

$$\underbrace{\sqrt{a} \cdot \sqrt{a}}_{\geq 0} = \underbrace{a}_{\geq 0}$$

$$\sqrt{a^2} = a = \sqrt{a} \cdot \sqrt{a} = a$$

$$\sqrt{a^2} = |a| \quad - \text{Betrag}$$

Rechnen mit $\sqrt{}$

$$\sqrt{9} + \sqrt{16} = 3 + 4 = 7$$

$$\sqrt{9+16} = \sqrt{25} = 5$$

$$\sqrt{100-64} = \sqrt{36} = 6$$

$$\sqrt{100} - \sqrt{64} = 10 - 8 = 2$$

Beachte also:

$$\sqrt{a} \pm \sqrt{b} \neq \sqrt{a \pm b} \quad - \text{Rechenverbot}$$

$\sqrt{3} \pm \sqrt{2}$ so stehen liegen!

Unterscheide:

$$(\sqrt{a} \pm \sqrt{b})^2 = \sqrt{a^2} \pm 2\sqrt{a}\sqrt{b} + \sqrt{b^2} = a \pm 2\sqrt{a}\sqrt{b} + b$$

$$\sqrt{a+b}^2 = a+b$$

$$\sqrt{a^2} \pm \sqrt{b^2} = a \pm b$$

$$\sqrt{a^2} + \sqrt{b^2} = |a| + |b|$$

$$\sqrt{(a \pm b)^2} = |a \pm b|$$

Multiplikation / Division von $\sqrt{}$

$$(\sqrt{a}\sqrt{b})^2 = (\sqrt{a}\sqrt{b})(\sqrt{a}\sqrt{b}) \stackrel{\text{AG}}{\underset{\text{KG}}{=}} \sqrt{a}\sqrt{b}\sqrt{a}\sqrt{b} = ab$$

$$\Rightarrow \sqrt{(\sqrt{a}\sqrt{b})^2} = \sqrt{ab}$$

$$\Rightarrow \underbrace{|\sqrt{a}\sqrt{b}|}_{\geq 0 \geq 0} = \sqrt{ab}$$

$$\Rightarrow \boxed{\sqrt{a}\sqrt{b} = \sqrt{ab}} \quad (\text{Multiplikationsregel})$$

analog gilt:

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	(Divisionsregel) ($b \neq 0$)
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Beispiele:

$$a) \sqrt{4,5} \cdot \sqrt{2} = \sqrt{4,5 \cdot 2} = \sqrt{9} = 3$$

$$b) \sqrt{\frac{4,5}{2}} = \sqrt{\frac{4,5}{2}} = \sqrt{2,25} = 1,5$$

$$c) \sqrt{3} \cdot \sqrt{5x} = \sqrt{3 \cdot 5x} = \sqrt{15x}$$

$$d) \sqrt{10x} : \sqrt{5} = \sqrt{\frac{10x}{5}} = \sqrt{2x}$$

$$e) \sqrt{2a} \cdot \sqrt{\frac{a}{2}} = \sqrt{2a \cdot \frac{a}{2}} = \sqrt{a^2} = |a| = a$$

$$f) \sqrt{a} : \sqrt{a^3} = \sqrt{\frac{a}{a^3}} = \sqrt{\frac{1}{a^2}} = \frac{\sqrt{1}}{\sqrt{a^2}} = \frac{1}{|a|} = \frac{1}{a} = a^{-1}$$

