



# ECE 470: Introduction to Robotics

## Lecture 17

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# Schedule Check

## • Lecture

O.	Overview	
	• Science & Engineering in Robotics	
I.	Spatial Representation & Transformation	<b>Fundamentals</b>
	• Coordinate Systems; Pose Representations; Homogeneous Transformations	Week 1-4
II.	Kinematics	
	• Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/Inverse Kinematics	Revision/ Quiz on Week 5
III.	Velocity Kinematics and Static Forces	
	• Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity	
IV.	Dynamics	<b>Essentials</b>
	• Acceleration of Body; Newton-Euler Approach; Lagrangian Formulation	
V.	Control	Week 6-9
	• Feedback Control, <u>Independent Joint Control</u> , <u>Force Control</u>	
Week 9 → VI.	Planning	Revision/ Quiz on Week 10
	• <u>Joint-based Motion Planning</u> ; <u>Cartesian-based Path Planning</u>	
VII.	Robot Vision (and Perception)	<b>Applied</b>
	• Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics	Week 11-13
		Revision/Reading Wk/ Exam on Week 14-16



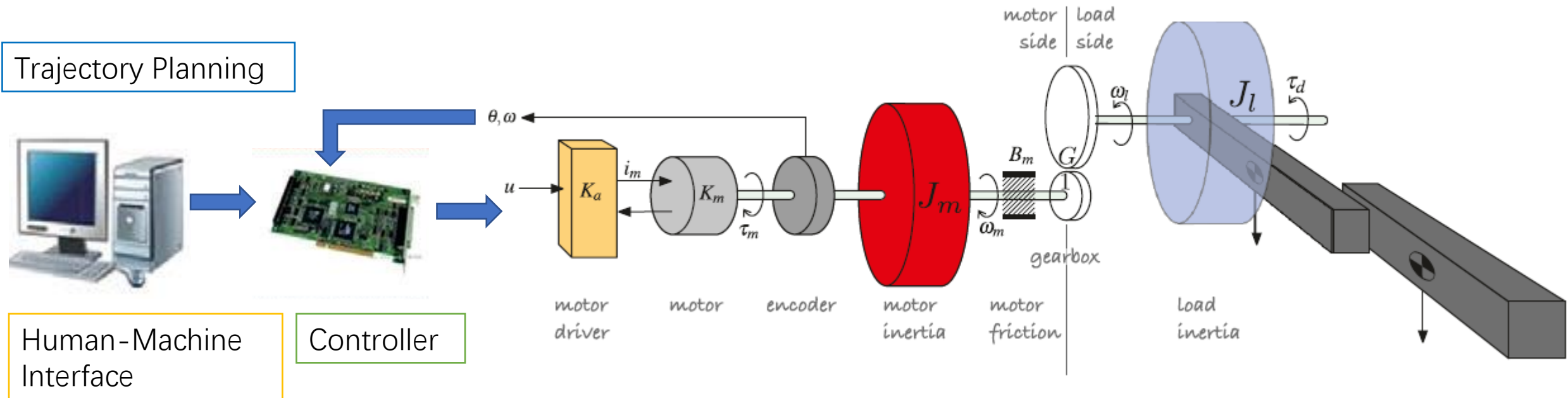
# Independent Joint Control

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# Independent Joint Control

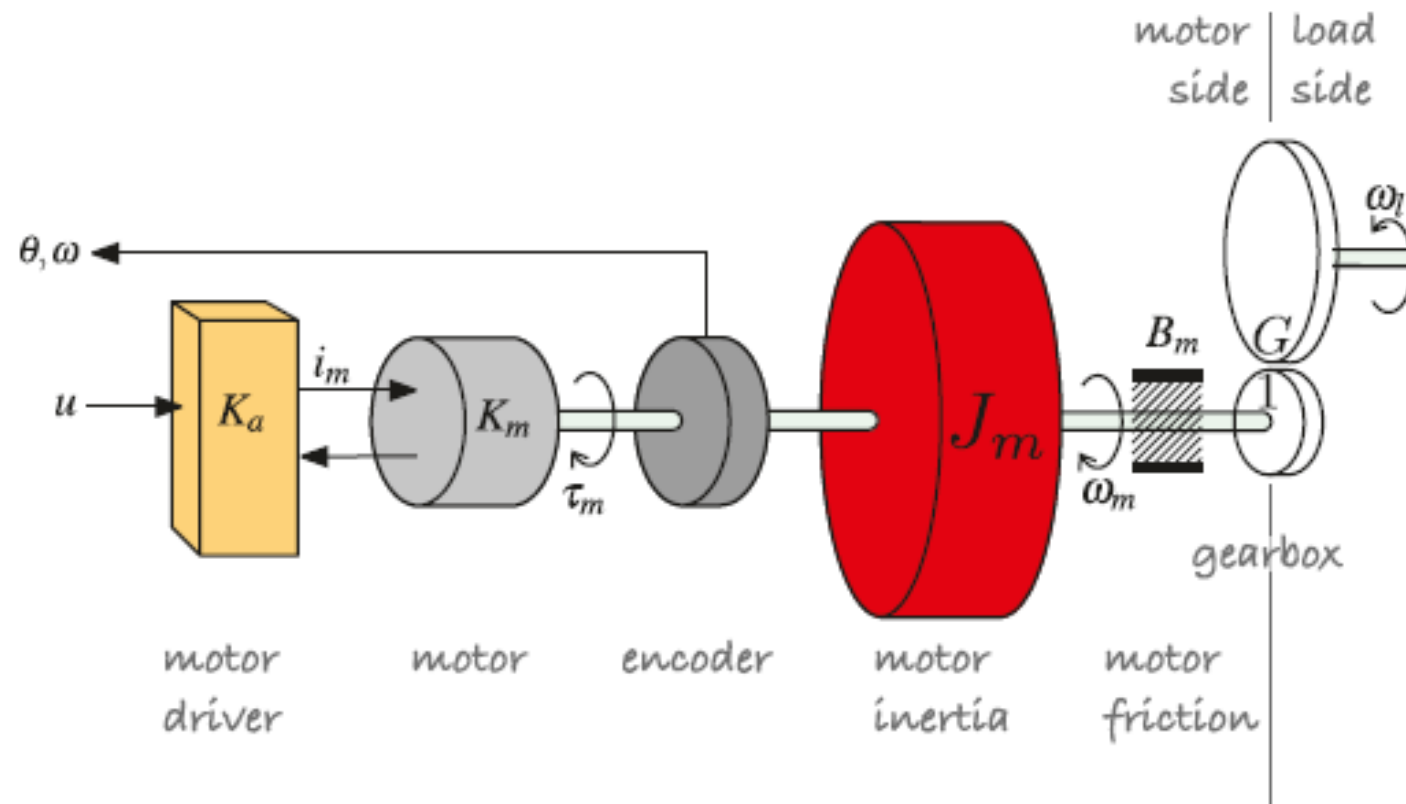
- The 2<sup>nd</sup> order system dealt with so far has single DOF
- Ultimately, we are interested in multibody robotic systems that involve Multi-Input, Multi-Output (MIMO) control systems
- We shall first adopt an **independent joint control** approach with  $N$  independent Single-Input Single-Output (SISO) control systems

# Joint Control in Robotic System



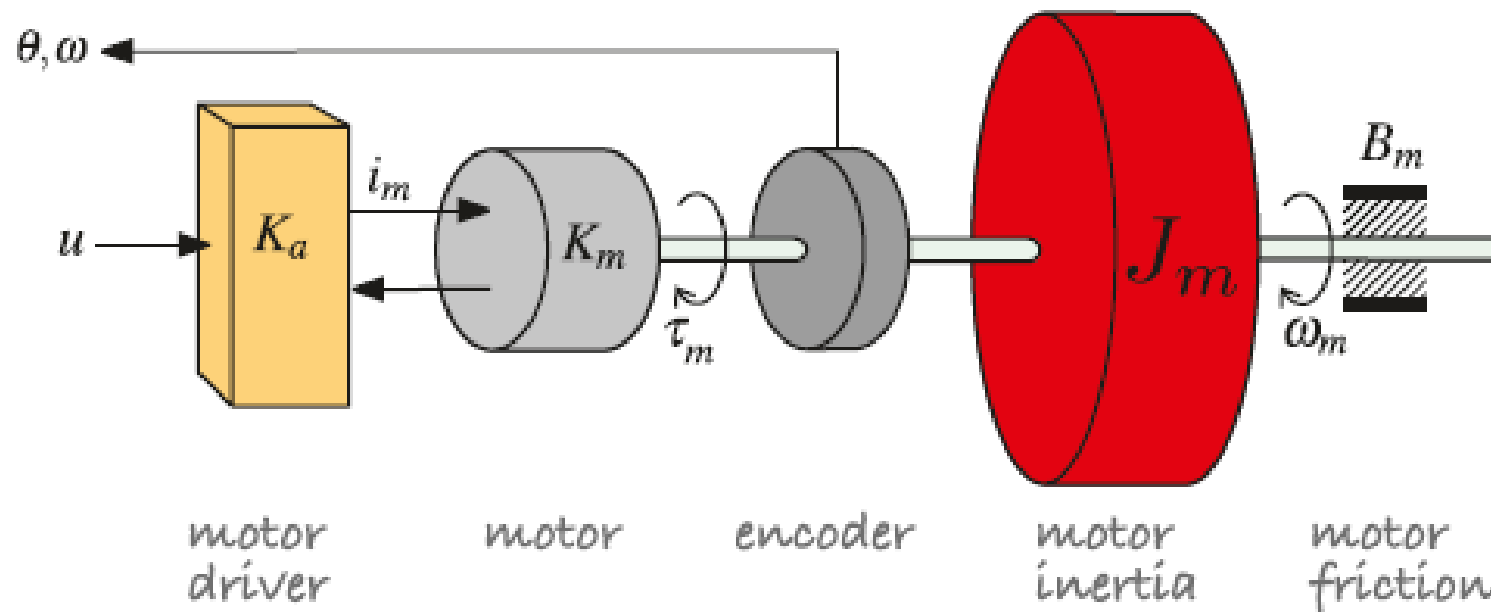
# Modeling Single Joint Control

## Integrated assembly



# Modeling Single Joint Control

A demand voltage  $u$  controls the current  $i_m$  flowing into the **motor (Actuator)** which generates a torque  $\tau_m$  that accelerates the rotational inertia  $J_m$  and is opposed by friction  $B_m \omega_m$ . The **encoder (Sensor)** measures rotational speed and angle



# Modeling Single Joint Control

- How to model the resultant system as a 2<sup>nd</sup> order linear system?
  - Combining the Dynamics of the electromechanical system



# Modeling Single Joint Control

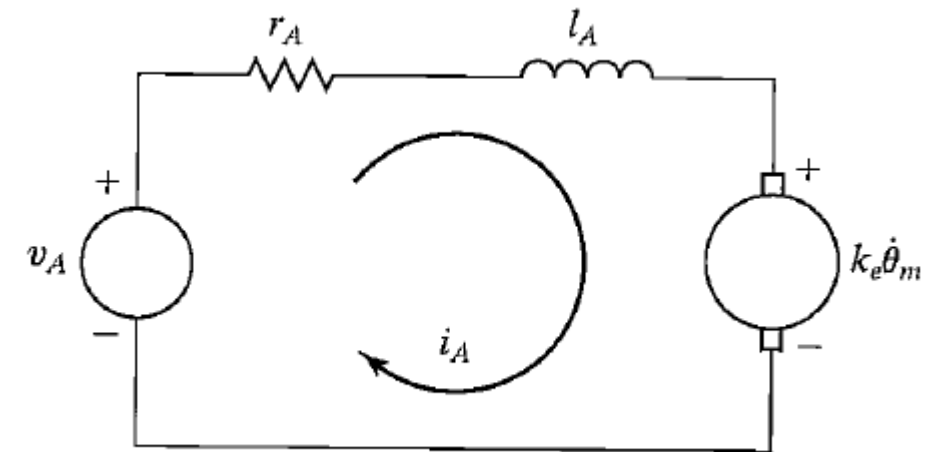
- How model the resultant system as a 2<sup>nd</sup> order linear system?

Physics Law:

Lenz Force,  $F = qV \times B$

Motor Torque,  $\tau_m = k_m i_a$

Back emf,  $v = k_e \dot{\theta}_m$



# Modeling Single Joint Control

- Model the Motor

Physics Law:

Lenz Force,  $F = qV \times B$

Current

Magnetic Field

Motor Torque,  $\tau_m = k_m i_A$

Back emf,  $v = k_e \dot{\theta}_m$

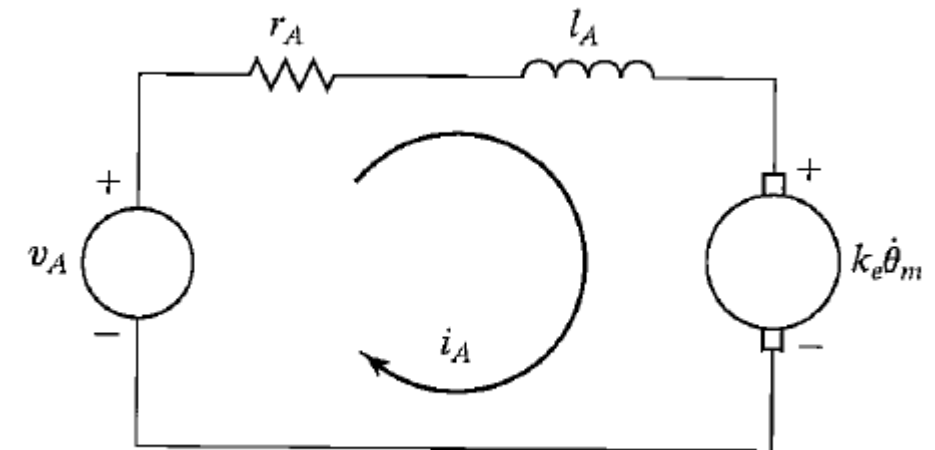
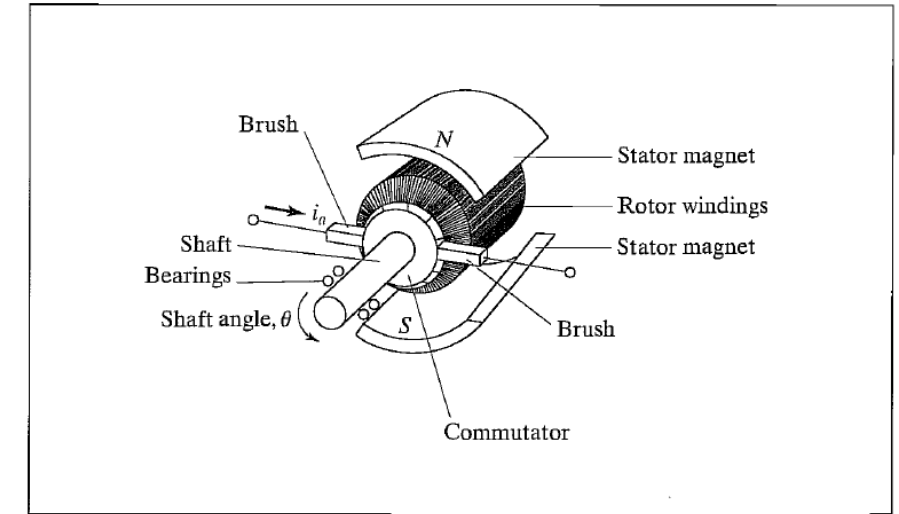
Torque Constant

Torque Constant

Kirchoff Law:

$$v_A = l_A \dot{i}_A + r_A i_A + k_e \dot{\theta}_m$$

$$v_A - k_e \dot{\theta}_m = l_A \dot{i}_A + r_A i_A$$



# Modeling Single Joint Control

Electromechanical System

- Model the Motor

Motor Torque,  $\tau_m = k_m i_A$

Back emf,  $v = k_e \dot{\theta}_m$

- Electrical

Kirchoff Law:

$$v_A = l_A \dot{i}_A + r_A i_A + k_e \dot{\theta}_m$$

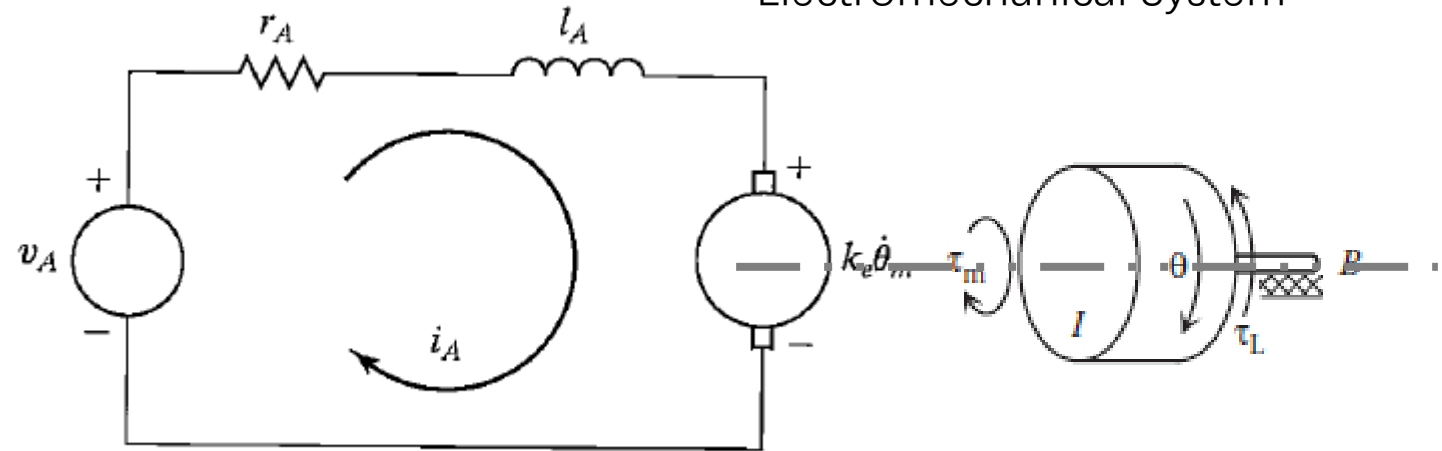
$$v_A - k_e \dot{\theta}_m = l_A \dot{i}_A + r_A i_A$$

- Mechanical

Newton (Euler) Law:

$$\tau_m - \tau_L - b \dot{\theta} = I \ddot{\theta}$$

$$k_m i_A - b \dot{\theta}_m - I \ddot{\theta}_m = \tau_L$$



# Modeling Single Joint Control

- Model the Motor-Gearing-Load

$$\tau_m = I_m \ddot{\theta}_m + b_m \dot{\theta}_m + \left(1/\eta\right) (I \ddot{\theta} + b \dot{\theta})$$

In terms of feedback control,  
what is the implication for large  $\eta$ ?

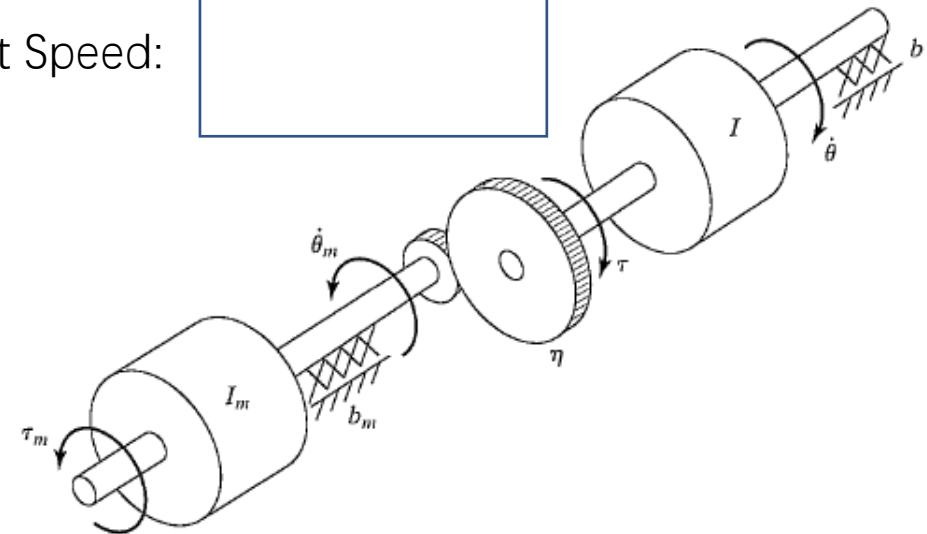
$$\tau_m = \boxed{\phantom{I_m}} \ddot{\theta}_m + \boxed{\phantom{b_m}} \dot{\theta}_m$$

$$\tau = \boxed{\phantom{I_m}} \ddot{\theta} + \boxed{\phantom{b_m}} \dot{\theta}$$

Gear Ratio,  $\eta$ :

Output Torque:

Output Speed:



# Nonlinear Control Scheme

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# Control Schemes so far.....

## Control-Law Partitioning

The control system consist of a **Model-based portion** and a **Servo portion**

Open loop equation:  $m\ddot{x} + b\dot{x} + kx = f$

Model-based portion is a control law in the form

$$f = \alpha f' + \beta$$

Hence, the system equation is written as

$$m\ddot{x} + b\dot{x} + kx = \alpha f' + \beta$$

To make the system unit mass,

$$\alpha = m, \quad \beta = b\dot{x} + kx$$

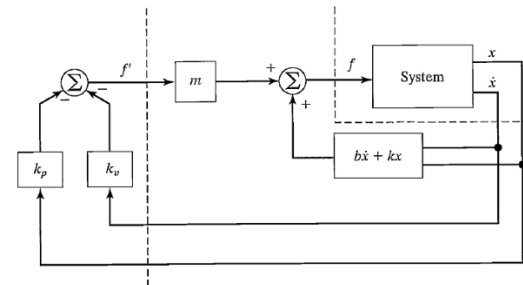
Therefore,  $\ddot{x} = f'$

Proceed with servo portion of control law

$$f' = -k_p x - k_v \dot{x}$$

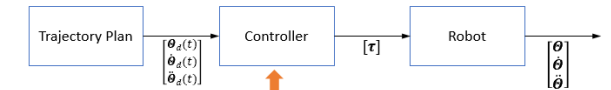
Since the model-based portion make  $\ddot{x} = f'$ ,

$$\ddot{x} + k_p x + k_v \dot{x} = 0$$



5

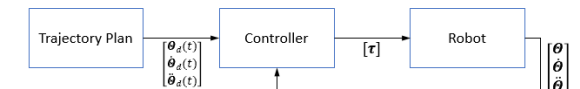
**Open-loop Control** computed input to the robot is independent of the actual measurement of the robot motion



One way is to model the dynamics of the system such that the controller determines the  $\tau$  to input with the dynamic equation

$$\tau = M(\theta_d)\ddot{\theta}_d + V(\theta_d, \dot{\theta}_d) + G(\theta_d)$$

**Closed-loop Control**



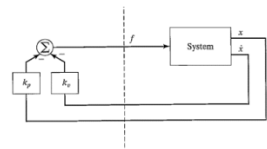
$$f(t) = -k_p x - k_v \dot{x}$$

$$m\ddot{x} + b\dot{x} + kx = -k_p x - k_v \dot{x}$$

$$m\ddot{x} + (b + k_v)\dot{x} + (k + k_p)x = 0,$$

$$m\ddot{x} + b\dot{x} + kx = 0,$$

Closed-loop Stiffness



4

# Nonlinear Controller

## General Approach:

1. Compute a nonlinear model-based control law that "cancels" the nonlinearities of the system to be controlled.
2. Reduce the system to a linear system that can be controlled with the simple linear servo law developed for the unit mass.

## Results in:

→ Linear closed-loop system

## Challenges:

- Knowing parameters and the structure of the nonlinear system
- Computational Cost

# Non-linearity

- Non-linear elasticity
- Non-linear friction
- Non-linear force-displacement





# Non-linearity

- Non-linear elasticity
- Non-linear friction
- Non-linear force-displacement

# Non-linearity

- Non-linear elasticity (E.g. 10.1, Craig 3<sup>rd</sup> Ed)

Consider the non-linear spring characteristic shown in the Figure. Construct a control law to keep the system critically damped and a stiffness of  $K_{CL}$

Open Loop Equation:

Model-based Portion:

Servo Portion:

Choose:  $k_p :=$

Since  $k_v =$

Control Block Diagram:

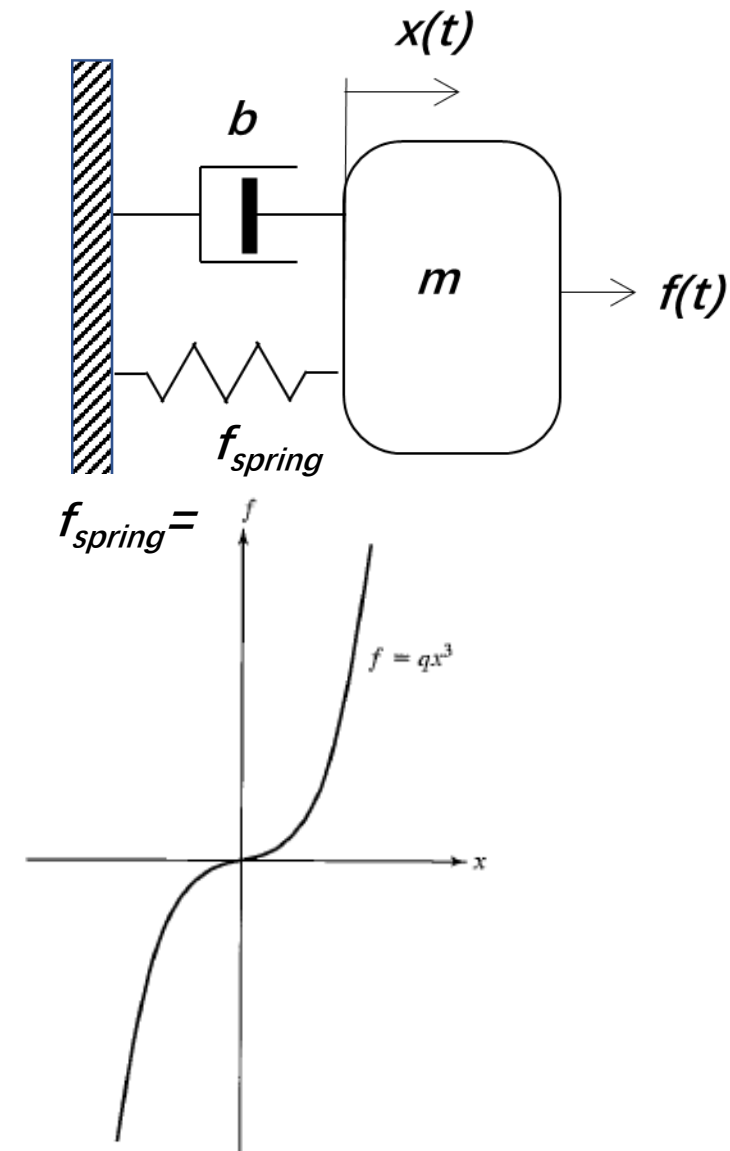
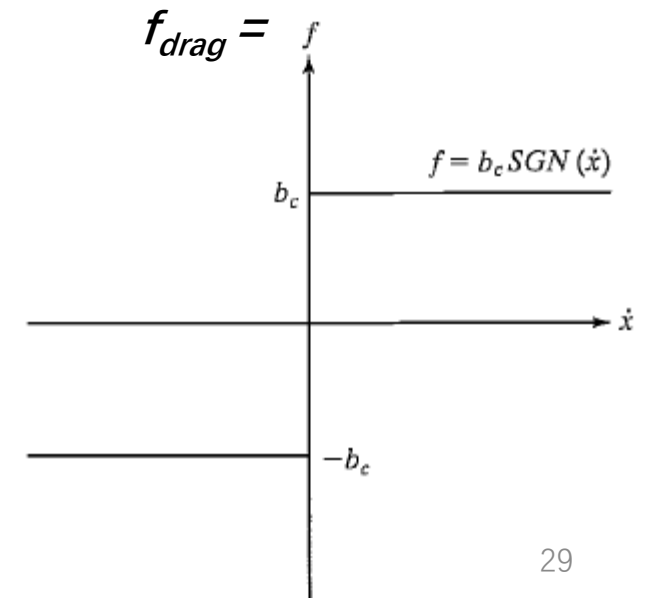
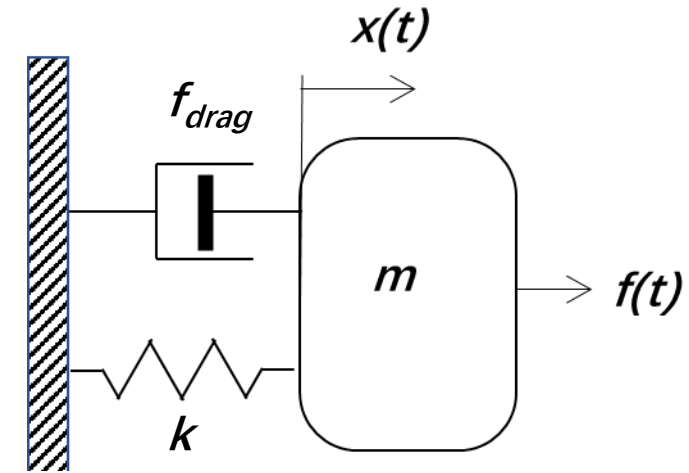


FIGURE 10.1: The force-vs.-distance characteristic of a nonlinear spring.

# Non-linearity

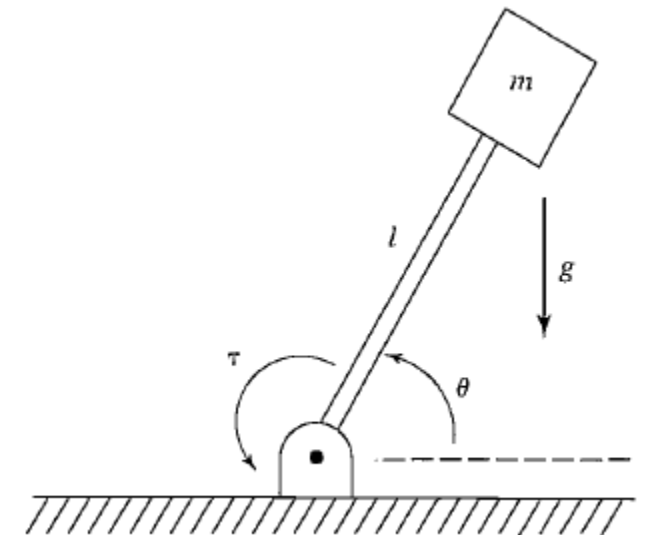
- Non-linear friction (E.g. 10.2, Craig 3<sup>rd</sup> Ed)

Consider the non-linear friction characteristic shown in the Figure. Construct a control law to keep the system critically damped and a stiffness of  $K_{CL}$



# Non-linearity

- Non-linear force-displacement (E.g. 10.3, Craig 3<sup>rd</sup> Ed)

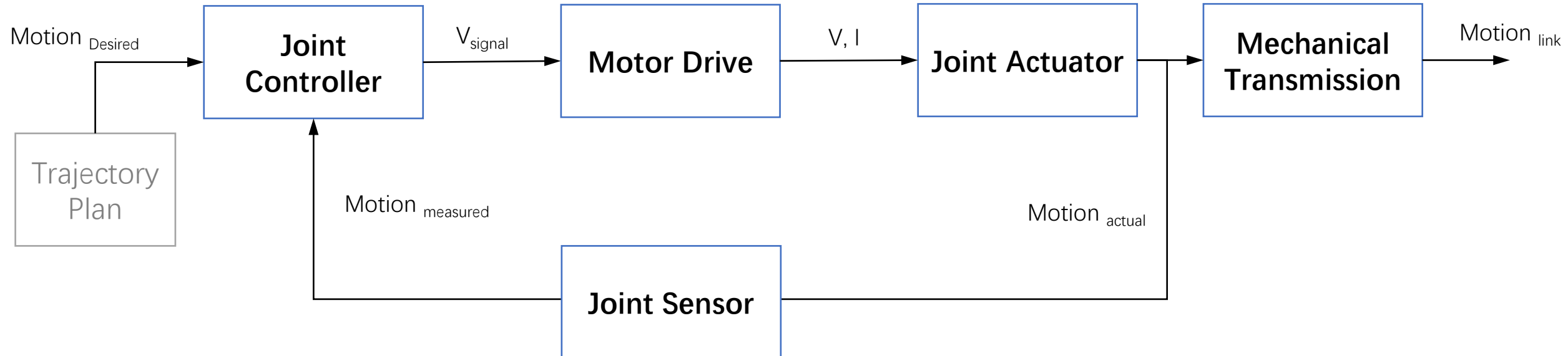
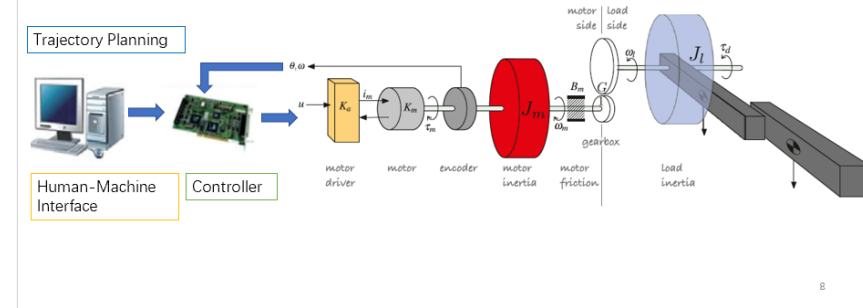


# Control Scheme for Manipulators

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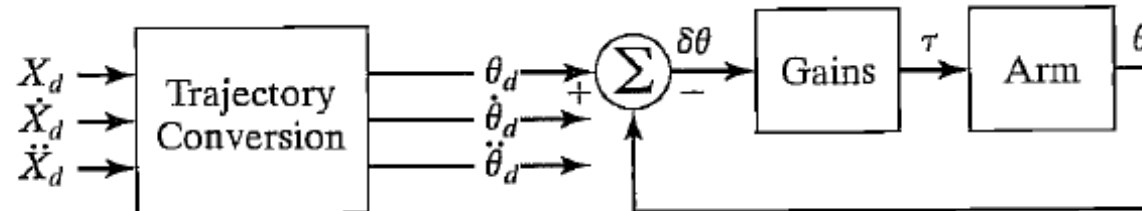
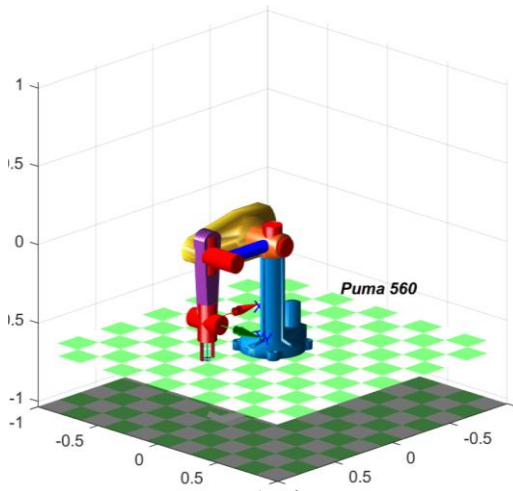
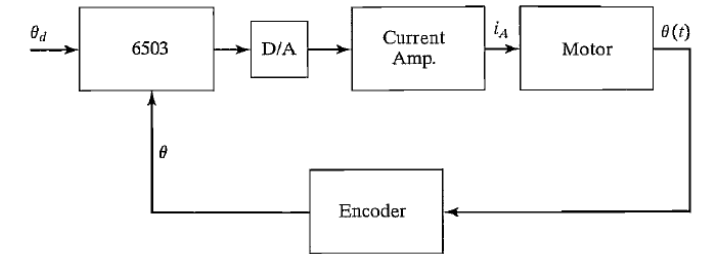
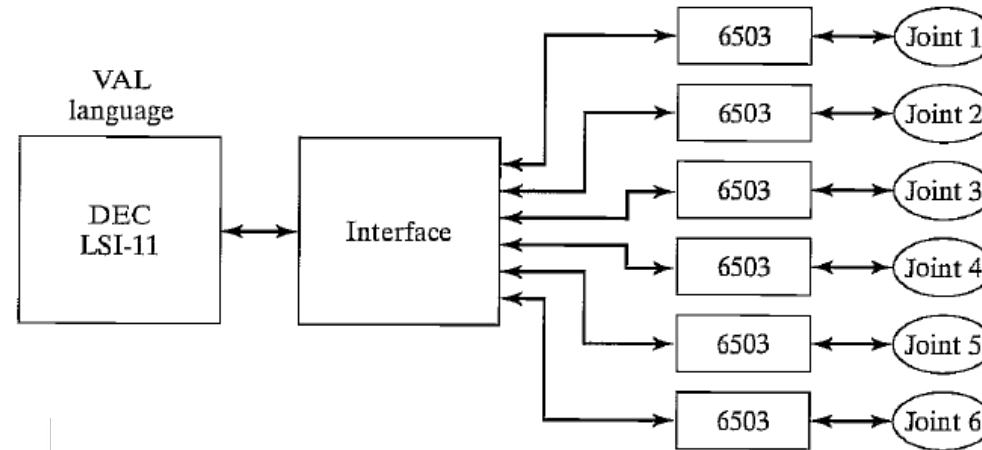
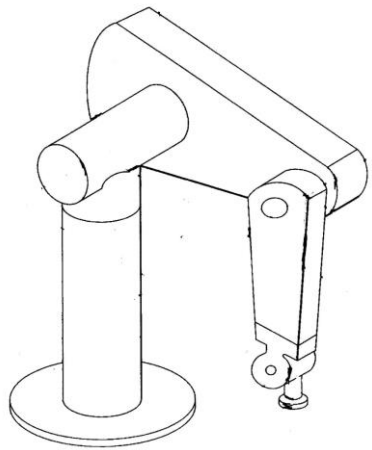
# Motion Control Scheme

## Independent Joint Control

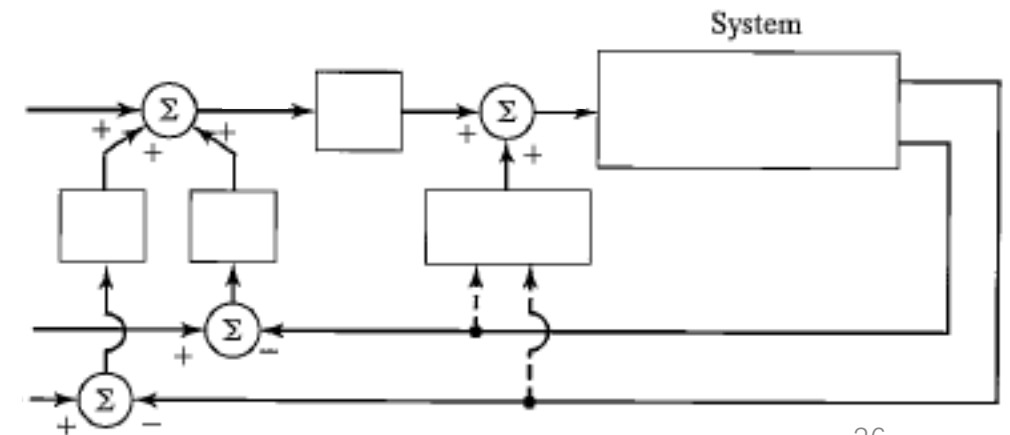


# Joint-Based Control for Robot

Example of industrial Robotic Arm: PUMA 560

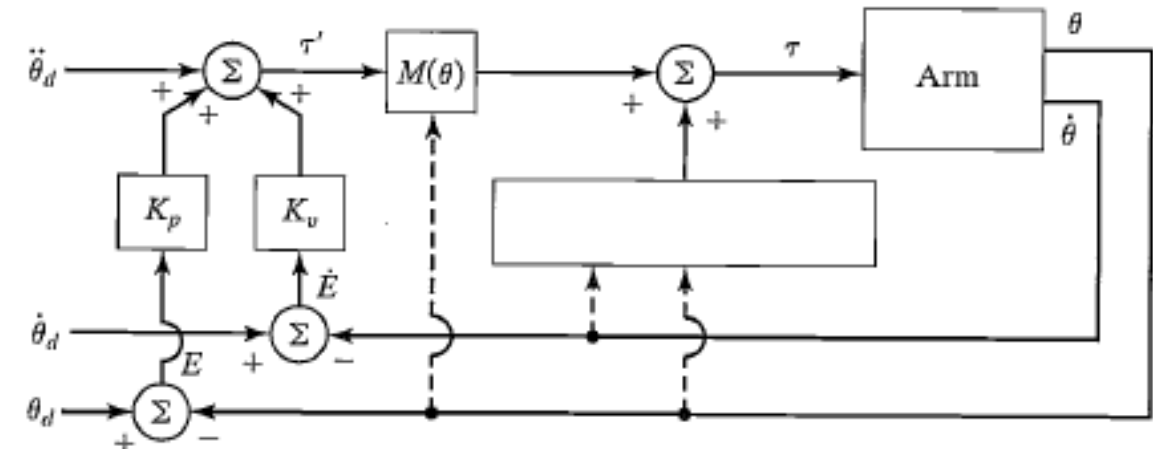


# Multi-Input, Multi-Output Control Systems





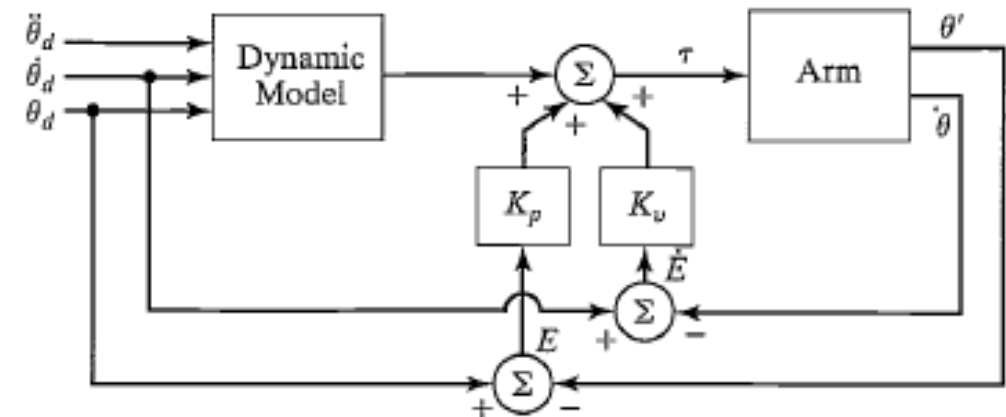
# Control of Manipulators



# Feedforward Nonlinear Control

- Performing computation of the nonlinear dynamics outside the feedback-loop
- Not required to be done at servo rate
- However, not completely decoupled.....

Error Equation:



# Joint based vs. Cartesian based

Textbok Chapter 10.8 (Craig 3<sup>rd</sup> Ed, 2005)

Computation before  
the control loop

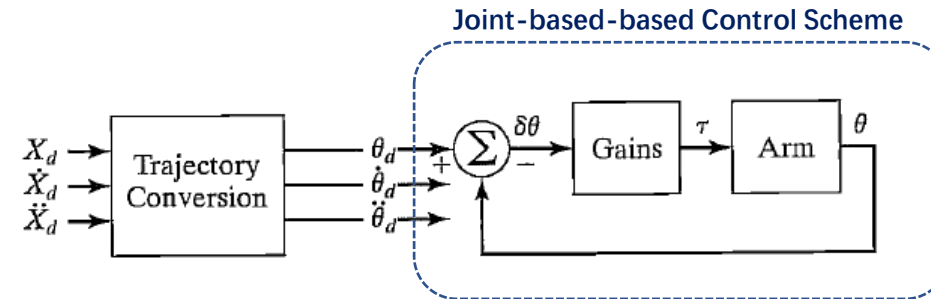


FIGURE 10.10: A joint-based control scheme with Cartesian-path input.

Computation within  
the control loop

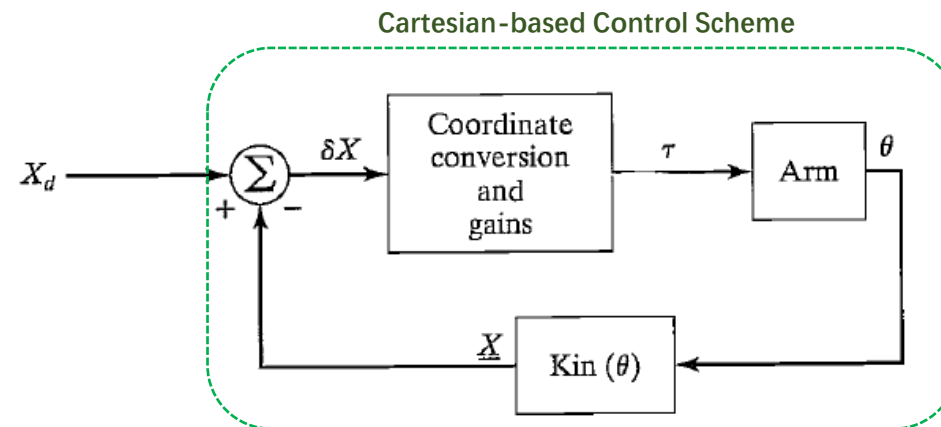


FIGURE 10.11: The concept of a Cartesian-based control scheme.

# Joint based vs. Cartesian based

Question: As an engineering designing the control scheme, which ones will you choose for the following robots?

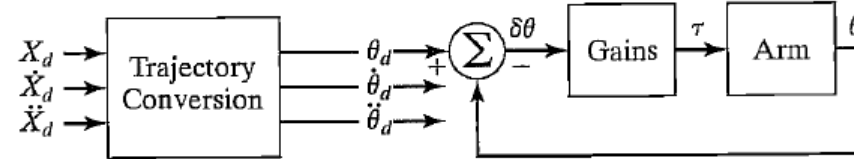


FIGURE 10.10: A joint-based control scheme with Cartesian-path input.

(A) Multi-joint Snake Robot

<https://www.gizbot.com/news/indian-scientists-developing-snake-robot-027613.html>

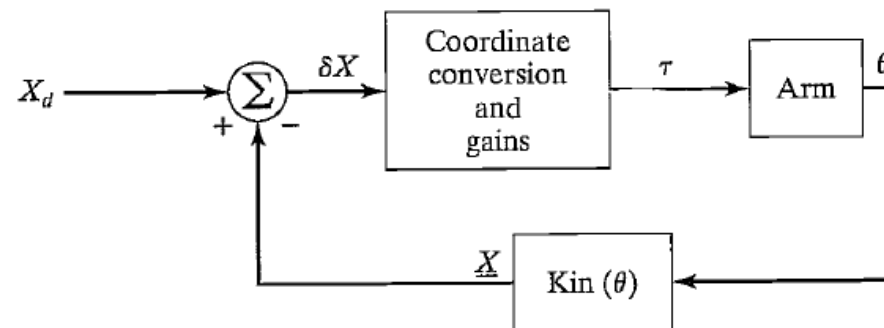


FIGURE 10.11: The concept of a Cartesian-based control scheme.

(B) Serial Arm with 2 long links



# Force Control

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# Force Control: Mass-Spring System

## Recall: Control Law Partitioning

Open loop equation:  $m\ddot{x} + k_e x + f_{dist} = f$

External Force applied on spring  $f_e = k_e x$

$$\ddot{f}_e k_e^{-1} = \ddot{x} \rightarrow m k_e^{-1} \ddot{f}_e + f_e + f_{dist} = f$$

Partitioning law

$$f = \alpha f' + \beta$$

Choosing the coefficient,

$$\alpha = m k_e^{-1}, \quad \beta = f_e + f_{dist}$$

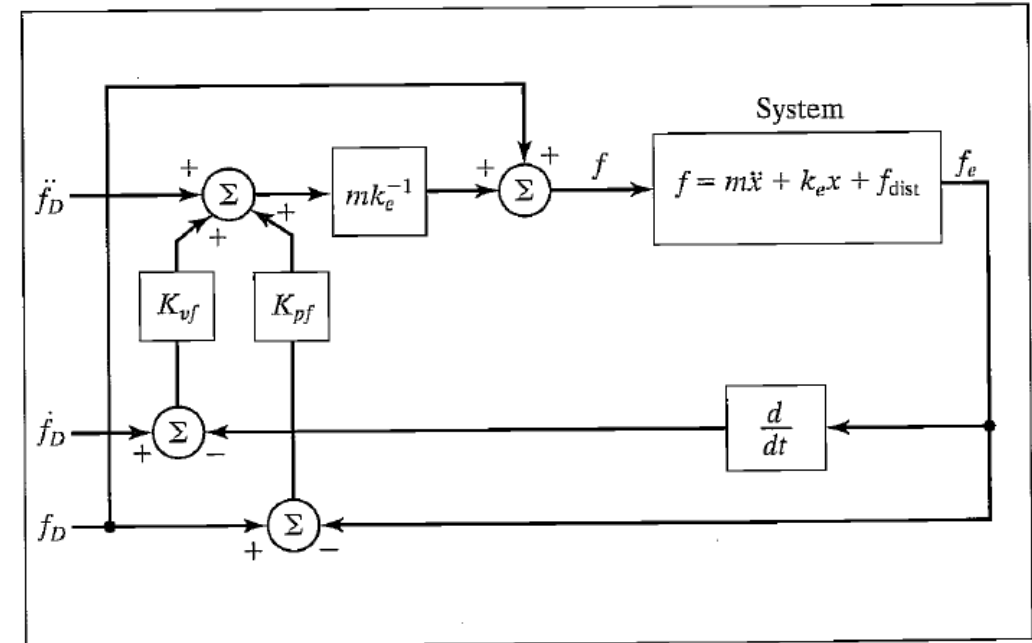
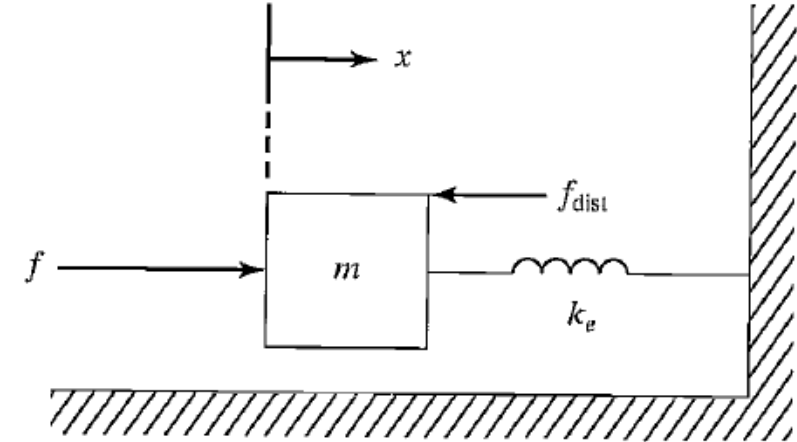
Therefore,  $\ddot{x} = f'$

Proceed with servo portion of control law

$$f' = \ddot{f}_d + k_{vf} \dot{e}_f + k_{pf} e_f$$

where  $e_f = f_d - f_e$ . Hence the control law,

$$f = m k_e^{-1} (\ddot{f}_d + k_{vf} \dot{e}_f + k_{pf} e_f) + f_e + f_{dist}$$



# Force Control: Mass-Spring System

## Recall: Control Law Partitioning

Open loop equation:  $m\ddot{x} + k_e x + f_{dist} = f$

External Force applied on spring  $f_e = k_e x$

$$\ddot{f}_e k_e^{-1} = \ddot{x} \rightarrow m k_e^{-1} \ddot{f}_e + f_e + f_{dist} = f$$

Partitioning law

$$f = \alpha f' + \beta$$

Choosing the coefficient,

$$\alpha = m k_e^{-1}, \quad \beta = f_e + f_{dist}$$

Therefore,  $\ddot{x} = f'$

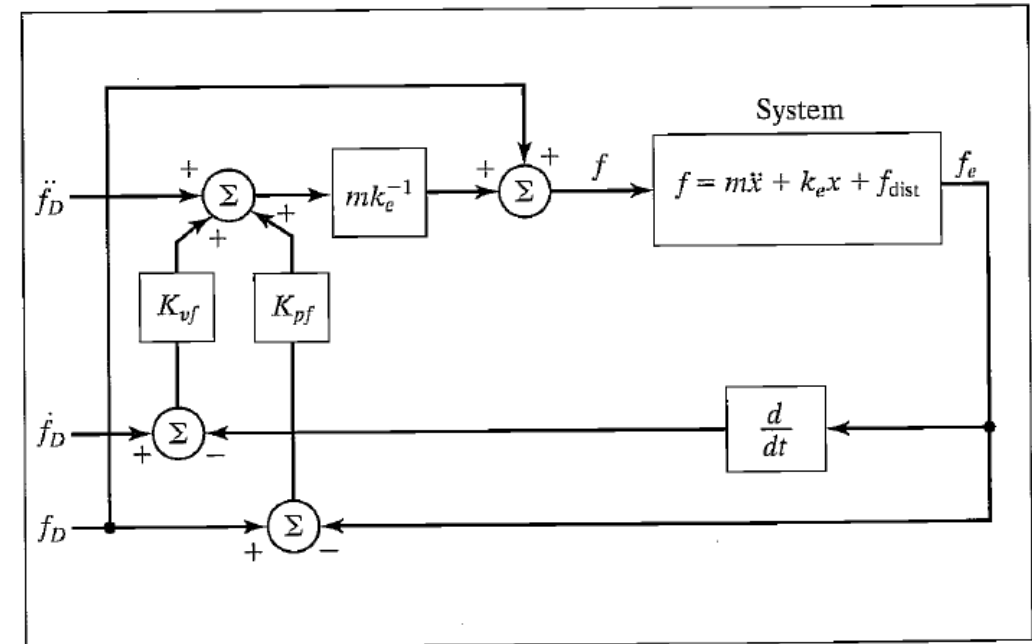
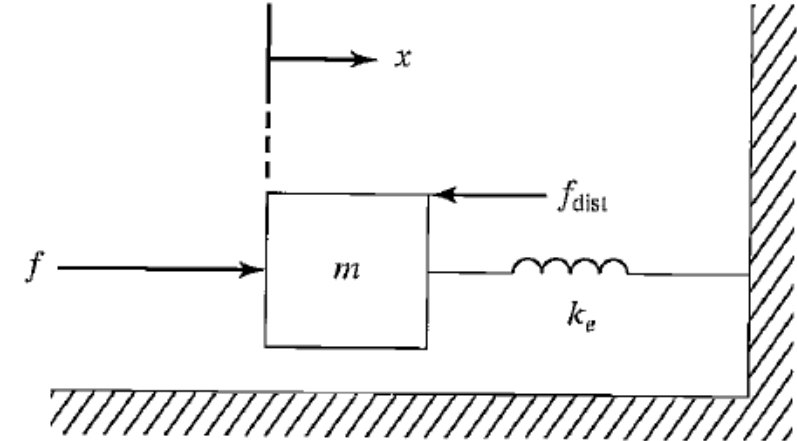
Proceed with servo portion of control law

$$f' = \ddot{f}_d + k_{vf} \dot{e}_f + k_{pf} e_f$$

where  $e_f = f_d - f_e$ . Hence the control law,  $f_d$

$$f = m k_e^{-1} (\ddot{f}_d + k_{vf} \dot{e}_f + k_{pf} e_f) + \boxed{f_e + f_{dist}}$$

$$f = m k_e^{-1} (\ddot{f}_d + k_{vf} \dot{e}_f + k_{pf} e_f) + f_d$$



# Force Control: Mass-Spring System

## Recall: Control Law Partitioning

Control law, in theory, can be chosen as

$$f = mk_e^{-1}(\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f) + f_d$$

In practice, force control has constant  $f_d$  i.e.

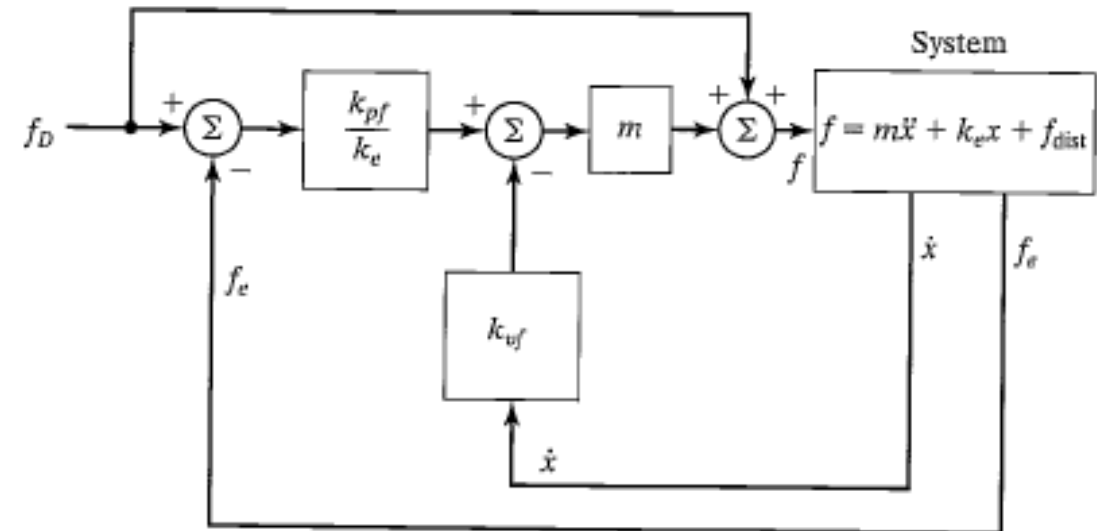
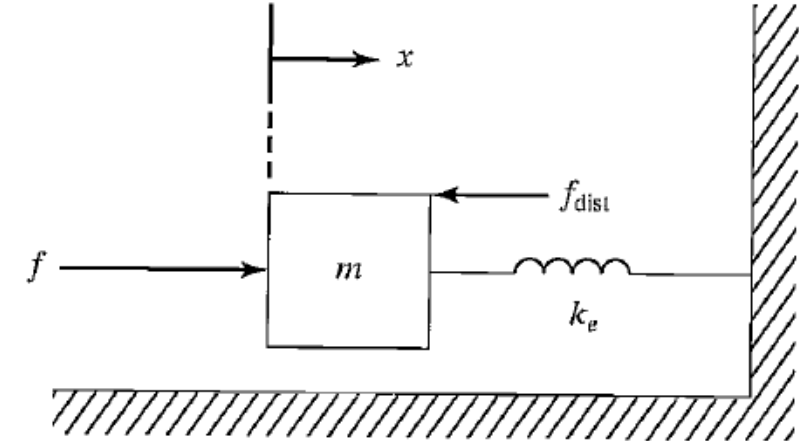
$$\ddot{f}_d = \dot{f}_d = 0$$

Sensing  $\dot{f}_e$  is not practical. Instead, use  $\dot{x}$  to get

$$\dot{f}_e = k_e \dot{x}$$

Hence the control law,

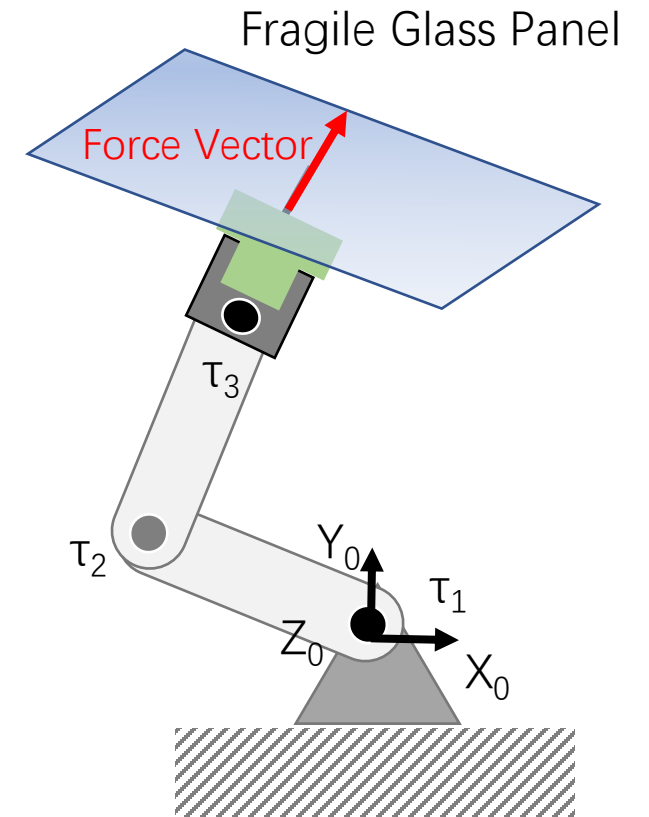
$$f = m(-k_{vf}\dot{x} + k_e^{-1}k_{pf}e_f) + f_d$$





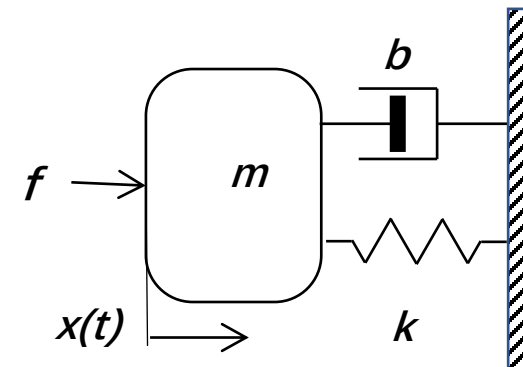
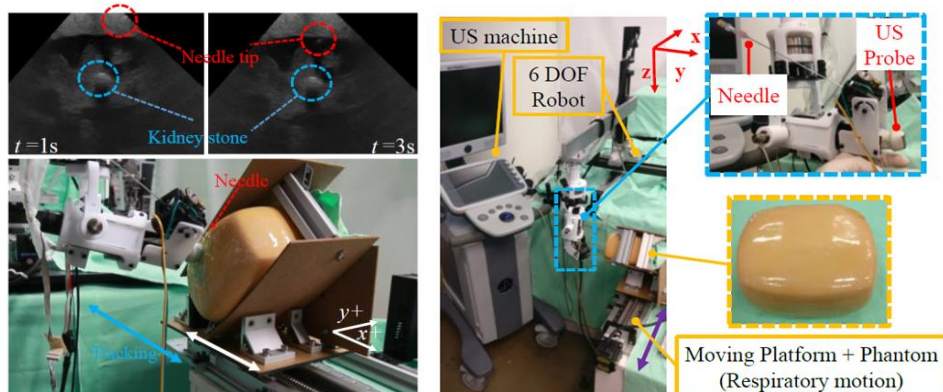
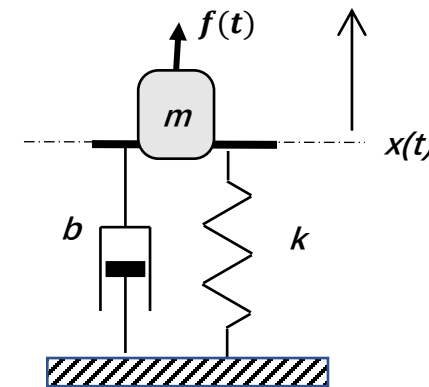
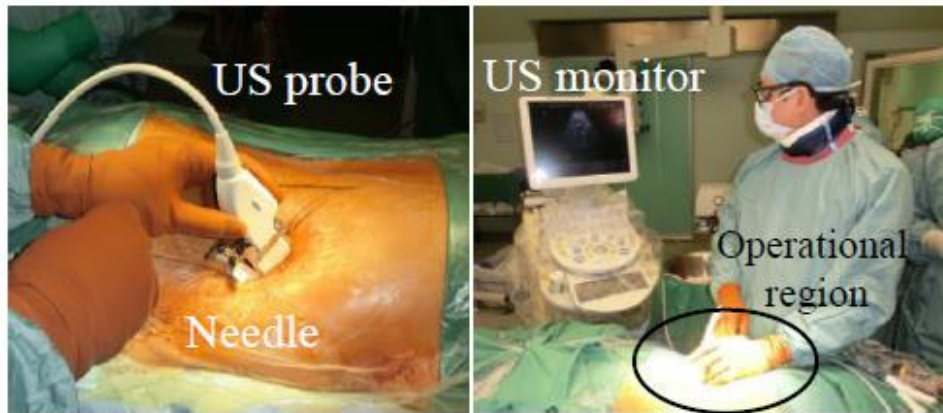
# Why is controlling forces important?

- Case examples
  - Cleaning the glass with the right amount of force so that it can clean but not break the glass
  - How do we know the relationship between the force vector and the joint torques?
    - Jacobian: mapping of the cartesian and joint coordinates
    - Then what's so difficult?



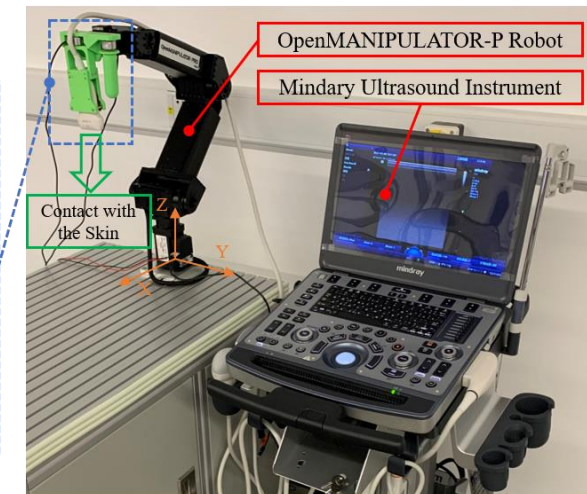
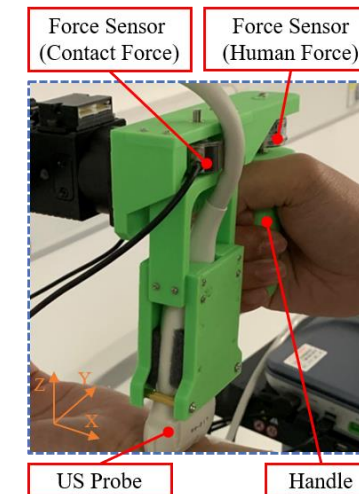
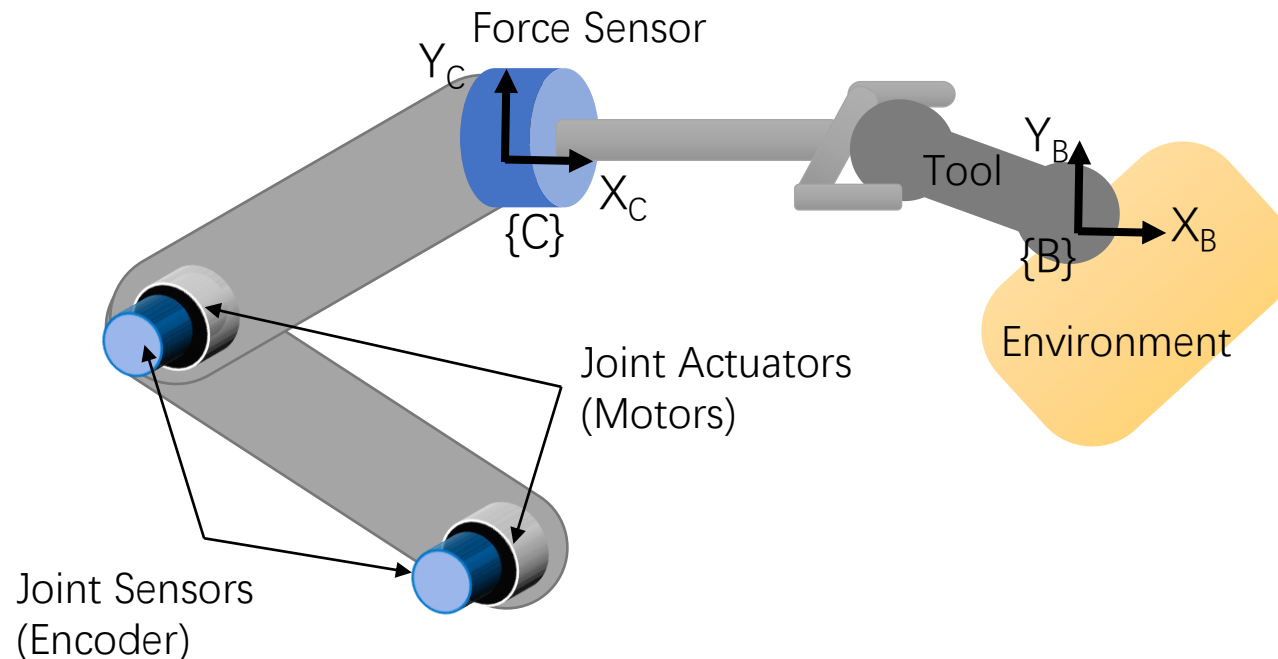
# Examples of Force Control

## Force Control in a Single Axis



# Examples of Force Control

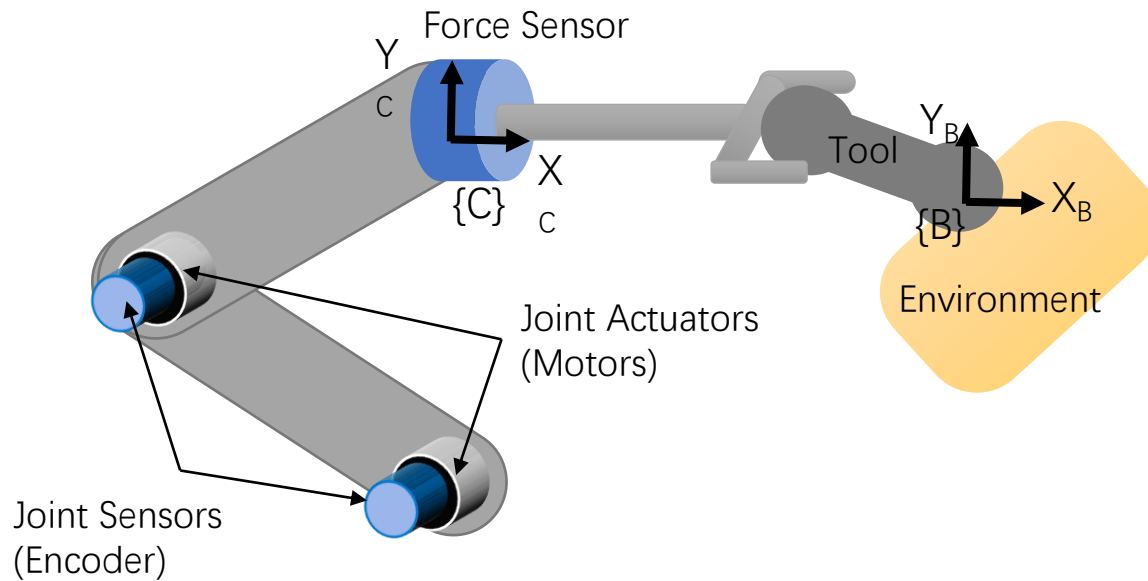
**Case Example:** Controlling a Robot to Track a Target on Ultrasound Image During Needle Insertion



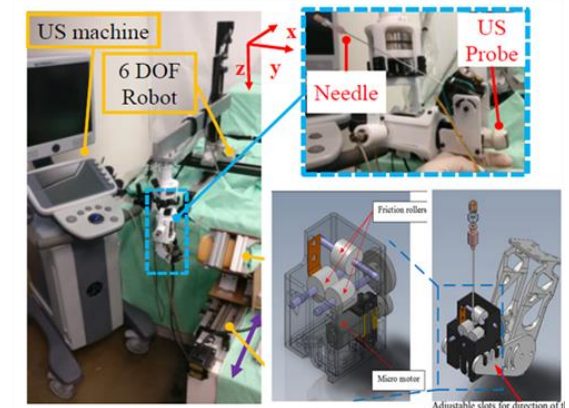
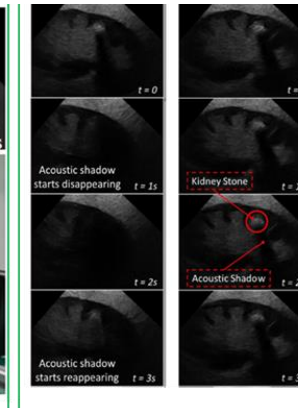
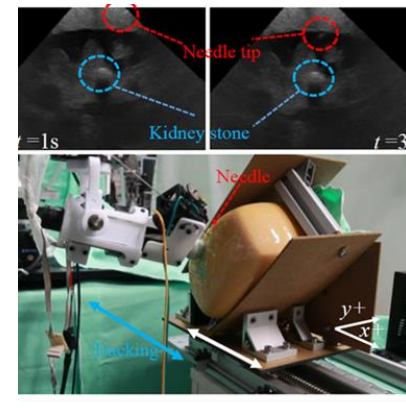
Example of an ultrasound transducer holding robot

# Examples of Force-Position Control

## Case Example: Controlling a Robot to Track a Target on Ultrasound Image During Needle Insertion

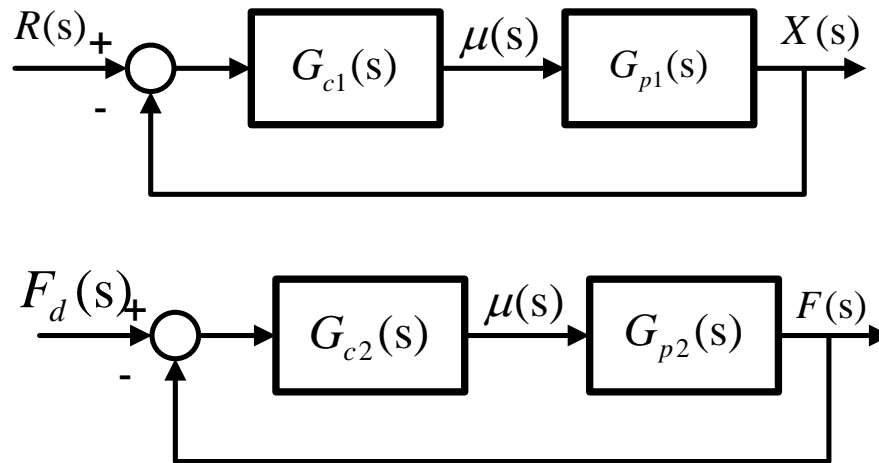


- There is a need to position the ultrasound probe with the right contact force when targeting the right position for needle insertion
- Hence, we are interested in sending in position and force commands



# Hybrid Position/ Force Control

- Obvious Decoupled Joints



$G_{p1,2}(s)$ : plant 1 & 2

$F(s)$ : Force output

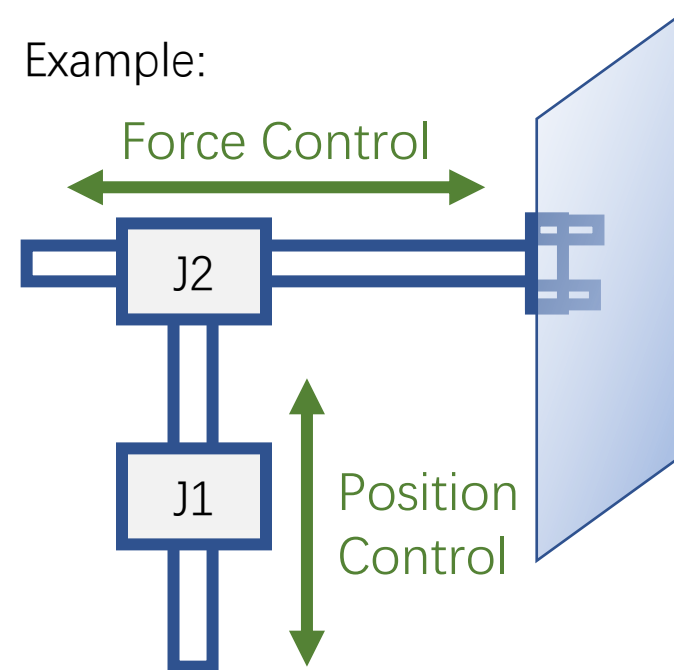
$F_d(s)$ : Force desired command

$X(s)$ : Position output

$R(s)$ : Position desired command

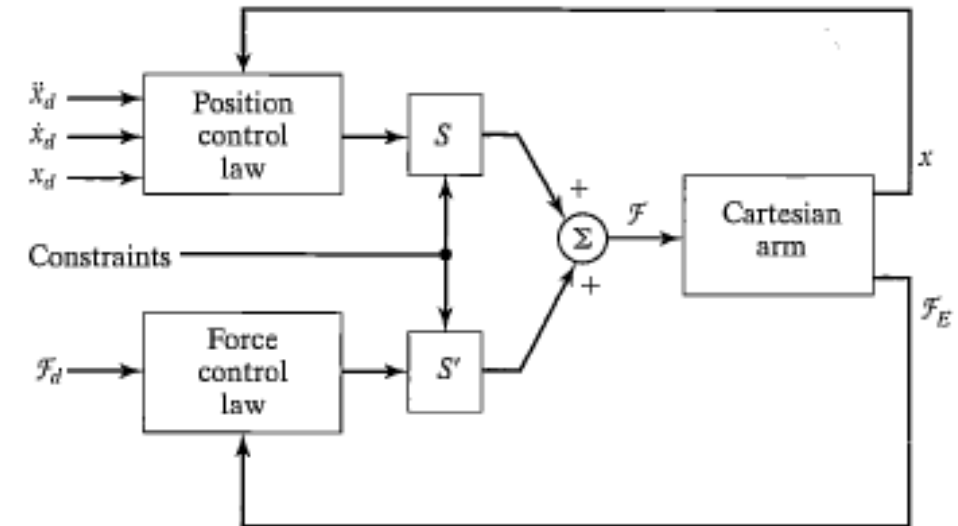
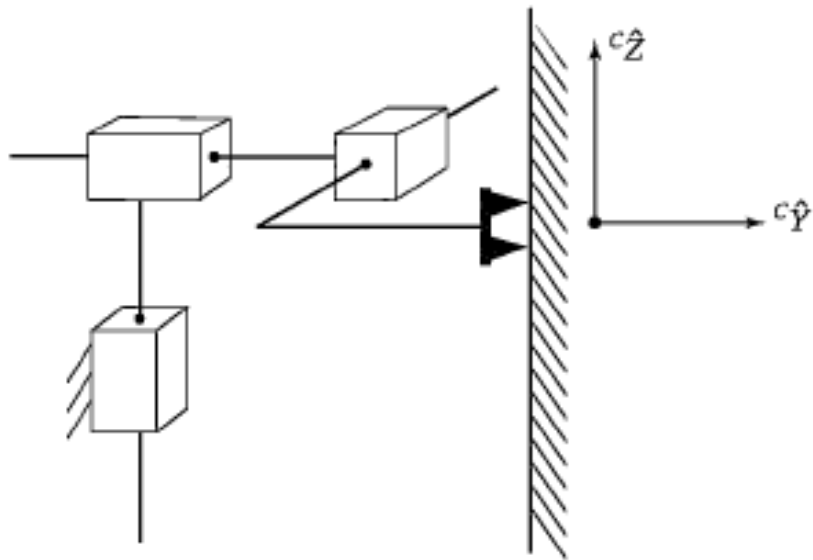
$G_{c1,2}(s)$ : Position Controller, Force Controller

$\mu(s)$ : Control signal



# Hybrid Position/ Force Control

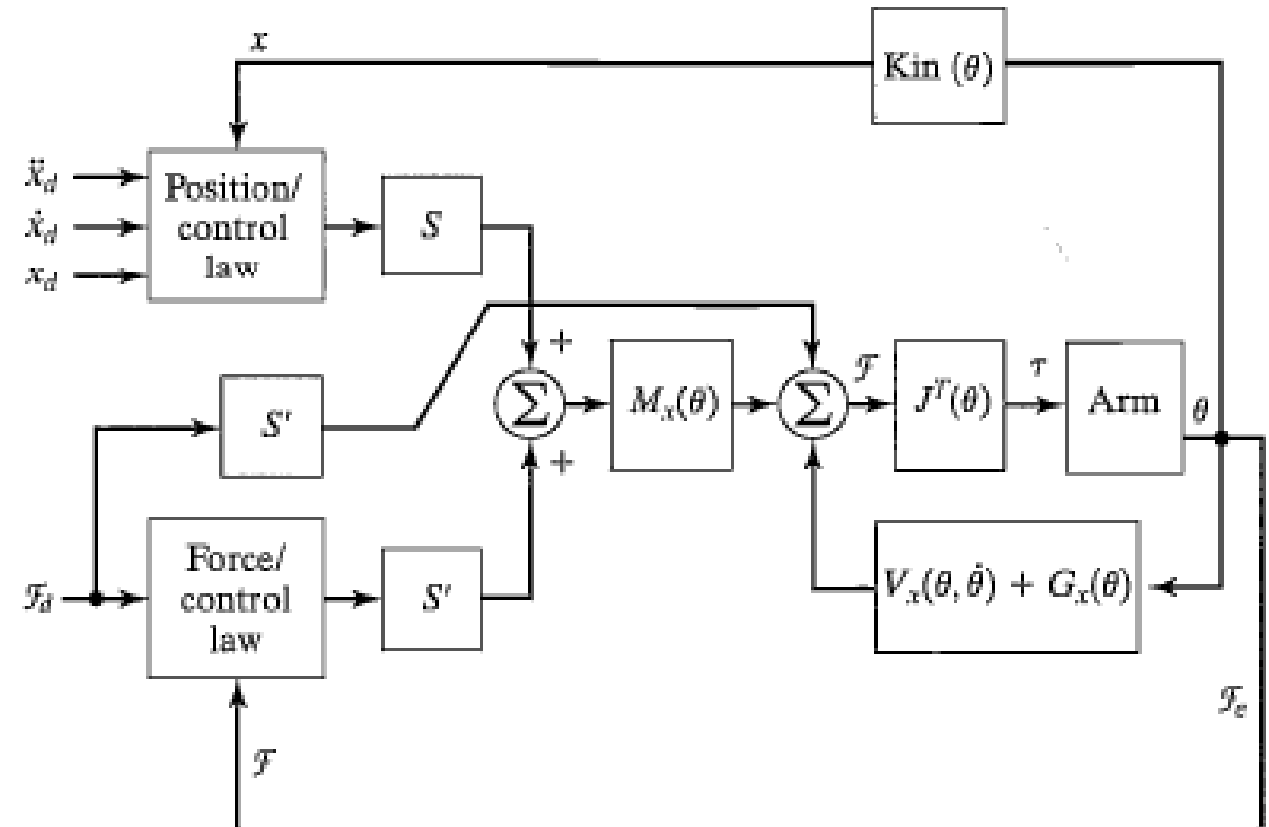
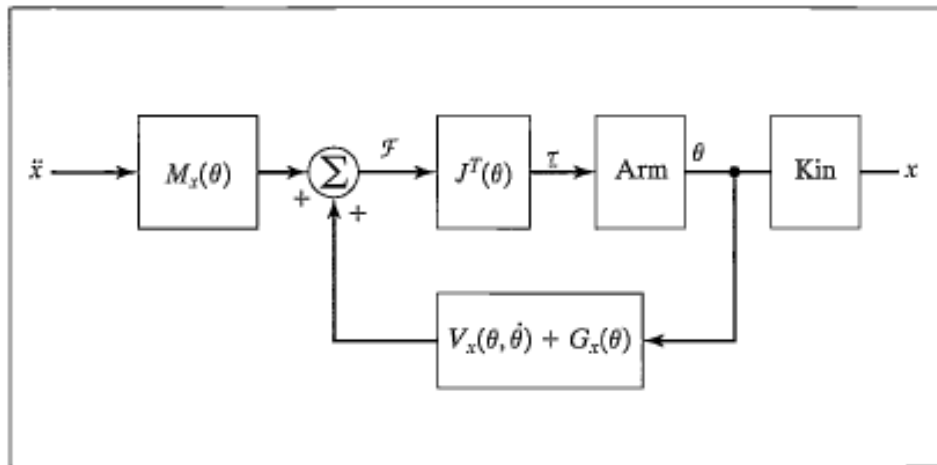
- Example (Decoupled Joints)





# Hybrid Position/ Force Control

- General Manipulator



# Application Example (Importance in HRI)

- Very different paradigm in modern robotics
- No longer isolated from human operators
- Modern robots may be working for, with, and on human



Example in Medical Robotics: Master-slave robot-assisted ultrasound-guided procedures