MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550
Applied forecasting

Ch5. The forecasters’ toolbox

OTexts.org/fpp3/

Outline

1. A tidy forecasting workflow
2. Some simple forecasting methods
3. Residual diagnostics
4. Distributional forecasts and prediction intervals
5. Forecasting with transformations
6. Forecasting and decomposition
7. Evaluating forecast accuracy
8. Time series cross-validation

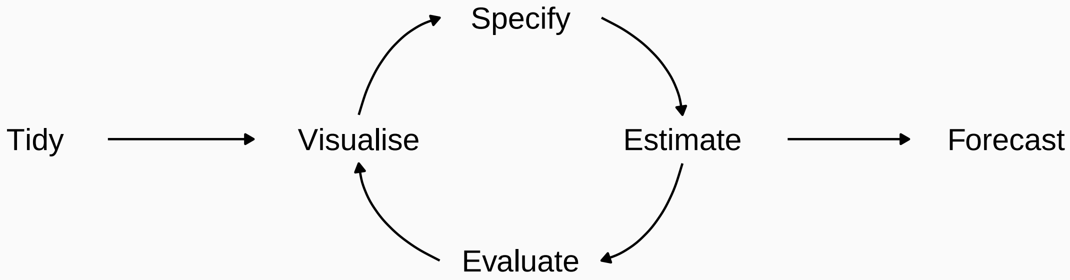
Outline

A tidy forecasting workflow

The process of producing forecasts can be split up into a few fundamental steps.

1. Preparing data
2. Data visualisation
3. Specifying a model
4. Model estimation
5. Accuracy & performance evaluation
6. Producing forecasts

A tidy forecasting workflow



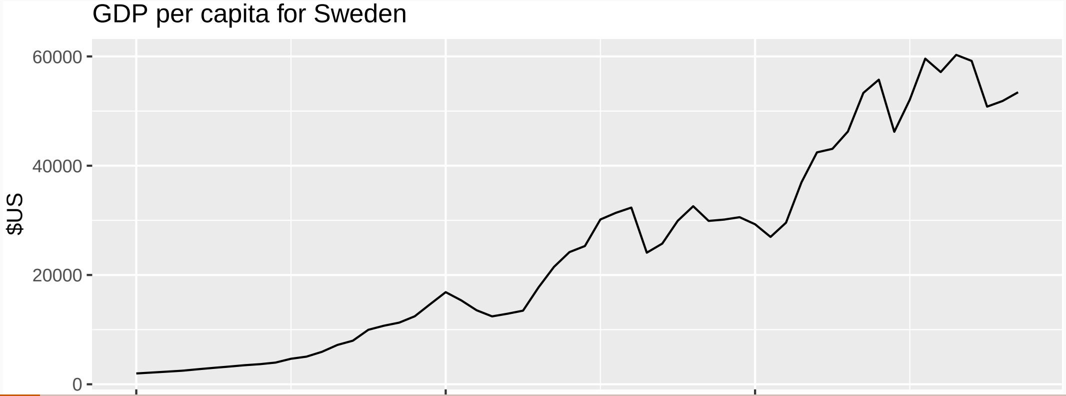
Data preparation (tidy)

```
gdppc <- global_economy %>%
  mutate(GDP_per_capita = GDP/Population) %>%
  select(Year, Country, GDP, Population, GDP_per_capita)
gdppc
```

```
## # A tsibble: 15,150 x 5 [1Y]
## # Key:       Country [263]
##     Year Country             GDP Population GDP_per_capita
##    <dbl> <fct>             <dbl>      <dbl>          <dbl>
##  1  1960 Afghanistan  537777811.    8996351           59.8
##  2  1961 Afghanistan  548888896.    9166764           59.9
##  3  1962 Afghanistan  546666678.    9345868           58.5
##  4  1963 Afghanistan  751111191.    9533954           78.8
##  5  1964 Afghanistan  800000044.    9731361           82.2
##  6  1965 Afghanistan 1006666638.    9938414          101.
```

Data visualisation

```
gdppc %>%
  filter(Country=="Sweden") %>%
  autoplot(GDP_per_capita) +
    labs(title = "GDP per capita for Sweden", y = "$US")
```



Model estimation

The `model()` function trains models to data.

```
fit <- gdppc %>%
  model(trend_model = TSLM(GDP_per_capita ~ trend()))
fit
```

```
## # A mable: 263 x 2
## # Key:     Country [263]
##    Country                  trend_model
##    <fct>                        <model>
##  1 Afghanistan                   <TSLM>
##  2 Albania                       <TSLM>
##  3 Algeria                       <TSLM>
##  4 American Samoa                <TSLM>
##  5 Andorra                       <TSLM>
```

Model estimation

The `model()` function trains models to data.

```r
fit <- gdppc %>%
  model(trend_model = TSLM(GDP_per_capita ~ trend()))
fit
```

```
## # A mable: 263 x 2
## # Key:     Country [263]
##    Country                    trend_model
##    <fct>                          <model>
##  1 Afghanistan                     <TSLM>
##  2 Albania                         <TSLM>
##  3 Algeria                         <TSLM>
##  4 American Samoa                  <TSLM>
##  5 Andorra                         <TSLM>
```

Producing forecasts

```
fit %>% forecast(h = "3 years")
```

```
## # A fable: 789 x 5 [1Y]
## # Key:     Country, .model [263]
##    Country     .model       Year   GDP_per_capita  .mean
##    <fct>       <chr>       <dbl>           <dist>  <dbl>
##  1 Afghanistan trend_model  2018     N(526, 9653)   526.
##  2 Afghanistan trend_model  2019     N(534, 9689)   534.
##  3 Afghanistan trend_model  2020     N(542, 9727)   542.
##  4 Albania     trend_model  2018  N(4716, 476419)  4716.
##  5 Albania     trend_model  2019  N(4867, 481086)  4867.
##  6 Albania     trend_model  2020  N(5018, 486012)  5018.
##  7 Algeria     trend_model  2018  N(4410, 643094)  4410.
##  8 Algeria     trend_model  2019  N(4489, 645311)  4489.
```

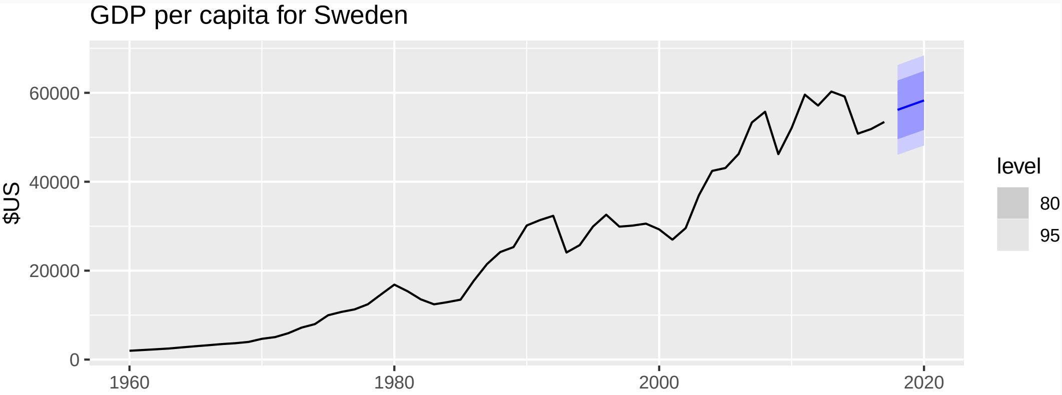
Producing forecasts

```r
fit %>% forecast(h = "3 years")
```

```
## # A fable: 789 x 5 [1Y]
## # Key:     Country, .model [263]
##    Country     .model       Year   GDP_per_capita  .mean
##    <fct>       <chr>       <dbl>           <dist>  <dbl>
##  1 Afghanistan trend_model  2018    N(526, 9653)   526.
##  2 Afghanistan trend_model  2019    N(534, 9689)   534.
##  3 Afghanistan trend_model  2020    N(542, 9727)   542.
##  4 Albania     trend_model  2018  N(4716, 476419)  4716.
##  5 Albania     trend_model  2019  N(4867, 481086)  4867.
##  6 Albania     trend_model  2020  N(5018, 486012)  5018.
##  7 Algeria     trend_model  2018  N(4410, 643094)  4410.
##  8 Algeria     trend_model  2019  N(4489, 645311)  4489.
```

Visualising forecasts

```
fit %>% forecast(h = "3 years") %>%
  filter(Country=="Sweden") %>%
  autoplot(gdppc) +
    labs(title = "GDP per capita for Sweden", y = "$US")
```

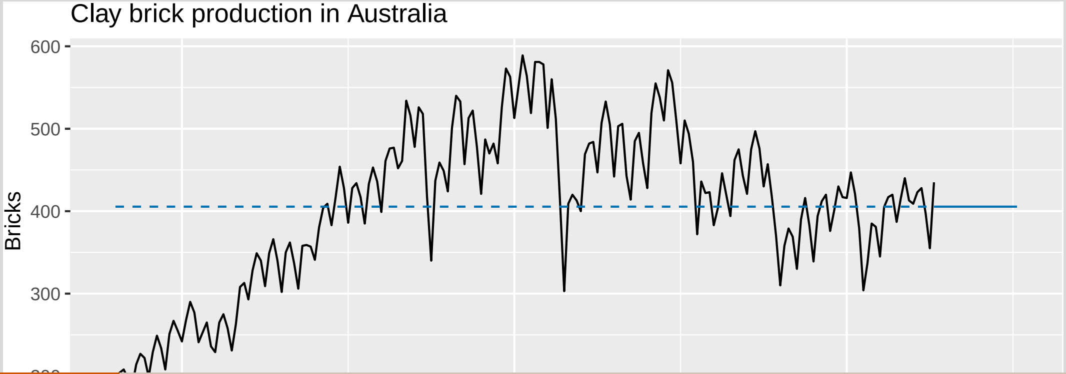


Outline

Some simple forecasting methods

`MEAN(y)`: Average method

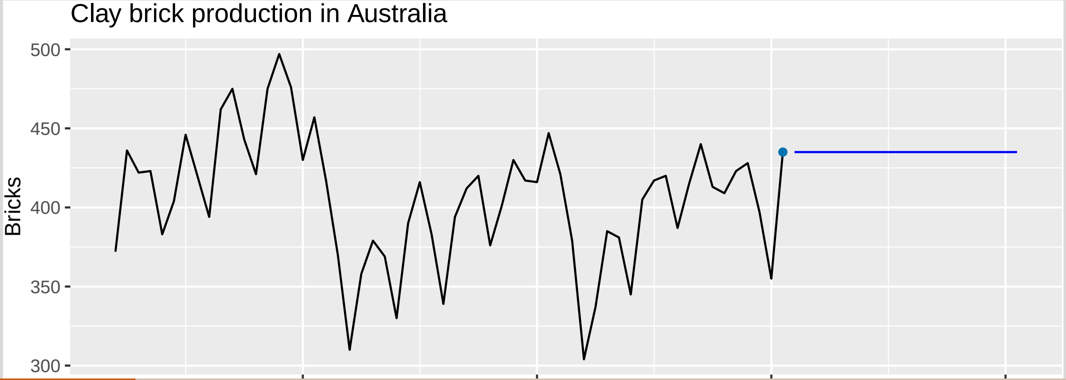
- Forecast of all future values is equal to mean of historical data $\{y_1, \ldots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$



Some simple forecasting methods

NAIVE(y): Naïve method

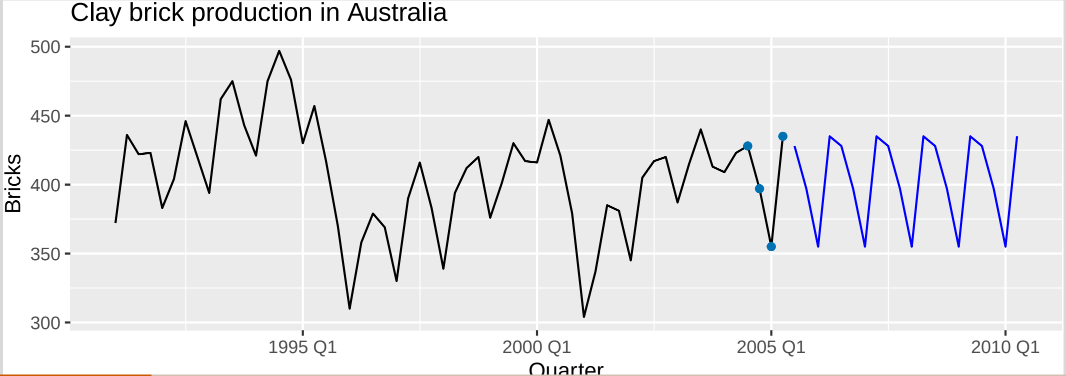
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

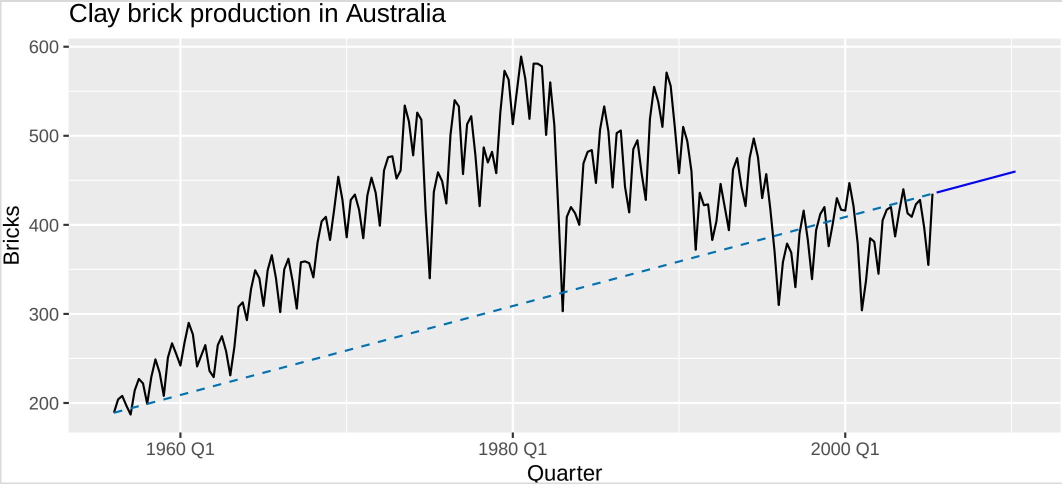
- Forecasts equal to last value plus average change.
- Forecasts:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1}(y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
brick_fit <-  aus_production %>%
  filter(!is.na(Bricks)) %>%
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

```
## # A mable: 1 x 4
##   Seasonal_naive   Naive          Drift    Mean
##          <model> <model>        <model> <model>
## 1       <SNAIVE> <NAIVE> <RW w/ drift>  <MEAN>
```

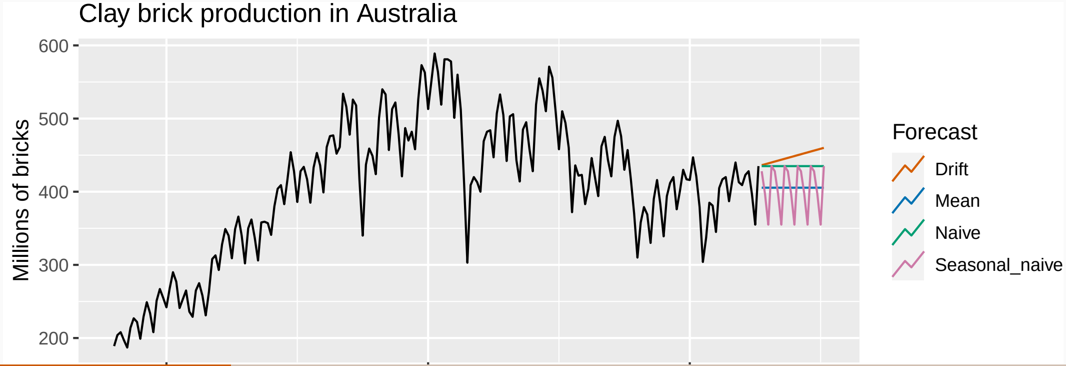
Producing forecasts

```r
brick_fc <- brick_fit %>%
  forecast(h = "5 years")
```

```
## # A fable: 80 x 4 [1Q]
## # Key:     .model [4]
##   .model         Quarter       Bricks .mean
##   <chr>            <qtr>       <dist> <dbl>
## 1 Seasonal_naive 2005 Q3 N(428, 2336)   428
## 2 Seasonal_naive 2005 Q4 N(397, 2336)   397
## 3 Seasonal_naive 2006 Q1 N(355, 2336)   355
## 4 Seasonal_naive 2006 Q2 N(435, 2336)   435
## # ... with 76 more rows
```

Visualising forecasts

```
brick_fc %>%
  autoplot(aus_production, level = NULL) +
  labs(title = "Clay brick production in Australia",
       y = "Millions of bricks") +
  guides(colour = guide_legend(title = "Forecast"))
```

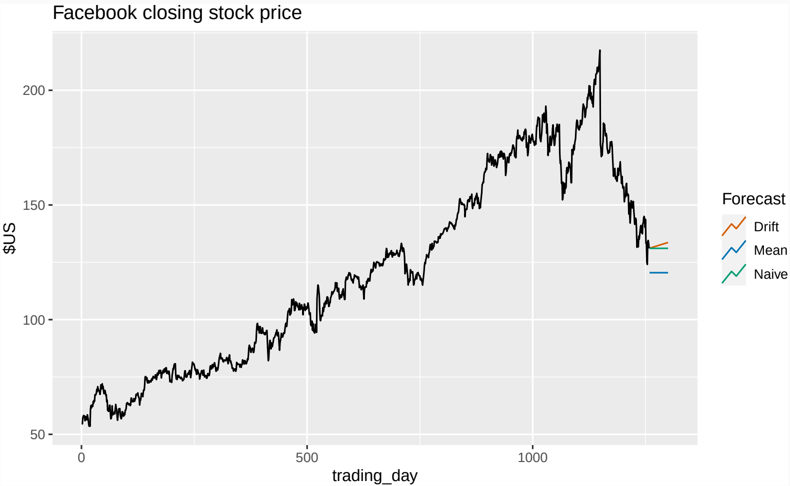


Facebook closing stock price

```
# Extract training data
fb_stock <- gafa_stock %>%
  filter(Symbol == "FB") %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE)

# Specify, estimate and forecast
fb_stock %>%
  model(
    Mean = MEAN(Close),
    Naive = NAIVE(Close),
    Drift = RW(Close ~ drift())
  ) %>%
  forecast(h=42) %>%
  autoplot(fb_stock, level = NULL) +
  labs(title = "Facebook closing stock price", y="$US") +
  guides(colour=guide_legend(title="Forecast"))
```

Facebook closing stock price



Outline

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_{t-1}$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

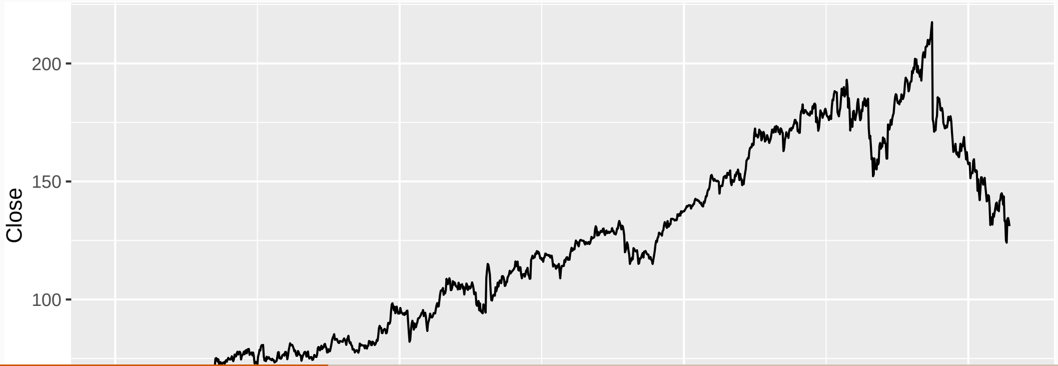
1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for distributions & prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Facebook closing stock price

```r
fb_stock <- gafa_stock %>%
  filter(Symbol == "FB") %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index = trading_day, regular = TRUE)
fb_stock %>% autoplot(Close)
```



Facebook closing stock price

```r
fit <- fb_stock %>% model(NAIVE(Close))
augment(fit)
```

```
## # A tsibble: 1,258 x 7 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid .innov
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA     NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64   2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01  -1.01
##  7 FB     NAIVE(Close)           7  57.9    57.2  0.720  0.720
##  8 FB     NAIVE(Close)           8  55.9    57.9 -2.03  -2.03
##  9 FB     NAIVE(Close)           9  57.7    55.9  1.83   1.83
## 10 FB     NAIVE(Close)          10  57.6    57.7 -0.140 -0.140
## # ... with 1,248 more rows
```

Facebook closing stock price

```r
fit <- fb_stock %>% model(NAIVE(Close))
augment(fit)
```

$\hat{y}_{t|t-1}$ e_t

```
## # A tsibble: 1,258 x 7 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid .innov
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA     NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64   2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01  -1.01
                                     7  57.9    57.2  0.720  0.720
                                     8  55.9    57.9 -2.03  -2.03
                                     9  57.7    55.9  1.83   1.83
                                    10  57.6    57.7 -0.140 -0.140
```

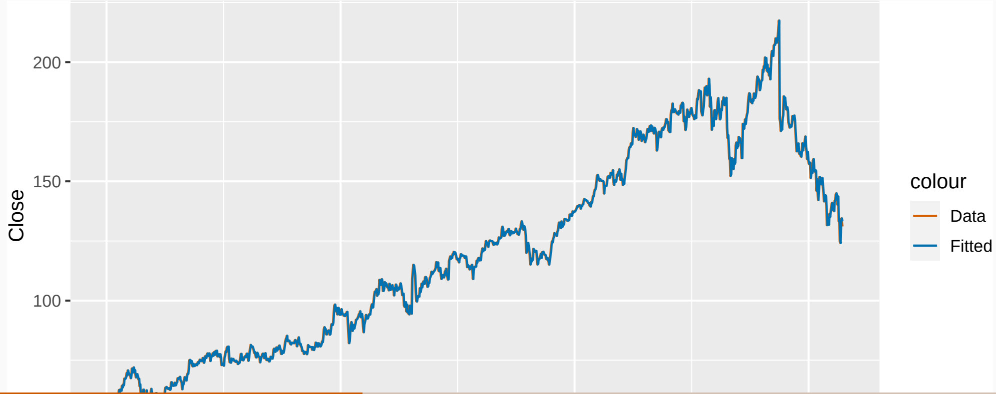
Naïve forecasts:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

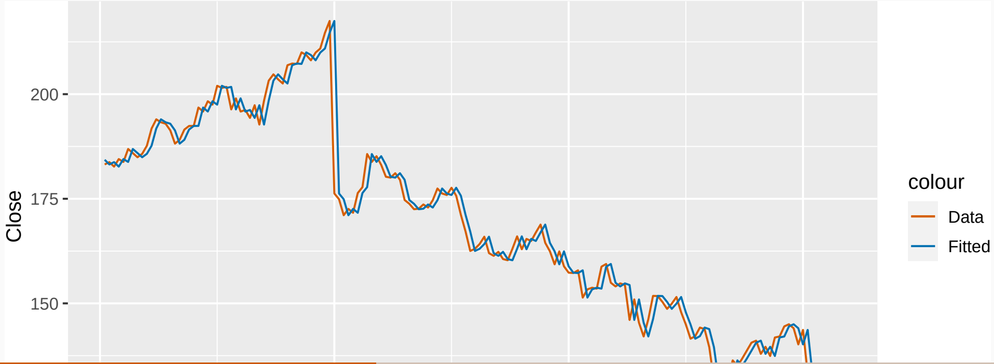
Facebook closing stock price

```
augment(fit) %>%
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



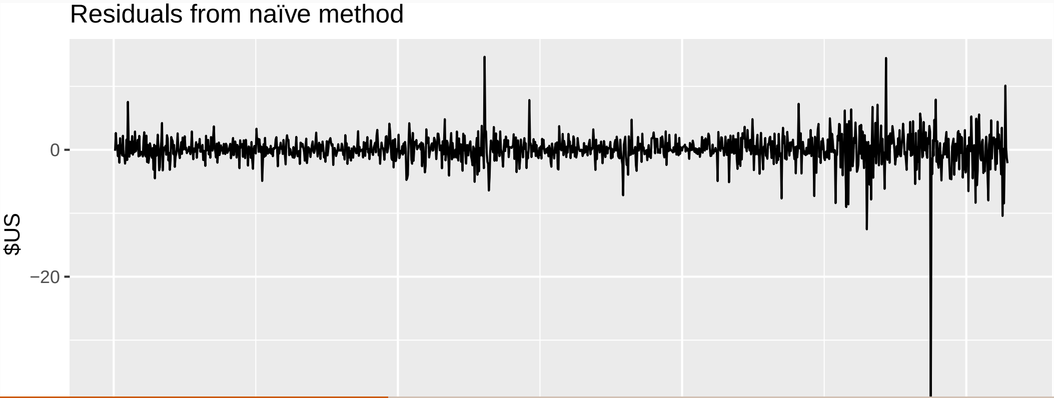
Facebook closing stock price

```
augment(fit) %>%
  filter(trading_day > 1100) %>%
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



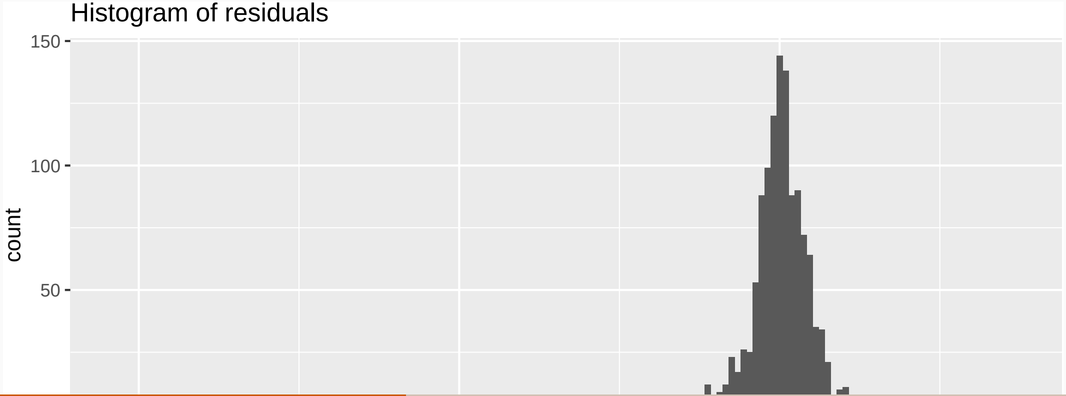
Facebook closing stock price

```
augment(fit) %>%
  autoplot(.resid) +
  labs(y = "$US",
       title = "Residuals from naïve method")
```



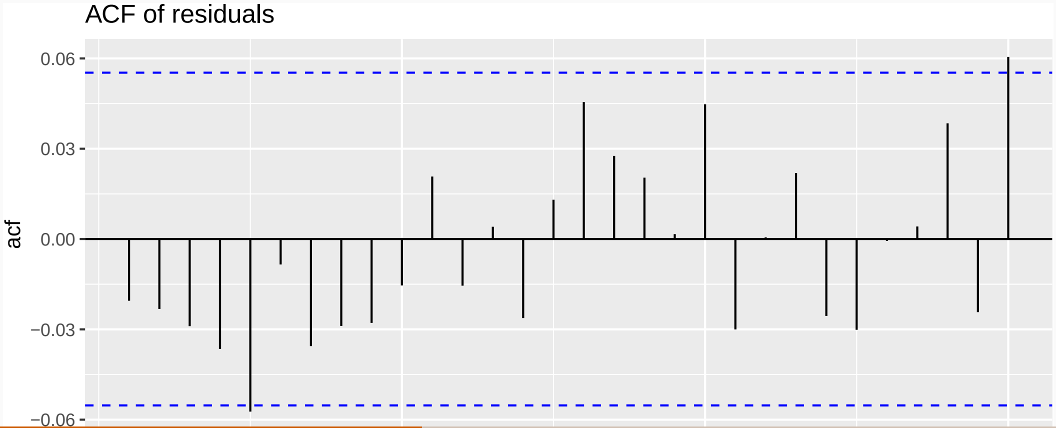
Facebook closing stock price

```
augment(fit) %>%
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 150) +
  labs(title = "Histogram of residuals")
```



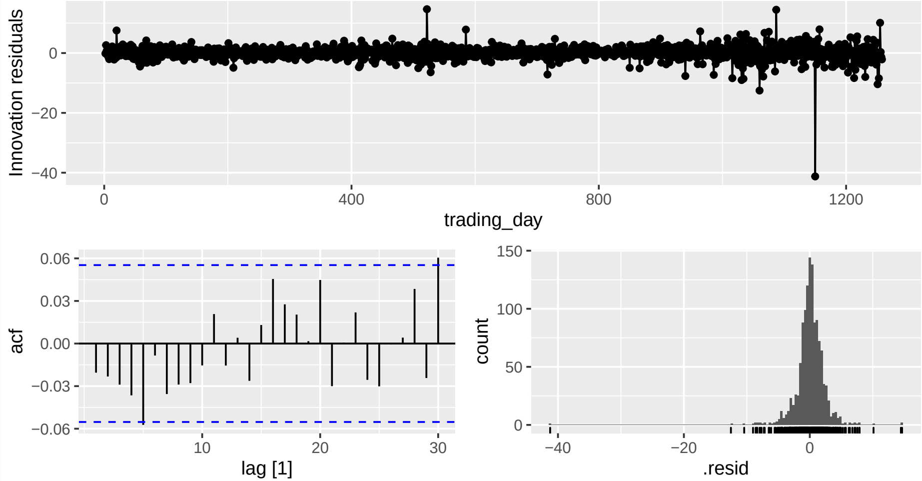
Facebook closing stock price

```
augment(fit) %>%
  ACF(.resid) %>%
  autoplot() + labs(title = "ACF of residuals")
```



gg_tsresiduals() function

```
gg_tsresiduals(fit)
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Box-Pierce test

$$Q = T\sum_{k=1}^{\ell} r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2)\sum_{k=1}^{\ell}(T-k)^{-1}r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- My preferences: $\ell = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(\ell - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.
- `lag` = ℓ, `dof` = K

```
augment(fit) %>%
  features(.resid, ljung_box, lag=10, dof=0)
```

```
## # A tibble: 1 x 4
##   Symbol .model        lb_stat lb_pvalue
##   <chr>  <chr>           <dbl>     <dbl>
## 1 FB     NAIVE(Close)     12.1     0.276
```

Outline

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean:	$\hat{y}_{T+h\|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$
Naïve:	$\hat{y}_{T+h\|T} \sim N(y_T, h\hat{\sigma}^2)$
Seasonal naïve:	$\hat{y}_{T+h\|T} \sim N(y_{T+h-m(k+1)}, (k + 1)\hat{\sigma}^2)$
Drift:	$\hat{y}_{T+h\|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h-step distribution.

- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```r
brick_fc %>% hilo(level = 95)
```

```
## # A tsibble: 80 x 5 [1Q]
## # Key:       .model [4]
##    .model         Quarter      Bricks .mean        `95%`
##    <chr>            <qtr>      <dist> <dbl>       <hilo>
##  1 Seasonal_naive 2005 Q3 N(428, 2336)   428 [333, 523]95
##  2 Seasonal_naive 2005 Q4 N(397, 2336)   397 [302, 492]95
##  3 Seasonal_naive 2006 Q1 N(355, 2336)   355 [260, 450]95
##  4 Seasonal_naive 2006 Q2 N(435, 2336)   435 [340, 530]95
##  5 Seasonal_naive 2006 Q3 N(428, 4672)   428 [294, 562]95
##  6 Seasonal_naive 2006 Q4 N(397, 4672)   397 [263, 531]95
##  7 Seasonal_naive 2007 Q1 N(355, 4672)   355 [221, 489]95
##  8 Seasonal_naive 2007 Q2 N(435, 4672)   435 [301, 569]95
##  9 Seasonal_naive 2007 Q3 N(428, 7008)   428 [264, 592]95
```

Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.

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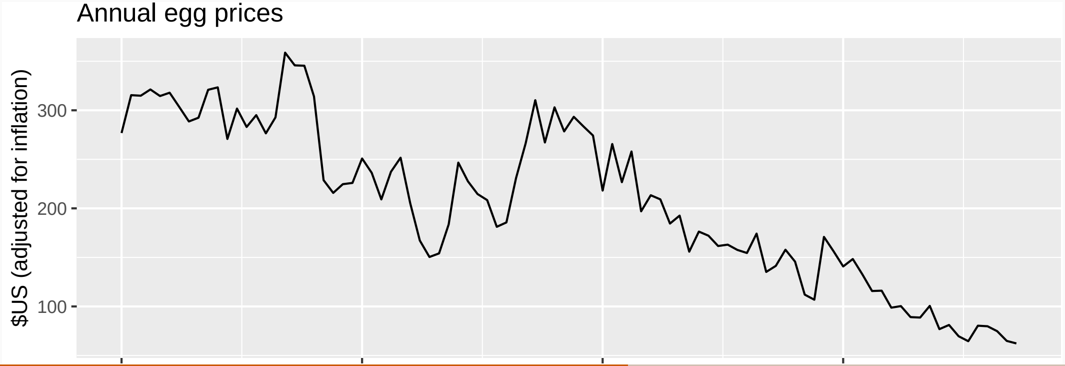
Modelling with transformations

```r
eggs <- prices %>%
  filter(!is.na(eggs)) %>% select(eggs)
eggs %>% autoplot() +
  labs(title="Annual egg prices",
       y="$US (adjusted for inflation)")
```



Modelling with transformations

Transformations used in the left of the formula will be automatically back-transformed. To model log-transformed egg prices, you could use:

```
fit <- eggs %>%
  model(RW(log(eggs) ~ drift()))
fit
```

```
## # A mable: 1 x 1
##   `RW(log(eggs) ~ drift())`
##                     <model>
## 1             <RW w/ drift>
```

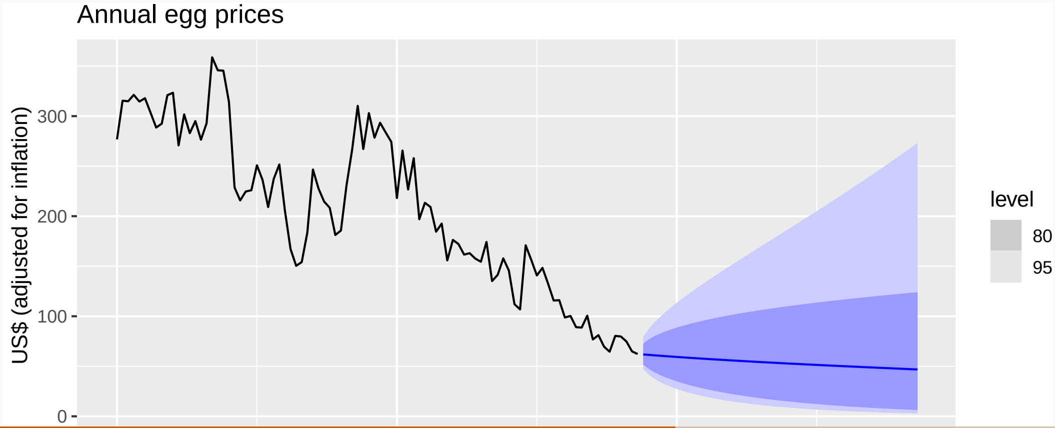
Forecasting with transformations

```r
fc <- fit %>%
  forecast(h = 50)
fc
```

```
## # A fable: 50 x 4 [1Y]
## # Key:       .model [1]
##    .model                       year             eggs .mean
##    <chr>                       <dbl>           <dist> <dbl>
##  1 RW(log(eggs) ~ drift())  1994 t(N(4.1, 0.018))  61.8
##  2 RW(log(eggs) ~ drift())  1995 t(N(4.1, 0.036))  61.4
##  3 RW(log(eggs) ~ drift())  1996 t(N(4.1, 0.054))  61.0
##  4 RW(log(eggs) ~ drift())  1997 t(N(4.1, 0.073))  60.5
##  5 RW(log(eggs) ~ drift())  1998 t(N(4.1, 0.093))  60.1
##  6 RW(log(eggs) ~ drift())  1999    t(N(4, 0.11))  59.7
##  7 RW(log(eggs) ~ drift())  2000    t(N(4, 0.13))  59.3
##  8 RW(log(eggs) ~ drift())  2001    t(N(4, 0.15))  58.9
```

Forecasting with transformations

```r
fc %>% autoplot(eggs) +
  labs(title="Annual egg prices",
       y="US$ (adjusted for inflation)")
```



Bias adjustment

- Back-transformed point forecasts are medians.
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Let X be have mean μ and variance σ^2.

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ:

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2 f''(\mu).$$

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$$\mathrm{E}[Y] = \mathrm{E}[f(X)] = f(\mu) + \tfrac{1}{2}\sigma^2 f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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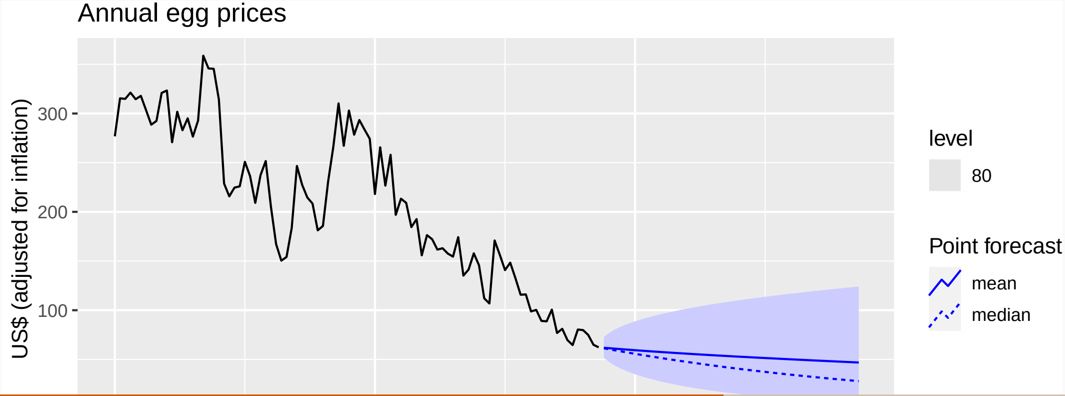
$$\mathrm{E}[Y] = \begin{cases} e^{\mu}\left[1 + \frac{\sigma^2}{2}\right] & \lambda = 0; \\ (\lambda\mu + 1)^{1/\lambda}\left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda\mu+1)^2}\right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
fc %>%
  autoplot(eggs, level = 80, point_forecast = lst(mean, median)) +
  labs(title="Annual egg prices",
       y="US$ (adjusted for inflation)")
```



Outline

Forecasting and decomposition

$$y_t = \hat{S}_t + \hat{A}_t$$

- $\hat{A}_t$ is seasonally adjusted component
- $\hat{S}_t$ is seasonal component.

- Forecast $\hat{S}_t$ using SNAIVE.
- Forecast $\hat{A}_t$ using non-seasonal time series method.
- Combine forecasts of $\hat{S}_t$ and $\hat{A}_t$ to get forecasts of original data.

US Retail Employment

```
us_retail_employment <- us_employment %>%
  filter(year(Month) >= 1990, Title == "Retail Trade") %>%
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
##       Month Title        Employed
##       <mth> <chr>           <dbl>
##  1 1990 Jan Retail Trade   13256.
##  2 1990 Feb Retail Trade   12966.
##  3 1990 Mar Retail Trade   12938.
##  4 1990 Apr Retail Trade   13012.
##  5 1990 May Retail Trade   13108.
##  6 1990 Jun Retail Trade   13183.
##  7 1990 Jul Retail Trade   13170.
##  8 1990 Aug Retail Trade   13160.
##  9 1990 Sep Retail Trade   13113.
```

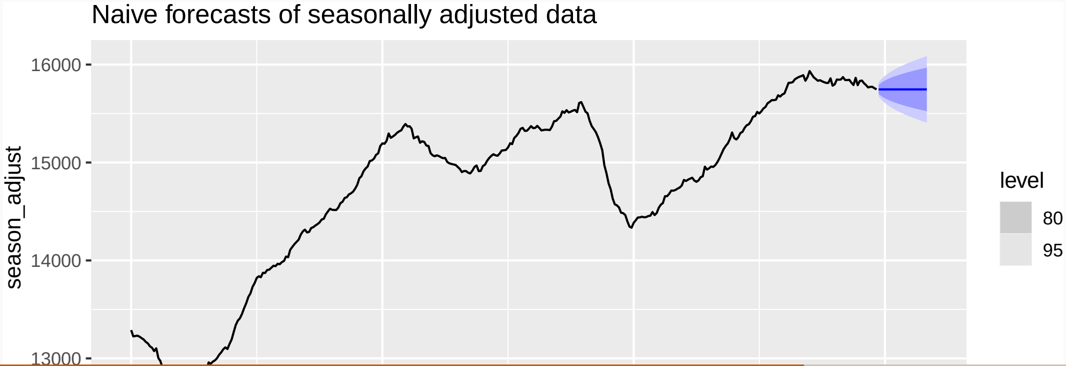
US Retail Employment

```r
dcmp <- us_retail_employment %>%
  model(STL(Employed)) %>%
  components() %>% select(-.model)
dcmp
```

```
## # A tsibble: 357 x 6 [1M]
##       Month Employed  trend season_year remainder season_adjust
##       <mth>    <dbl>  <dbl>       <dbl>     <dbl>         <dbl>
##  1 1990 Jan   13256. 13288.       -33.0     0.836        13289.
##  2 1990 Feb   12966. 13269.      -258.    -44.6          13224.
##  3 1990 Mar   12938. 13250.      -290.    -22.1          13228.
##  4 1990 Apr   13012. 13231.      -220.      1.05         13232.
##  5 1990 May   13108. 13211.      -114.     11.3          13223.
##  6 1990 Jun   13183. 13192.       -24.3    15.5          13207.
##  7 1990 Jul   13170. 13172.       -23.2    21.6          13193.
##  8 1990 Aug   13160. 13151.        -9.52   17.8          13169.
```

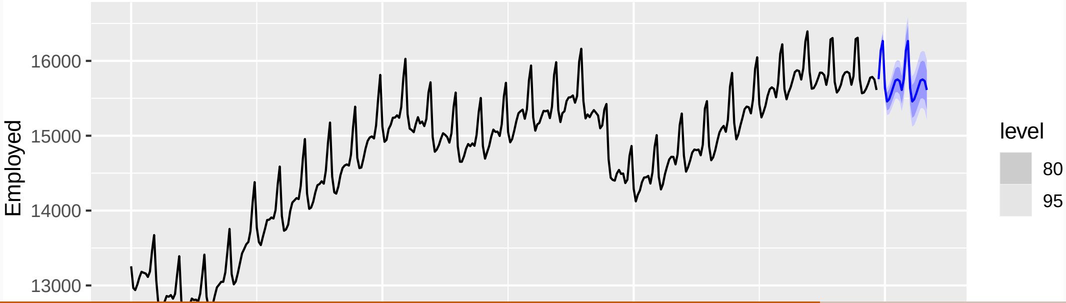
US Retail Employment

```
dcmp %>%
  model(NAIVE(season_adjust)) %>%
  forecast() %>%
  autoplot(dcmp) +
  labs(title = "Naive forecasts of seasonally adjusted data")
```



US Retail Employment

```
us_retail_employment %>%
  model(stlf = decomposition_model(
    STL(Employed ~ trend(window = 7), robust = TRUE),
    NAIVE(season_adjust)
  )) %>%
  forecast() %>%
  autoplot(us_retail_employment)
```



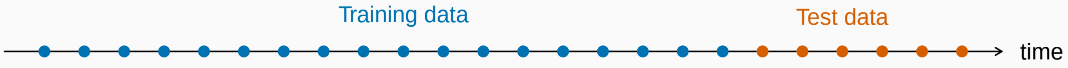
Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

Outline

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

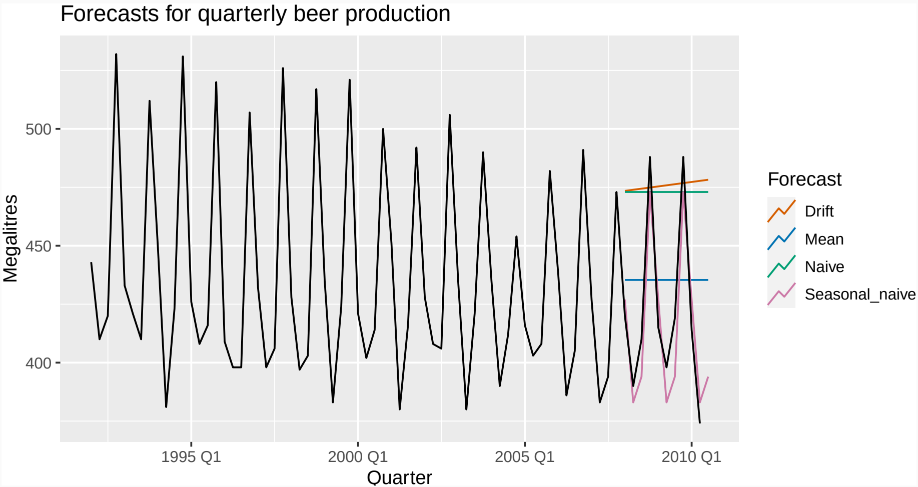
Forecast "error": the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

$$\begin{aligned} y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\ \hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\ e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T} \end{aligned}$$

$$\begin{aligned} \text{MAE} &= \text{mean}(|e_{T+h}|) \\ \text{MSE} &= \text{mean}(e^2_{T+h}) \qquad\qquad \text{RMSE} = \sqrt{\text{mean}(e^2_{T+h})} \\ \text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|) \end{aligned}$$

Measures of forecast accuracy

$$
\begin{aligned}
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\end{aligned}
$$

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\end{aligned}
$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

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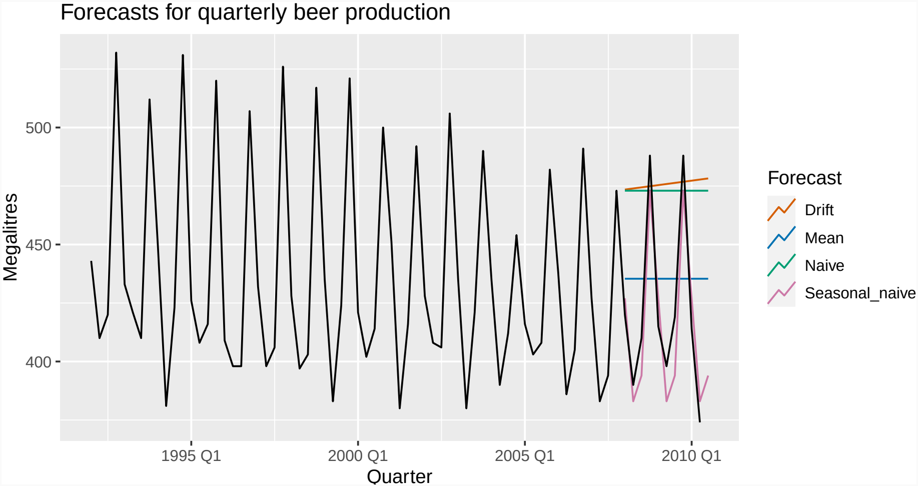
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T-m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy



Measures of forecast accuracy

```r
recent_production <- aus_production %>%
  filter(year(Quarter) >= 1992)
train <- recent_production %>%
  filter(year(Quarter) <= 2007)
beer_fit <- train %>%
  model(
    Mean = MEAN(Beer),
    Naive = NAIVE(Beer),
    Seasonal_naive = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fc <- beer_fit %>%
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit)
```

```
## # A tibble: 4 x 6
##   .model         .type     RMSE   MAE  MAPE  MASE
##   <chr>          <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Training  65.3  54.8 12.2   3.83
## 2 Mean           Training  43.6  35.2  7.89  2.46
## 3 Naive          Training  65.3  54.7 12.2   3.83
## 4 Seasonal_naive Training  16.8  14.3  3.31  1
```

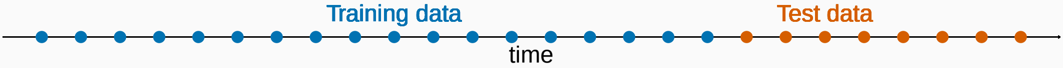
```
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 x 6
##   .model         .type  RMSE   MAE  MAPE  MASE
##   <chr>          <chr> <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Test   64.9  58.9 14.6  4.12
## 2 Mean           Test   38.4  34.8  8.28 2.44
## 3 Naive          Test   62.7  57.4 14.2  4.01
## 4 Seasonal_naive Test   14.3  13.4  3.17 0.937
```

Outline

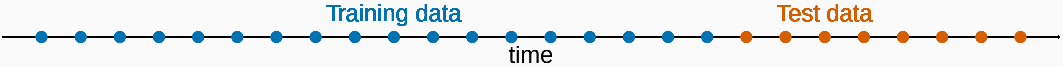
Time series cross-validation

Traditional evaluation

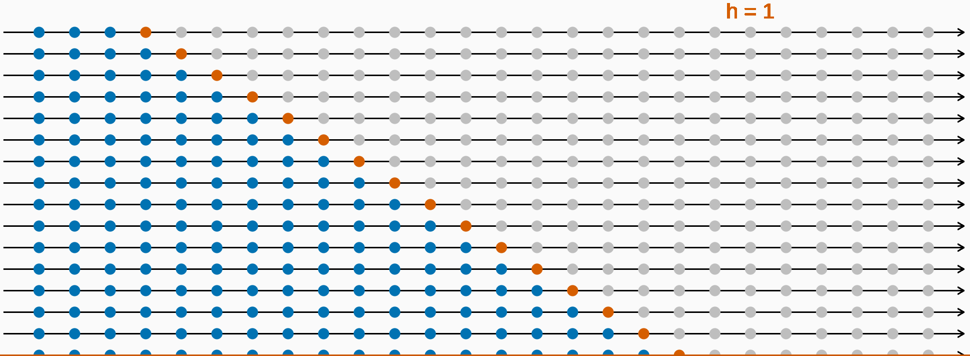


Time series cross-validation

Traditional evaluation

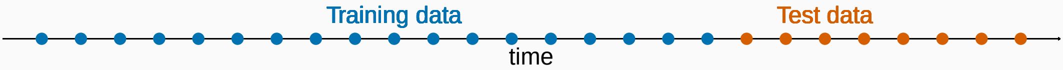


Time series cross-validation

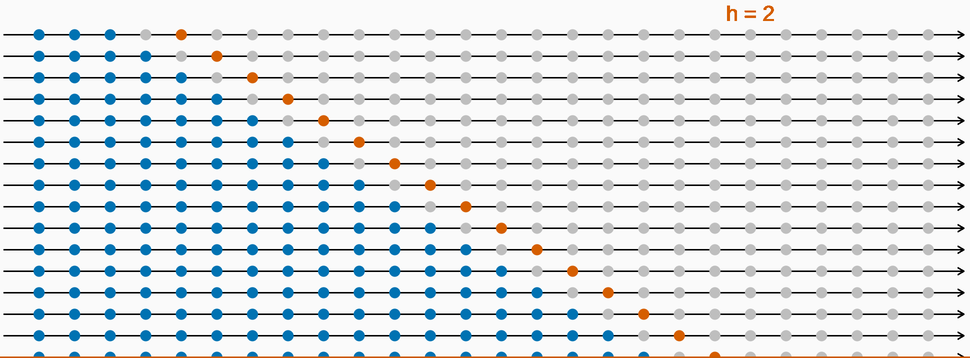


Time series cross-validation

Traditional evaluation



Time series cross-validation



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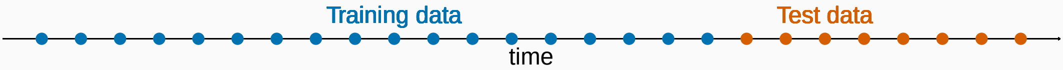
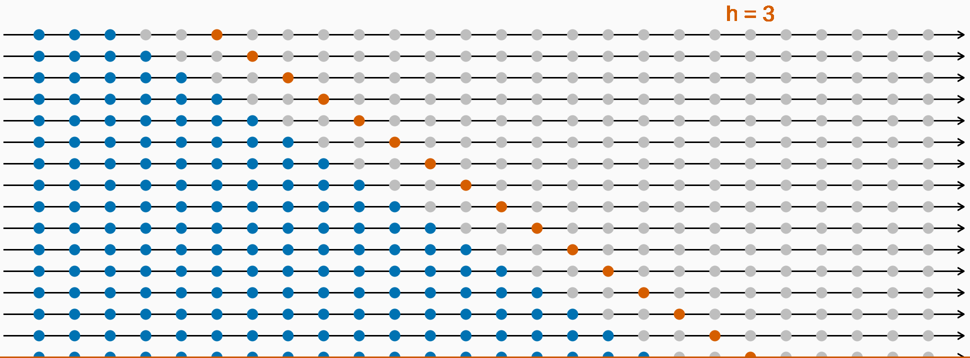


Time series cross-validation



Time series cross-validation

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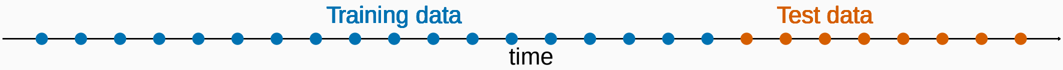
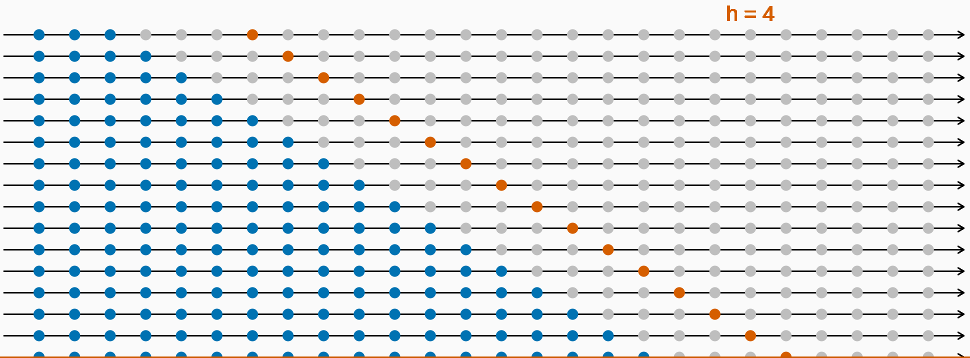
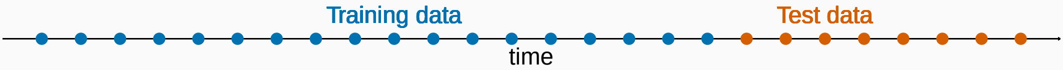


Time series cross-validation

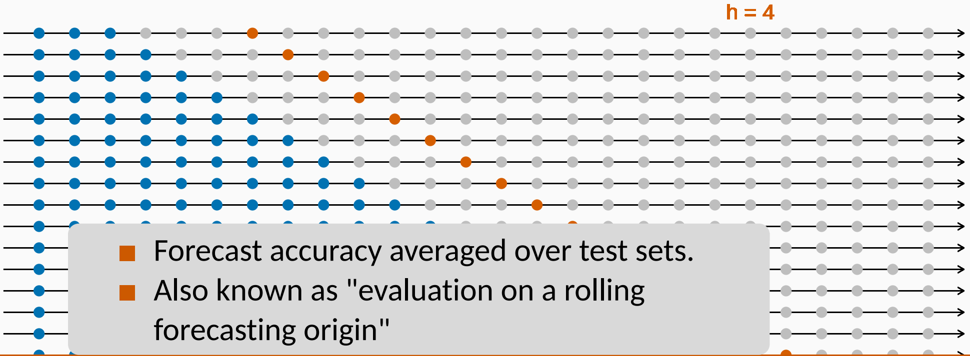


Time series cross-validation

Traditional evaluation



Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"

Time series cross-validation

Stretch with a minimum length of 3, growing by 1 each step.

```
fb_stretch <- fb_stock %>%
  stretch_tsibble(.init = 3, .step = 1) %>%
  filter(.id != max(.id))
```

```
## # A tsibble: 790,650 x 4 [1]
## # Key:       .id [1,255]
##   Date       Close trading_day   .id
##   <date>     <dbl>       <int> <int>
## 1 2014-01-02  54.7           1     1
## 2 2014-01-03  54.6           2     1
## 3 2014-01-06  57.2           3     1
## 4 2014-01-02  54.7           1     2
## 5 2014-01-03  54.6           2     2
## 6 2014-01-06  57.2           3     2
## 7 2014-01-07  57.9           4     2
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```
fit_cv <- fb_stretch %>%
  model(RW(Close ~ drift()))
```

```
## # A mable: 1,255 x 3
## # Key:     .id, Symbol [1,255]
##      .id Symbol `RW(Close ~ drift())`
##    <int> <chr>                <model>
## 1      1 FB               <RW w/ drift>
## 2      2 FB               <RW w/ drift>
## 3      3 FB               <RW w/ drift>
## 4      4 FB               <RW w/ drift>
## # ... with 1,251 more rows
```

Time series cross-validation

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv %>%
  forecast(h=1)
```

```
## # A fable: 1,255 x 5 [1]
## # Key:     .id, Symbol [1,255]
##      .id Symbol trading_day      Close .mean
##    <int> <chr>        <dbl>     <dist> <dbl>
## 1      1 FB               4 N(58, 3.9)  58.4
## 2      2 FB               5   N(59, 2)  59.0
## 3      3 FB               6 N(59, 1.5)  59.1
## 4      4 FB               7 N(58, 1.8)  57.7
## # ... with 1,251 more rows
```

Time series cross-validation

```
# Cross-validated
fc_cv %>% accuracy(fb_stock)
# Training set
fb_stock %>% model(RW(Close ~ drift())) %>% accuracy()
```

	RMSE	MAE	MAPE
Cross-validation	2.418	1.469	1.266
Training	2.414	1.465	1.261

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.