ECE 470: Introduction to Robotics

Week 09

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Robot Planning

ECE 470 Introduction to Robotics

Schedule Check

- **Lecture**

O. Overview
- Science & Engineering in Robotics

I. Spatial Representation & Transformation
- Coordinate Systems; Pose Representations; Homogeneous Transformations

II. Kinematics
- Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/Inverse Kinematics

III. Velocity Kinematics and Static Forces
- Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity

Fundamentals
Week 1-4
Revision/ ~~Quiz on Week 5~~

IV. Dynamics
- Acceleration of Body; Newton-Euler Equations of Motion; Lagrangian Formulation

V. Control
- Closed-Loop Control and Feedback, Control of 2nd order system, Independent Joint Control, Force Control

VI. Planning
- Joint-Based Scheme; Cartesian-Based Scheme; Collision Free Path Planning

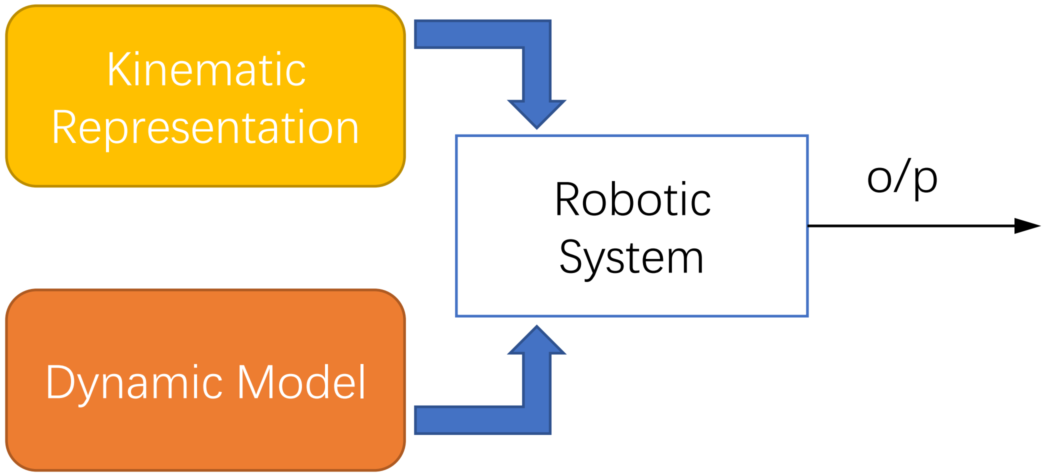
Essentials
Week 6-9
Revision/ Quiz on Week 10

VII. Robot Vision (and Perception)
- Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics

Applied
Week 11-14
Reading Wk/ Exam on Week 15-16

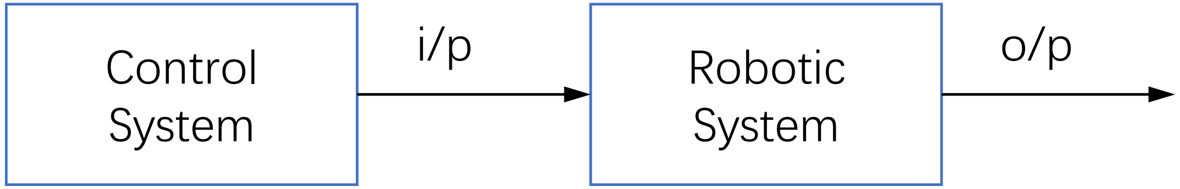
Till Now……

- Model **kinematics** and **dynamics** of the robotic system
  - hence, able to predict outcomes based on the model



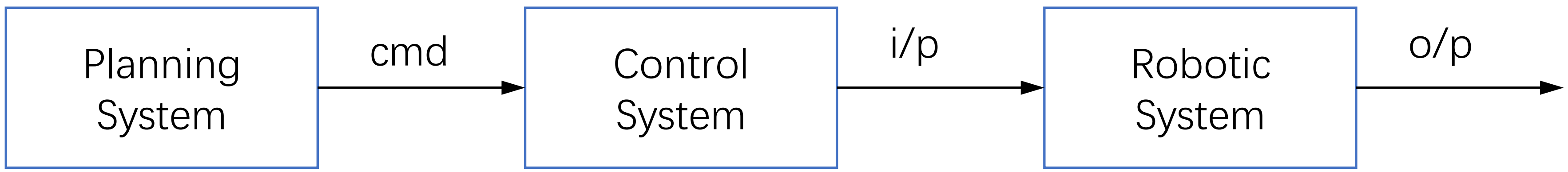
Till Now……

- Model **kinematics** and **dynamics** of the robotic system
- Design **control** for appropriate input to achieve desired outcome



Extending our View

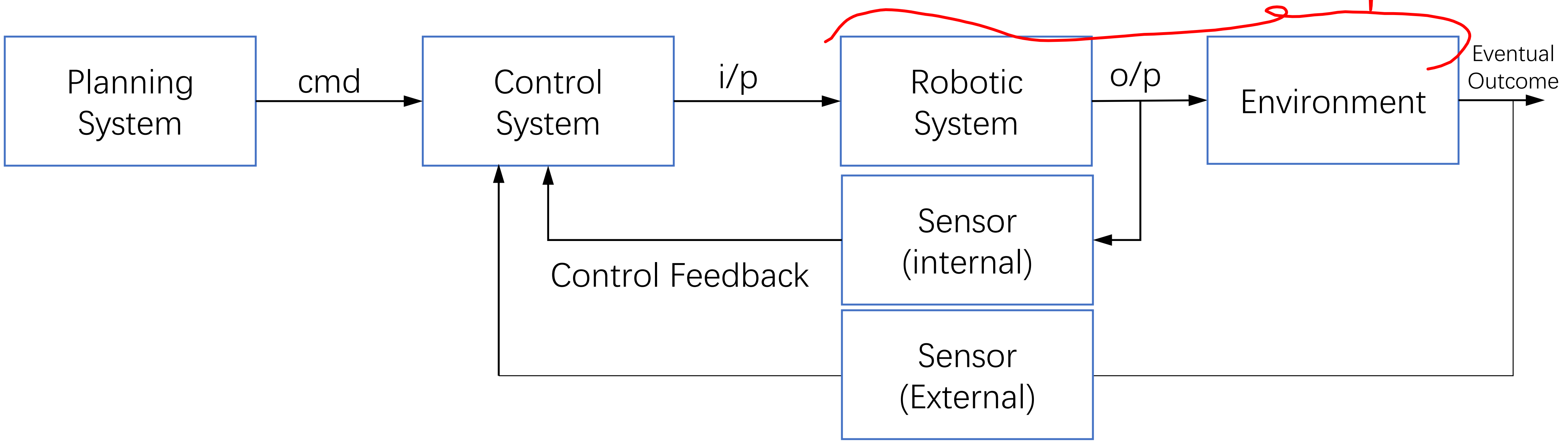
- **Planning system** to send the command to **control** system



The bigger picture

- Model **kinematics** and **dynamics** of the robotic system
- Design **control** for appropriate input to achieve desired outcome
- **Planning system** to send the command to **control** system

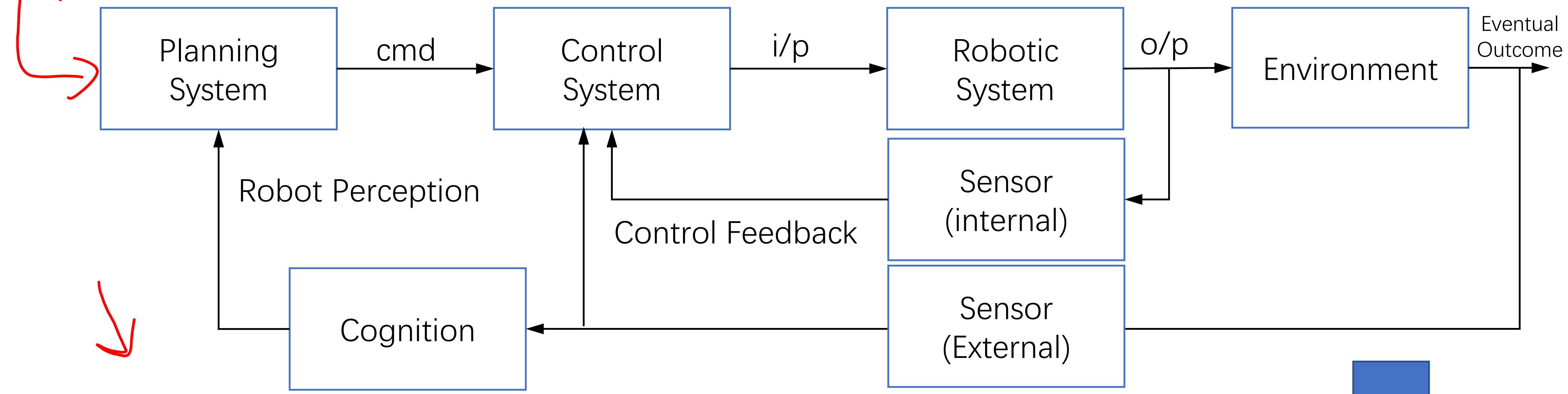
plant



The bigger picture: Yet to come ……

- **Planning system** to send the command to **control** system
  - in absence of accurate modeling information, but with perfect control, online motion planning algorithms are used to find a strategy that is guaranteed to get the robot from start to goal.

User



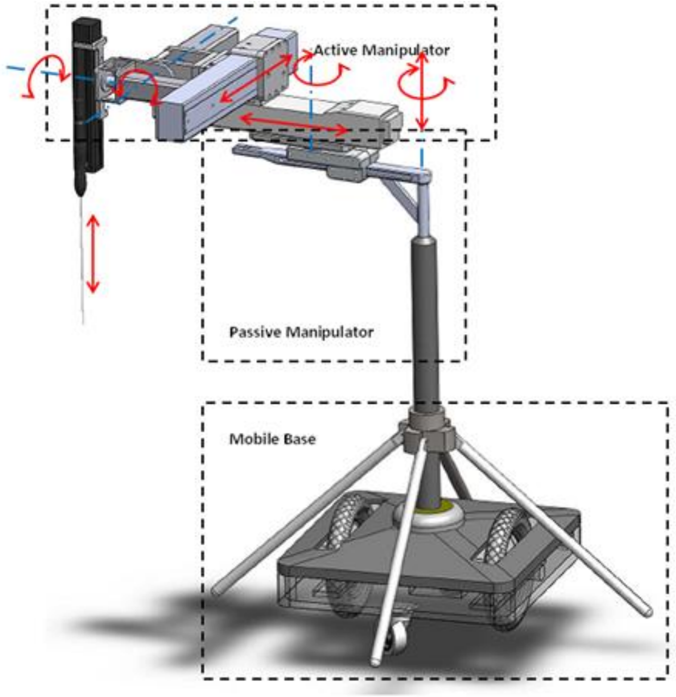
- **Perceive** and interact with environment to achieve goal

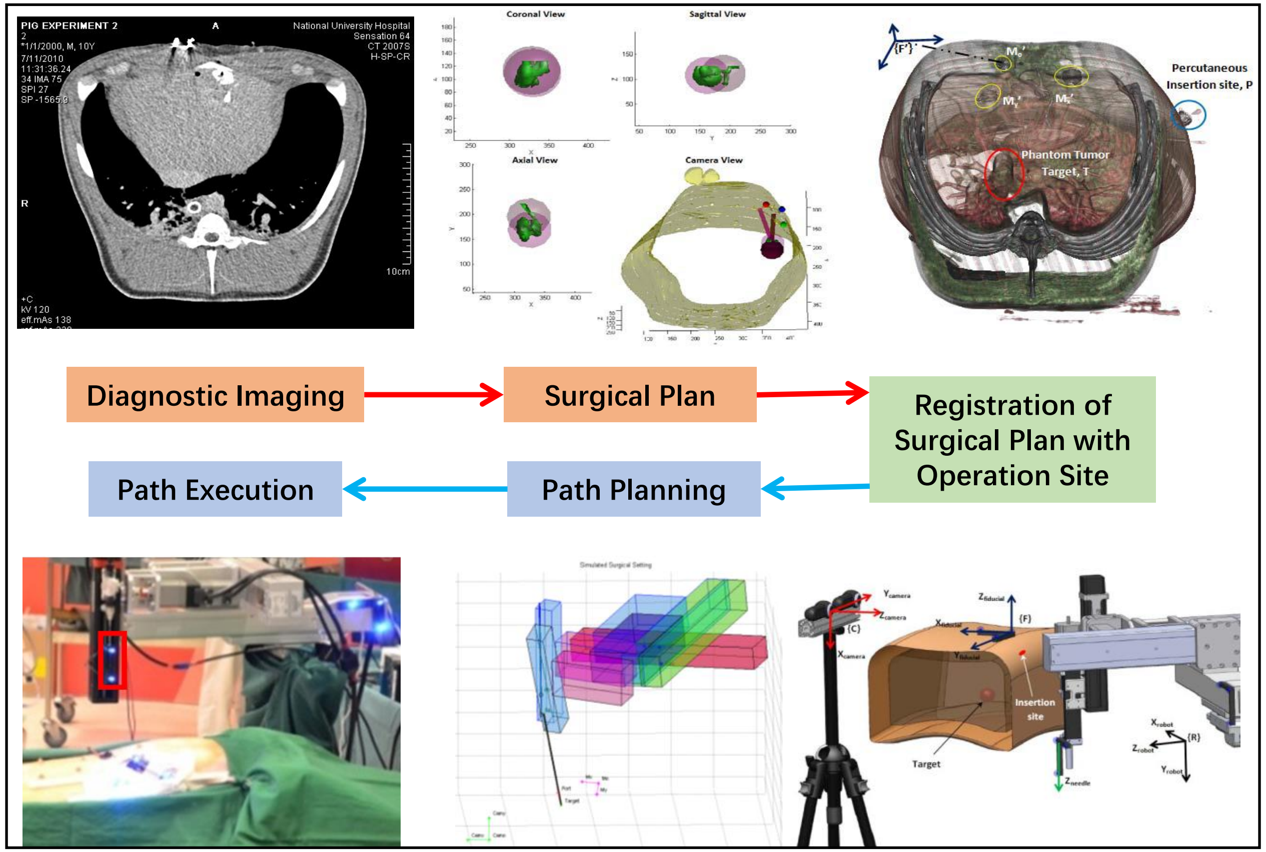
The bigger picture

- Kinematics
  - relating joint and operational coordinates with spatial representation
- Dynamics
  - relating forces and motions of the multibody robotic system
- Control
  - designing control systems that generate the appropriate inputs for the robotic system to achieve a desired outcome in a dynamical environment with a specified performance
- Planning
  - Changing from an initial state to a goal state

Case Example

- Planning Needle Path





Overview in Planning

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Path vs Trajectory

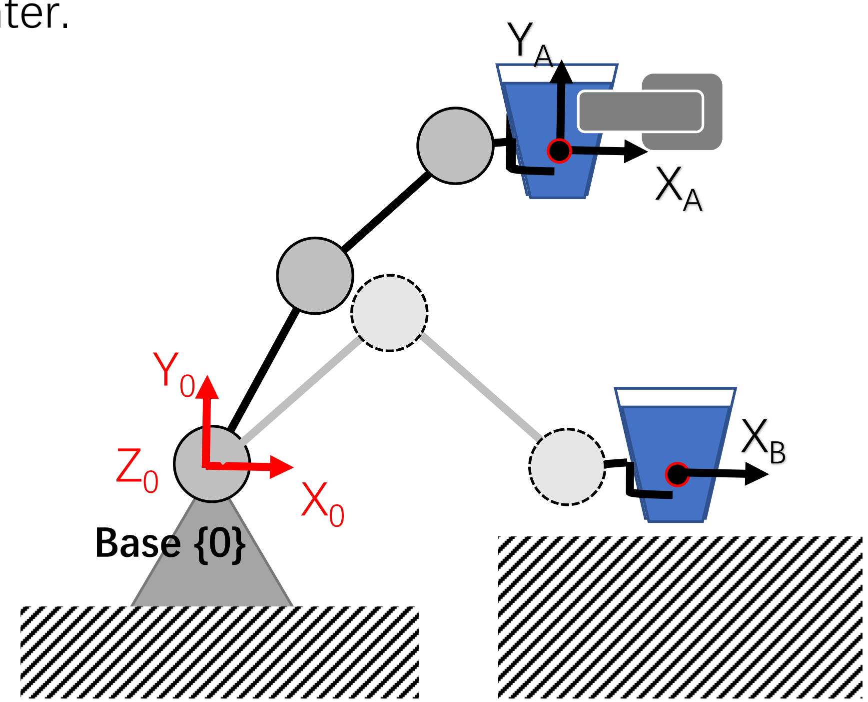
- **Path**: representation of motion in geometrical locus
  - robot's locations/configuration in space (joint or operation) along the executing motion
  - purely spatial variable e.g. $f(x, y), f(s)$
- **Trajectory**: Path with timing specification
  - robot's motion specific to time
  - involve time derivatives like velocity & acceleration e.g. $f(x(t), y(t)), f(\dot{x}(t), \dot{y}(t))$

The Planning Problem

- **Path** planning involves generating the space curve that robot moves along from the initial to the final location/pose
- **Trajectory** planning includes interpolation or approximation of the desired path by a class of polynomial functions
  - Generates a sequence of time based control set points
- **Motion** planning (in the context of most literature) account for obstacle in the operation space hence is concerned with collision-free movement

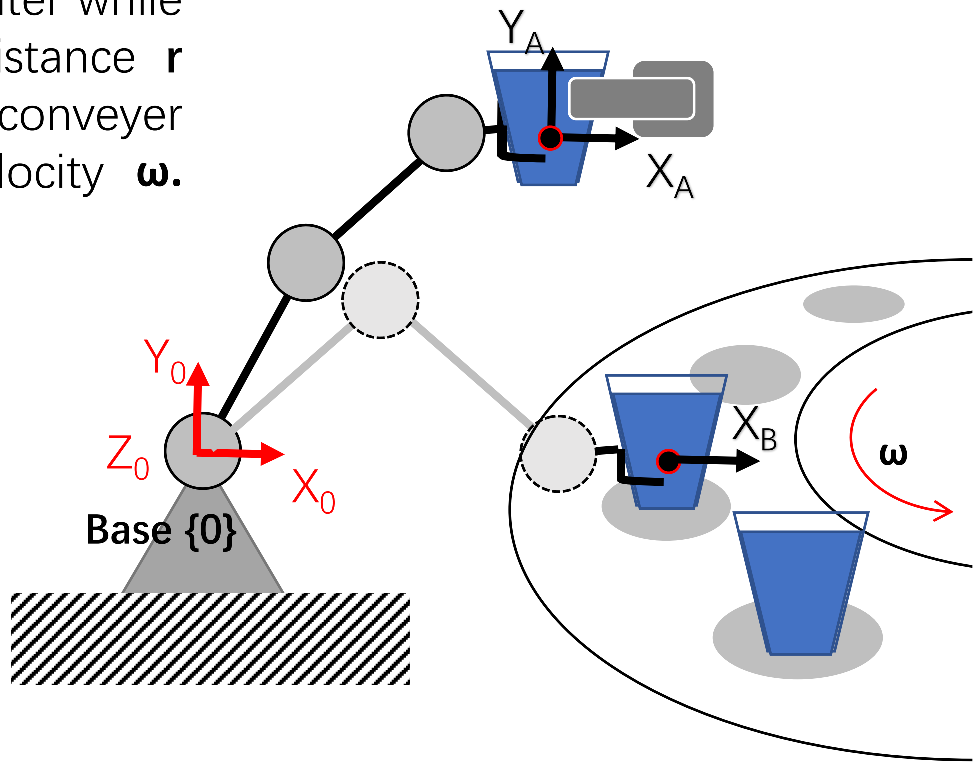
Concept Check

A robot is tasked to bring a cup from A to B.
The requirement is not to spill the water.
What kind of planning is required?



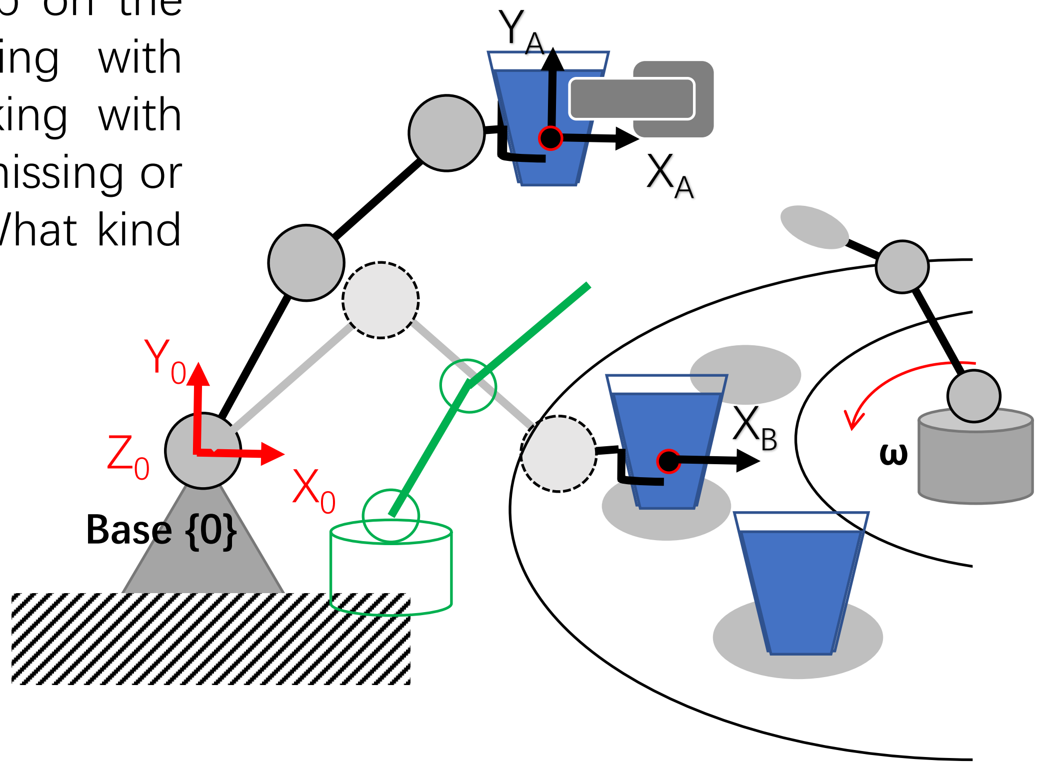
Concept Check

A robot is tasked to bring a cup from A to B. The requirement is not to spill the water while placing the cup on the saucers distance **r** away from the revolution center of a conveyer belt moving with angular with velocity **ω.** What kind of planning is required?



Concept Check

A robot is tasked to bring a cup from A to B. Not to spill the water, place the cup on the saucers on a conveyer belt moving with angular with velocity **ω** while working with other robots that replace and clean missing or dirty in the same operation space. What kind of planning is required?



Formulation of the planning problem

- Moving a robot from one configuration to another, while avoiding obstacles
  - Assume perfect modeling of robot and environment and control capability to achieve control setpoint
  - Represent state of the robot as a point in configuration space, moving from start to goal
  - may be viewed as finding a curve in free-space connecting two points

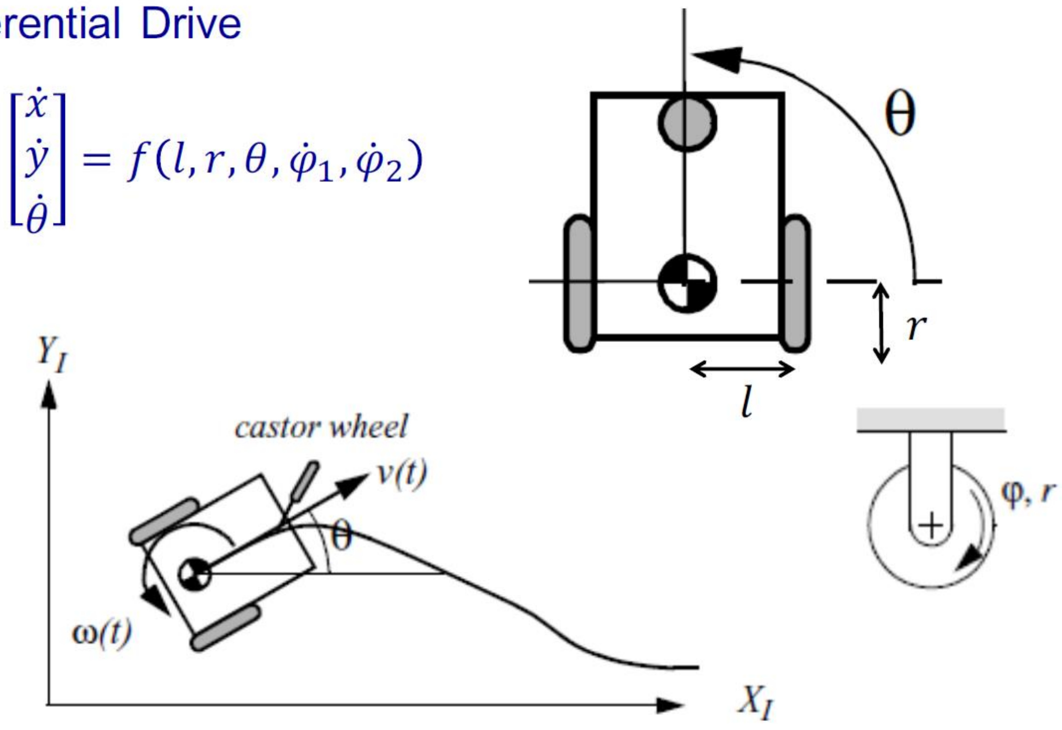
Practical Issues

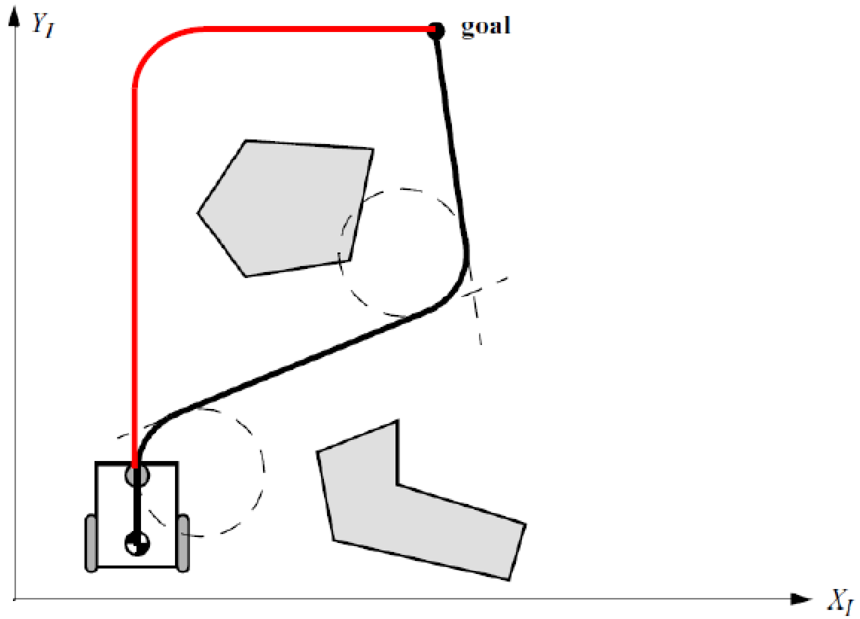
- In practice, there are imperfection in model and uncertainty in control
  - Noise in sensors, uncertainty in actuators and miscalibration can cause error
  - Models of the world and robots maybe inaccurate

Example cases in mobile wheeled robot

Differential Drive

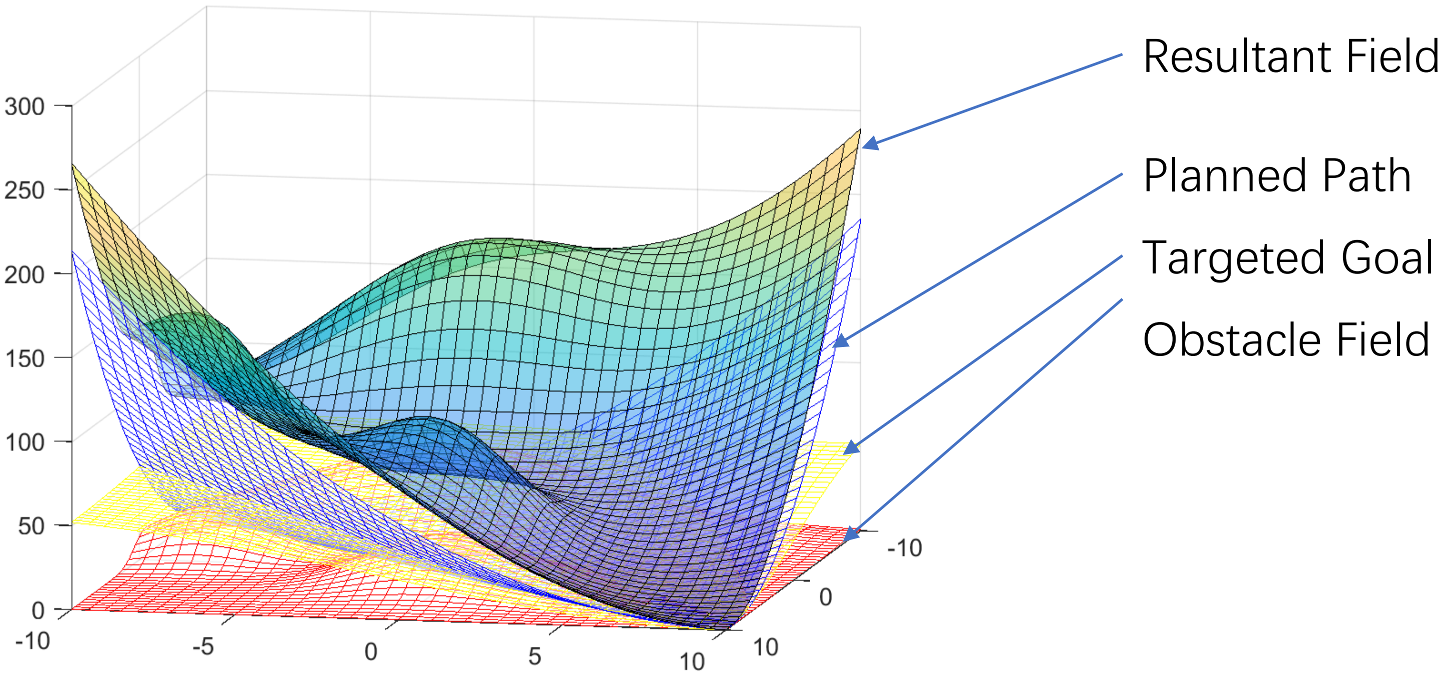
$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$





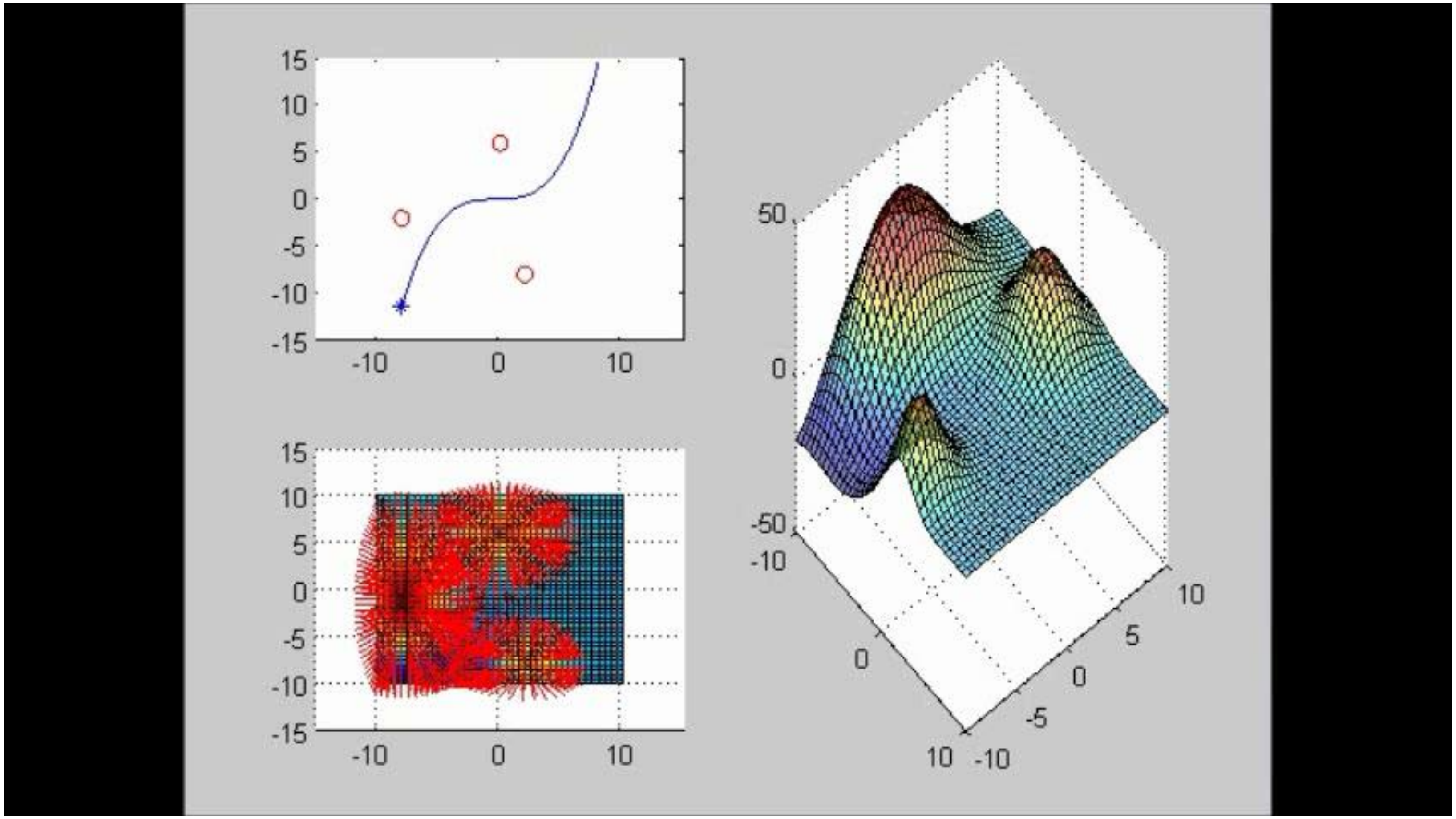
Example of abstraction for planning

- Potential Field
  - Identified obstacles as source
  - Targeted goal as sink
  - Planned path as continuous valley to generate stiffness control



Example of abstraction for planning

Potential Field



Joint Space Scheme

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Joint and Cartesian Schemes

- Joint-Space Schemes: specified in terms of joint coordinates
- Cartesian-Space Schemes: specified in terms of pose

Joint-Space Scheme

- Path generations: motion in terms in terms of functions of joint angles
- Procedural overview
  - Each path point is usually specified in terms of a desired position and orientation of the tool frame *{T}* relative to the station frame *{S}*.
  - Each of these points is converted into a set of desired joint angles by inverse kinematics.
  - Finally, a smooth function is found for each of the *n*-joints which pass through the via points and ends at the goal point.

Joint-Space Scheme

- To move the tool from its initial position to a goal position in a given amount of time
  - The initial position of the manipulator is known as a set of joint angles.
  - Using the inverse kinematics, the set of joint angles corresponding to the goal position and orientation can be obtained.
- Find a function for each joint whose value at t_0 is the **initial position** of the joint, and whose value at t_f is the **desired goal position**

Joint-Space Scheme

For smooth function, 4 constraints

Known Initial and Final Value

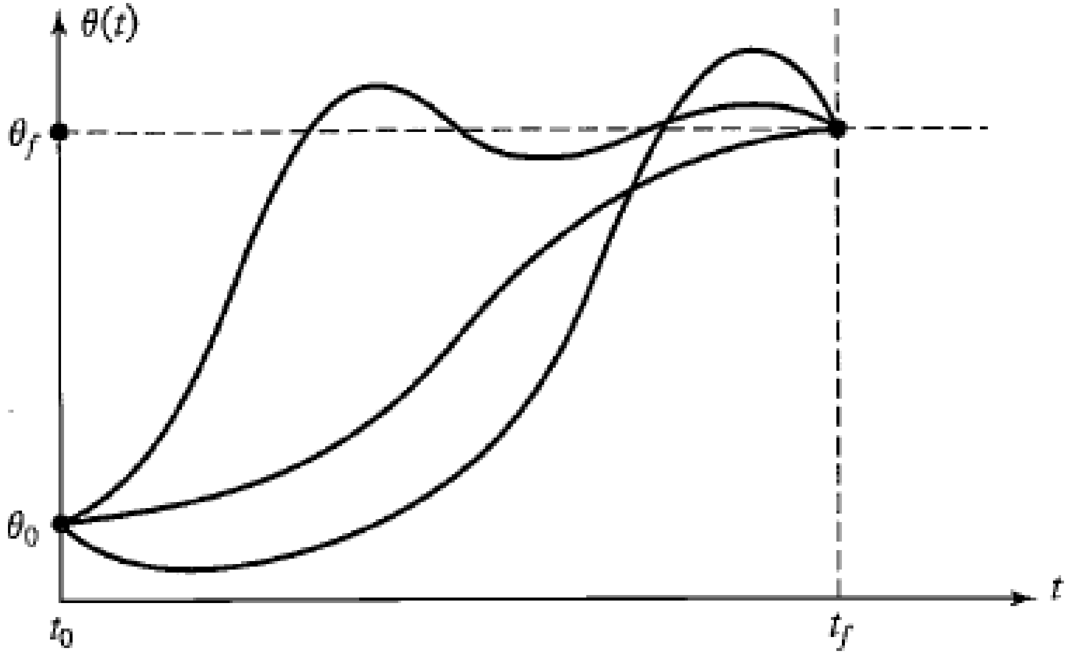
$\theta(t_0) = \square$ $\theta(t_f) = \square$

For Smooth Condition: Velocity must be continuous (differentiable) resulting in boundary condition:

$\dot{\theta}(t_0) = \square$ $\dot{\theta}(t_f) = \square$

Cubic polynomial with 4 coefficients

$\theta(t) = \square$



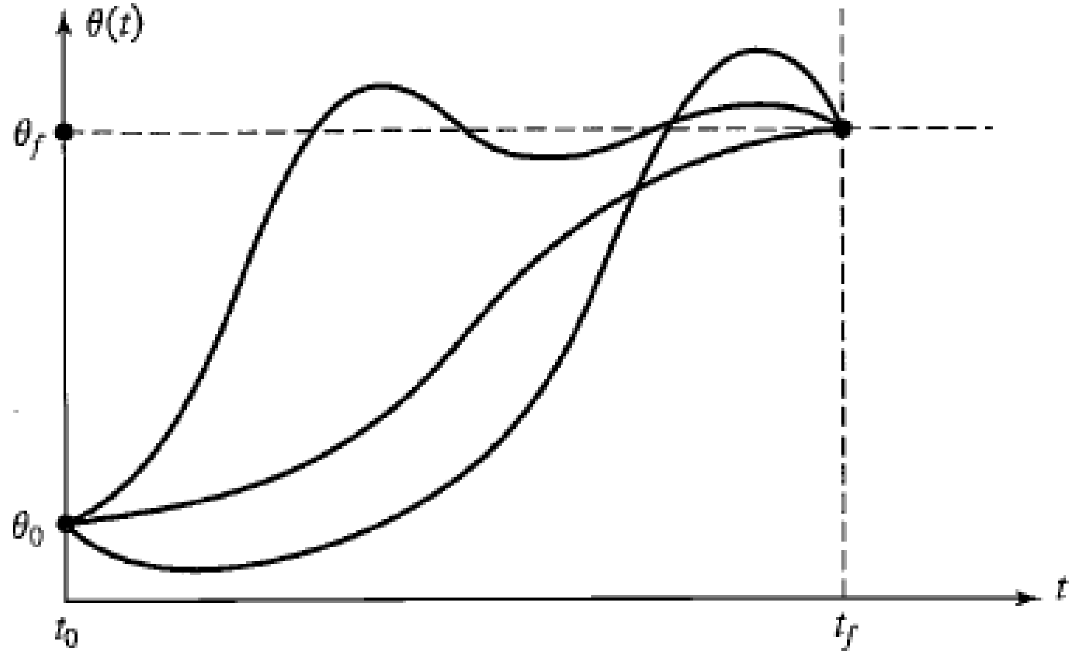
Joint-Space Scheme

Cubic polynomial with 4 coefficients

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = \boxed{}$$

$$\ddot{\theta}(t) = \boxed{}$$



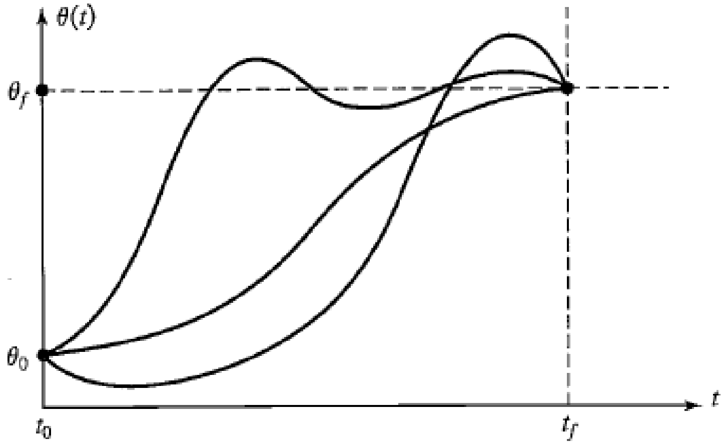
Joint-Space Scheme

Cubic polynomial with 4 coefficients

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$



Including the boundary conditions

$$\theta_0 = \theta(0) = a_0$$

$$\theta_f = \theta(t_f) = a_0 + a_1 t_f + a_2 {t_f}^2 + a_3 {t_f}^3$$

$$0 = a_1$$

$$0 = a_1 + 2a_2 t_f + 3a_3 {t_f}^2$$

Solving Parameter

$$a_0 =$$

$$a_1 =$$

$$a_2 =$$

$$a_3 =$$

Example: Cubic Polynomial

EXAMPLE 7.1

A single-link robot with a rotary joint is motionless at $\theta = 15$ degrees. It is desired to move the joint in a smooth manner to $\theta = 75$ degrees in 3 seconds. Find the coefficients of a cubic that accomplishes this motion and brings the manipulator to rest at the goal. Plot the position, velocity, and acceleration of the joint as a function of time.

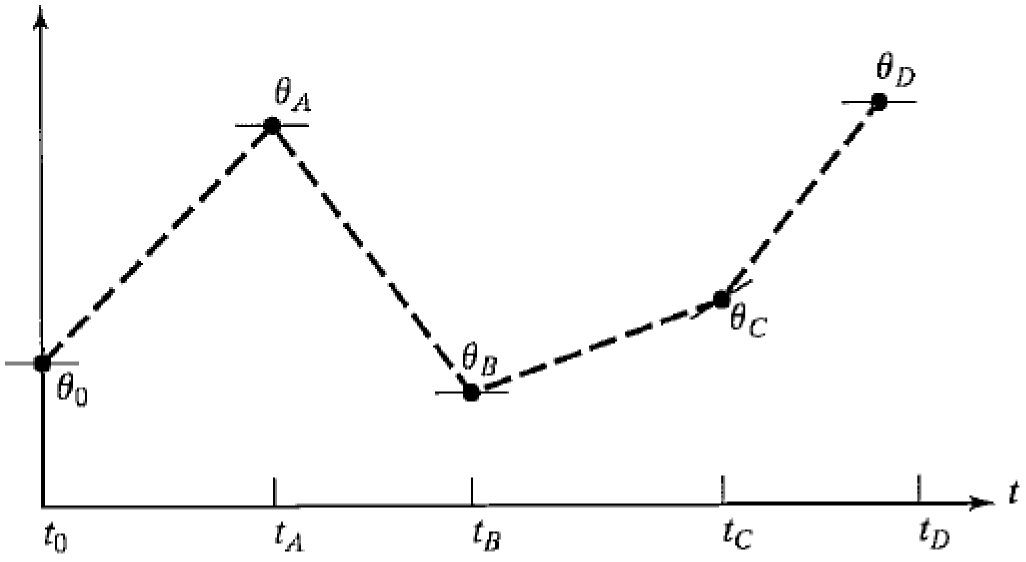
Textbook pg. 205, Craig. 3rd Ed.

Joint-Space Scheme

$\theta(t) =$

$\dot{\theta}(t) =$

$\ddot{\theta}(t) =$



Including the boundary conditions

$$\theta_0 = \theta(0) = a_0$$

$$\theta_f = \theta(t_f) = a_0 + a_1 t_f + a_2 {t_f}^2 + a_3 {t_f}^3$$

$$\dot{\theta}_0 = a_1$$

$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 {t_f}^2$$

Solving Parameter

$a_0 =$

$a_1 =$

$a_2 =$

$a_3 = -$

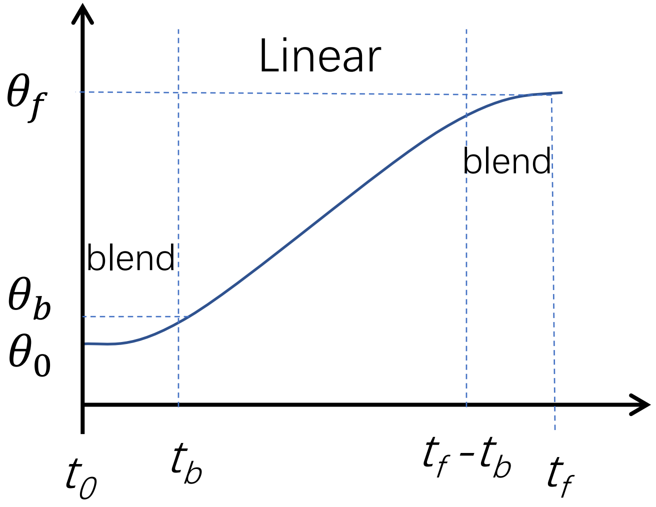
Example: Via Point

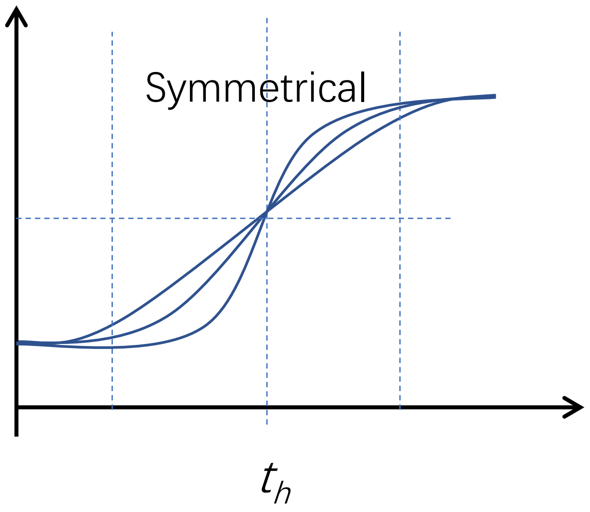
EXAMPLE 7.2

Solve for the coefficients of two cubics that are connected in a two-segment spline with continuous acceleration at the intermediate via point. The initial angle is θ_0, the via point is θ_v, and the goal point is θ_g.

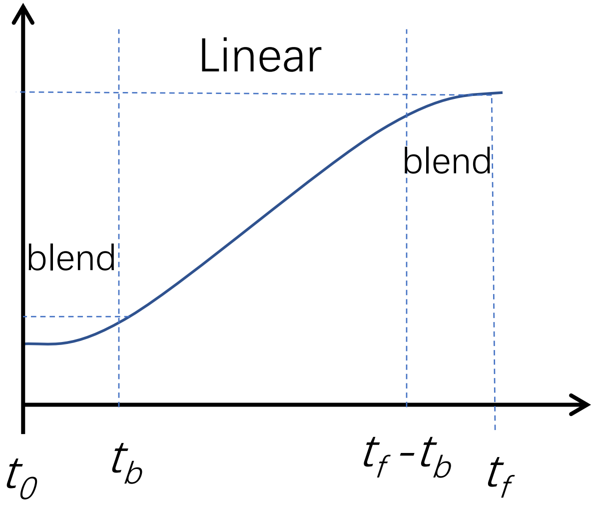
Textbook pg. 208, Craig. 3rd Ed.

Linear segment with parabolic blends





Linear segment with parabolic blends



continuity b/w segments: equal gradient

constant acceleration: parabolic curve

$$\ddot{\theta} t_b = \frac{\theta_h - \theta_b}{t_h - t_b} \qquad \theta_b = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2$$

$$\theta_h = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2 + \ddot{\theta} t_b (t_h - t_b)$$

Symmetrical

$$\theta_h = \theta_0 - \frac{1}{2}\ddot{\theta} t_b^2 - \ddot{\theta} t_b (t_h - t_b)$$

Combining

$$\ddot{\theta} t_b^2 - \ddot{\theta} t_f t_b + \theta_f - \theta_0 = 0$$

Linear segment with parabolic blends

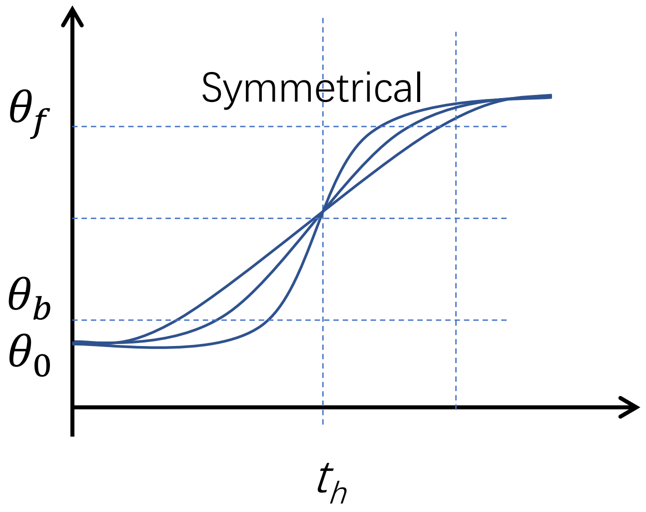
$$\boxed{\ddot{\theta} t_b^2 - \ddot{\theta} t_f t_b + \theta_f - \theta_0 = 0}$$

Usually, an acceleration, is chosen and the above equation is solved for the corresponding t_b.

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta} t_b^2 - 4\ddot{\theta}\,(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

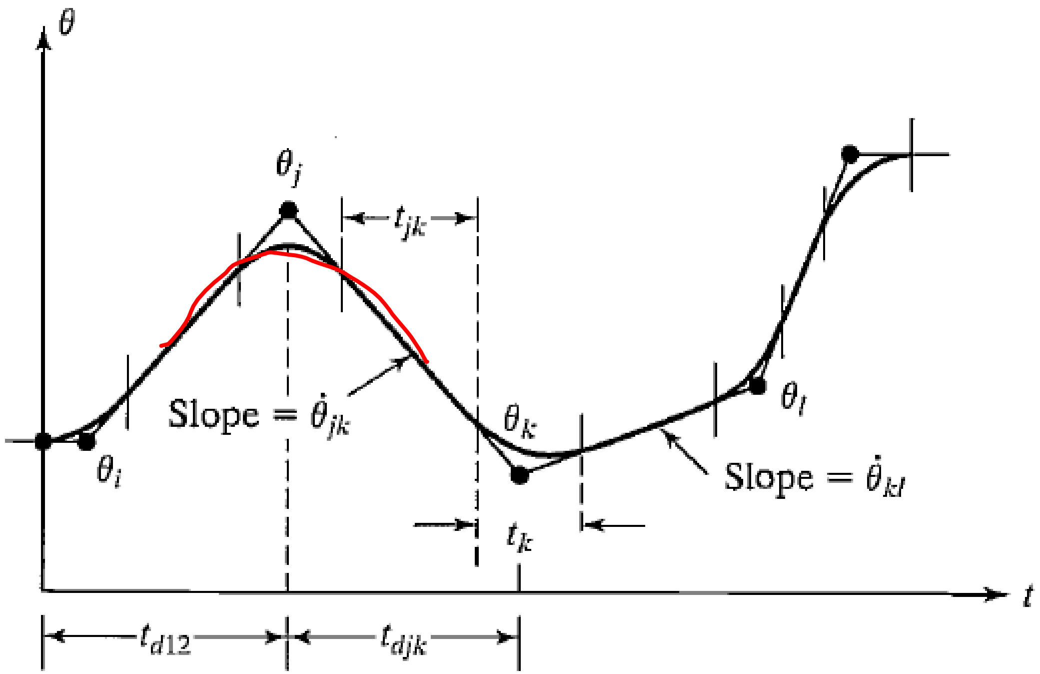
For real solutions to exist, acceleration need to meet the criteria

$$\ddot{\theta} \geq \frac{4\,(\theta_f - \theta_0)}{{t_f}^2}$$



Linear-parabolic blends: Path with via point

- Linear segment with parabolic blends at adjacent via points



Cartesian Space Scheme

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