Matrices and Vectors

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Special cases

column vector $\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$

row vector $\underline{b} = [b_1 \; b_2 \; \cdots \; b_n]$

Product of Two Matrices

$$C = A \, B$$

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1l} \\ c_{21} & c_{22} & \cdots & c_{2l} \\ & & & \\ c_{n1} & c_{n2} & \cdots & c_{nl} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & & a_{nm} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{ml} \end{bmatrix}$$

Example

$$\begin{bmatrix} 2 & -1 & 6 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 4 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Transpose of Matrix

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad A^T_{n\times m} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \Rightarrow \underline{a}^T =$$

Dot Product of Two Vectors

$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\underline{a} \cdot \underline{b} = \underline{a}^T \underline{b} =$$

Determinant

- only defined for
- Notation :

Case $n = 2$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$=$$

Case $n=3$

$$A = \begin{bmatrix} a_{11} & {}_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det(A) = a_{11} \underbrace{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}}_{} + a_{12} \underbrace{\left(- \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \right)}_{} + a_{13} \underbrace{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}_{}$$

Example

$$A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}, \quad |A| =$$

Matrix of Co-Factors

$$A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}, \quad C =$$

Adjoint of Matrix

$$\operatorname{adj}(A) = C^T =$$

Inverse of Matrix

- well-defined only for A with
- A^{-1} satisfies :

$$I_{n\times n} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{n\times n}$$

Result

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

Example

$$A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 3 & 9 \\ 8 & -11 & -34 \\ -5 & 7 & 21 \end{bmatrix}$$

$$\det(A) = |A| = 1$$

$$\operatorname{adj}(A) = \begin{bmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{bmatrix} =$$

Check :

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix} \begin{bmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{bmatrix} =$$

Binary Hypothesis Testing (ECE 313)

X :

H_0 : plane is

H_1 : plane is

Goal: Decide which hypothesis is true based on X

Prior information $P(H_0) = \pi_0$, $P(H_1) = \pi_1 =$

Observation pdf/pmf $p(x|H_0)$, $p(x|H_1)$

Likelihood Ratio $\Lambda(x) =$

Performance

Probability of False Alarm $P_{FA} =$

Probability of Miss $P_M =$

Overall probability of error $P_e =$

MAP Rule decide 1 if

decide 0 if

Example $\pi_0 = 0.5, \quad \pi_1 = 0.5.$

$$p(x|H_0) = \begin{cases} 0.1 & x=0 \\ 0.2 & x=1 \\ 0.3 & x=2 \\ 0.4 & x=3 \\ 0 & \text{otherwise} \end{cases} \qquad p(x|H_1) = \begin{cases} 0.25 & x=0 \\ 0.25 & x=1 \\ 0.2 & x=2 \\ 0.3 & x=3 \\ 0 & \text{otherwise} \end{cases}$$

(a) MAP Rule:

$$\Lambda(x) = \frac{p(x|H_1)}{p(x|H_0)} \underset{0}{\overset{1}{\gtrless}} \frac{\pi_0}{\pi_1}$$

$\Lambda(x) >$

$\Lambda(x) <$

MAP rule decides $\begin{cases} 1 & \text{if} \\ 0 & \text{if} \end{cases}$

(b) Performance:

$P_{FA} = P(\text{decide } H_1 | H_0) =$

$P_M = P(\text{decide } H_0 | H_1) =$

$P_e =$