UNIDIMENSIONAL COMMUNITY DETECTION: A MONTE CARLO SIMULATION, GRID SEARCH, AND COMPARISON

PSYARXIV PREPRINT

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January 1, 2024

This preprint is under review.
This latest version has benefited from one round of reviewer feedback.

ABSTRACT

Unidimensionality is fundamental to psychometrics. Despite the focus on dimensionality assessment in network psychometrics, unidimensionality assessment remains a challenge. Community detection algorithms are the most common approach to estimate dimensionality in networks. Many community detection algorithms optimize an objective criterion called modularity. A limitation of modularity is that it penalizes unidimensional structures in networks, favoring two or more communities (dimensions). In this study, this penalization is discussed and a solution is offered. After, a Monte Carlo simulation using one and two factor models is performed. Based on previous simulation studies, several community detection algorithms that have performed well with unidimensional structures (Leading Eigenvalue, Leiden, Louvain, and Walktrap) were compared. A grid search was performed on the tunable parameters of these algorithms to determine the optimal trade-off between unidimensional and bidimensional recovery. The best performing parameters for each algorithm were then compared against each other and parallel analysis with mean and 95th percentile eigenvalues. The Louvain and Leiden algorithms with modularity performed as well as or better than the other community detection algorithms and on par with parallel analysis. This study advances unidimensionality assessment in network psychometrics, further developing the psychometric toolbox of network analysis in psychology.

Keywords unidimensionality · network psychometrics · community detection · modularity

1 Introduction

Network analysis has become a popular psychometric modeling approach in psychology (Epskamp, Maris, Waldrop, & Borsboom, 2018). A network is comprised of nodes (circles) representing observable variables and edges (lines) representing (partial) correlations. The network itself represents one or more psychological constructs (Christensen, Golino, & Silvia, 2020; Cramer, 2012). Dimensionality assessment is a common psychometric analysis performed using networks. In networks, communities—sets of densely connected nodes—are consistent with latent factors in factor models (Golino & Epskamp, 2017). To date, psychometric network approaches are often as accurate than factor analytic methods in many conditions (Christensen, Garrido, Guerra-Peña, & Golino, 2023; Cosemans, Rosseel, & Gelper, 2021; Golino et al., 2020) but are less accurate when there are substantial cross-loadings (Brandenburg & Papenberg, 2022) and few variables per factor with large correlations (Haslbeck & van Bork, 2022). Despite their accurate recovery of multidimensional structures, studies examining their effectiveness to recover unidimensional structures are lacking.

Unidimensionality is fundamental to psychometrics. Many psychometric models (e.g., item response theory) and analyses (e.g., internal consistency) assume that scales are unidimensional. The validity and interpretation of scale scores are often based on the measurement of a single, unitary construct (Montoya & Edwards, 2020). Networks, representing a scale, are often quantified as if all variables belong to a single construct—the network itself represents a single dimension. Centrality measures, for example, are based on a node's relative position in the network—their position is based on their connection to all other nodes in the network (e.g., sum of a node's connections to other nodes; Bringmann et al., 2019). The psychometric interpretation of these measures is confounded in the presence of multiple underlying causes (e.g., multidimensionality; Hallquist, Wright, & Molenaar, 2021).

The goal of the present study is to explain and resolve the limited capacity of community detection algorithms to recover unidimensional structures. The paper starts by reviewing previous simulation studies that examined multidimensional and unidimensional structures with community detection algorithms. Next, several algorithms are introduced with specific attention paid to a common objective criterion, modularity, that underlies many of these algorithms. The limits of modularity to recover unidimensional structures are explained using an empirical example. Based on these limitations, the use of correlation matrices, rather than networks, is proposed to be a more effective method to detect unidimensional structures using community detection algorithms. A Monte Carlo simulation and grid search are performed to identify optimal parameters that are used in these algorithms to minimize the trade-off between the detection of unidimensional and bidimensional (i.e., two factor) structures. After, the most accurate parameters for each community detection algorithm are compared against parallel analysis.

2 Psychometric Networks and Community Detection

In psychology, most networks involve estimating relationships between variables because their relationships between variables are not known (in contrast to social networks where links between people can be determined directly; Bringmann & Eronen, 2018). The Gaussian graphical model (Lauritzen, 1996) is one of the more common models to estimate, representing the relationships between nodes as conditionally dependent given all other nodes in the network. A common method to estimate a Gaussian graphical model is the so-called EBICglasso (Epskamp, Cramer, Waldorp, Schmittmann, & Borsboom, 2012; Epskamp & Fried, 2018; Foygel & Drton, 2010), which applies the graphical least absolute shrinkage and selection operator (GLASSO; Friedman, Hastie, & Tibshirani, 2008; Friedman, Hastie, & Tibshirani, 2014). The least absolute shrinkage and selection operator (LASSO; Tibshirani, 1996) of the GLASSO is a regularization technique that reduces parameter estimates (i.e., partial correlations) with some estimates becoming exactly zero.

The EBICglasso has two tunable parameters: lambda (λ) and gamma (γ). λ controls the sparsity of the network with larger values leading to sparser (fewer edges) networks. Usually, several values (e.g., 100) that exist between a logarithmic range of λ are used to estimate networks. The default range is defined by a minimum-maximum ratio typically set to 0.01 (Epskamp & Fried, 2018). The γ parameter is used in the extended Bayesian information criterion (Chen & Chen, 2008), which is used for model selection on the networks estimated across the range of λ's. γ controls the extent to which simpler models (i.e., networks with fewer edges) are preferred to more complex models (i.e., networks with more edges). Larger γ values lead to denser networks, smaller γ values lead to sparser networks. The default setting for this parameter is typically 0.50.

Community detection algorithms are used to identify communities that form meaningful groups of variables (Fortunato, 2010). In psychology, an approach called exploratory graph analysis (EGA; Golino & Epskamp, 2017) was first applied with the EBICglasso and the Walktrap community detection algorithm (Pons & Latapy, 2006). Golino and Epskamp (2017) formalized communities in networks as statistically consistent with latent factors in factor models. They conducted a simulation study that demonstrated EGA's effectiveness at recovering multidimensional structures, relative to common factor analytic approaches (e.g., eigenvalues greater than 1), and found EGA was among the most accurate dimension recovery methods.

Since their seminal simulation study, several simulation studies have been conducted and continue to demonstrate EGA's effectiveness across different data conditions. Studies have demonstrated that EGA is highly accurate when the data are dichotomous and skewed (Cosemans et al., 2021; Golino et al., 2020). Other studies have examined different network estimation methods (Golino et al., 2020), different community detection algorithms, polytomous data (Christensen, Garrido, et al., 2023), text data, intensive longitudinal data (Golino, Christensen, Moulder, Kim, & Boker, 2022), data with wording effects (Garcia-Pardina, Abad, Christensen, Golino, & Garrido, 2022), and hierarchical or bi-factor structured data (Jiménez et al., 2023). Across all of these studies, EGA was among the top methods to recover the number of simulated dimensions in the data. Other studies have found that EGA tends to be less accurate than factor analytic methods when there are substantial cross-loadings (Brandenburg & Papenberg, 2022) or few variables per

factor with large correlations (Haslbeck & van Bork, 2022). Most of these studies, however, focused exclusively on multidimensional data (i.e., data with two or more underlying latent factors).

2.1 Community Detection and Unidimensional Structures

Of these simulation studies, only a few have examined unidimensional structures. Golino et al.'s (2020) simulation was one of the first to examine continuous and dichotomous data with unidimensional and multidimensional structures using EGA. They point out that the Walktrap algorithm is ineffective at recovering unidimensional structures on its own. They identified the problem was with the objective criterion of the Walktrap algorithm called modularity (Newman, 2006b). Modularity seeks to maximize the edges within communities and minimize edges between communities, with greater modularity suggesting a greater proportion of edges within communities relative to between. Based on how modularity is computed, which is discussed in more detail in the next section, there is a penalty for unidimensional structures such that they have a modularity of zero. To overcome this limitation, Golino and colleagues proposed a so-called "unidimensionality adjustment" by generating four variables from a single factor model that are (close to) orthogonal to the empirical data. If the Walktrap algorithm returned two communities (one empirical community and the other generated data), then the empirical data are determined to be unidimensional; otherwise, the Walktrap algorithm was applied to the empirical data only. Their approach worked well, boasting 96% accuracy in the unidimensional conditions.

The approach employed by Golino and colleagues (2020) was effective but accurate recovery of unidimensional structures decreased as the number of variables increased (i.e., from four variables to twelve). Another simulation study by Christensen et al. (2023) sought to examine whether applying community detection algorithms to the correlation matrices could be used instead, a common approach used in network neuroscience (e.g., Gates, Henry, Steinley, & Fair, 2016). They found that applying community detection algorithms to the correlation matrices vastly improved their ability to detect unidimensional structures. This improvement in performance came at a cost: The algorithms could detect unidimensional and bidimensional structures as well as or better than the same algorithms applied to networks estimated by the EBICglasso, but they failed to accurately recover multidimensional structures with more than two factors. Of the algorithms tested, the Leading Eigenvalue (Newman, 2006a) and Louvain (Blondel, Guillaume, Lambiotte, & Lefebvre, 2008) algorithms were the most accurate for unidimensional and bidimensional structures. They proposed an updated unidimensionality approach for psychometric networks: apply the Leading Eigenvalue algorithm to the correlation matrix, if one community is identified, then the structure is unidimensional; otherwise, apply the community detection algorithm to the estimated network. Relative to Golino and colleagues' (2020) unidimensionality adjustment, this approach saw a slight decrease in accuracy for unidimensional structures (about 3%) and a slight increase in accuracy for bidimensional structures (about 5%). Although these studies have hinted at ways community detection algorithms can recover unidimensional structures, none of them have explicitly defined why recovery of unidimensional structures in networks is problematic.

2.2 Modularity and Community Detection

Before addressing the limits of unidimensional recovery with community detection algorithms, their inner workings must be understood. Modularity is one of the most common objective criterion used in community detection algorithms (Fortunato, 2010). Its premise is simple: Maximize the within-community connections and minimize the between-community connections. The formal definition of this premise is:

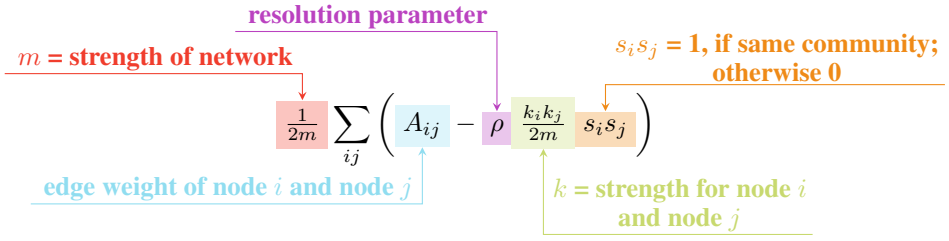


Figure 1: Equation for modularity (Newman, 2006b) with resolution parameter (Reichardt & Bornholdt, 2006)

where the term $\frac{k_i k_j}{2m}$ represents the expected edge weights between node i and j if edges are placed at random (Newman, 2006b). The term ρ is a so-called resolution parameter that can be tuned to prefer fewer ($0 < \rho < 1$) or more ($1 < \rho < \infty$) communities (Reichardt & Bornholdt, 2006).

Community detection algorithms use modularity in different ways. The Walktrap algorithm uses modularity to identify the community placements for nodes that maximize modularity after applying an agglomerative hierarchical clustering random walk method (Pons & Latapy, 2006). The Leading Eigenvalue and Louvain algorithms also use modularity. The Leading Eigenvalue algorithm uses the so-called modularity matrix ($A_{ij} - \frac{k_i k_j}{2m}$) where elements of the matrix represent relative within- and between-community strength gained (or lost). Taking from spectral clustering, the eigenvector of the largest eigenvalue in the modularity matrix is used to determine the number of communities and node placements in those communities (Newman, 2006a).

The Louvain algorithm applies a greedy approach: The algorithm starts with nodes in singleton (or their own) communities and merges nodes one-by-one based on the maximum (positive) change in modularity (Blondel et al., 2008). After the algorithm has made a pass over all nodes, the communities are merged by summing the edge weights of the nodes in their respective communities, forming "latent" nodes. The process starts over on these "latent" nodes until modularity can no longer be increased. The Louvain algorithm is one of the most popular algorithms in network science and is often among the most accurate algorithms to recover simulated and known community structures (e.g., Christensen, Garrido, et al., 2023; Gates et al., 2016; Yang, Algesheimer, & Tessone, 2016).

A recently developed algorithm called Leiden was developed to overcome some of the limitations of the Louvain algorithm (Traag, Waltman, & Van Eck, 2019). First, rather than using modularity, an alternative objective criterion called the Constant Potts Model (Traag, Van Dooren, & Nesterov, 2011) can be used to overcome the resolution limit (i.e., failure to detect smaller communities; Fortunato & Barthelemy, 2007). Similar to modularity, the Constant Potts Model uses a resolution parameter but slightly differs in that it is used as a threshold: communities should have a density (number of edges within the community divided by the total number of edges) greater than the resolution parameter while the density between communities should be less than the resolution parameter. Another improvement on the Louvain algorithm is the so-called "smart local move," which allows nodes to move to different communities as a refinement of the original merging and does so in random order of the nodes (Waltman & Van Eck, 2013). This last improvement underlies the more deterministic nature of the Leiden algorithm relative to the Louvain algorithm. The Louvain algorithm is stochastic such that its final placement of nodes in communities are affected by the initial ordering of the nodes (Blondel et al., 2008). Approaches such as consensus clustering can ameliorate this limitation, but it is more computationally intensive relative to the Leiden algorithm (Lancichinetti & Fortunato, 2012).

2.3 Modularity in Networks and its Limits for Unidimensional Structures

Community detection algorithms are usually applied directly to networks in the psychological literature. Often researchers are investigating multidimensional constructs; however, given the importance of unidimensional structures in psychology, identifying unidimensional structures in networks is fundamental to their psychometric utility. In the factor analytic tradition, principal component analysis (PCA) and exploratory factor analysis (EFA) are useful tools in this endeavor, with different approaches performing well in different conditions (Garrido, Abad, & Ponsoda, 2013; Slocum-Gori & Zumbo, 2011). One approach to resolve the unidimensional problem in network psychometrics would be to combine factor analytic approaches with network analysis approaches, which has been proposed to solve various other measurement issues such as measurement error (Epskamp, Rhemtulla, & Borsboom, 2017). One aim of this paper, however, is to explain why unidimensionality recovery in network models is a challenge in the first place.

Modularity is at the heart of many community detection algorithms. As Newman and Girvan (2004) point out, modularity equals zero in a network when the within-community edges are no better than random. This case can occur in networks where there is a single community. In contrast, singleton communities often have modularity less than zero. To provide an example, the `bfi` dataset from the {psychTools} package (version 2.3.9; Revelle, 2019) in R (version 4.3.1; R Core Team, 2022) is used. The `bfi` dataset has 2800 people who completed the 25-item Big Five Inventory (John, Donahue, & Kentle, 1991). There are five items per Big Five factor: openness to experience, conscientiousness, extraversion, agreeableness, and neuroticism. For the example, only the extraversion items are used.

To check whether the data are unidimensional, a one factor model using confirmatory factor analysis was fit with the WLSMV estimator via the {lavaan} package (version 0.6.12; Rosseel, 2012) in R. The five extraversion items demonstrated good fit for a single factor model: $\chi^2(5) = 73.380, p < .001$, CFI $= 0.992$, TLI $= 0.984$, RMSEA $= 0.071, p = .007$, 95% CI[0.057, 0.086], SRMR $= 0.035$. Using dynamic fit cut-offs from the {dynamic} package (version 1.1.0; Wolf & McNeish, 2022) in R, level-1 misspecification (fitted model plus one-third of variables with 0.30 residual correlations) for the model was CFI $= 0.968$, RMSEA $= 0.097$, and SRMR $= 0.033$ and level-2 misspecification (fitted model plus two-third of variables with 0.30 residual correlations) was CFI $= 0.950$, RMSEA

$= 0.120$, and SRMR $= 0.038$ (McNeish & Wolf, 2022). Based on these dynamic fit measures, the one factor model demonstrated minimal misspecification.

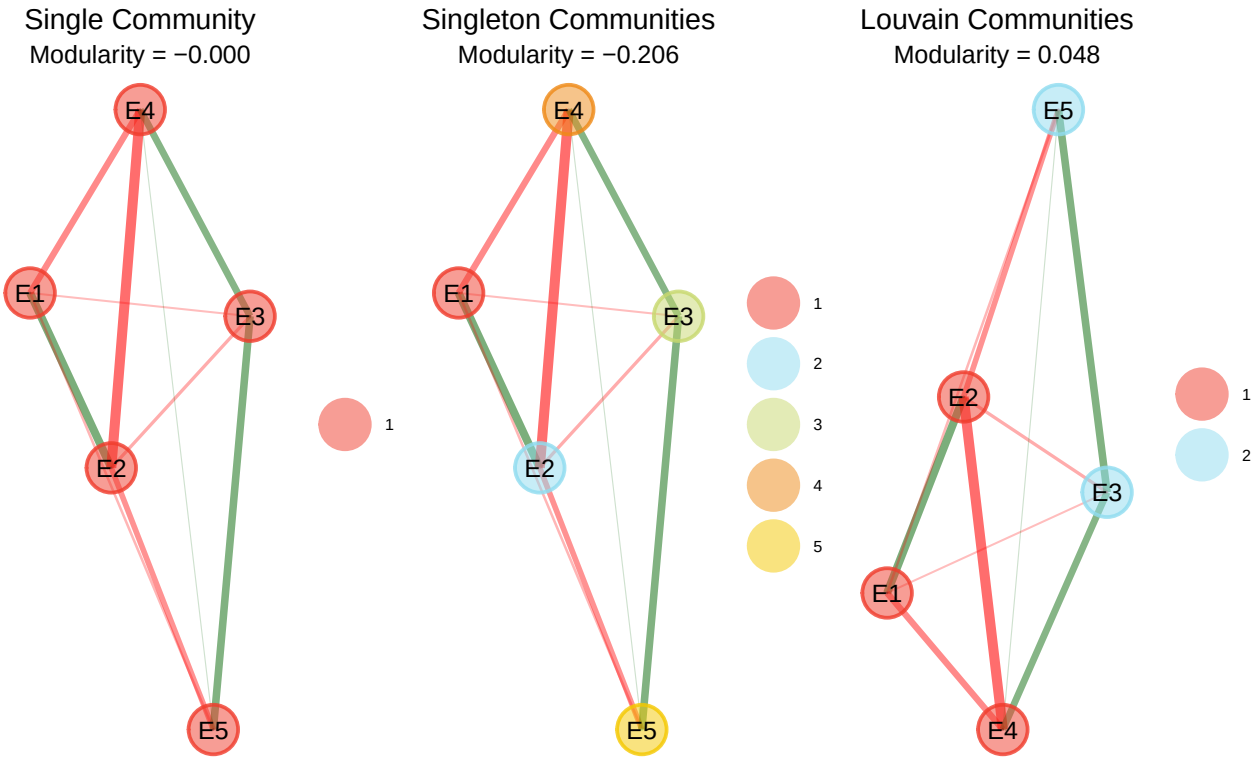


Figure 2: Example of modularity in networks (regularized partial correlations). Single community modularity (left), singleton communities modularity (middle), and Louvain communities modularity (right).

The modularity for a single community, singleton communities, and Louvain communities are displayed in Figure 2. As expected, modularity for the single community was equal to zero. For the singleton communities, modularity was negative (-0.21), while for the Louvain communities, modularity was positive (0.05). Despite strong evidence for a single community (factor), the Louvain algorithm's two community structure had the highest modularity.

There are a few things to point out. First, in psychology, community detection algorithms are applied to the absolute values of the network. Some algorithms can handle signed networks (e.g., spinglass; Reichardt & Bornholdt, 2006), but negative signed edges are often interpreted as repulsive, rather than attractive, forces for community cohesion (Gómez, Jensen, & Arenas, 2009). Second, negative modularity values do not reflect negative edges in the network but rather greater between-community strength than within-community strength. Consider singleton communities where between-community strength is the sum of a node's edges (to all other nodes) whereas within-community strength is zero (no reciprocal connection to itself). Third, the extraversion network is fully connected (i.e., every node is connected to all other nodes), which is often *not* the case for networks in psychology.

This last point deserves more attention. Waldorp and Marsman (2021) showed that unidimenisonal latent variable models induce non-zero partial correlations between variables (Holland & Rosenbaum, 1986) and only as the number of variables increase to infinity do its partial correlations shrink to zero (Guttman, 1953). With regularization, some of these partial correlations may be (inappropriately) set to zero, leading to more distinct connectivity patterns between nodes that biases modularity away from selecting a single community.

Many networks have zeros in their (partial) correlation matrix—that is, networks are a sparse representation of the covariance between variables. This sparsity is often induced by network estimation approaches such as regularization in the GLASSO (Epskamp, Kruis, & Marsman, 2017). Sparsity may facilitate the detection of multiple communities in network models because there is often a lack of connection between many variables, promoting a more modular covariance structure (Christensen, Garrido, et al., 2023; Golino & Epskamp, 2017; Golino et al., 2020). Sparsity therefore could be a culprit for why unidimensional structures are not often detected in networks. This rationale, however, does not hold for this example.

Instead, community detection algorithms, particularly those that use modularity, are affected by the weights between the nodes. (Regularized) partial correlations condition associations between two nodes on all other nodes in the network, leading to (often) smaller associations between nodes. Even without regularization, the partial correlations of a unidimensional factor model cannot be stronger than its corresponding zero-order correlation (van Bork et al., 2019). This statement is reflected in the left and middle heatmaps of Figure 3.

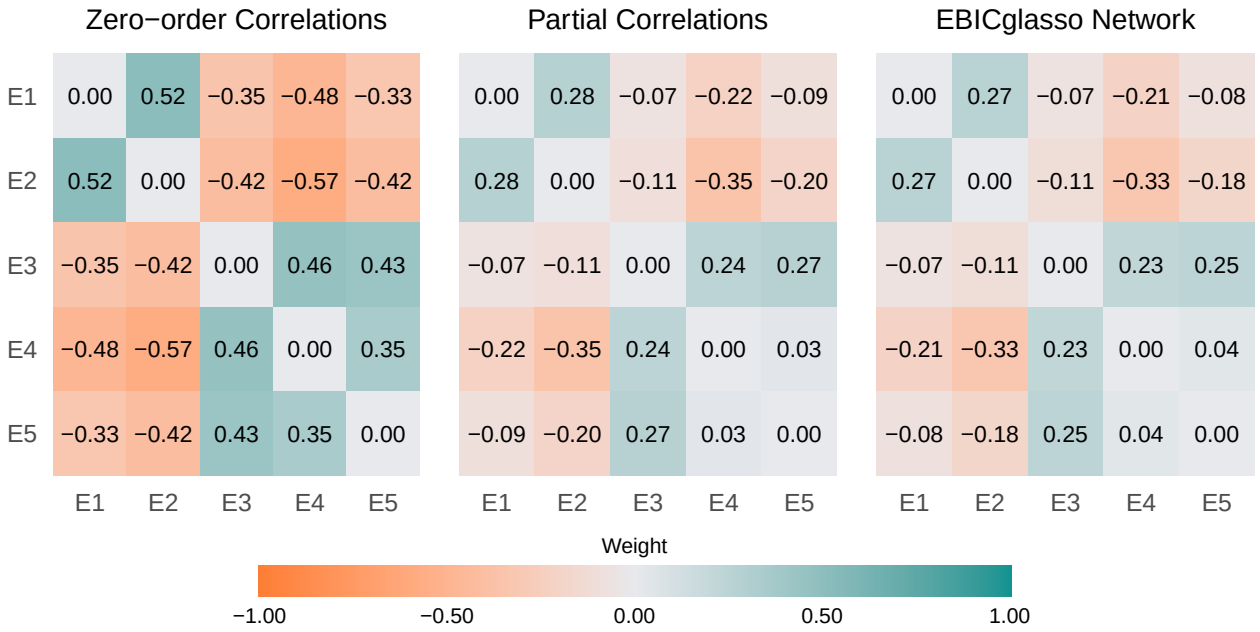


Figure 3: Comparison of zero-order (left), partial (middle), and regularized partial correlations (right) for the Big Five Inventory extraversion scale. Positive (partial) correlations are teal and negative (partial) correlations are orange.

When moving from zero-order correlations to partial correlations, the change in the magnitude of the associations is unevenly distributed with some partial correlations becoming very small relative to their original zero-order correlation (e.g., E5–E4 in Figure 3). The (absolute) zero-order correlations for E3 to all other variables, for example, are roughly the same size, ranging from 0.35-0.46. The partial correlations for E3, however, have a larger range of (absolute) values (0.03-0.35) with a greater change in the magnitude of association (absolute change in parentheses) for E1 (0.28) and E2 (0.31) than for E4 (0.22) and E5 (0.16). These differences remain when the partial correlations are regularized (right heatmap in Figure 3).

The consequences of these differences are double-edged. On the one hand, there is a tendency for the association structure to differentiate the within-community strengths from the between-community strengths, which can be advantageous when trying to recover more multiple communities. On the other hand, community detection algorithms that use modularity partition the network into two or more communities due to this more modular structure—even when the underlying structure of the data are unidimensional.

The implication is that unidimensional structures are unlikely to be detected in network models (particularly with community detection algorithms using modularity). To detect unidimensional structures using community detection algorithms, the zero-order correlation matrix may be more fruitful. Although the modularity of unidimensional structures will still equal zero, there should be a greater penalty on modularity for structures with more than one community. Such a penalty is due to the larger magnitude of the associations between all variables.

To illuminate why, consider the Louvain communities with reference to Figure 3: For the zero-order correlations, E5 has an absolute between-community strength of 1.10 to community one (E1, E2, and E4) and an absolute within-community strength of 0.43 (E3); for the EBICglasso network, its absolute between-community strength is 0.30 (E1, E2, and E4) and its absolute within-community strength is 0.25 (E3). With modularity seeking to maximize within-community strength and minimize between-community strength, there will be a greater preference to merge E5 with community one (E1, E2, and E4) for the zero-order correlations than the EBICglasso network. In contrast, the between-community strength for E5 in the EBICglasso network is nearly equal to its within-community strength.

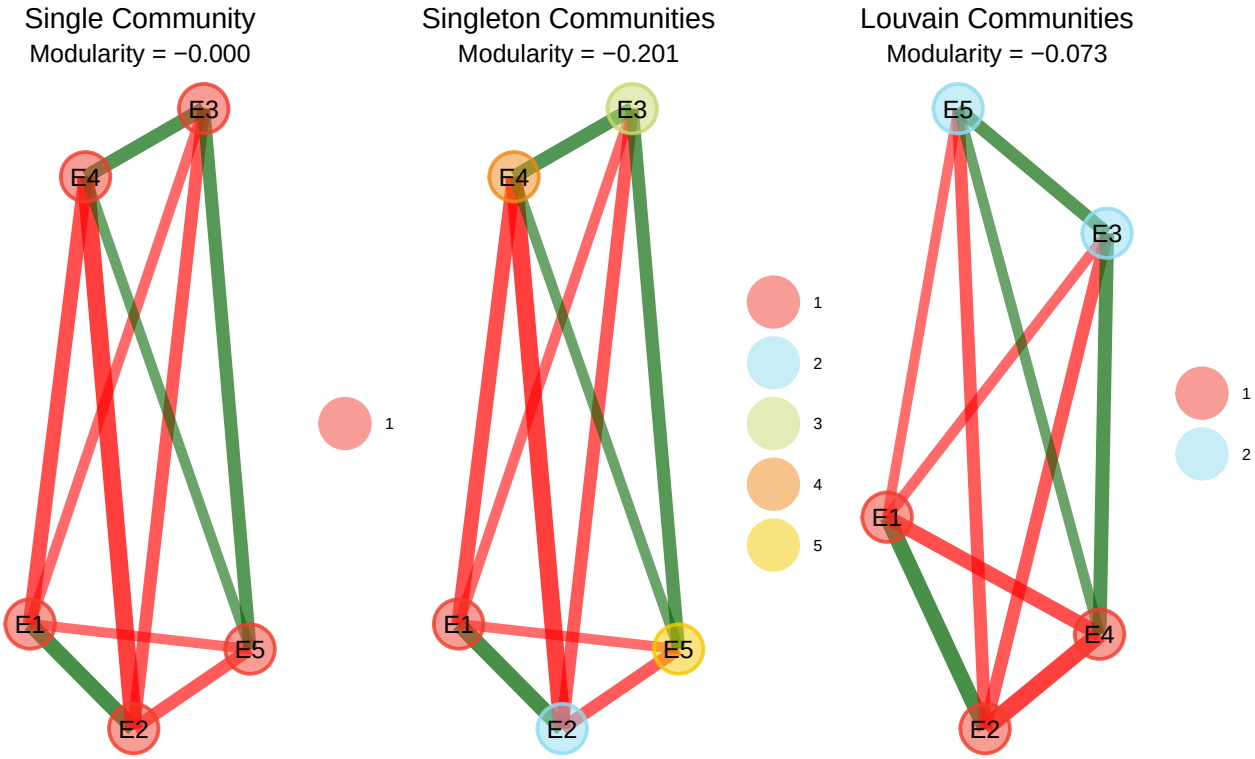


Figure 4: Example of modularity in zero-order correlations. Single community modularity (left), singleton communities modularity (middle), and the original Louvain communities modularity (right).

In the example, the penalty of zero-order correlations on multidimensional structures is observed in modularity (Figure 4): the single community equals zero and the singleton communities still had a negative modularity (-0.20), while the original Louvain communities that had the highest (positive) modularity in the EBICglasso network (0.05; Figure 2) now have a negative modularity (-0.07) in the zero-order correlation matrix—lower modularity than the single community. This example demonstrates the crux of the problem to recover unidimensional structures in psychometric networks: partial correlations reduce the associations between variables (often unevenly), creating wider discrepancies between the within- and between-community strengths that are used to maximize modularity. This discrepancy works well to detect multidimensional structures in networks, but limits their ability to recover unidimensional structures.

3 Present Research

Research examining the recovery of unidimensional structures with community detection algorithms is sparse. Simulation studies that have examined single community structures are limited in their design. Gates and colleagues (2016) examined one unidimensional condition with correlation matrices of 1,000 variables. Two other studies used unidimensional structures of four, eight, and twelve variables (Christensen, Garrido, et al., 2023; Golino et al., 2020). The number of variables in the former are unlikely to occur in most psychometric settings, while the number of variables in the latter only evaluate the lower end of the spectrum for psychological constructs that may be unidimensional. The overarching goal of this study was to evaluate the efficacy of community detection algorithms to recover unidimensional structures in a more representative set of conditions.

To achieve this goal, the simulation conducted in this study set out with three aims. The first aim was to examine community detection algorithms on zero-order and partial correlation matrices when data were generated from unidimensional and bidimensional factor structures. Partial correlation matrices, rather than networks, were used to demonstrate that what limits community detection algorithms from detecting unidimensional structures is mainly due to association discrepancies between zero-order and partial correlations rather than sparse network structures.

The second aim was to perform a grid search along the tunable parameters in community detection algorithms to determine the optimal trade-off between the recovery of unidimensional and bidimensional structures. This study examined the Walktrap, Leading Eigenvalue, Louvain, and Leiden algorithms. The Walktrap was selected because of its

common use in the psychometric network literature (e.g,. EGA; Golino & Epskamp, 2017; Golino et al., 2020). The Leading Eigenvalue and Louvain algorithms were chosen because of their high accuracy recovering unidimensional and bidimensional structures in a previous simulation (Christensen, Garrido, et al., 2023). The Leiden algorithm was chosen because it introduces several improvements to the Louvain algorithm, potentially increasing its efficacy (Traag et al., 2019). The Louvain and Leiden algorithm can tune the resolution parameter in modularity to select for fewer communities (resolution ≤ 1). The Walktrap algorithm can tune the number of steps it takes in its random walk. Finally, the Leiden algorithm can use the Constant Potts Model as an alternative objective criterion, which may overcome some of the aforementioned limitations of modularity.

The final aim was to compare the most accurate algorithm and parameter pairings with parallel analysis. Parallel analysis represents the benchmark standard to recover unidimensional and bidimensional structures (Garrido et al., 2013; Guo & Choi, 2022). Previous simulation studies have demonstrated that parallel analysis with PCA often outperforms other factor analytic and network psychometric methods when recovering unidimensional structures, particularly when the data are continuous (Christensen, Garrido, et al., 2023; Golino et al., 2020).

The design of this simulation study was preregistered on the Open Science Framework (OSF): https://osf.io/2p5hq. There were no explicit hypotheses for this study other than the main goal of demonstrating the (in)effectiveness of community detection algorithms using zero-order (and partial) correlations. Given previous work examining the efficacy of the Leiden algorithm, however, it might be expected that this algorithm will be the most accurate (Traag et al., 2019). Similarly, because the Constant Potts Model is reported to not be affected by the resolution limit (Reichardt & Bornholdt, 2006; Traag et al., 2019), it might be expected that this objective criterion would be the most accurate.

4 Methods

4.1 Simulation Design

The preregistered simulation design was as follows: Continuous multivariate random variables were generated from unidimensional (one factor) and bidimensional (two factors) factor models. The total number of variables were 8, 12, 16, 20, 30, and 40 where the two factor models had half the total number of variables loading on each of its factors (i.e., 4, 6, 8, 10, 15, and 20).

For both the unidimensional and bidimensional models, there were three levels of factor loadings representing low (0.40), moderate (0.55), and high (0.70) that were randomly drawn from a uniform distribution with a range ± 0.10 (i.e., 0.30-0.50, 0.45-0.65, and 0.60-0.80, respectively). There were three levels of sample size representing small (250), moderate (500), and large (1000). Zero-order correlations were computed using Pearson's correlation (W). Partial correlations $W_{XY|\mathbf{Z}}$ were obtained by following:

$$
\begin{aligned}
W &= \frac{\Sigma}{\sigma_X \sigma_Y}, \\
K &= W^{-1}, \\
W_{XY|\mathbf{Z}} &= -\frac{K}{\sqrt{IK}\sqrt{IK}},
\end{aligned}
$$

where Σ is the covariance matrix, σ_X and σ_Y are the standard deviations of variables X and Y (respectively), K is the inverse of the zero-order correlation matrix, and $\sqrt{IK}$ is the square root of the inverse zero-order correlation matrix's diagonal.

For the bidimensional models, there were four levels of correlations between factors representing orthogonal (0.00), small (0.30), moderate (0.50), and large (0.70). In addition, there were cross-loadings generated randomly from a normal distribution with a mean of 0.00 and standard deviation of 0.10 (García-Garzón, Abad, & Garrido, 2019).

The mixed factorial design for the unidimensional models was $6 \times 3 \times 3$ (number of variables $\times$ loadings $\times$ sample size) for a total of 54 conditions. The mixed factorial design for the bidimensional models was $6 \times 3 \times 3 \times 4$ (number of variables $\times$ loadings $\times$ sample size $\times$ correlations between factors) for a total of 216 conditions. For each condition, 100 replicates were generated for a total of 27,000 replicates (i.e., 5,400 and 21,600 from unidimensional and bidimensional, respectively).

4.2 Grid Search for the Optimal Community Detection Algorithms

The community detection algorithms were described in the Introduction. Here, the preregistered parameters for the grid search are reported. For Louvain and Leiden algorithms with modularity as the objective criterion, the range of

the resolution parameter (ρ) was between 0.60 to 1.00 in increments of 0.05. For the Leiden algorithm with Constant Potts Model as the objective criterion, the range of the resolution parameter was between 0.00 to 0.40 in increments of 0.05. For the Walktrap algorithm, the range of the steps parameter was between 3 to 8 in increments of 1. The Leading Eigenvalue algorithm did not have a tunable parameter to search.

4.3 Parallel Analysis

Parallel analysis was used as a benchmark to compare against the most accurate community detection algorithm and parameter pairs derived from the grid search. PCA was applied because of its extensive use in the literature (Garrido et al., 2013). Parallel analysis was applied directly to the data, rather than correlation matrix, so that it could generate random datasets by resampling from the original data (Horn, 1965). Two criteria were used to select the number of components: eigenvalues of the original dataset that were (1) greater than the mean and (2) greater than the 95$^{\text{th}}$ percentile. The mean criterion has a notable bias to select at minimum one dimension as the first eigenvalue is almost always greater than the mean of all eigenvalues. It is therefore appropriate and recommended to use the 95$^{\text{th}}$ percentile criterion for the first eigenvalue and the mean criterion for the rest of the eigenvalues. For the purposes of this study, the mean eigenvalue criterion did *not* use this strategy to avoid mixing the parallel analysis criteria.

4.4 Data Generation

The data generation procedure generally followed the same approach as Golino et al. (2020). First, the reproduced population correlation matrix was computed:

$$\mathbf{R_R} = \mathbf{\Omega\Phi\Omega}',$$

where $\mathbf{R_R}$ is the reproduced population correlation matrix, omega ($\mathbf{\Omega}$) is the k (variables) $\times$ r (factors) factor loading matrix, and $\mathbf{\Phi}$ is the $r \times r$ correlation matrix. The population correlation matrix, $\mathbf{R_P}$, was then obtained by putting the unities on the diagonal of $\mathbf{R_R}$. Next, Cholesky decomposition was performed on the correlation matrix such that:

$$\mathbf{R_P} = \mathbf{U}'\mathbf{U}.$$

If the population correlation matrix was not positive definite (i.e., at least one eigenvalue ≤ 0) or any single item's communality was greater than 0.90, then $\mathbf{\Omega}$ was re-generated and the same procedure was followed until these criteria are met. Finally, the sample data matrix of continuous variables was computed:

$$\mathbf{X} = \mathbf{ZU},$$

where $\mathbf{Z}$ is a matrix of random multivariate normal data with rows equal to the sample size and columns equal to the number of variables. All generated data were continuous.

4.5 Performance Measures

For the grid search, accuracy was the sole performance metric. Accuracy is defined as '1' if the estimated number of dimensions (e.g., 1) equal the simulated number of dimensions (e.g., 1) and '0' otherwise. For the comparison between the best performing algorithm and parameter pairings, accuracy and false discovery rate were used. False discovery rate (FDR) was defined by the total number of false positives divided by the total number of positives (false positives plus true positives). FDR represents the extent to which the estimated result can be trusted as true. An FDR of 0.05, for example, represents 5 false positives for every 95 true positives. Accuracy is provided for the number of simulated factors whereas FDR is provided for each method's estimated factors. For FDR, a true positive was when the estimated number of dimensions (e.g., 1) equaled the number of simulated dimensions (e.g., 1). A false positive was when the estimated number of dimensions (e.g., 1) did not equal the number of simulated dimensions (e.g., 2).

Analysis of variances (ANOVAs) were computed for accuracy using the most accurate algorithm-parameter pairs and parallel analysis criteria. Separate ANOVAs were conducted on the unidimensional and bidimensional structures. The dependent variable of the ANOVAs were the accuracy of '1' or '0' for each condition and replicate, resulting in 5,400 cases for the unidimensional and 21,600 cases for the bidimensional structures. For both structures, a fully factorial design was used to allow for all possible interactions. Partial eta squared (η_p^2) was used for effect size. As reported in the preregistration, only large effect sizes ($\eta_p^2 = 0.14$, according to Cohen's (1992) guidelines) are discussed.

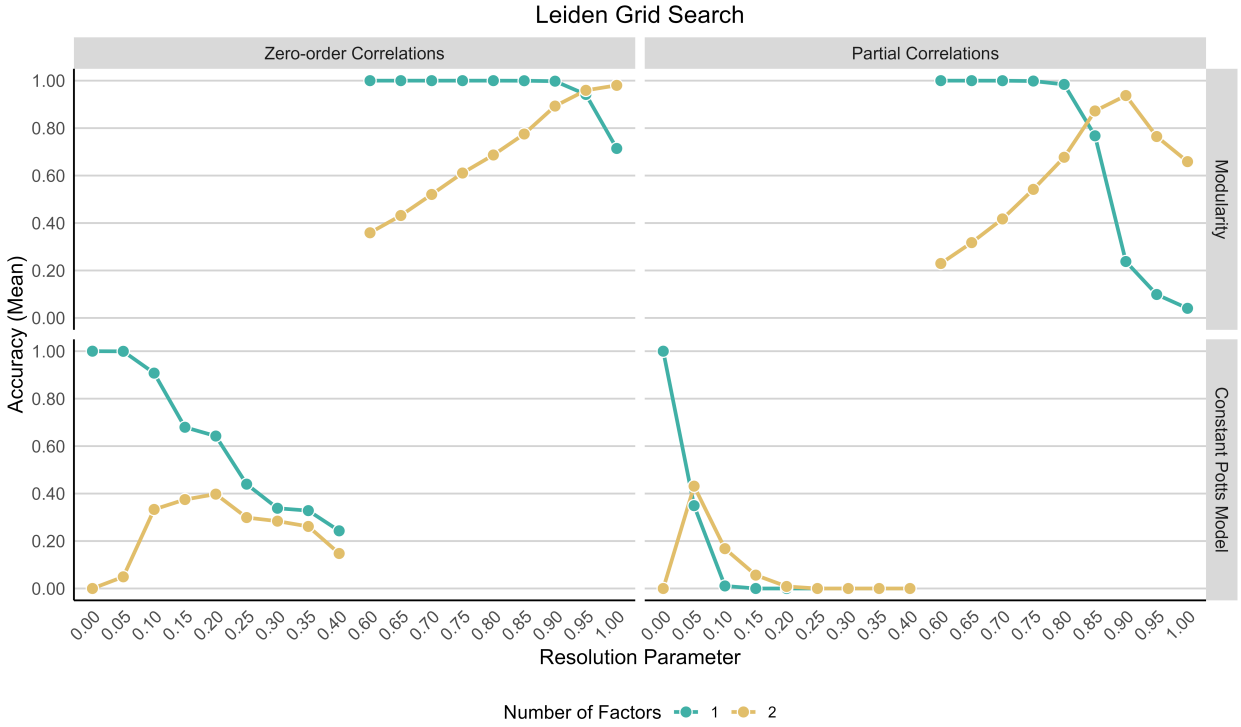
4.6 Software Implementation

All data generation and manipulation, methods, statistical analyses, and visualizations were performed in R. The {latentFactoR} package (version 0.0.6; Christensen, Nieto Canaveras, et al., 2023) was used to generate the data, {igraph} package (version 1.6.0; Csardi & Nepusz, 2006) was used to implement the community detection algorithms, and {ggplot2} package (version 3.4.4; Wickham, 2016) was used to visualize all results. All R scripts and data used in the analyses are available on the OSF: https://osf.io/zn3hs/.

5 Preregistered Results

5.1 Grid Search

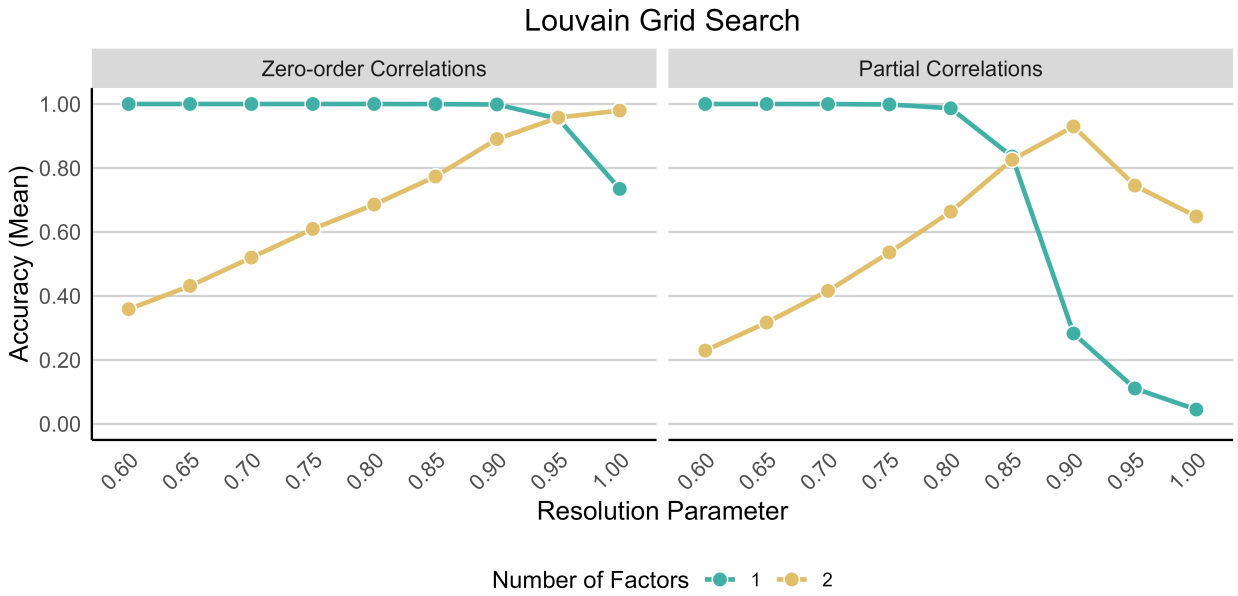
Figure 5: Grid Search for the Leiden algorithm



The Leiden algorithm grid search is presented in Figure 5. Between the two objective criteria, modularity and Constant Potts Model, the former outperformed the latter. For both criteria, the zero-order correlations outperformed the partial correlations within their corresponding resolution parameters. For modularity, there was a clear trend where bidimensional structures accuracy increased and the unidimensional structures accuracy decreased as the resolution parameter increased. The most accurate objective criterion and resolution parameter for the Leiden algorithm was with modularity and 0.95, respectively. For unidimensional structures, its accuracy was 94.2%; for bidimensional structures, its accuracy was 95.9%.

The Louvain algorithm grid search is presented in Figure 6. Consistent with the Leiden algorithm, the zero-order correlations outperformed the partial correlations within their corresponding resolution parameters. Similarly, as the resolution parameter increased, the accuracy for unidimensional structures decreased and the accuracy for bidimensional structures increased.

Figure 6: Grid Search for the Louvain algorithm



The Louvain algorithm with zero-order correlations and a resolution parameter of 0.95 was the most accurate across structures. For unidimensional structures, its accuracy was 95.4%; for bidimensional structures, its accuracy was 95.8%.

As with the Leiden and Louvain algorithms, the zero-order correlations outperformed the partial correlations across the number of steps in the Walktrap algorithm. In terms of the steps parameter, there was no clear pattern of effect—that is, the steps parameter did not appear to affect the recovery of unidimensional or bidimensional structures.

There was a negligible linear increase in accuracy occurred as the number of steps increased for bidimensional structures (from 97.6% to 97.8%). Conversely, there was a negligible linear decrease in accuracy as the number of steps increased for unidimensional structures (from 78.8% to 77.5%). Given the nominal differences between the step parameters, the default step parameter for {igraph}'s `cluster_walktrap` function was used for further comparisons (i.e., 4).

There were no parameters to search over the Leading Eigenvalue algorithm. Instead, only zero-order and partial correlations were compared. Once again, zero-order correlations outperformed the partial correlations. The accuracy for the zero-order correlations were 75.7% and 98.4% and partial correlations were 6.0% and 80.0% for unidimensional and bidimensional structures, respectively.

In sum, the Leiden and Louvain algorithms with modularity as its objective criterion were the most accurate algorithms. The Walktrap algorithm was hardly affected by the choice in steps parameter. The Leading Eigenvalue algorithm was more accurate with zero-order correlations. Across all algorithms, zero-order correlations outperformed partial correlations, supporting the interpretation that the difficulty of recovering unidimensional structures with community detection algorithms that attempt to maximize modularity is primarily due to the differential reductions in the associations between variables.

5.2 Benchmark Comparison

The accuracy and FDR for both parallel analysis criteria are presented along side the most accurate zero-order correlation algorithm and parameter pairings (Table 1).

Factors	Accuracy		FDR	
	1	2	1	2
Leiden	0.942	0.959	0.040	0.058
Louvain	0.954	0.958	0.041	0.045
Walktrap	0.781	0.977	0.017	0.201
Leading Eigenvalue	0.757	0.984	0.014	0.228
Parallel Analysis (Mean)	0.999	0.954	0.043	0.001
Parallel Analysis (Quantile)	1.000	0.935	0.064	0.000

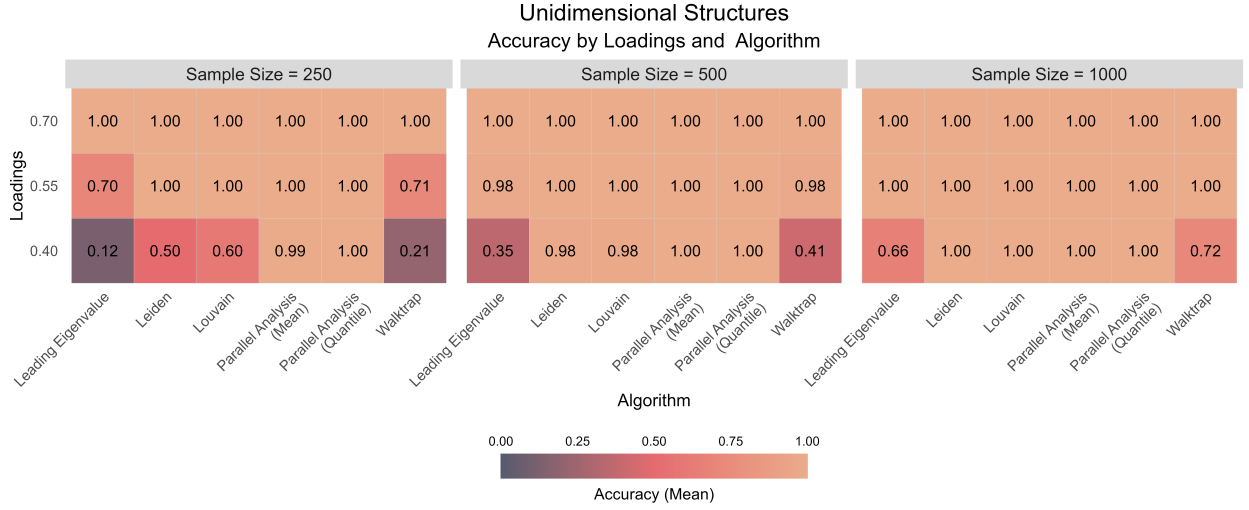
Table 1: Best Performing Algorithm and Parameter Pairings

Both mean and 95th percentile criteria of parallel analysis achieved near perfect recovery of unidimensional structures (99.9% and 100%, respectively) followed by Louvain (95.4%) and Leiden (94.2%) algorithms. For bidimensional structures, Leading Eigenvalue (98.4%) and Walktrap (97.7$) had the highest accuracy. All methods examined were highly accurate (>90%) at recovering bidimensional structures. FDR told a more nuanced story. The Leading Eigenvalue and Walktrap algorithms were biased toward bidimensional structures (FDR = 0.228 and 0.201, respectively) relative to unidimensional structures (FDR = 0.014 and 0.017, respectively). The mean and 95th percentile criteria for parallel analysis rarely estimated two dimensions when there was a single dimension (FDR = 0.001 and 0.000, respectively). The Louvain and Leiden algorithms, and particularly the Louvain algorithm, had relatively balanced FDRs showing the least relative bias toward unidimensional or bidimensional structures.

5.3 ANOVAs for Accuracy

For the unidimensional structures, there were two interactions that reached large effect sizes across the methods. The Leading Eigenvalue ($\eta_p^2 = 0.265$) and Walktrap ($\eta_p^2 = 0.153$) algorithms had a three-way interaction between number of variables, loading size, and sample size such that accuracy decreased as the number of variables increased but the loading size and sample size decreased. The Leiden ($\eta_p^2 = 0.316$) and Louvain ($\eta_p^2 = 0.218$) algorithms had an interaction between loading size and sample size such that accuracy decreased as they both decreased (Figure 7). Neither parallel analysis criteria reached a large effect size across conditions.

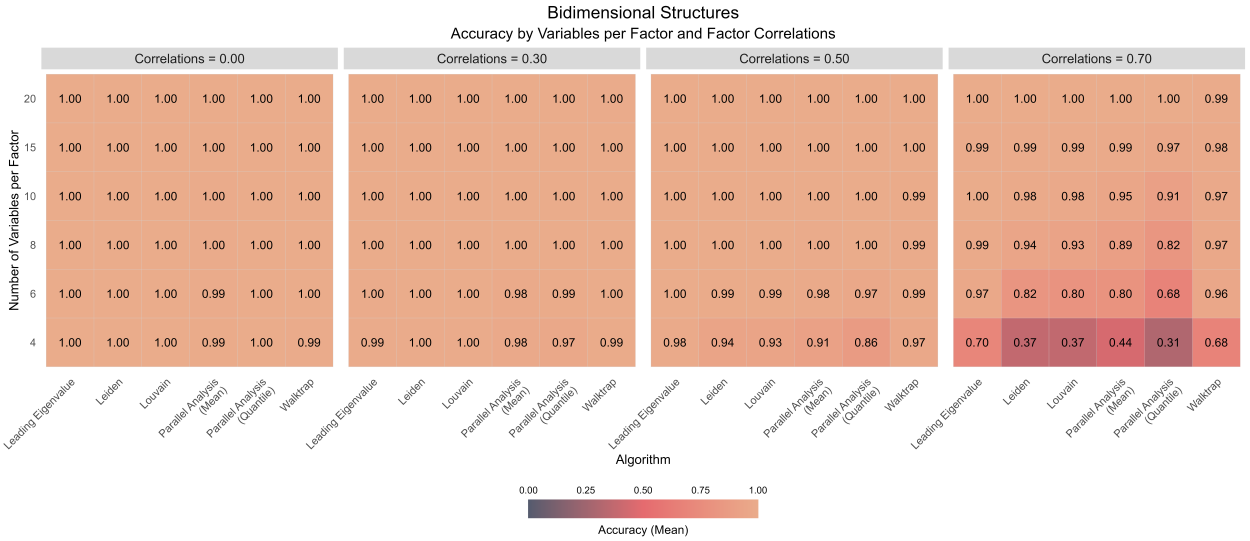
Figure 7: Accuracy broken down by loadings and sample size for each method in unidimensional structures



For the bidimensional structures, the interaction between the number of variables and correlations between factors was the only effect size to reach at least a moderate-to-large effect size across all methods: Leading Eigenvalue ($\eta_p^2 = 0.148$), Leiden ($\eta_p^2 = 0.284$), Louvain ($\eta_p^2 = 0.275$), parallel analysis with mean criterion ($\eta_p^2 = 0.187$), and 95th criterion ($\eta_p^2 = 0.241$; Figure 8). Only the Walktrap algorithm had a moderate effect size ($\eta_p^2 = 0.110$). In this interaction,

accuracy decreased as the number of variables decreased and the correlations between factors increased. There were no other effects that reached a large effect size for any method.

Figure 8: Accuracy broken down by number of variables per factor and correlation between factors (Correlations) for each method in bidimensional structures



For unidimensional structures, all community detection algorithms struggled to recover a single community when the loadings were low and sample size was small. The Leiden and Louvain algorithms performed as well as parallel analysis except in these conditions, while the Walktrap and Leading Eigenvalue algorithm did not perform as well when the loadings were small in general (Figure 7). For bidimensional structures, all methods struggled when there were few variables and large correlations between factors. All community detection algorithms performed about as well as the parallel analysis. In terms of FDR, there were a mix of biases. The Leading Eigenvalue and Walktrap algorithms were biased more toward two dimensions relative to one, both parallel analysis criteria were biased more toward one dimension relative to two, and the Leiden and Louvain algorithms had relatively similar biases between these structures.

For the Leading Eigenvalue and Walktrap algorithms, the pattern of the interactions across the structures were particularly concerning. For unidimensional structures, the accuracy decreased as the number of variables increased; for bidimensional structures, the accuracy decreased as the number of variables decreased. The pattern of this decrease in accuracy was the same for both algorithms: for unidimensional structures, as the number of variables increased, the more likely they would overestimate the number of dimensions; for bidimensional structures, as the number of variables decreased, the more likely they would underestimate the number of dimensions.

6 Exploratory Approach and Analysis

6.1 Consensus Clustering

Based on the results, the Leiden and Louvain algorithms were the most accurate community detection algorithms and were comparable to the parallel analysis criteria. Of the two, the Louvain algorithm was at least as accurate as the Leiden algorithm and demonstrated relatively lower bias. This result is surprising given that the Leiden algorithm includes several improvements on top of the base Louvain algorithm (Traag et al., 2019). One limitation of the Louvain algorithm is that the ordering of the nodes can affect the resulting community solution (the result is stochastic). To overcome this limitation, the Leiden algorithm incorporates different node switching procedures between communities during a refinement phase that leads to a more deterministic result (Traag et al., 2019).

The Louvain algorithm is not alone in its stochastic nature as many community detection algorithms are stochastic rather than deterministic (Fortunato, 2010; Gates et al., 2016). One approach to overcome this limitation is called consensus clustering (Lancichinetti & Fortunato, 2012). Consensus clustering applies a community detection algorithm to a network (or correlation matrix) repeatedly (e.g., 1,000). The community memberships are then used to compute the proportion of times two nodes are placed into the same community, resulting in a symmetric matrix. Some threshold

(e.g., 0.30) is then applied to the matrix such that nodes that are assigned to a community 30% or less of the time have their proportion set to zero. This process starts over, applying the community detection algorithm repeatedly (e.g., 1,000) to this new matrix. This procedure continues until all proportions reach 1, resulting in a block diagonal matrix. The block diagonal matrix represents the "consensus" community structure.

To explore whether the Louvain algorithm's recovery of unidimensional and bidimensional structures could be improved, the consensus clustering approach using 1,000 iterations was applied to the same zero-order correlation matrices and the best performing resolution parameter (i.e., 0.95). Two additional approaches based on the traditional consensus clustering approach were also evaluated. The first approach was to use the most common community structure identified across the first pass of 1,000 iterations. This approach will be called the "most common" approach. The second approach was to obtain the structure that had the highest modularity across the first pass of 1,000 iterations. This approach will be called the "highest modularity" approach.

For the unidimensional structures, the most common approach had the highest accuracy (95.8%) followed by the traditional consensus clustering (94.3%) and highest modularity (93.8%) approaches. The most common approach demonstrated nominal improvement relative to the standard Louvain (95.4%). For the bidimensional structures, the differences between approaches were similarly nominal: highest modularity (96.0%), most common (95.8%), and traditional consensus clustering (95.8%). All approaches were comparable with the standard Louvain (95.8%). Based on these results, there was not a substantial improvement over and above the single-shot application of the Louvain algorithm.

7 Discussion

This study evaluated the extent to which community detection algorithms can recover unidimensional structures. Previous simulations in this area are limited, examining only a few conditions with few, if any, providing a systematic analysis on the trade-offs between unidimensional and bidimensional structures. One issue with identifying unidimensional structures using community detection algorithms is that the objective criterion of modularity penalizes single community solutions. This simulation explored whether this penalty could be overcome by using zero-order correlations rather than partial correlations. Using a Monte Carlo simulation, a grid search was performed over the tunable parameters of the Leiden, Louvain, and Walktrap algorithms. Confirming expectations, zero-order correlations were found to be more effective at recovering unidimensional (and bidimensional) structures than partial correlations. Using the most accurate parameters from the grid search, these community detection algorithms, and the Leading Eigenvalue algorithm, were compared against a factor analytic benchmark of parallel analysis with PCA. The Leiden (using modularity) and Louvain algorithms performed on par with parallel analysis whereas the Walktrap and Leading Eigenvalue algorithms were much less accurate at recovering unidimensional structures.

Unidimensional structures are fundamental to psychometrics but problematic for community detection algorithms used in network analysis. The root cause of this issue has been discussed (e.g., modularity penalizes unidimensional structures; Golino et al., 2020; Newman, 2006b) but rarely examined. An empirical example was provided to demonstrate that partial correlations are responsible for the limited utility of modularity to recover unidimensional structures in using community detection algorithms. With zero-order correlations, the penalty of modularity on single community solutions is mitigated, yielding more effective recovery of unidimensional structures. The grid search demonstrated this point explicitly: Zero-order correlations outperformed partial correlations for every community detection algorithm. Taken together, this evidence points to partial correlations, despite capturing common covariance (Waldorp & Marsman, 2021), as a major limiting factor to the accurate recovery of unidimensional structures using community detection algorithms that seek to maximize modularity.

These results build on the sparse literature using community detection algorithms to recover unidimensional structures. In Gates et al.'s (2016) study, only one condition was examined: a unidimensional structure with 1,000 variables. In traditional psychometrics, instances of 1,000 variables being collected, let alone having a single, underlying unidimensional structure, will be rare. In this condition, two algorithms not used in this study (Infomap and Label Propagation) were the most accurate. Two other simulation studies examined unidimensional structures with four, eight, and twelve variables (Christensen, Garrido, et al., 2023; Golino et al., 2020). Golino and colleagues' (2020) offered a novel approach of expanding the network matrix with an orthogonal unidimensional factor to identify these structures. Christensen and colleagues (2023) compared this expansion approach and the zero-order correlation approach used in this study, finding that the Leading Eigenvalue and Louvain algorithms using the zero-order correlations were the most accurate.

This study extends the latter study but makes unidimensional and bidimensional structures its focus. Two limitations of the Christensen et al. (2023) study were that only a few number of variables were used (i.e., four, eight, and twelve) and the number of variables per factor in the two factor condition were not matched to be the same total number of

variables between the unidimensional and bidimensional conditions. This simulation addressed these limitations directly by including several different conditions for the number of variables (i.e., eight, twelve, sixteen, twenty, thirty, and forty) and equating the total number of variables between the unidimensional and bidimensional structures. These design choices lead to substantial differences in the conclusions drawn by Christensen et al. (2023). Specifically, they suggested that the Leading Eigenvalue algorithm should be favored. This simulation demonstrates that the Leading Eigenvalue algorithm is actually among the least accurate and most biased algorithms tested in this study. The results from this simulation make clear that the Leiden and Louvain algorithms should be favored.

The Leiden and Louvain algorithms were about as accurate as parallel analysis with mean and 95th percentile criteria. Although the accuracy differences for both unidimensional and bidimensional structures were negligible, the methods demonstrated different biases. Based on FDR, the parallel analysis criteria tended toward unidimensional structures relative to bidimensional structures whereas Louvain and Leiden algorithms were generally balanced in their bias with the Leiden algorithm relatively more toward bidimensional structures.

There are a few key limitations that constrain the generalizability of the results of this study. First, only continuous data without skew were generated. Polytomous and dichotomous data with skew are known to be detrimental to dimension recovery methods (Christensen, Garrido, et al., 2023; Garrido et al., 2013; Golino et al., 2020). Similarly, population error can introduce subtle deviance on structures that are unnoticeable in contemporary measures of fit but can hinder the accurate recovery of dimensions (Cudeck & Browne, 1992; Jiménez et al., 2023). Second, as unidimensional structures were the primary concern of this study, the identification of no common factors is equally as important as bidimensional structures. Parallel analysis with the mean of the eigenvalues criterion, for example, would be expected to identify at least one dimension often—even when there are no common factors. For the community detection algorithms, the expectation would be that they would also break down and identify dimensions when there are none. One solution to this issue is to perform common factor adequacy test, such as Bartlett's test of sphericity (Bartlett, 1951), to determine that there is sufficient covariance between variables to identify any dimensions in the data.

Another potential issue is whether there are variables that are "orphaned" such that they do not strongly covary with any variables. These so-called orphaned variables can lead to low loadings or worse increase the number of dimensions identified without much notice (e.g., eigenvalues would appear to be consistent with the number of extra dimensions due to the variable having an eigenvalue around 1). In networks, such variables often appear as disconnected from the rest of the network due to regularization or sparsity induction (i.e., removal of small or negligible associations). How these variables would be treated by community detection algorithms on the zero-order correlation matrix is an open question. Some expected outcomes might be that these variables would be identified as singleton communities or they would be placed in a dimension with other variables. For the former, the EGA approach treats singleton communities as missing and does not include them as part of their dimension recovery result, thereby mitigating their consequences. The latter, however, is more insidious and warrants investigation.

Network analysis is a burgeoning area in psychometrics. Despite its progress over the last decade, studies that set out to establish its footing in traditional psychometric theory have been sparse (Hallquist et al., 2021). Given that unidimensionality is an assumption of many psychometric models (e.g., item response theory) and underlies the validity of the measurement and interpretation of scores, establishing how network science approaches can estimate undimensionality is fundamental to advancing network psychometrics as a field (Montoya & Edwards, 2020). The consequences of unidimensionality are especially important for network measures that are commonly used in psychology such as centrality which tend to treat networks as unidimensional (Bringmann et al., 2019). Unless researchers use measures that account for the community structure of their networks, their centrality measures are likely to biased at best and invalid at worst (Blanken et al., 2018; Hallquist et al., 2021).

8 Acknowledgments

The author would like to thank Luis Eduardo Garrido for his thoughtful conversations and feedback on previous versions of this draft. Similarly, the author would like to thank the anonymous reviewer who has provided constructive comments that have substantially improved the quality of this research.

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