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ECE 470: Introduction to Robotics Homework 1

Question 1.

In Figure 1, Frame {A} and {B} are not connected.

a) Determine the transformation matrix ${}^A_B T_1$ after {B} rotates 45° about its axis X_B.

b) Determine the inverse matrix ${}^A_B T_1{}^{-1}$ in (a)

c) Determine the transformation matrix ${}^A_B T_2$ if new {B} revolve 45° about Y_A.

d) Determine the transformation matrix ${}^A_B T_3$ if {A} rotates -90° about its X_A

(10 Points)

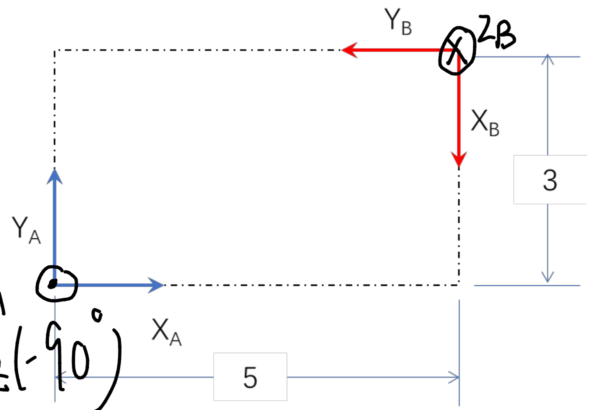

Figure 1

a) $${}^A R_B = R_x(180^\circ) R_z(-90^\circ)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x(45^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

~~Assume the frame before {B} rotates 45° is {B'}~~

$${}^A R_{B_1} = {}^A R_B \, R_x(45^\circ)$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$${}^A P_{B_1} = {}^A P_B = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$$

$$\therefore {}^A T_{B_1} = \left[\begin{array}{c|c} {}^A R_{B_1} & {}^A P_{B_1} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 5 \\ 1 & 0 & 0 & 3 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. b) $${}^{A}_{B_1}T^{-1} = {}^{B_1}_{A}T = \begin{bmatrix} {}^{A}_{B_1}R^{T} & -{}^{A}_{B_1}R^{T} \cdot {}^{A}P_{B_1} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & -3 \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & -\frac{5\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & -\frac{5\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-{}^{A}_{B_1}R^{T} \cdot {}^{A}P_{B_1} = -\begin{bmatrix} 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ -\frac{5\sqrt{2}}{2} \\ -\frac{5\sqrt{2}}{2} \end{bmatrix}$$

$${}^{A}_{B_1}R = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

c) $$R_Y(45^\circ) = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$${}^{A}_{B_2}T = {}^{A}_{B_1}T\, R_Y(45^\circ) = \begin{bmatrix} 0 & 0 & 1 & \frac{5\sqrt{2}}{2} \\ 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $${}^{A}_{B}T_3 = R_x(-90^\circ)\, {}^{A}_{B}T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & \frac{5\sqrt{2}}{2} \\ 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & \frac{5\sqrt{2}}{2} \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2.

A cuboid with Frame {M} and Frame {C} attached rigidly is shown in Figure 2. The universe frame of reference {U} serves as an absolute frame that is always fixed. The cuboid motion is described by the series of transformation operations.

1> Rotation about the z axis of Frame C by 30°, then
2> Translation of (1, 2, 3) along Frame C, then
3> Rotation about the x axis of Frame M by 45°, and then
4> Rotation about the y axis of Frame U by 60°.

Let ${}^{U}T_{C_i}$ and ${}^{U}T_{M_i}$ be the 4×4 homogeneous transformation matrices that describes the position and orientation of Frames C and M, respectively, in U after motion i.

Find
i. ${}^{U}T_{C_1}$
ii. ${}^{U}T_{C_2}$
iii. ${}^{U}T_{C_3}$
iv. ${}^{U}T_{C_4}$
v. ${}^{U}T_{M_4}$

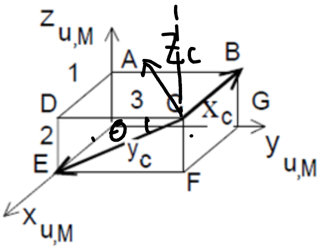

Figure 2

line segment lengths:
AD=1
DC=3
DE=2

(10 Points)

$${}^{U}T_C = {}^{M}T_C$$

$$\cos\theta = \frac{3}{\sqrt{2^2+3^2}} = \frac{3}{\sqrt{13}}$$

$$\sin\theta = \frac{2}{\sqrt{2^2+3^2}} = \frac{2}{\sqrt{13}}$$

$$P_c = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

i) $${}^{U}R_C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ 0 & -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix}$$

$$R_z(30^\circ) = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore {}^{U}R_{C_1} = {}^{U}R_C \cdot R_z(30^\circ) = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{3}{2\sqrt{13}} & -\frac{3\sqrt{3}}{2\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ -\frac{1}{\sqrt{13}} & -\frac{\sqrt{3}}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix}$$

$${}^{U}T_{C_1} = \begin{bmatrix} {}^{U}R_{C_1} & {}^{U}P_{C_1} \\ 000 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 1 \\ -\frac{3}{2\sqrt{13}} & -\frac{3\sqrt{3}}{2\sqrt{13}} & -\frac{2}{\sqrt{13}} & 3 \\ -\frac{1}{\sqrt{13}} & -\frac{\sqrt{3}}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.866 & 0.5 & 0 & 1 \\ -0.416 & -0.721 & -0.555 & 3 \\ -0.277 & -0.48 & 0.832 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii) $${}^{U}T_{C_2} = \begin{bmatrix} {}^{U}R_C & {}^{U}P_{C_1} + {}^{U}R_C \cdot P_c \\ 000 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.866 & 0.5 & 0 & 1.134 \\ -0.416 & -0.721 & -0.555 & -0.519 \\ -0.277 & -0.48 & 0.832 & 3.257 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{U}R_C \cdot P_c = \begin{bmatrix} 1 - \frac{\sqrt{3}}{2} \\ -\frac{3\sqrt{3}}{\sqrt{13}} - \frac{3}{2\sqrt{13}} - \frac{6}{\sqrt{13}} \\ -\frac{2\sqrt{3}}{\sqrt{13}} + \frac{9}{\sqrt{13}} - \frac{1}{\sqrt{13}} \end{bmatrix}$$

$$= \begin{bmatrix} 0.134 \\ -3.519 \\ 1.257 \end{bmatrix}$$

iii) $R_x(45^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

$$\begin{cases} {}^{U}T_{M_2} = {}^{U}T_{C_2}\,{}^{C_2}T_{M_2} \\ {}^{U}T_{M_3} = {}^{U}T_{M_2}\,R_x(45^\circ) \\ {}^{U}T_{C_3} = {}^{U}T_{M_3}\,{}^{M_3}T_{C_3} \end{cases} \Rightarrow {}^{U}T_{C_3} = {}^{U}T_{C_2}\,{}^{C_2}T_{M_2}\,R_x(45^\circ)\,{}^{M_3}T_{C_3}$$

$$= \begin{bmatrix} -0.866 & 0.354 & 0.354 & 1.662 \\ -0.416 & -0.117 & -0.901 & -2.696 \\ -0.277 & -0.928 & -0.249 & 4.872 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv) $${}^{U}T_{C_4} = R_y(60^\circ)\cdot{}^{U}T_{C_3} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}{}^{U}T_{C_3} = \begin{bmatrix} -0.673 & -0.627 & 0.392 & 5.051 \\ -0.416 & -0.117 & -0.901 & -2.696 \\ 0.611 & -0.772 & -0.182 & 0.997 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v) $${}^{U}T_{M_4} = {}^{U}T_{C_4}\,{}^{C}T_{M} = \begin{bmatrix} 0.6732 & 0.304 & 0.674 & 2.111 \\ 0.416 & 0.598 & -0.0685 & -3.535 \\ -0.611 & 0.742 & 0.276 & -1.169 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$