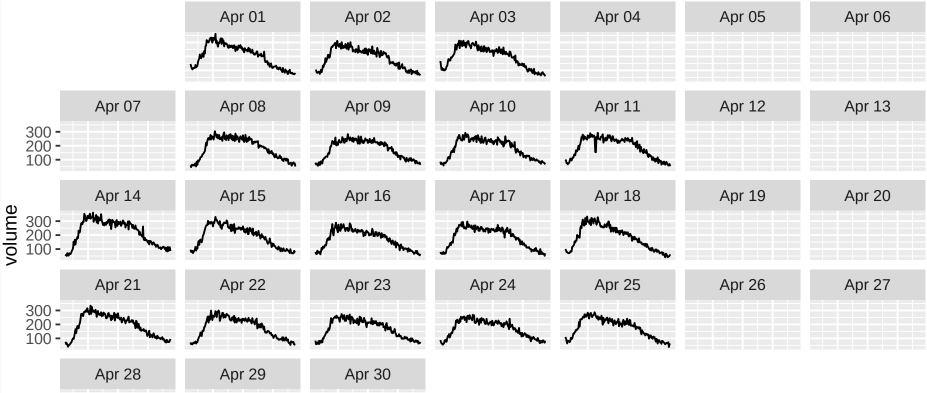
5-minute call centre volume

```
calls %>% fill_gaps() %>%
  gg_season(volume, period = "day", alpha = 0.1) +
  guides(colour = FALSE)
```



5-minute call centre volume

```r
library(sugrrants)
calls %>% filter(month(date, label = TRUE) == "Apr") %>%
  ggplot(aes(x = time, y = volume)) +
  geom_line() + facet_calendar(date)
```



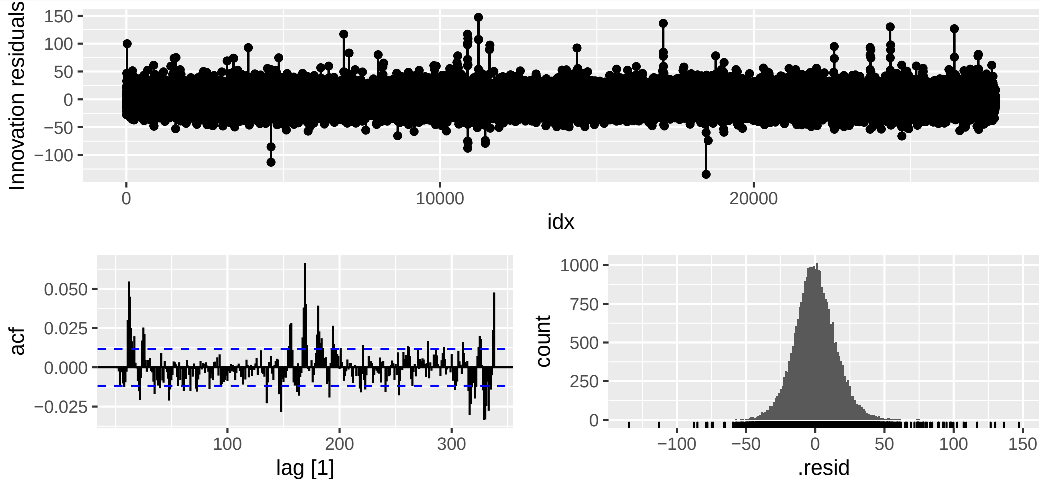
5-minute call centre volume

```r
calls_mdl <- calls %>%
  mutate(idx = row_number()) %>%
  update_tsibble(index = idx)
fit <- calls_mdl %>%
  model(ARIMA(volume ~ fourier(169, K = 10) + pdq(d=0) + PDQ(0,0,0)))
report(fit)
```

```
## Series: volume
## Model: LM w/ ARIMA(1,0,3) errors
##
## Coefficients:
##          ar1      ma1      ma2      ma3  fourier(169, K = 10)C1_169
##        0.989  -0.7383  -0.0333  -0.0282                       -79.1
## s.e.   0.001   0.0061   0.0075   0.0060                         0.7
##        fourier(169, K = 10)S1_169  fourier(169, K = 10)C2_169
##                            55.298                     -32.361
## s.e.                        0.701                       0.378
##        fourier(169, K = 10)S2_169  fourier(169, K = 10)C3_169
##                            13.742                      -9.318
## s.e.                        0.379                       0.273
##        fourier(169, K = 10)S3_169  fourier(169, K = 10)C4_169
```

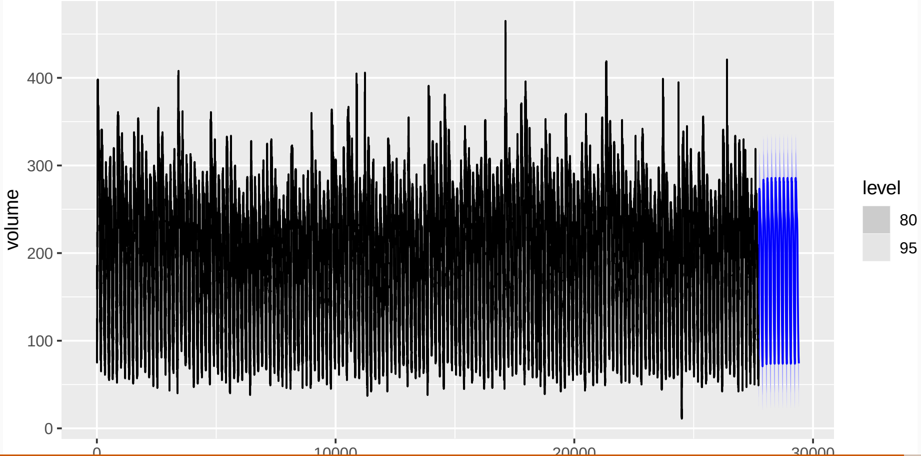
5-minute call centre volume

```r
gg_tsresiduals(fit, lag = 338)
```



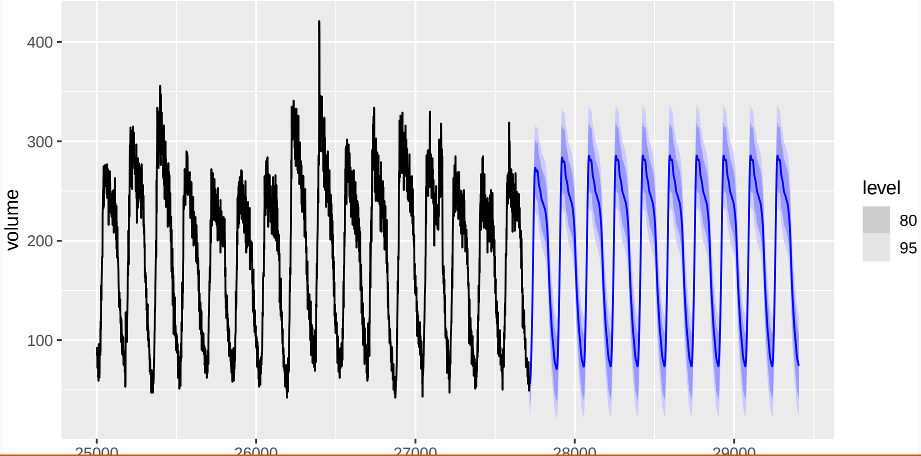
5-minute call centre volume

```r
fit %>% forecast(h = 1690) %>%
  autoplot(calls_mdl)
```



5-minute call centre volume

```r
fit %>% forecast(h = 1690) %>%
  autoplot(filter(calls_mdl, idx > 25000))
```



Outline

1 Regression with ARIMA errors

2 Stochastic and deterministic trends

3 Dynamic harmonic regression

4 Lagged predictors

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

Lagged predictors

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- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

- These are dynamic systems with input (x_t) and output (y_t).
- x_t is often a leading indicator.
- There can be multiple predictors.

Lagged predictors

The model include present and past values of predictor:

$x_t, x_{t-1}, x_{t-2}, \dots.$

$$y_t = a + \gamma_0 x_t + \gamma_1 x_{t-1} + \dots + \gamma_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

Lagged predictors

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where η_t is an ARIMA process.

Rewrite model as

$$\begin{aligned} y_t &= a + (\gamma_0 + \gamma_1 B + \gamma_2 B^2 + \cdots + \gamma_k B^k) x_t + \eta_t \\ &= a + \gamma(B) x_t + \eta_t. \end{aligned}$$

Lagged predictors

The model include present and past values of predictor: $x_t, x_{t-1}, x_{t-2}, \ldots$.

$$y_t = a + \gamma_0 x_t + \gamma_1 x_{t-1} + \cdots + \gamma_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

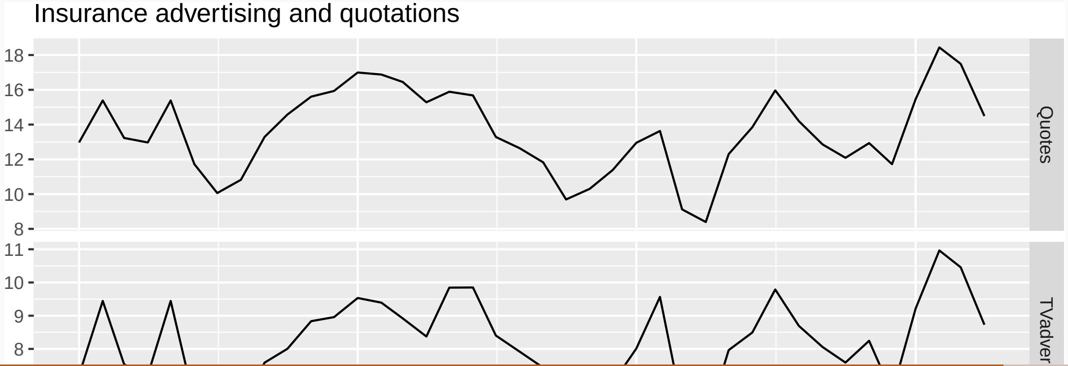
Rewrite model as

$$\begin{aligned} y_t &= a + (\gamma_0 + \gamma_1 B + \gamma_2 B^2 + \cdots + \gamma_k B^k) x_t + \eta_t \\ &= a + \gamma(B) x_t + \eta_t. \end{aligned}$$

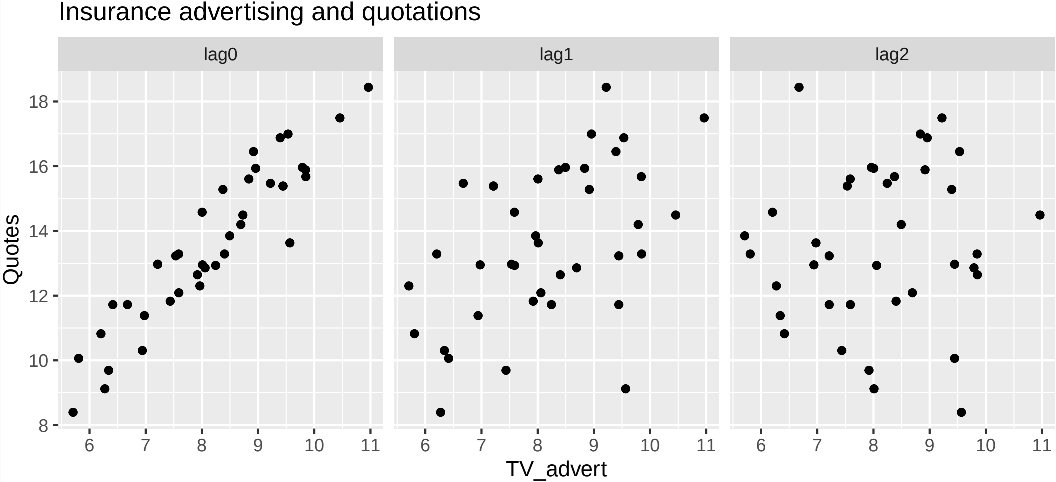
- $\gamma(B)$ is called a *transfer function* since it describes how change in x_t is transferred to y_t.

Example: Insurance quotes and TV adverts

```
insurance %>%
  pivot_longer(Quotes:TVadverts) %>%
  ggplot(aes(x = Month, y = value)) + geom_line() +
  facet_grid(vars(name), scales = "free_y") +
  labs(y = NULL, title = "Insurance advertising and quotations")
```



Example: Insurance quotes and TV adverts



Example: Insurance quotes and TV adverts

```
fit <- insurance %>%
  # Restrict data so models use same fitting period
  mutate(Quotes = c(NA,NA,NA,Quotes[4:40])) %>%
  # Estimate models
  model(
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts) +
            lag(TVadverts, 2)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts) +
            lag(TVadverts, 2) + lag(TVadverts, 3))
  )
```

Example: Insurance quotes and TV adverts

```
glance(fit)
```

Lag order	sigma2	log_lik	AIC	AICc	BIC
0	0.265	-28.3	66.6	68.3	75.0
1	0.209	-24.0	58.1	59.9	66.5
2	0.215	-24.0	60.0	62.6	70.2
3	0.206	-22.2	60.3	65.0	73.8

Example: Insurance quotes and TV adverts

```r
fit_best <- insurance %>%
  model(ARIMA(Quotes ~ pdq(d=0) + TVadverts + lag(TVadverts)))
report(fit_best)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##         ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
##       0.512  0.917  0.459     1.2527          0.1464       2.16
## s.e.  0.185  0.205  0.190     0.0588          0.0531       0.86
##
## sigma^2 estimated as 0.2166:  log likelihood=-23.9
## AIC=61.9   AICc=65.4   BIC=73.7
```

Example: Insurance quotes and TV adverts

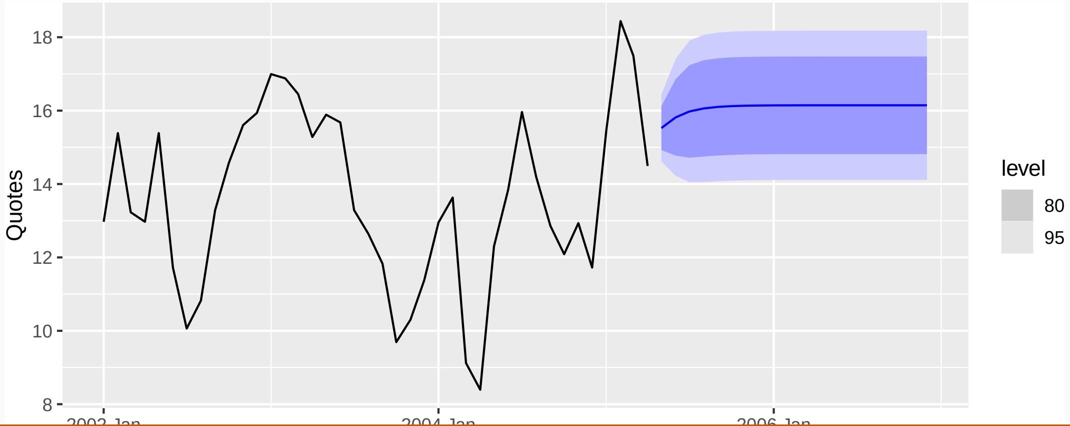
```r
fit_best <- insurance %>%
  model(ARIMA(Quotes ~ pdq(d=0) + TVadverts + lag(TVadverts)))
report(fit_best)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##          ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
##        0.512  0.917  0.459     1.2527          0.1464       2.16
## s.e.   0.185  0.205  0.190     0.0588          0.0531       0.86
##
## sigma^2 estimated as 0.2166:  log likelihood=-23.9
## AIC=61.9   AICc=65.4   BIC=73.7
```

$$
\begin{aligned}
y_t &= 2.155 + 1.253x_t + 0.146x_{t-1} + \eta_t, \\
\eta_t &= 0.512\eta_{t-1} + \varepsilon_t + 0.917\varepsilon_{t-1} + 0.459\varepsilon_{t-2},
\end{aligned}
$$

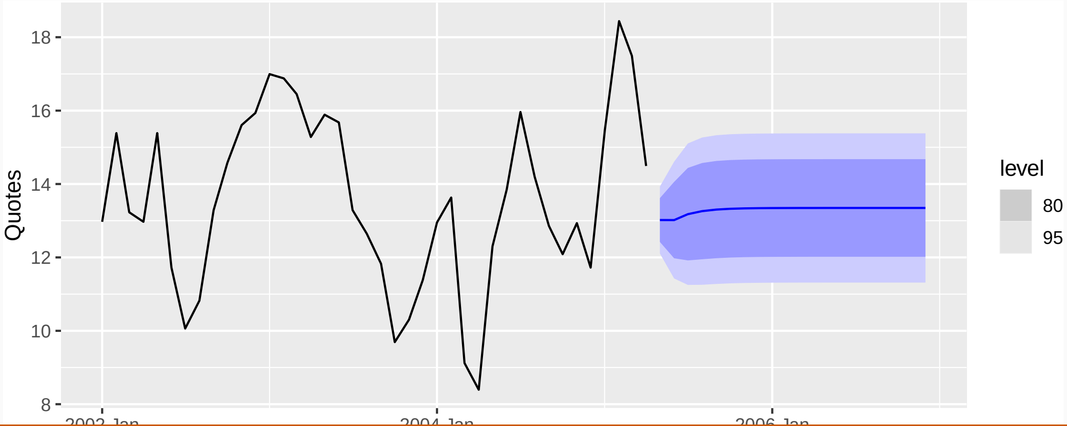
Example: Insurance quotes and TV adverts

```r
advert_a <- new_data(insurance, 20) %>%
  mutate(TVadverts = 10)
forecast(fit_best, advert_a) %>% autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_b <- new_data(insurance, 20) %>%
  mutate(TVadverts = 8)
forecast(fit_best, advert_b) %>% autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_c <- new_data(insurance, 20) %>%
  mutate(TVadverts = 6)
forecast(fit_best, advert_c) %>% autoplot(insurance)
```