



# ECE 470: Introduction to Robotics

## Lecture 25

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# Overview of Robot Vision

## O. Introduction to Robot Vision

- What is Robot Vision?

### I. Image Formation

- The science behind machine vision (+ represent as a form of signal)

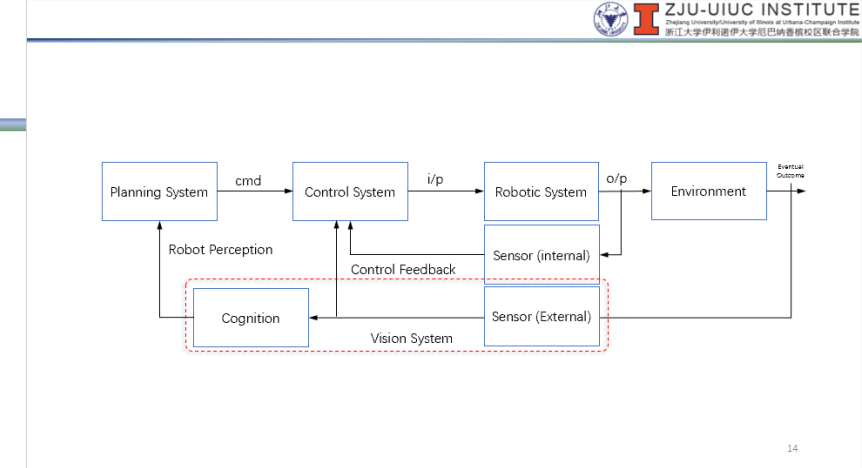
### II. Image Processing

- Common techniques to manipulate, enhance & analyse images



### III. Robot Vision Applications

- 3D Vision; Photogrammetry; Vision-based techniques in robotics- visual servo, pose estimation, localization, mapping, navigation



# Image Processing

- ☒ Thresholding & Histogram Processing
- ☒ Filtering
- ☒ Edge & Corner Detection
- Interest Points/ Feature Description
- Lines & Shapes



# Recap: Canny Edge Detection

- Involves 5 steps:
  1. Apply Gaussian filter: remove the noise
  2. Find the intensity gradients of the image
  3. Apply non-maximum suppression: remove spurious response
  4. Apply edge detection using two threshold value
  5. Finalize edge detection by hysteresis

## Canny Edge Detection

- Step (1): Apply Gaussian filter to smooth the image in order to remove the noise
  - Edge detection are easily affected by image noise
  - A Gaussian filter is applied to convolve with the image
    - $h(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$
  - This will smooth the image to reduce the effects of obvious noise on the edge detector

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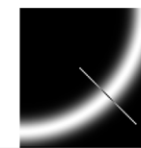
## Canny Edge Detection

- Step (2): Find the intensity gradients of the image
  - $\frac{\partial I(x, y)}{\partial x} = \frac{I(x+1, y) - I(x-1, y)}{2}$
  - $\frac{\partial I(x, y)}{\partial y} = \frac{I(x, y+1) - I(x, y-1)}{2}$
  - Gradient Vector:  $\nabla I(x, y) = \left[ \frac{\partial I(x, y)}{\partial x}, \frac{\partial I(x, y)}{\partial y} \right]^T$ 
    - $|\nabla I(x, y)| = \sqrt{\left( \frac{\partial I(x, y)}{\partial x} \right)^2 + \left( \frac{\partial I(x, y)}{\partial y} \right)^2}$
    - $\theta(x, y) = \tan^{-1} \left( \frac{\partial I(x, y)}{\partial y} / \frac{\partial I(x, y)}{\partial x} \right)$

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## Canny Edge Detection

- Step (3): Apply non-maximum suppression to get rid of spurious response to edge detection
  - compare values along the gradient vector to retain only the maximum



## Canny Edge Detection

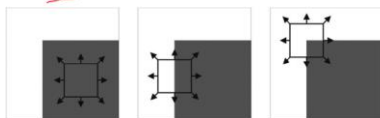
- Step (4): Apply edge detection using two threshold value  $K_H$  and  $K_L$ 
  - $Edge(x, y) = \begin{cases} E_{strong} & \text{if } |\nabla I(x, y)| > K_H \\ E_{average} & \text{if } K_L \leq |\nabla I(x, y)| \leq K_H \\ E_{weak} & \text{if } |\nabla I(x, y)| < K_L \end{cases}$
- Step (5): Finalize edge detection by hysteresis thresholding
  - Small  $K$  means more details
  - High  $K$  includes more noise
  - Hysteresis Thresholding allows us to apply both
    - Keep both high threshold  $K_H$  and low threshold  $K_L$
    - Any edges with magnitude  $< K_L$  are discarded
    - Any edges with magnitude  $> K_H$  are kept
    - An edge with magnitude between the two threshold values is kept if there is a path of edges with magnitude  $> K_L$  connecting the edge to another edge with magnitude  $> K_H$

# Recap: Harris Corner Detection

- Involves 5 steps:
  1. Apply Gaussian filter: remove the noise
  2. Find the intensity gradients of the image
  3. Apply non-maximum suppression: remove spurious response
  4. Apply edge detection using two threshold value
  5. Finalize edge detection by hysteresis

## Corner Detection

- Basic idea of corner detection: large change in appearance
  - Flat region: no change
  - Edge: no change along edge
  - Corner: significant **change in more than one** direction



Flat region Edge Corner  
Images taken from Introduction to Autonomous Mobile Robot

## Harris Corner Detector

- Consider taking an image patched centered on  $(u, v)$  and shifting it by  $(x, y)$ , the sum of square differences SSD between these two patches is:

$$SSD(x, y) = \sum_u \sum_v [I(u, v) - I(u + x, v + y)]^2$$

- Using first-order Taylor expansion,  
 $I(u + x, v + y) \approx I(u, v) + I_x(u, v)x + I_y(u, v)y$
- Hence, SSD becomes

$$SSD(x, y) \approx \sum_u \sum_v [I_x(u, v)x + I_y(u, v)y]^2 = \sum_u \sum_v [I_x^2 x^2 + 2xy I_x I_y + I_y^2 y^2]$$

## Harris Corner Detector

$$SSD(x, y) = \sum_u \sum_v [I_x^2 x^2 + 2xy I_x I_y + I_y^2 y^2]^2 = \sum_u \sum_v [x \ y] \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [x \ y] \sum_u \sum_v \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- $SSD(x, y) = [x \ y] M \begin{bmatrix} x \\ y \end{bmatrix}$
- Since  $M$  is symmetric, we can rewrite the matrix as:  
 $M = A^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} A$
- where  $\lambda_1$  and  $\lambda_2$  are the eigenvalues of  $M$

## Harris Corner Detector

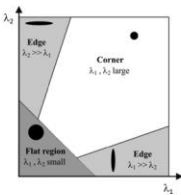
- As mentioned, a corner is characterized by a large variation of SSD in all direction, the larger the variation in that direction

- Both  $\lambda$  are small means flat region
- One strong and one weak  $\lambda$  means edge
- Two strong  $\lambda$  means corner

Quick way of Calculating Corner Response:

$$R = \lambda_1 \lambda_2 - k \cdot (\lambda_1 + \lambda_2)^2 = \det(M) - k \cdot \text{tr}(M)^2$$

where  $k$  is an empirically determined constant:  $k \in [0.04, 0.06]$ .



# Feature Extraction

- (Local) Features
  - (Locally) Unique structures e.g. Edges, Corners, Blobs
- Detector
  - Methods for deciding if a pixel is associated with particular a feature e.g. Canny, Harris, LoG
- Descriptor
  - Representations of (local) features

# Feature Extraction

- Ideal features should be
  - Repeatable
    - Robustly detectable in different viewpoints and imaging conditions (e.g. noise)
  - Distinctive
    - Uniquely representable for comparison
  - Localizable
    - Provide spatial information

# Feature Extraction

- Detector
  - Methods for deciding if a pixel is associated with particular a feature e.g. Canny, Harris, LoG

Detector	Feature Type	Function	Scale Independent
FAST [1]	Corner	<a href="#">detectFASTFeatures</a>	No
Minimum eigenvalue algorithm [4]	Corner	<a href="#">detectMinEigenFeatures</a>	No
Corner detector [3]	Corner	<a href="#">detectHarrisFeatures</a>	No
SURF [11]	Blob	<a href="#">detectSURFFeatures</a>	Yes
KAZE [12]	Blob	<a href="#">detectKAZEFeatures</a>	Yes
BRISK [6]	Corner	<a href="#">detectBRISKFeatures</a>	Yes
MSER [8]	Region with uniform intensity	<a href="#">detectMSERFeatures</a>	Yes
ORB [13]	Corner	<a href="#">detectORBFeatures</a>	No



# Feature Extraction

- Descriptor
  - Representations of (local) features

Descriptor	Binary	Function and Method	Invariance		Typical Use	
			Scale	Rotation	Finding Point Correspondences	Classification
HOG	No	<code>extractHOGFeatures(I, ...)</code>	No	No	No	Yes
LBP	No	<code>extractLBPFeatures(I, ...)</code>	No	Yes	No	Yes
SURF	No	<code>extractFeatures(I,points,'Method','SURF')</code>	Yes	Yes	Yes	Yes
KAZE	No	<code>extractFeatures(I,points,'Method','KAZE')</code>	Yes	Yes	Yes	Yes
FREAK	Yes	<code>extractFeatures(I,points,'Method','FREAK')</code>	Yes	Yes	Yes	No
BRISK	Yes	<code>extractFeatures(I,points,'Method','BRISK')</code>	Yes	Yes	Yes	No
ORB	Yes	<code>extractFeatures(I,points,'Method','ORB')</code>	No	Yes	Yes	No
<ul style="list-style-type: none"><li>• Block</li><li>• Simple pixel neighborhood around a keypoint</li></ul>	No	<code>extractFeatures(I,points,'Method','Block')</code>	No	No	Yes	Yes

# Feature Extraction Practical Consideration

- Select appropriate types of feature-detector-descriptor
  - Based on
  - Nature of scenes
  - Application context
  - Operation requirement

# Feature Matching

- Given views of the same scene, how to match the features?
  - Detect; Describe; Match; Transform (for image mapping) or Tracking (Visual tracking)

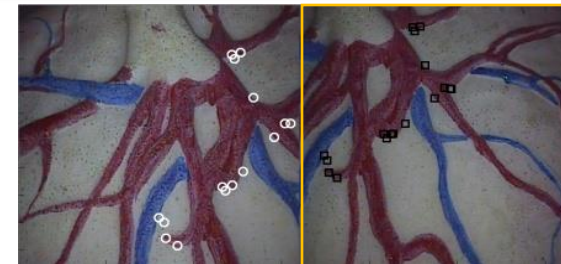
Feature Detection, Description,  
& Matching

Feature Matches

Detect



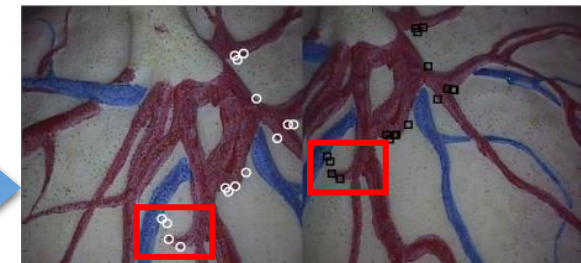
Describe



Match



Tracking



# Similarity Scores between image patches

- How do we measure the similarity/ difference?

# Similarity Score

- Sum of Squared Difference (SSD)

SSD between two image patches  $I_1$  and  $I_2$

$$w_{SSD}(x, y) = \sum_p^P \sum_q^Q [I_1(p, q) - I_2(p, q)]^2$$

Recall in corner detection, we look at change of patch  $I$  centered on  $(p, q)$  and itself when shifted by  $(x, y)$ , we used SSD to represent the change

$$w_{SSD}(x, y) = \sum_p^P \sum_q^Q [I(p, q) - I(p + x, q + y)]^2$$

# Similarity Score

- Normalized Cross-correlation (NCC)

For a  $U \times V$  image and  $P \times Q$  patch, the cross-correlation  $w_{cc}(u, v)$  at a particular image coordinates  $(u, v)$  of a template patch  $g(p, q)$  and the image  $f(p, q)$  is

$$w_{cc}(u, v) = \sum_{p=0}^P \sum_{q=0}^Q g(p, q) f(p+u, q+v)$$

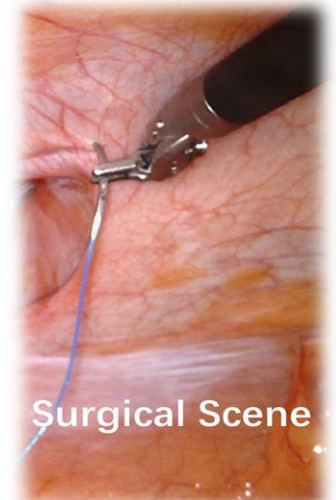
To reduce the over sensitivity toward intensity variant, normalized cross-correlation coefficient (Similarity Score),

$$w_{ncc}(u, v) = \frac{\sum_{p=0}^P \sum_{q=0}^Q (g(p, q) - \bar{g})(f(p+u, q+v) - \bar{f}(u, v))}{\left[ \left( \sum_{p=0}^P \sum_{q=0}^Q (g(p, q) - \bar{g})^2 \right) \left( \sum_{p=0}^P \sum_{q=0}^Q (f(p+u, q+v) - \bar{f}(u, v))^2 \right) \right]^{0.5}}$$

is used where  $\bar{g}$  and  $\bar{f}$  denotes the mean intensity in the template and overlapping region, respectively.

# Detection of Line and Shape

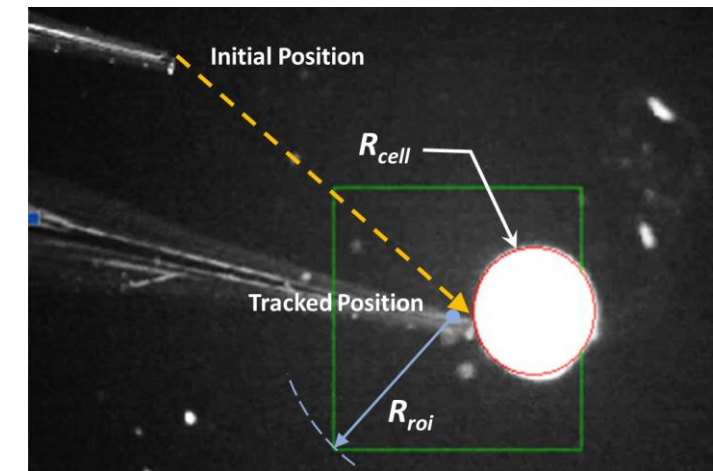
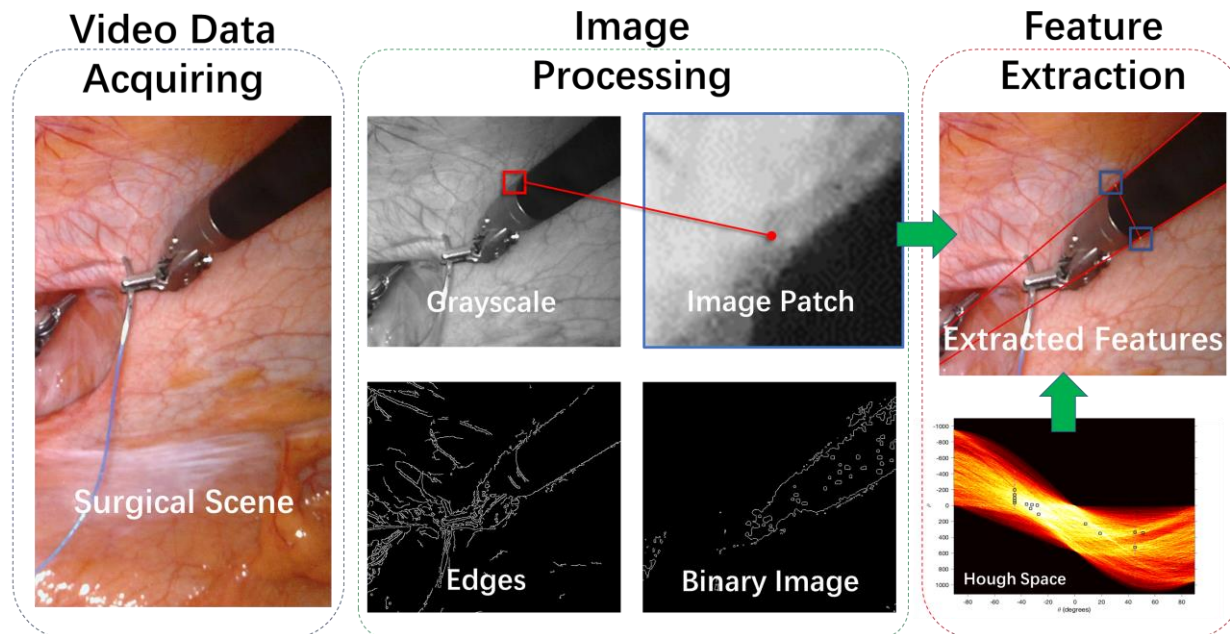
- After detecting the edges and local interest points, how do we detect lines and other geometries?
- A problem of pattern recognition





# Detection of Line and Shape

- After detecting the edges and local interest points, how do we detect lines and other geometries?
- A problem of pattern recognition

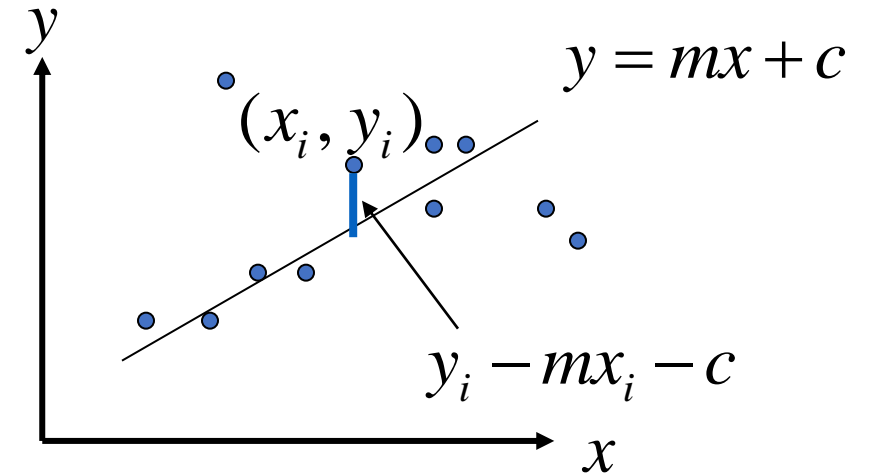


Circle detection



# Fitting lines to edges using least square

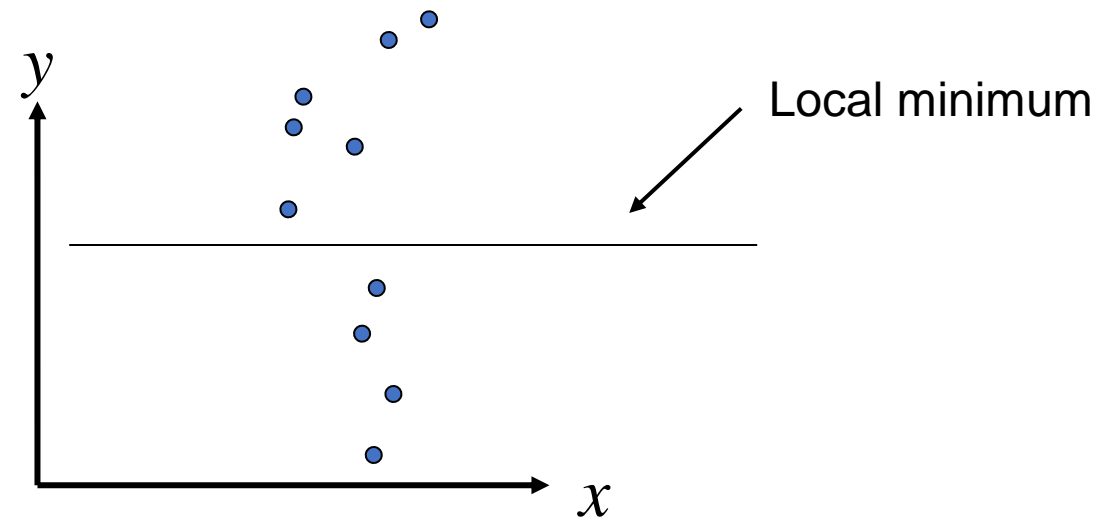
- Given:
  - Many edges  $(x_i, y_i)$
  - Equation of line:  $y = mx + c$
  - Parameters:  $m, c$
- Objective function is to minimize average square distance:
  - $E = \sum \frac{(y_i - mx_i - c)^2}{N}$
- Using  $\frac{\partial E}{\partial m} = 0$  &  $\frac{\partial E}{\partial c} = 0$



$$c = \bar{y} - m \bar{x}$$

$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

# Fitting lines to edges using least square



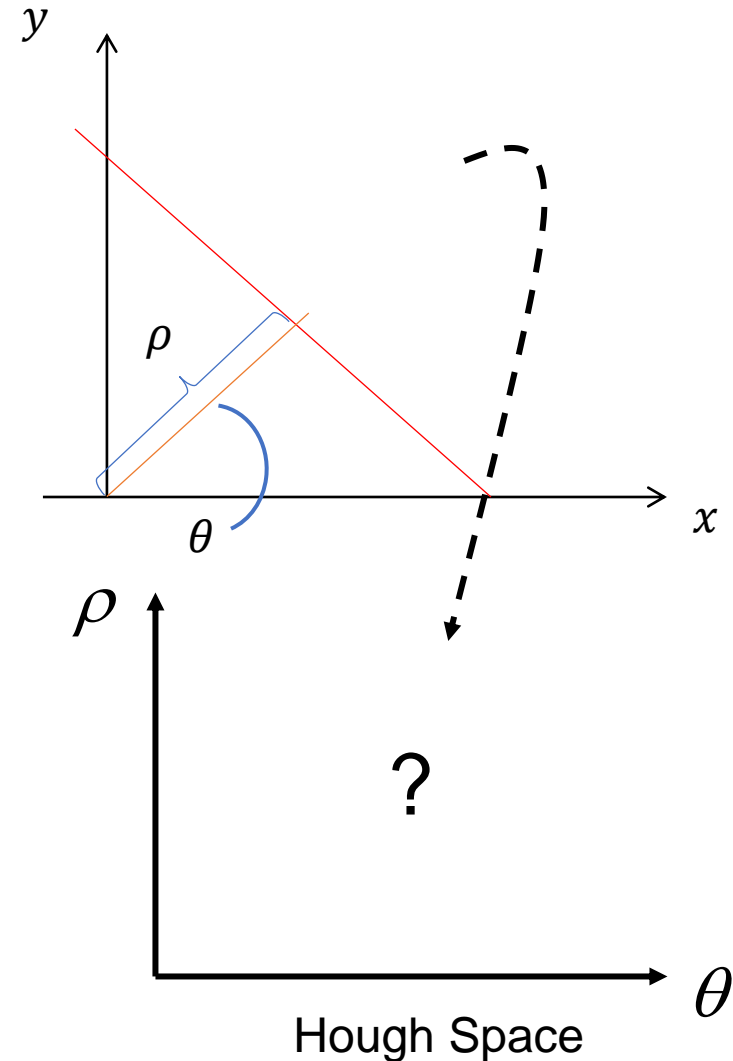
- Possible Solution: Hough Transformation

# Hough Transform

- Ability to detect edges
- What about lines? Shapes (eg: circle)?
- Hough Transform
  - Elegant method for direct object recognition
  - Edges need not be connected
  - Key idea: edges “vote” for the possible model

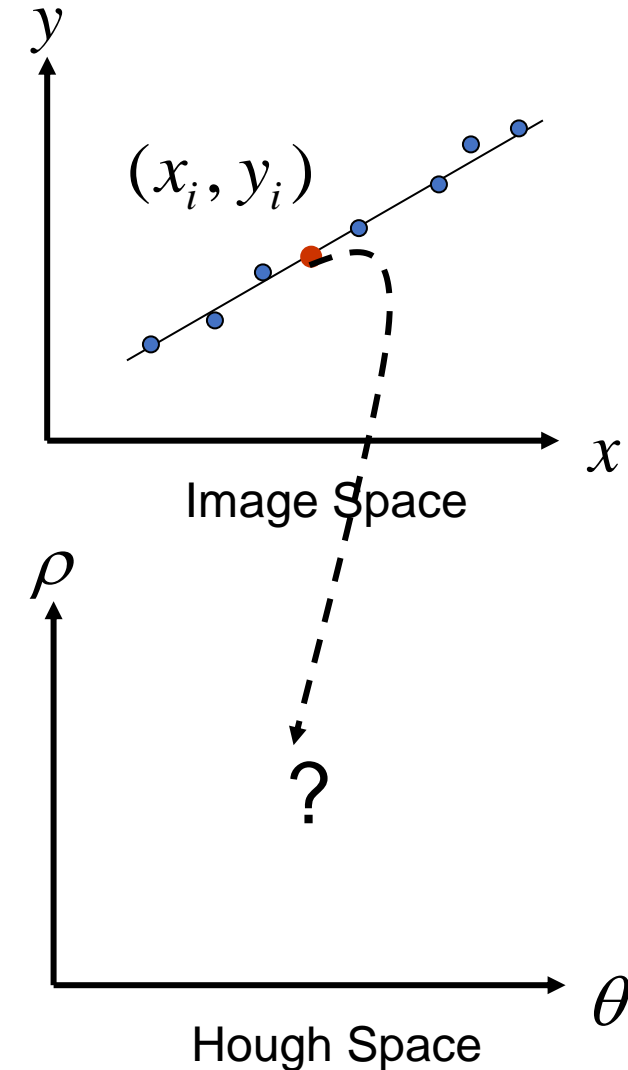
# Hough Space – $(\rho, \theta)$ space

- $\rho = x \cos \theta + y \sin \theta$
- $y = \frac{\rho}{\sin \theta} - \frac{x}{\tan \theta}$
- Line equation:  $\rho = x \cos \theta + y \sin \theta$ 
  - Parameters:  $\rho$  and  $\theta$
  - Where  $0 \leq \theta \leq 2\pi$



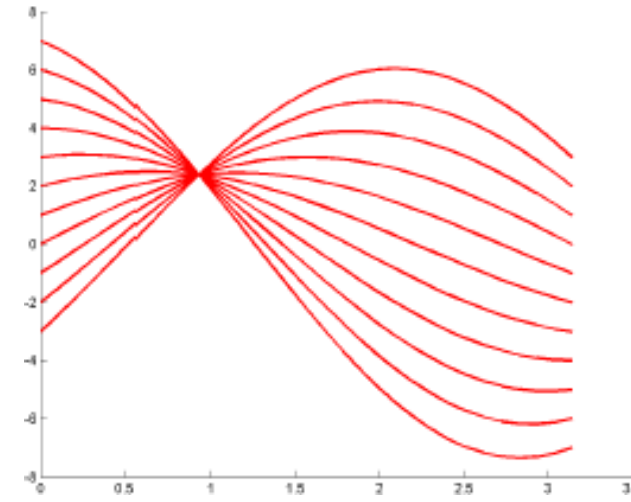
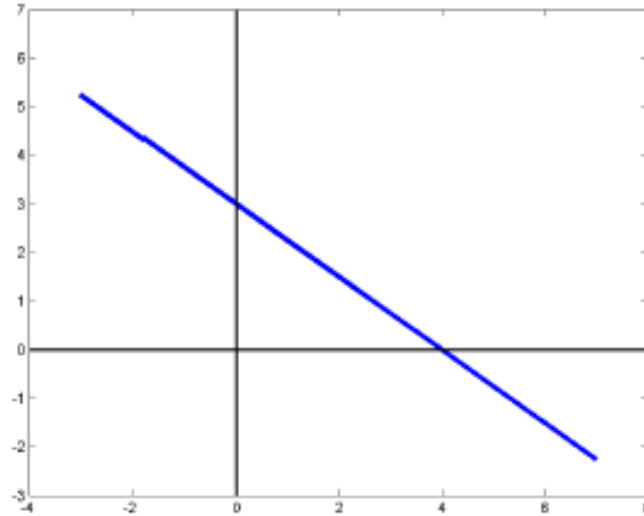
# Hough Space – $(\rho, \theta)$ space

- Line equation:  $\rho = x \cos \theta + y \sin \theta$ 
  - Parameters:  $\rho$  and  $\theta$
  - Where  $0 \leq \theta \leq 2\pi$
- How to map onto the Hough Space (sometimes also known as parameter space)?
  - $\rho = x_i \cos \theta + y_i \sin \theta$
  - Sinusoid curve



# Hough Space – $(\rho, \theta)$ space

- $\rho = x \cos \theta + y \sin \theta$
- Points in picture  $\rightarrow$  sinusoids in parameter space
- Points in parameter space  $\rightarrow$  lines in picture
- There will be a unique intersection point if the points in the picture form a straight line



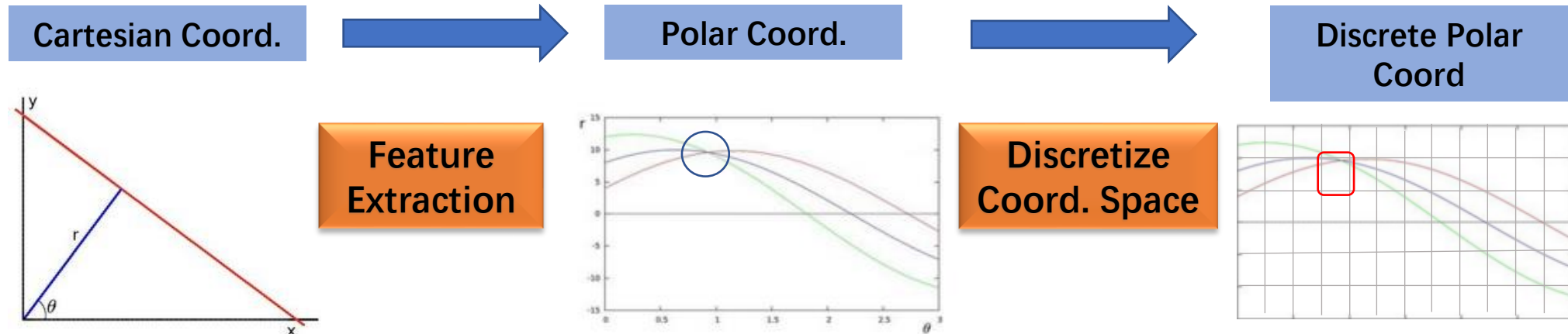
But how to choose the solution if there are noise?

# Quantizing parameter space and Voting

- Discretize the  $(\rho, \theta)$  space
  - For each point  $(x_i, y_i)$ , compute only for a finite set of angles  $\theta = \theta_1, \theta_2, \dots, \theta_N$
  - For each  $\theta_j$ , obtain  $\rho_{ij} = x_i \cos \theta_j + y_i \sin \theta_j$
- Create a matrix, called the accumulator matrix
  - Each column corresponds to angles  $\theta = \theta_1, \theta_2, \dots, \theta_N$
  - Each row corresponds to the “bins” (intervals) of the resulting distance  $\rho$
- Voting:
  - For each point in the image and for each  $\theta_j$ , compute the  $\rho_{ij}$ , and increment the corresponding element of the accumulator matrix
  - Highest value means highest “vote”

# Quantizing parameter space and Voting

- Voting (continue)
  - If more than one line, you can set a threshold value (number of vote) to obtain more lines
  - For example, if number of votes (ie value in that element) is more than the threshold value, then consider that  $(\rho, \theta)$  to be a line





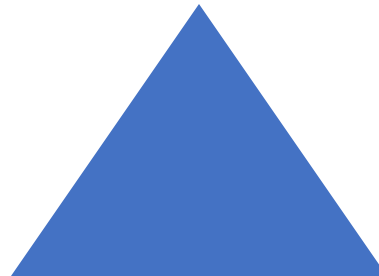
# Case Problem (1)

Using what you have learnt, design an algorithm that is capable of identifying the following simple blocks on white background.

1)Rectangle



2)Triangle



**Solution**

# Case Problem (1) Solution

- Stage (1): **Edge detection** using Canny edge detection
  - Apply Gaussian filter to smoothen the image in order to remove the noise
  - Find the intensity gradients vector of the image
    - $|\nabla I(x, y)| = \sqrt{\left(\frac{\partial I(x, y)}{\partial x}\right)^2 + \left(\frac{\partial I(x, y)}{\partial y}\right)^2}$
    - $\theta(x, y) = \tan^{-1} \left( \frac{\partial I(x, y)}{\partial y} / \frac{\partial I(x, y)}{\partial x} \right)$
  - Apply non-maximum suppression to get rid of spurious response to edge detection
  - Apply edge detection using two threshold value  $K_H$  and  $K_L$ 
    - $Edge(x, y) = \begin{cases} E_{strong} & \text{if } |\nabla I(x, y)| > K_H \\ E_{average} & \text{if } K_L \leq |\nabla I(x, y)| \leq K_H \\ E_{weak} & \text{if } |\nabla I(x, y)| < K_L \end{cases}$  Finalize edge detection by hysteresis
    - Any edges with magnitude  $< K_L$  are discarded
    - Any edges with magnitude  $> K_H$  are kept
    - An edge with magnitude between the two threshold values is kept if there is a path of edges with magnitude  $> K_L$  connecting the edge to another edge with magnitude  $> K_H$

# Case Problem (1) Solution

- Stage (2): Hough Transformation – **line detection**
  - Use  $\rho = x \cos \theta + y \sin \theta$
  - Map it onto the Hough space –  $(\rho, \theta)$  space
  - Intersection point exist if the points in the picture form a straight line
  - As more than one line is involved, various  $(\rho, \theta)$  are considered to be a line number if number of votes is more than the threshold value

# Case Problem (1) Solution

- Stage (2): Hough Transformation - **voting**
  - Discretize the  $(\rho, \theta)$  space
    - For each point  $(x_i, y_i)$ , compute only for a finite set of angles  $\theta = \theta_1, \theta_2, \dots, \theta_N$
    - For each  $\theta_j$ , obtain  $\rho_{ij} = x_i \cos \theta_j + y_i \sin \theta_j$
  - Create a matrix, called the accumulator matrix
    - Each column corresponds to angles  $\theta = \theta_1, \theta_2, \dots, \theta_N$
    - Each row corresponds to the “bins” (intervals) of the resulting distance  $\rho$
  - Voting:
    - For each point in the image and for each  $\theta_j$ , compute the  $\rho_{ij}$ , and increment the corresponding element of the accumulator matrix
    - Highest value means highest “vote”

# Case Problem (1) Solution

- Stage (3) – Shape identification
  - Rectangle:
    - 4 vertices
    - All angles are  $\sim 90$  degrees apart
      - Checked using  $\theta$
  - Triangle
    - 3 vertices



# FYI: Digit Recognition

- Contour is used in the digit recognition program

1AB

# Detection of other shapes

- Hough Transform can be generalized to find other shapes like circles and ellipses.
- However, the computational complexity increases
  - More computational time is required

# Hough circle transform

- Equation of circle:

$$(x - a)^2 + (y - b)^2 = R^2$$

- $(a, b)$  is the center of the circle
- $R$  is the radius of the circle

- Parameters:  $a, b, R$

- Rewriting the equation:

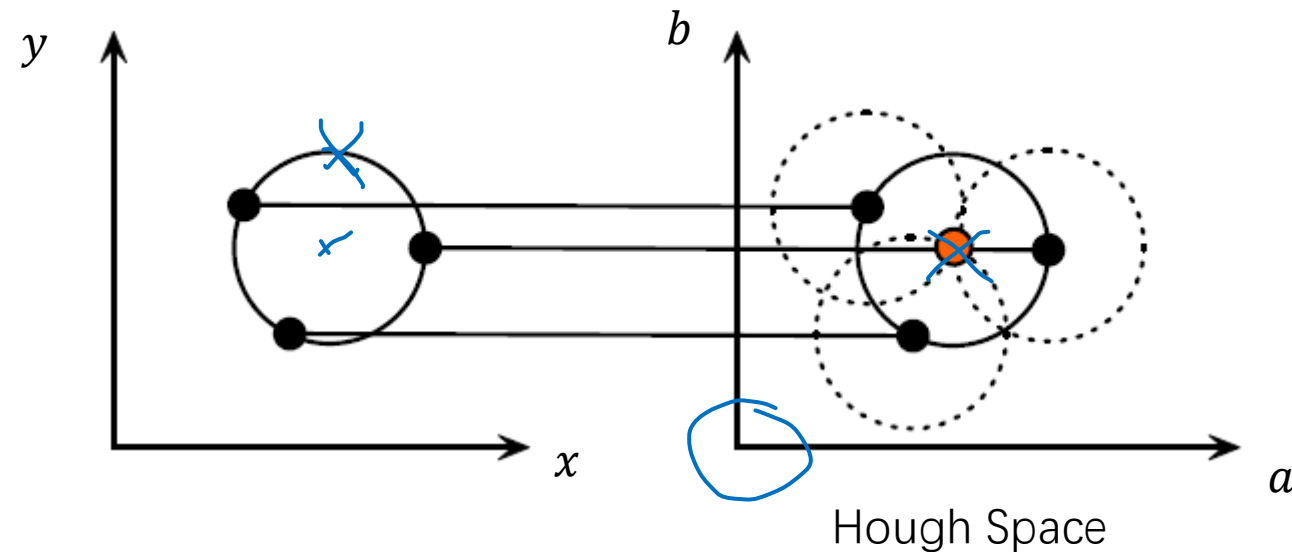
$$(a - x)^2 + (b - y)^2 = R^2$$



# Circle with known $R$

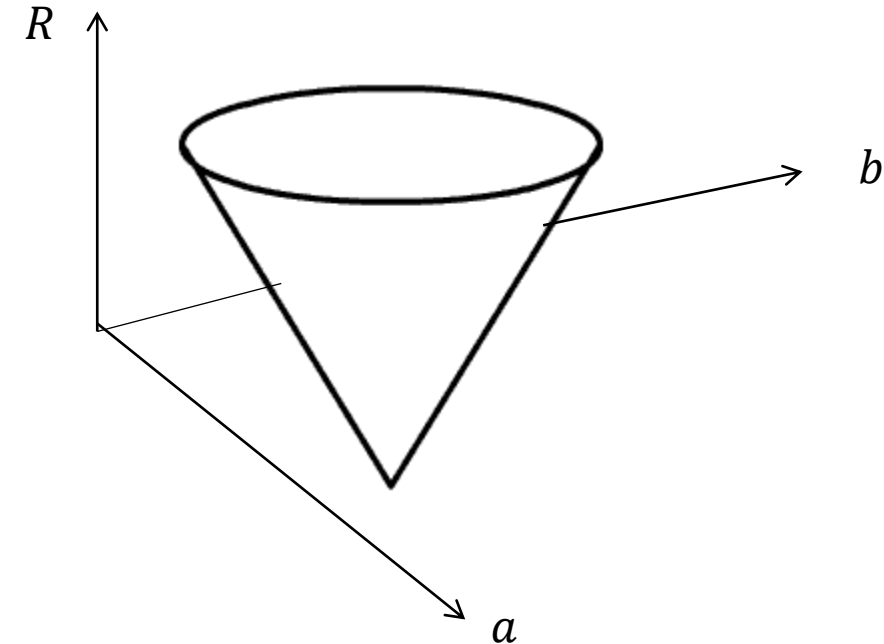
$$(a - x)^2 + (b - y)^2 = R^2$$

- Parameters:  $a, b$
- Points (edge) in picture  $\rightarrow$  circle in parameter space
- Points in parameter space  $\rightarrow$  circle in picture
- Like line detection, discretization and voting are used
- Example:



# Circle with unknown $R$

- Parameters:  $a, b, R$
- Points (edge) in picture  $\leftrightarrow$  conical surface in parameter space
- Additional computational time

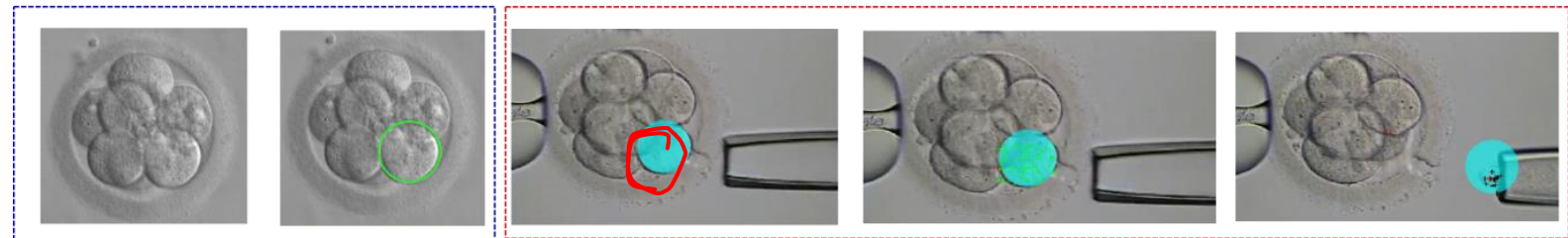


# Application: Extraction of Blastomere

- Embryo Biopsy
- 8-cell stage
- Tracking of the extraction of blastomere for biopsy



First Frame



Circle detection

Blastomere tracking