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ECE 470: Introduction to Robotics Homework 4

Question 1.

You are tasked to design the joint control for the manipulator arm analyzed in Homework 3. Using the control partitioning law, and the formulated dynamics in Homework 3, fill in the required expression (a)-(d) in the block diagram shown in Figure 1. *(6 Points)*

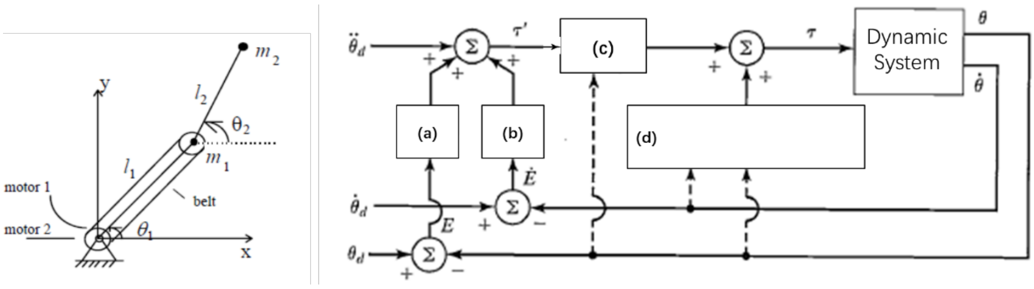


Figure 1.

Question 2.

a) Consider a single-link manipulator arm shown Figure 2.

i) Formulate the dynamic equation factoring coulomb and viscous force acting on the joint and loading due to gravity. You may assume that viscous and coulomb frictions result in resisting moment $M_v = b_v\dot{\theta}$ and $M_c = b_c \text{sgn}\,(\dot{\theta})$, respectively. *(2 Points)*

ii) Draw the control block diagram illustrating the use of control law partitioning for the non-linearities in the system. *(8 Points)*

You may assume negligible mass distribution for the link and assumed lumped mass at the distal time as shown.

b) A DC motor is installed to drive the arm with motor torque τ_m input to a gear transmission ratio η. Redraw the control diagram incorporating the dynamics of the actuator. *(4 Points)*

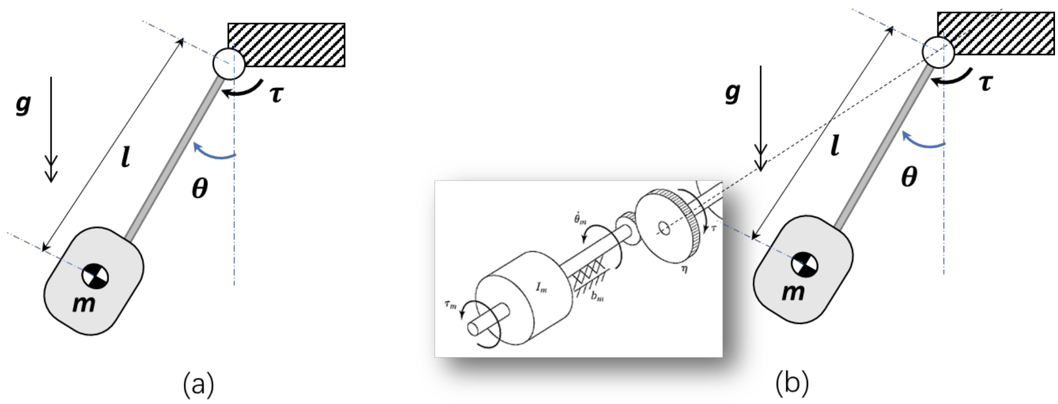


Figure 2 (a)-(b).

Q1. $\tau = \begin{bmatrix} (m_1+m_2)l_1^2 & m_2 l_1 l_2 \cos(\theta_2-\theta_1) \\ m_2 l_1 l_2 \cos(\theta_2-\theta_1) & m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -m_2 l_1 l_2 \sin(\theta_2-\theta_1)\dot{\theta}_2^2 \\ m_2 l_1 l_2 \sin(\theta_2-\theta_1)\dot{\theta}_1^2 \end{bmatrix}$

$+ \begin{bmatrix} (m_1+m_2) g l_1 \cos\theta_1 \\ m_2 g l_2 \cos\theta_2 \end{bmatrix} = M_x(\theta)\ddot{\theta} + V(\theta)\dot{\theta} + G(\theta) = m\ddot{x} + b\dot{x} + kx$

$\begin{cases} f = \alpha f' + \beta \\ f' = -k_p x - k_v \dot{x} \end{cases}$

a) k_p

b) k_v

c) $\begin{bmatrix} (m_1+m_2)l_1^2 & m_2 l_1 l_2 \cos(\theta_2-\theta_1) \\ m_2 l_1 l_2 \cos(\theta_2-\theta_1) & m_2 l_2^2 \end{bmatrix}$

d) $\begin{bmatrix} -m_2 l_1 l_2 \sin(\theta_2-\theta_2)\dot{\theta}_2^2 \\ m_2 l_1 l_2 \sin(\theta_2-\theta_1)\dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} (m_1+m_2) g l_1 \cos\theta_1 \\ m_2 g L_2 \cos\theta_2 \end{bmatrix}$

Q2 a) $\tau = M(\theta)\ddot{\theta} + V(\theta)\dot{\theta} + G(\theta) + F(\theta, \dot{\theta})$

i. $= ml^2\ddot{\theta} + V(\theta)\dot{\theta} + mgl\cos\theta + M_v + M_c$

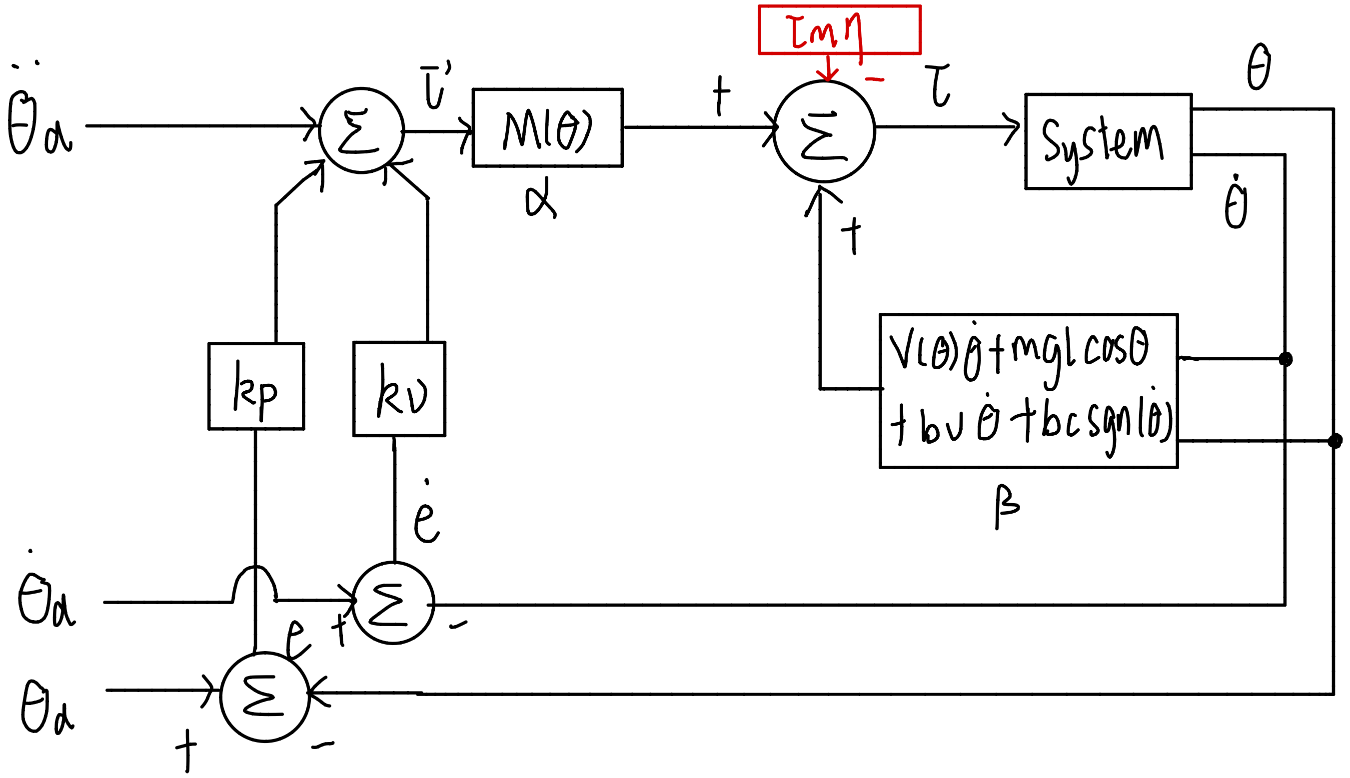
where $M_v = b_v\dot{\theta}$, $M_c = b_c \operatorname{sgn}(\dot{\theta})$

ii. By control raw, $\tau = \alpha\tau' + \beta$

$\Rightarrow \begin{cases} \alpha = M(\theta) = ml^2 \\ \beta = V(\theta)\dot{\theta} + mgl\cos\theta + b_v\dot{\theta} + b_c \operatorname{sgn}(\dot{\theta}) \end{cases}$

$\tau' = \ddot{\theta}_d + k_v \dot{e} + k_p e$, where $e = \theta_d - \theta$

$\dot{e} = \dot{\theta}_d - \dot{\theta}$



b) Changes drawn in red in the figure diagram of part a.

$$\tau = ml^2\ddot{\theta} + V(\theta)\dot{\theta} + mgl\cos\theta + b_v\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) - \tau_m\eta$$