ECE 470: Introduction to Robotics

Lecture 19

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Quick Recap

- Last week
  - The overview of Robot Planning
    - Path
    - Trajectory
    - Motion
  - Trajectory Generation
    - Joint-Space Scheme
    - Cartesian-Space Scheme
  - Issues and Challenges in Motion Planning

This Lecture

Joint Scheme: Polynomial Function

Cubic Function

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

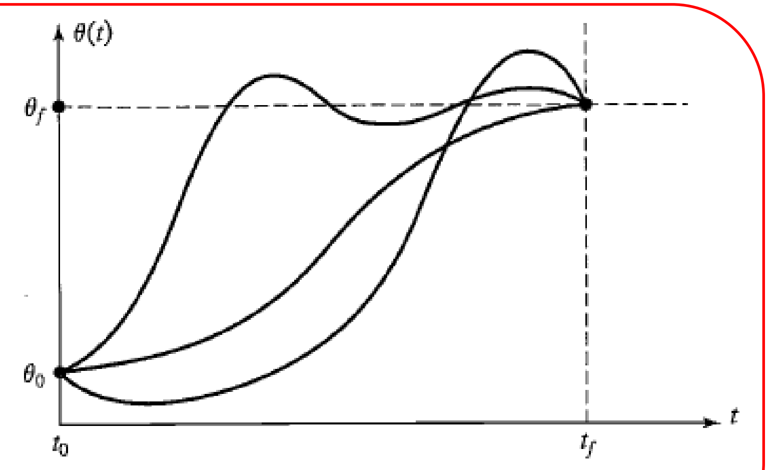
Parameter

$$a_0 = \theta_0$$

$$a_1 = \dot{\theta}_0$$

$$a_2 = \frac{3}{t^2{}_f}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$a_3 = -\frac{2}{t^3{}_f}(\theta_f - \theta_0) + \frac{1}{t^2{}_f}(\dot{\theta}_f - \dot{\theta}_0)$$



Boundary conditions

$$\theta_0 = \theta(0) = a_0$$

$$\theta_f = \theta(t_f) = a_0 + a_1 t_f + a_2 t_f{}^2 + a_3 t_f{}^3$$

$$\dot{\theta}_0 = a_1$$

$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f{}^2$$

Parameter

$$a_0 = \theta_0$$

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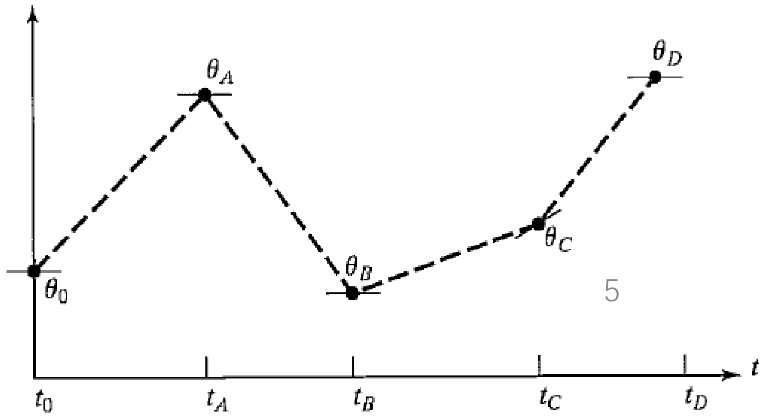


Linear segment with parabolic blends

continuity b/w segments: equal gradient

constant acceleration: parabolic curve

$$\ddot{\theta} t_b = \frac{\theta_h - \theta_b}{t_h - t_b} \qquad \theta_b = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2$$

$$\theta_h = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2 + \ddot{\theta} t_b (t_h - t_b)$$

Symmetrical

$$\theta_h = \theta_0 - \frac{1}{2}\ddot{\theta} t_b^2 - \ddot{\theta} t_b (t_h - t_b)$$

Combining

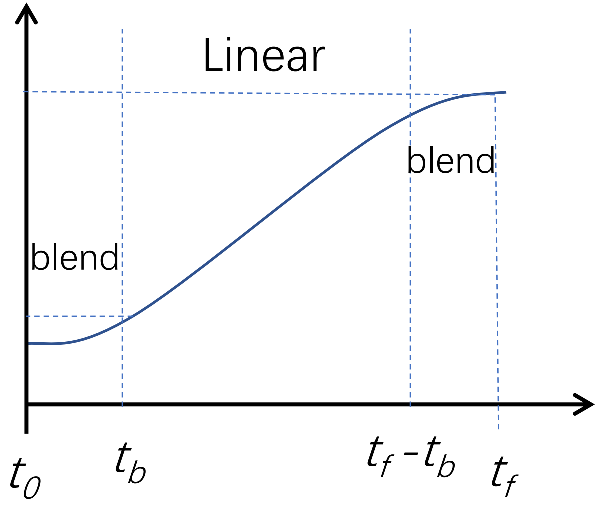
$$\ddot{\theta} t_b^2 - \ddot{\theta} t_f t_b + \theta_f - \theta_0 = 0$$

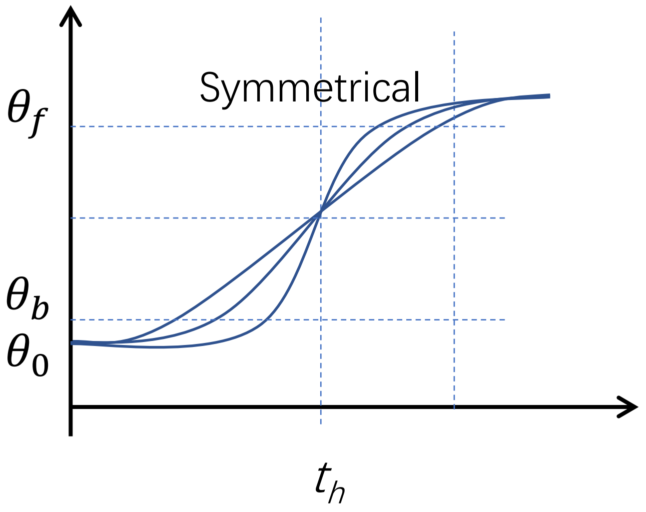
Usually, an acceleration, is chosen and the above equation is solved for the corresponding t_b.

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta} t_b^2 - 4\ddot{\theta}\,(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

For real solutions to exist, acceleration need to meet the criteria

$$\ddot{\theta} \geq \frac{4\,(\theta_f - \theta_0)}{{t_f}^2}$$





Cartesian Space Scheme

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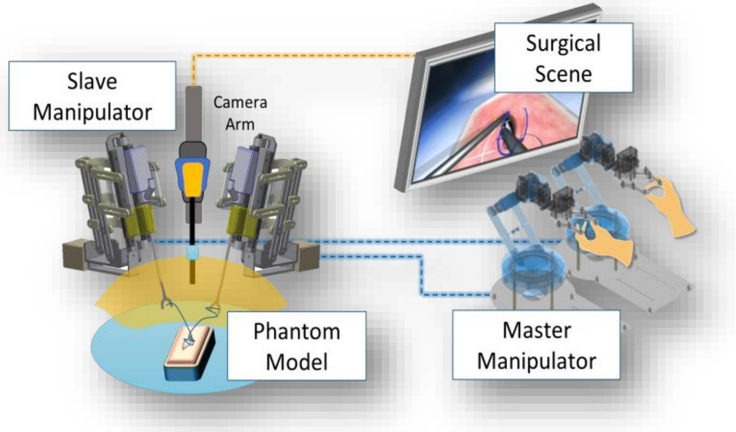
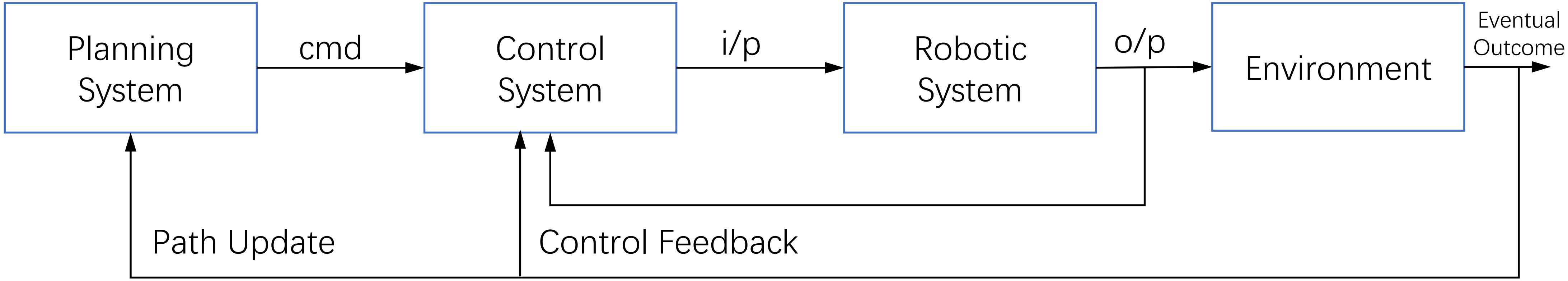
Cartesian Space Scheme

- Specified in terms of pose
- Path points in cartesian coordinates as a function of time
  - planned directly from the user's definition of path without performing inverse kinematics (i.e. may not be preplanned)
  - inverse kinematics solved at the path update rate
  - thus, more computationally expensive.

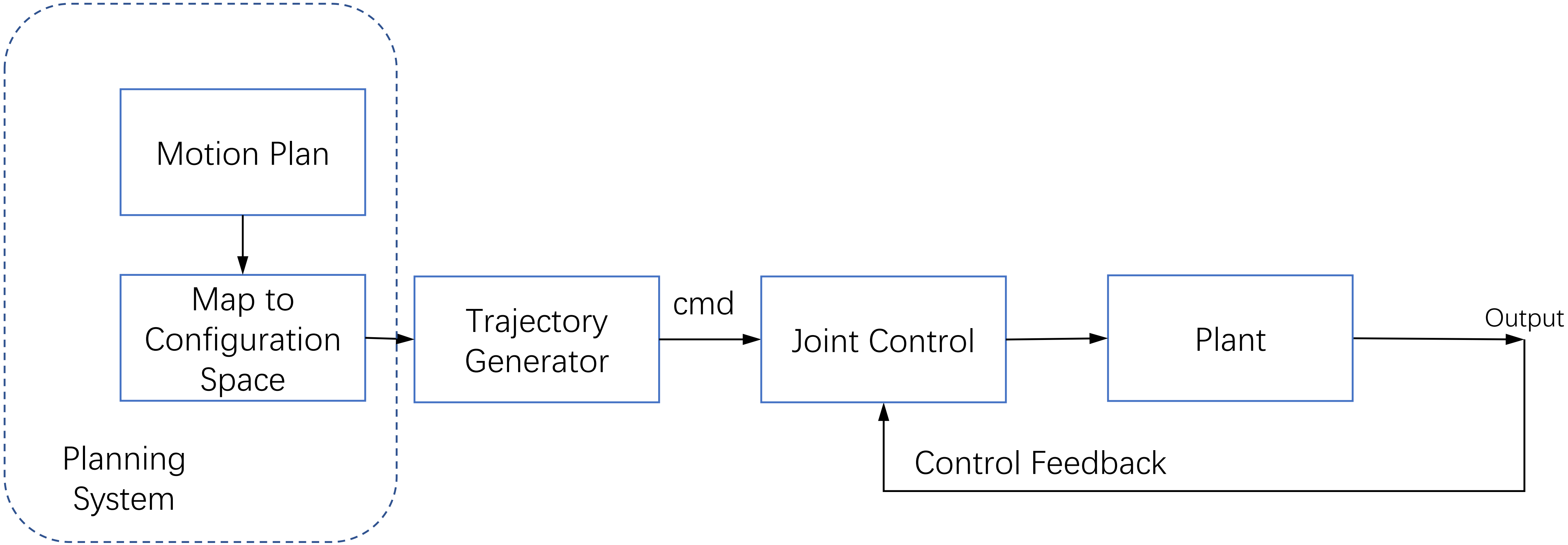


Teleoperating a Robot for Example

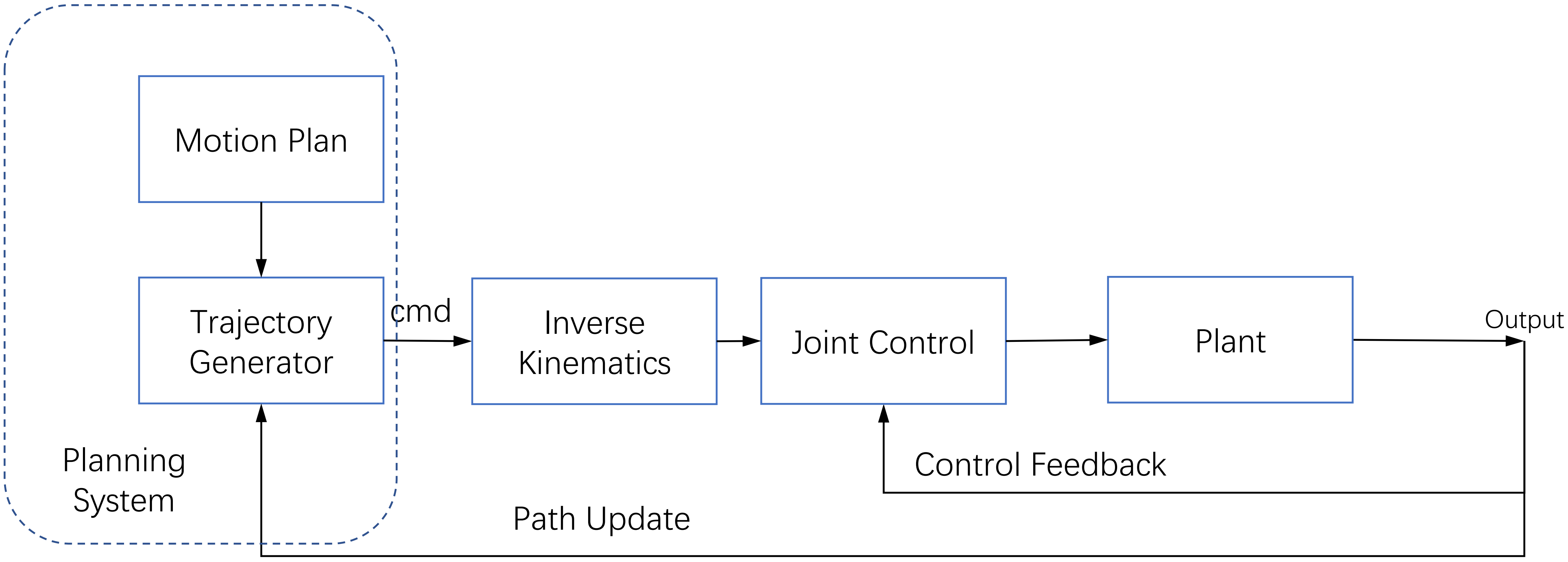
Recall the big picture



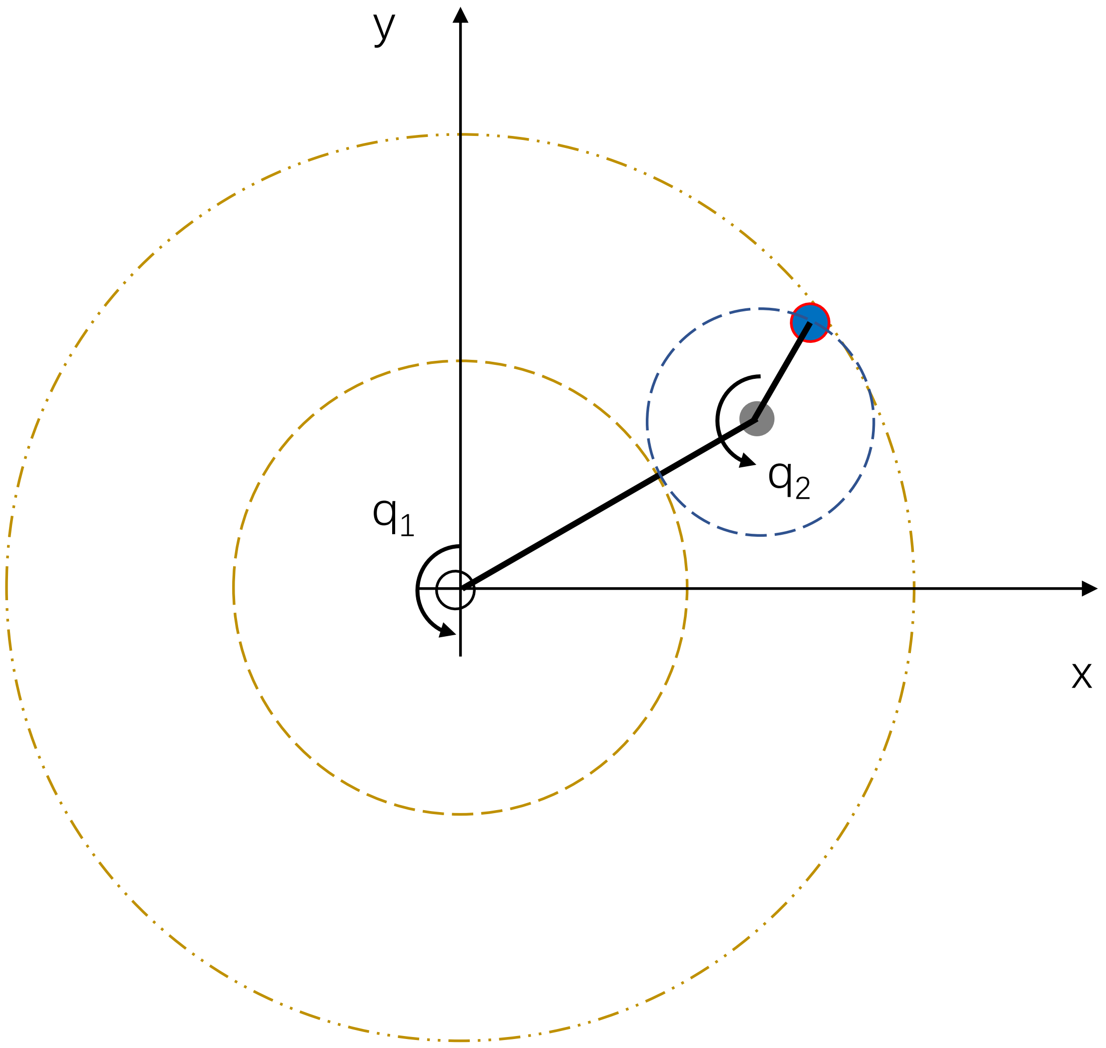
In Joint Space

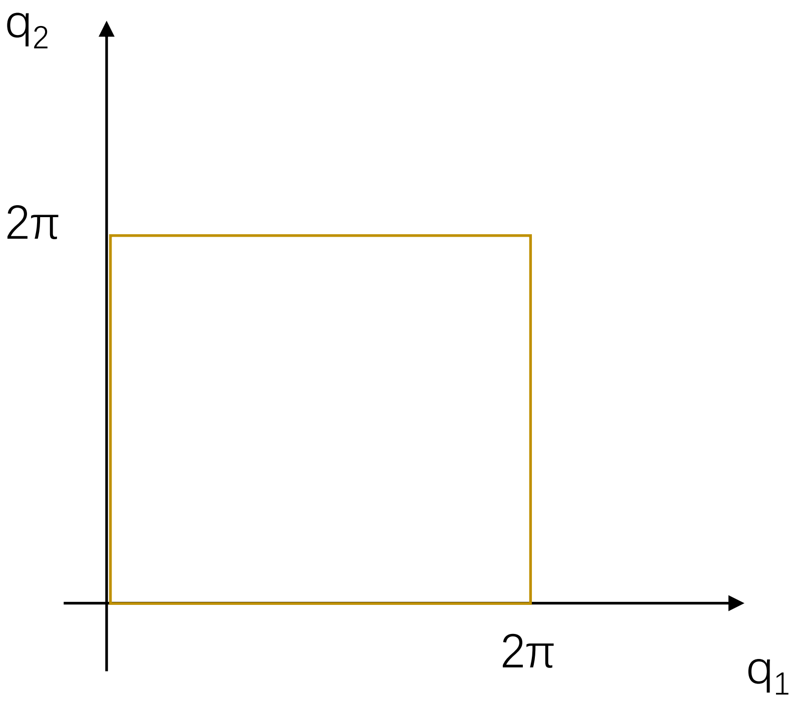


In Cartesian Space



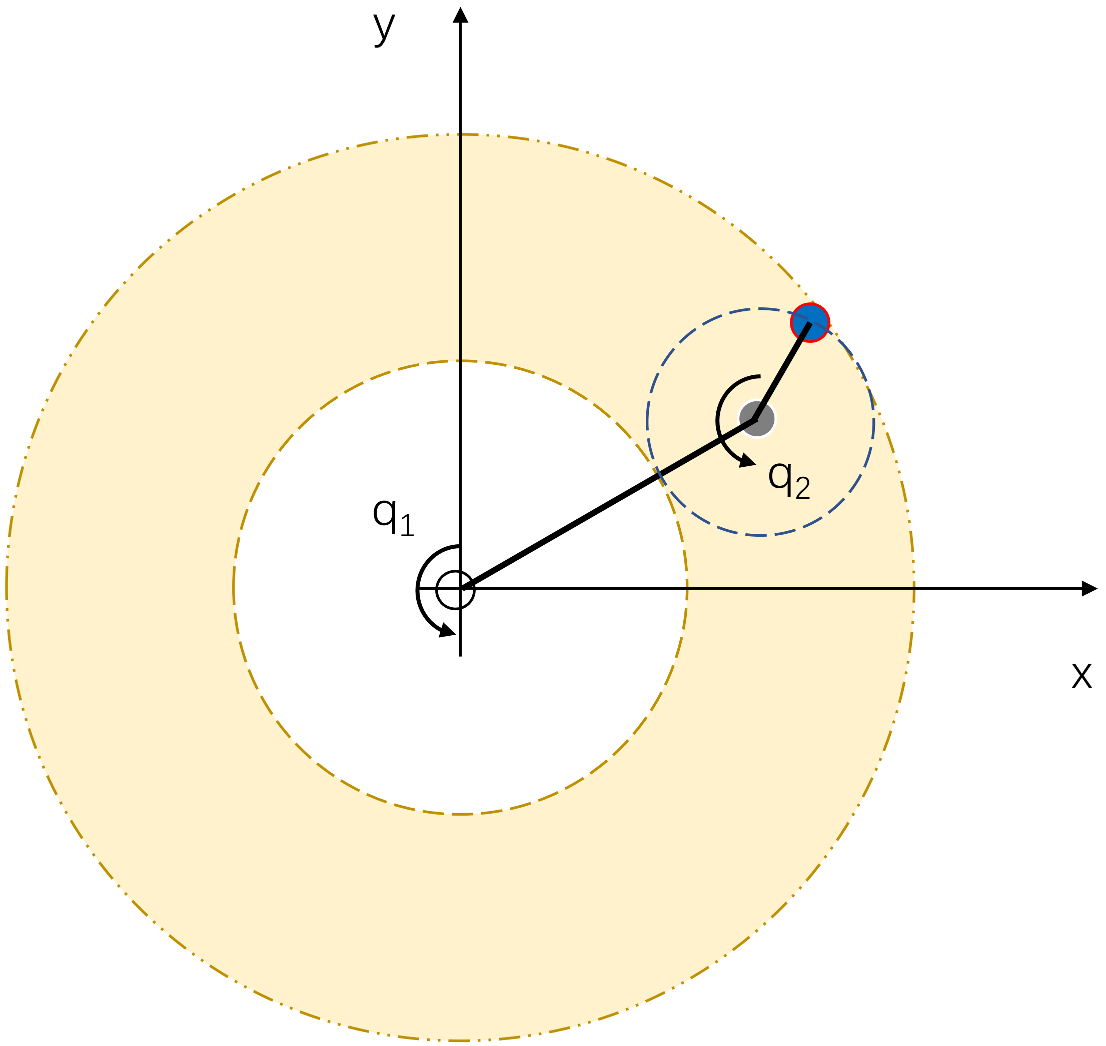
Cartesian VS. Configuration Space

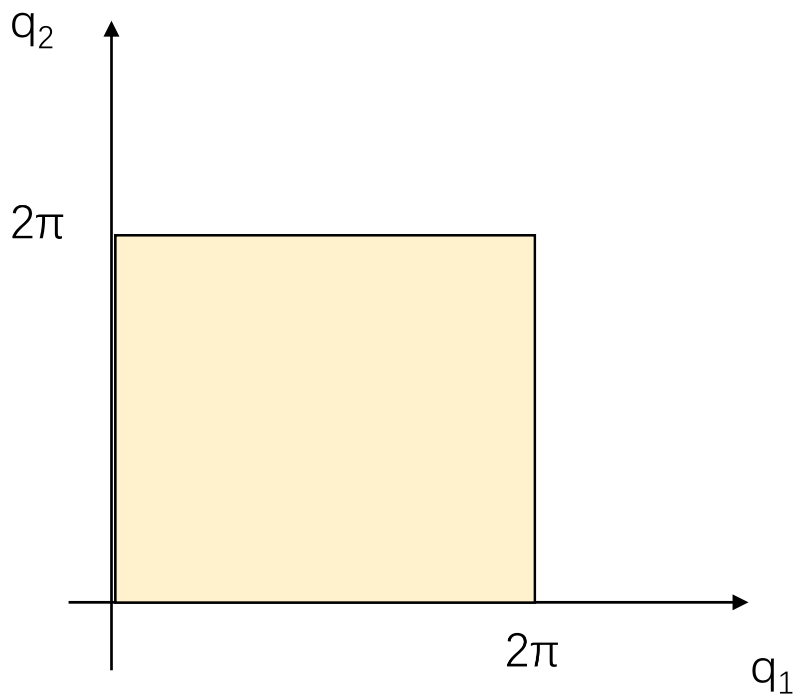




Workspace Boundary and Joint Limits

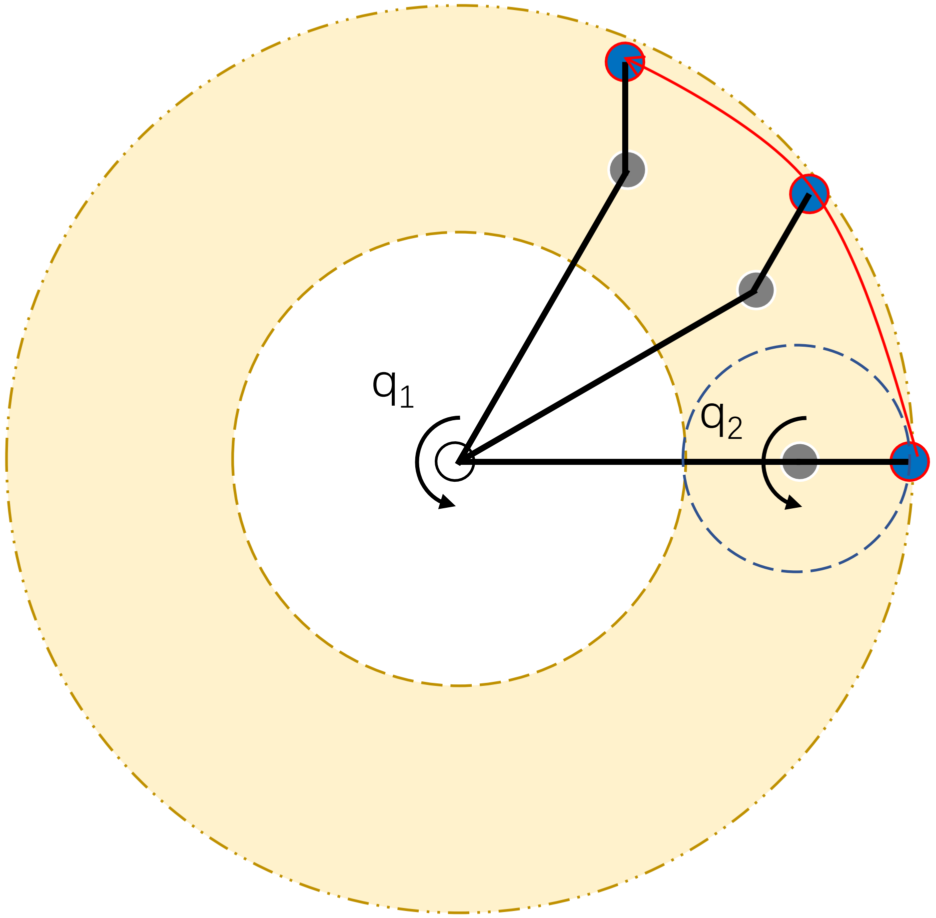
Cartesian VS. Configuration Space

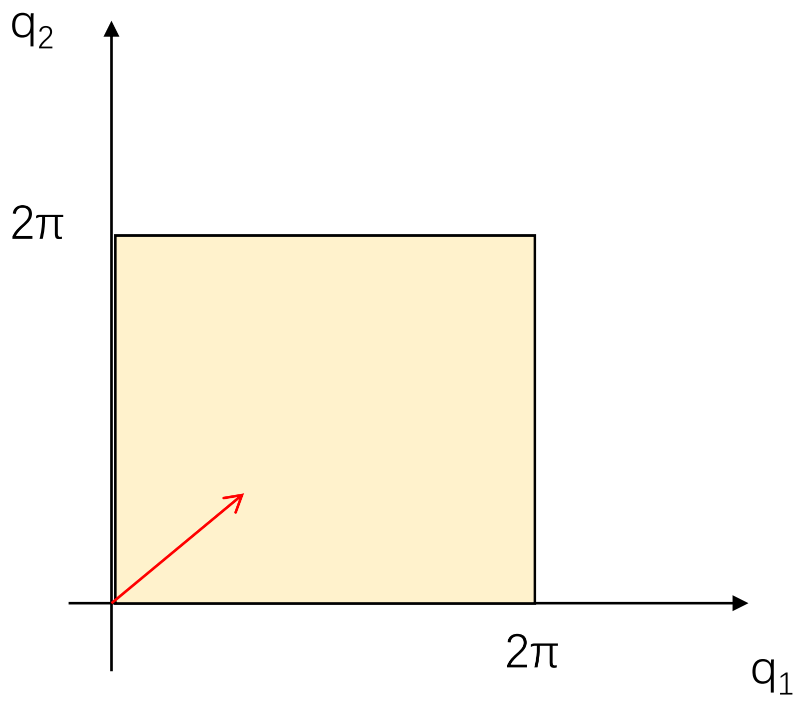




Workspace and Configuration Space

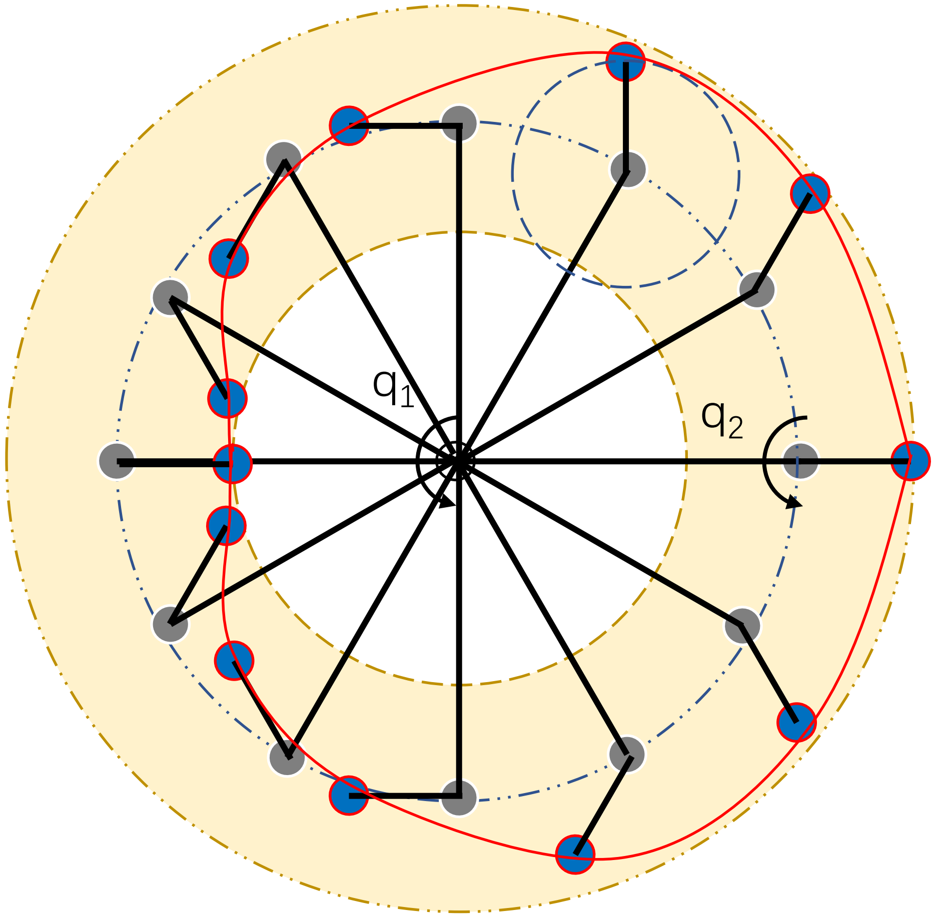
Cartesian VS. Configuration Space

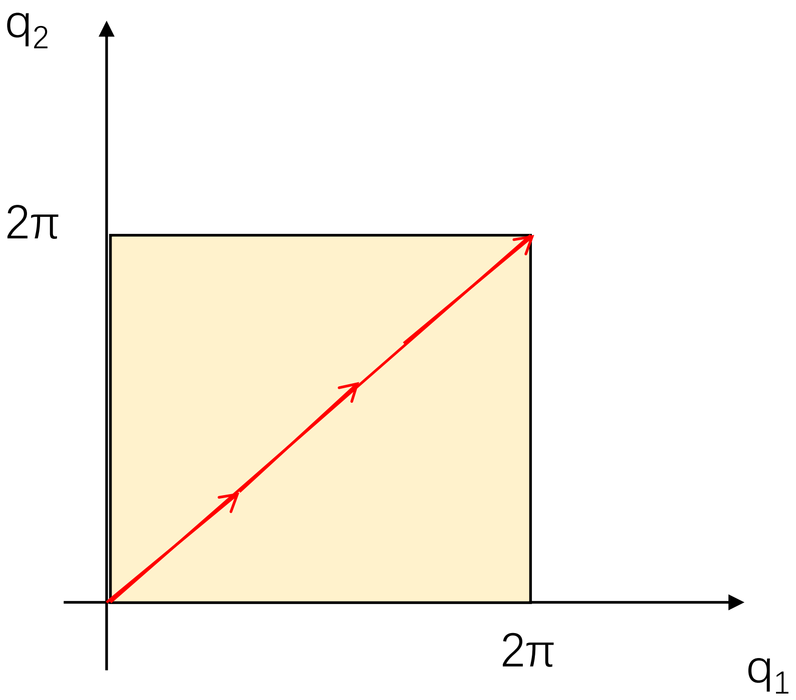




Path in Workspace and Configuration Space

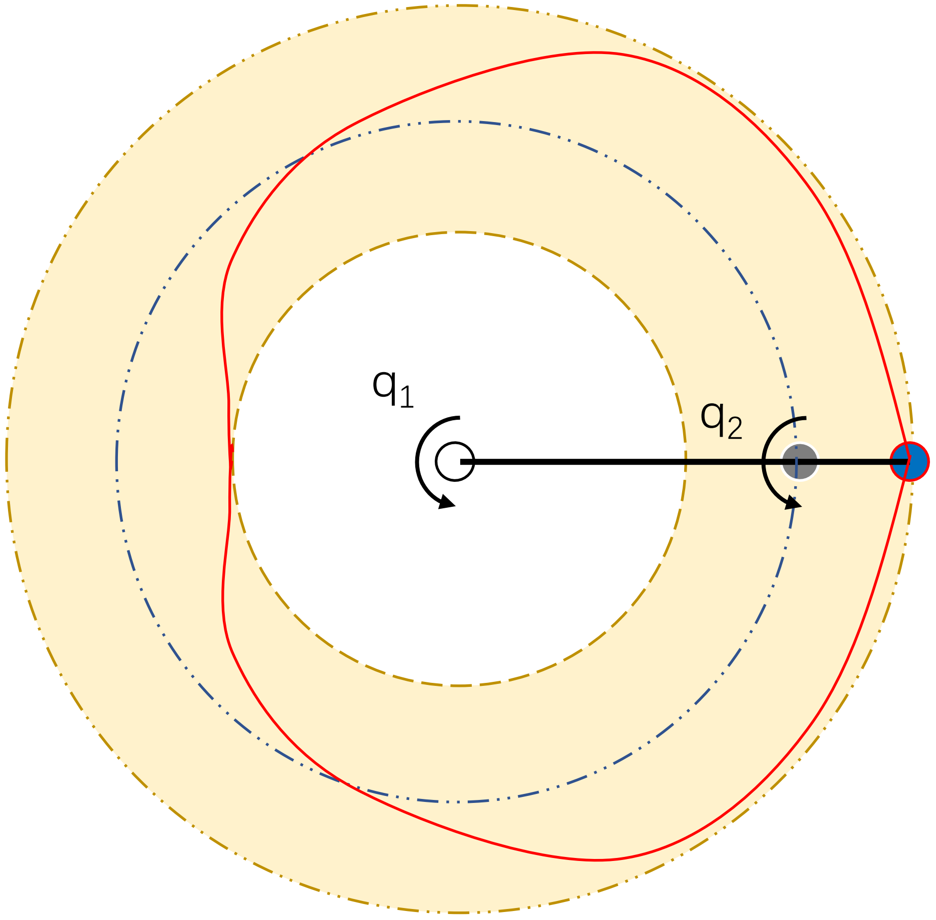
Cartesian VS. Configuration Space

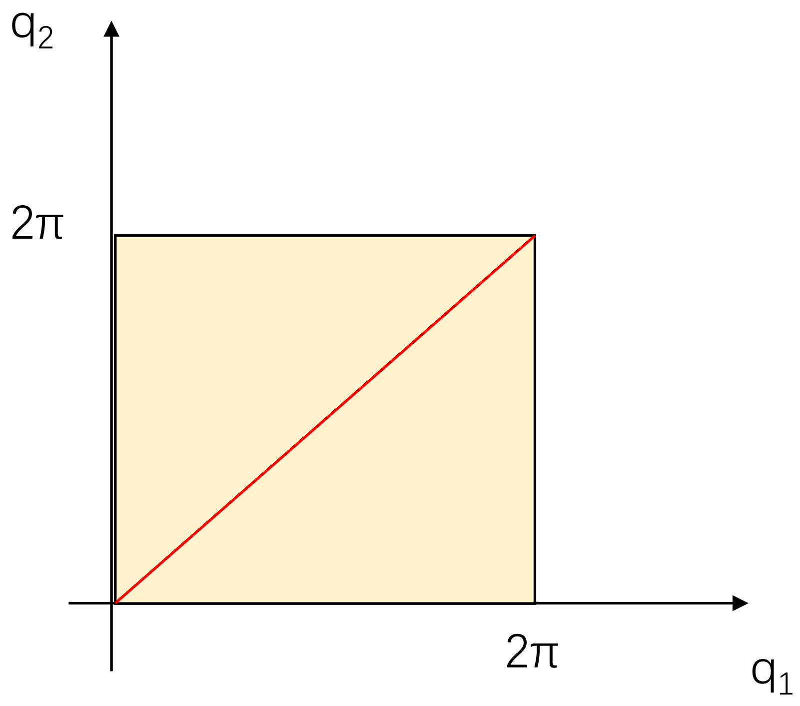




Path in Workspace and Configuration Space

Cartesian VS. Configuration Space





Types of Problems in Cartesian Scheme

- Intermediate points unreachable
- High joint rates near singularity
- Start and goal reachable in different solution

Types of Problems in Cartesian Scheme

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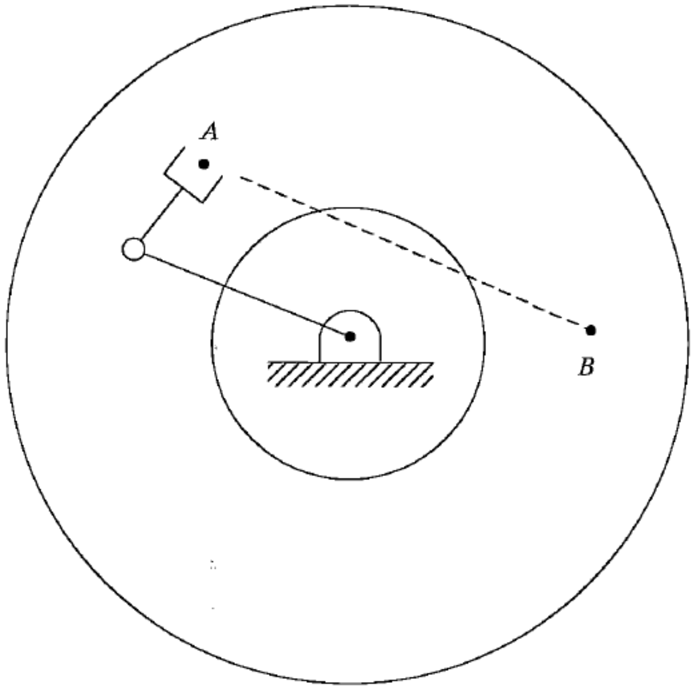

Types of Problems in Cartesian Scheme

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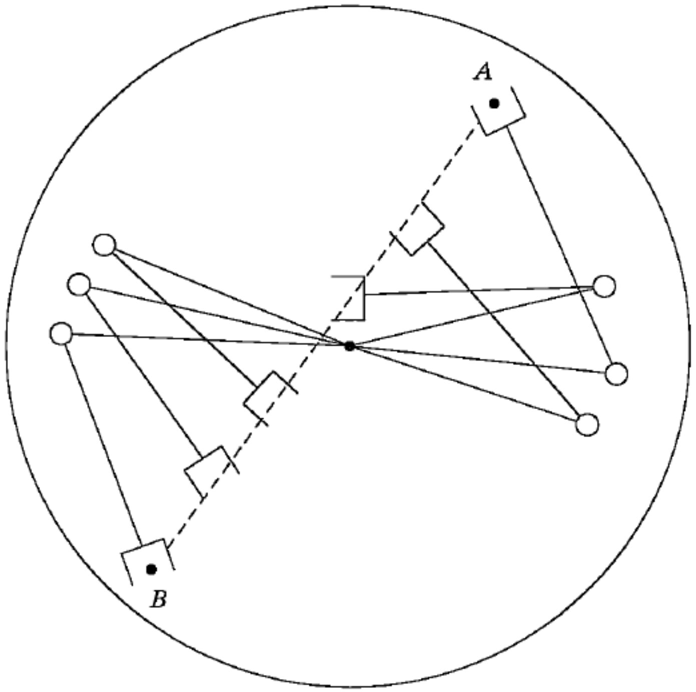
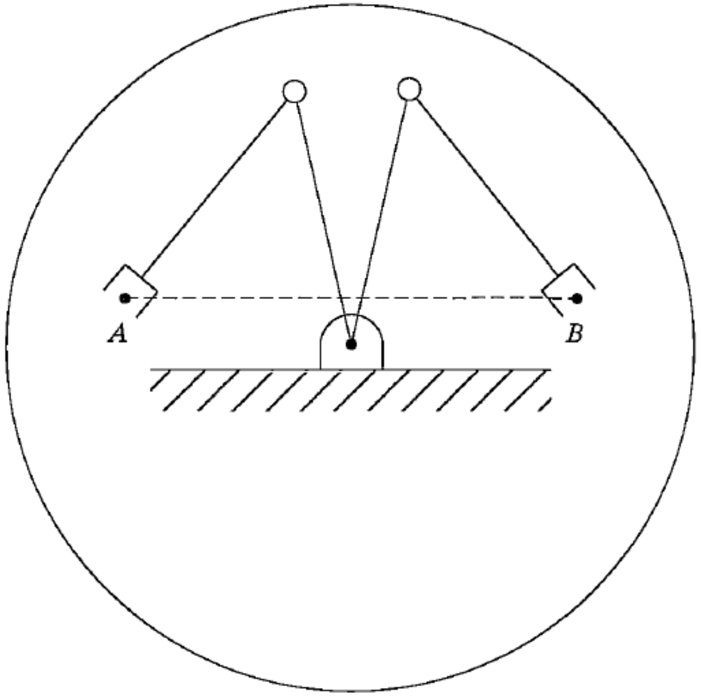
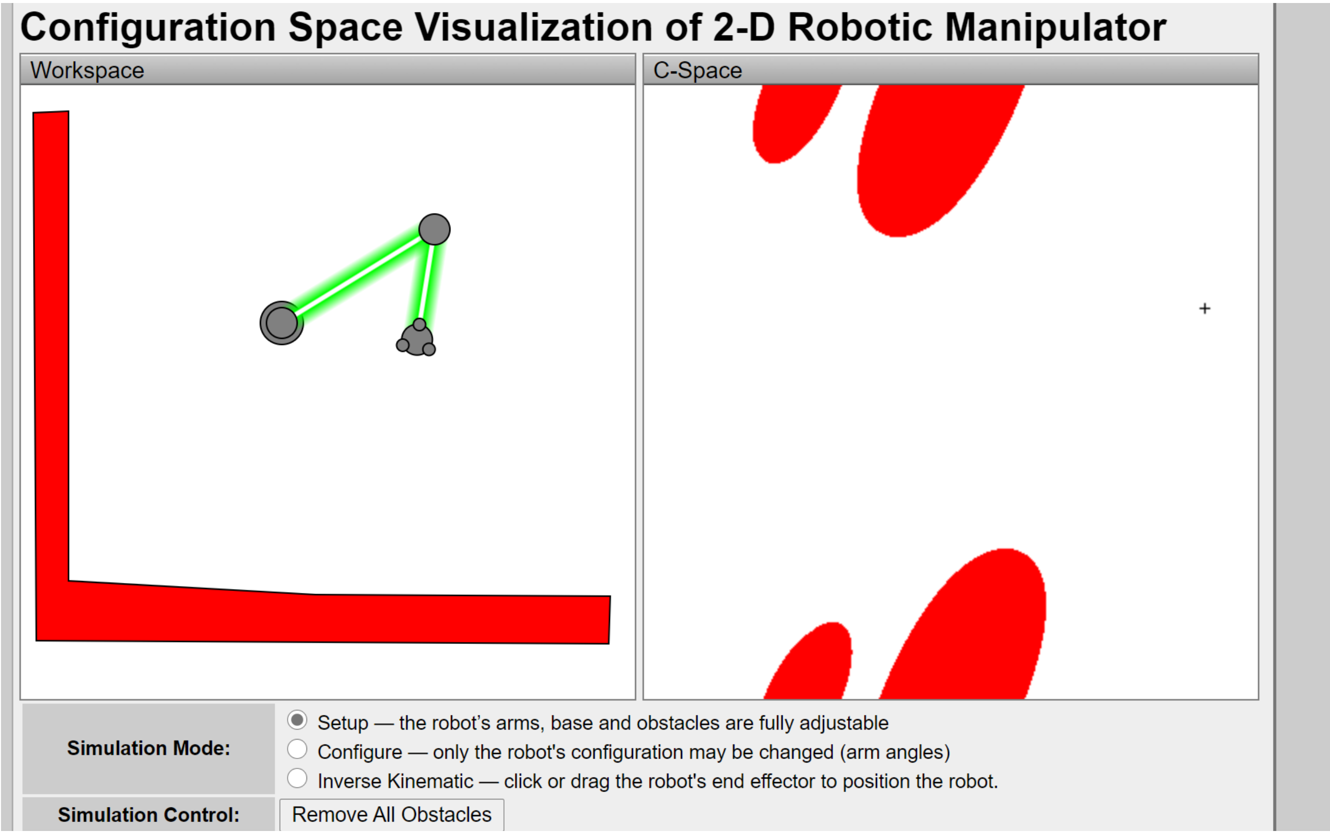


Types of Problems in Cartesian Scheme

- Start and goal reachable in different solution



Hands on Simulation

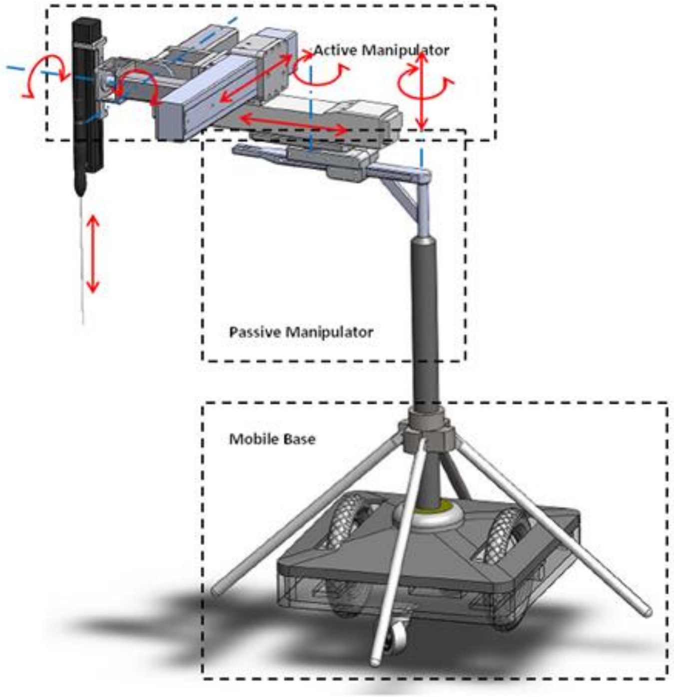


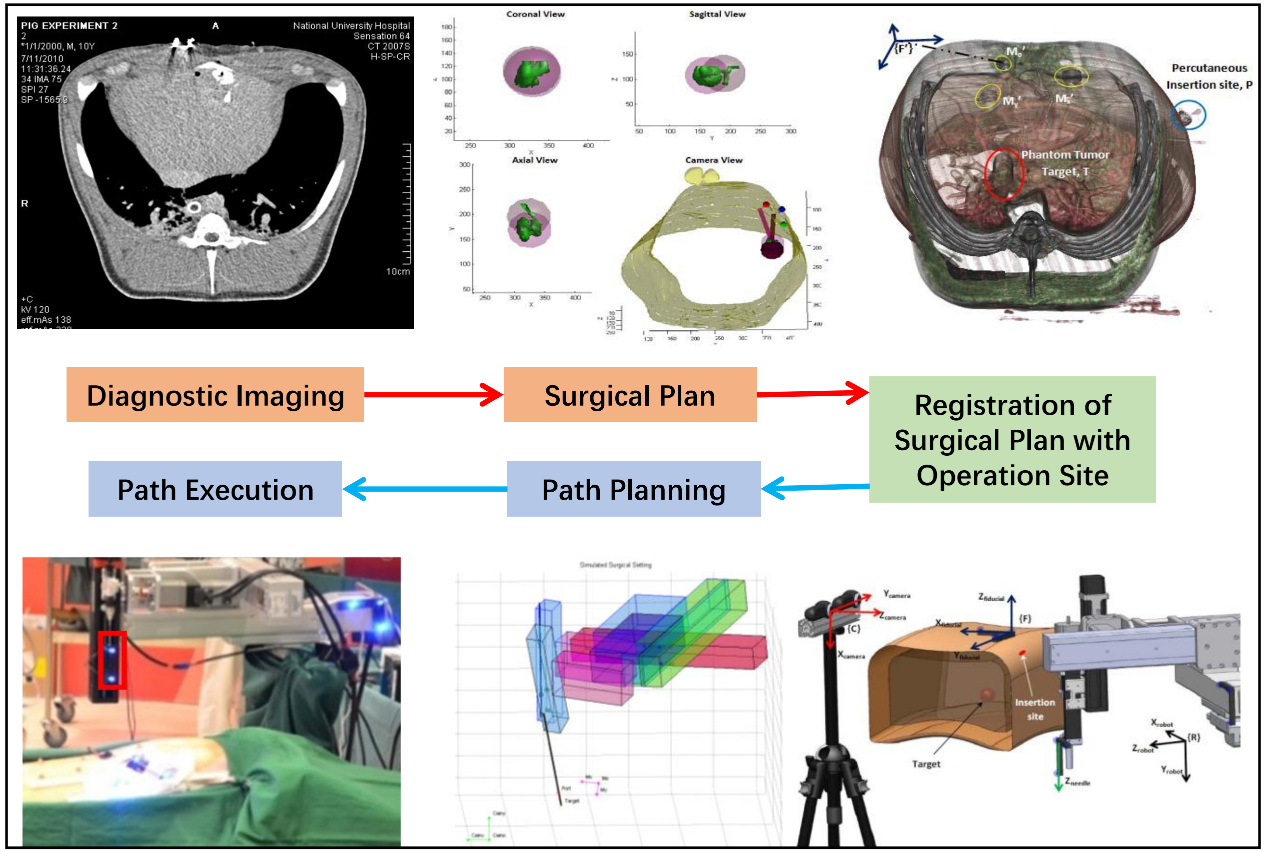
Go to:
https://www.cs.unc.edu/~jeffi/c-space/robot.xhtml

By Professor Ron Alterovitz, University of North Carolina at Chapel Hill

Case Examples

- Relook this example of needle insertion

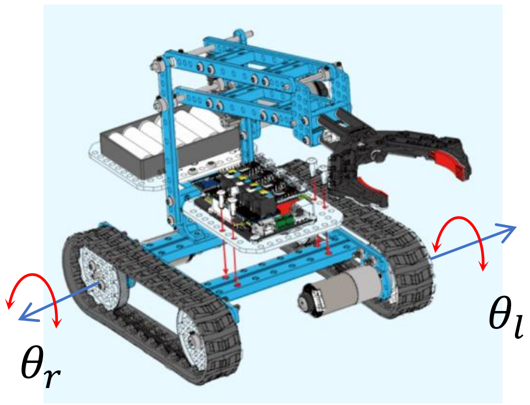


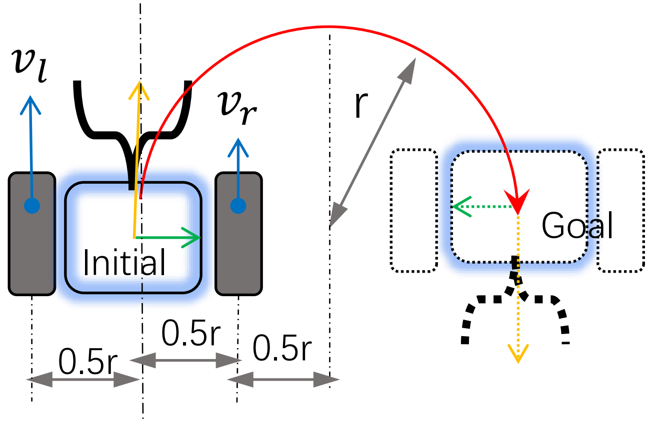


Joint-Space VS. Task-space

Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors





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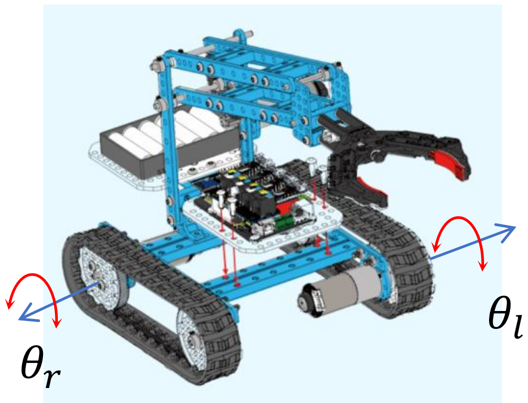

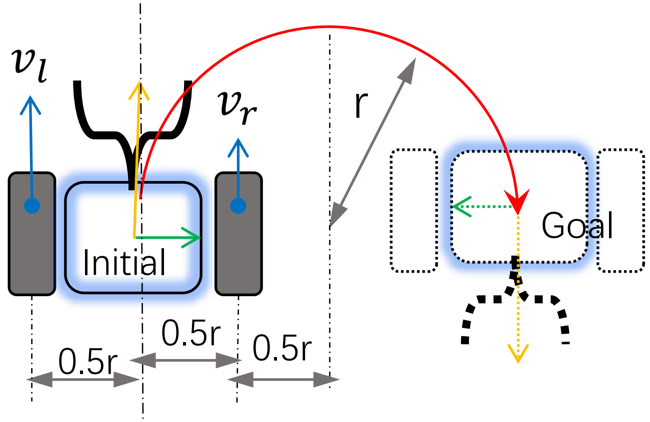

Path plan in task-space:

Recognizing the constraint:

Relating constraint to joint-space:

Joint-Space VS. Task-space

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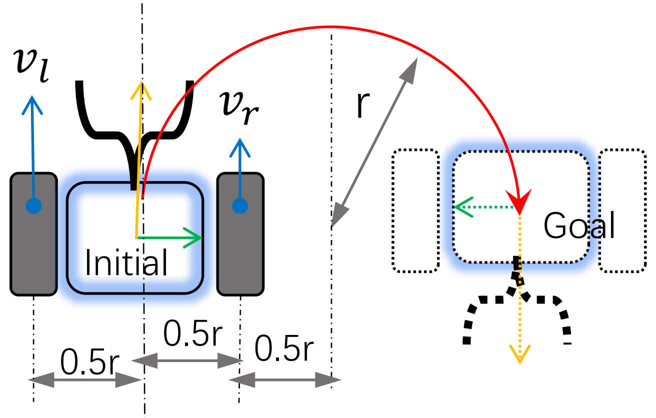

A) Given that the angular displacement of the right wheel θ_r follows a cubic trajectory from θ_0 to θ_f, describe angular trajectory of the left motor to maintain the distance r from the center of rotation. Assume that the coefficients are a0= a1= a2= a3=1.

B) Describe a control scheme to accomplish this trajectory-following application.

Joint-Space VS. Task-space

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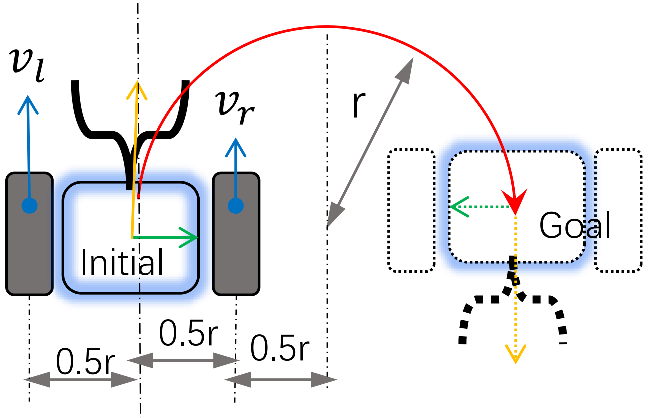

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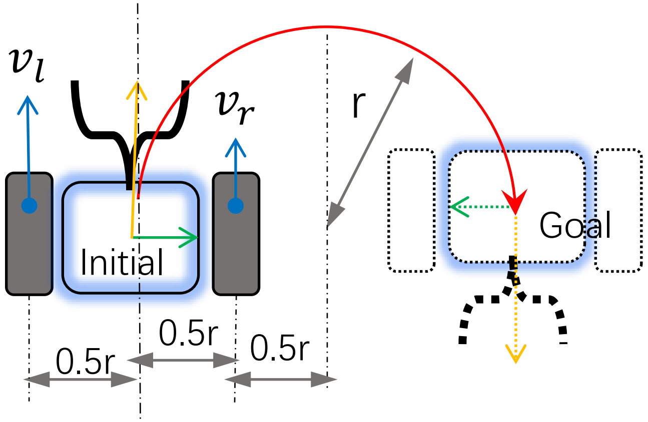
Joint-Space VS. Task-space

Mobile Robot Example



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Revision

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