

ETC3550/ETC5550

Applied forecasting

Ch5. The forecasters' toolbox

OTexts.org/fpp3/



Outline

- 1 A tidy forecasting workflow
- 2 Some simple forecasting methods
- 3 Residual diagnostics
- 4 Distributional forecasts and prediction intervals
- 5 Forecasting with transformations
- 6 Forecasting and decomposition
- 7 Evaluating forecast accuracy

Outline

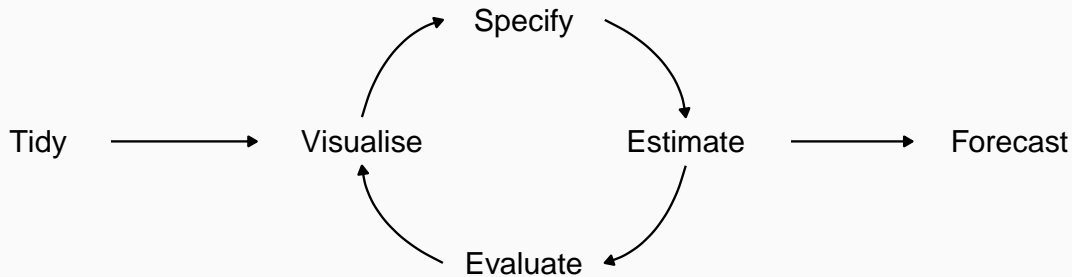
- 1 A tidy forecasting workflow
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A tidy forecasting workflow

The process of producing forecasts can be split up into a few fundamental steps.

- 1 Preparing data
- 2 Data visualisation
- 3 Specifying a model
- 4 Model estimation
- 5 Accuracy & performance evaluation
- 6 Producing forecasts

A tidy forecasting workflow



Data preparation (tidy)

```
gdppc <- global_economy %>%  
  mutate(GDP_per_capita = GDP/Population) %>%  
  select(Year, Country, GDP, Population, GDP_per_capita)  
gdppc
```

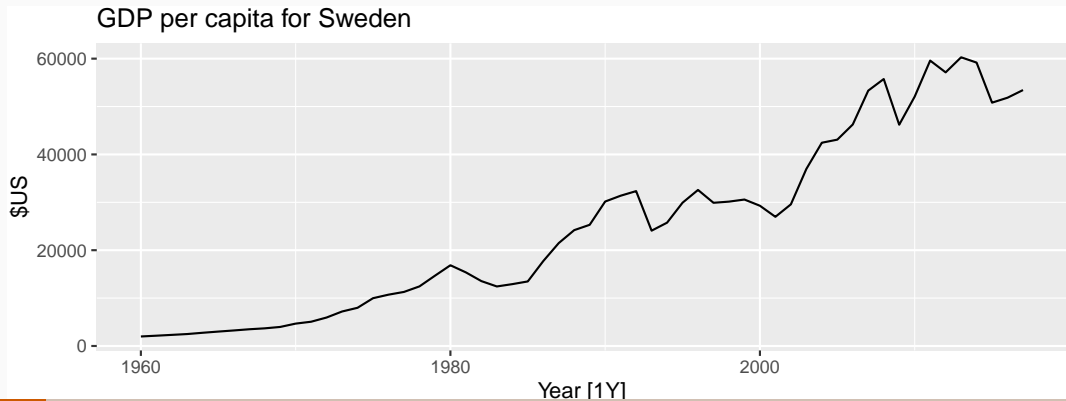
```
## # A tibble: 15,150 x 5 [1Y]
```

```
## # Key:      Country [263]
```

| ## | Year | Country | GDP | Population | GDP_per_capita |
|------|-------|-------------|-------------|------------|----------------|
| ## | <dbl> | <fct> | <dbl> | <dbl> | <dbl> |
| ## 1 | 1960 | Afghanistan | 537777811. | 8996351 | 59.8 |
| ## 2 | 1961 | Afghanistan | 548888896. | 9166764 | 59.9 |
| ## 3 | 1962 | Afghanistan | 546666678. | 9345868 | 58.5 |
| ## 4 | 1963 | Afghanistan | 751111191. | 9533954 | 78.8 |
| ## 5 | 1964 | Afghanistan | 800000044. | 9731361 | 82.2 |
| ## 6 | 1965 | Afghanistan | 1006666638. | 9938414 | 101. |

Data visualisation

```
gdppc %>%  
  filter(Country=="Sweden") %>%  
  autoplot(GDP_per_capita) +  
    labs(title = "GDP per capita for Sweden", y = "$US")
```



Model estimation

The `model()` function trains models to data.

```
fit <- gdppc %>%  
  model(trend_model = TSLM(GDP_per_capita ~ trend()))  
fit
```

```
## # A mable: 263 x 2  
## # Key:      Country [263]  
##   Country      trend_model  
##   <fct>        <model>  
## 1 Afghanistan <TSLM>  
## 2 Albania     <TSLM>  
## 3 Algeria     <TSLM>  
## 4 American Samoa <TSLM>  
## 5 Andorra     <TSLM>
```


Model estimation

The `model()` function trains models to data.

```
fit <- gdppc %>%  
  model(trend_model = TSLM(GDP_per_capita ~ trend()))  
fit
```

```
## # A mable: 263 x 2  
## # Key:      Country [263]  
##   Country      trend_model  
##   <fct>         <model>  
## 1 Afghanistan <TSLM>  
## 2 Albania      <TSLM>  
## 3 Algeria      <TSLM>  
## 4 American Samoa <TSLM>  
## 5 Andorra      <TSLM>
```

A mable is a model table, each cell corresponds to a fitted model.

Producing forecasts

```
fit %>% forecast(h = "3 years")
```

```
## # A tibble: 789 x 5 [1Y]
```

```
## # Key:   Country, .model [263]
```

| ## | Country | .model | Year | GDP_per_capita | .mean |
|----|---------------|-------------|-------|-----------------|-------|
| ## | <fct> | <chr> | <dbl> | <dist> | <dbl> |
| ## | 1 Afghanistan | trend_model | 2018 | N(526, 9653) | 526. |
| ## | 2 Afghanistan | trend_model | 2019 | N(534, 9689) | 534. |
| ## | 3 Afghanistan | trend_model | 2020 | N(542, 9727) | 542. |
| ## | 4 Albania | trend_model | 2018 | N(4716, 476419) | 4716. |
| ## | 5 Albania | trend_model | 2019 | N(4867, 481086) | 4867. |
| ## | 6 Albania | trend_model | 2020 | N(5018, 486012) | 5018. |
| ## | 7 Algeria | trend_model | 2018 | N(4410, 643094) | 4410. |
| ## | 8 Algeria | trend_model | 2019 | N(4489, 645311) | 4489. |

Producing forecasts

```
fit %>% forecast(h = "3 years")
```

A fable is a forecast table with point forecasts and distributions.

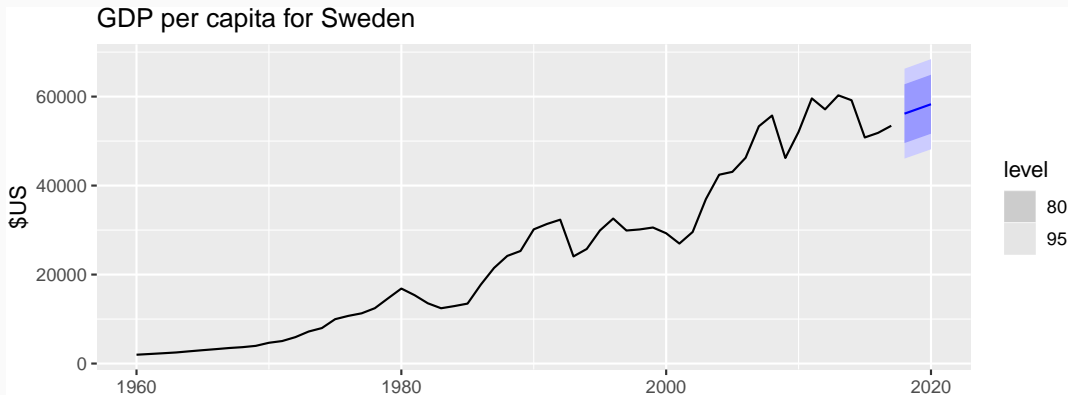
```
## # A fable: 789 x 5 [1Y]
```

```
## # Key:      Country, .model [263]
```

| ## | Country | .model | Year | GDP_per_capita | .mean |
|----|---------------|-------------|-------|-----------------|-------|
| ## | <fct> | <chr> | <dbl> | <dist> | <dbl> |
| ## | 1 Afghanistan | trend_model | 2018 | N(526, 9653) | 526. |
| ## | 2 Afghanistan | trend_model | 2019 | N(534, 9689) | 534. |
| ## | 3 Afghanistan | trend_model | 2020 | N(542, 9727) | 542. |
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| ## | 5 Albania | trend_model | 2019 | N(4867, 481086) | 4867. |
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| ## | 8 Algeria | trend_model | 2019 | N(4489, 645311) | 4489. |

Visualising forecasts

```
fit %>% forecast(h = "3 years") %>%  
  filter(Country=="Sweden") %>%  
  autoplot(gdppc) +  
    labs(title = "GDP per capita for Sweden", y = "$US")
```



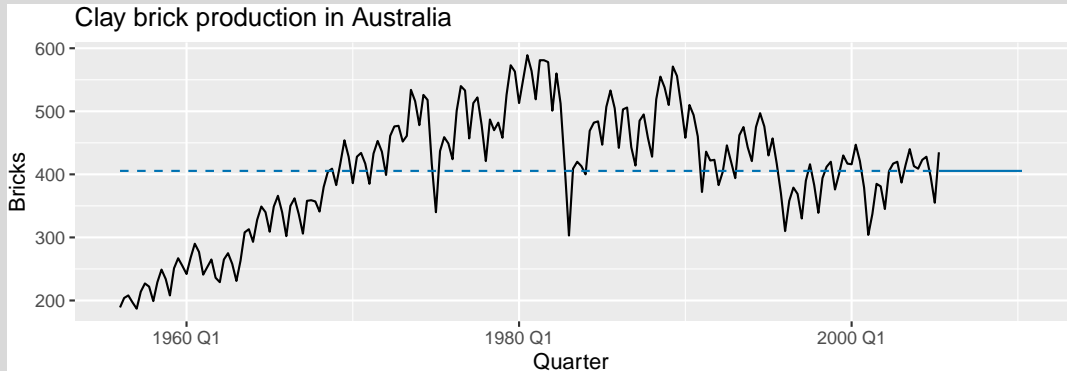
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Some simple forecasting methods

MEAN(y): Average method

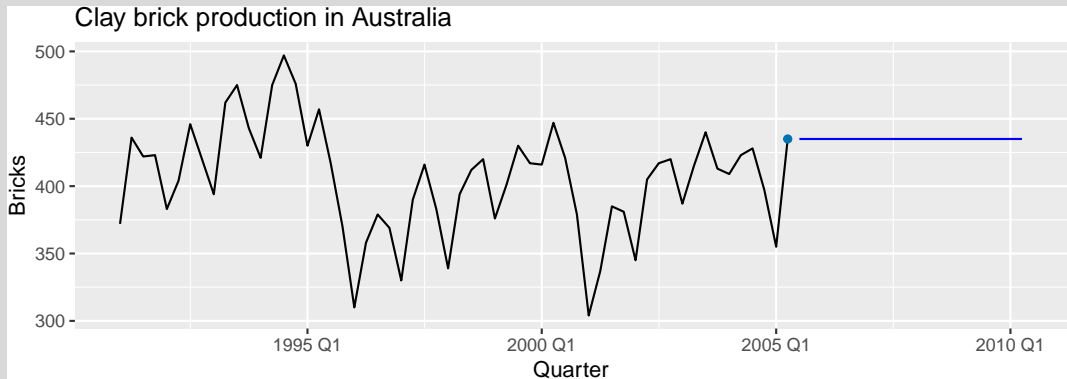
- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$



Some simple forecasting methods

NAIVE(y): Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

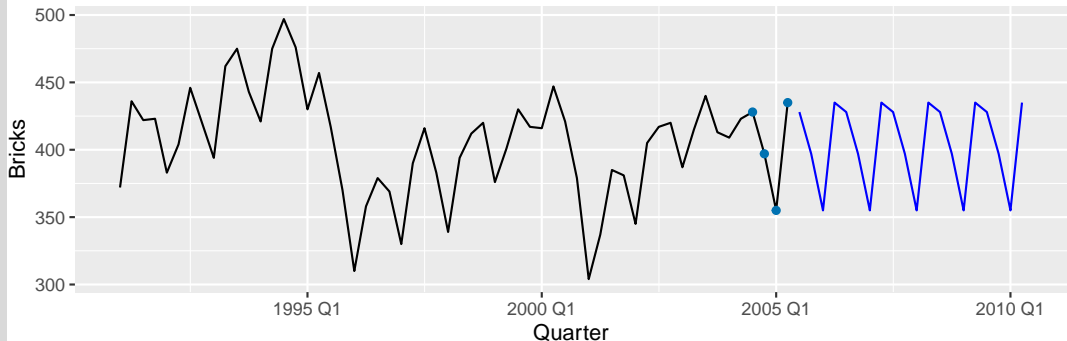


Some simple forecasting methods

SNAIVE($y \sim \text{lag}(m)$): Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.

Clay brick production in Australia



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

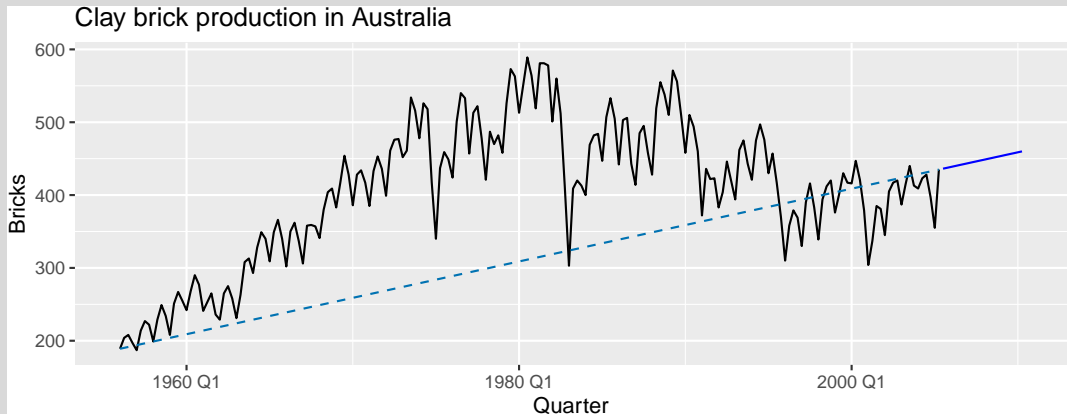
- Forecasts equal to last value plus average change.
- Forecasts:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
brick_fit <- aus_production %>%  
  filter(!is.na(Bricks)) %>%  
  model(  
    Seasonal_naive = SNAIVE(Bricks),  
    Naive = NAIVE(Bricks),  
    Drift = RW(Bricks ~ drift()),  
    Mean = MEAN(Bricks)  
  )
```

```
## # A mable: 1 x 4  
##   Seasonal_naive   Naive        Drift    Mean  
##           <model> <model>      <model> <model>  
## 1           <SNAIVE> <NAIVE> <RW w/ drift> <MEAN>
```

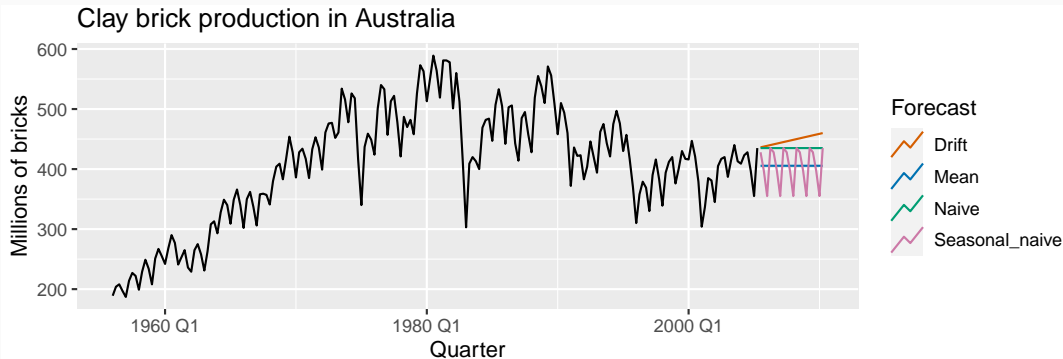
Producing forecasts

```
brick_fc <- brick_fit %>%  
  forecast(h = "5 years")
```

```
## # A fable: 80 x 4 [1Q]  
## # Key:      .model [4]  
##   .model      Quarter      Bricks .mean  
##   <chr>        <qtr>        <dist> <dbl>  
## 1 Seasonal_naive 2005 Q3 N(428, 2336) 428  
## 2 Seasonal_naive 2005 Q4 N(397, 2336) 397  
## 3 Seasonal_naive 2006 Q1 N(355, 2336) 355  
## 4 Seasonal_naive 2006 Q2 N(435, 2336) 435  
## # ... with 76 more rows
```

Visualising forecasts

```
brick_fc %>%  
  autoplot(aus_production, level = NULL) +  
  labs(title = "Clay brick production in Australia",  
        y = "Millions of bricks") +  
  guides(colour = guide_legend(title = "Forecast"))
```



Facebook closing stock price

```
# Extract training data
fb_stock <- gafa_stock %>%
  filter(Symbol == "FB") %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE)

# Specify, estimate and forecast
fb_stock %>%
  model(
    Mean = MEAN(Close),
    Naive = NAIVE(Close),
    Drift = RW(Close ~ drift())
  ) %>%
  forecast(h=42) %>%
  autoplot(fb_stock, level = NULL) +
  labs(title = "Facebook closing stock price", y="$US") +
  guides(colour=guide_legend(title="Forecast"))
```

Facebook closing stock price



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Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations y_1, \dots, y_{t-1} .
- We call these “fitted values”.
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

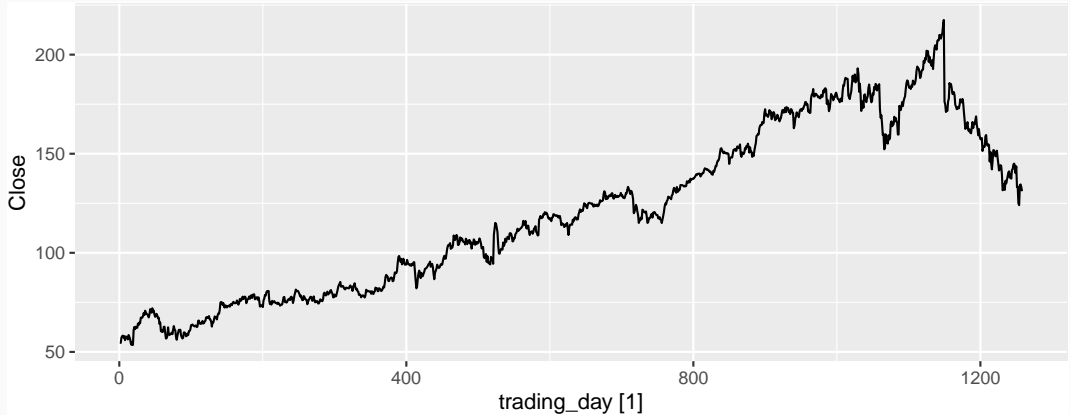
- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for distributions & prediction intervals)

- 3 $\{e_t\}$ have constant variance.
- 4 $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock %>% autoplot(Close)
```



Facebook closing stock price

```
fit <- fb_stock %>% model(NAIVE(Close))  
augment(fit)
```

```
## # A tsibble: 1,258 x 7 [1]  
## # Key:      Symbol, .model [1]  
##   Symbol .model      trading_day Close .fitted .resid .innov  
##   <chr>  <chr>          <int> <dbl>  <dbl>  <dbl>  <dbl>  
## 1 FB    NAIVE(Close)      1  54.7   NA    NA    NA  
## 2 FB    NAIVE(Close)      2  54.6   54.7 -0.150 -0.150  
## 3 FB    NAIVE(Close)      3  57.2   54.6  2.64   2.64  
## 4 FB    NAIVE(Close)      4  57.9   57.2  0.720  0.720  
## 5 FB    NAIVE(Close)      5  58.2   57.9  0.310  0.310  
## 6 FB    NAIVE(Close)      6  57.2   58.2 -1.01  -1.01  
## 7 FB    NAIVE(Close)      7  57.9   57.2  0.720  0.720  
## 8 FB    NAIVE(Close)      8  55.9   57.9 -2.03  -2.03  
## 9 FB    NAIVE(Close)      9  57.7   55.9  1.83   1.83  
## 10 FB   NAIVE(Close)     10  57.6   57.7 -0.140 -0.140  
## # ... with 1,248 more rows
```

Facebook closing stock price

```
fit <- fb_stock %>% model(NAIVE(Close))  
augment(fit)
```

```
## # A tsibble: 1,258 x 7 [1]  
## # Key:      Symbol, .model [1]  
##   Symbol .model      trading_day Close .fitted .resid .innov  
##   <chr>   <chr>          <int> <dbl>   <dbl>   <dbl>   <dbl>  
## 1 FB     NAIVE(Close)         1  54.7    NA     NA      NA  
## 2 FB     NAIVE(Close)         2  54.6    54.7  -0.150  -0.150  
## 3 FB     NAIVE(Close)         3  57.2    54.6   2.64    2.64  
## 4 FB     NAIVE(Close)         4  57.9    57.2   0.720   0.720  
## 5 FB     NAIVE(Close)         5  58.2    57.9   0.310   0.310  
## 6 FB     NAIVE(Close)         6  57.2    58.2  -1.01   -1.01  
## 7 FB     NAIVE(Close)         7  57.9    57.2   0.720   0.720  
## 8 FB     NAIVE(Close)         8  55.9    57.9  -2.03   -2.03  
## 9 FB     NAIVE(Close)         9  57.7    55.9   1.83    1.83  
## 10 FB    NAIVE(Close)        10  57.6    57.7  -0.140  -0.140
```

 $\hat{y}_{t|t-1}$ e_t

Naïve forecasts:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

```
## # ... with 1,248 more rows
```

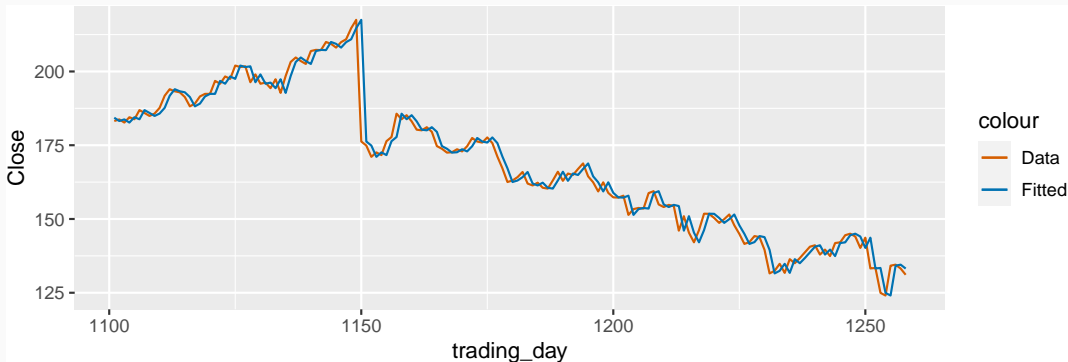
Facebook closing stock price

```
augment(fit) %>%  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



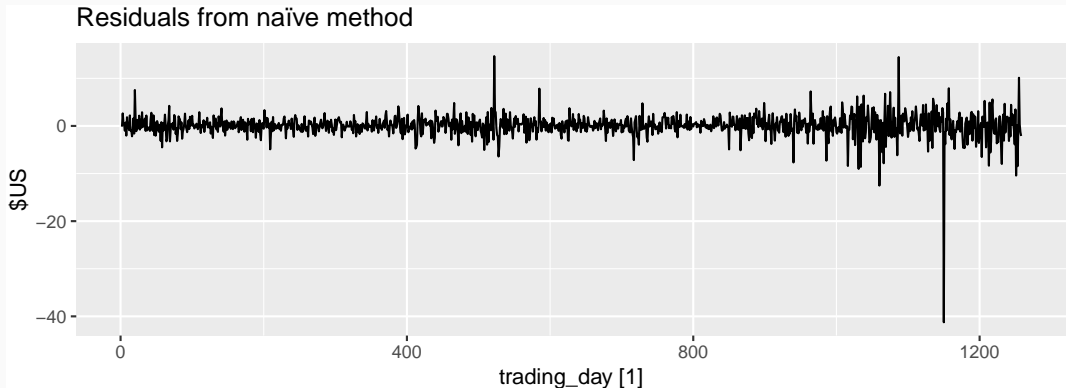
Facebook closing stock price

```
augment(fit) %>%  
  filter(trading_day > 1100) %>%  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



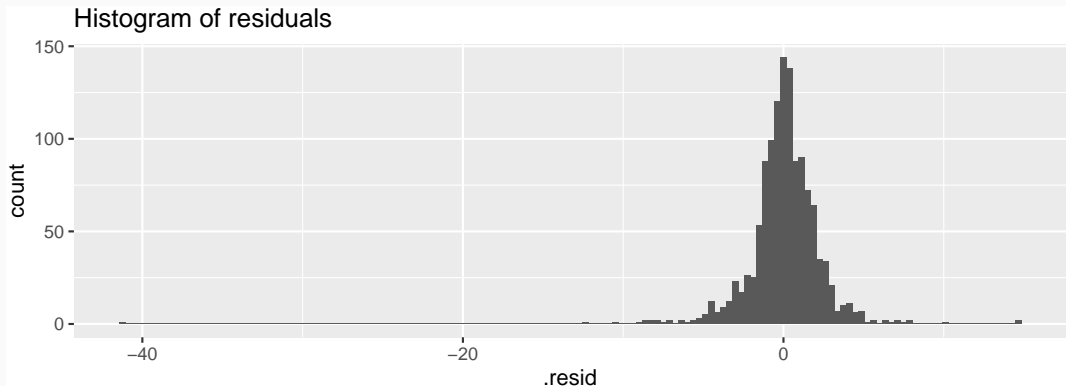
Facebook closing stock price

```
augment(fit) %>%  
  autoplot(.resid) +  
  labs(y = "$US",  
       title = "Residuals from naïve method")
```



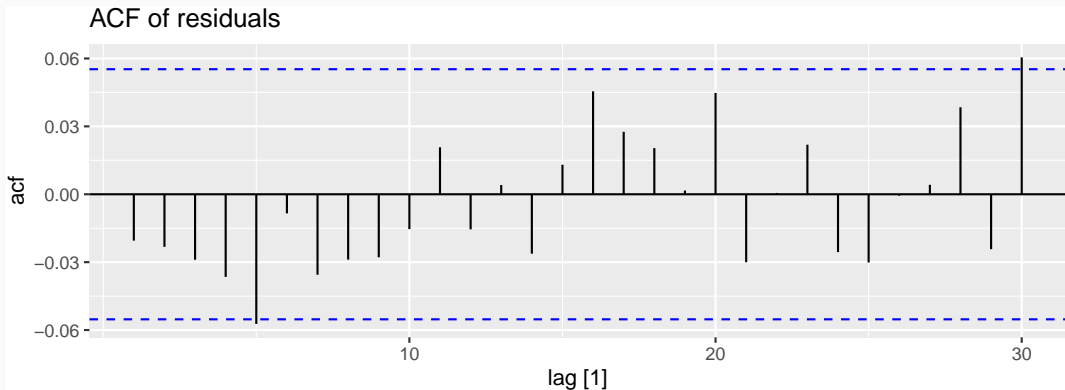
Facebook closing stock price

```
augment(fit) %>%  
  ggplot(aes(x = .resid)) +  
  geom_histogram(bins = 150) +  
  labs(title = "Histogram of residuals")
```



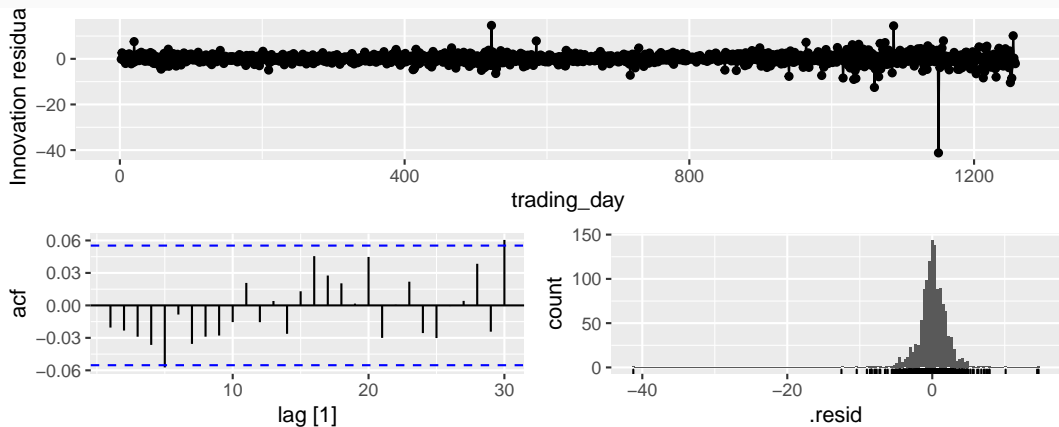
Facebook closing stock price

```
augment(fit) %>%  
  ACF(.resid) %>%  
  autoplot() + labs(title = "ACF of residuals")
```



gg_tsresiduals() function

```
gg_tsresiduals(fit)
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

r_k = autocorrelation of residual at lag k

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Portmanteau tests

r_k = autocorrelation of residual at lag k

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Box-Pierce test

$$Q = T \sum_{k=1}^{\ell} r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

r_k = autocorrelation of residual at lag k

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2) \sum_{k=1}^{\ell} (T-k)^{-1} r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- My preferences: $\ell = 10$ for non-seasonal data, $h = 2m$ for seasonal data (where m is seasonal period).
- Better performance, especially in small samples.

Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(\ell - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.
- $\text{lag} = \ell$, $\text{dof} = K$

```
augment(fit) %>%  
  features(.resid, lbjung_box, lag=10, dof=0)
```

```
## # A tibble: 1 x 4  
##   Symbol .model      lb_stat lb_pvalue  
##   <chr>   <chr>      <dbl>    <dbl>  
## 1 FB     NAIVE(Close)    12.1     0.276
```

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Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, $\text{sd} = \hat{\sigma}$:

Mean: $y_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

Naïve: $y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

Seasonal naïve: $y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$

Drift: $y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h -step distribution.

- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```
brick_fc %>% hilo(level = 95)
```

```
## # A tsibble: 80 x 5 [1Q]
```

```
## # Key:           .model [4]
```

| ## | .model | Quarter | Bricks | .mean | `95%` |
|----|------------------|---------|--------------|-------|--------------|
| ## | <chr> | <qtr> | <dist> | <dbl> | <hilo> |
| ## | 1 Seasonal_naive | 2005 Q3 | N(428, 2336) | 428 | [333, 523]95 |
| ## | 2 Seasonal_naive | 2005 Q4 | N(397, 2336) | 397 | [302, 492]95 |
| ## | 3 Seasonal_naive | 2006 Q1 | N(355, 2336) | 355 | [260, 450]95 |
| ## | 4 Seasonal_naive | 2006 Q2 | N(435, 2336) | 435 | [340, 530]95 |
| ## | 5 Seasonal_naive | 2006 Q3 | N(428, 4672) | 428 | [294, 562]95 |
| ## | 6 Seasonal_naive | 2006 Q4 | N(397, 4672) | 397 | [263, 531]95 |
| ## | 7 Seasonal_naive | 2007 Q1 | N(355, 4672) | 355 | [221, 489]95 |
| ## | 8 Seasonal_naive | 2007 Q2 | N(435, 4672) | 435 | [301, 569]95 |
| ## | 9 Seasonal_naive | 2007 Q3 | N(428, 7008) | 428 | [264, 592]95 |

Prediction intervals

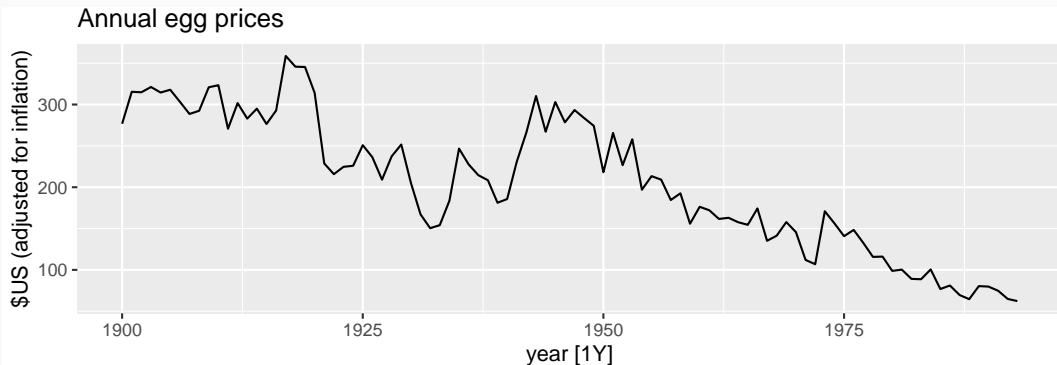
- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.

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Modelling with transformations

```
eggs <- prices %>%  
  filter(!is.na(eggs)) %>% select(eggs)  
eggs %>% autoplot() +  
  labs(title="Annual egg prices",  
        y="$US (adjusted for inflation)")
```



Modelling with transformations

Transformations used in the left of the formula will be automatically back-transformed. To model log-transformed egg prices, you could use:

```
fit <- eggs %>%  
  model(RW(log(eggs) ~ drift()))  
fit
```

```
## # A mable: 1 x 1  
##   `RW(log(eggs) ~ drift())`  
##                               <model>  
## 1                             <RW w/ drift>
```

Forecasting with transformations

```
fc <- fit %>%  
  forecast(h = 50)  
fc
```

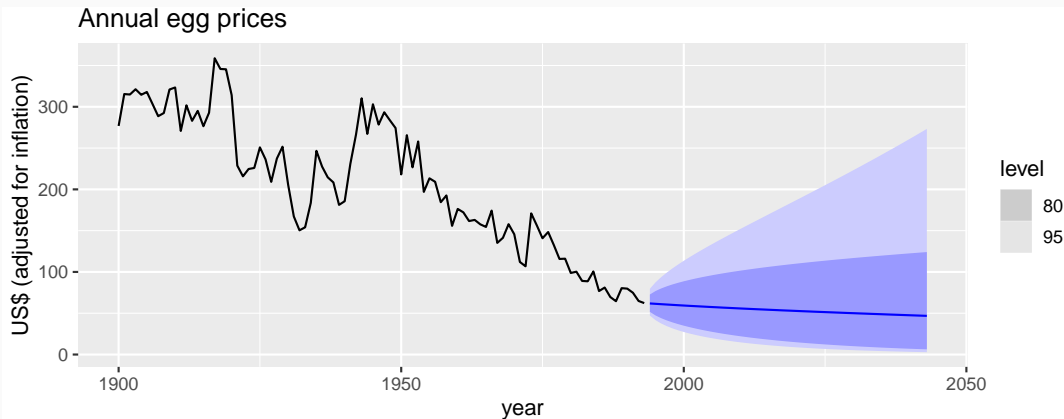
```
## # A tibble: 50 x 4 [1Y]
```

```
## # Key:           .model [1]
```

| ## | .model | year | eggs | .mean |
|----|---------------------------|-------|------------------|-------|
| ## | <chr> | <dbl> | <dist> | <dbl> |
| ## | 1 RW(log(eggs) ~ drift()) | 1994 | t(N(4.1, 0.018)) | 61.8 |
| ## | 2 RW(log(eggs) ~ drift()) | 1995 | t(N(4.1, 0.036)) | 61.4 |
| ## | 3 RW(log(eggs) ~ drift()) | 1996 | t(N(4.1, 0.054)) | 61.0 |
| ## | 4 RW(log(eggs) ~ drift()) | 1997 | t(N(4.1, 0.073)) | 60.5 |
| ## | 5 RW(log(eggs) ~ drift()) | 1998 | t(N(4.1, 0.093)) | 60.1 |
| ## | 6 RW(log(eggs) ~ drift()) | 1999 | t(N(4, 0.11)) | 59.7 |
| ## | 7 RW(log(eggs) ~ drift()) | 2000 | t(N(4, 0.13)) | 59.3 |
| ## | 8 RW(log(eggs) ~ drift()) | 2001 | t(N(4, 0.15)) | 58.8 |

Forecasting with transformations

```
fc %>% autoplot(eggs) +  
  labs(title="Annual egg prices",  
        y="US$ (adjusted for inflation)")
```



Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

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Let X be have mean μ and variance σ^2 .

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ :

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2f''(\mu).$$

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$$E[Y] = E[f(X)] = f(\mu) + \frac{1}{2}\sigma^2f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

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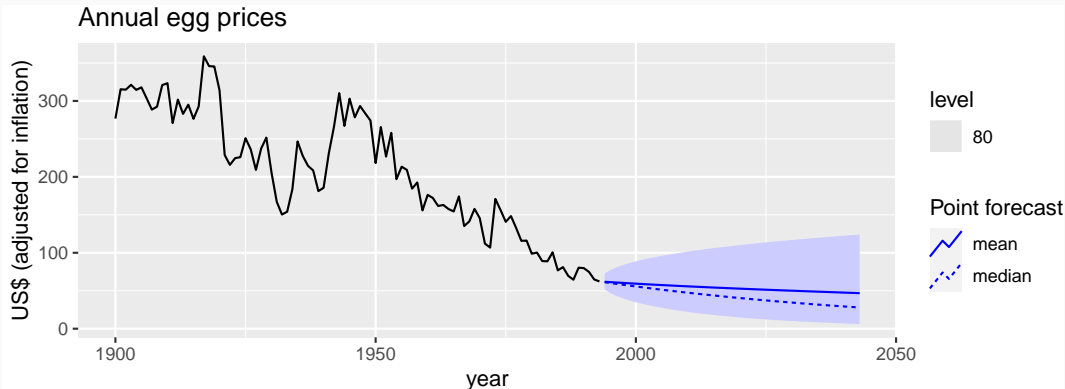
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$$E[Y] = \begin{cases} e^\mu \left[1 + \frac{\sigma^2}{2} \right] & \lambda = 0; \\ (\lambda \mu + 1)^{1/\lambda} \left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda \mu + 1)^2} \right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
fc %>%  
  autoplot(eggs, level = 80, point_forecast = lst(mean, median)) +  
  labs(title="Annual egg prices",  
        y="US$ (adjusted for inflation)")
```



Outline

- 1 A tidy forecasting workflow
- 2 Some simple forecasting methods
- 3 Residual diagnostics
- 4 Distributional forecasts and prediction intervals
- 5 Forecasting with transformations
- 6 Forecasting and decomposition
- 7 Evaluating forecast accuracy

Forecasting and decomposition

$$y_t = \hat{S}_t + \hat{A}_t$$

- \hat{A}_t is seasonally adjusted component
 - \hat{S}_t is seasonal component.
-
- Forecast \hat{S}_t using SNAIVE.
 - Forecast \hat{A}_t using non-seasonal time series method.
 - Combine forecasts of \hat{S}_t and \hat{A}_t to get forecasts of original data.

US Retail Employment

```
us_retail_employment <- us_employment %>%  
  filter(year(Month) >= 1990, Title == "Retail Trade") %>%  
  select(-Series_ID)  
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]  
##       Month Title      Employed  
##       <mth> <chr>      <dbl>  
## 1 1990 Jan Retail Trade 13256.  
## 2 1990 Feb Retail Trade 12966.  
## 3 1990 Mar Retail Trade 12938.  
## 4 1990 Apr Retail Trade 13012.  
## 5 1990 May Retail Trade 13108.  
## 6 1990 Jun Retail Trade 13183.  
## 7 1990 Jul Retail Trade 13170.  
## 8 1990 Aug Retail Trade 13160.  
## 9 1990 Sep Retail Trade 13112.
```

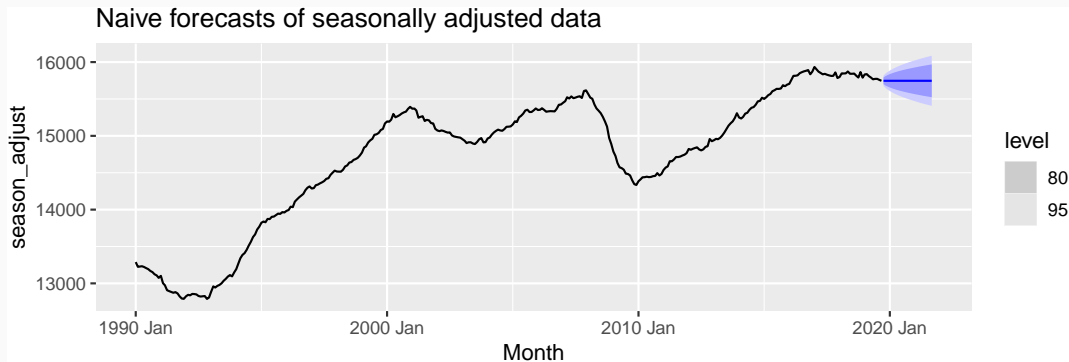
US Retail Employment

```
dcmp <- us_retail_employment %>%  
  model(STL(Employed)) %>%  
  components() %>% select(-.model)  
dcmp
```

```
## # A tsibble: 357 x 6 [1M]  
##       Month Employed trend season_year remainder season_adjust  
##       <mth>    <dbl> <dbl>      <dbl>      <dbl>      <dbl>  
## 1 1990 Jan   13256. 13288.    -33.0      0.836     13289.  
## 2 1990 Feb   12966. 13269.   -258.     -44.6     13224.  
## 3 1990 Mar   12938. 13250.   -290.     -22.1     13228.  
## 4 1990 Apr   13012. 13231.   -220.      1.05     13232.  
## 5 1990 May   13108. 13211.   -114.     11.3     13223.  
## 6 1990 Jun   13183. 13192.   -24.3     15.5     13207.  
## 7 1990 Jul   13170. 13172.   -23.2     21.6     13193.  
## 8 1990 Aug   13160. 13151.    -9.52     17.8     13169.  
## 9 1990 Sep   13112. 13131.    20.5     22.0     13152.
```

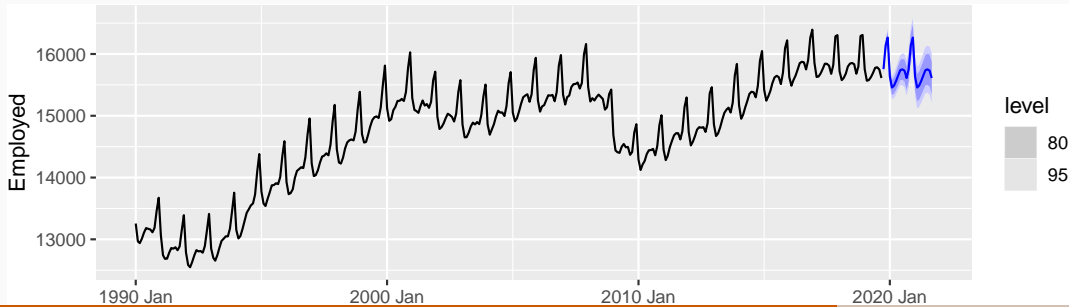
US Retail Employment

```
dcmp %>%  
  model(NAIVE(season_adjust)) %>%  
  forecast() %>%  
  autoplot(dcmp) +  
  labs(title = "Naive forecasts of seasonally adjusted data")
```



US Retail Employment

```
us_retail_employment %>%  
  model(stlf = decomposition_model(  
    STL(Employed ~ trend(window = 7), robust = TRUE),  
    NAIVE(season_adjust)  
  )) %>%  
  forecast() %>%  
  autoplot(us_retail_employment)
```



Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

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Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

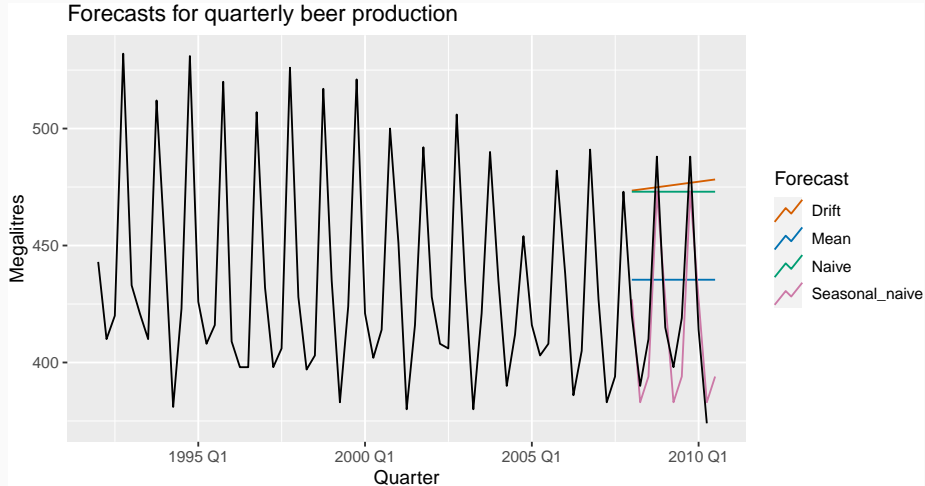
Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

y_{T+h} = $(T + h)$ th observation, $h = 1, \dots, H$

$\hat{y}_{T+h|T}$ = its forecast based on data up to time T .

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$$

$$\text{MAE} = \text{mean}(|e_{T+h}|)$$

$$\text{MSE} = \text{mean}(e_{T+h}^2)$$

$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

$$\text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)}$$

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$$\text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)}$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t , and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^T |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

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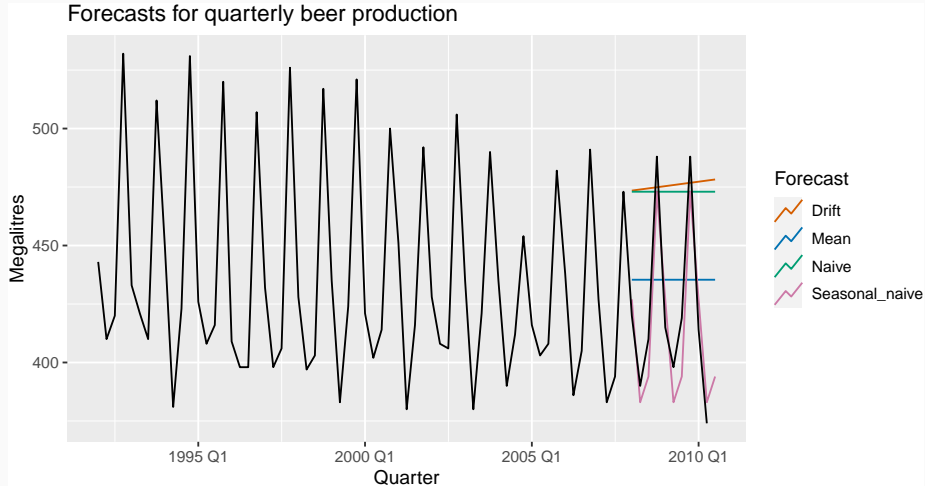
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy



Measures of forecast accuracy

```
recent_production <- aus_production %>%  
  filter(year(Quarter) >= 1992)  
train <- recent_production %>%  
  filter(year(Quarter) <= 2007)  
beer_fit <- train %>%  
  model(  
    Mean = MEAN(Beer),  
    Naive = NAIVE(Beer),  
    Seasonal_naive = SNAIVE(Beer),  
    Drift = RW(Beer ~ drift())  
  )  
beer_fc <- beer_fit %>%  
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit)
```

```
## # A tibble: 4 x 6
##   .model      .type    RMSE    MAE    MAPE    MASE
##   <chr>      <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift      Training  65.3   54.8  12.2   3.83
## 2 Mean      Training  43.6   35.2   7.89   2.46
## 3 Naive      Training  65.3   54.7  12.2   3.83
## 4 Seasonal_naive Training  16.8   14.3   3.31   1
```

```
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 x 6
##   .model      .type    RMSE    MAE    MAPE    MASE
##   <chr>      <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift      Test     64.9   58.9  14.6   4.12
## 2 Mean      Test     38.4   34.8   8.28   2.44
## 3 Naive      Test     62.7   57.4  14.2   4.01
## 4 Seasonal_naive Test     14.3   13.4   3.17   0.937
```

Time series cross-validation

Traditional evaluation

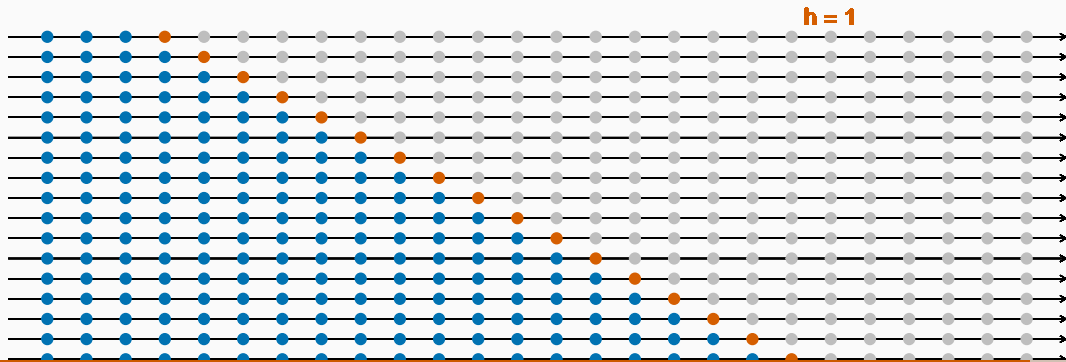


Time series cross-validation

Traditional evaluation



Time series cross-validation

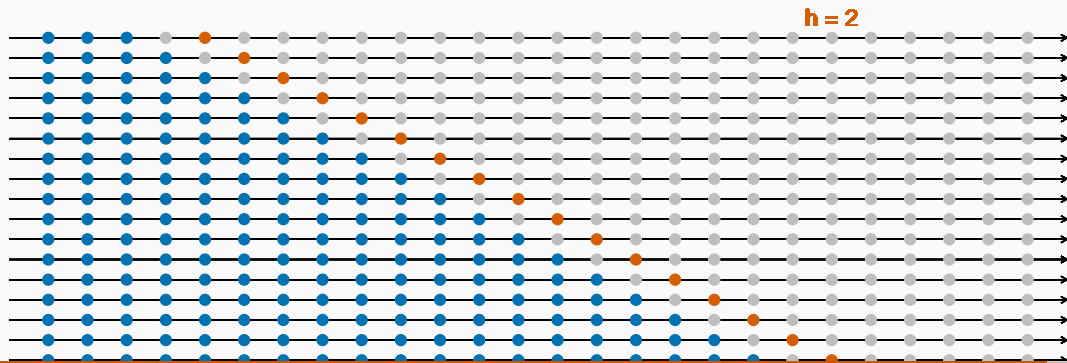


Time series cross-validation

Traditional evaluation



Time series cross-validation

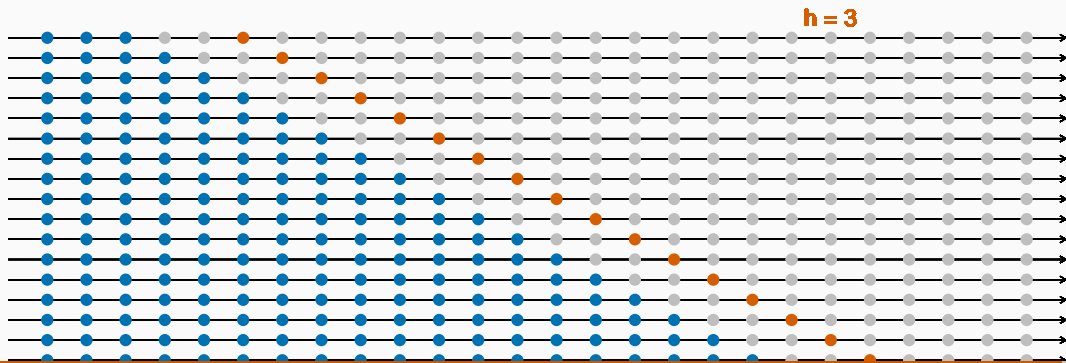


Time series cross-validation

Traditional evaluation



Time series cross-validation

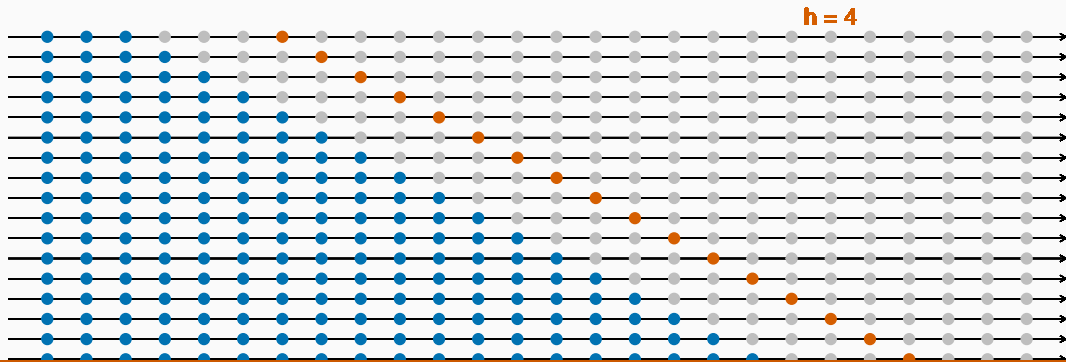


Time series cross-validation

Traditional evaluation



Time series cross-validation

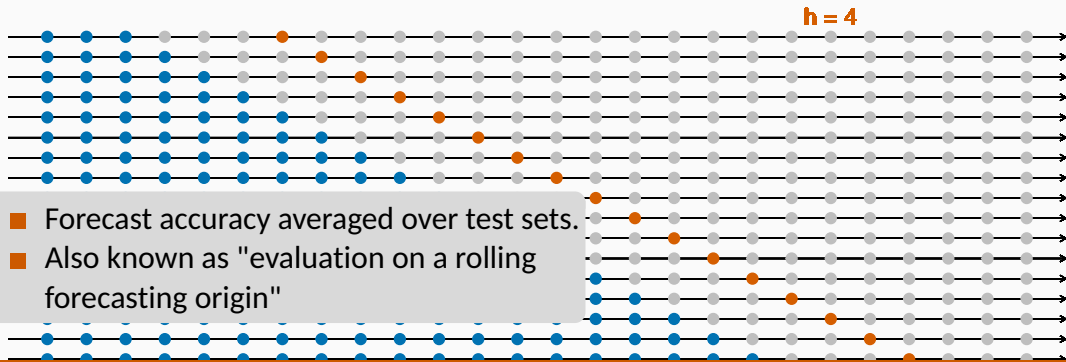


Time series cross-validation

Traditional evaluation



Time series cross-validation



Time series cross-validation

Stretch with a minimum length of 3, growing by 1 each step.

```
fb_stretch <- fb_stock %>%  
  stretch_tsibble(.init = 3, .step = 1) %>%  
  filter(.id != max(.id))
```

```
## # A tsibble: 790,650 x 4 [1]  
## # Key:      .id [1,255]  
##   Date      Close trading_day  .id  
##   <date>    <dbl>      <int> <int>  
## 1 2014-01-02  54.7          1     1  
## 2 2014-01-03  54.6          2     1  
## 3 2014-01-06  57.2          3     1  
## 4 2014-01-02  54.7          1     2  
## 5 2014-01-03  54.6          2     2  
## 6 2014-01-06  57.2          3     2  
## 7 2014-01-07  57.9          4     2
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```
fit_cv <- fb_stretch %>%  
  model(RW(Close ~ drift()))
```

```
## # A mable: 1,255 x 3  
## # Key:      .id, Symbol [1,255]  
##      .id Symbol `RW(Close ~ drift())`  
##      <int> <chr>                <model>  
## 1      1 FB                      <RW w/ drift>  
## 2      2 FB                      <RW w/ drift>  
## 3      3 FB                      <RW w/ drift>  
## 4      4 FB                      <RW w/ drift>  
## # ... with 1,251 more rows
```

Time series cross-validation

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv %>%  
  forecast(h=1)
```

```
## # A tibble: 1,255 x 5  
## #   Key:      .id, Symbol [1,255]  
##   .id Symbol trading_day      Close .mean  
##   <int> <chr>      <dbl>      <dist> <dbl>  
## 1     1  FB          4 N(58, 3.9)  58.4  
## 2     2  FB          5 N(59, 2)   59.0  
## 3     3  FB          6 N(59, 1.5)  59.1  
## 4     4  FB          7 N(58, 1.8)  57.7  
## # ... with 1,251 more rows
```

Time series cross-validation

```
# Cross-validated  
fc_cv %>% accuracy(fb_stock)  
# Training set  
fb_stock %>% model(RW(Close ~ drift())) %>% accuracy()
```

| | RMSE | MAE | MAPE |
|------------------|-------|-------|-------|
| Cross-validation | 2.418 | 1.469 | 1.266 |
| Training | 2.414 | 1.465 | 1.261 |

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.