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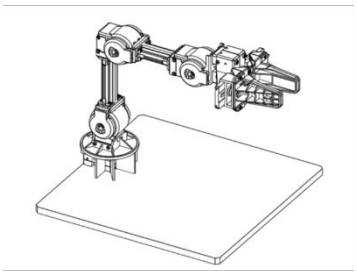
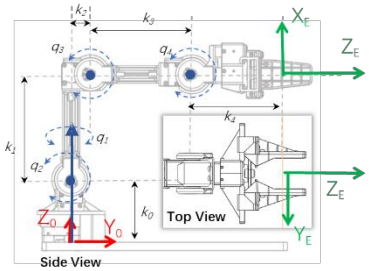
ECE 470: Introduction to Robotics Homework 2

Question 1. *(12 marks)*

A 4-DOF (excluding gripper) robotic serial manipulator arm is shown in Figure 1.





Using the D-H convention learned in class,

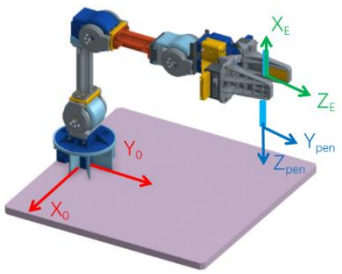
a) Assign frames to the links on a schematic diagram that represents the robot arm

b) Tabulate the D-H parameters

c) Obtain the forward kinematics representing the pose of end-effector frame {E} referenced from base frame {0}.

Question 2. *(8 marks)*

The serial manipulator arm is tasked to write on the board plane Z_o, with a pen attached to the gripper {E}. For the ink to flow, ${}^0Z_{pen}$ has to be $(0\ 0\ -1)^T$ i.e. vertically downwards. As shown in the diagram, axis X_E and Z_E are parallel to Z_{pen} and $Y_{pen.}$ respectively. The distance between Z_E and Y_{pen} is k_0.



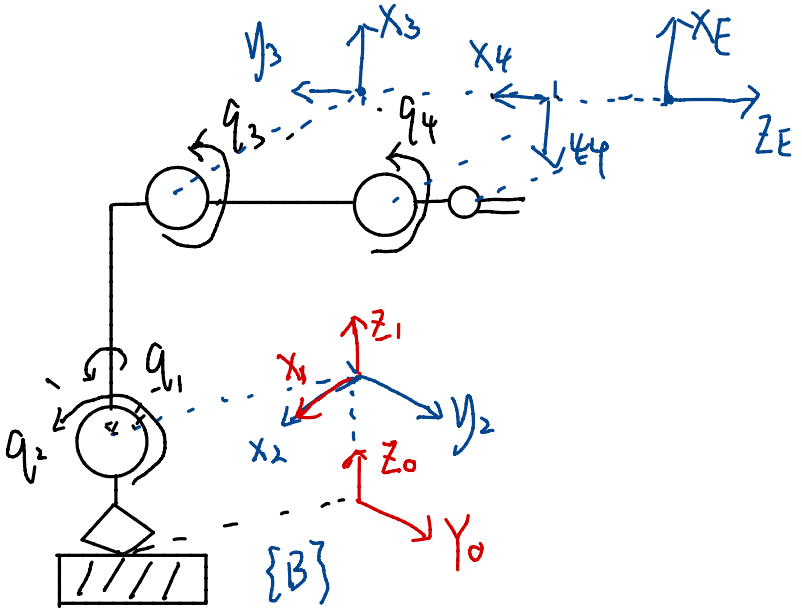
State any assumption or condition while working on the following:

a) Write down the transformation matrix ${}^{E}_{pen}T$

b) If the pen tip is to be place on the board with coordinates ${}^0(u,v)$, find the expressions describing the joint variable $\boldsymbol{q}$ in terms of $k_{0\text{-}4}$, u and v.

c) Describe the workspace of the writing task if the distance between Z_E and Y_{pen} is now change to $k_0/2$. Assume that q_2 can only move its link in a range of 0 to 180° from the plane.

1. a)



b)

	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	$\frac{\pi}{2}$	k_0
2	$-\frac{\pi}{2}$	0	0	0
3	0	$-k_2$	$-\frac{\pi}{2}$	0
4	0	k_1	0	0
5	0	0	$\frac{\pi}{2}$	0
6	0	$-k_3$	0	0
7	0	$-k_4$	$\frac{\pi}{2}$	0
E	$\frac{\pi}{2}$	0	0	0

where ${}^{i-1}_{i}T = R_x(\alpha_{i-1}) D_x R_z(\theta_i) D_z(d_i)$

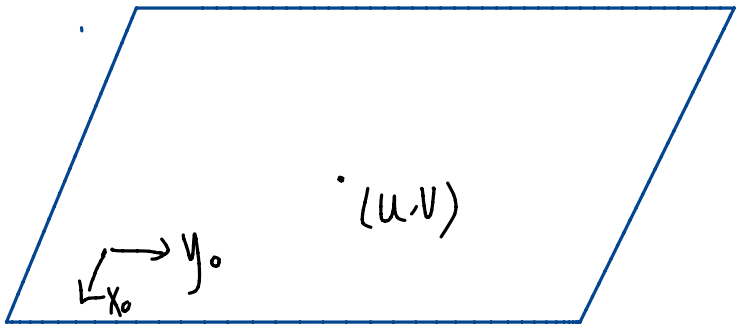
c) ${}^0T_E = F(Q) = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_5T\,{}^5_6T\,{}^6_7T\,{}^7_ET$.

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & k_2+k_3+k_4 \\ 1 & 0 & 0 & k_0+k_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. a) ${}^{E}_{pen}T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

b) $q = IGM({}^{0}T_{E})$ with $q = [q_1, q_2, q_3, q_4]^T$

$\therefore {}^{0}p = (u, v)^T$



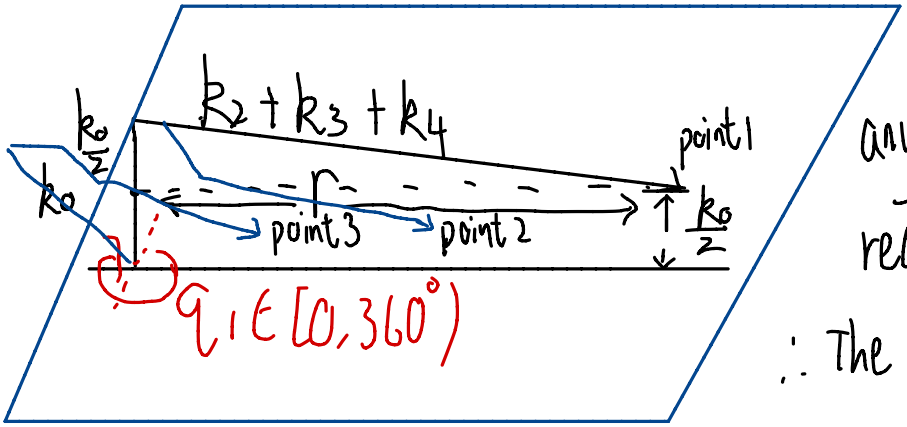
The pen tip is to be place on $(u, v, 0)$

${}^{0}_{E}T = \begin{bmatrix} -\cos(\varphi) & -\sin(\varphi) & 0 & u \\ \sin(\varphi) & \cos(\varphi) & 0 & v \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

where $\begin{cases} \cos(\varphi) = C_{1234} \\ \sin(\varphi) = S_{1234} \\ u = k_0 C_1 + (k_1 + k_2) C_{12} + k_3 C_{123} \\ v = k_0 S_1 + (k_1 + k_2) S_{12} + k_3 S_{123} \end{cases}$

c) $r = \sqrt{(k_2 + k_3 + k_4)^2 - \frac{k_0^2}{4}}$

Due to the three DOF q_2, q_3, q_4, the pen tip can be moved onto any point inside its maximum reached outline (a circle)



$\therefore$ The work space:

$S = \pi r^2 = \pi \left[(k_2 + k_3 + k_4)^2 - \frac{k_0^2}{4} \right]$