ECE 470: Introduction to Robotics
Lecture 15

Liangjing Yang
Assistant Professor, ZJU-UIUC Institute
liangjingyang@intl.zju.edu.cn

Wechat ID: Liangjing_Yang

Schedule Check

- **Lecture**

O. Overview
- Science & Engineering in Robotics

I. Spatial Representation & Transformation
- Coordinate Systems; Pose Representations; Homogeneous Transformations

II. Kinematics
- Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/Inverse Kinematics

III. Velocity Kinematics and Static Forces
- Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity

Fundamentals

Week 1-4

Revision/ Quiz on Week 5

IV. Dynamics
- Acceleration of Body; Newton-Euler Approach; Lagrangian Formulation

Week 8

V. Control
- Feedback Control, Independent Joint Control, Force Control

VI. Planning
- Joint-based Motion Planning; Cartesian-based Path Planning

Essentials

Week 6-9

Revision/ Quiz on Week 10

VII. Robot Vision (and Perception)
- Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics

Applied

Week 11-13

Revision/Reading Wk/ Exam on Week 14-16

Summary Recap of Robot Dynamics

- **Dynamics**: Concern with the forces (/torque) on bodies that cause motion
- **Representation**: Equation of Motion/ Dynamic Equation
- **Approaches**: (1) Newton-Euler & (2) Lagrangian Formulation

Recap N-E Method: Iteration through Links

Outwards Iteration

$$^{i+1}\omega^{0}_{i+1} = {}^{i+1}_{i}R\ {}^{i}\omega^{0}_{i} + \dot\theta_{i+1}\,{}^{i+1}\hat Z_{i+1}$$

$$^{i+1}\dot\omega^{0}_{i+1} = {}^{i+1}_{i}R\dot\omega + {}^{i+1}_{i}R\ {}^{i}\omega^{0}_{i} \times \dot\theta_{i+1}\,{}^{i+1}\hat Z_{i+1} + \ddot\theta_{i+1}\,{}^{i+1}\hat Z_{i+1}$$

$$^{i+1}v^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}v^{0}_{i} + {}^{i}\omega^{0}_{i} \times {}^{i}P_{i+1}\right)$$

$$^{i+1}\dot v^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot\omega^{0}_{i} \times {}^{i}P_{i+1} + {}^{i}\omega^{0}_{i} \times \left({}^{i}\omega^{0}_{i} \times {}^{i}P_{i+1}\right) + {}^{i}\dot v^{0}_{i}\right) + 2\,{}^{i+1}\omega^{0}_{i+1} \times \dot d_{i+1}\,{}^{i+1}\hat Z_{i+1} + \ddot d_{i+1}\,{}^{i+1}\hat Z_{i+1}$$

$$^{i+1}\dot v^{0}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot\omega^{0}_{i} \times {}^{i}P_{i+1} + {}^{i}\omega^{0}_{i} \times \left({}^{i}\omega^{0}_{i} \times {}^{i}P_{i+1}\right) + {}^{i}\dot v^{0}_{i}\right)$$

Newton Equation $\quad {}^{i+1}F_{i+1} = m_{i+1}\,{}^{i+1}\dot v_{Ci+1}$

Euler Equation $\quad {}^{i+1}N_{i+1} = {}^{Ci+1}I_{i+1}\,{}^{i+1}\dot\omega_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{Ci+1}I_{i+1}\,{}^{i+1}\omega_{i+1}$

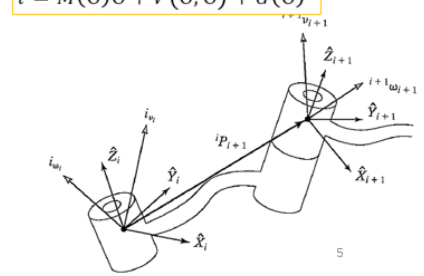
Inwards Iteration

$$^{i}f_{i} = {}_{i+1}^{i}R\ {}^{i+1}f_{i+1} + {}^{i}F_{i}$$

$$^{i}n_{i} = {}^{i}N_{i} + {}_{i+1}^{i}R\ {}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_{i} + {}^{i}P_{i+1} \times \left({}_{i+1}^{i}R\ {}^{i+1}f_{i+1}\right)$$

$$\tau_i = {}^{i}n_i^{T}\ {}^{i}\hat Z_i$$

$$\boxed{\tau = M(\Theta)\ddot\Theta + V(\Theta,\dot\Theta) + G(\Theta)}$$



5

Recap: Velocity & Acceleration

$$\vec V_1 = \vec V_{B,ORG} + \dot x\,\hat\imath_B + \dot y\,\hat\jmath_B + \dot z\,\hat k_B + \vec\omega \times \vec P_{1/B}$$

Differentiate w.r.t. time

$$\dot{\vec V}_1 = \dot{\vec V}_{B,ORG} + \ddot x\,\hat\imath_B + \ddot y\,\hat\jmath_B + \ddot z\,\hat k_B + \vec\omega \times \vec V_{1/B} + \vec\omega \times (\vec V_{1/B} + \vec\omega \times \vec P_{1/B}) + \dot{\vec\omega} \times \vec P_{1/B}$$

$$\dot{\vec V}_1 = \dot{\vec V}_{B,ORG} + \ddot x\,\hat\imath_B + \ddot y\,\hat\jmath_B + \ddot z\,\hat k_B + \underbrace{2\vec\omega \times \vec V_{1/B}}_{\text{coriolis acceleration}} + \underbrace{\vec\omega \times (\vec\omega \times \vec P_{1/B})}_{\text{centrifugal acceleration}} + \underbrace{\dot{\vec\omega} \times \vec P_{1/B}}_{\text{tangential acceleration}}$$

Recap: Robot Dynamics

- **Dynamics**: Concern with the forces (/torque) on bodies that cause motion

Dynamic equation:
- $\tau = M(\Theta)\ddot\Theta + V(\Theta,\dot\Theta) + G(\Theta)$
- $M(\Theta)$ is n x n mass matrix of the manipulator
- $V(\Theta,\dot\Theta)$ is an n x 1 vector of centrifugal and Coriolis terms
- $G(\Theta)$ is an n x 1 vector of gravity terms

Cartesian Space: $\quad \mathcal{F} = M_x(\Theta)\ddot{\mathcal{X}} + V_x(\Theta,\dot\Theta) + G_x(\Theta),$

4

Recap: Lagrangian Approach

Method (2) Lagrangian: Energy-based approach

Potential energy of ith link:

$$u_i = -m_i\,{}^{0}g^{T}\,{}^{0}P_{C_i} + u_{ref}$$

- ^{0}g is 3 x 1 gravity vector
- $^{0}P_{C_i}$ is the vector locating the center of mass of ith link
- u_{ref} is the reference

Total potential energy is: $u = \sum_{i=1}^{n} u_i$

$^{0}P_{C_i}$ is a function of Θ, $\sum_{i=1}^{n} u_i = u(\Theta)$

Kinetic energy of the ith link is:
- $k_i = \frac{1}{2} m_i v_{C_i}^T v_{C_i} + \frac{1}{2}\,{}^{i}\omega_i^T\,{}^{C_i}I_i\,{}^{i}\omega_i$
- Must be positive

Kinetic energy of the manipulator is: $k = \sum_{i=1}^{n} k_i$

v_{C_i} and $^{i}\omega_i$ are functions of Θ and $\dot\Theta$

$k(\Theta,\dot\Theta) = \frac{1}{2}\dot\Theta^T M(\Theta)\,\dot\Theta$

- Lagrangian dynamic formulation provides a means of deriving the equations of motion from a scalar function called the Lagrangian
- Lagrangian is defined as the difference between the kinetic and potential energy of a mechanical system
- $L(\Theta,\dot\Theta) \equiv k(\Theta,\dot\Theta) - u(\Theta)$
- Equations of motion are then given by:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot\Theta} - \frac{\partial L}{\partial\Theta} = \tau$$

$$\frac{d}{dt}\frac{\partial k}{\partial\dot\Theta} - \frac{\partial k}{\partial\Theta} + \frac{\partial u}{\partial\Theta} = \tau$$

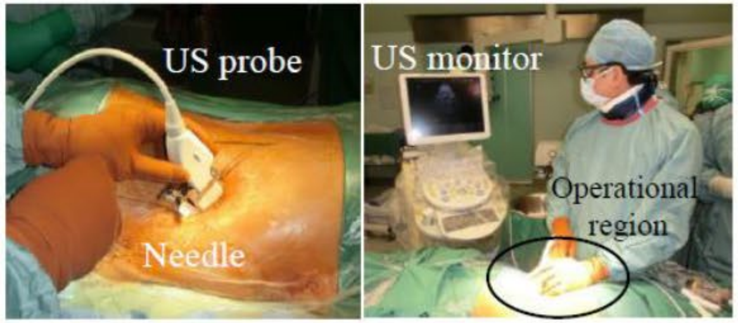
$$\boxed{\tau = M(\Theta)\ddot\Theta + V(\Theta,\dot\Theta) + G(\Theta)}$$

6

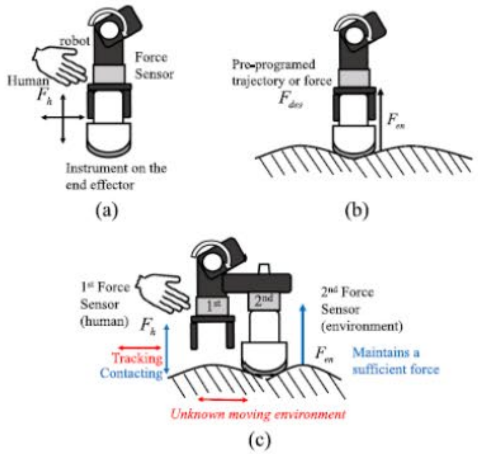
Robot Control

ECE 470: Introduction to Robotics

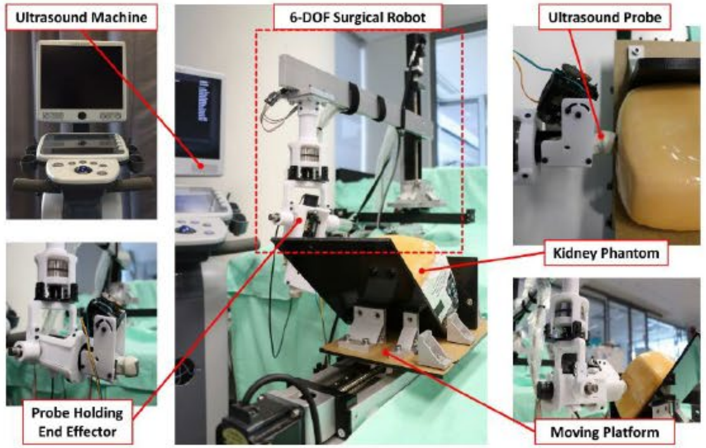
Achieving Goals in Dynamic Systems



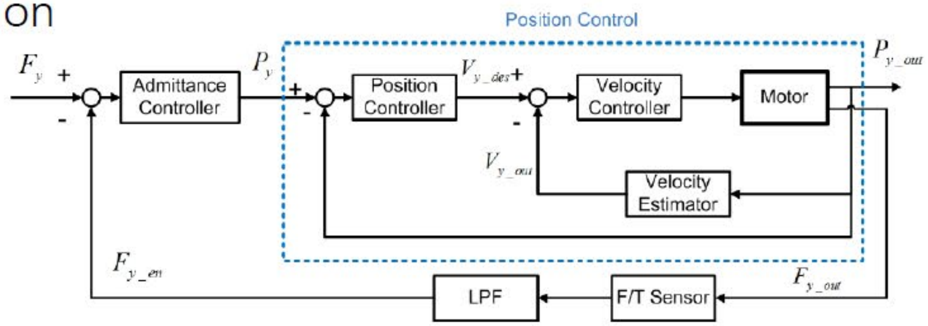
Understanding the physical problem



Modeling the systems



Design the solution



Control: Robotic Application

In robotics, we need control to perform motion

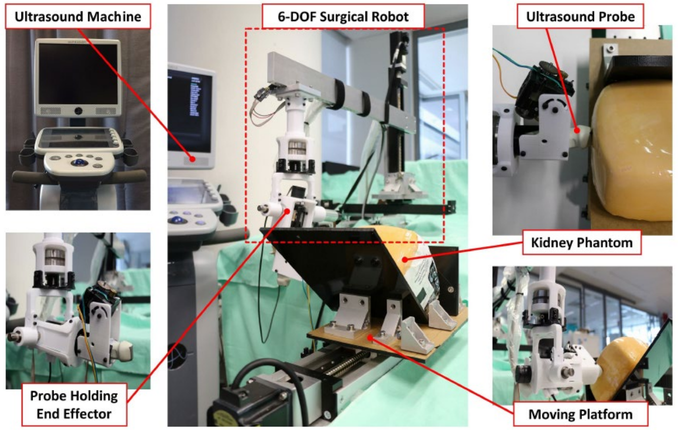
Drones

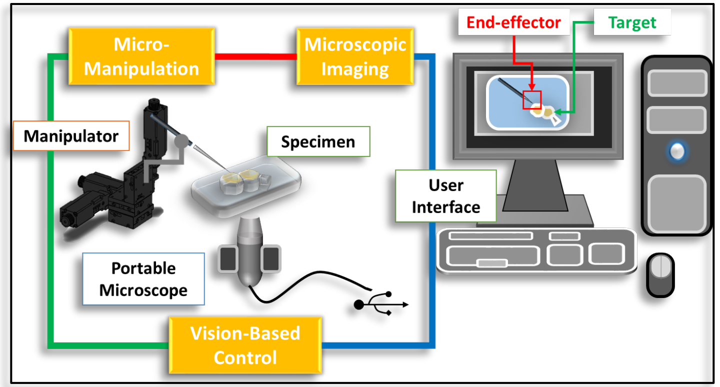
Legged Robots

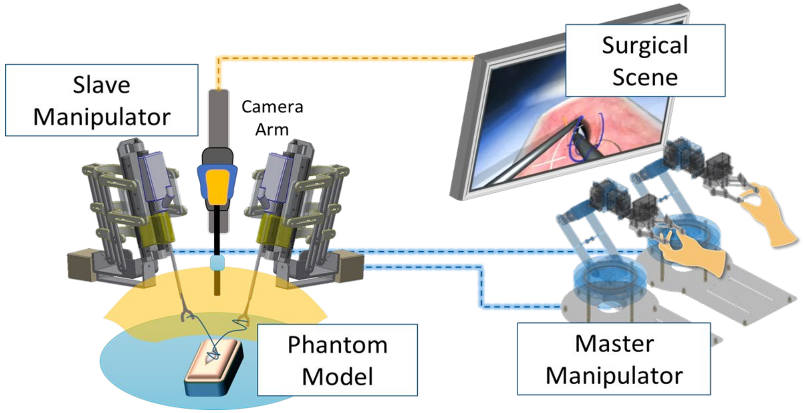
Manipulator

Mobile Robots

Control: Robotic Application







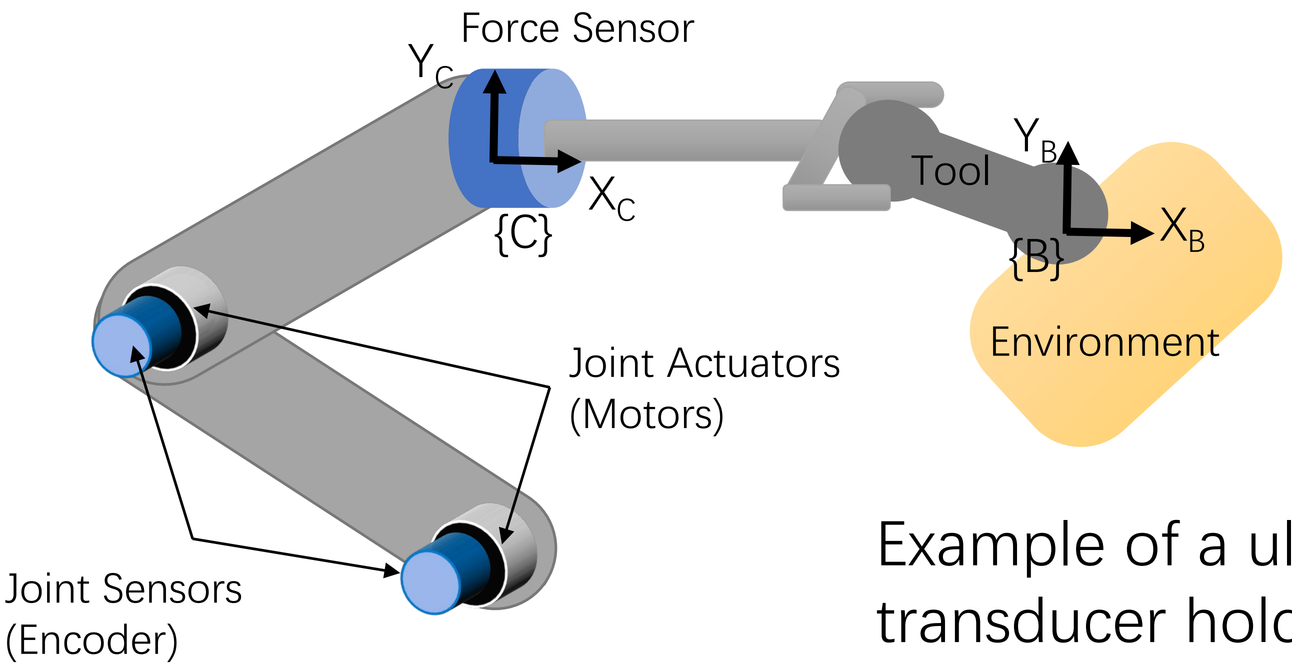
US-Guided Needle
Insertion Robot

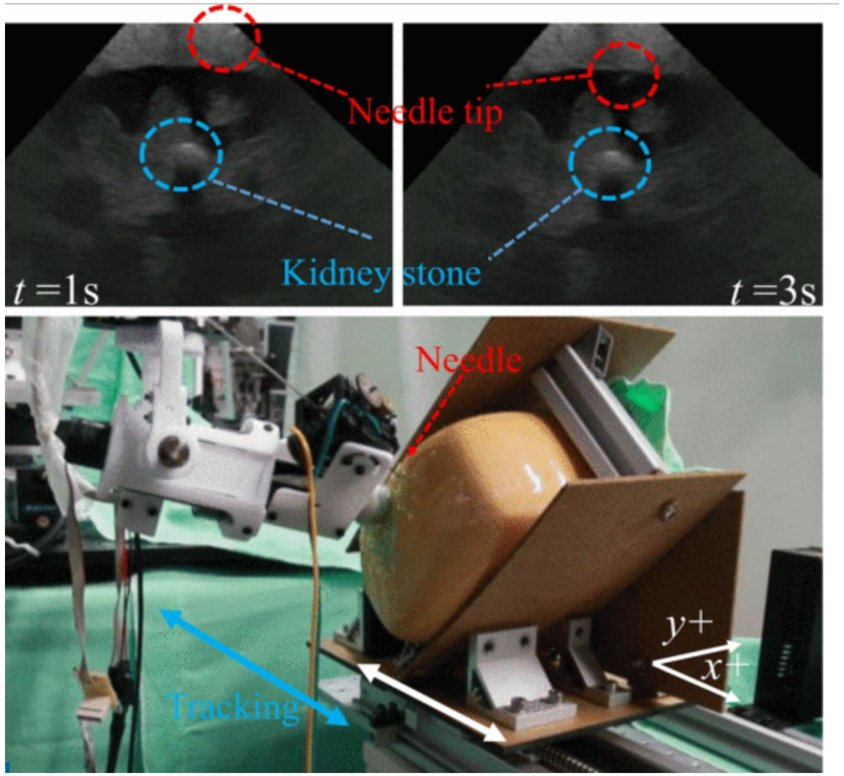
Vision-based Control Cell
Manipulation

Remote Controcfor
Tele-operation

Application Examples of Robotic Control

Case Example: Controlling a Robot to Track a Target on Ultrasound Image During Needle Insertion

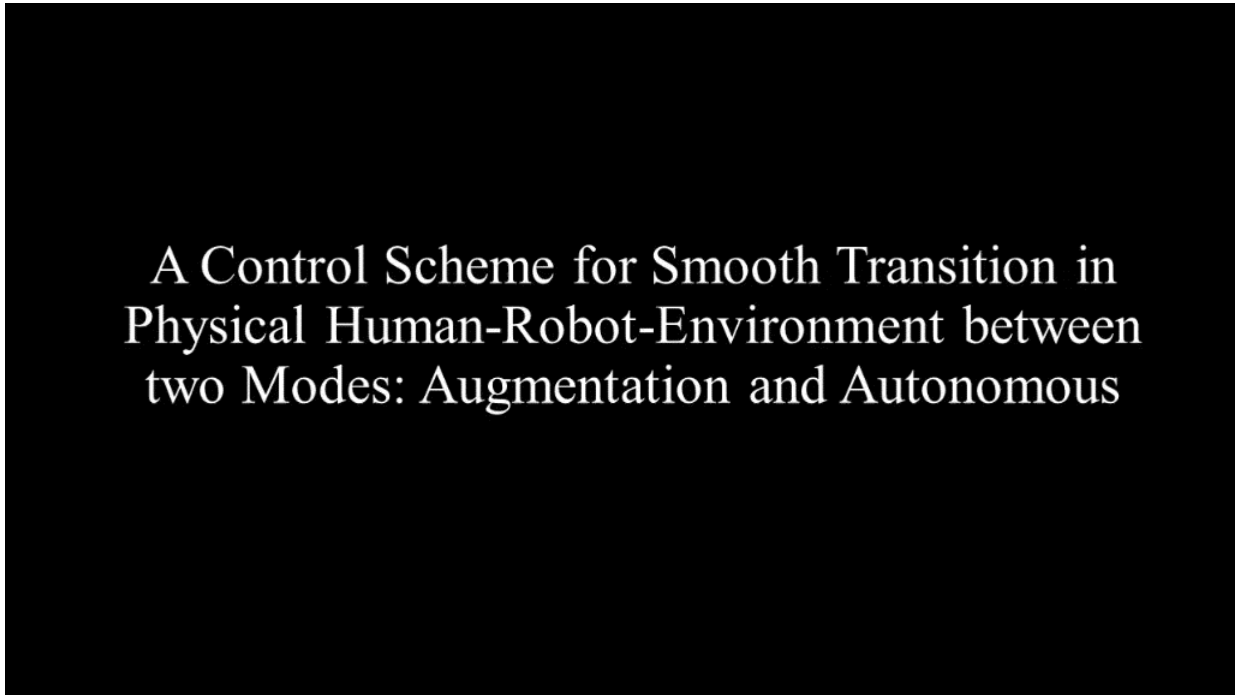


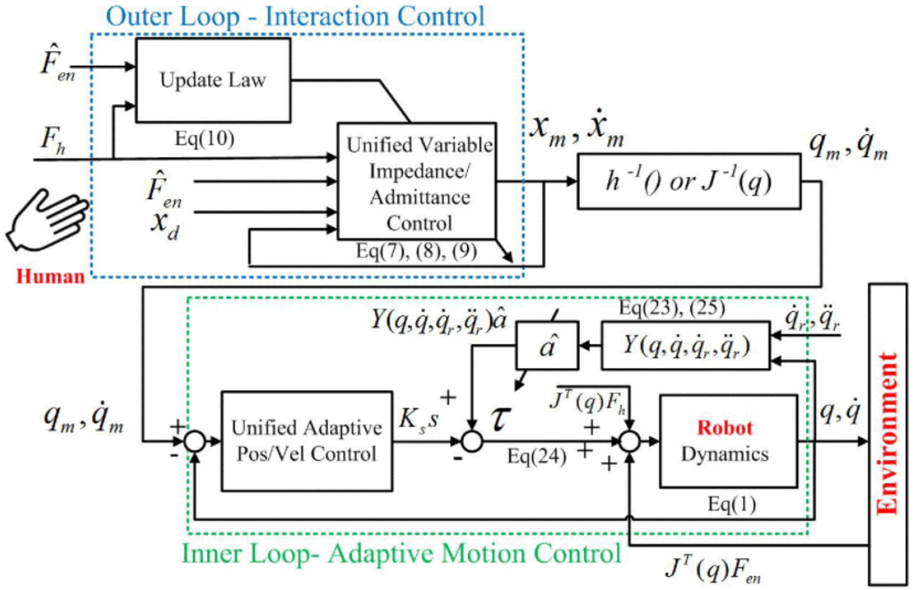


Example of a ultrasound transducer holding robot

Robot Control: Case Example

- A Control Scheme for Smooth Transition in Physical Human-Robot-Environment Between Two Modes: Augmentation and Autonomous





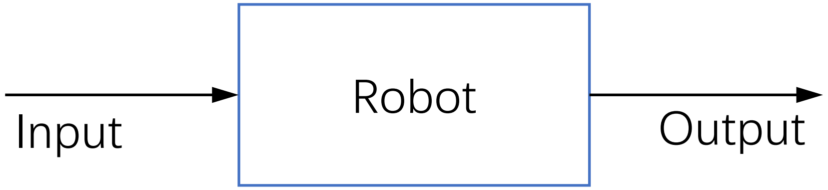
Li, Hsieh-Yu, Liangjing Yang, and U-Xuan Tan. "A Control Scheme for Smooth Transition in Physical Human-Robot-Environment Between Two Modes: Augmentation and Autonomous." IEEE Robotics and Automation Letters 5.4 (2020): 5803-5810.

Robot Control

- Typically, we are interested in designing control systems that generate the **appropriate inputs** for the robotic system to achieve a **desired outcome** in a **dynamic environment** with a **specified performance**

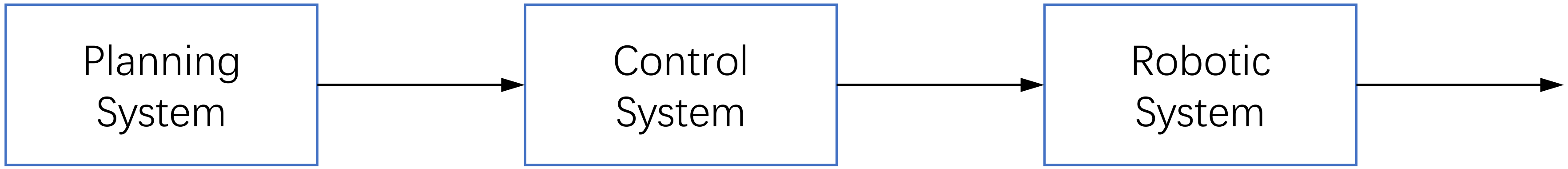
Robot Control

- **Robot Control**: Generate the input command to the robot to achieve a desired outcome



Deferred Topic: Planning

- The control system may take in command from a **planning system**

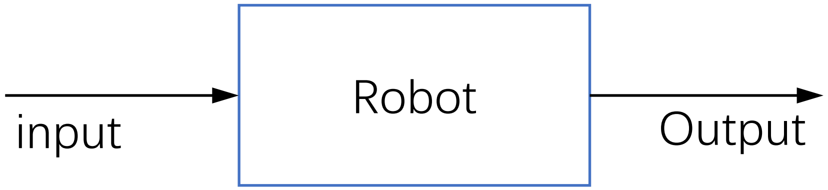


Feedback Control

ECE 470: Introduction to Robotics

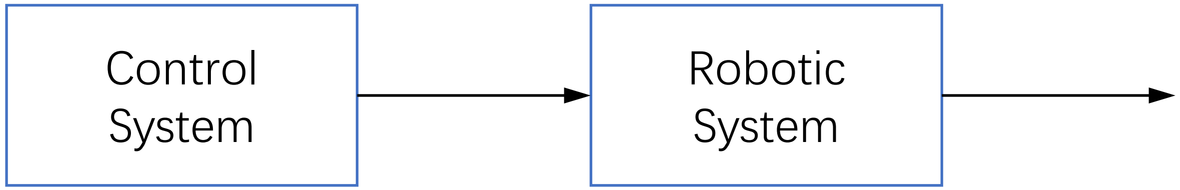
Robot Control

- The **control** problem is concerned with generating the appropriate inputs for the robotic system to achieve the desired outcome



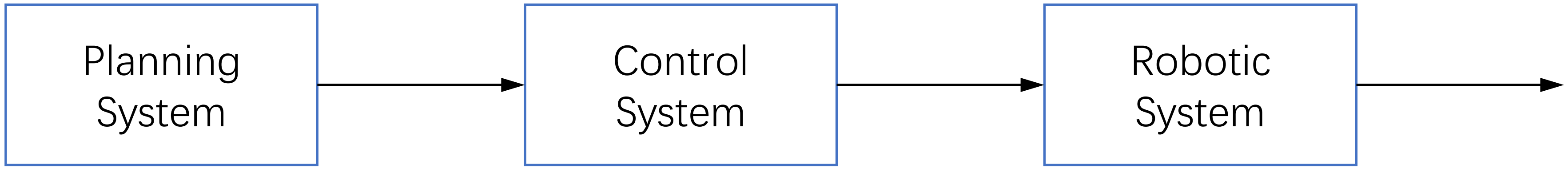
The big picture of robot control

- Design a control system that will generate the input to the robotic system so as to achieve a planned outcome



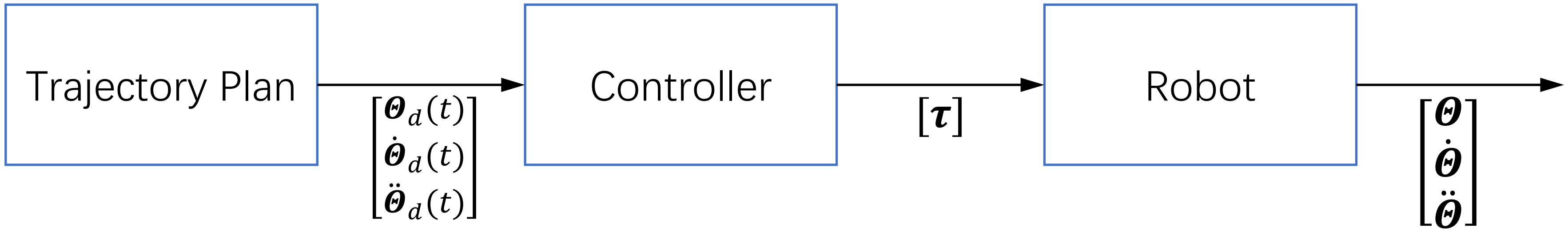
The big picture of robot control

- Design a control system that will generate the input to the robotic system so as to achieve a planned outcome
- The control system may take in command from a planning system



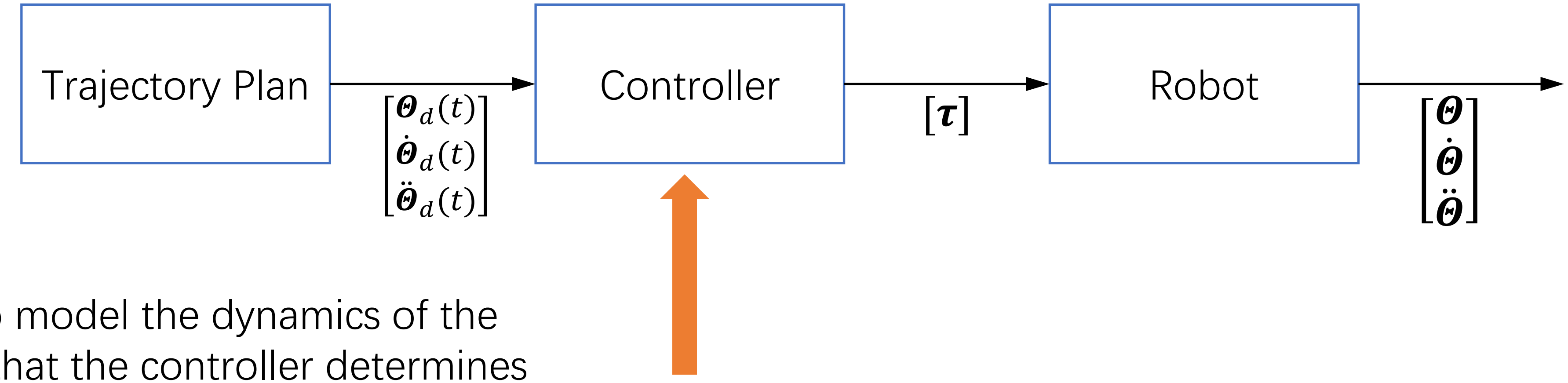
Robot Control: High-level Representation

Example: Control the robot to assume a desired motion profile by inputting the appropriate joint torque



Robot Control: High-level Representation

Example: Control the robot to assume a desired motion profile by inputting the appropriate joint torque



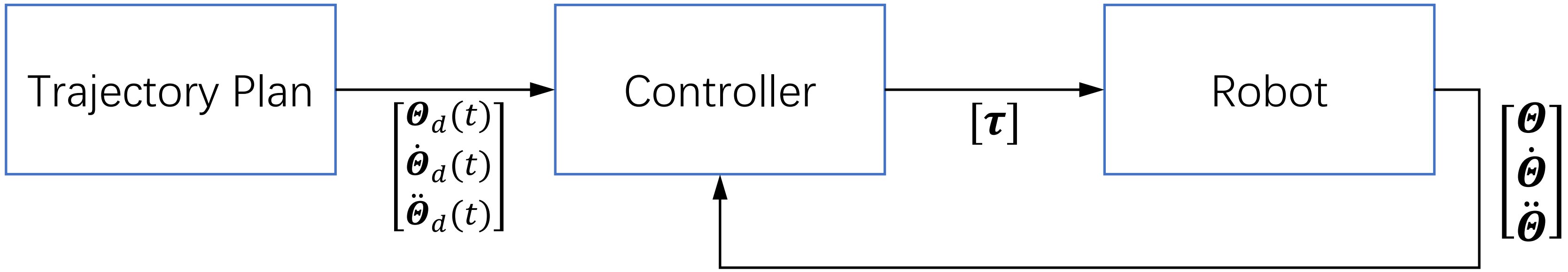
One way is to model the dynamics of the system such that the controller determines the $\boldsymbol{\tau}$ to input with the dynamic equation

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\Theta}_d)\ddot{\boldsymbol{\Theta}}_d + \boldsymbol{V}(\boldsymbol{\Theta}_d, \dot{\boldsymbol{\Theta}}_d) + \boldsymbol{G}(\boldsymbol{\Theta}_d)$$

This is referred to as **open-loop system** as the computed input to the robot is independent of the actual measurement of the robot motion

Robot Control: High-level Representation

Example: Control the robot to assume a desired motion profile by inputting the appropriate joint torque



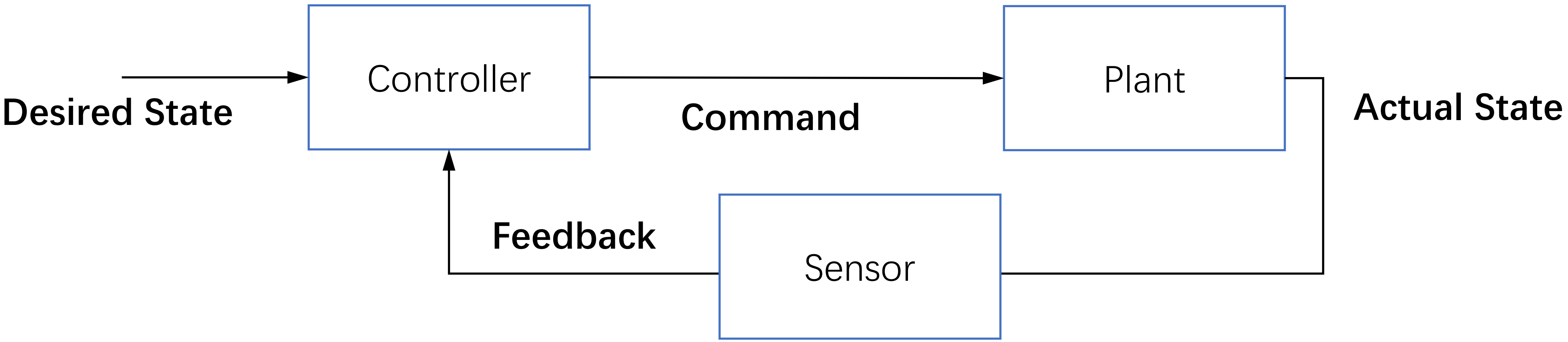
Through sensors we are able to feedback the measurement to produce the value of $[\boldsymbol{\tau}]$ that will minimize the error between desired and actual targeted profile. This is known as a **closed-loop system**.

Feedback Control

Open-loop

Through preestablished model, generate the command that will achieve the desired state.

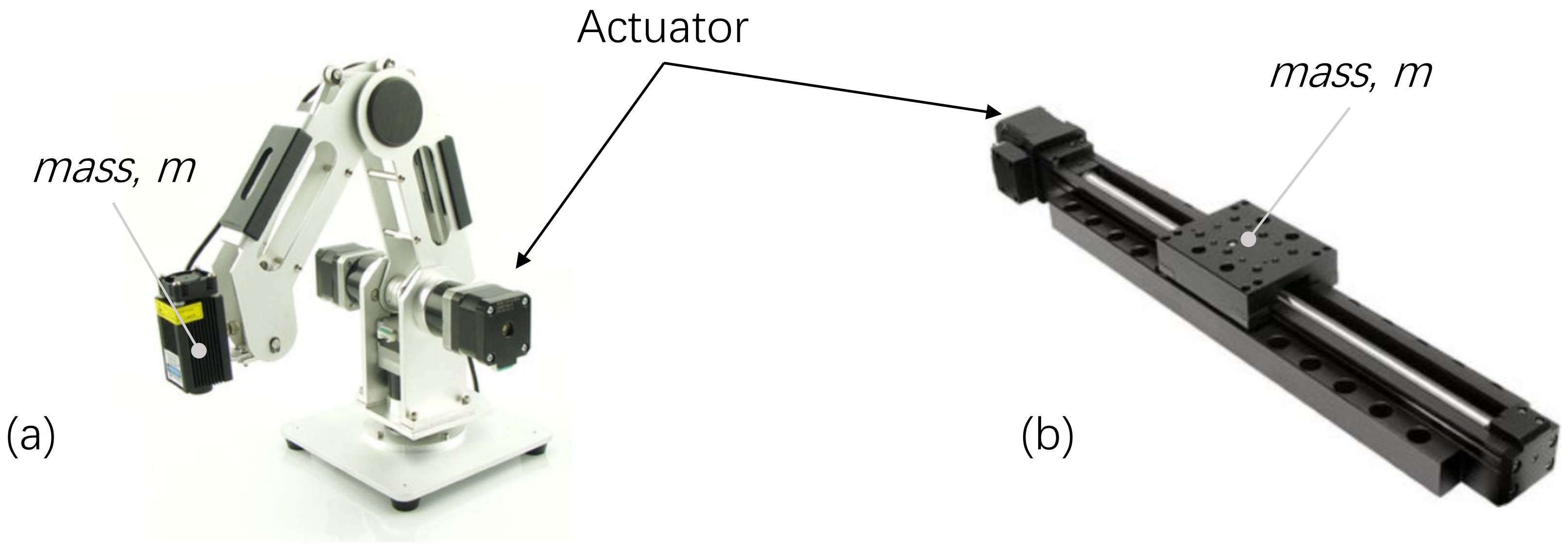
Closed-loop



Through sensors, we are able to **feedback** the measurement to produce the command that will minimize the error between desired and actual targeted profile.

Open vs. Closed loop

- Imagine you are controlling the position of the mass, *m* using a stepper motor by inputting the number of steps.
- Which mechanism (a) or (b) will open loop control be more suitable for? Discuss.

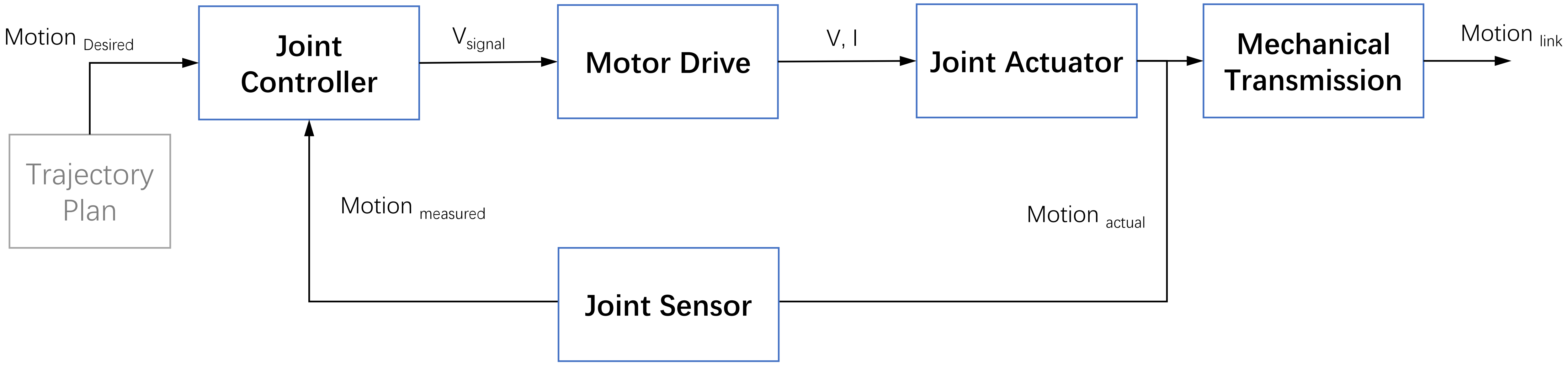


Joint Control

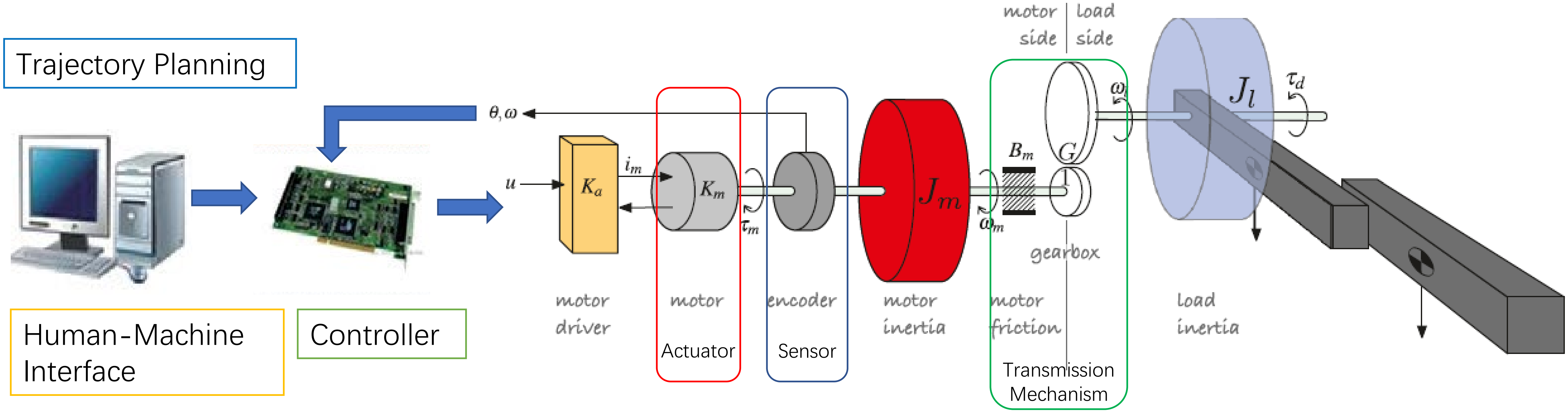
ECE 470 Introduction to Robotics

Joint Control in Robotic System

Components of a robot joint control system

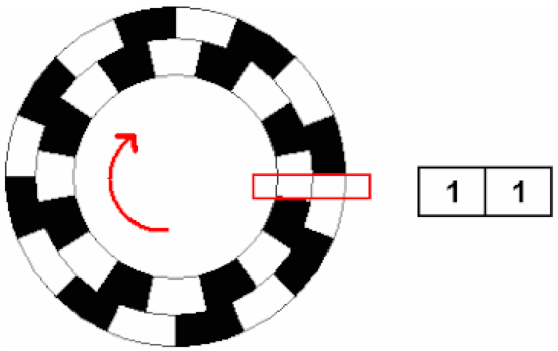


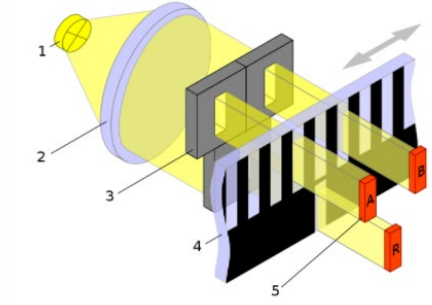
Joint Control in Robotic System

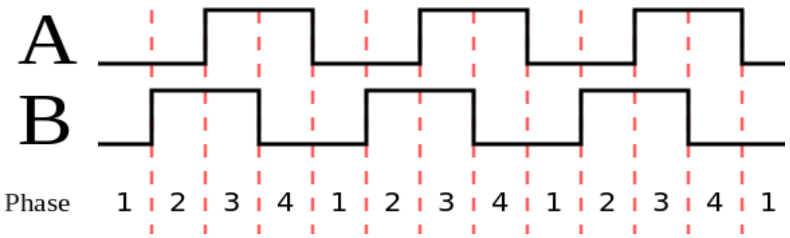


Robot Joint Sensor

- Position Sensing
  - Range Sensor (Linear)
  - Quadrature Encoder (Angular)







By Sidehack at English Wikibooks - Transferred from en.wikibooks to Commons., Public Domain, https://commons.wikimedia.org/w/index.php?curid=61795514

Robot Joint Sensor

- Force Sensing
  - Strain Gauge
  - Load Cell
  - Hall Sensor for Current Measure

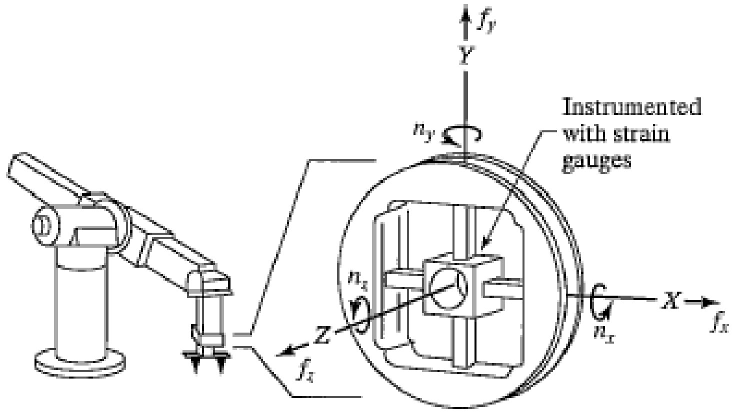
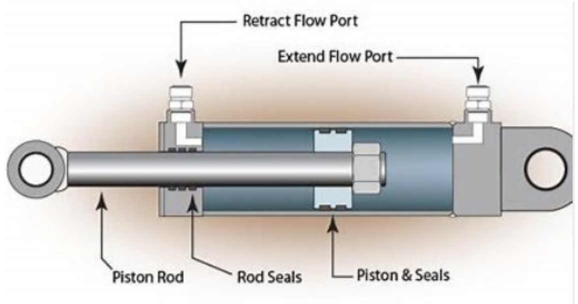


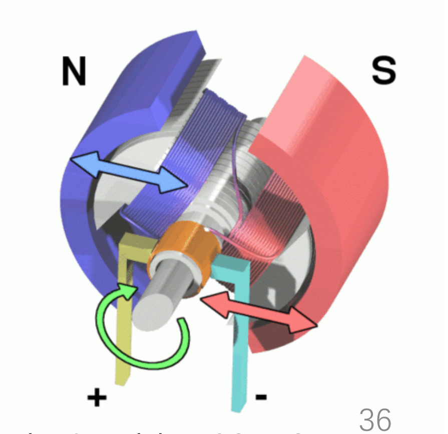
Figure 8.19 textbook (Craig, 3rd ed.)

Robot Joint Actuator

- Devices that power motion

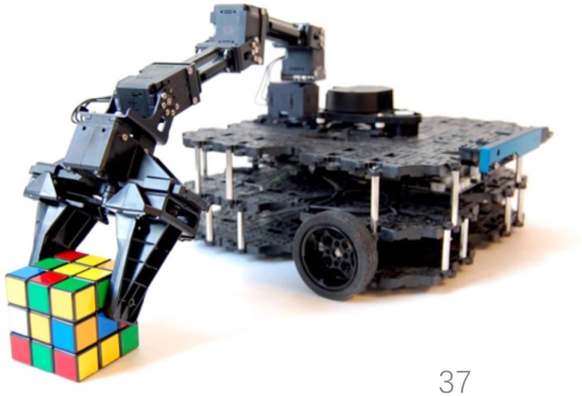
- Principle of force/torque generation
  - Hydraulic
  - Pneumatic
  - Electric
  - ……
- Type of motion
  - Linear
  - Rotational
  - Vibration
  - Reciprocation
  - Free form
  - ……





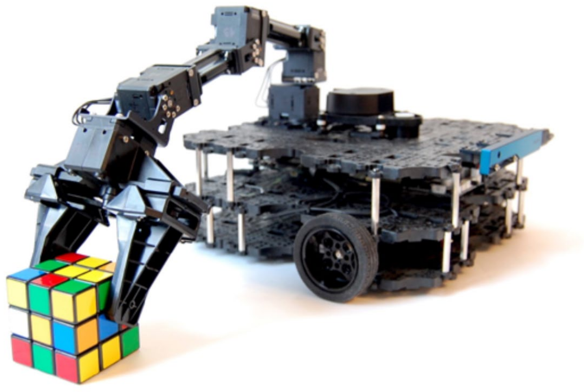
Robot Joint Actuator

- Different actuation principles for different robotics applications
- For examples:
  - Hydraulic for industrial robots→ high torque on direct drive
  - Pneumatic for soft robotics → flexible configuration
  - Electric for relatively lightweight robotic platforms → requiring ease of interface



Robot Joint Actuator

- Different nature of motion for different applications
- For examples:
  - Linear for positioning in cartesian workspace
  - Rotational for continuous circular motion wheeled robot

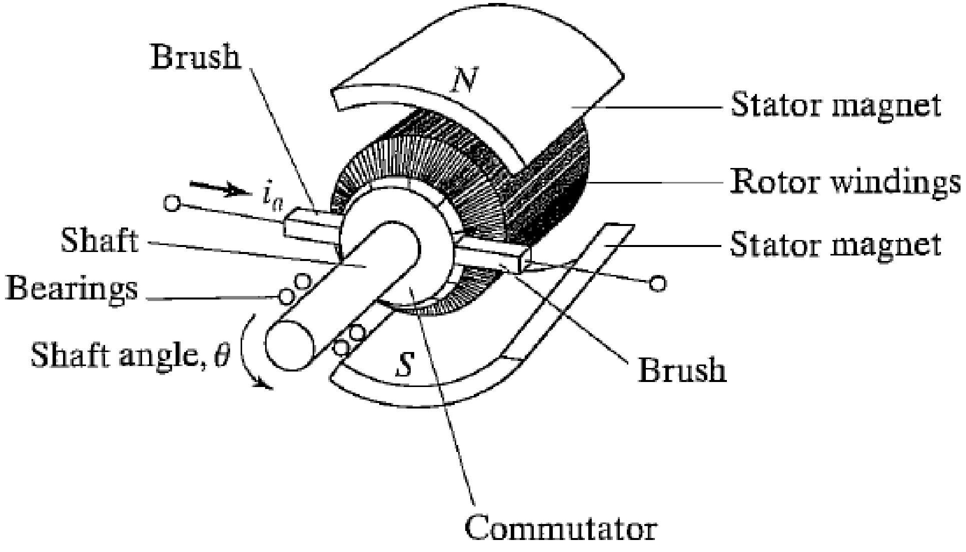


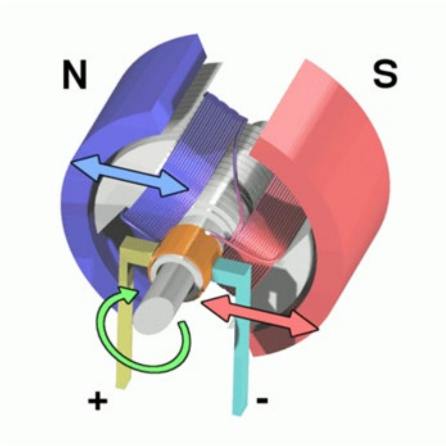
Robot Joint Actuator

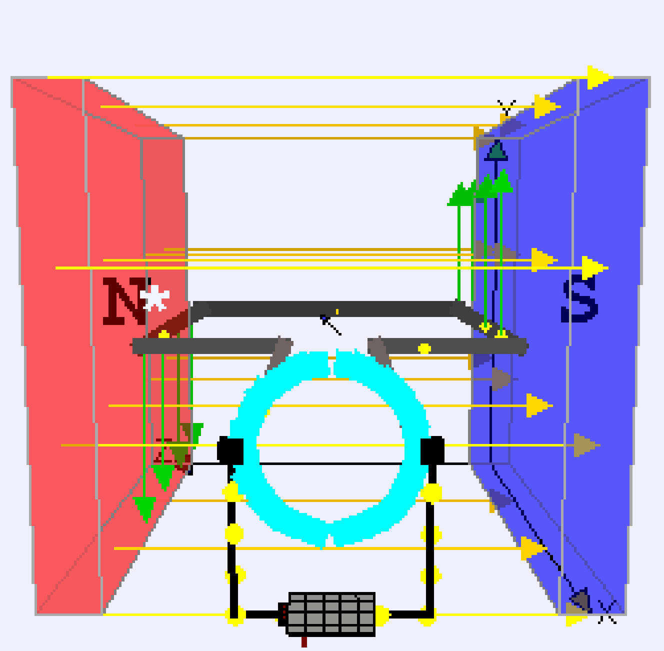
- Electric motor
    - Direct-Current (DC) motor
    - Stepper motor
    - Alternating Current (AC) motor
    - Ultrasonic
    - Piezoelectic

Robot Joint Actuator

- In this course, we look at DC motor





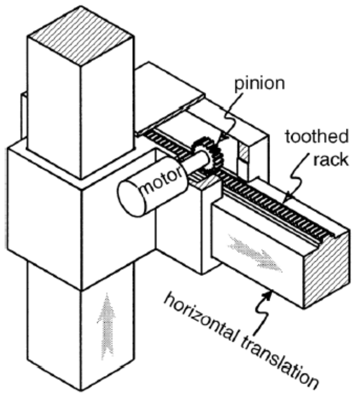


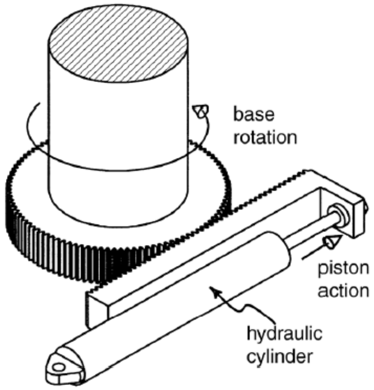
By Lookang many thanks to Fu-Kwun Hwang and author of Easy Java Simulation = Francisco Esquembre - Own work http://iwant2study.org/ospsg/index.php/interactive-resources/physics/05-electricity-and-magnetism/09-electromagnetic-induction/313-ejs-model-dcmotor10, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=15736309

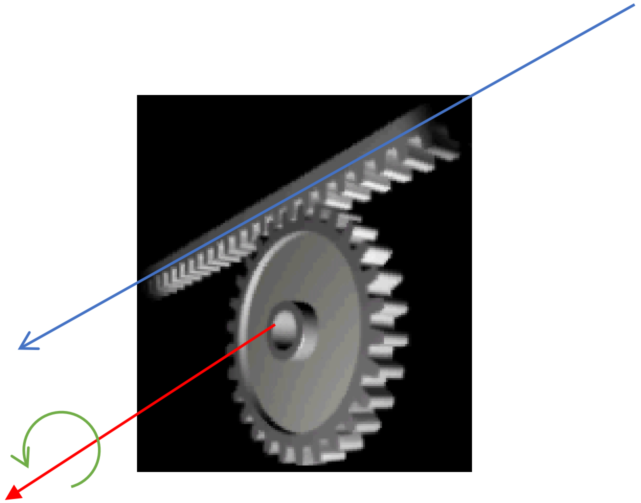
Robot Joint Actuator

By Bcraig15 at English Wikipedia - Transferred from en.wikipedia to Commons., Public Domain, https://commons.wikimedia.org/w/index.php?curid=86179881

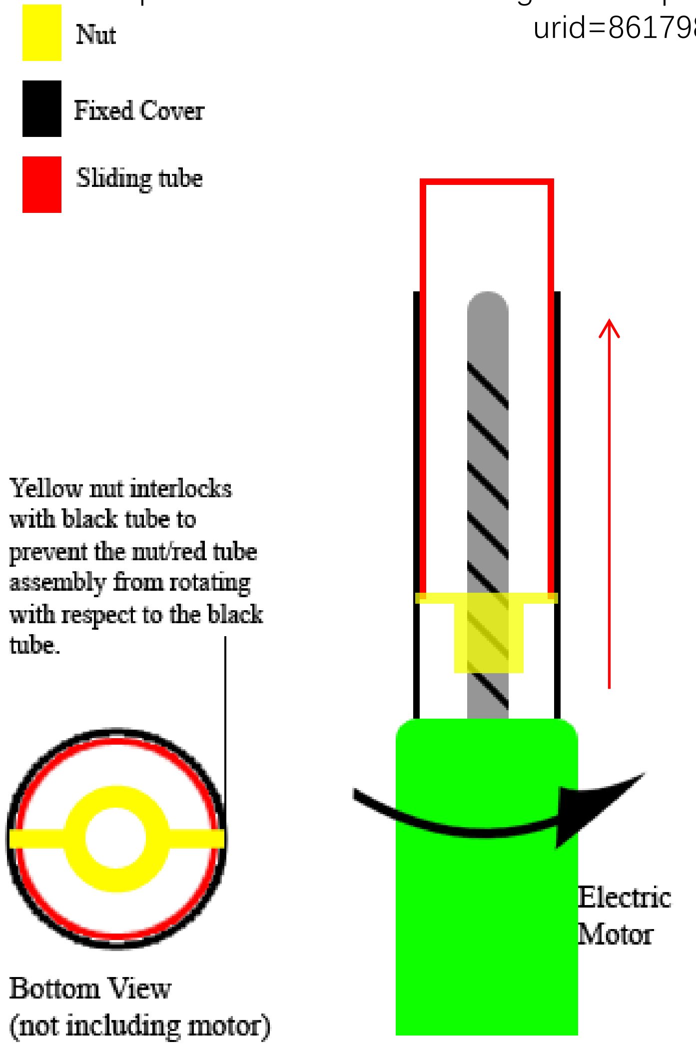
- Transmission System







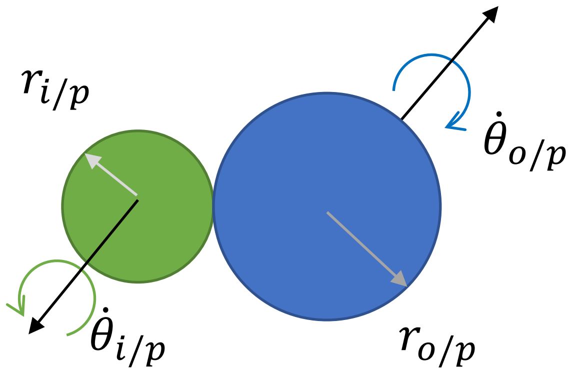
By OSHA Directorate of Technical Support and Emergency Management - Point of Contact Between a Rack and Pinion. The original uploader was Brian0918 at English Wikipedia., Public Domain, https://commons.wikimedia.org/w/index.php?curid=186765



Robot Joint Actuator

- Transmission System for
  - Speed-Reducing & Torque-Increasing, or Vice-Versa
  - Remote Driving of Joint
  - Convert motion type

Robot Joint Actuator



- Transmission System
  - Speed-Reducing & Torque-Increasing, or Vice-Versa
  - Remote Driving of Joint
  - Convert motion type

Speed-Reduction $\dot{\theta}_{o/p} = \frac{1}{\eta}\dot{\theta}_{i/p}$

Torque-Increment $\tau_{o/p} = \eta\tau_{i/p}$

Gear Ratio $\eta = \frac{r_{o/p}}{r_{i/p}} = \frac{\tau_{o/p}}{\tau_{i/p}} = \frac{\dot{\theta}_{i/p}}{\dot{\theta}_{o/p}}$

Control of 2^{nd} Order System

ECE 470: Introduction to Robotics

Designing Robot Control System

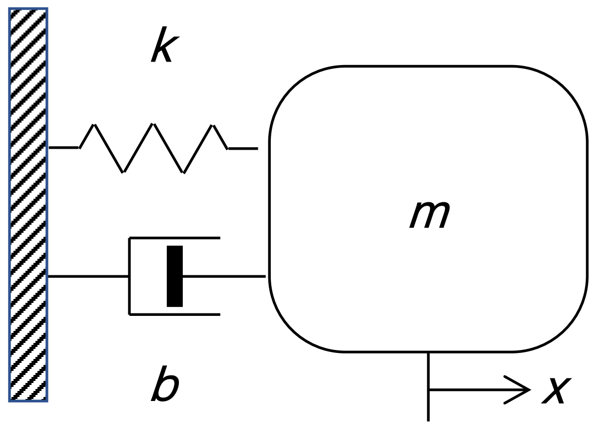
- The central question in designing a control system is the performance specification
  - System response
  - Stability analysis

Second-Order System

Equation of Motion (EOM),

$$\sum F = -F_{drag} - F_{spring}$$

$$m\ddot{x}(t) = -b\dot{x}(t) - kx(t)$$
$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = 0$$



Taking Laplace Transform,

$$m[s^2X(s) - sx(0) - \dot{x}(0)] + b[sX(s) - x(0)] + kX(s) = 0$$

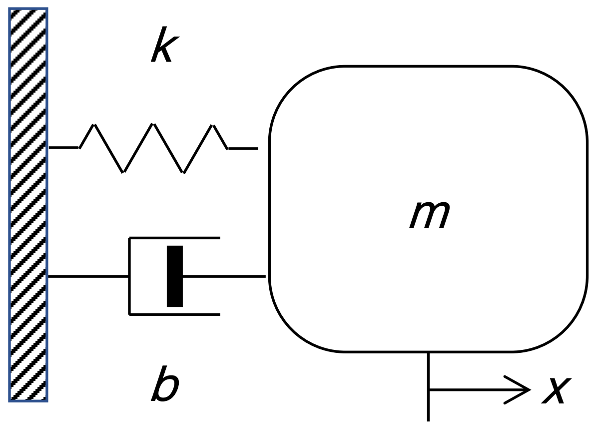
$$(ms^2 + bs + k)X(s) = m[sx(0) - \dot{x}(0)] + bx(0)$$
$$X(s) = \frac{(ms - b)x(0) - m\dot{x}(0)}{ms^2 + bs + k}$$

Characteristic Equation: $ms^2 + bs + k = 0$

Second-Order System

Solving the 2rd order ODE,

$$s_1 = -\frac{b}{2m} + \frac{\sqrt{b^2 - 4mk}}{2m}, \qquad s_2 = -\frac{b}{2m} - \frac{\sqrt{b^2 - 4mk}}{2m}$$



System Response,

Overdamped	$b^2 - 4mk > 0$	Real and unequal roots
Underdamped	$b^2 - 4mk < 0$	Complex roots
Critically damped	$b^2 - 4mk = 0$	Real and Equal roots

The Natural response in time domain can be obtained

Second-Order System

General Solutions

Overdamped $b^2 - 4mk > 0$
Real and unequal roots: $s_1 \neq s_2$

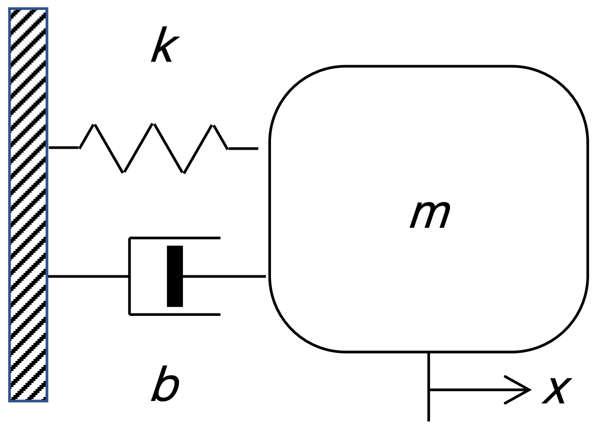
$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

Underdamped $b^2 - 4mk < 0$
Complex roots: $s_{1,2} = \lambda \pm \mu i$

$$x(t) = c_1 e^{\lambda t} \cos(\mu t) + c_2 e^{\lambda t} \sin(\mu t)$$

Critically damped $b^2 - 4mk = 0$
Real and Equal roots $s_1 = s_2$

$$x(t) = (c_1 + c_2 t) e^{s_{1,2} t}$$



where c_1 and c_2 are constants determined by initial conditions
i.e. initial position and velocity

Example 6.2: Overdamped

Example 9.1 in textbook (Craig, 3rd ed.)

The block mass is released from x=-1 at rest, m=1, b=5 and k=6

Characteristic Equation $s^2 + 5s + 6 = 0$

with real and unequal root $s_1 = -2$ and $s_2 = -3$

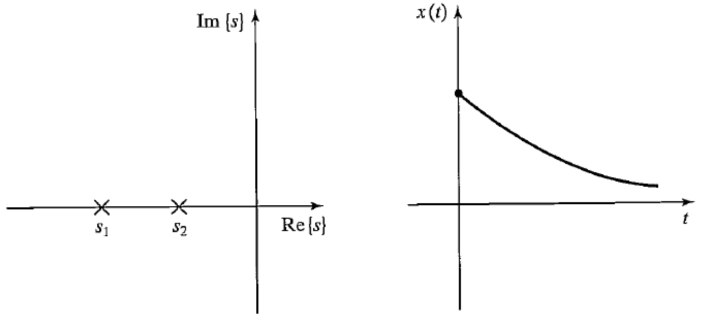
System response $x(t) = c_1 e^{-2t} + c_2 e^{-3t}$

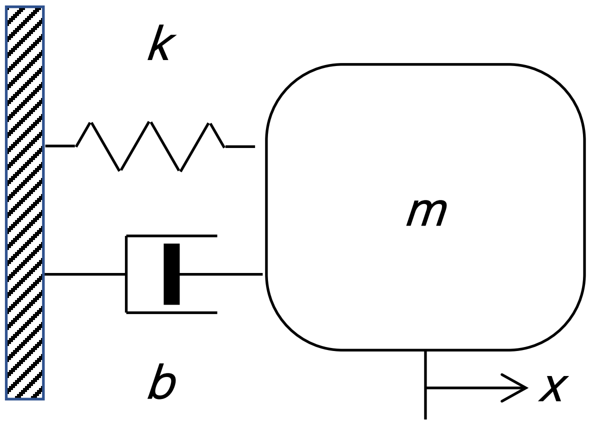
using $x(0) = -1$ $c_1 + c_2$=-1

using $\dot{x}(0) = 0$ $-2c_1 - 3c_2 = 0$

Satisfying the initial conditions: $c_1 = -3$ and $c_2 = 2$

Motion for t $\geq$ 0 $x(t) = -3e^{-2t} + 2e^{-3t}$



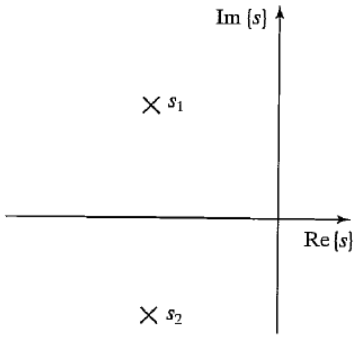


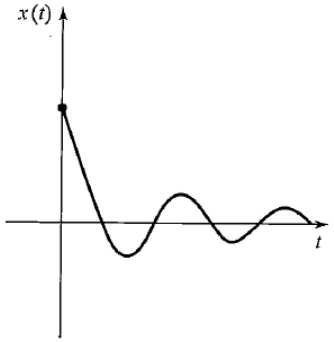
Example 6.3 Underdamped

Example 9.2 in textbook (Craig, 3rd ed.)

The block mass is released from x=-1 at rest, m=1, b=1 and k=1

Characteristic Equation $s^2 + s + 1 = 0$





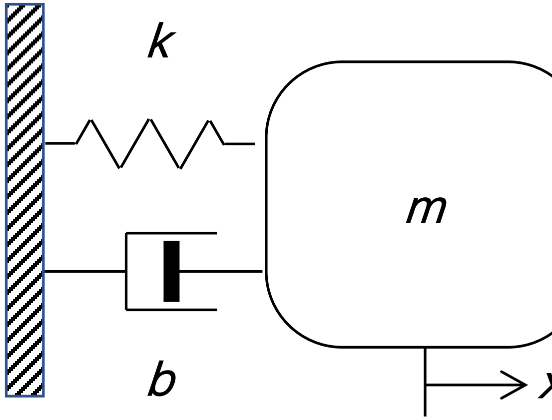
with complex roots: $s_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

System response:

$$x(t) = e^{-\frac{t}{2}}\left(c_1\cos(\frac{\sqrt{3}}{2}t) + c_2\sin(\frac{\sqrt{3}}{2}t)\right)$$

using $x(0) = -1$ $\quad c_1 = -1$

using $\dot{x}(0) = 0$ $\quad -\frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 = 0$



Satisfying the initial conditions: $c_1 = -1$ and $c_2 = -\frac{\sqrt{3}}{3}$

Motion for t ≥ 0

$$x(t) = e^{-\frac{t}{2}}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{\sqrt{3}}{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$$

$$x(t) = \frac{\sqrt{3}}{3}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t + \frac{5\pi}{6}\right)$$

Recall R-formula

$$x(t) = re^{\lambda t}\cos(\mu t - \delta),$$

$$r = \sqrt{c_1^2 + c_2^2},$$

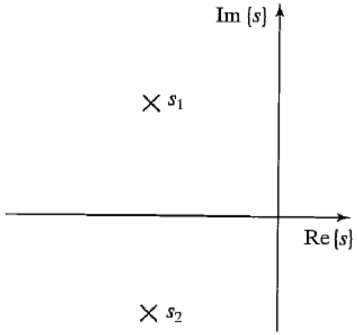
$$\delta = \mathrm{Atan2}(c_2, c_1).$$

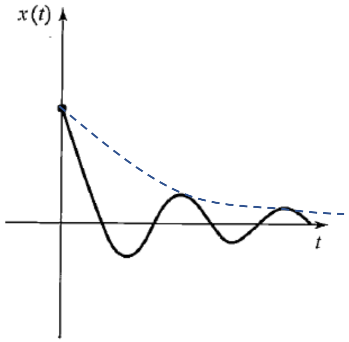
Example 6.3 Underdamped

Example 9.2 in textbook (Craig, 3rd ed.)

The block mass is released from x=-1 at rest, m=1, b=1 and k=1

Characteristic Equation $s^2 + s + 1 = 0$

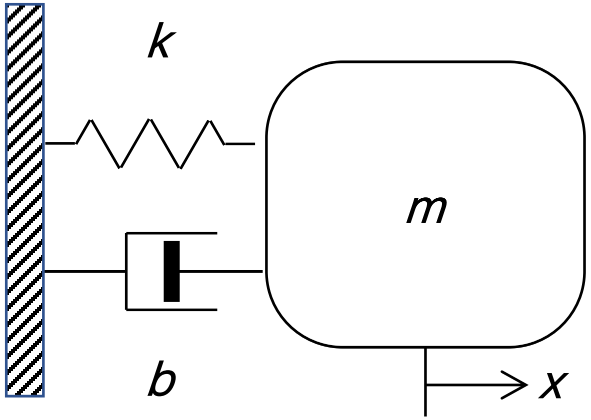


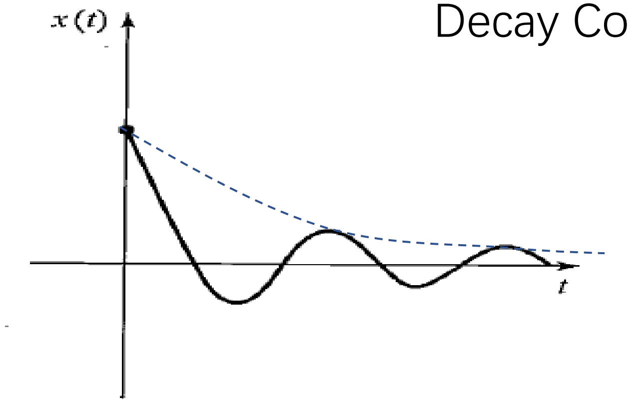


Motion for t $\geq$ 0

$$x(t) = e^{-\frac{t}{2}}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{\sqrt{3}}{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$$

$$x(t) = \frac{\sqrt{3}}{3}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t + \frac{5\pi}{6}\right)$$





Decay Constant

Damped Frequency

Example 6.4 Critically Damped

The block mass is released from x=-1 at rest, m=1, b=4 and k=4

Characteristic Equation $s^2 + 4s + 4 = 0$

with real and equal root $s_1 = s_2 = -2$

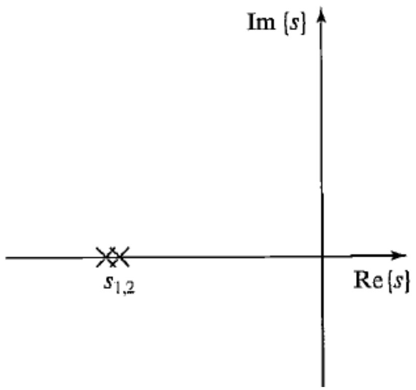
System response $x(t) = (c_1 + c_2 t)e^{-2t}$

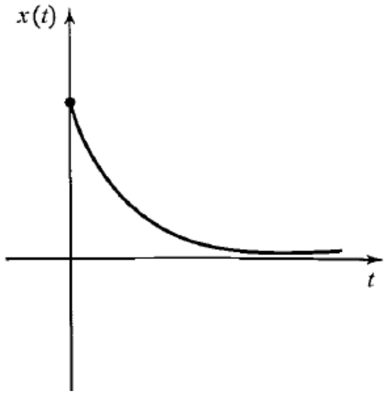
using $x(0) = -1$ c_1=-1

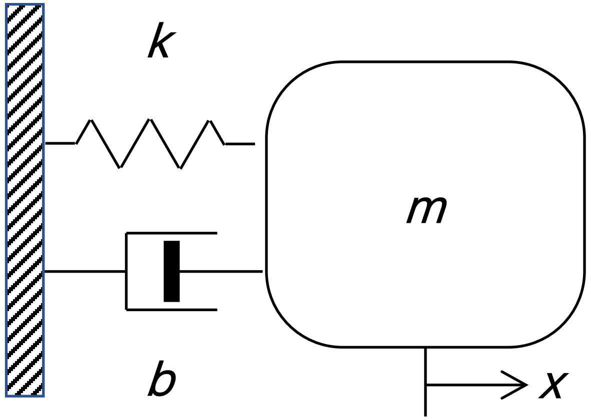
using $\dot{x}(0) = 0$ $-2c_1 + c_2 = 0$

Satisfying the initial conditions: $c_1 = -1$ and $c_2 = -2$

Motion for t $\geq$ 0 $x(t) = (-1 - 2t)e^{-2t}$







Second-Order System

From Ex 6.2-4, the systems are all **stable** where <u>*m, b and k* >0</u>

In free oscillation, the system has <u>natural response</u>

In control applications, the system has <u>forced response</u>

Second-Order System

The Dynamics of the Mass-Spring-Damper System

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = 0$$

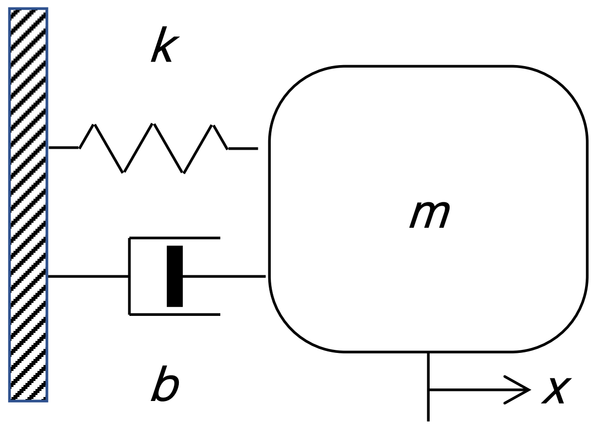
can be described as an oscillatory 2nd order system

$$\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = 0$$

characterized by

Natural frequency: $\omega_n = \sqrt{\frac{K}{m}}$

Damping Ratio: $\zeta = \frac{1}{2}\sqrt{\frac{b^2}{mk}}$



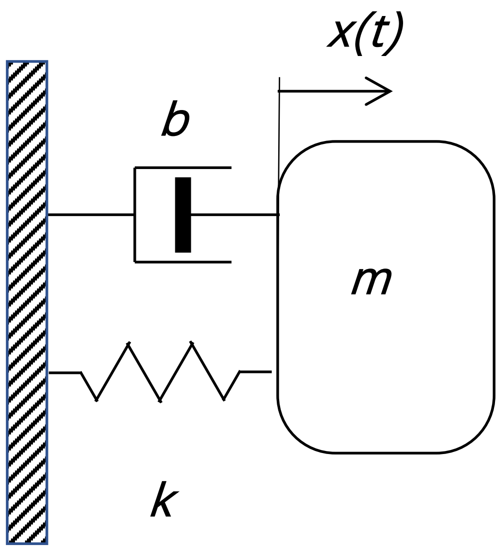
Second-Order System

Oscillatory 2nd order system with free response

Homogenous ODE $\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = 0$

Free response

Overdamped	$\zeta > 1$	Real and unequal roots: $s_1 = s_2$
Critically damped	$\zeta = 1$	Real and Equal roots $s_1 \neq s_2$
Underdamped	0< ζ <1	Complex roots: $s_{1,2} = \lambda \pm \mu i$
No damping	$\zeta = 0$	Imaginary roots: $s_{1,2} = \pm\mu i$



Simulation Demo