ZJU-UIUC Institute
Zhejiang University / University of Illinois at Urbana-Champaign Institute

ECE 470: Introduction to Robotics

Lecture 10

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Dynamics

ECE 470 Introduction to Robotics

Syllabus and Schedule

- **Lecture**

O. Overview
- Science & Engineering in Robotics

I. Spatial Representation & Transformation
- Coordinate Systems; Pose Representations; Homogeneous Transformations

II. Kinematics
- Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/Inverse Kinematics

III. Velocity Kinematics and Static Forces
- Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity

Fundamentals

Week 1-4

Revision/ Quiz on Week 5

IV. Dynamics
- Lagrangian Formulation; Newton-Euler Equations of Motion

V. Control
- Independent Joint/Feedforward/Inverse Dynamics Controls; Controller Architectures

VI. Planning
- Joint-based Motion Planning; Cartesian-based Path Planning

Essentials

Week 6-9

Revision/ Quiz on Week 10

VII. Robot Vision (and Perception)
- Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics

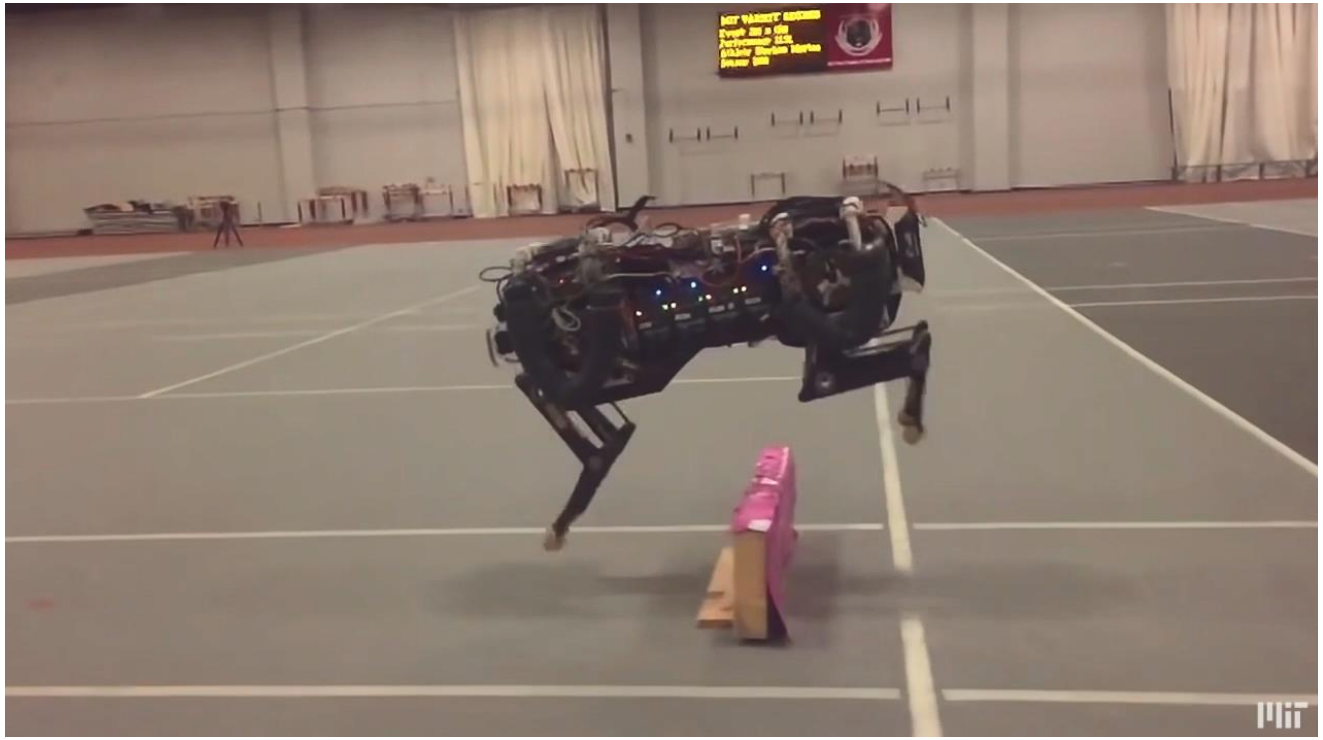
Applied

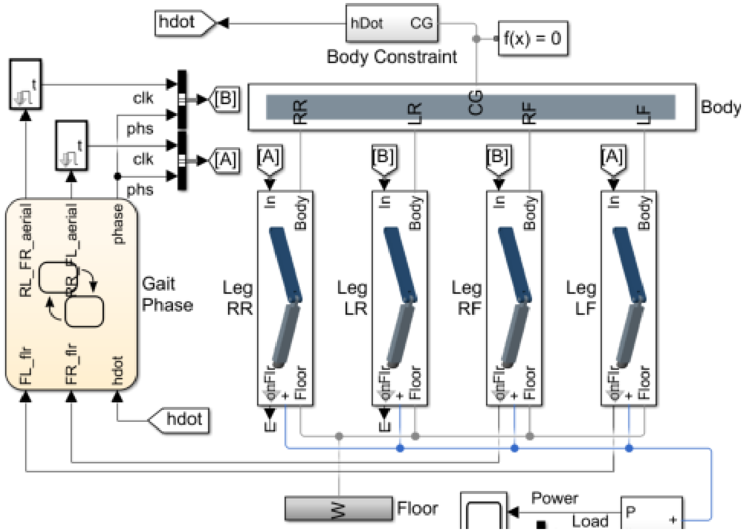
Week 11-13

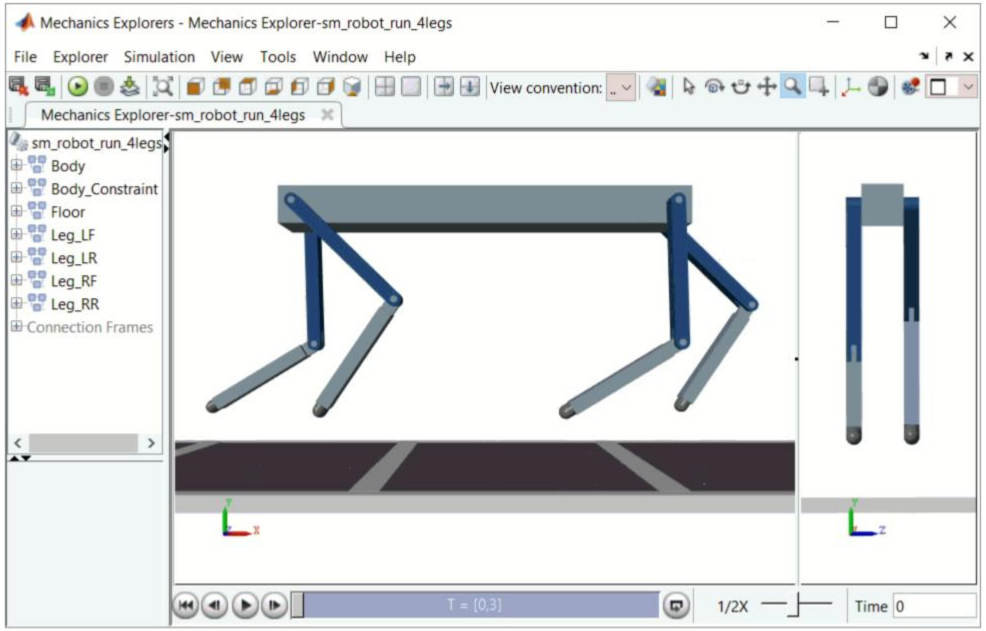
Revision/Reading Wk/ Exam on Week 14-16

Robot Dynamics

- Example: Quadrupedal Robot

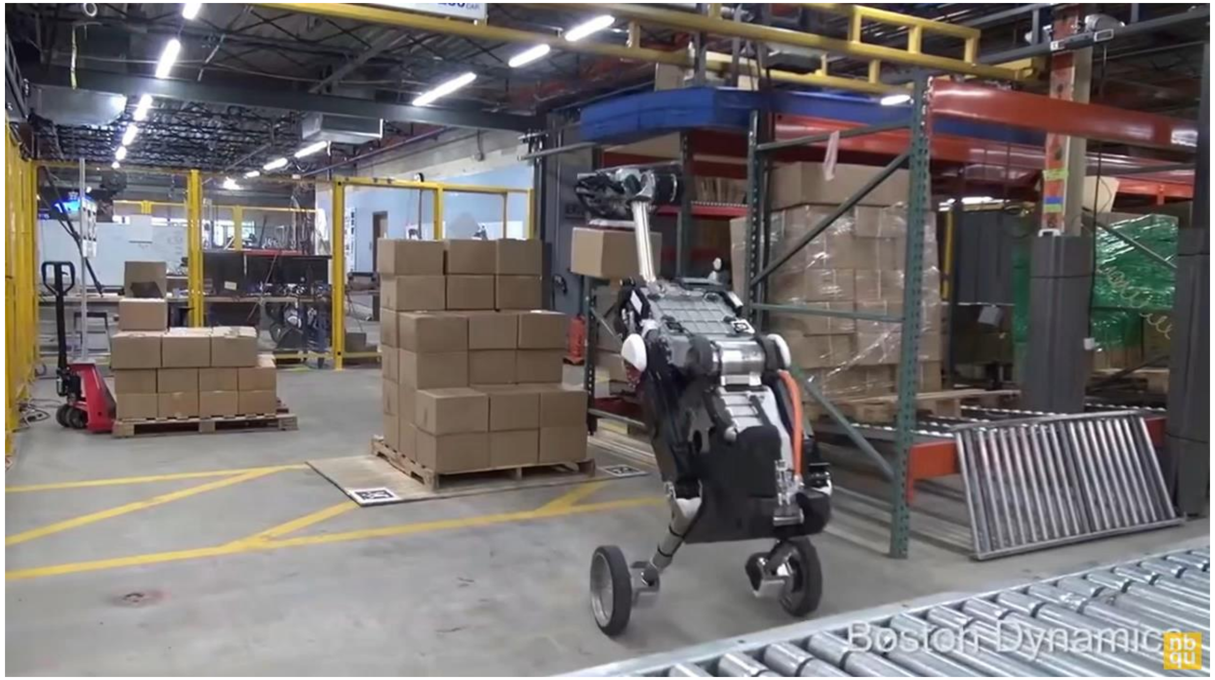






Importance of Simulation

- Why do we need more simulation before real test?

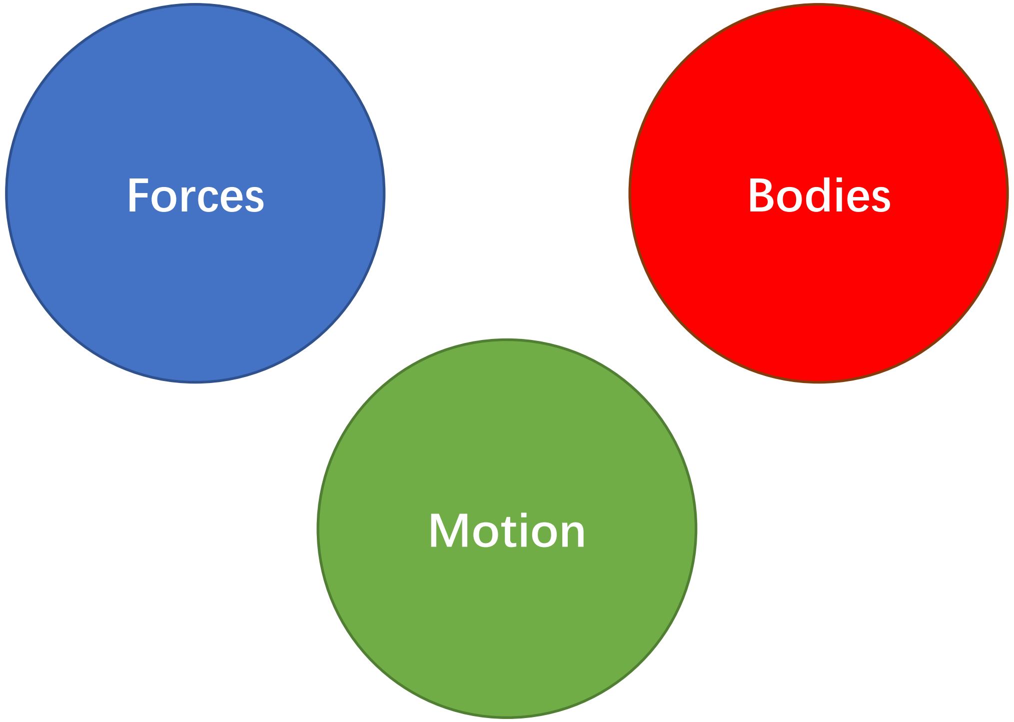


Robot Mechanics

- **Kinematics**: The science of motion without regards to the forces that cause it
  - Pose of the manipulator in static situations
  - Analyze (linear and angular) motion of bodies (/linkages)
- **Statics**: Bodies in equilibrium and force (/moment) relationship
- **Dynamics**: Concern with the forces (/torque) on bodies that cause motion
  - In ECE 470, we are interested in relating forces (/torque) and motion
  - i.e. Dynamic Equation

Robot Mechanics: Dynamics

- **Dynamics**: Concern with the forces (/torque) on bodies that cause motion



Robot Mechanics: Dynamics

- **Dynamics**: Concern with the forces (/torque) on bodies that cause motion

Dynamic equation:

- $\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta)$
- $M(\Theta)$ is n x n mass matrix of the manipulator
- $V(\Theta, \dot{\Theta})$ is an n x 1 vector of centrifugal and Coriolis terms
- $G(\Theta)$ is an n x 1 vector of gravity terms

Cartesian Space: $\mathcal{F} = M_x(\Theta)\ddot{\chi} + V_x(\Theta, \dot{\Theta}) + G_x(\Theta),$

Formulating Dynamic Equations

Method (1) Newton-Euler: “Force balance”

Method (2) Lagrangian: Energy-based approach

Acceleration of Rigid Bodies

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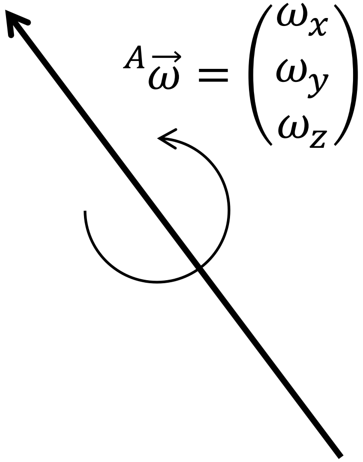
In deriving velocity expression……

$$\vec{P}_1 = \vec{P}_{B,ORG} + x\,\hat{i}_B + y\,\hat{j}_B + z\hat{k}_B$$

- Differentiating with respecting to time,

$$\vec{V}_1 = \vec{V}_{B,ORG} + \dot{x}\,\hat{i}_B + \dot{y}\,\hat{j}_B + \dot{z}\hat{k}_B + \underbrace{x\dot{\hat{i}}_B + y\dot{\hat{j}}_B + z\dot{\hat{k}}_B}_{\vec{\omega} \times (x\,\hat{i}_B + y\,\hat{j}_B + z\hat{k}_B)}$$



$${}^{A}\vec{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Skew-symmetric matrices

Similarly for acceleration ……

$$\vec{V}_1 = \vec{V}_{B,ORG} + \dot{x}\,\hat{i}_B + \dot{y}\,\hat{j}_B + \dot{z}\,\hat{k}_B + \vec{\omega} \times \vec{P}_{1/B}$$

Differentiate w.r.t. time

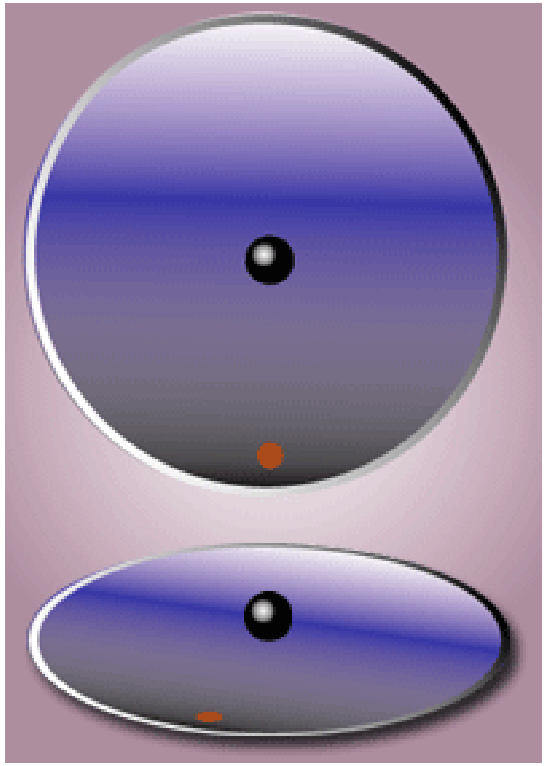
$$\dot{\vec{V}}_1 = \dot{\vec{V}}_{B,ORG} + \ddot{x}\,\hat{i}_B + \ddot{y}\,\hat{j}_B + \ddot{z}\,\hat{k}_B + \vec{\omega} \times \vec{V}_{1/B} + \vec{\omega} \times (\vec{V}_{1/B} + \vec{\omega} \times \vec{P}_{1/B}) + \dot{\vec{\omega}} \times \vec{P}_{1/B}$$

$$\dot{\vec{V}}_1 = \dot{\vec{V}}_{B,ORG} + \ddot{x}\,\hat{i}_B + \ddot{y}\,\hat{j}_B + \ddot{z}\,\hat{k}_B + 2\vec{\omega} \times \vec{V}_{1/B} + \vec{\omega} \times \vec{\omega} \times \vec{P}_{1/B} + \dot{\vec{\omega}} \times \vec{P}_{1/B}$$

coriolis acceleration

tangential acceleration

centrifugal acceleration



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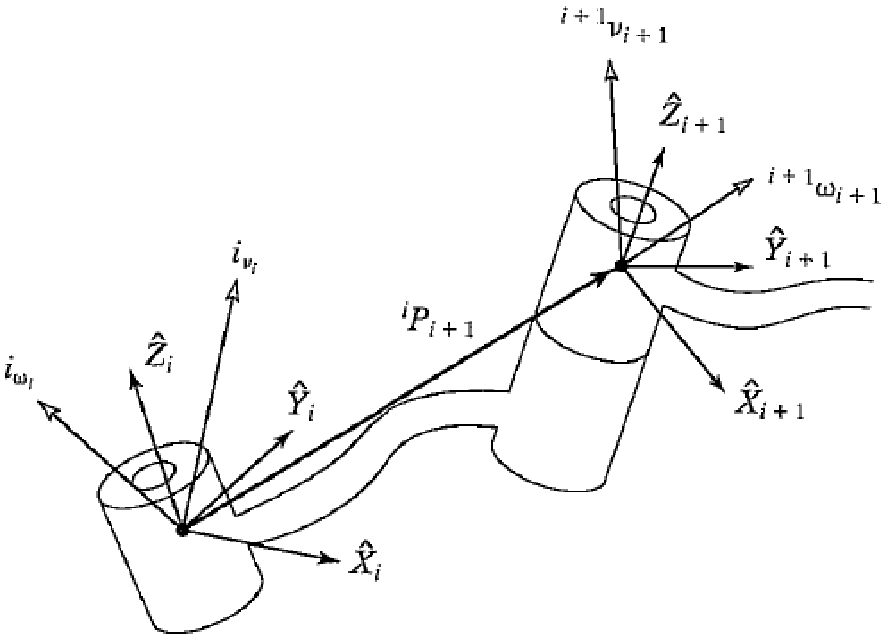
In Velocity “Propagation” from link to link

- Rotational velocities can be added when both ω vectors are written with respect to the same frame

$$ {}^{i}\omega^{0}_{i+1} = {}^{i}\omega^{0}_{i} + {}^{i}_{i+1}R\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} $$

$$ \boxed{{}^{i+1}\omega^{0}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega^{0}_{i} + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}} $$

*$\hat{Z}$ in the direction of joint
Notation: In consideration of robot links, frame {0} is used as the reference frame. Meaning to say, ${}^{i+1}\omega_{i+1}$ is the absolute angular velocity of {i+1} expressed in frame {i+1}



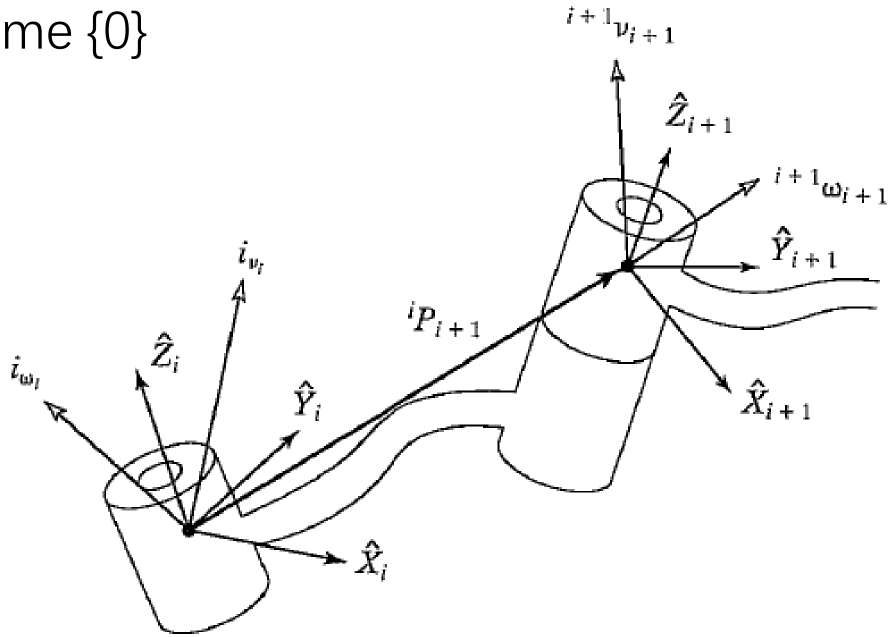
In Velocity "Propagation" from link to link

- Linear velocities

$$^{i}v^{0}{}_{i+1} = {}^{i}v^{0}{}_{i} + {}^{i}\omega^{0}{}_{i} \times {}^{i}P_{i+1}$$

Differentiate w.r.t to frame {0}

$$\boxed{^{i+1}v^{0}{}_{i+1} = {}^{i+1}_{i}R\left({}^{i}v^{0}{}_{i} + {}^{i}\omega^{0}{}_{i} \times {}^{i}P_{i+1}\right)}$$



$*\hat{Z}$ in the direction of joint
Notation: In consideration of robot links, frame {0} is used as the reference frame. Meaning to say, $^{i+1}v_{i+1}$ is the absolute velocity of {i+1} origin expressed in frame {i+1}

Similarly in Acceleration for "Propagation" from link to link

$$ {}^{0}\omega_{i+1} = {}^{0}\omega_i + {}_{i+1}^{0}R\ \dot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} $$

Differentiate wrt to time

$$ {}^{0}\dot{\omega}^{0}{}_{i+1} = {}^{0}\dot{\omega}_i + {}_{i+1}^{0}\dot{R}\ \dot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} + {}_{i+1}^{0}R\ \ddot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} $$

$$ = {}^{0}\dot{\omega}_i + {}_{i+1}^{0}\dot{R}\ {}_{i+1}^{0}R^{T}\ \ {}_{i+1}^{0}R\ \dot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} + {}_{i+1}^{0}R\ \ddot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} $$

$$ = {}^{0}\dot{\omega}_i + {}^{0}\Omega_i\ \ {}_{i+1}^{0}R\ \dot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} + {}_{i+1}^{0}R\ \ddot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} $$

$$ = {}^{0}\dot{\omega}_i + {}^{0}\omega_i \times {}_{i+1}^{0}R\ \dot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} + {}_{i+1}^{0}R\ \ddot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} $$

$$ \boxed{ {}^{i+1}\dot{\omega}^{0}{}_{i+1} = {}_{i}^{i+1}R\ \ {}^{i}\dot{\omega}{}_{i} + {}_{i}^{i+1}R\ {}^{i}\omega_i \times \dot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\ {}^{i+1}\hat{Z}_{i+1} } $$

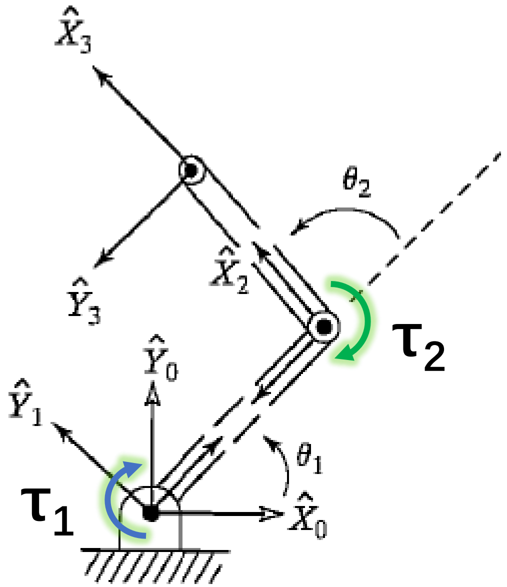
- For prismatic joint, ${}^{i+1}\dot{\omega}^{0}{}_{i+1} = {}_{i}^{i+1}R\ \ {}^{i}\dot{\omega}^{0}{}_{i}$

Since $\dot{\vec{V}}_1 = \dot{\vec{V}}_{B,ORG} + \ddot{x}\,\hat{i}_B + \ddot{y}\,\hat{j}_B + \ddot{z}\hat{k}_B + 2\vec{\omega} \times \vec{V}_{1/B} + \vec{\omega} \times \vec{\omega} \times \vec{P}_{1/B} + \dot{\vec{\omega}} \times \vec{P}_{1/B}$

$$ \boxed{ {}^{i+1}\dot{v}^{0}{}_{i+1} = {}_{i}^{i+1}R\ \left({}^{i}\dot{v}^{0}{}_{i} + 0 + 0 + {}^{i}\omega^{0}{}_{i} \times {}^{i}\omega^{0}{}_{i} \times {}^{i}P_{i+1} + {}^{i}\dot{\omega}^{0}{}_{i} \times {}^{i}P_{i+1} \right) } $$

Q5.1 Example of acceleration

- Given the following 2-link planar manipulator in Q3.4, determine for each link the a) absolute angular acceleration, and b) joint absolute linear acceleration. Express the answer in their own frame.
- $^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $^2_3T = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- Assume acceleration due to gravity to be g
  - *i.e.* $^0\dot{v}_0 = g\hat{Y}_0$



$$^0\omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ ^0v_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ ^0\dot{\omega}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ ^0\dot{v}_0 = \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}$$

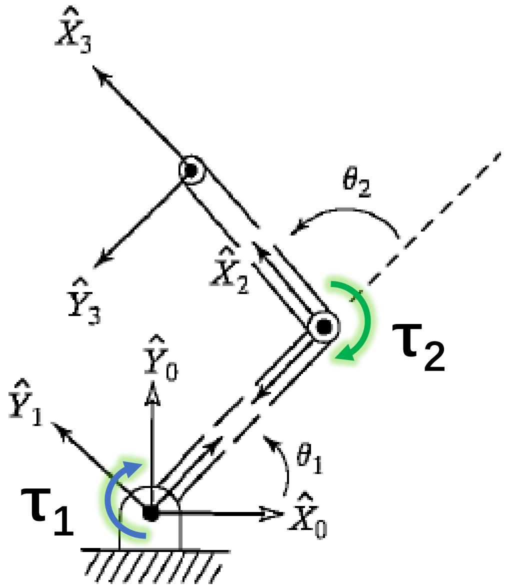
Q5.1 Example of acceleration

Assume ${}^0\dot{v}_0 = g\,\hat{Y}_0$

$${}^0_1R = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2R = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0\omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad {}^0v_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad {}^0\dot{\omega}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad {}^0\dot{v}_0 = \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}$$



Q5.1 Example of acceleration

$$\boxed{{}^{i+1}\omega^{0}_{i+1} = {}^{i+1}_{i}R\ {}^{i}\omega^{0}_{i} + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}}$$

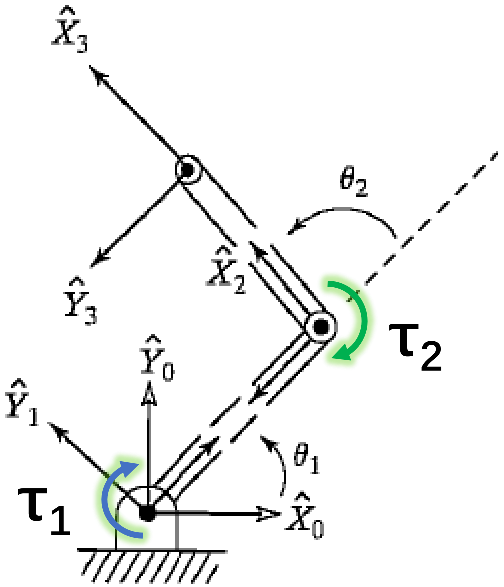
$$\boxed{{}^{i+1}\dot{\omega}^{0}_{i+1} = {}^{i+1}_{i}R\ {}^{i}\dot{\omega}_{i} + {}^{i+1}_{i}R\ {}^{i}\omega_{i} \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}}$$

$$\boxed{{}^{i+1}v^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}v^{0}_{i} + {}^{i}\omega^{0}_{i} \times {}^{i}P_{i+1}\right)}$$

$$\boxed{{}^{i+1}\dot{v}^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot{v}^{0}_{i} + {}^{i}\Omega_{i}\ {}^{i}\Omega_{i}\ {}^{i}P_{i+1} + {}^{i}\dot{\Omega}_{i}\ {}^{i}P_{i+1}\right)}$$

- i=0

$${}^{1}\omega_{1} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix},\quad {}^{1}\dot{\omega}_{1} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix},\quad {}^{1}\dot{v}_{1} = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix} = \begin{bmatrix} g\ s_1 \\ g\ c_1 \\ 0 \end{bmatrix}$$



Q5.1 Example of acceleration

$$\boxed{{}^{i+1}\omega^{0}_{i+1} = {}^{i+1}_{i}R\ {}^{i}\omega^{0}_{i} + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}}$$

$$\boxed{{}^{i+1}\dot{\omega}^{0}_{i+1} = {}^{i+1}_{i}R\ {}^{i}\dot{\omega}_{i} + {}^{i+1}_{i}R\ {}^{i}\omega_{i} \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}}$$

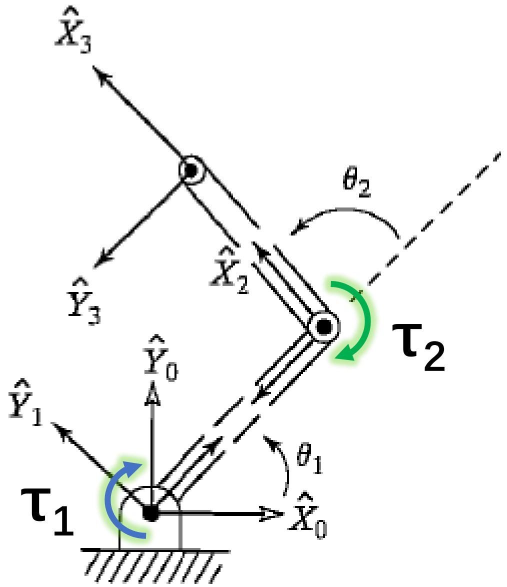
$$\boxed{{}^{i+1}v^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}v^{0}_{i} + {}^{i}\omega^{0}_{i} \times {}^{i}P_{i+1}\right)}$$

$$\boxed{{}^{i+1}\dot{v}^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot{v}^{0}_{i} + {}^{i}\Omega_{i}\ {}^{i}\Omega_{i}\ {}^{i}P_{i+1} + {}^{i}\dot{\Omega}_{i}\ {}^{i}P_{i+1}\right)}$$

- i=1

$$ {}^{2}\omega_{2} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix},\quad {}^{2}\dot{\omega}_{2} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 + \ddot{\theta}_2 \end{bmatrix}, $$

$$ {}^{2}\dot{v}_{2} = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_1\dot{\theta}_1^2 + g\, s_1 \\ l_1\ddot{\theta}_1 + g\, c_1 \\ 0 \end{bmatrix} = \begin{bmatrix} l_1\ddot{\theta}_1^2 s_2 - l_1\dot{\theta}_1^2\, c_2 + g\, s_{12} \\ l_1\ddot{\theta}_1^2 c_2 + l_1\dot{\theta}_1^2\, s_2 + g\, c_{12} \\ 0 \end{bmatrix} $$



Newton-Euler Formulation

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