ECE 470: Introduction to Robotics Homework 7

1) In Canny edge detection algorithm,
   a) What happens if Gaussian filter is not applied in the first step?
   b) Which steps cause the thinning effect of the edge? Explain.
   c) What happens of the first and second thresholds are very close to each other in the hysteresis thresholding step?

(6 Points)

2) In trying to detect lines represented by equation $\begin{pmatrix} y \\ 1 \end{pmatrix} = \begin{pmatrix} A & B \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix}$ in the cartesian space with coordinates (x, y), *we transform the points (x_i,y_i) to a parameter space (A, B).*
   a) How will a point (x_i,y_i) look like when transformed to the (A, B) space? *(1 Points)*
   b) How is a point on the (A, B) space represented in the (x,y) space? *(1 Points)*
   c) Describe graphically how collinear points P1 to P4 can be identified in Fig. 1?

   (4 Points)

   d) What will be the problem in detecting lines in Fig. 1 using (A, B) as parameter space?

   (2 Points)

   e) Describe a method you learn in class that could deal with the problem.

   (6 Points)

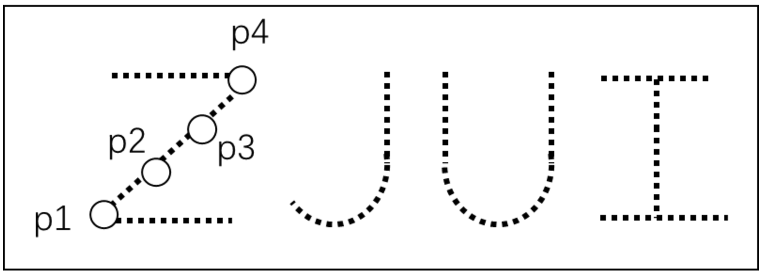


Fig. 2

3) Fig. 2 shows the orientation and position of a camera frame {C} with respect to the world reference frame {W}.

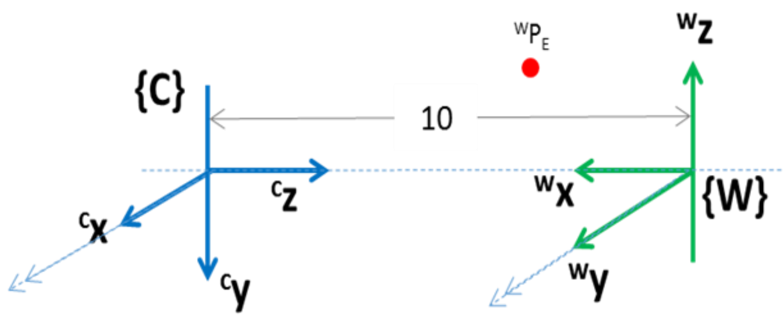


Fig. 2

a) Write down the rotation matrix representing the orientation of the world frame {C} with respect to the camera frame {W} i.e. ${}^{W}R_{C}$. *(1 Points)*

b) Write down the 3x4 extrinsic matrix of the camera. *(1 Points)*

c) A point referenced from the world frame $(15, 30, 15)^T$ is observed to have image coordinates (600, 300). Given that $fx{=}fy$ and $ic{=}jc$ and assuming skew coefficient a=0, solve for the intrinsic camera matrix

$$K = \begin{bmatrix} fx & a & ic \\ 0 & fy & jc \\ 0 & 0 & 1 \end{bmatrix}$$

(8 Points)

Solution

1)

a) The intensity gradient-based approach will unwittingly enhance the noise without first doing noise reduction like Gaussian filtering.

b) Non-maximum suppression step removes spurious response by retaining only strongest intensity change along the gradient vector.

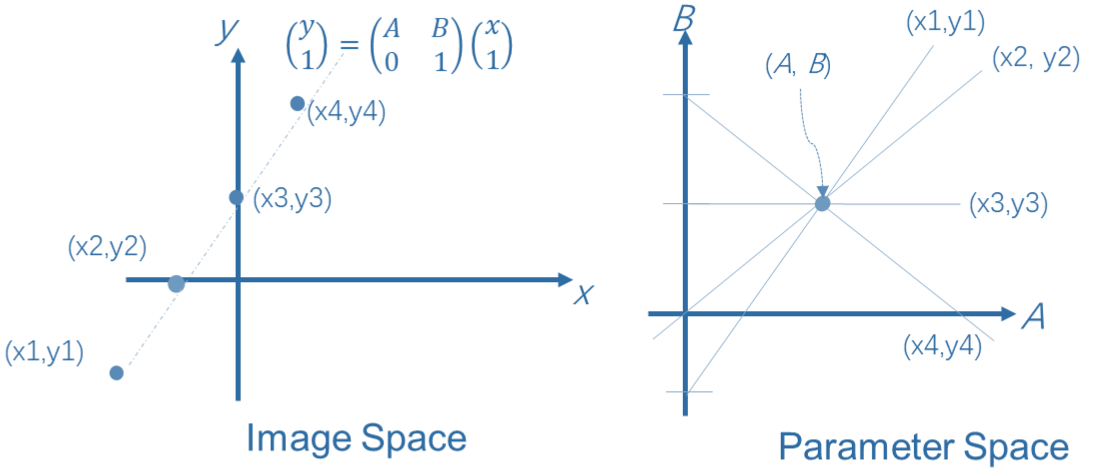
c) If first and second thresholds get very close, the marginal cases, which require the checking of continuity, diminishes making like close to just having a single threshold. Continuity of the edges are less likely to be checked for in deciding if a pixel lies on an edge or not.

2)

a) It is represented as a line in (A, B) space. The line has equation B = -Ax+ y with -x as the gradient and y as the vertical intercept.

b) It is represented as a line in (x, y) space.

c) The intersection points between the 4 lines in the parameter space is the coordinates (A, B) that represents the line in image space.



d) Vertical lines associate with the "U, J and I" are not possible when using (A, B) as the Hough-space. This is because the gradient B will result in an infinite value.

e) See Lecture Notes on Hough Transform for line detection and list the steps.

3)

a) $^{W}R_{c}=\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

b) $^{C}[R|t]_{W}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 10 \end{bmatrix}$

c) $s\,[u\ v\ 1]^{T}=K\ \ ^{c}[R\mid t]_{W}\,^{W}P_{E}$

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}=\begin{bmatrix} fx & a & ic \\ 0 & fy & jc \\ 0 & 0 & 1 \end{bmatrix}[R|\quad t]\ \ ^{W}P_{E}$$

Since $fx=fy$ and $ic=jc$, let $f=fx=fy$ and $c=ic=jc$. Also substitute $a=0$.

$$s\begin{bmatrix} 600 \\ 300 \\ 1 \end{bmatrix}=\begin{bmatrix} f & 0 & c \\ 0 & f & c \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} 15 \\ 30 \\ 15 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 600s \\ 300s \\ s \end{bmatrix}=\begin{bmatrix} f & 0 & c \\ 0 & f & c \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 30 \\ -15 \\ -5 \end{bmatrix}$$

From 3rd row of equation, $s=-5$

$$-3000=30f-5c$$
$$-1500=-15f-5c$$
$$f=-100/3;\ c=400$$

$$\therefore K=\begin{bmatrix} -100/3 & 0 & 400 \\ 0 & -100/3 & 400 \\ 0 & 0 & 1 \end{bmatrix}$$