ECE 470: Introduction to Robotics
Lecture 03

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Schedule Check

- **Lecture**

WK 02

O. Overview
- Science & Engineering in Robotics

I. Spatial Representation & Transformation
- Coordinate Systems; Pose Representations; Homogeneous Transformations

II. Kinematics
- Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/**Inverse Kinematics**

III. Velocity Kinematics and Static Forces
- Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity

Fundamentals

Week 1-4

Revision/ Quiz on Week 5

IV. Dynamics
- Lagrangian Formulation; Newton-Euler Equations of Motion

V. Planning
- Joint-based Motion Planning; Cartesian-based Path Planning

VI. Control
- Independent Joint/Feedforward/Inverse Dynamics Controls; Controller Architectures

Essentials

Week 6-9

Revision/ Quiz on Week 10

VII. Robot Vision (and Perception)
- Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics

Applied

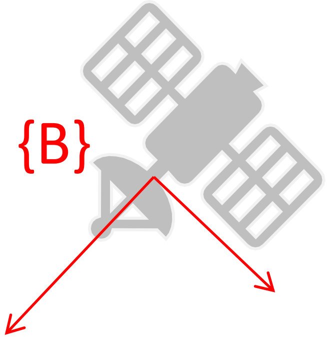
Week 11-14

Reading Wk/ Exam on Week 15-16

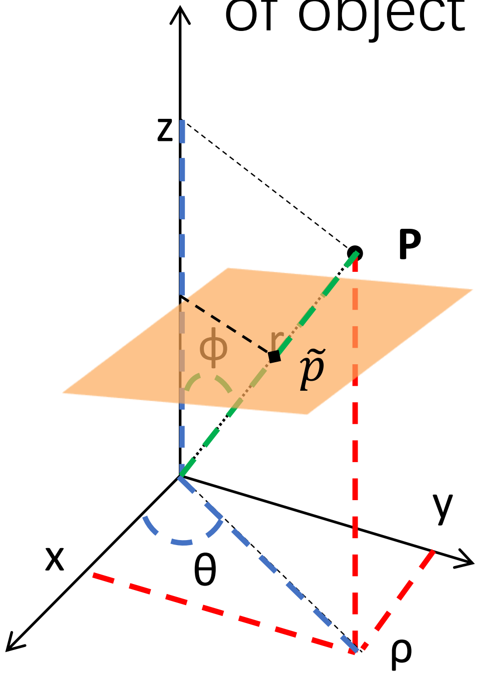
Recap on Week 01

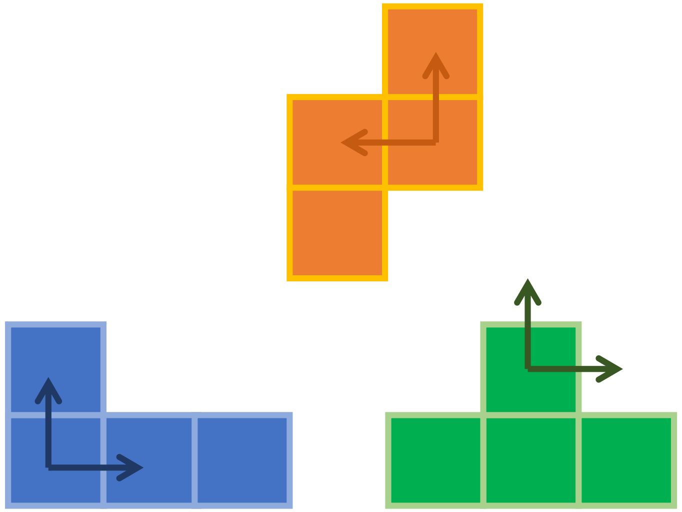
- Define robotics
  - Robot: A machine/agent designed to execute task(s) while interacting with the environment
- Appreciate the relevance and scope of robotics
  - Robots designed in various forms for different tasks while operating in different environment
  - Important aspects include: Robot kinematics, dynamics, planning, control and perception
- Familiarize with mathematical representations for spatial description and transformation
  - Coordinate systems and frames can be assigned to describe poses and transformations
  - The homogenous transformation matrix encompasses information on orientation/rotation and position/translation

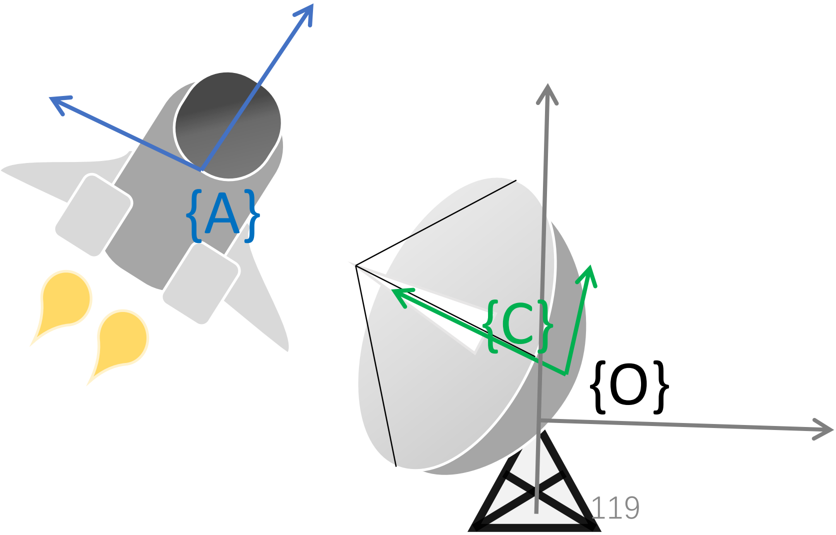
Recap on Week 01



- Coordinate systems and frames
  - **Frame** is a **coordinate system** usually specified in position and orientation relative to other assigned coordinate systems
  - **Reference frames** can be assigned to rigid bodies for the description of object poses and transformation







Recap on Week 01

- Homogenous transformation matrix

$$ {}^{A}_{B}T = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ \boldsymbol{0} & \mathbf{1} \end{bmatrix} $$

Coordinates of P in {B}

Coordinates of P in {A}

$$ {}^{A}P = {}^{A}_{B}R\, {}^{B}P + {}^{A}P_{BORG} $$

Coordinates of origin of {B} in {A}

$$ {}^{A}\tilde{P} = {}^{A}_{B}T\ {}^{B}\tilde{P} $$

$$ \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} & {}^{A}_{B}R & & {}^{A}P_{BORG} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} $$

Homogeneous Coordinates in {A}

Homogeneous Transformation

Homogeneous Coordinates in {B}

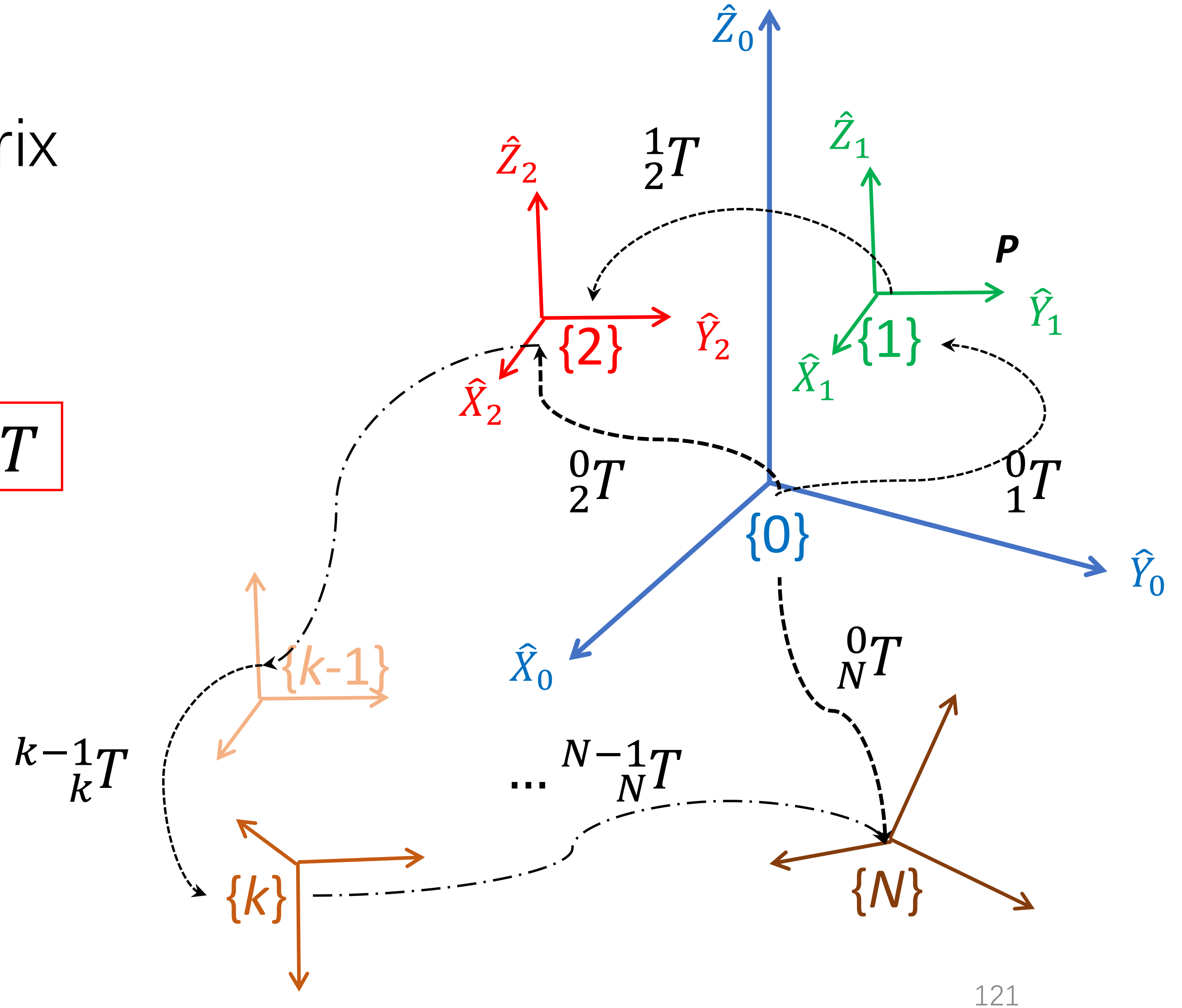
Can be seen as an multiplication operator of 4 x 4 matrix in 3D space

Recap on Week 01

- Composite transformation matrix

$$ {}^0_2T = {}^0_1T\,{}^1_2T $$

$$ \boxed{{}^0_NT = {}^0_{\boxed{1}}T\,{}^{\boxed{1}}_2T \;\ldots\; \underbrace{{}^{k-1}_{\boxed{k}}T\;\;{}^{\boxed{k}}_{k+1}T}_{{}^{k-1}_{k+1}T} \;\ldots\; {}^{N-1}_{N}T} $$



Recap on Week 01

- Inverse of a homogenous transformation, ${}^{A}T_{B}^{-1}$
- ${}^{A}T_{B}^{-1} = {}^{B}T_{A}$
  - Reversing the order of reference

- $${}^{B}\tilde{P} = \left[\begin{array}{ccc|c} & {}^{A}_{B}R^{\mathrm{T}} & & -{}^{A}_{B}R^{\mathrm{T}}\,{}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] {}^{A}\tilde{P}$$

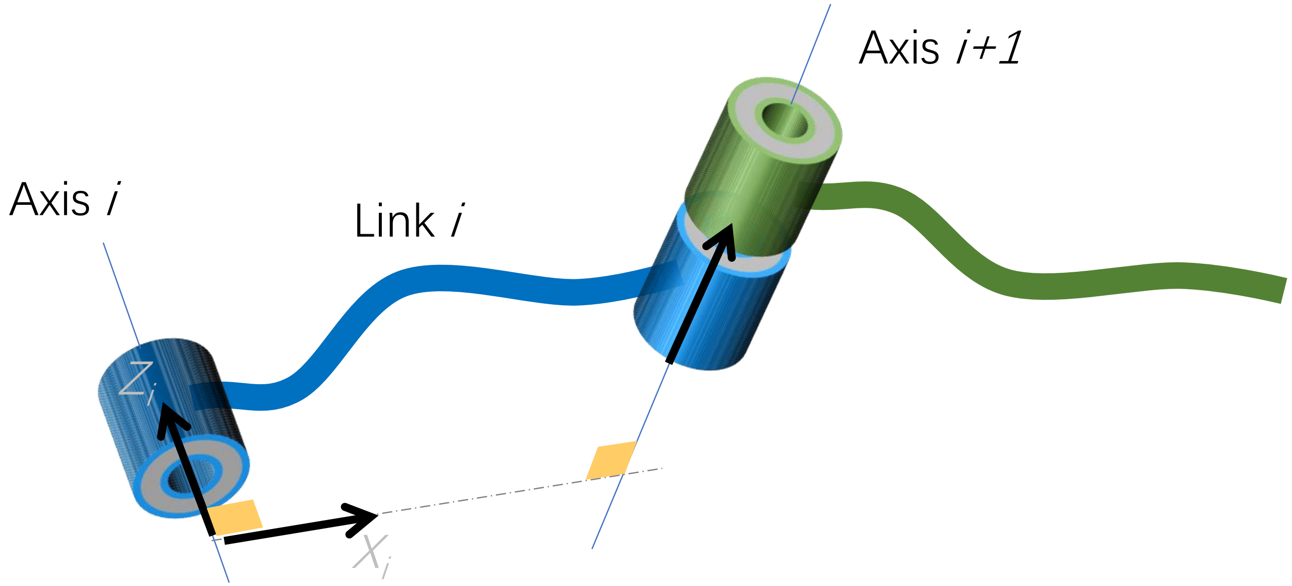
- $${}^{A}T_{B}^{-1} = {}^{B}T_{A} = \left[\begin{array}{ccc|c} & {}^{A}_{B}R^{\mathrm{T}} & & -{}^{A}_{B}R^{\mathrm{T}}\,{}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Recap on Week 01

- Kinematics: The science of motion (a branch of mechanics) without regards to the forces that cause it
  - Pose (/configuration) of the manipulator in static situations
  - Analyze (linear and angular) motion of bodies (/linkages)
- D-H Method: A systematic way to represent the configuration of the serial manipulator
  1. Frame assignment
  2. D-H parameters and tables
  3. Homogenous transformation matrix
- Forward Kinematics: mapping from joint coordinates, or robot configuration to end-effector pose
  - $^{0}_{\mathrm{E}}\mathrm{T} = {}^{0}_{1}\mathrm{T}(q_1) \cdot {}^{1}_{2}\mathrm{T}(q_2) \cdot {}^{2}_{3}\mathrm{T}(q_3) \cdot \cdots {}^{N-1}_{\mathrm{N}}\mathrm{T}(q_{\mathrm{N}}) \cdot {}^{N}_{\mathrm{E}}\mathrm{T}$

Recap: Summary of DH Frame Assignment

1. Identify the joint axes and attach infinite lines along them. For neighboring pair (*i* and *i*+1)
2. Identify the common perpendicular or point of intersection. At the point of intersection, or at the point where the common perpendicular meets i^{th} axis, assign the link-frame origin.
3. Assign the Z_i axis pointing along the i^{th} joint axis.
4. Assign the X_i axis pointing along the direction normal to the two neighboring Z-axes.
5. Assign the Y_i axis to complete a right-hand coordinate system.
6. Assign {0} to match {1}. For {N}, choose an origin location and X direction freely, but generally so as to cause as many linkage parameters as possible to become zero.



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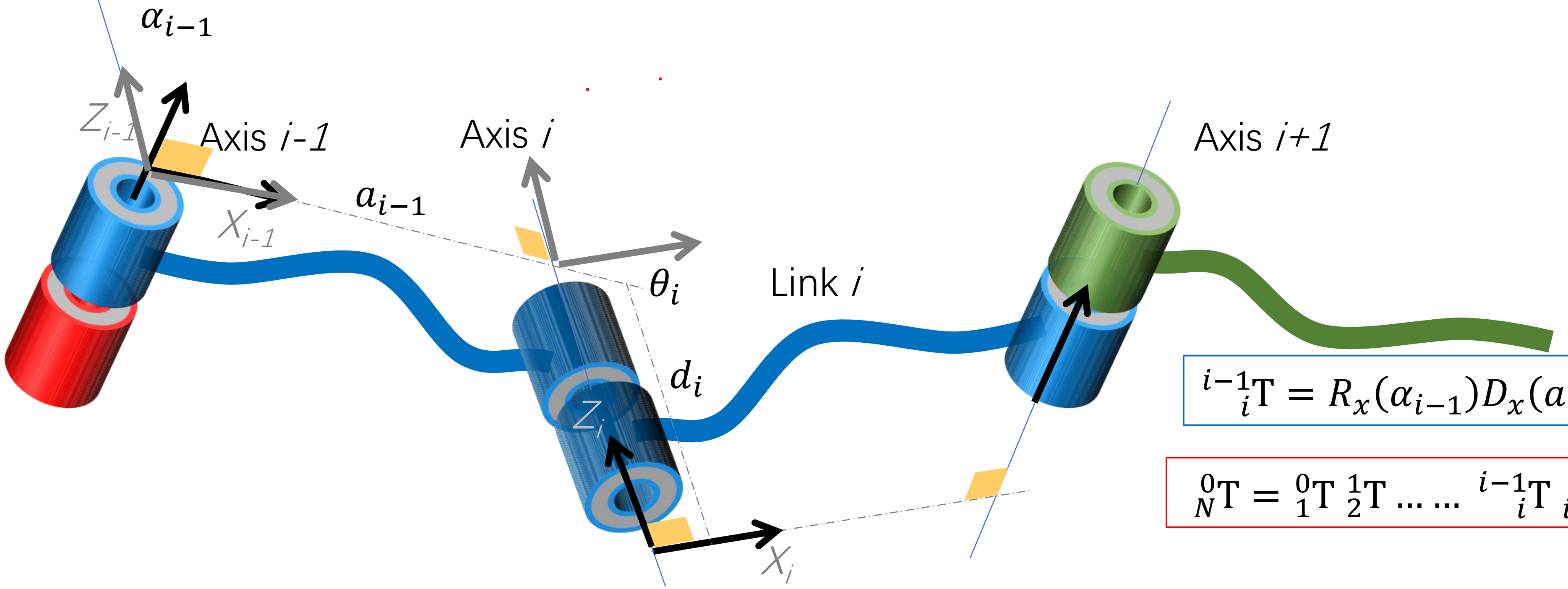


$$ {}^{i-1}_{\ \ i}T = R_x(\alpha_{i-1}) D_x(a_{i-1})\, R_z(\theta_i) D_z(d_i) $$

$$ {}^{0}_{N}T = {}^{0}_{1}T\, {}^{1}_{2}T \dots \dots {}^{i-1}_{\ \ i}T\, {}^{\ \ i}_{i+1}T \dots \dots {}^{N-2}_{N-1}T\, {}^{N-1}_{\ \ N}T $$

Recap: Q2.3: Example on an RRR Kinematics

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

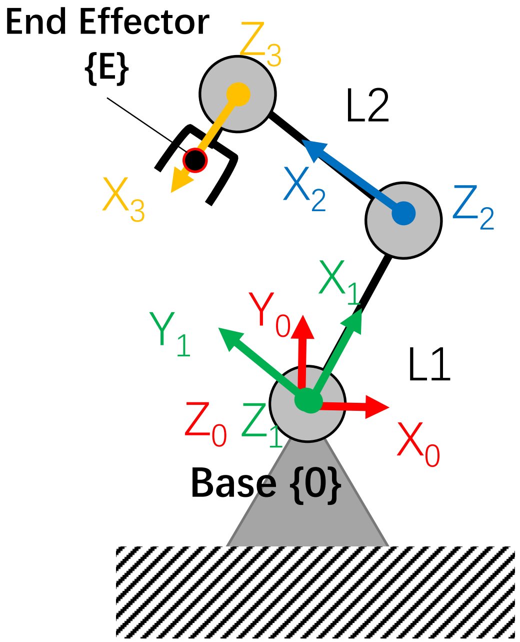
$$
{}^{3}_{E}T = \begin{bmatrix} 1 & 0 & 0 & L3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

5. Forward Kinematics

$$
{}^{0}_{E}\mathrm{T} = {}^{0}_{1}\mathrm{T}(\theta_1) \cdot {}^{1}_{2}\mathrm{T}(\theta_2) \cdot {}^{2}_{3}\mathrm{T}(\theta_3) \cdot {}^{3}_{E}\mathrm{T}
$$

$$
{}^{0}_{E}\widetilde{\mathrm{P}} = {}^{0}_{1}\mathrm{T}(\theta_1) \cdot {}^{1}_{2}\mathrm{T}(\theta_2) \cdot {}^{2}_{3}\mathrm{T}(\theta_3) \cdot {}^{3}_{E}\widetilde{\mathrm{P}}
$$

1. Schematic Diagram



2. Frame Assignment

3. DH Parameters & Table

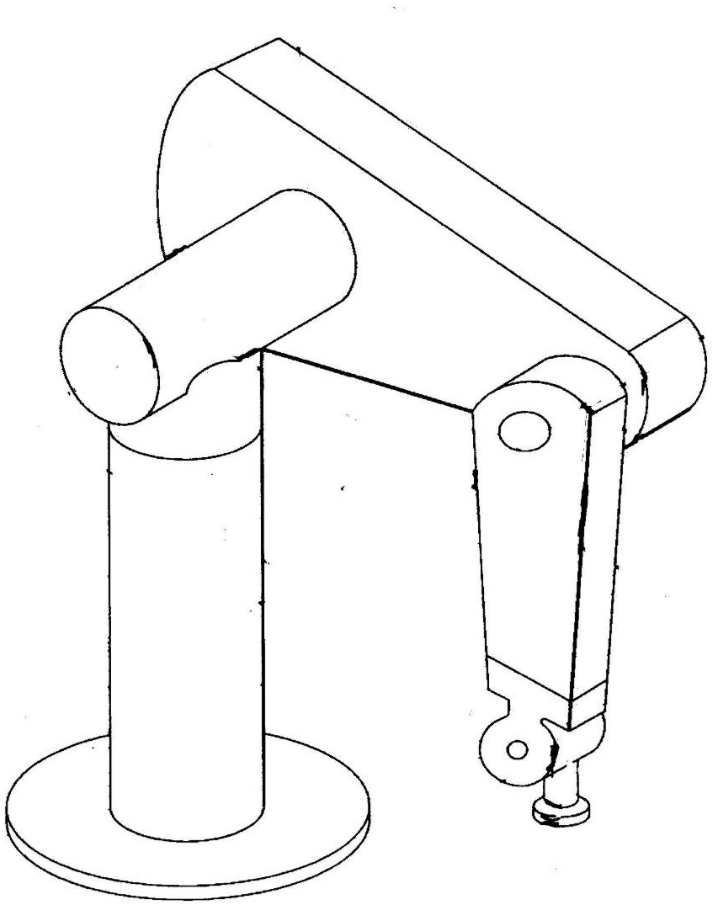
	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	$Q1 = \theta_1$	0
2	0	$L1$	$Q2 = \theta_2$	0
3	0	$L2$	$Q3 = \theta_3$	0

4. Homogenous Transformation

$$
{}^{i-1}_{i}\mathrm{T} = R_x(\alpha_{i-1})D_x(\alpha_{i-1})\, R_z(\theta_i)D_z(d_i)
$$

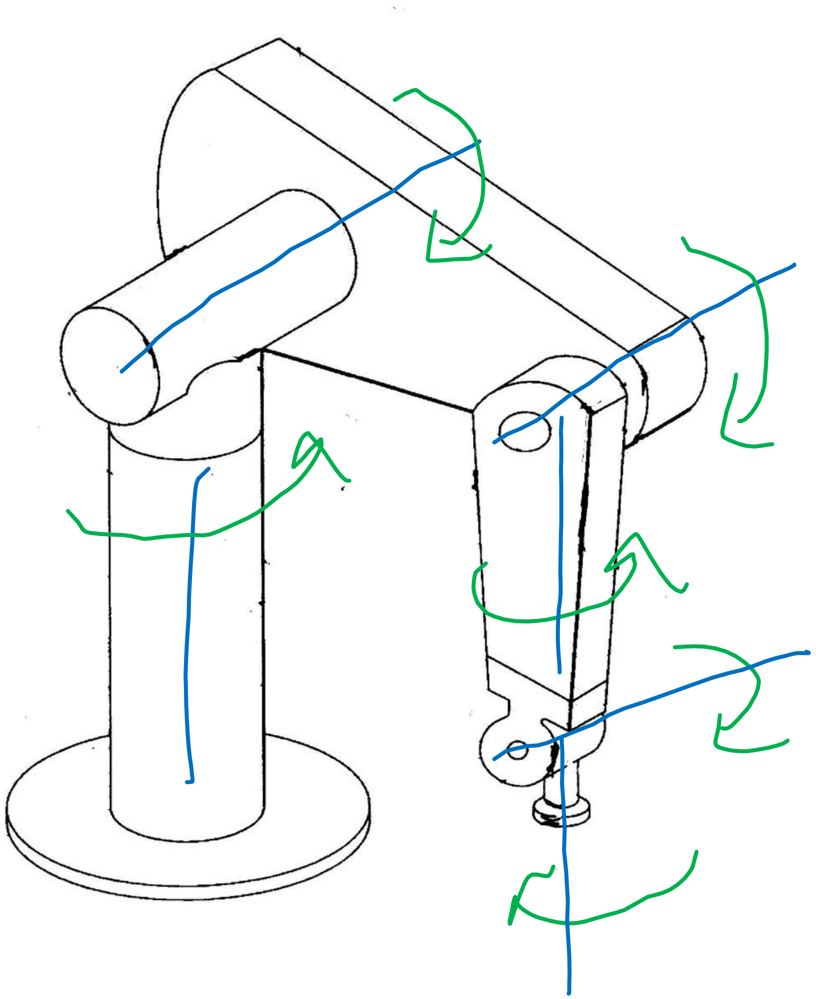
$$
{}^{0}_{3}\mathrm{T} = {}^{0}_{1}\mathrm{T}\, {}^{1}_{2}\mathrm{T}\, {}^{2}_{3}\mathrm{T}
$$

Q2.4: Example of Puma 560



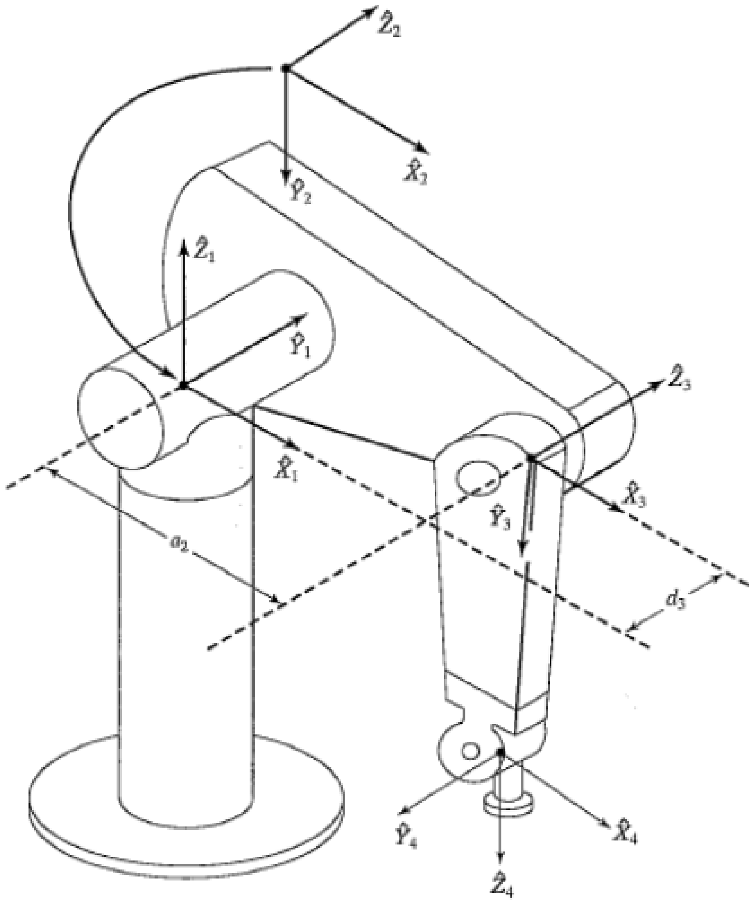
Q2.4: Example of Puma 560

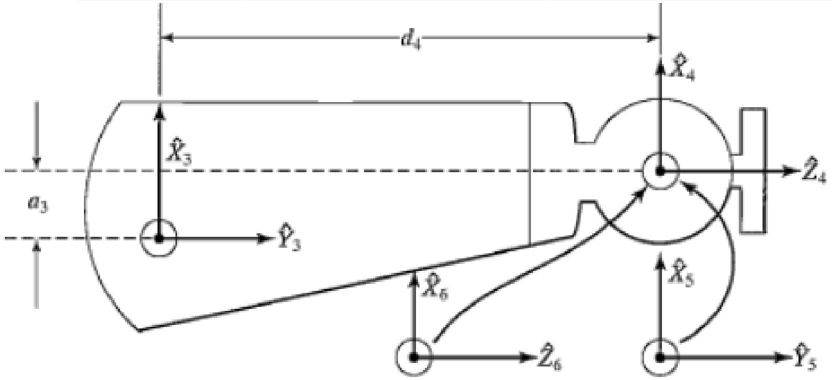
Try it



Q2.4: Example of Puma 560

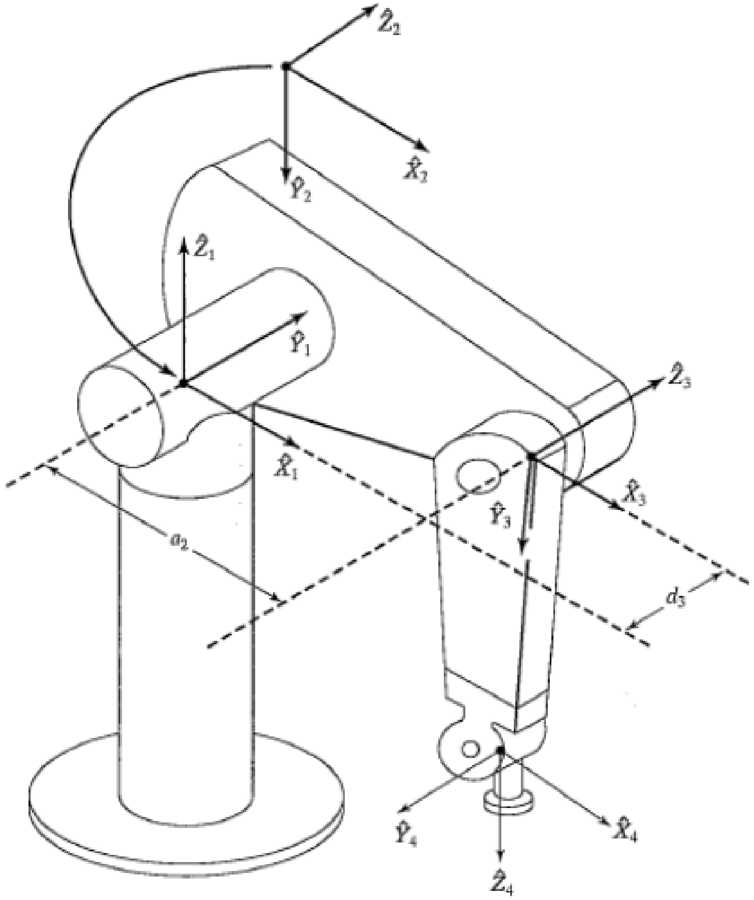
	α_{i-1}	a_{i-1}	θ_i	d_i
1				
2				
3				
4				
5				
6				

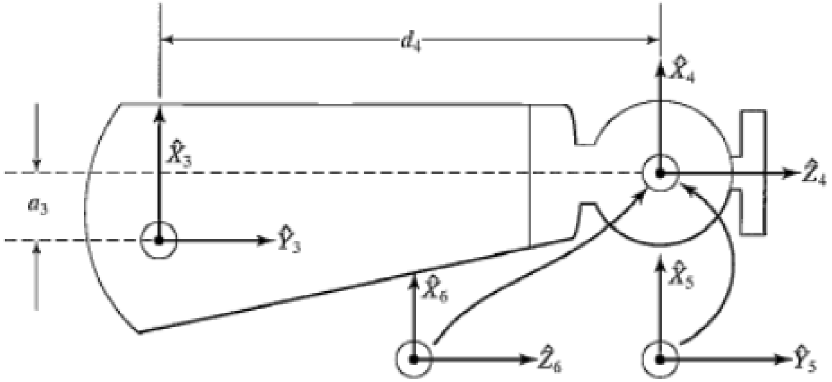




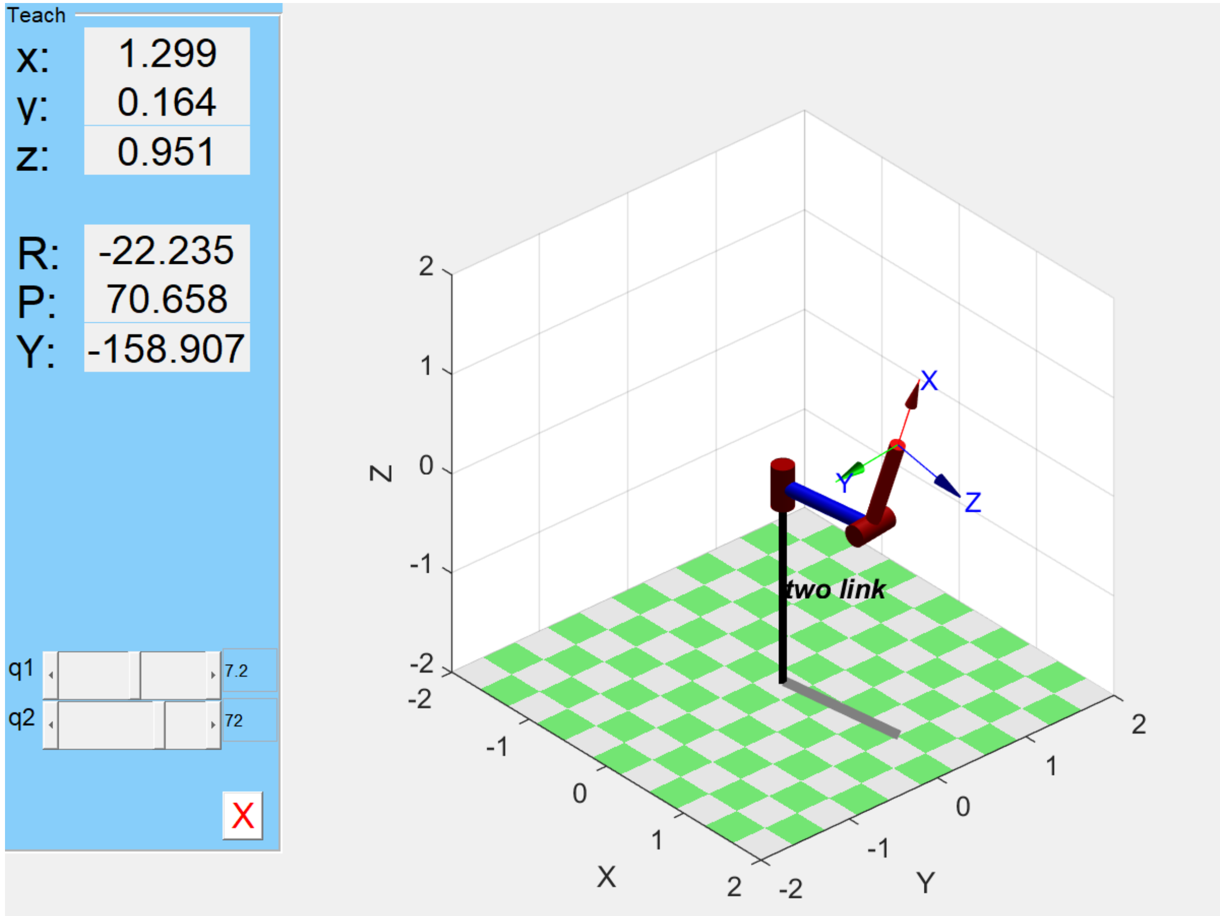
Q2.4: Example of Puma 560

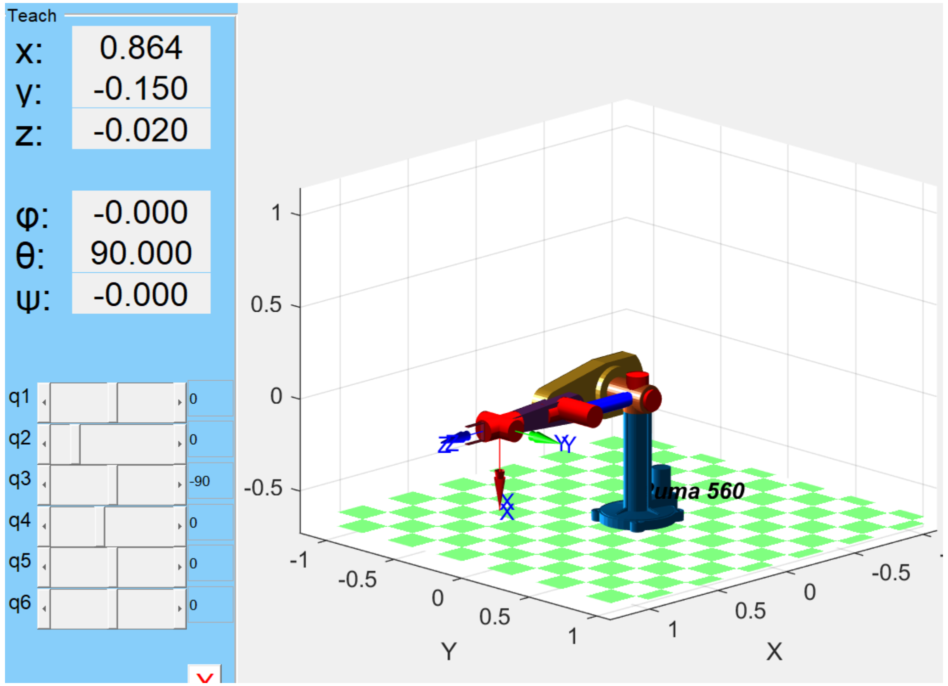
	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	$q_1 = 0$	0
2	$-\frac{\pi}{2}$	0	$q_2 = 0$	0
3	0	a_2	$q_3 = 0$	d_2
4	$-\frac{\pi}{2}$	a_3	$q_4 = 0$	d_3
5	$\frac{\pi}{2}$	0	$q_5 = 0$	0
6	$-\frac{\pi}{2}$	0	$q_6 = 0$	0





Demo on Matlab Robotics Toolbox

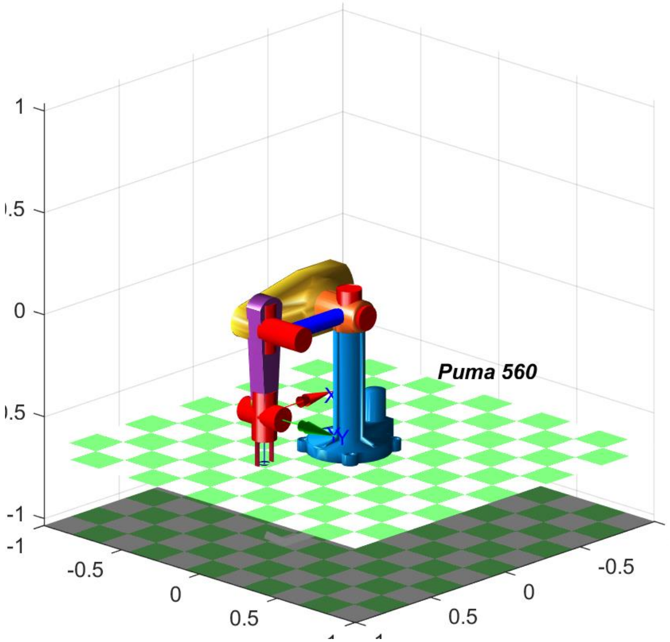




Q2.4: Example of Puma 560

$$^{i-1}_{i}\mathrm{T} = R_x(\alpha_{i-1})D_x(\alpha_{i-1})\ R_z(\theta_i)D_z(d_i)$$

- $^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



$$^0_\mathrm{E}\mathrm{T} = {}^0_1\mathrm{T}(\theta_1)\cdot {}^1_2\mathrm{T}(\theta_2)\cdot {}^2_3\mathrm{T}(\theta_3)\cdot {}^3_4\mathrm{T}(\theta_4)\cdot {}^4_5\mathrm{T}(\theta_5)\cdot {}^5_6\mathrm{T}(\theta_6)$$

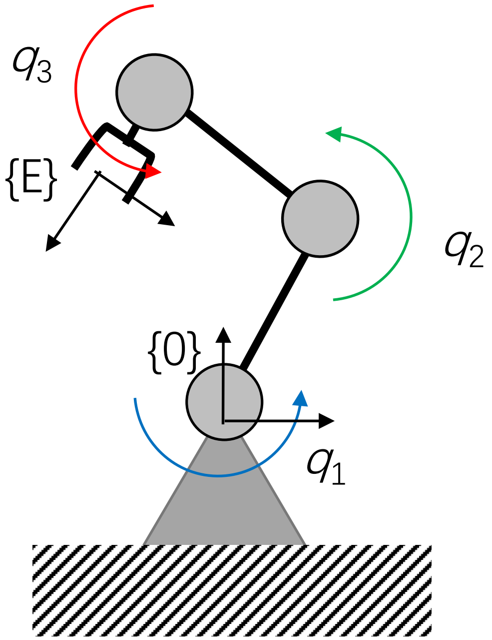
See Textbook Section 3.7, Craig 3rd Ed

Inverse Kinematics

Introduction to Robotics: Fundamentals

Inverse Kinematics

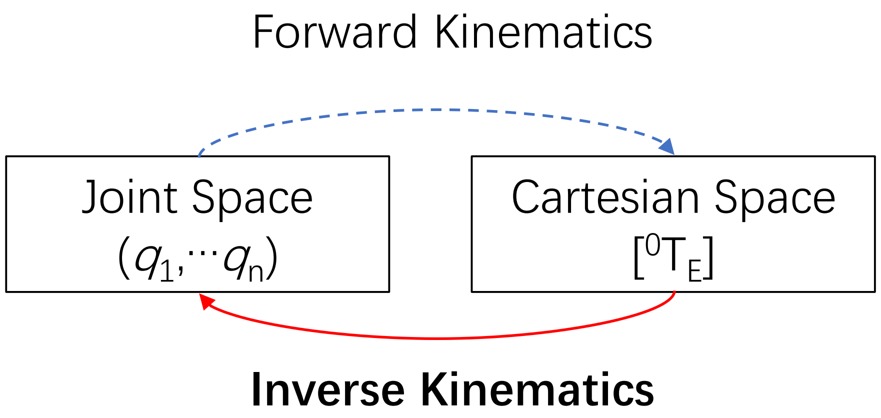
- **Inverse kinematics** is concerned with obtaining the joint coordinates for a desired end-effector pose



For a particular serial arm ${}^0T_E(\mathbf{Q})$,
solve the joint coordinates $\mathbf{Q} = (q_1, \cdots q_n)$
for desired pose ${}^0T_E$

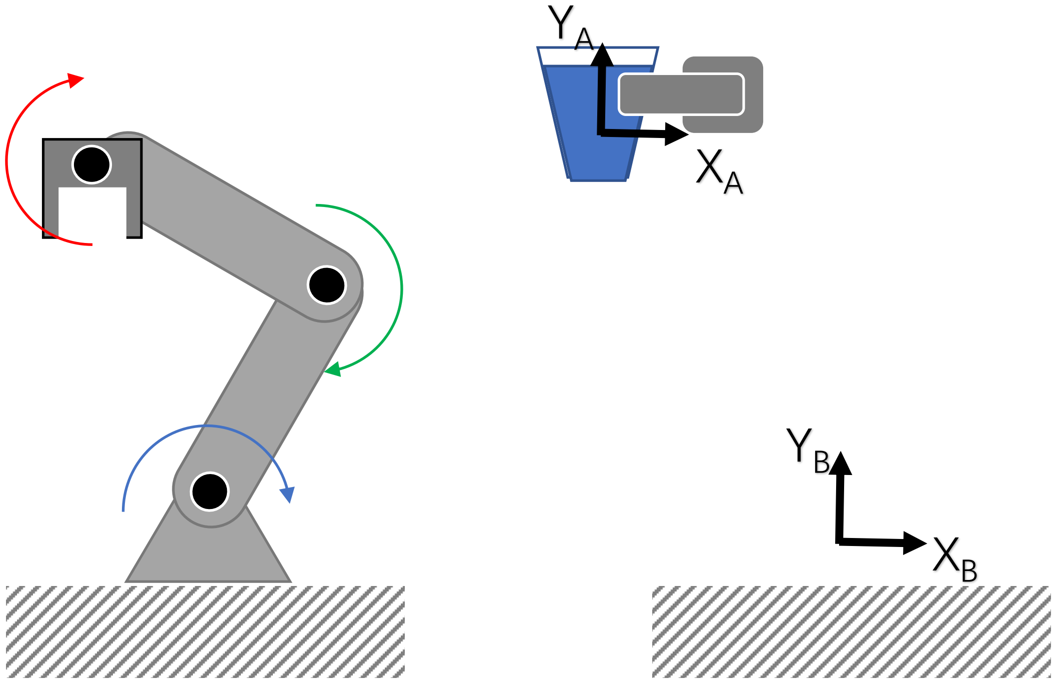
Mapping between Kinematics Description

- **Inverse kinematics** is concerned with obtaining the joint coordinates for a desired end-effector pose



Q3.1: RRR example for Inverse Kinematics

Task 1: Grab the cup at {A} → Required joint variables $\mathbf{Q}_A$?
Task 2: Place the cup on {B} → Required joint variables $\mathbf{Q}_B$?



Q3.1: RRR example for Inverse Kinematics

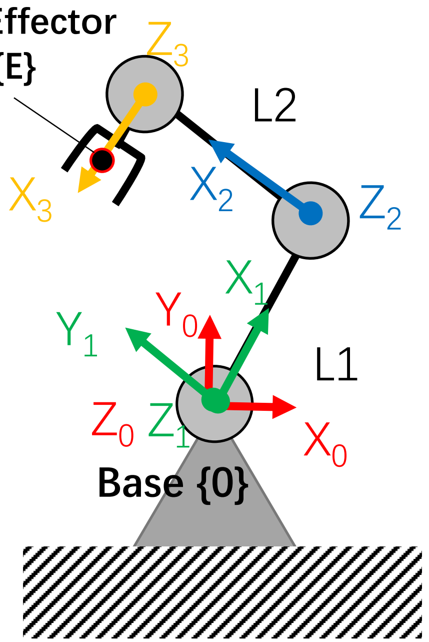
$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{\mathrm{E}}T = \begin{bmatrix} 1 & 0 & 0 & L3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

1. Schematic Diagram



2. Frame Assignment

3. DH Parameters & Table

	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	$Q1 = \theta_1$	0
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3	0	$L2$	$Q3 = \theta_3$	0

4. Homogenous Transformation

$$ {}^{i-1}_{i}\mathrm{T} = R_x(\alpha_{i-1})D_x(a_{i-1})\ R_z(\theta_i)D_z(d_i) $$

$$ {}^{0}_{3}\mathrm{T} = {}^{0}_{1}\mathrm{T}\ {}^{1}_{2}\mathrm{T}\,{}^{2}_{3}\mathrm{T} $$

5. Forward Kinematics

$$ {}^{0}_{\mathrm{E}}\mathrm{T} = {}^{0}_{1}\mathrm{T}(\theta_1) \cdot {}^{1}_{2}\mathrm{T}(\theta_2) \cdot {}^{2}_{3}\mathrm{T}(\theta_3) \cdot {}^{3}_{\mathrm{E}}\mathrm{T} $$

$$ {}^{0}_{\mathrm{E}}\widetilde{\mathrm{P}} = {}^{0}_{1}\mathrm{T}(\theta_1) \cdot {}^{1}_{2}\mathrm{T}(\theta_2) \cdot {}^{2}_{3}\mathrm{T}(\theta_3) \cdot {}^{3}_{\mathrm{E}}\widetilde{\mathrm{P}} $$

Q3.1: RRR example for Inverse Kinematics

Forward Kinematics

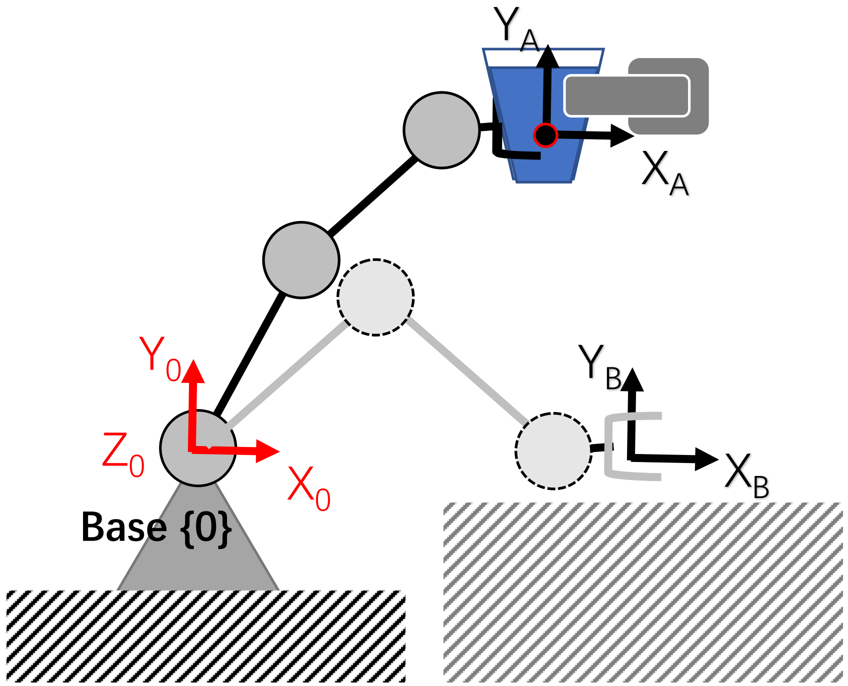
$$ {}^{0}_{E}T = {}^{0}_{1}T(\theta_1) \cdot {}^{1}_{2}T(\theta_2) \cdot {}^{2}_{3}T(\theta_3) \cdot {}^{3}_{E}T $$

$$ {}^{0}_{E}\widetilde{P} = {}^{0}_{1}T(\theta_1) \cdot {}^{1}_{2}T(\theta_2) \cdot {}^{2}_{3}T(\theta_3) \cdot {}^{3}_{E}\widetilde{P} $$

Inverse Kinematics

a) Solve $\theta_1\, \theta_2\, \theta_3$, such that ${}^{0}_{E}T = {}^{0}_{A}T$

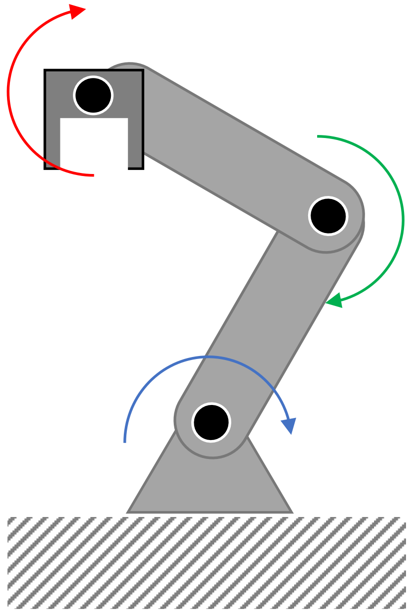
b) Solve $\theta_1\, \theta_2\, \theta_3$, such that ${}^{0}_{E}T = {}^{0}_{B}T$



Solvability

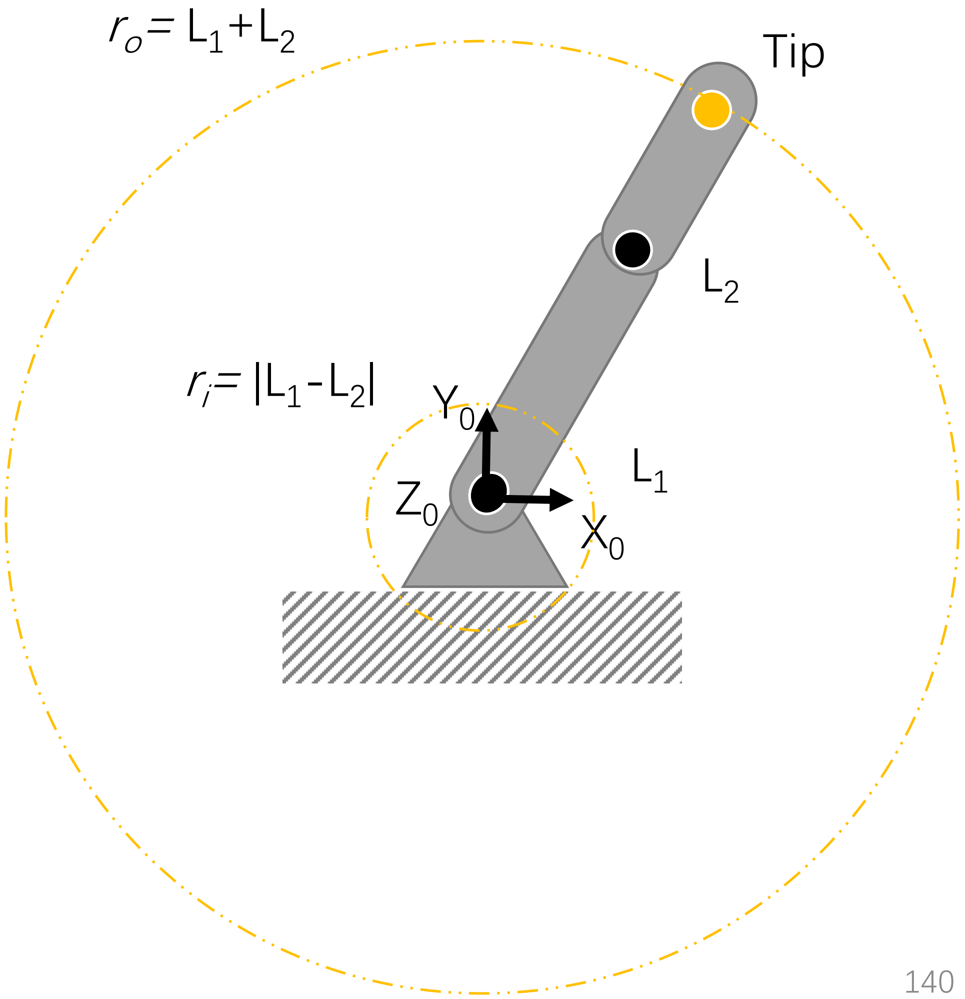
- 3 Joint Angles
- 3 Unknowns (2 Position, 1 Orientation)

- Solutions can be obtained numerically or analytically
    - Numerical solution
    - Closed-form solution



Existence of Solution

- Workspace
  - **Reachable**: Region where the end-effector can be located
  - **Dexterous**: Region where the end-effector can be located with all orientations



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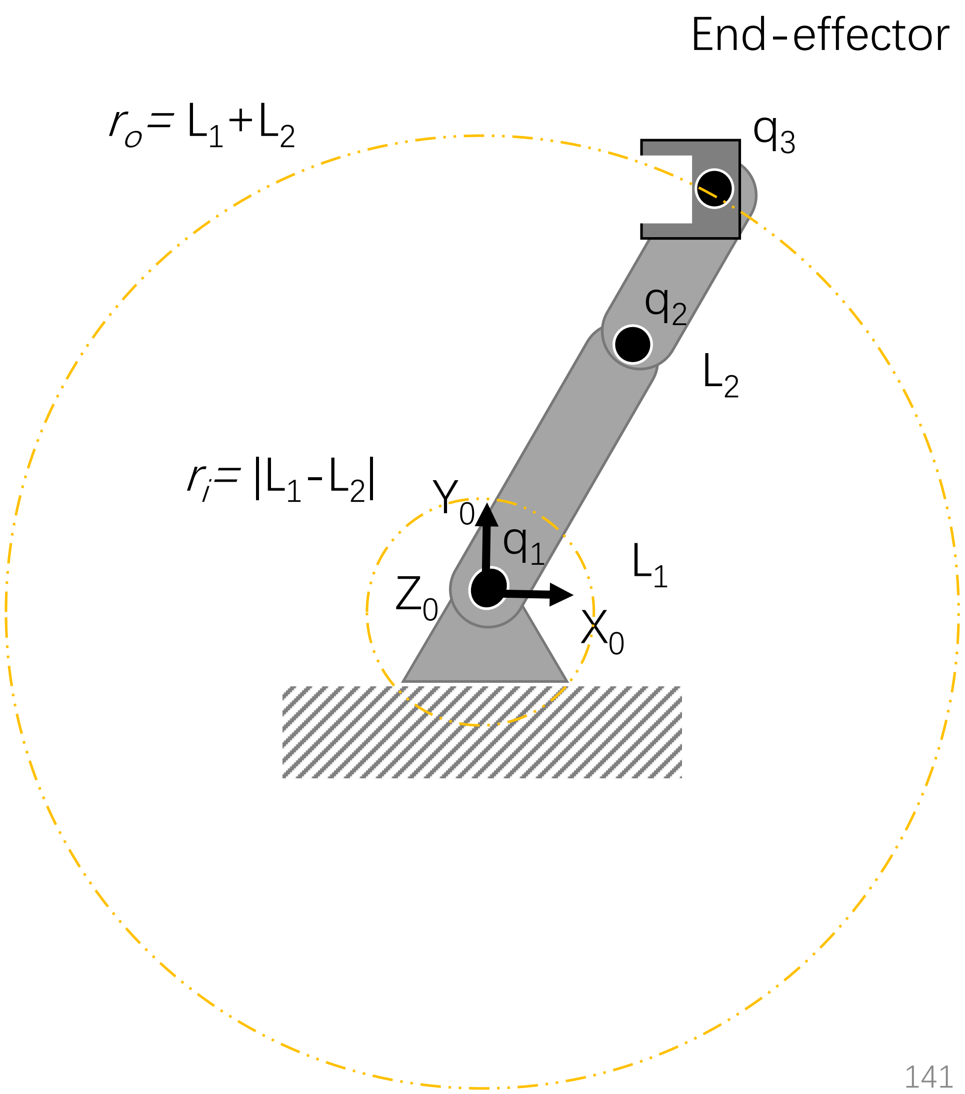

Existence of Solution

- Workspace
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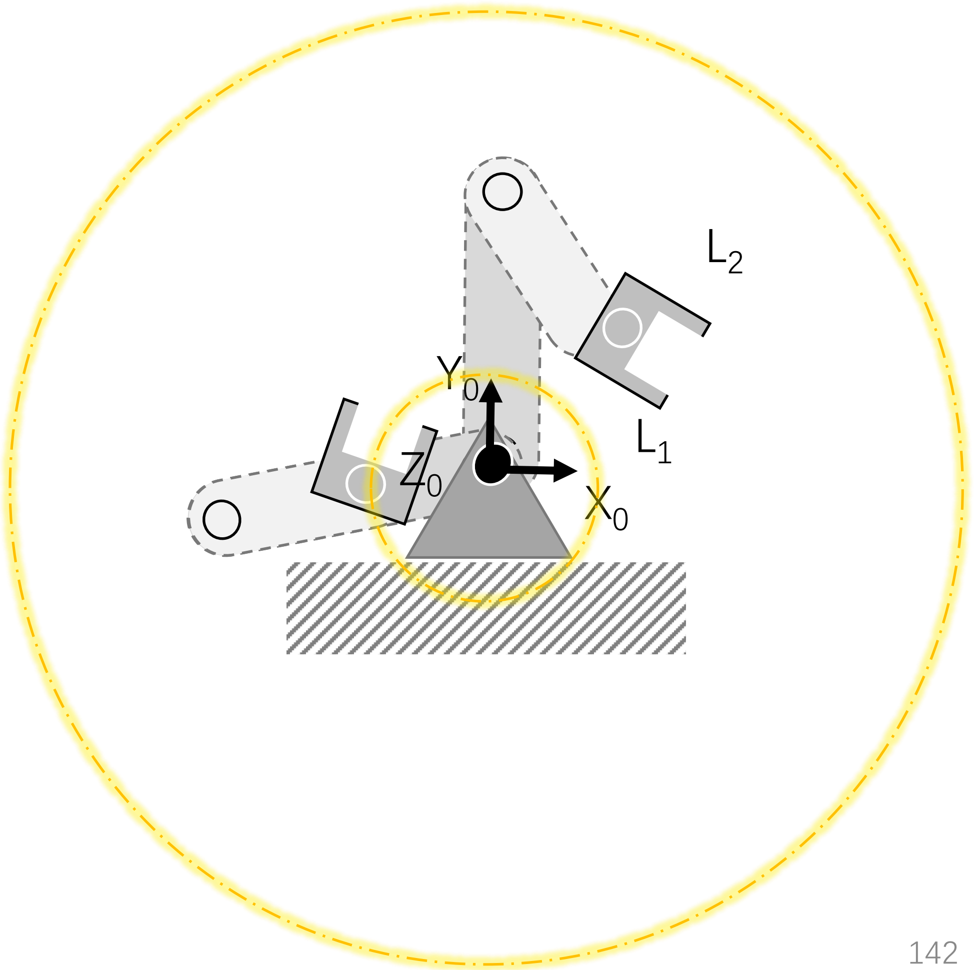
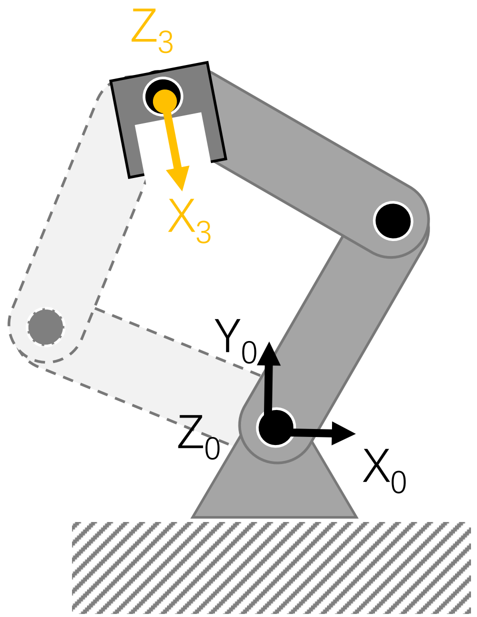
  - **Dexterous**: Region where the end-effector can be located with all orientations



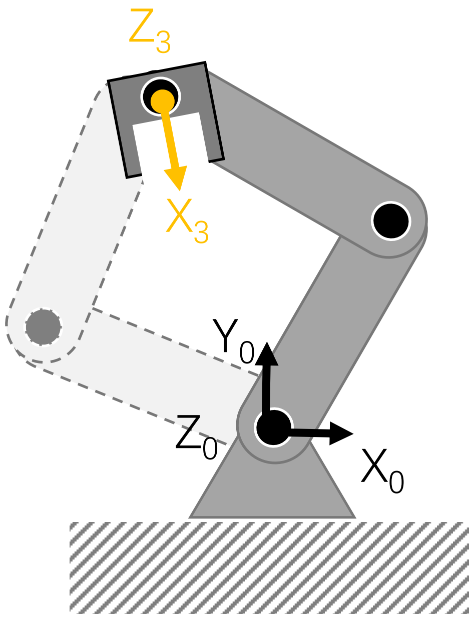
Multiple Solutions

- For the same end-effector pose, there could be 2 possible solutions



Q 3.2 Concept Check

- When will the solution be unique?



Q3.1: RRR example for Inverse Kinematics

Forward Kinematics

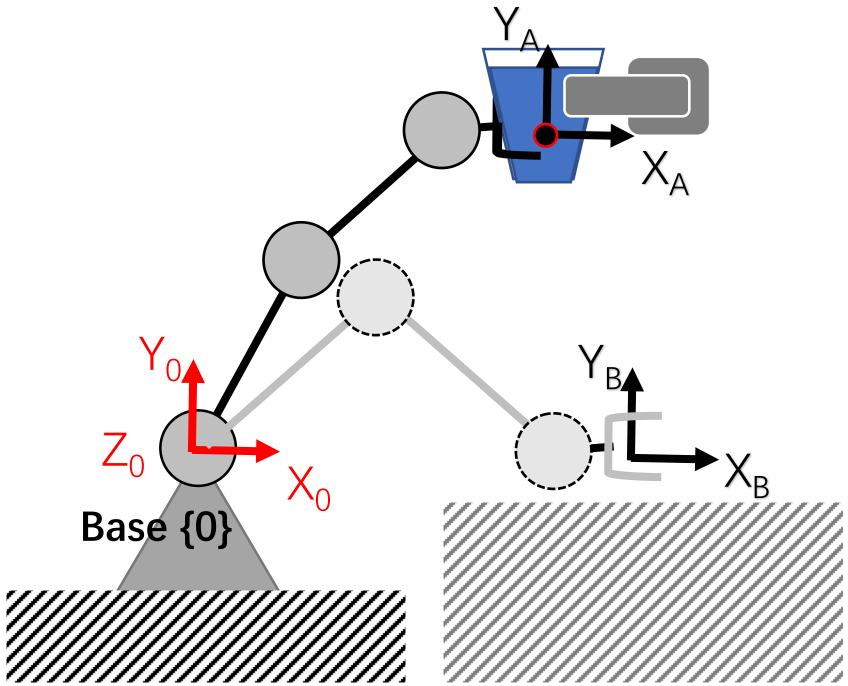
$$ {}^{0}_{E}T = {}^{0}_{1}T(\theta_1) \cdot {}^{1}_{2}T(\theta_2) \cdot {}^{2}_{3}T(\theta_3) \cdot {}^{3}_{E}T $$

$$ {}^{0}_{E}\widetilde{P} = {}^{0}_{1}T(\theta_1) \cdot {}^{1}_{2}T(\theta_2) \cdot {}^{2}_{3}T(\theta_3) \cdot {}^{3}_{E}\widetilde{P} $$

Inverse Kinematics

a) Solve $\theta_1\,\theta_2\,\theta_3$, such that ${}^{0}_{E}T = {}^{0}_{A}T$

b) Solve $\theta_1\,\theta_2\,\theta_3$, such that ${}^{0}_{E}T = {}^{0}_{B}T$



Q3.1: RRR example for Inverse Kinematics

Inverse Kinematics

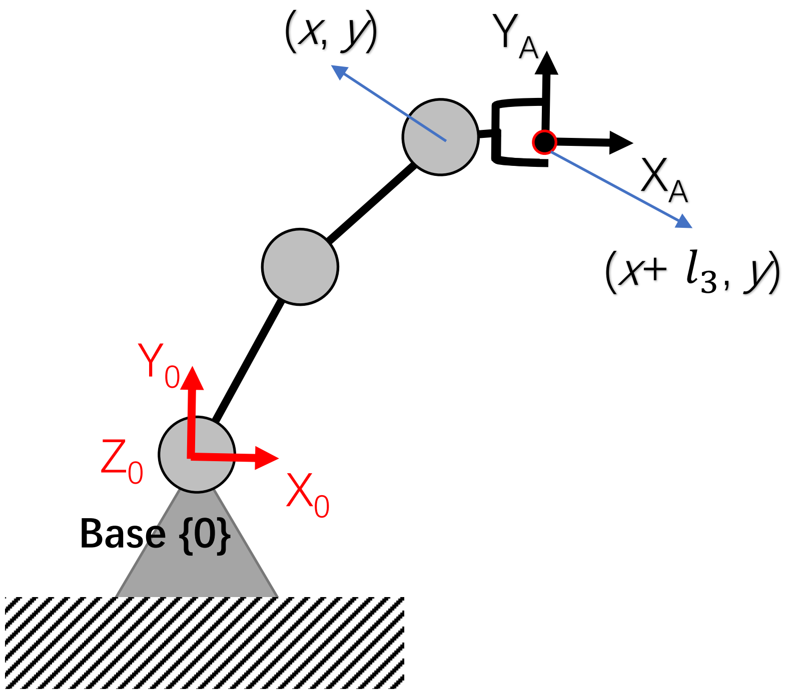
a) Solve $\theta_1\,\theta_2\,\theta_3$, such that ${}^0_E\mathrm{T} = {}^0_A\mathrm{T}$

a) Solve $\theta_1, \theta_2, \theta_3$, such that:

$$ {}^0_3\mathrm{T} = \begin{bmatrix} c_{123} & -s_{123} & 0 & \mathrm{L1}c_1 + \mathrm{L2}c_{12} \\ s_{123} & c_{123} & 0 & \mathrm{L1}s_1 + \mathrm{L2}s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Generally

$$ \begin{bmatrix} c_{123} & -s_{123} & 0 & l_1c_1 + l_2c_{12} \\ s_{123} & c_{123} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 & x \\ \sin(\varphi) & \cos(\varphi) & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



See textbook Section 4.4

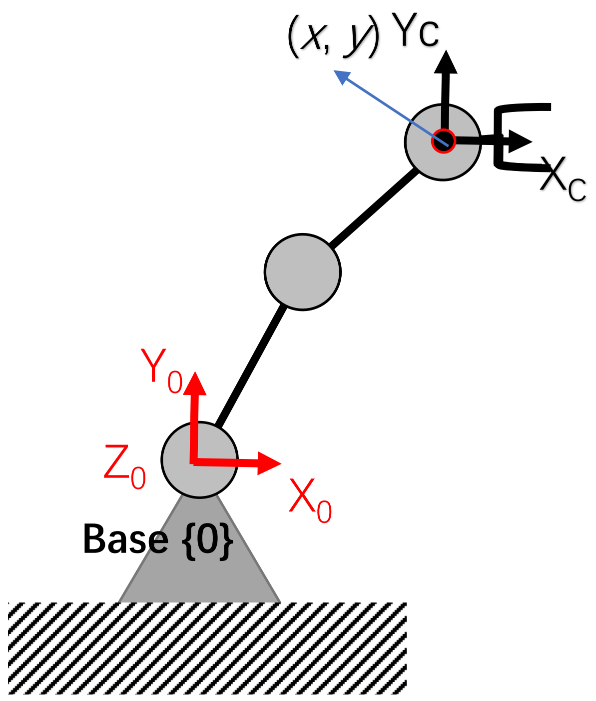
Q3.1: RRR example for Inverse Kinematics

Inverse Kinematics

a) Solve $\theta_1\, \theta_2\, \theta_3$, such that ${}^0_E\mathrm{T} = {}^0_A\mathrm{T}$

Generally

$$\begin{bmatrix} c_{123} & -s_{123} & 0 & l_1c_1+l_2c_{12} \\ s_{123} & c_{123} & 0 & l_1s_1+l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 & x \\ \sin(\varphi) & \cos(\varphi) & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Three unknowns:

Four nonlinear equations:

$$\cos(\varphi) = c_{123} \qquad (1)$$
$$\sin(\varphi) = s_{123} \qquad (2)$$
$$x = l_1c_1 + l_2c_{12} \qquad (3)$$
$$y = l_1s_1 + l_2s_{12} \qquad (4)$$

See textbook Section 4.4

Q3.1: RRR example for Inverse Kinematics

Inverse Kinematics

Algebraic Approach

Three unknowns:
Four nonlinear equations:

$\cos(\varphi) = c_{123}$ (1)

$\sin(\varphi) = s_{123}$ (2)

$x = l_1 c_1 + l_2 c_{12}$ (3)

$y = l_1 s_1 + l_2 s_{12}$ (4)

$C_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$

(3) & (4): $x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 c_2$

$^* c_{12} = c_1 c_2 - s_1 s_2$
$s_{12} = s_1 c_2 + c_1 s_2$

$c_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}$

- Right hand side must be between -1 and 1, else out of workspace
- θ_2 is solved

(3): $x = l_1 c_1 + l_2 c_{12} = l_1 c_1 + l_2 c_1 c_2 - l_2 s_1 s_2$

$(l_1 + l_2 c_2) c_1 - l_2 s_2 s_1 = R \sin(\theta_1 + \gamma)$ ***a* sin *ϑ* ± *b* cos *ϑ* ≡*R* sin(*ϑ* ± *α*)**

Where $R = \sqrt{(l_1 + l_2 c_2)^2 + l_2^2 s_2^2}$ and $\gamma = -\tan^{-1} \frac{(l_1 + l_2 c_2)}{l_2 s_2}$

- θ_1 is solved

From (1) or (2), $\theta_1 + \theta_2 + \theta_3 = \phi$

- θ_3 is solved

See textbook Section 4.4

Q3.1: RRR example for Inverse Kinematics

Inverse Kinematics

Geometrical Approach?

See textbook Section 4.4

Velocity Kinematics and Static Forces

Introduction to Robotics: Fundamentals

Velocity and Jacobian

Introduction to Robotics: Fundamentals

Revisit Orientation/Rotation

- Representation:

1. Euler Angle
2. Rotation Matrix
3. Rotation Vector
4. Unit Quaternion

Revisit Orientation/Rotation

- Representation:

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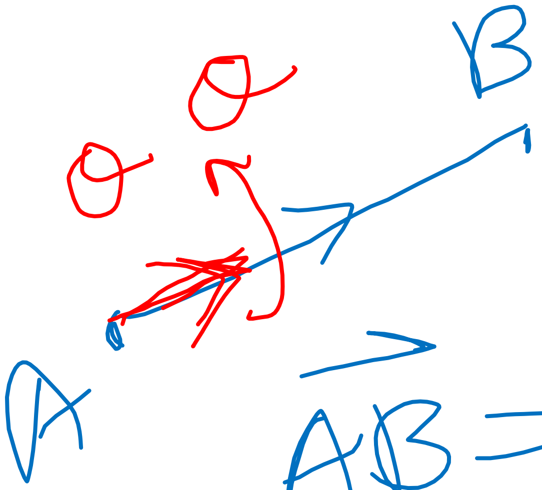
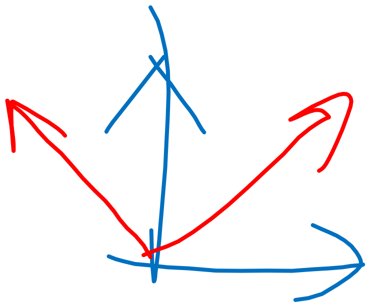


$$\vec{AB} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

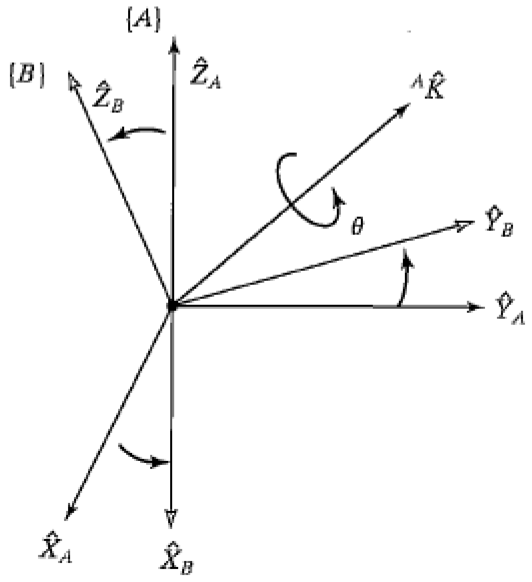


$$k\,\hat{AB} = \vec{AB}$$

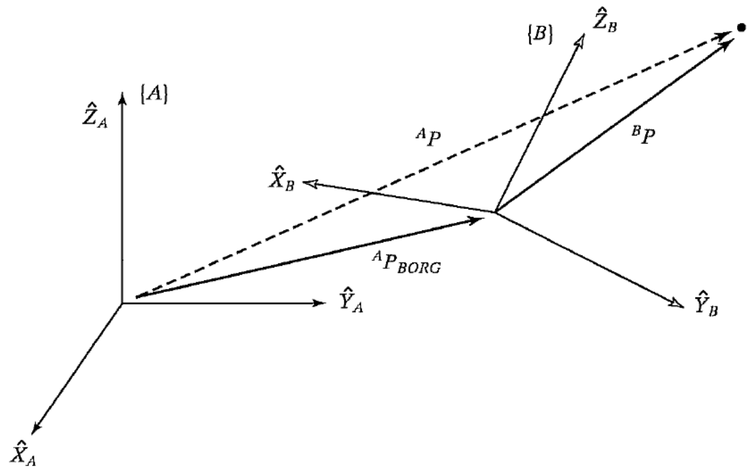
$$\theta\,\hat{k} = \boxed{K}$$

Rotation

- Any rotation can be expressed as:
    1. Rotation of angle θ
    2. About some rotation axis $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$



Review: Position



$$^{A}P = {}^{A}P_{B,ORG} + {}^{A}_{B}R \, . \, {}^{B}P$$

Concept Check

- It is given that the point is stationary in frame {B}. Can we conclude that the velocity of the point is zero in frame {A}?