

Deriving forecast variances

Rob J Hyndman

1 March 2021

Mean

Assume ε_t has mean 0 and variance σ^2

$$y_t = \mu + \varepsilon_t$$

$$y_{T+h} = \mu + \varepsilon_{T+h}$$

$$E[y_{T+h}|T] = \mu$$

$$\begin{aligned} V[y_{T+h}|T] &= V(\hat{\mu}) + V(\varepsilon_t) \\ &= \sigma^2/T + \sigma^2 \\ &= \sigma^2(1 + 1/T) \end{aligned}$$

Naive

$$y_t = y_{t-1} + \varepsilon_t$$

$$y_{T+h} = y_{T+h-1} + \varepsilon_{T+h}$$

$$= y_{T+h-2} + \varepsilon_{T+h-1} + \varepsilon_{T+h}$$

$$\vdots$$

$$= y_T + \sum_{i=0}^{h-1} \varepsilon_{T+h-i}$$

$$E[y_{T+h}|T] = y_T$$

$$V[y_{T+h|T}] = h\sigma^2$$