ECE 470: Introduction to Robotics

Lecture 19

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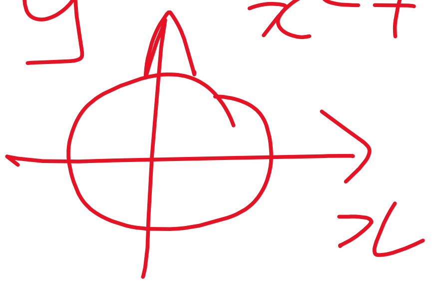
Quick Recap

- Last week
  - The overview of Robot Planning
    - Path
    - Trajectory
    - Motion
  - Trajectory Generation
    - Joint-Space Scheme
    - Cartesian-Space Scheme
  - Issues and Challenges in Motion Planning

This Lecture

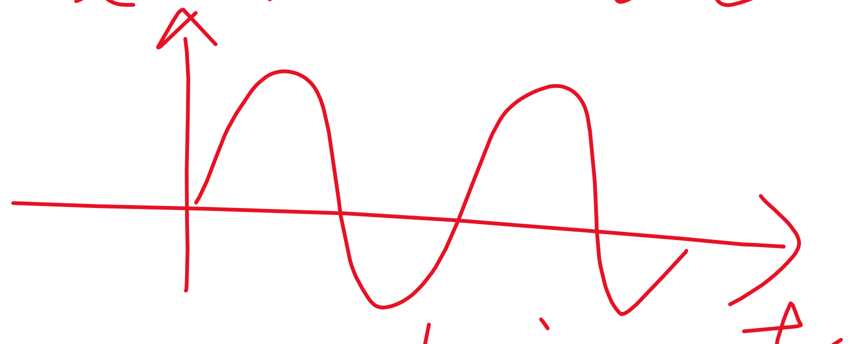
Quick Recap

$x^2 + y^2 = r^2$



path

$x = \cos\omega t$



trajectory

- Last week
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    - Joint-Space Scheme
    - Cartesian-Space Scheme
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This Lecture

q θ

Joint space

x y

Cartesian

Joint Scheme: Polynomial Function

Cubic Function

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

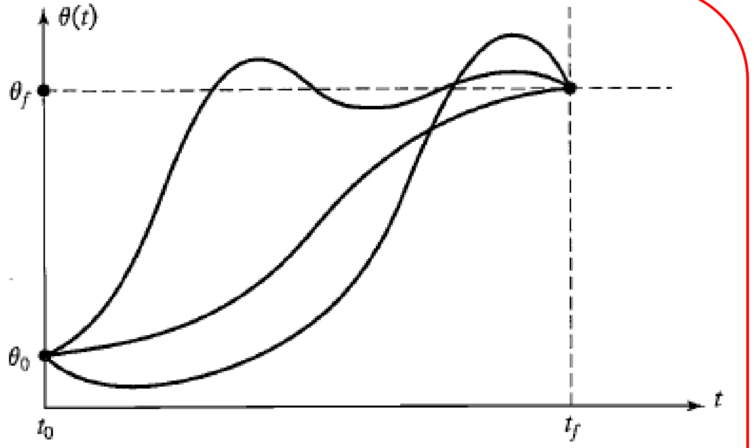
Parameter

$$a_0 = \theta_0$$

$$a_1 = \dot{\theta}_0$$

$$a_2 = \frac{3}{t^2{}_f}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$a_3 = -\frac{2}{t^3{}_f}(\theta_f - \theta_0) + \frac{1}{t^2{}_f}(\dot{\theta}_f - \dot{\theta}_0)$$



Boundary conditions

$$\theta_0 = \theta(0) = a_0$$

$$\theta_f = \theta(t_f) = a_0 + a_1 t_f + a_2 t_f{}^2 + a_3 t_f{}^3$$

$$\dot{\theta}_0 = a_1$$

$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f{}^2$$

Parameter

$$a_0 = \theta_0$$

$$a_1 = \dot{\theta}_0$$

$$a_2 = \frac{3}{t^2{}_f}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

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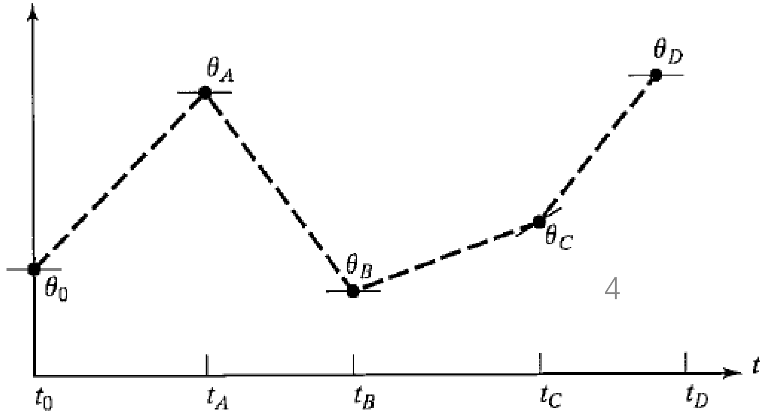


Linear segment with parabolic blends

continuity b/w segments: equal gradient

constant acceleration: parabolic curve

$$\ddot{\theta} t_b = \frac{\theta_h - \theta_b}{t_h - t_b} \qquad \theta_b = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2$$

$$\theta_h = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2 + \ddot{\theta} t_b (t_h - t_b)$$

Symmetrical

θ_f

$$\theta_h = \cancel{\theta_0}\, f - \frac{1}{2}\ddot{\theta} t_b^2 - \ddot{\theta} t_b (t_h - t_b)$$

error in class

Combining

$$\ddot{\theta} t_b^2 - \ddot{\theta} t_f t_b + \theta_f - \theta_0 = 0$$

Usually, an acceleration, is chosen and the above equation is solved for the corresponding t_b.

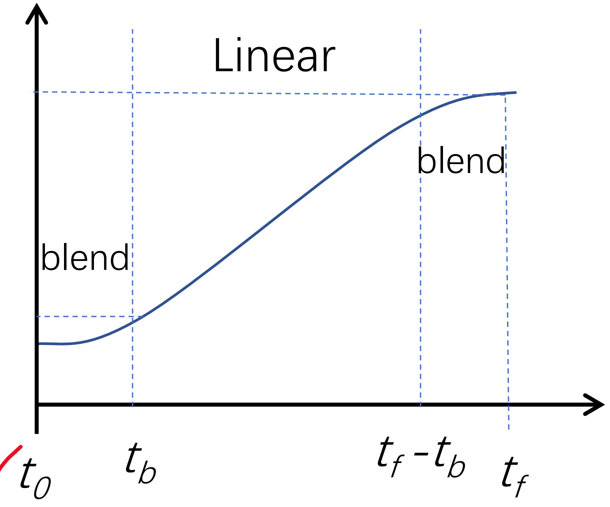
$(\ddot{\theta})^2$

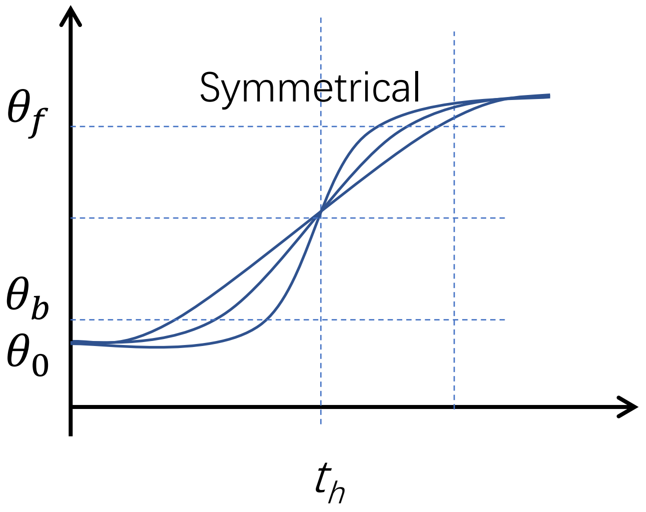
$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta} t_b^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

error

For real solutions to exist, acceleration need to meet the criteria

$$\ddot{\theta} \geq \frac{4(\theta_f - \theta_0)}{{t_f}^2}$$





Linear segment with parabolic blends

continuity b/w segments: equal gradient

constant acceleration: parabolic curve

$$\ddot{\theta} t_b = \frac{\theta_h - \theta_b}{t_h - t_b} \qquad \theta_b = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2$$

parabolic

1 $$\theta_h = \theta_0 + \underbrace{\frac{1}{2}\ddot{\theta} t_b^2}_{\text{parabolic}} + \underbrace{\ddot{\theta} t_b (t_h - t_b)}_{\text{linear}}$$ t

Symmetrical

2 $$\theta_h = \theta_f - \frac{1}{2}\ddot{\theta} t_b^2 - \ddot{\theta} t_b (t_h - t_b)$$

$t_f = 2t_h$ parabolic

Combining

2 - 1 $$\ddot{\theta} t_b^2 - \ddot{\theta} t_f t_b + \theta_f - \theta_0 = 0$$

Usually, an acceleration, is chosen and the above equation is solved for the corresponding t_b.

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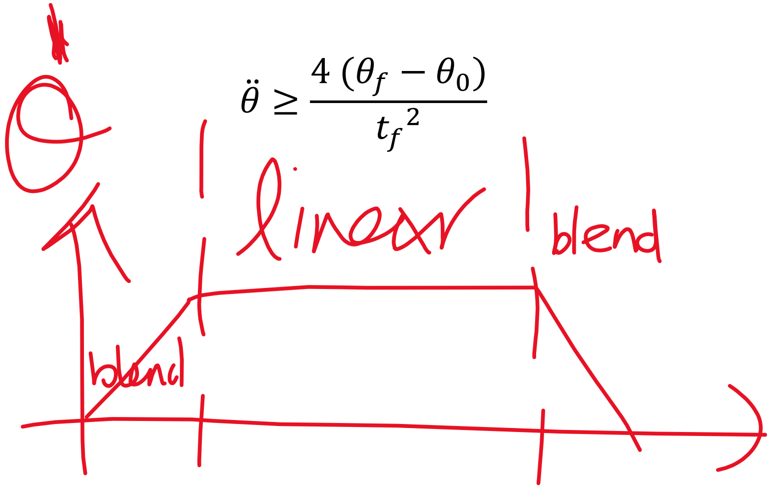

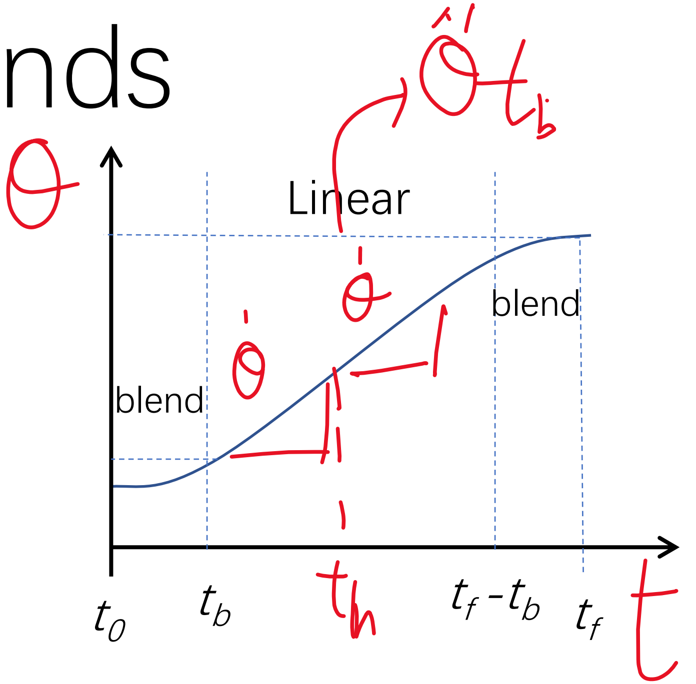

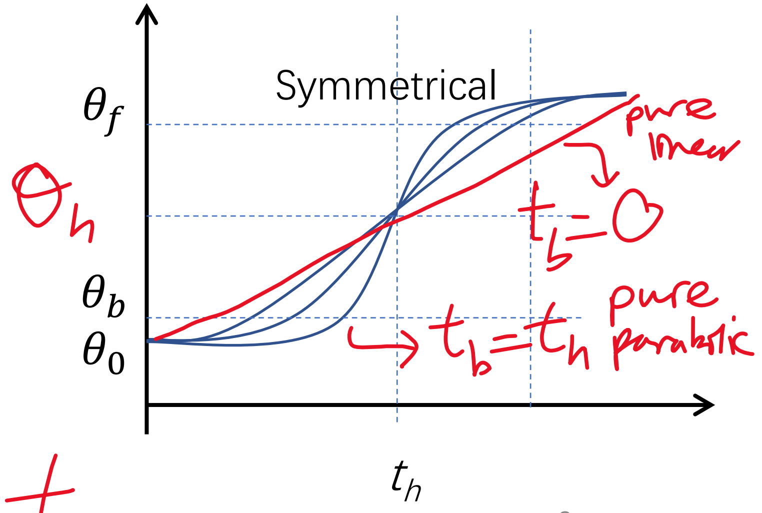

Consider the single-link manipulator arm as shown also in Figure 10.4 (Craig, Introduction to Robotics 3rd Ed.)..

a) Given that the revolute joint moves the link over 2 cubic segments in **6s** from an initial angle $\boldsymbol{\theta_0 = 15\ \mathrm{deg}}$ to rest at a final position $\boldsymbol{\theta_f = 90\ \mathrm{deg}}$ through a via point $\boldsymbol{\theta_v = 30\ \mathrm{deg}}$ at $\boldsymbol{t_v = 3s}$ with a velocity of $\boldsymbol{\dot{\theta}_v = 15\ \mathrm{deg/s}}$, obtain the 8 parameters of the 2-segment cubic polynomial.

Cubic Function

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

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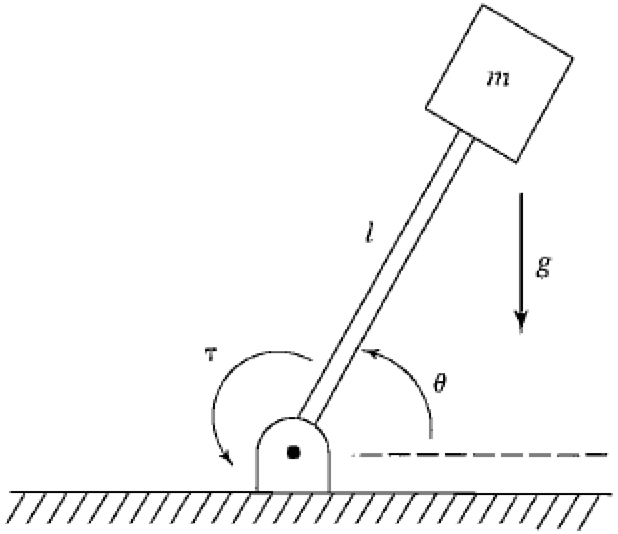

$\tau = ml^2\ddot{\theta} + v\dot{\theta} + c\,sgn(\dot{\theta}) + mlg\cos(\theta).$ (a)

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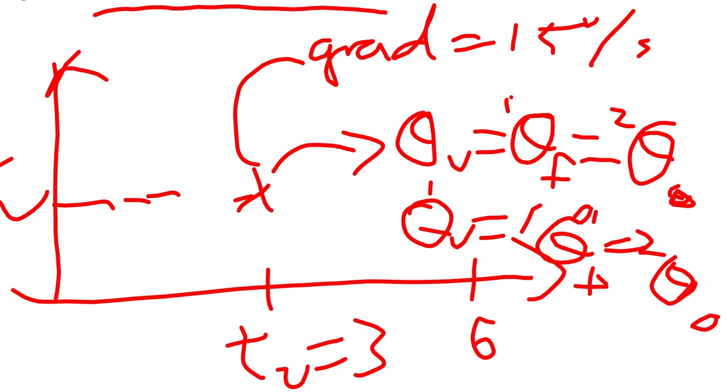
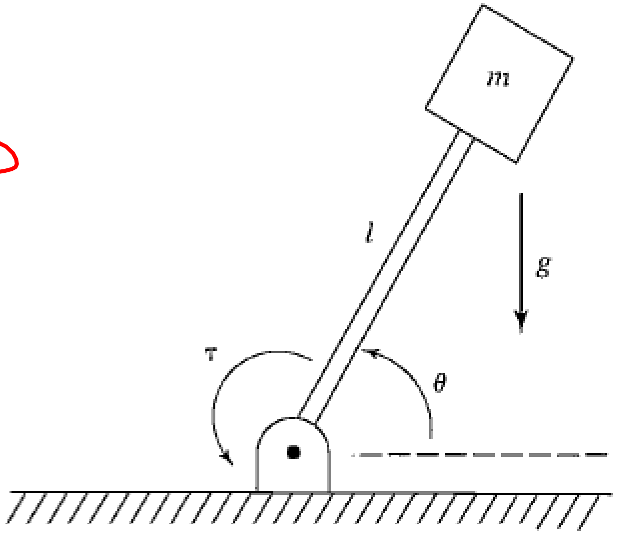
Cubic Function	Parameter
$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = a_0$	$a_0 = \theta_0 = 15^\circ$
$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$	$a_1 = \dot{\theta}_0 = 0$ → use directly
$\ddot{\theta}(t) = 2a_2 + 6a_3 t$	$a_2 = \frac{3}{t^2{}_f}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$
error in class 30 15 ~~25~~	$a_3 = -\frac{2}{t^3{}_f}(\theta_f - \theta_0) + \frac{1}{t^2{}_f}(\dot{\theta}_f - \dot{\theta}_0)$



1st seg $a_2 = \frac{3}{9}(\cancel{90}\ 30 - 15) - \frac{2}{3}(0) - \frac{1}{3}(15) = 20$

$a_3 = -\frac{2}{27}(90-15) + \frac{1}{9}(15-0)$



$\tau = ml^2\ddot{\theta} + v\dot{\theta} + c\,sgn(\dot{\theta}) + mlg\cos(\theta).$ (a)

Cartesian Space Scheme

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Cartesian Space Scheme

Pose

$$\begin{bmatrix} \vec{x} \\ \vec{\alpha} \end{bmatrix}$$

cartesian (x, y, z)

q_n

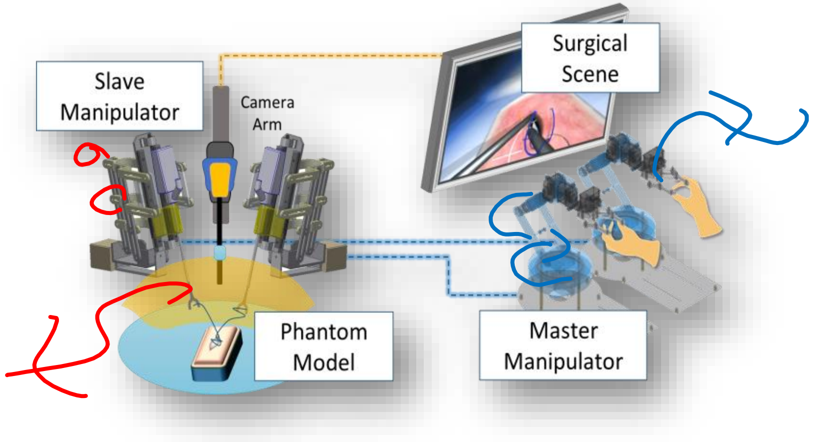
q_1

config

- Specified in terms of pose
- Path points in cartesian coordinates as a function of time
  - planned directly from the user's definition of path without performing inverse kinematics (i.e. may not be preplanned)
  - inverse kinematics solved at the path update rate
  - thus, more computationally expensive.

Slave

Master



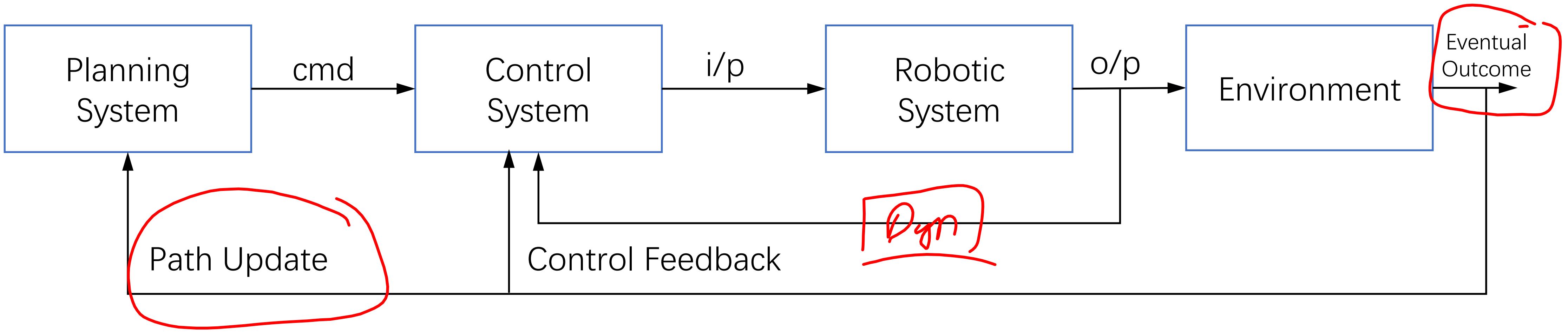
Teleoperating a Robot for Example

Cartesian Space Scheme

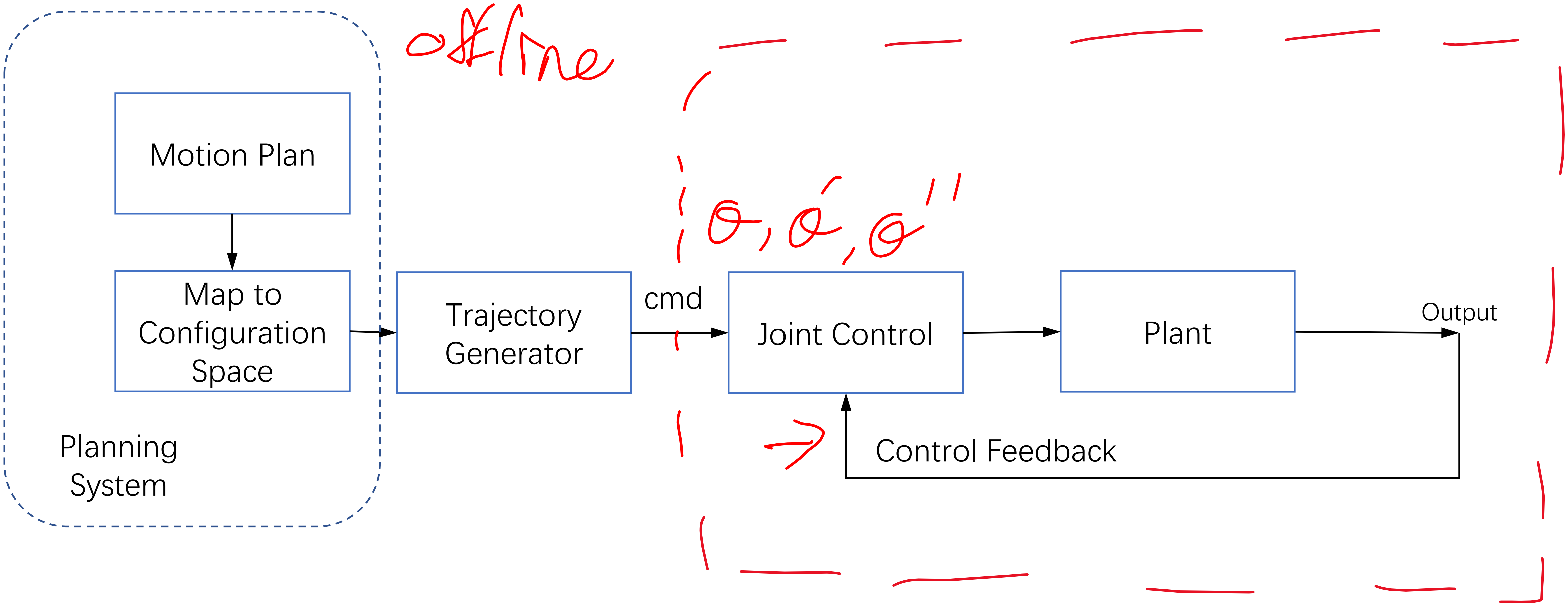
- Specified in terms of pose
- Path points in cartesian coordinates as a function of time
  - planned directly from the user's definition of path without performing inverse kinematics.
  - inverse kinematics solved at the path update rate
  - thus, more computationally expensive.

Teleoperating a Robot for Example

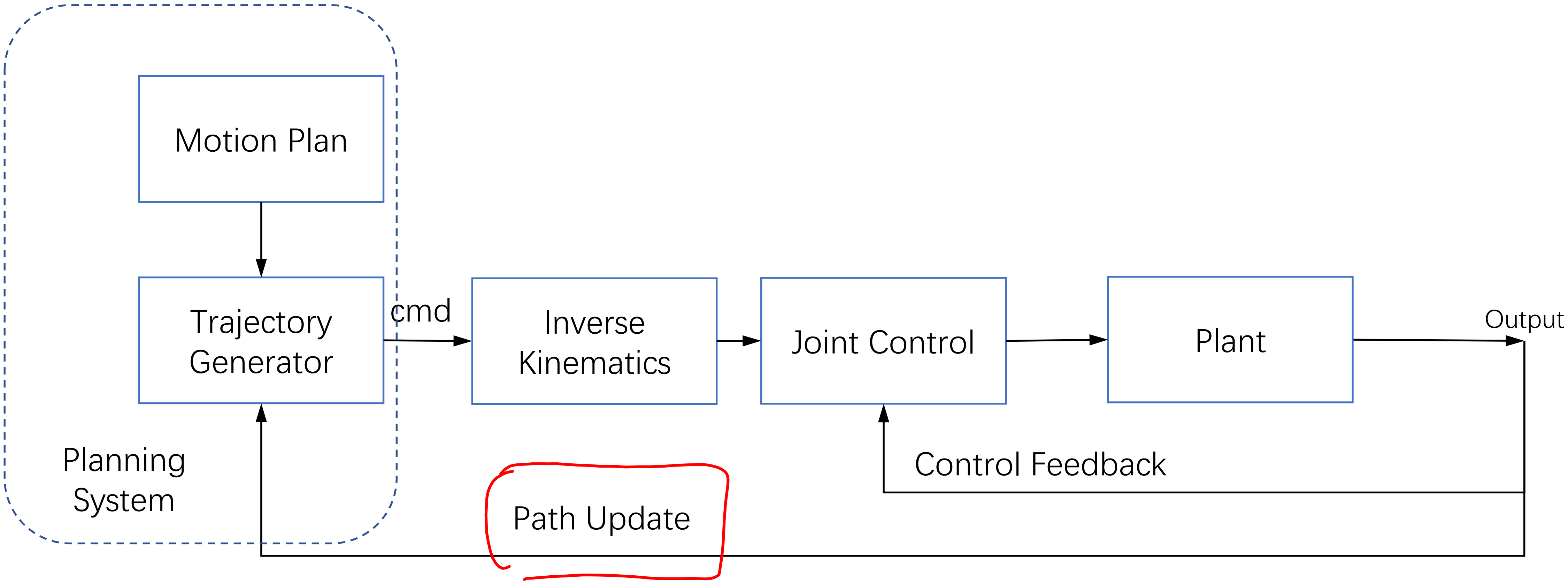
Recall the big picture



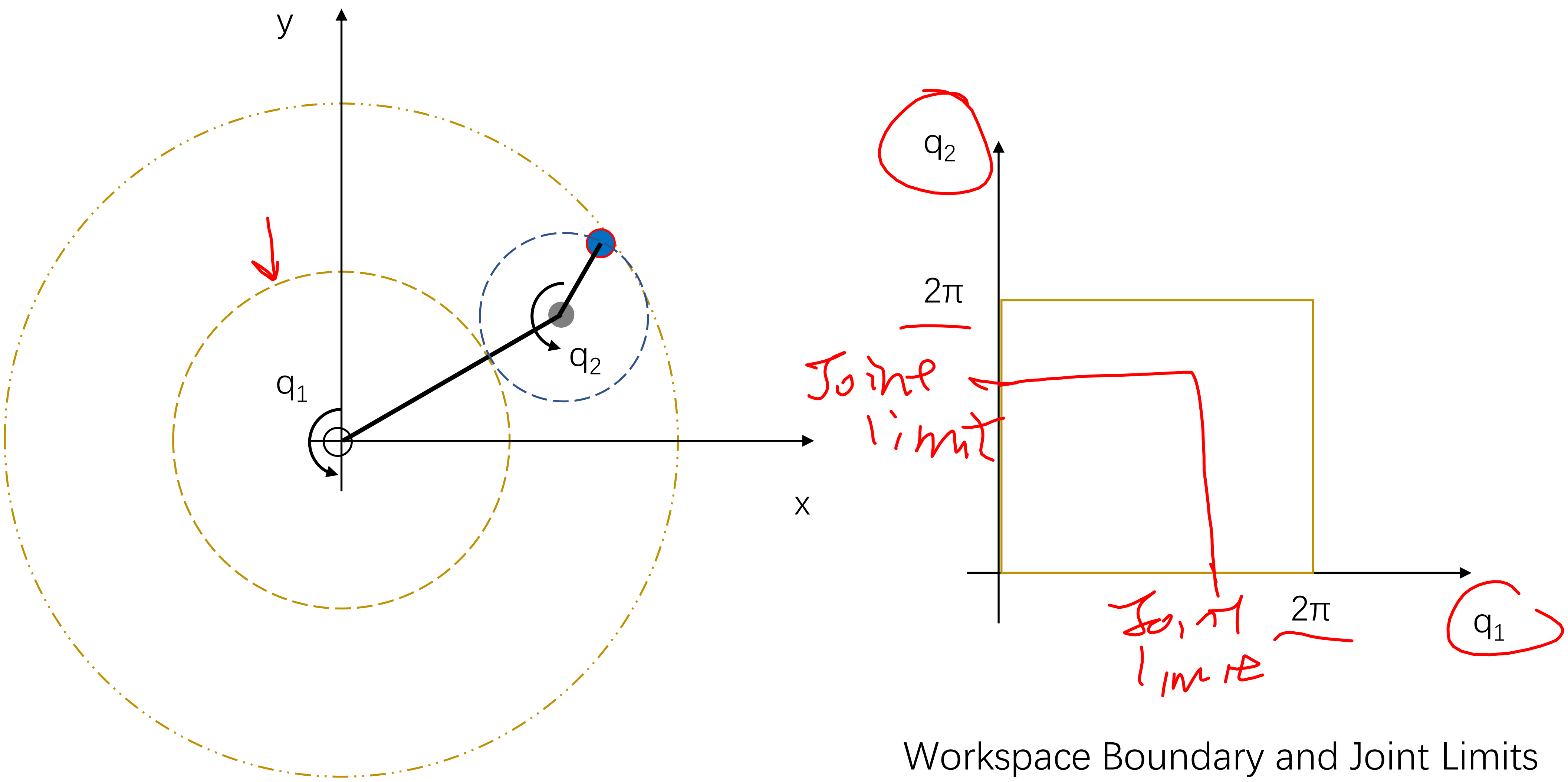
In Joint Space



In Cartesian Space

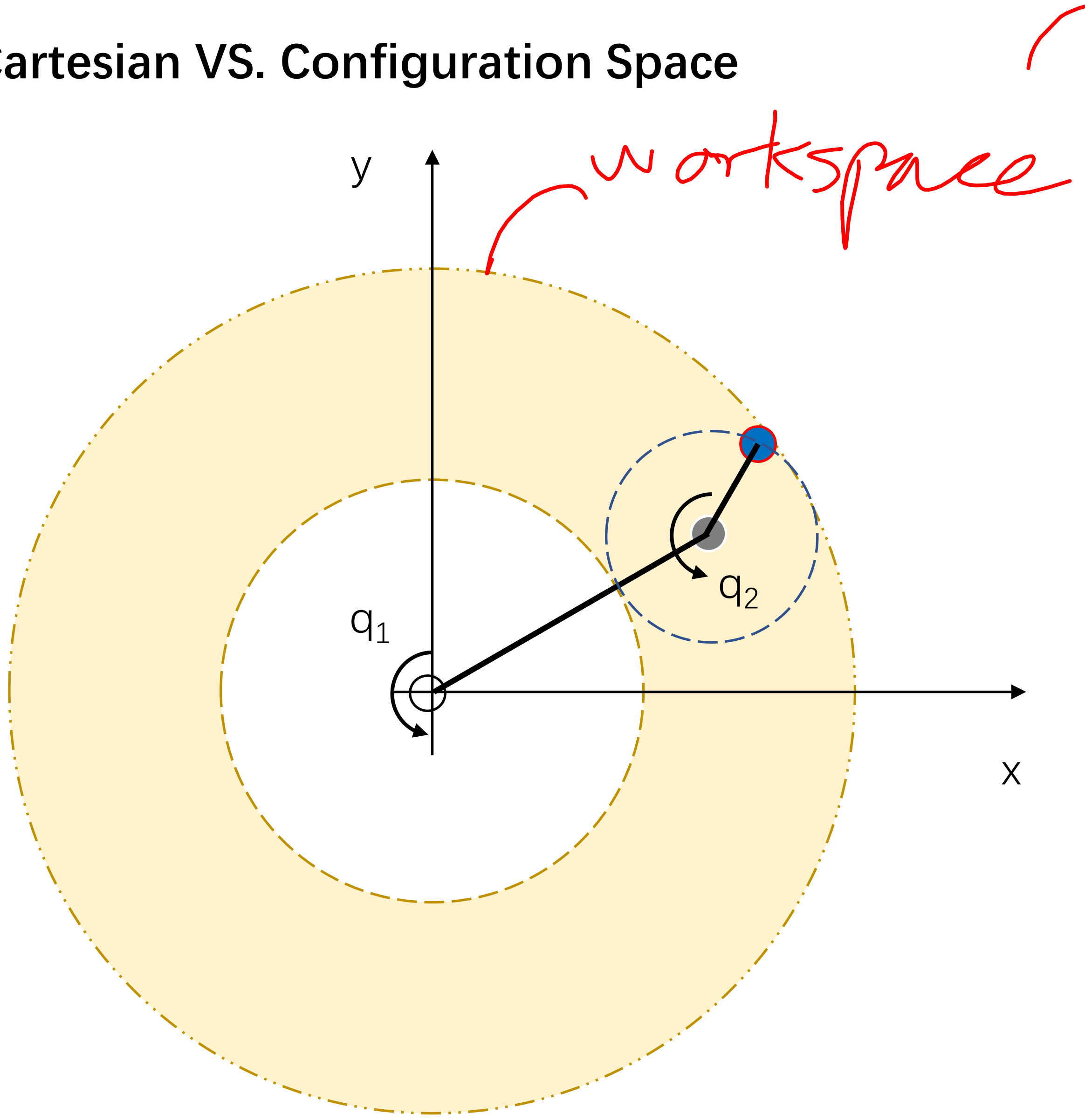


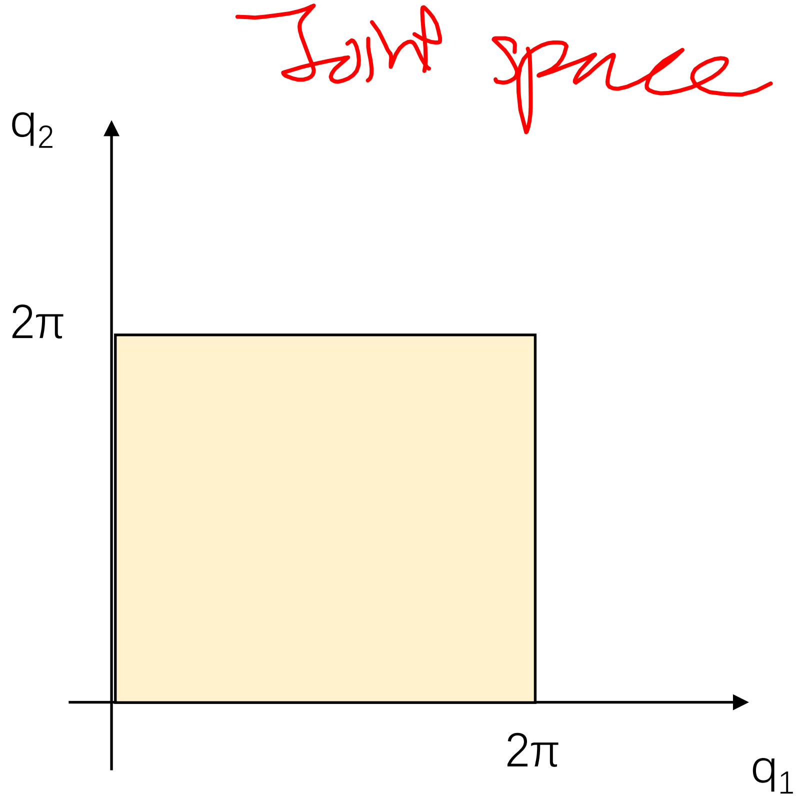
Cartesian VS. Configuration Space



Workspace Boundary and Joint Limits

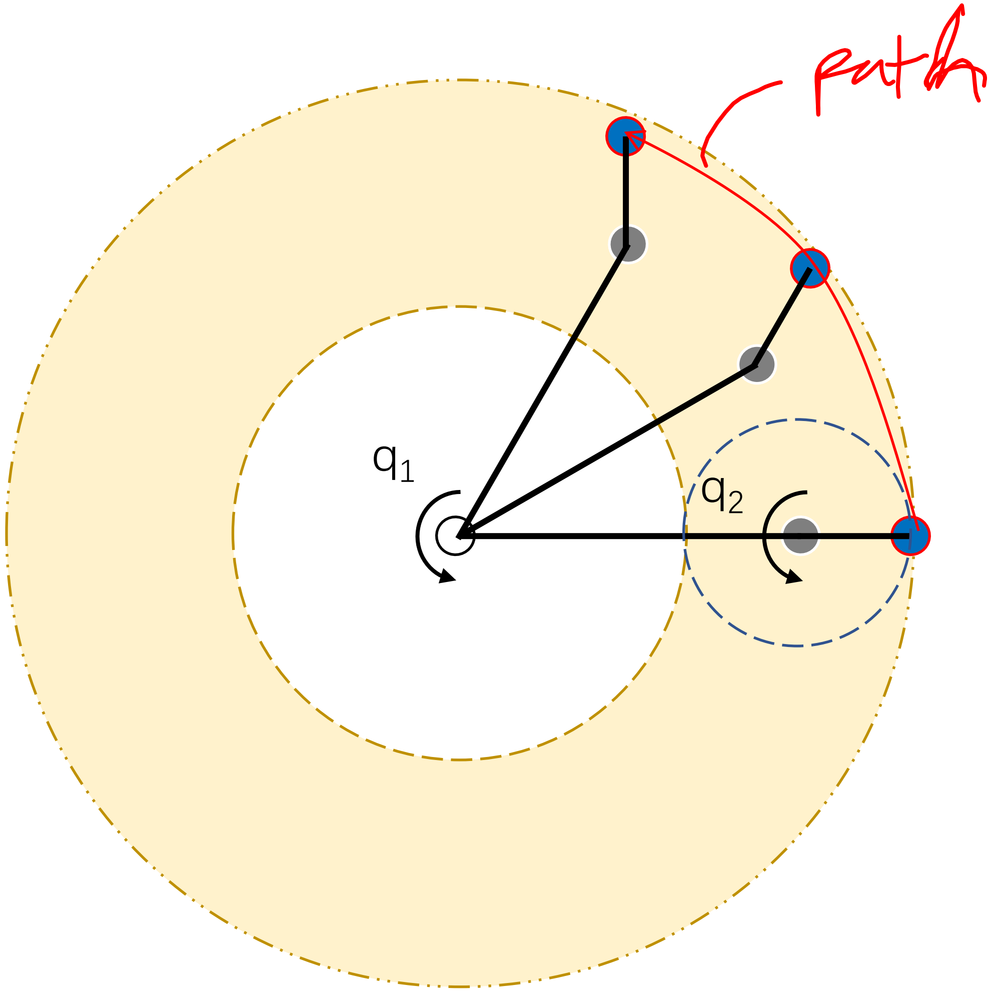
Cartesian VS. Configuration Space

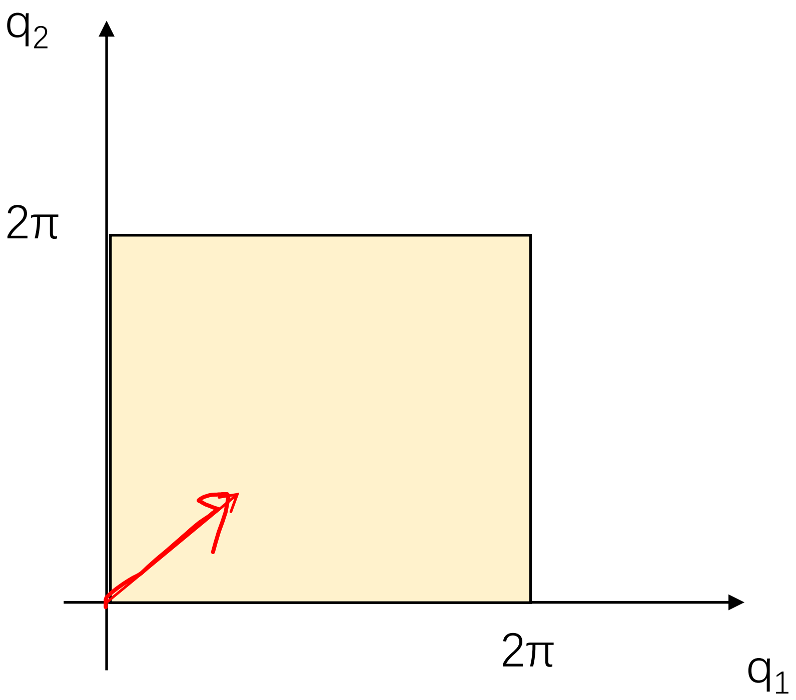




Workspace and Configuration Space

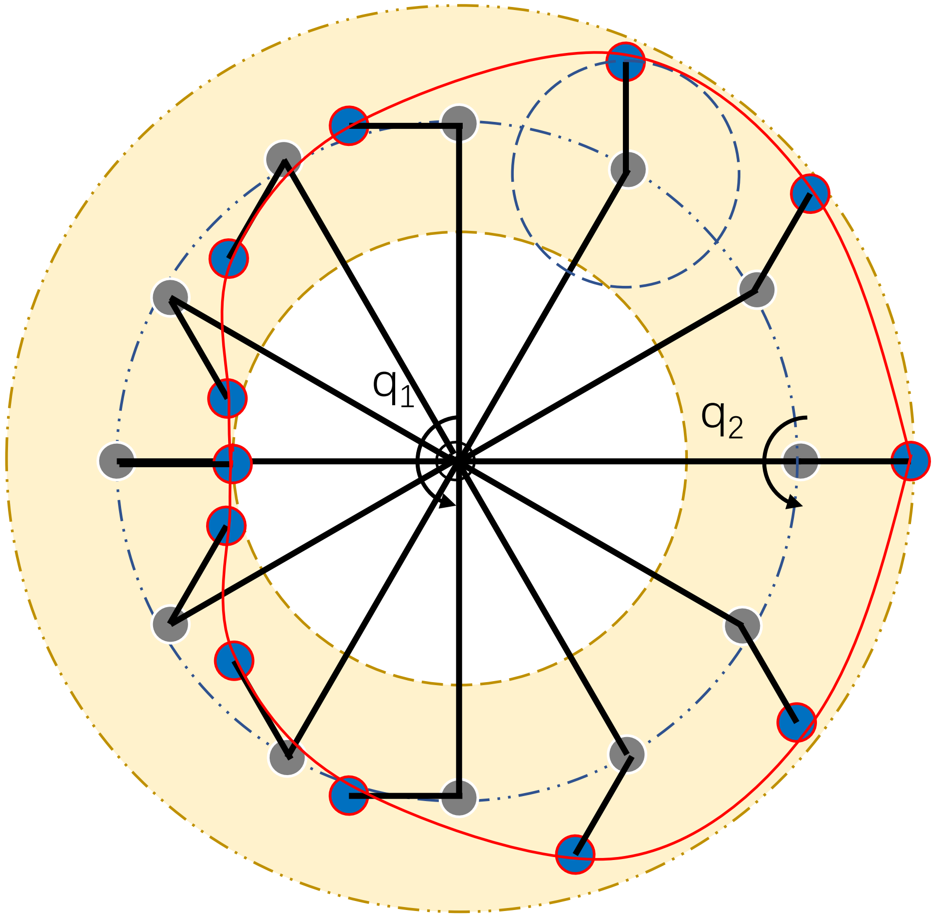
Cartesian VS. Configuration Space

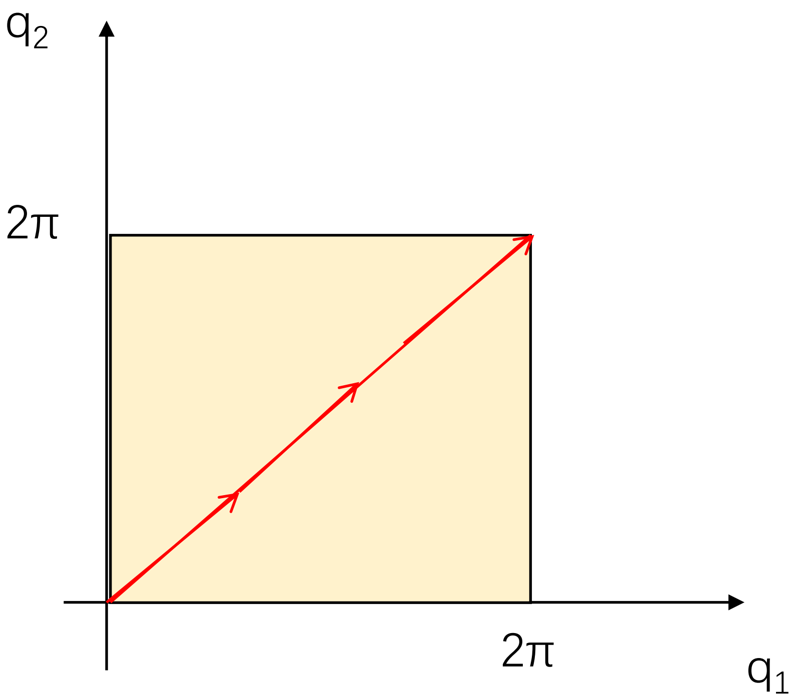




Path in Workspace and Configuration Space

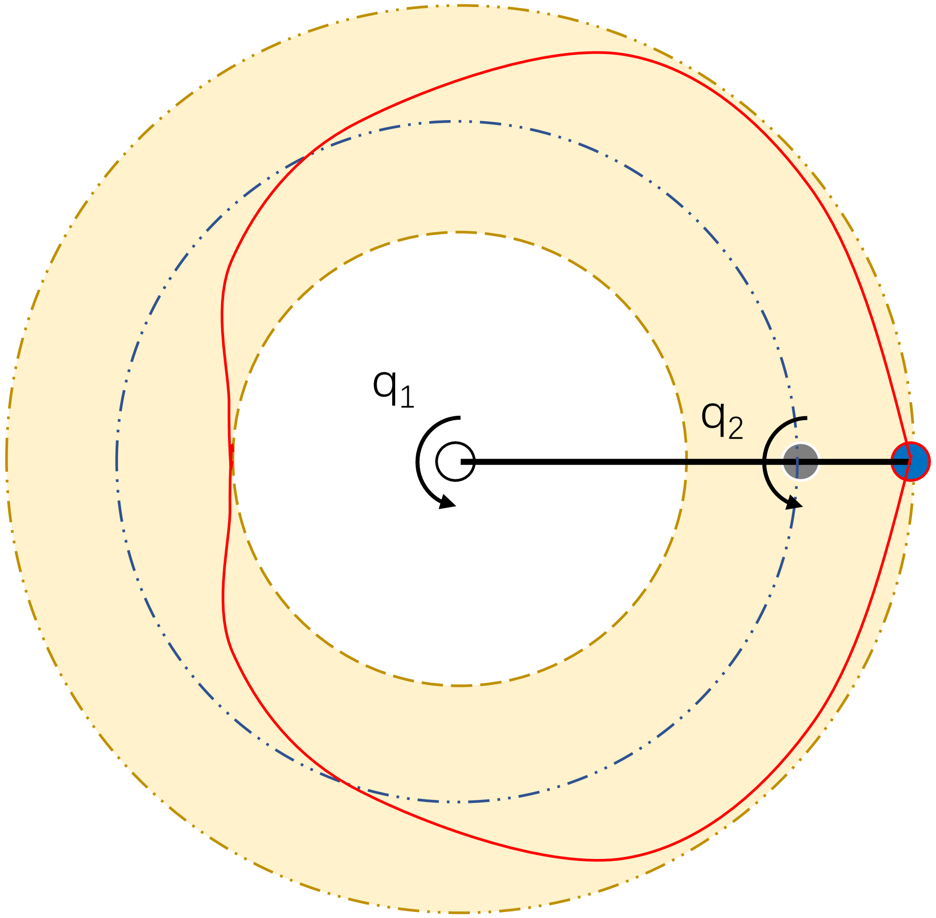
Cartesian VS. Configuration Space

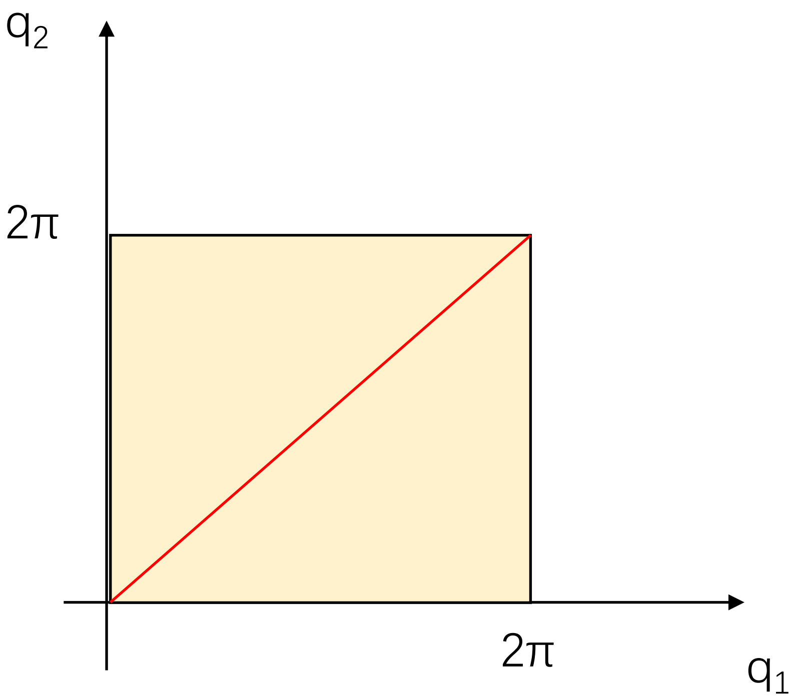




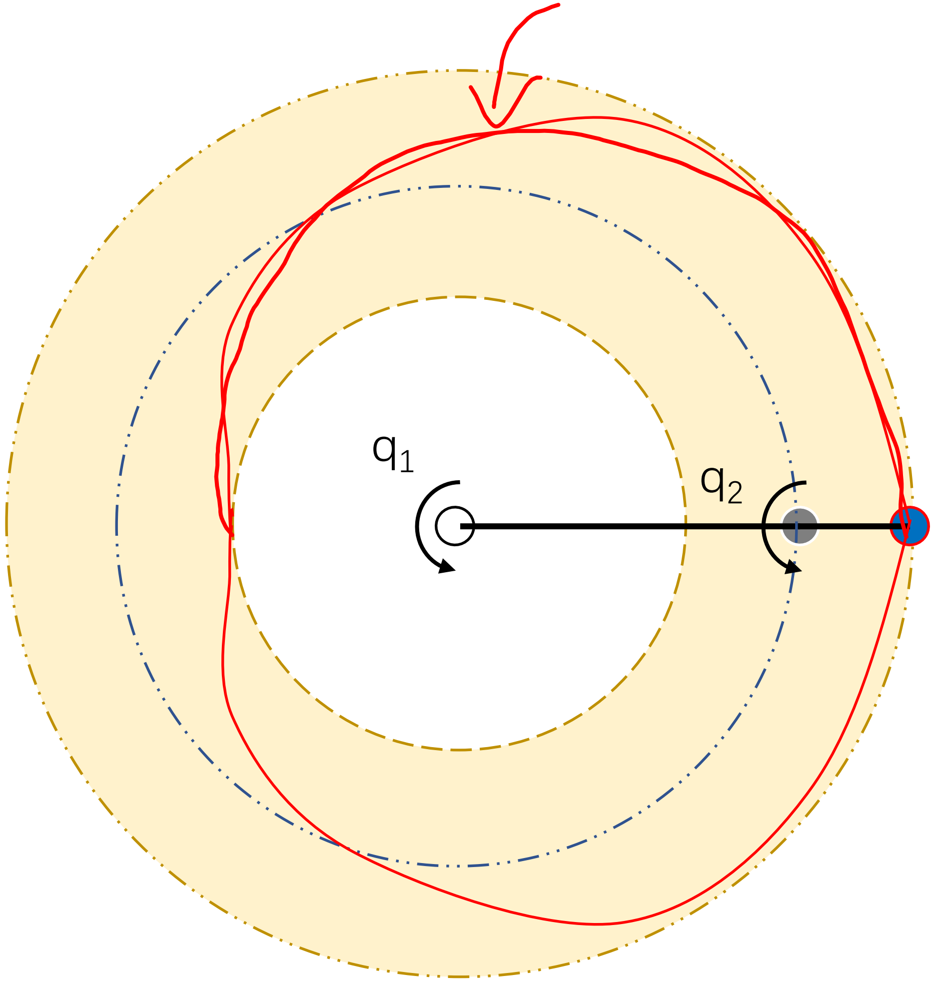
Path in Workspace and Configuration Space

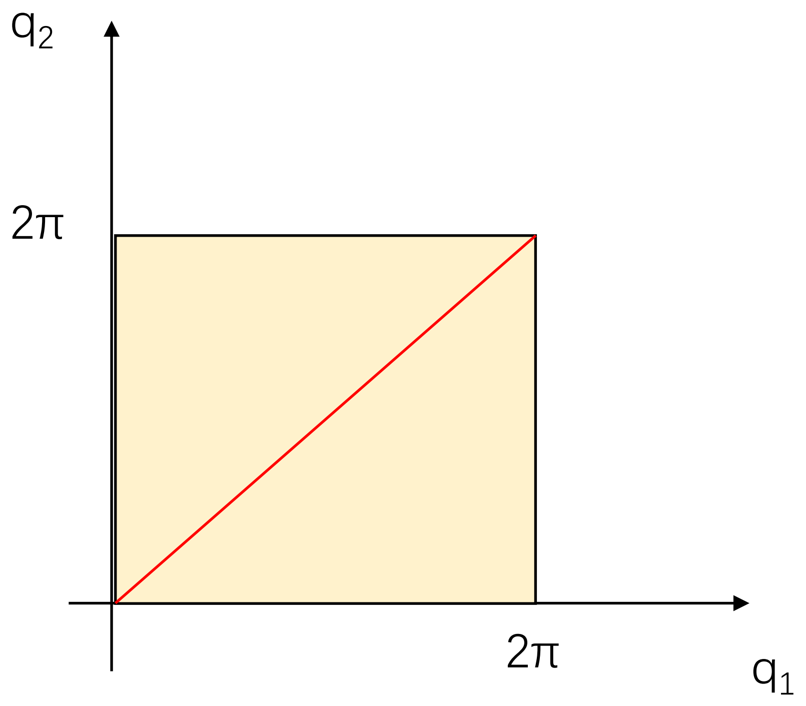
Cartesian VS. Configuration Space





Cartesian VS. Configuration Space



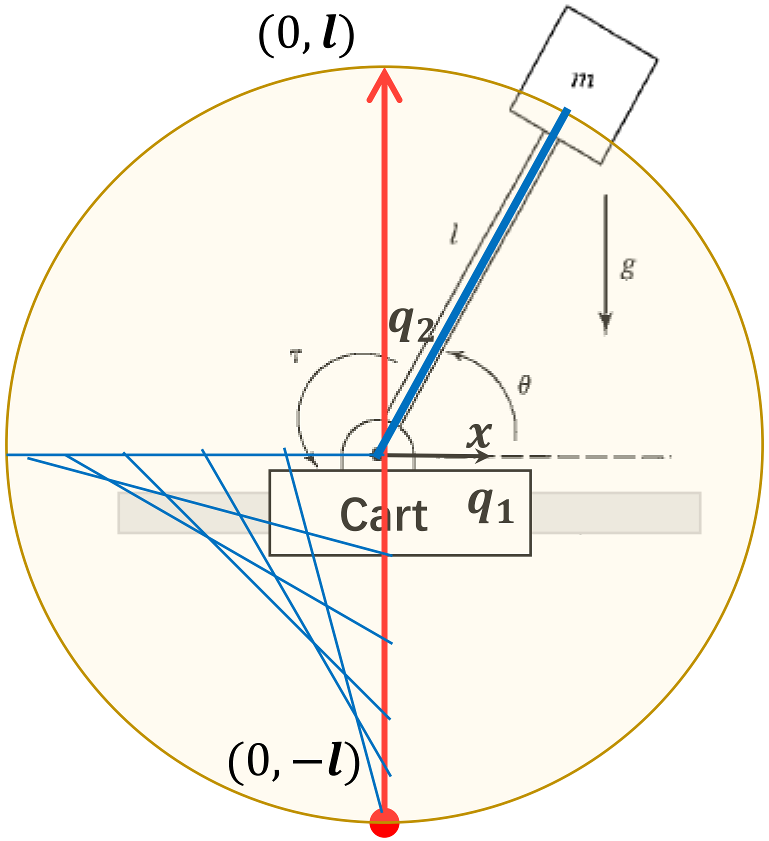


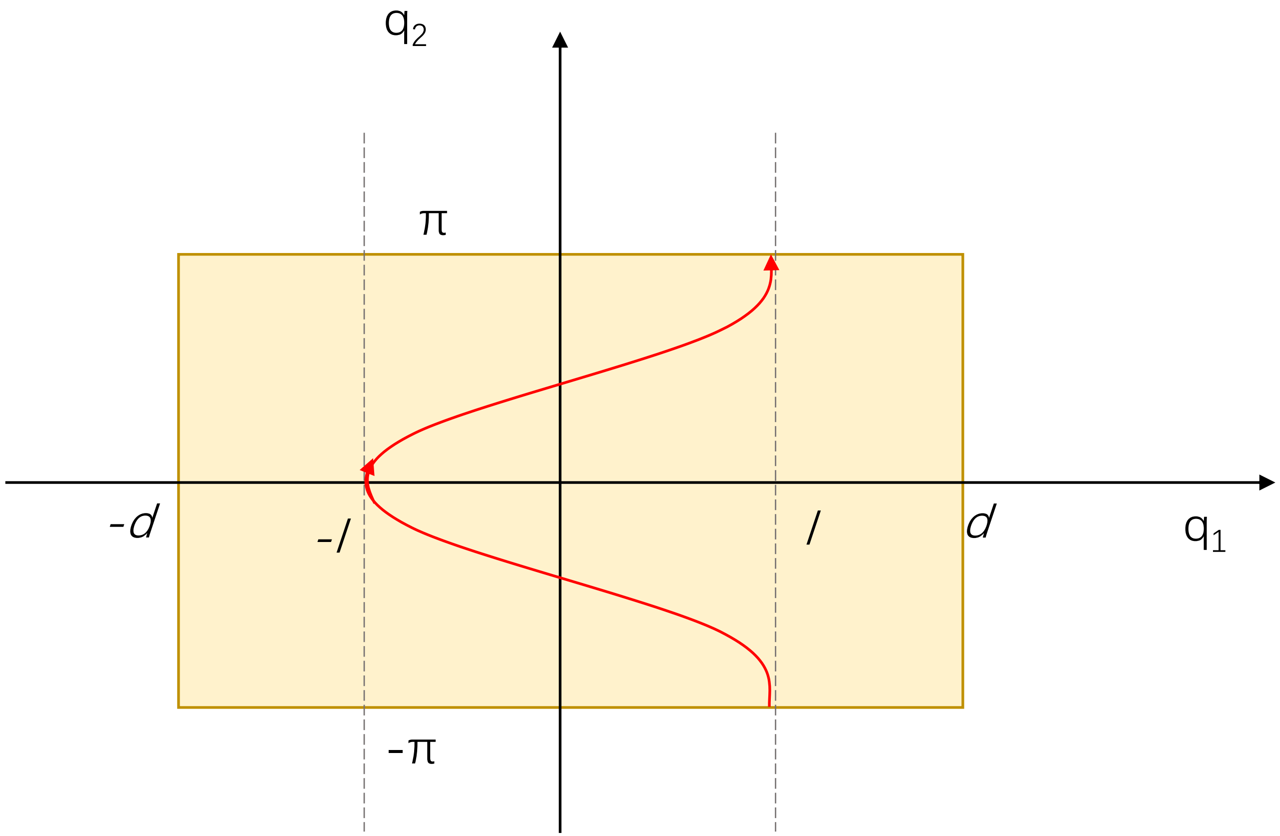
Imagine you decided to create a prismatic-revolute (PR) 2-dof robotic manipulator and mounted the single-link on a horizontally moving cart as shown in Figure 1(b),

I. Illustrate the configuration space of this PR manipulator with a sketch given that the joint limits are $x \subset [-d, d]$ and $\theta \subset [-\pi, \pi]$. (2 Points)

II. Describe a possible path in the configuration space if a vertical straight path is desired from point $(0, -\boldsymbol{l})$ to $(0, \boldsymbol{l})$ in the workspace of point *m*. (*Hint: circular motion projects to orthogonal axes as sinusoidal motion*)

III. Assuming the motor at $\boldsymbol{q_2}$ rotates at a constant speed of $\boldsymbol{\omega}$, suggest a trajectory for $\boldsymbol{q_1}$





HW5

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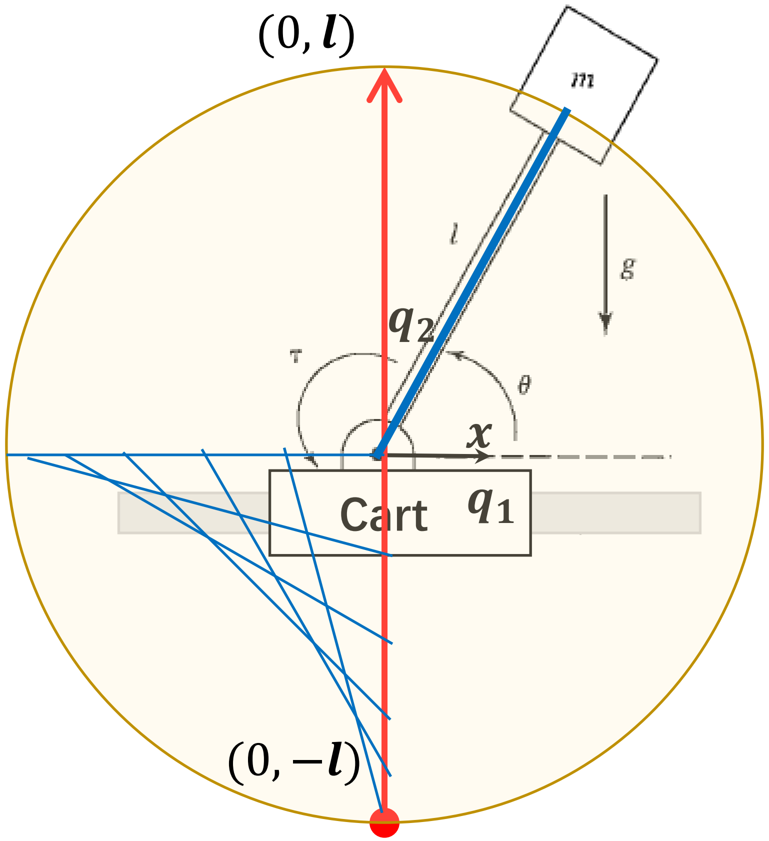

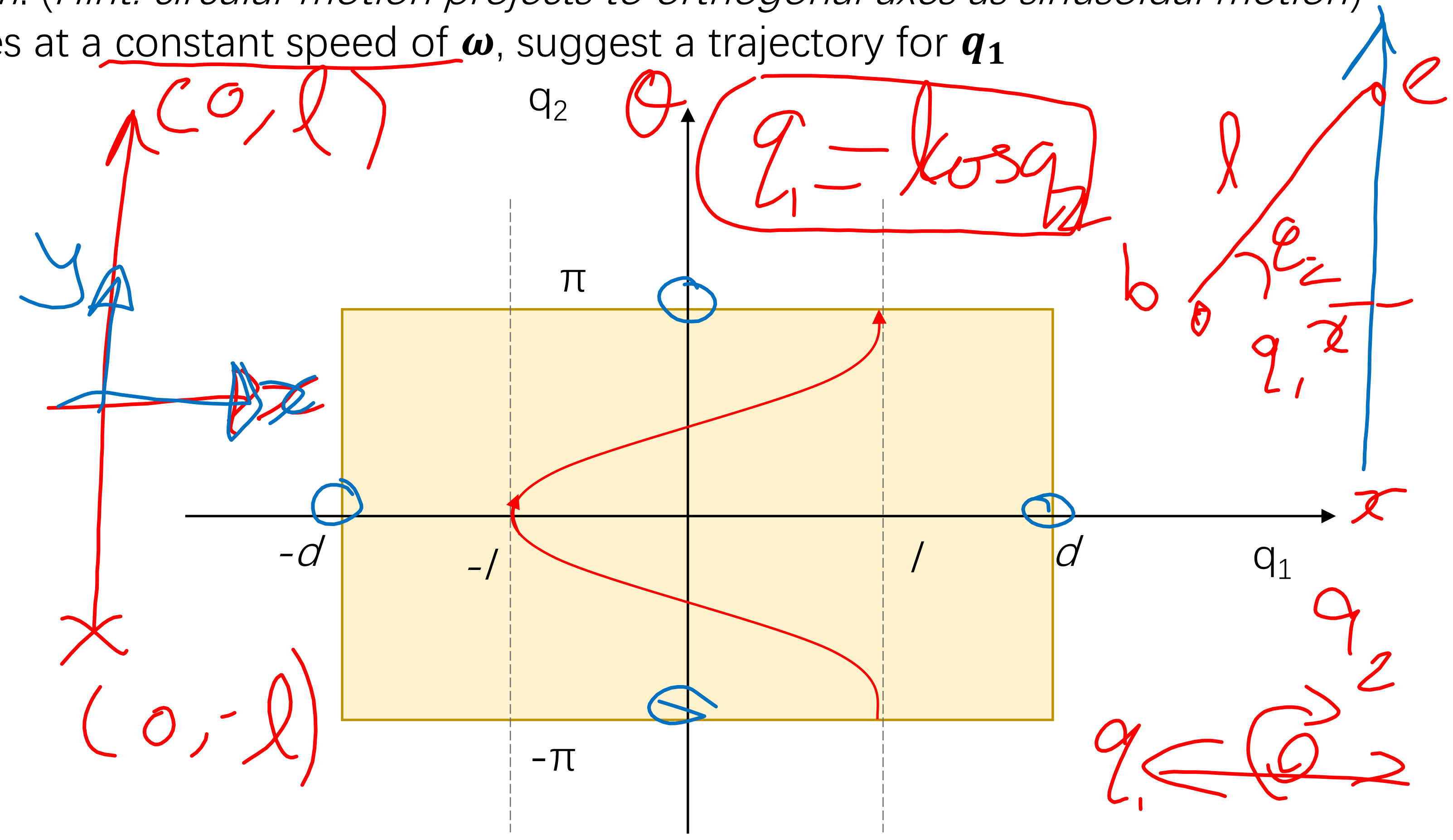

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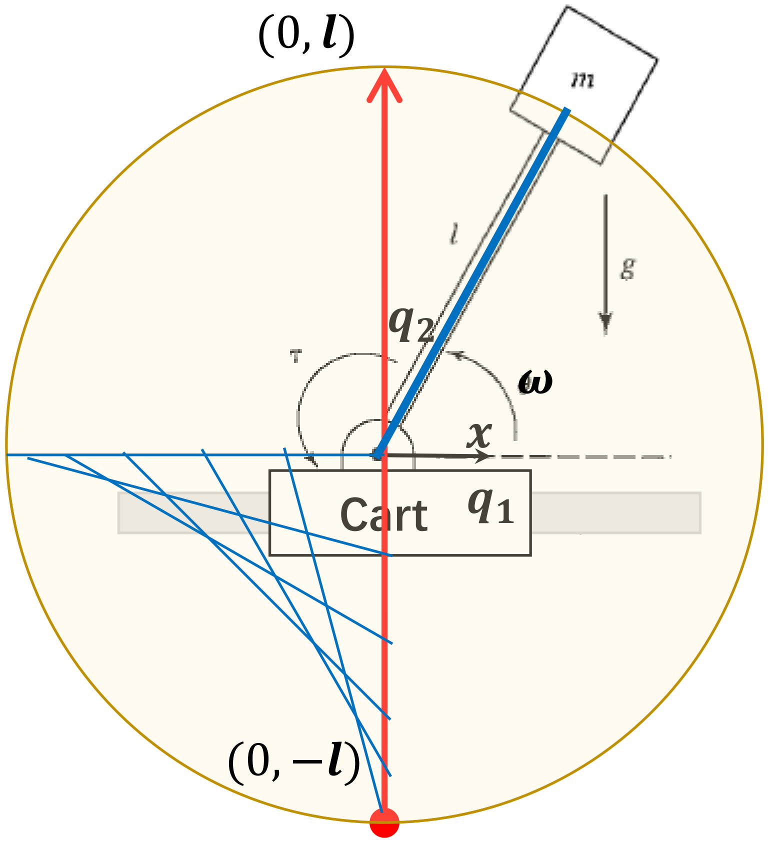

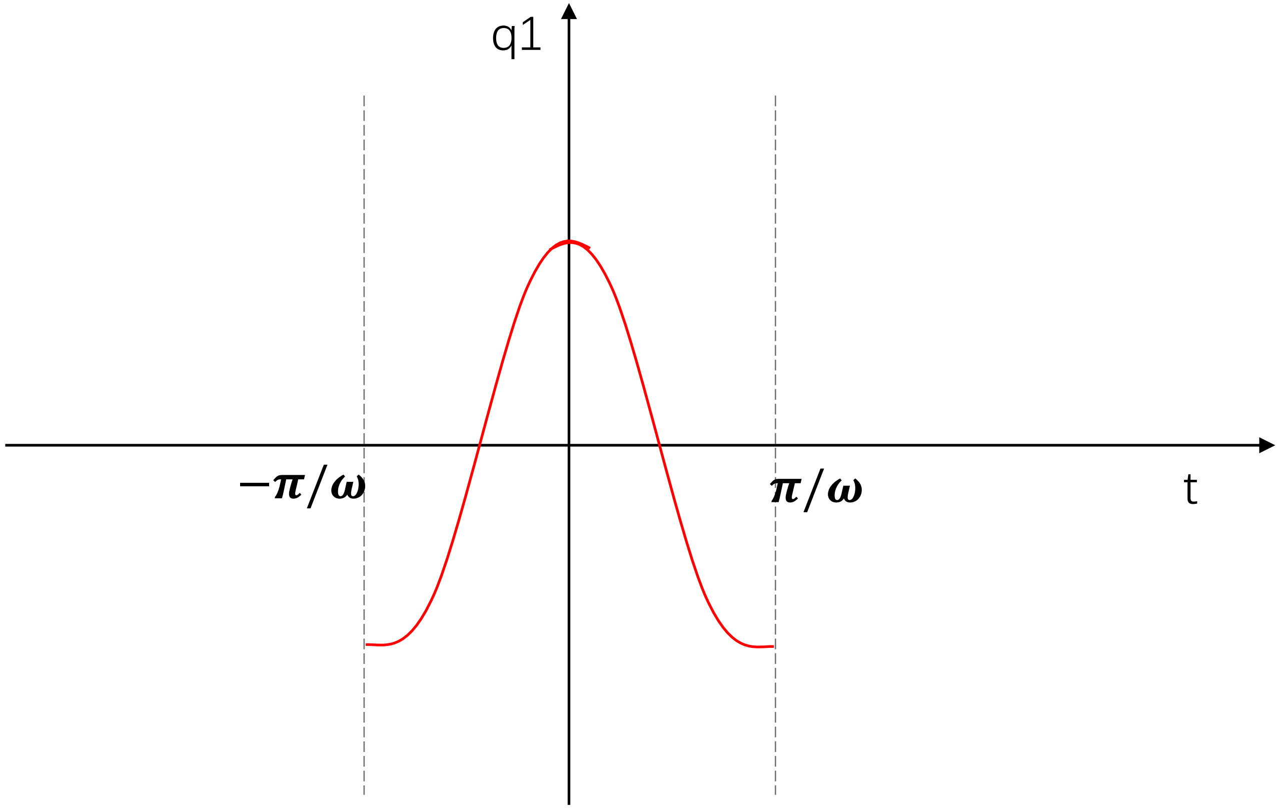

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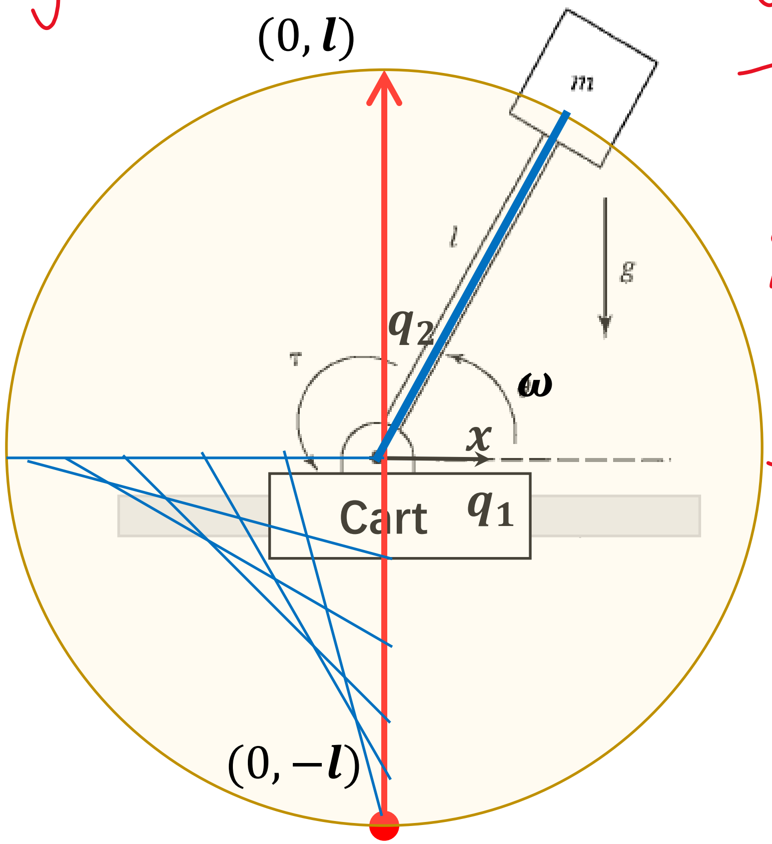
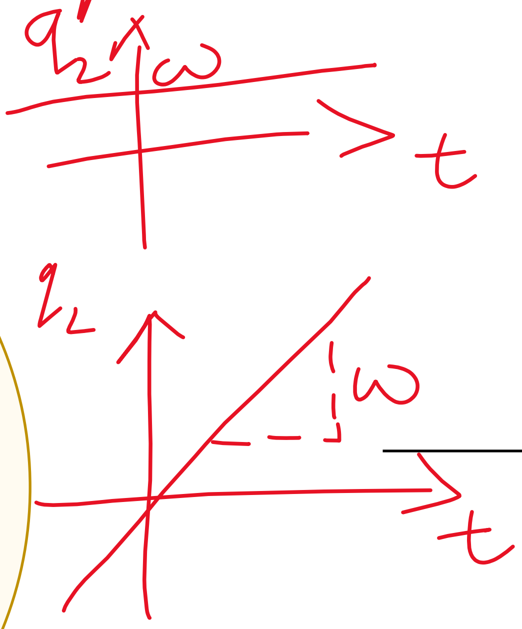
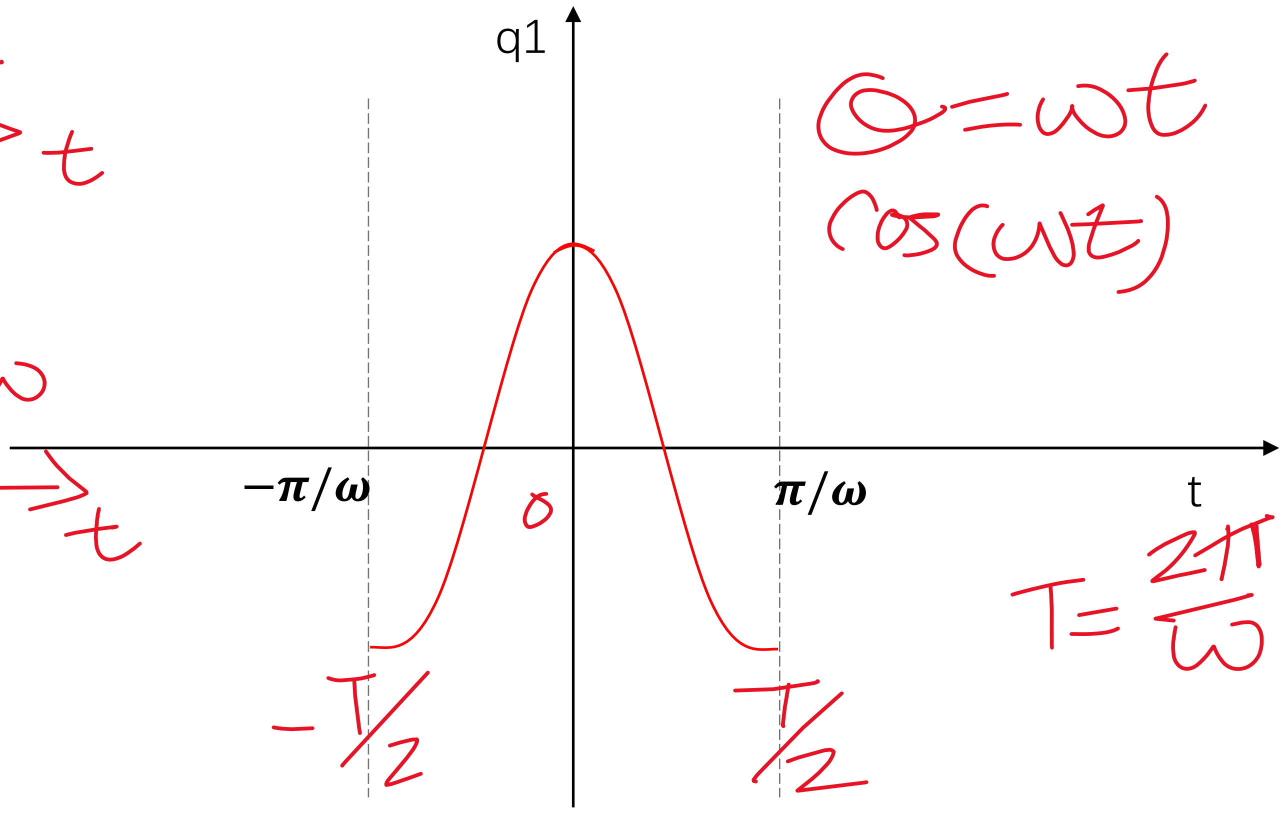
Path II. Describe a possible path in the configuration space if a vertical straight path is desired from point $(0, -\boldsymbol{l})$ to $(0, \boldsymbol{l})$ in the workspace of point *m*. (*Hint: circular motion projects to orthogonal axes as sinusoidal motion*)

Traj III. Assuming the motor at $\boldsymbol{q_2}$ rotates at a constant speed of $\boldsymbol{\omega}$, suggest a trajectory for $\boldsymbol{q_1}$







$\theta = \omega t$

$\cos(\omega t)$

$T = \frac{2\pi}{\omega}$

Types of Problems in Cartesian Scheme

- Intermediate points unreachable
- High joint rates near singularity
- Start and goal reachable in different solution

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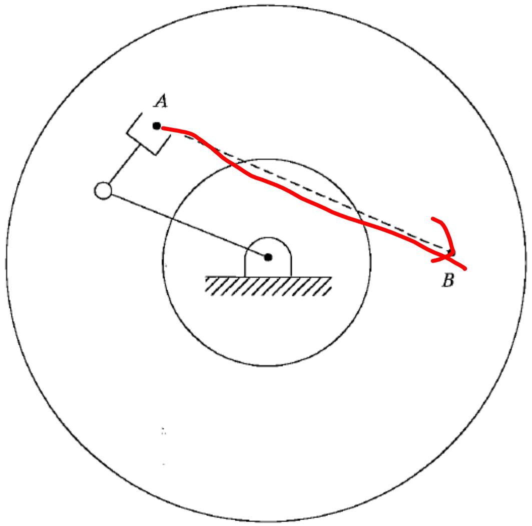
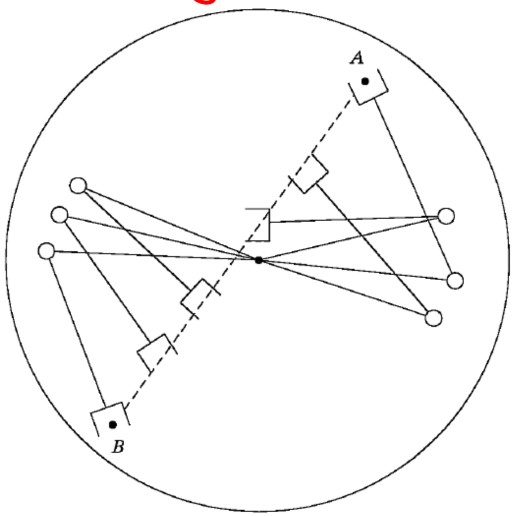
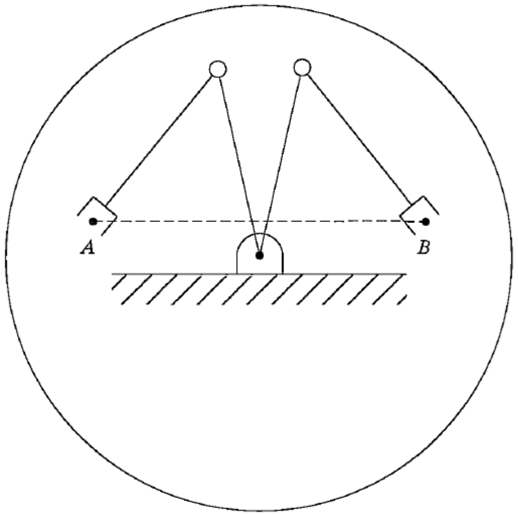
no solu　　　　sngle　　　　more soln







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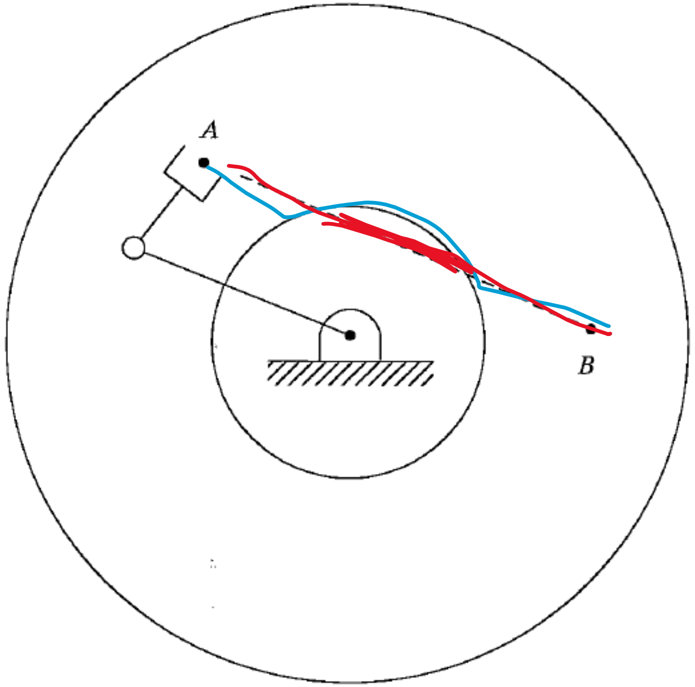
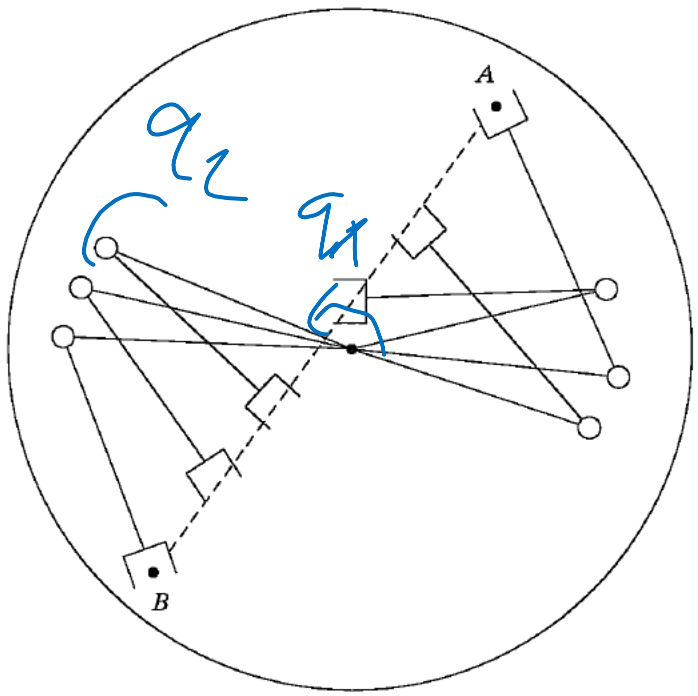


No solution for some points

Types of Problems in Cartesian Scheme

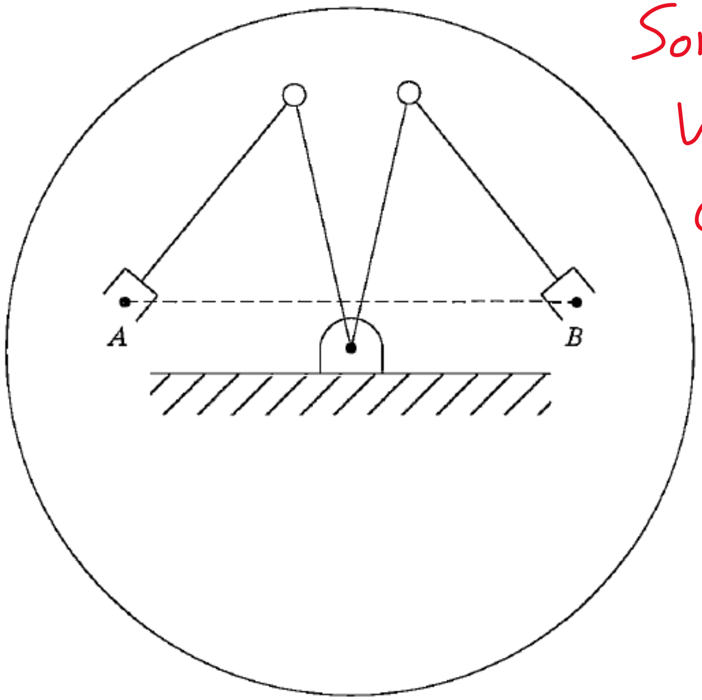
- High joint rates near singularity



$\dot{q}_2 \uparrow$ very high

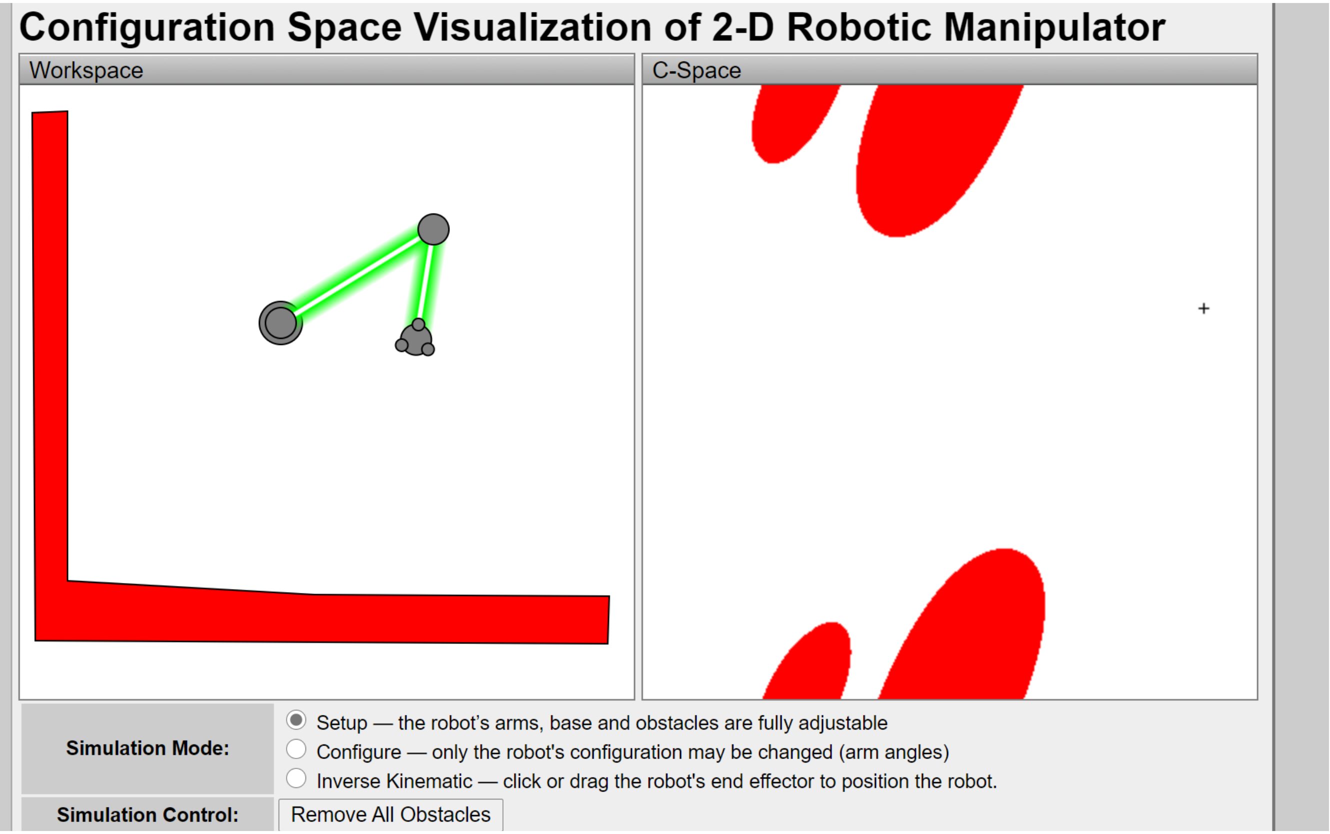
Types of Problems in Cartesian Scheme

- Start and goal reachable in different solution



Sometime useful for obstacles avoidance

Hands on Simulation



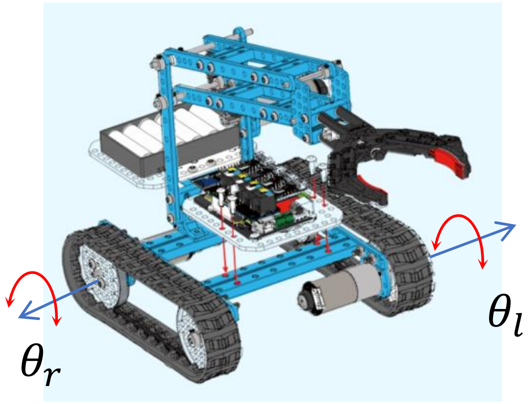
Go to:
https://www.cs.unc.edu/~jeffi/c-space/robot.xhtml

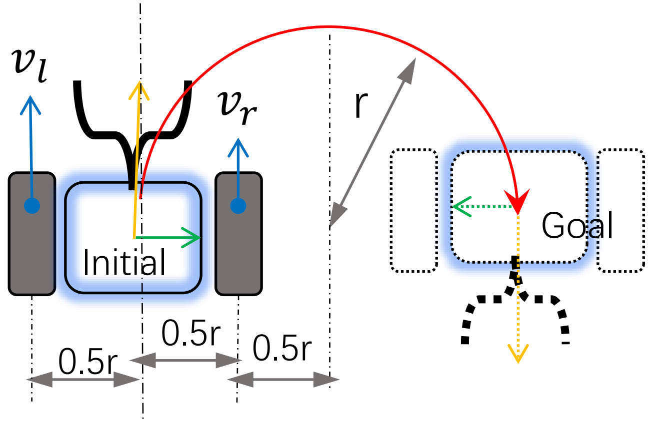
By Professor Ron Alterovitz, University of North Carolina at Chapel Hill

Joint-Space VS. Task-space

Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors

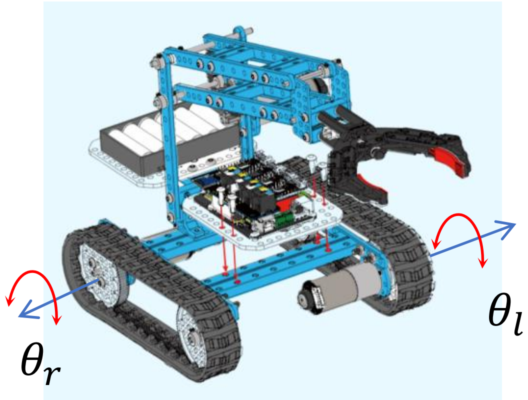


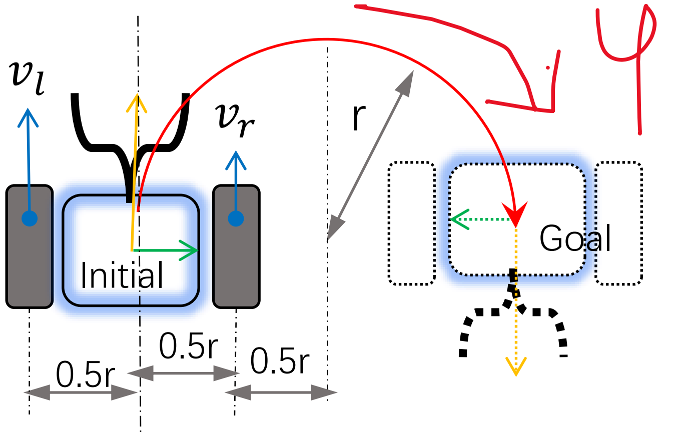


Joint-Space VS. Task-space

Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors





Path plan in task-space:

Recognizing the constraint:

Relating constraint to joint-space:

Joint-Space VS. Task-space

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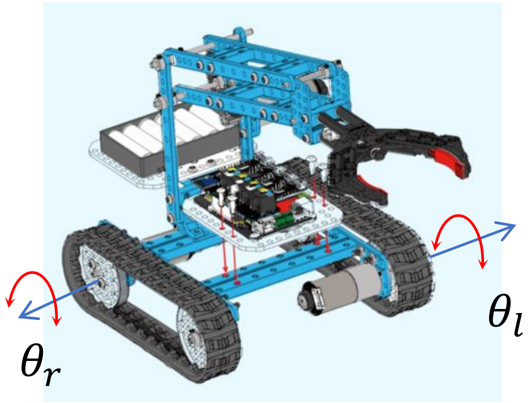
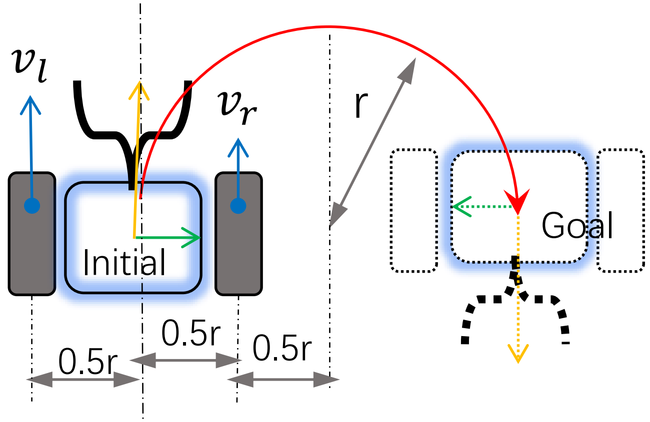




Path plan in task-space: to move the robot from the initial position to the goal position as shown (a circular motion)

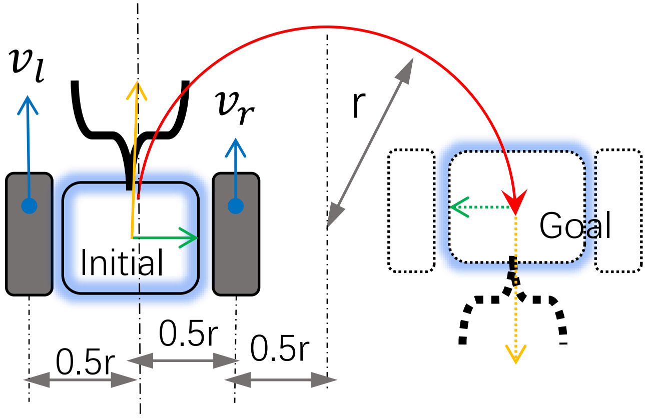
Recognizing the constraint: distances from center of rotation to left and right wheel are $1.5r$ and $0.5r$ respectively.

Relating constraint to joint-space: ratio of linear velocities of the wheels equals to the ratio of their distances to the rotation center and assuming same wheel radius for left and right, $\frac{v_l}{v_r} = \frac{r_l}{r_r} = \frac{\dot{\theta}_l}{\dot{\theta}_r}$

Joint-Space VS. Task-space

Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors



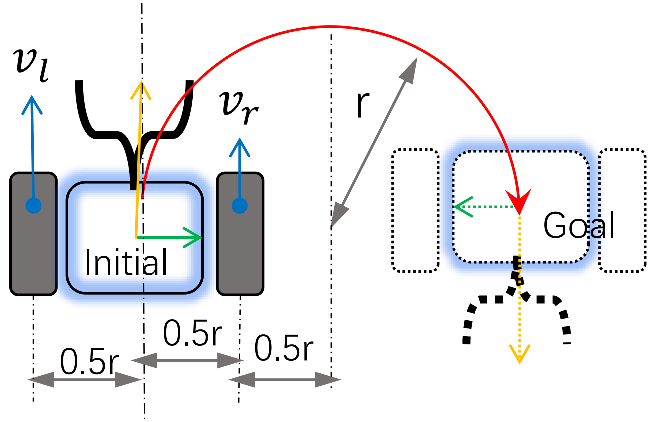
A) Given that the angular displacement of the right wheel θ_r follows a cubic trajectory from θ_0 to θ_f, describe angular trajectory of the left motor to maintain the distance *r* from the center of rotation. Assume that the coefficients are a0= a1= a2= a3=1.

B) Describe a control scheme to accomplish this trajectory-following application.

Joint-Space VS. Task-space

Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors



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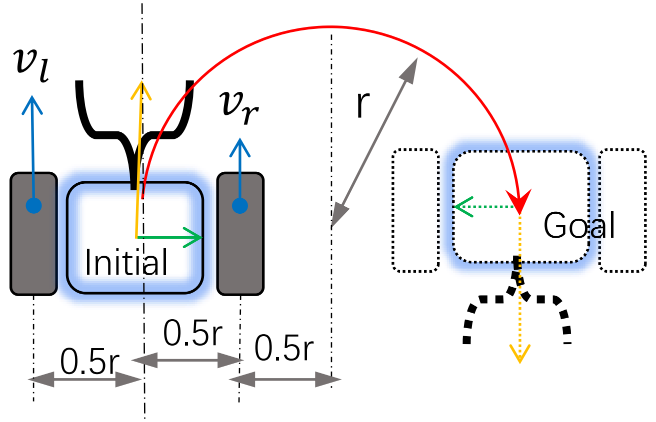
Joint-Space VS. Task-space



Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors

A) Given that the angular displacement of the right wheel θ_r follows a cubic trajectory from θ_0 to θ_f, describe angular trajectory of the left motor to maintain the distance *r* from the center of rotation. Assume that the coefficients are a0= a1= a2= a3=1.

To maintain the circular motion, the speed of the left wheel should satisfy $\dot{\theta}_l = 3\dot{\theta}_r$.

From the cubic polynomial, $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ the velocity profile can be obtained as $\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$.

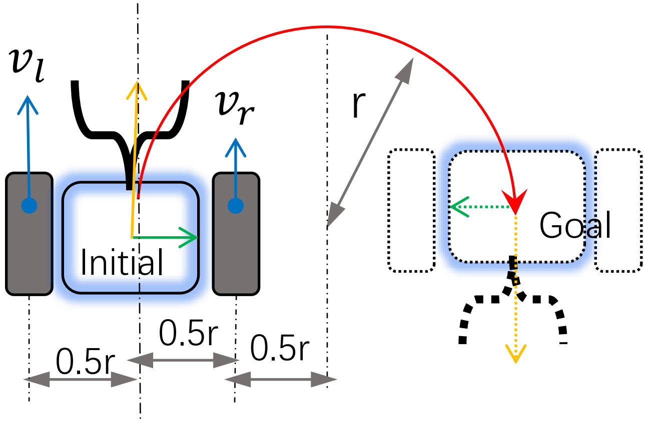
For a1= a2= a3=1, $\dot{\theta}_r(t) = 1 + 2t + 3t^2$

The left wheel $\dot{\theta}_l(t) = 3 + 6t + 9t^2$

Joint-Space VS. Task-space

Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors

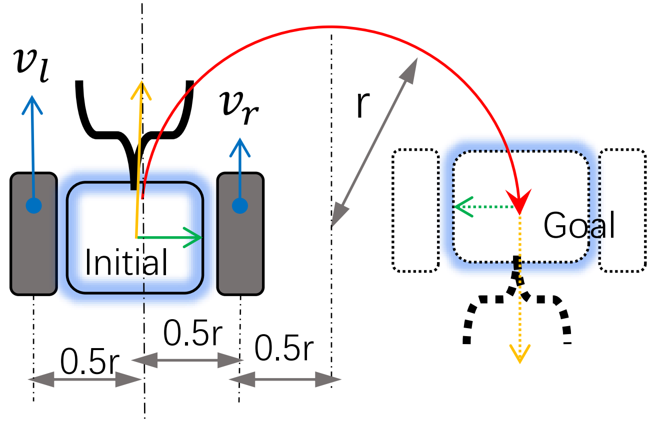


B) Describe a <u>control scheme</u> to accomplish this <u>trajectory-following</u> application.

Joint-Space VS. Task-space

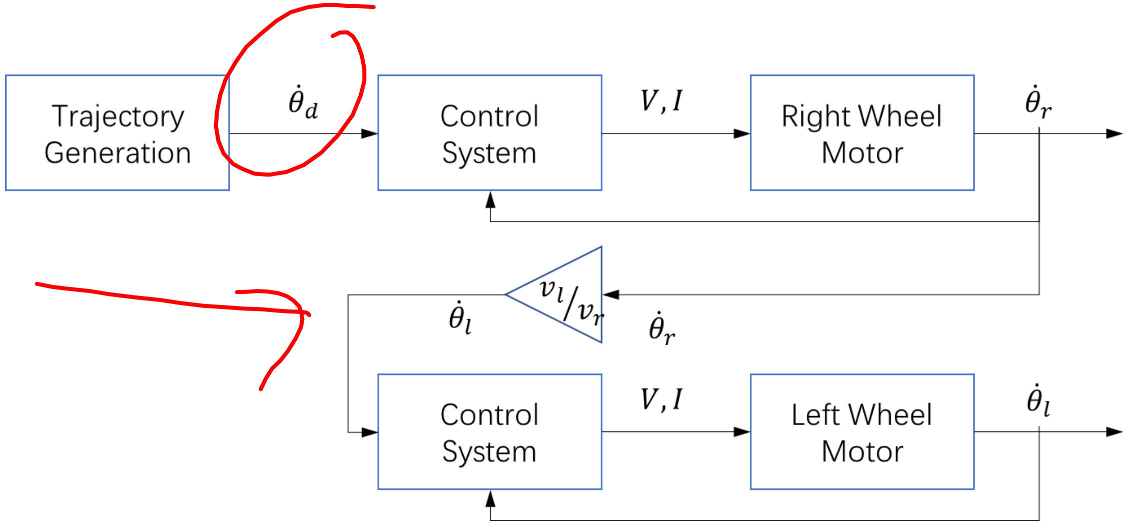
Mobile Robot Example

Differential drive robot that can steer in direction using the velocity difference between the left and right wheels driven by controlled motors



B) Describe a control scheme to accomplish this trajectory-following application.

One way is to coordinate the direction to **satisfy the constraint $\dot{\boldsymbol{\theta}}_l = {}^{v_l}/_{v_r}\ \dot{\boldsymbol{\theta}}_r$.** This can be done by feeding the **encoder reading of the right wheel to the controller for the left wheel**, with a **gain ${}^{v_l}/_{v_r}$,** as shown.



But, is there a problem?

Revision

ECE 470 Introduction to Robotics

Schedule Check

- **Lecture**

O. Overview
- Science & Engineering in Robotics

I. Spatial Representation & Transformation
- Coordinate Systems; Pose Representations; Homogeneous Transformations

II. Kinematics
- Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/Inverse Kinematics

III. Velocity Kinematics and Static Forces
- Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity

Fundamentals

Week 1-4

Revision/ ~~Quiz on Week 5~~

IV. Dynamics
- Acceleration of Body; Newton-Euler Equations of Motion; Lagrangian Formulation

V. Control
- Closed-Loop Control and Feedback, Control of 2nd order system, Independent Joint Control, Force Control

VI. Planning
- ~~Joint-Based Scheme; Cartesian-Based Scheme; Collision Free Path Planning~~

Essentials

Week 6-9

Revision/ Quiz on Week 10

VII. Robot Vision (and Perception)
- Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics

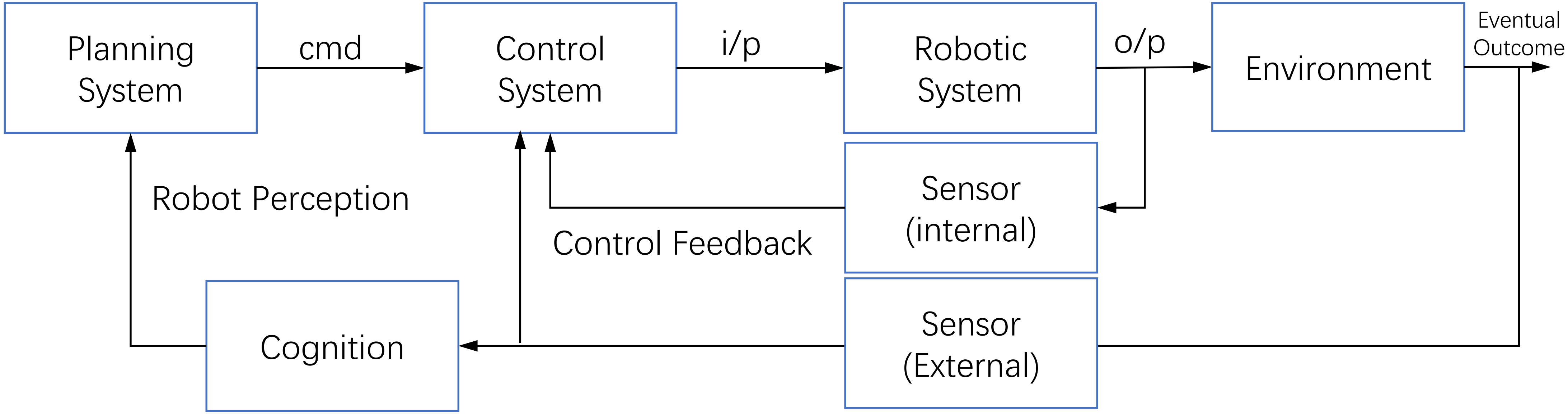
Applied

Week 11-14

Reading Wk/ Exam on Week 15-16

Robotic Systems

- Model **kinematics** and **dynamics** of the robotic system
- Design **control** for appropriate input to achieve desired outcome
- **Planning system** to send the command to **control** system



- **Perceive** and interact with environment to achieve goal

Summary of Topics so Far

- Kinematics
  - relating joint and operational coordinates with spatial representation
- Dynamics
  - relating forces and motions of the multibody robotic system
- Control
  - designing control systems that generate the appropriate inputs for the robotic system to achieve a desired outcome in a dynamical environment with a specified performance
- Planning
  - Strategize series of appropriate commands for the robot to execute the desired action
- Perception

Wk06-08 Impt Take Away: Dynamics & Control

- Dynamics
  - interested in relating forces and motions of the multibody robotic system
  - Newtonian & Lagrangian Formulation
- Control
  - interested in designing control systems that generate the appropriate inputs for the robotic system to achieve a desired outcome in a dynamical environment with a specified performance

Impt Take Away: Dynamics

- **Dynamics**: Concern with the forces on bodies that cause motion
- In this course, we are interested in relating forces and motions of the multibody robotic system

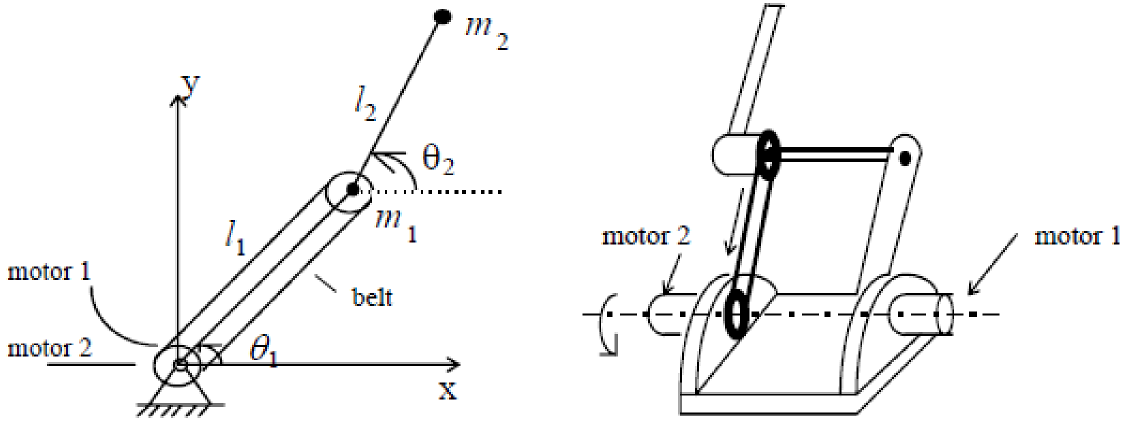
Dynamic equation:

- $\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta)$
- $M(\Theta)$ is n x n mass matrix of the manipulator
- $V(\Theta, \dot{\Theta})$ is an n x 1 vector of centrifugal and Coriolis terms
- $G(\Theta)$ is an n x 1 vector of gravity terms

Cartesian Space: $\mathcal{F} = M_x(\Theta)\ddot{\chi} + V_x(\Theta, \dot{\Theta}) + G_x(\Theta)$

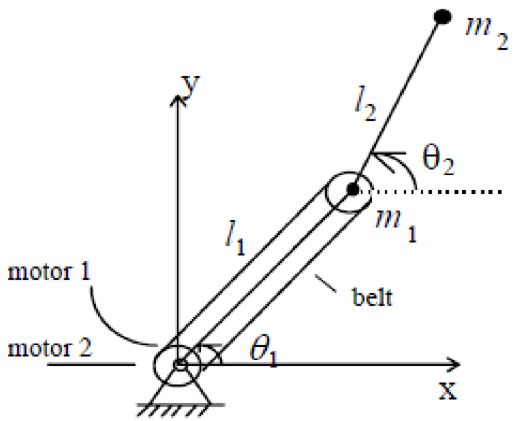
Review: Dynamics

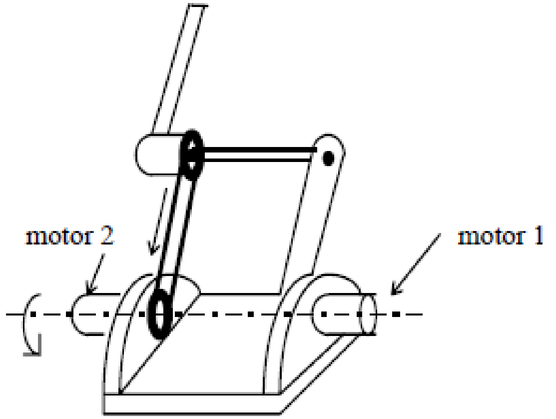
- Example HW3



Review: Dynamics

- Example HW3





a) $\begin{cases} x_1 = l_1 \cos\theta_1 \\ y_1 = l_1 \sin\theta_1 \end{cases}$ $\begin{cases} x_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2 \\ y_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \end{cases}$

b) $\begin{cases} \dot{x}_1 = -l_1 \sin\theta_1 \dot{\theta}_1 \\ \dot{y}_1 = l_1 \cos\theta_1 \dot{\theta}_1 \end{cases}$ $\begin{cases} \dot{x}_2 = -(l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin\theta_2 \dot{\theta}_2) \\ \dot{y}_2 = l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2 \end{cases}$

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2 = l_1^2 \dot{\theta}_1^2$$

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2 = l_1^2 \dot{\theta}_1^2 + 2 l_1 l_2 \cos(\theta_2 - \theta_1) \dot{\theta}_1 \dot{\theta}_2 + l_2^2 \dot{\theta}_2^2$$

c)

$$K = \frac{1}{2} m_1 {v_1}^2 + \frac{1}{2} m_2 {v_2}^2$$

$$= \frac{1}{2}(m_1 {l_1}^2 + m_2 {l_1}^2) {\dot{\theta}_1}^2 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\, \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 {l_2}^2 {\dot{\theta}_2}^2$$

d)

$$U = [m_1 g l_1 \sin\theta_1 + m_2 g (l_1 \sin\theta_1 + l_2 \sin\theta_2)]$$

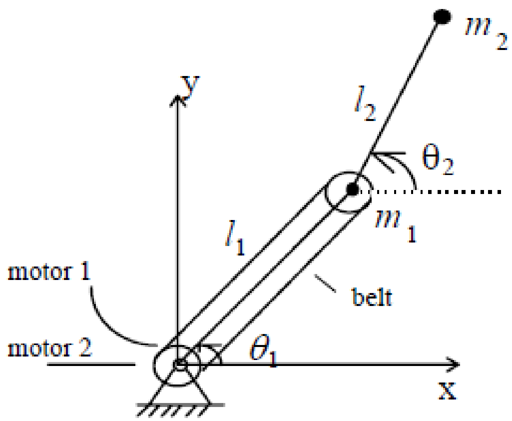
Lagrangian $L = K - U$

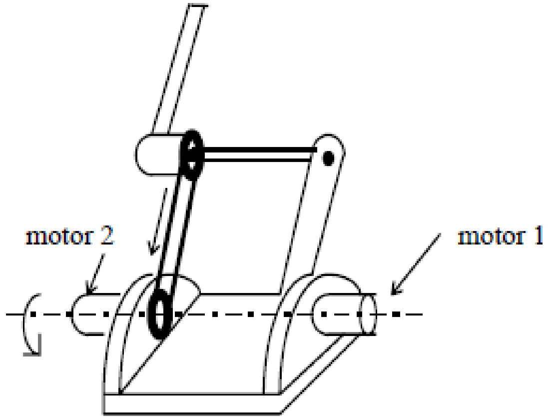
$$= \left[\frac{1}{2}(m_1 {l_1}^2 + m_2 {l_1}^2) {\dot{\theta}_1}^2 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\, \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 {l_2}^2 {\dot{\theta}_2}^2\right]$$

$$- [m_1 g l_1 \sin\theta_1 + m_2 g (l_1 \sin\theta_1 + l_2 \sin\theta_2)]$$

Review: Dynamics

- Example HW3





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Lagrangian $L = K - U$

$$= \left[\frac{1}{2}(m_1 {l_1}^2 + m_2 {l_1}^2) {\dot{\theta}_1}^2 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1) \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 {l_2}^2 {\dot{\theta}_2}^2\right]$$

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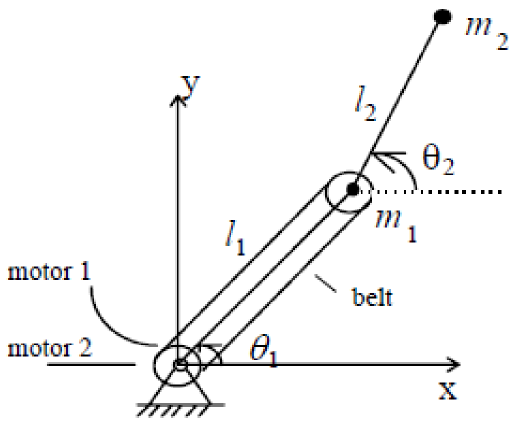
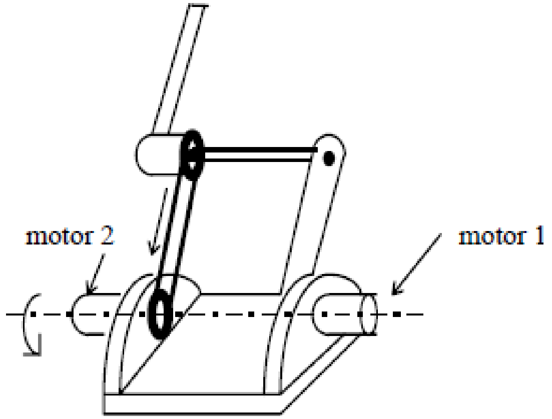
Review: Dynamics

- Example HW3





Lagrangian $L = K - U$

$$= \left[\frac{1}{2}(m_1 l_1{}^2 + m_2 l_1{}^2)\dot{\theta}_1{}^2 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\,\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2} m_2 l_2{}^2 \dot{\theta}_2{}^2\right]$$
$$- [m_1 g l_1 \sin\theta_1 + m_2 g(l_1 \sin\theta_1 + l_2 \sin\theta_2)]$$

$$\tau_1 = \left[(m_1 + m_2) l_1{}^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\,\ddot{\theta}_2 - m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_2{}^2 + (m_1 + m_2) g l_1 \cos\theta_1\right]$$

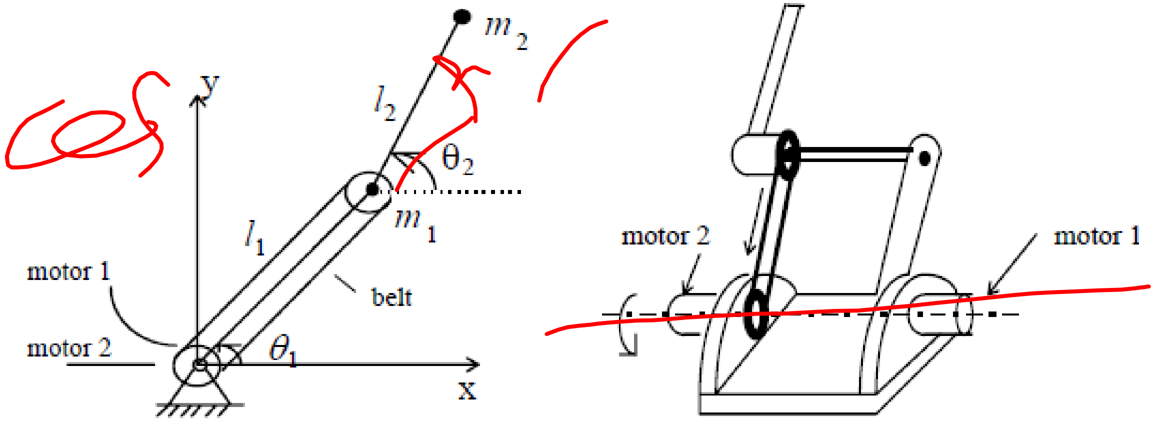
$$\tau_2 = \left[m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\,\ddot{\theta}_1 + m_2 l_2{}^2 \ddot{\theta}_2 + m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_1{}^2 + m_2 g l_2 \cos\theta_2\right]$$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}(m_1 + m_2) l_1{}^2 & m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\\ m_2 l_1 l_2 \cos(\theta_2 - \theta_1) & m_2 l_2{}^2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix}-m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_2{}^2\\ m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_1{}^2\end{bmatrix} + \begin{bmatrix}(m_1 + m_2) g l_1 \cos\theta_1\\ m_2 g l_2 \cos\theta_2\end{bmatrix}$$

Review: Dynamics

→ Generalized forces

- Example HW3



Lagrangian $L = K - U$

$$= \left[\frac{1}{2}(m_1 l_1{}^2 + m_2 l_1{}^2)\dot{\theta}_1{}^2 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\,\dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 l_2{}^2 \dot{\theta}_2{}^2\right]$$
$$- [m_1 g l_1 \sin\theta_1 + m_2 g(l_1 \sin\theta_1 + l_2 \sin\theta_2)]$$

$$\tau_1 = \left[(m_1 + m_2) l_1{}^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\,\ddot{\theta}_2 - m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_2{}^2 + (m_1 + m_2) g l_1 \cos\theta_1\right]$$

$$\tau_2 = \left[m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\,\ddot{\theta}_1 + m_2 l_2{}^2 \ddot{\theta}_2 + m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_1{}^2 + m_2 g l_2 \cos\theta_2\right]$$

τ_a τ_b

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}(m_1 + m_2) l_1{}^2 & m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\\ m_2 l_1 l_2 \cos(\theta_2 - \theta_1) & m_2 l_2{}^2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix}-m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_2{}^2\\ m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\,\dot{\theta}_1{}^2\end{bmatrix} + \begin{bmatrix}(m_1 + m_2) g l_1 \cos\theta_1\\ m_2 g l_2 \cos\theta_2\end{bmatrix}$$

Review: Dynamics

How is it that the Dynamics is so different?
Kinematically Identical Mechanism

Recall Example 5.2

Extracting the $\hat{Z}$ components of the ${}^i n_i$, we find the joint torques:

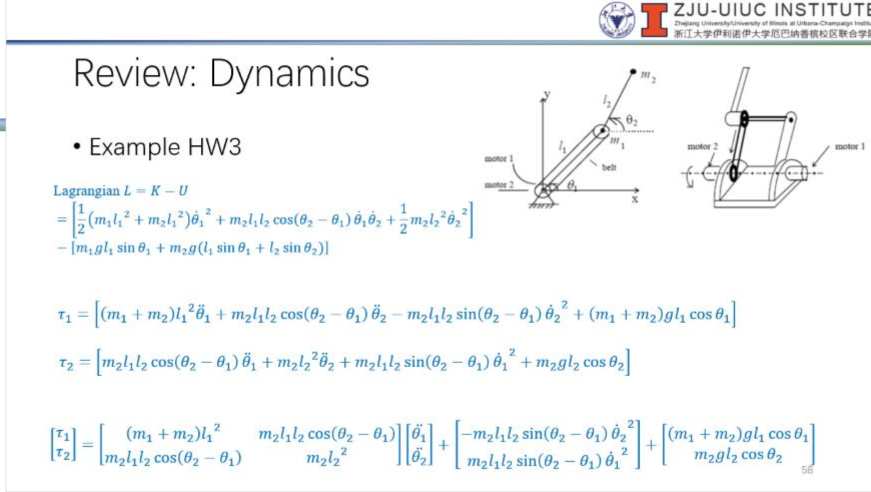
$$\tau_1 = \boxed{m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2)} + \boxed{m_2 l_1 l_2 c_2 (2\ddot{\theta}_1} + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2$$
$$- 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + \boxed{m_2 l_2 g c_{12}} + (m_1 + m_2) l_1 g c_1,$$
$$\tau_2 = \boxed{m_2 l_1 l_2 c_2 \ddot{\theta}_1} + \underline{m_2 l_1 l_2 s_2 \dot{\theta}_1^2} + \boxed{m_2 l_2 g c_{12}} + \boxed{m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2)}.$$

$$M(\Theta) = \begin{bmatrix} l_2^2 m_2 + 2 l_1 l_2 m_2 c_2 + l_1^2(m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix}$$

$$V(\Theta, \dot{\Theta}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix}$$

$$G(\Theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}$$

$$\boxed{\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta)}$$



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Review: Dynamics

- Example HW3

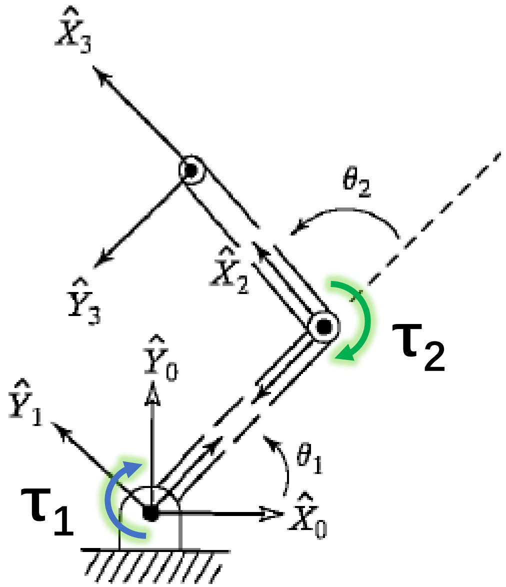
Lagrangian $L = K - U$
$$= \left[\frac{1}{2}(m_1 l_1^2 + m_2 l_1^2)\dot{\theta}_1^2 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2\right]$$
$$- [m_1 g l_1 \sin\theta_1 + m_2 g(l_1 \sin\theta_1 + l_2 \sin\theta_2)]$$

$$\tau_1 = \left[(m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\ddot{\theta}_2 - m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\dot{\theta}_2^2 + (m_1 + m_2) g l_1 \cos\theta_1\right]$$

$$\tau_2 = \left[m_2 l_1 l_2 \cos(\theta_2 - \theta_1)\ddot{\theta}_1 + m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\dot{\theta}_1^2 + m_2 g l_2 \cos\theta_2\right]$$

$$\begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix} = \begin{bmatrix} (m_1 + m_2) l_1^2 & m_2 l_1 l_2 \cos(\theta_2 - \theta_1) \\ m_2 l_1 l_2 \cos(\theta_2 - \theta_1) & m_2 l_2^2 \end{bmatrix}\begin{bmatrix}\ddot{\theta}_1 \\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix} -m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\dot{\theta}_2^2 \\ m_2 l_1 l_2 \sin(\theta_2 - \theta_1)\dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} (m_1 + m_2) g l_1 \cos\theta_1 \\ m_2 g l_2 \cos\theta_2 \end{bmatrix}$$

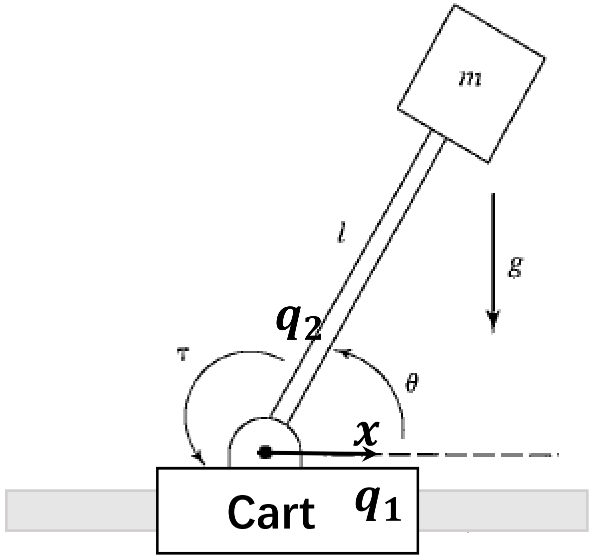
58

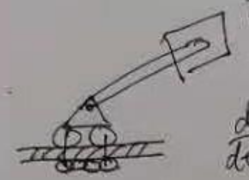


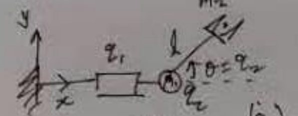
Review: Dynamics

- Example HW5

Find the new equation of motion relating $\boldsymbol{f}$ and $\boldsymbol{\tau}$ to $\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}, \ddot{\boldsymbol{\theta}}, \dot{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}$ if the single-link manipulator is mounted on a horizontally moving cart as shown in the Figure.







m_1: ${}^0P_1 = \begin{pmatrix} q_1 \\ 0 \\ 0 \end{pmatrix}$ ${}^0v_1 = \begin{pmatrix} \dot q_1 \\ 0 \\ 0 \end{pmatrix}$

m_2: ${}^0P_2 = \begin{pmatrix} l\cos q_2 + q_1 \\ l\sin q_2 \\ 0 \end{pmatrix}$

${}^0v_2 = \begin{pmatrix} -l\sin q_2 \cdot \dot q_2 + \dot q_1 \\ l\cos q_2 \dot q_2 \end{pmatrix}$

$$K = \frac{1}{2} m_1 \dot q_1^2 + \frac{1}{2} m_2 \left[l^2 \cos^2 q_2 \dot q_2^2 + (\dot q_1 - l\sin q_2 \dot q_2)^2 \right]$$
$$= \frac{1}{2} m_1 \dot q_1^2 + \frac{1}{2} m_2 \left[l^2 \dot q_2^2 \cos^2 q_2 + \dot q_1^2 - 2\dot q_1 l \sin q_2 \dot q_2 + l^2 \sin^2 q_2 \dot q_2^2 \right]$$
$$= \frac{1}{2} m_1 \dot q_1^2 + \frac{1}{2} m_2 \left[l^2 \dot q_2^2 + \dot q_1^2 - 2\dot q_1 \dot q_2 l \sin q_2 \right]$$
$$= \frac{\dot q_1^2}{2}(m_1 + m_2) + \frac{m_2 l^2}{2} \dot q_2^2 - m_2 \dot q_1 \dot q_2 l \sin q_2$$
$$= \frac{(m_1 + m_2)}{2} \dot q_1^2 + \frac{m_2 l^2}{2} \dot q_2^2 - m_2 \dot q_1 \dot q_2 l \sin q_2$$

$$P = m_2 g l \sin q_2$$

$$L = K - P = \left(\frac{m_1 + m_2}{2}\right) \dot q_1^2 + \frac{m_2 l^2}{2} \dot q_2^2 - m_2 \dot q_1 \dot q_2 l \sin q_2 - m_2 g l \sin q_2$$

$$\frac{\partial L}{\partial q} = \begin{bmatrix} 0 \\ -m_2 \dot q_1 \dot q_2 l \cos q_2 - m_2 l g \cos q_2 \end{bmatrix}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q_{i=1}} = (m_1 + m_2) \ddot q_1 - m_2 \ddot q_2 l \sin q_2 - m_2 \dot q_2 l \cos q_2 \dot q_2$$
$$= (m_1 + m_2)\ddot q_1 - m_2 l \sin q_2 \ddot q_2 - m_2 \dot q_2^2 l \cos q_2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q_{i=2}} = m_2 l^2 \ddot q_2 - m_2 \ddot q_1 l \sin q_2 - m_2 \dot q_1 l \cos q_2 \dot q_2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q_{i=1,2}} = \begin{bmatrix} (m_1 + m_2) \ddot q_1 - m_2 l \sin q_2 \ddot q_2 - m_2 \dot q_2^2 l \cos q_2 \\ m_2 l^2 \ddot q_2 - m_2 l \sin q_2 \ddot q_1 - m_2 l \cos q_2 \dot q_1 \dot q_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} (m_1 + m_2)\ddot q_1 - m_2 l \sin q_2 \ddot q_2 - m_2 \dot q_2^2 l \cos q_2 \\ m_2 l^2 \ddot q_2 - \underbrace{m_2 l \sin q_2 \ddot q_1}_{\text{interactions}} - m_2 l g \cos q_2 \end{bmatrix}}$$

Review: Dynamics

- Example HW5

Find the new equation of motion relating $\boldsymbol{f}$ and $\boldsymbol{\tau}$ to $\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}, \ddot{\boldsymbol{\theta}}, \dot{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}$ if the single-link manipulator is mounted on a horizontally moving cart as shown in the Figure.

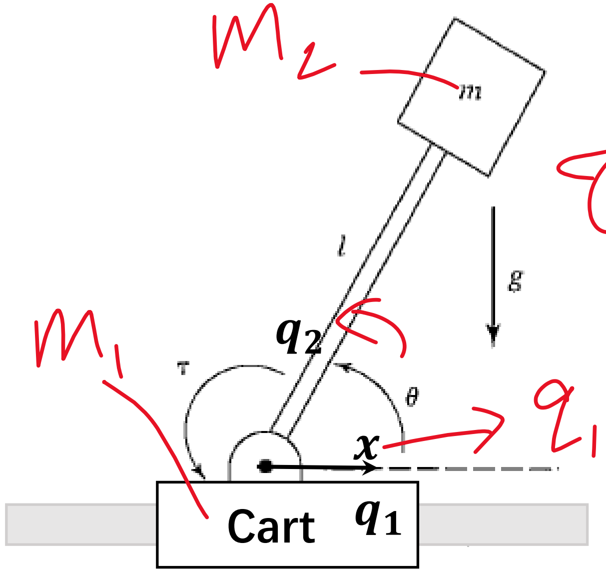
$$\tau = m l^2 \ddot{\theta} - m l g \cos\theta$$

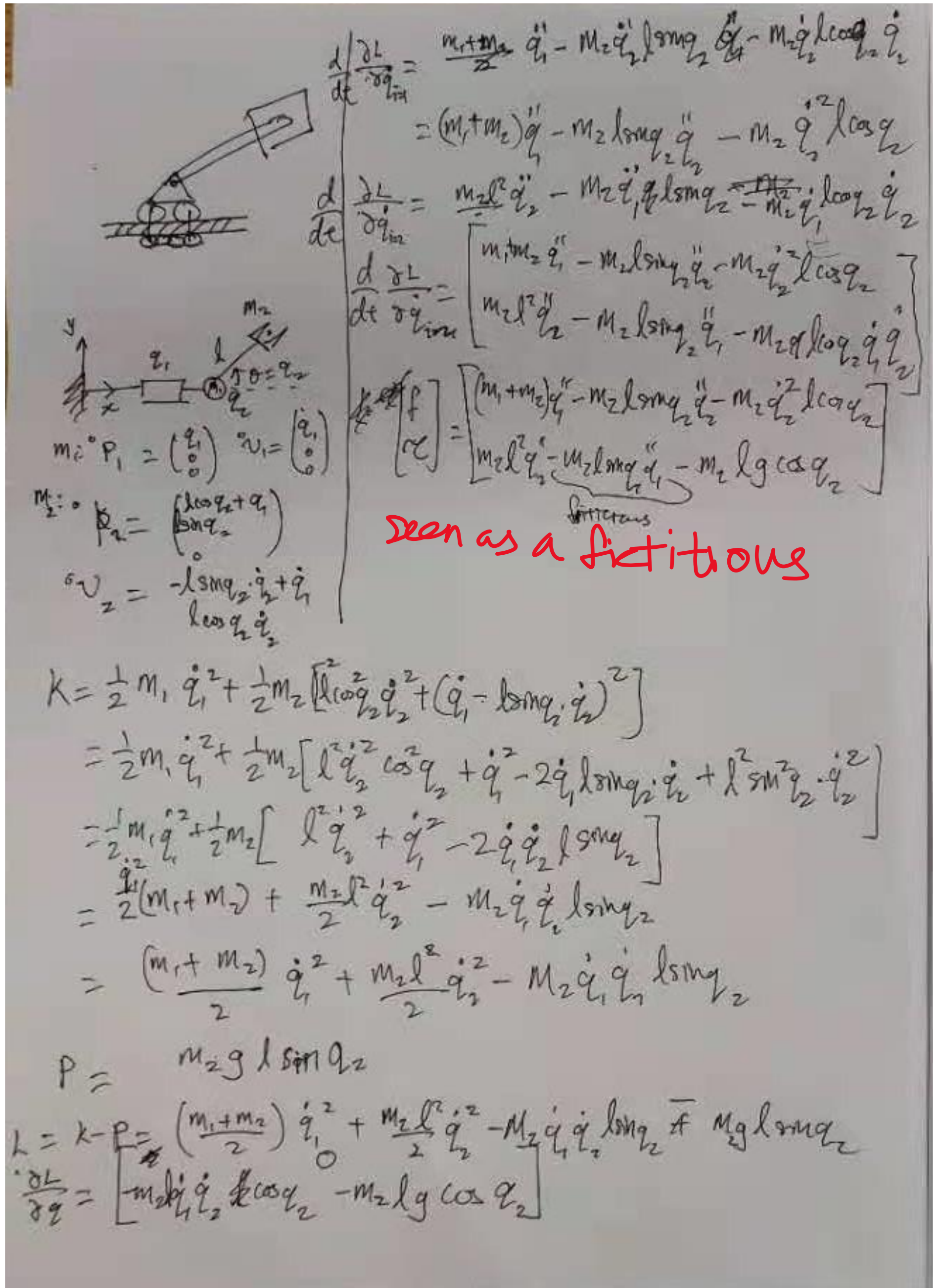
new

$$\tau' = \tau + \tau_{fict}$$

$$m_2 \quad r \times F_{fict}$$

$$l \sin q_2 (m_2 \ddot{q}_1)$$

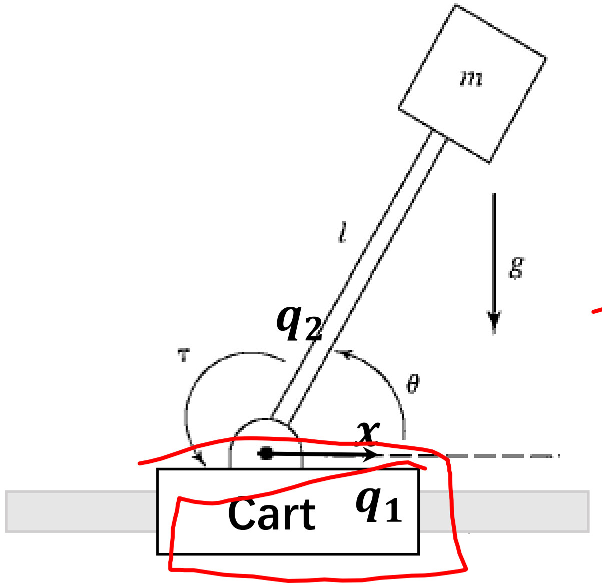


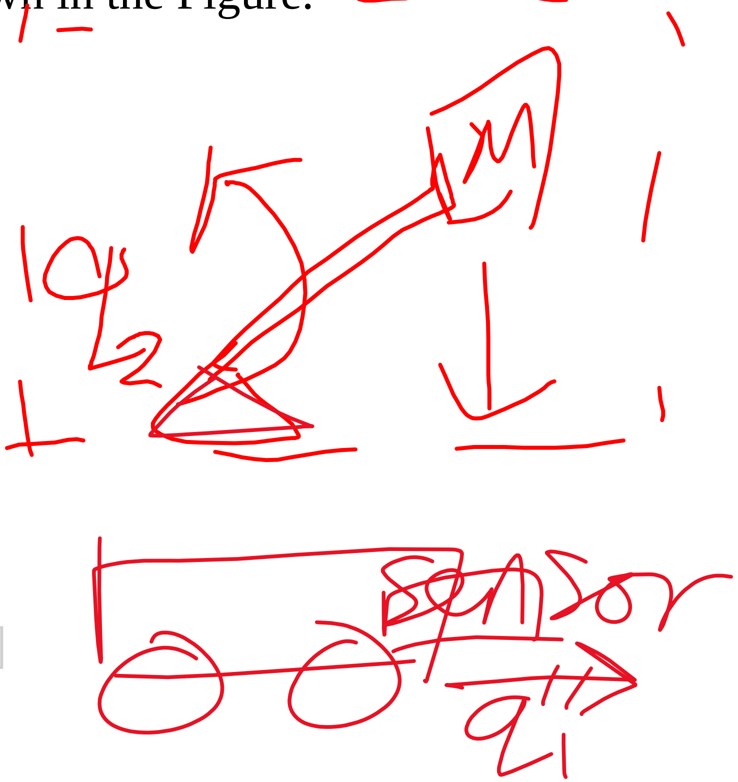


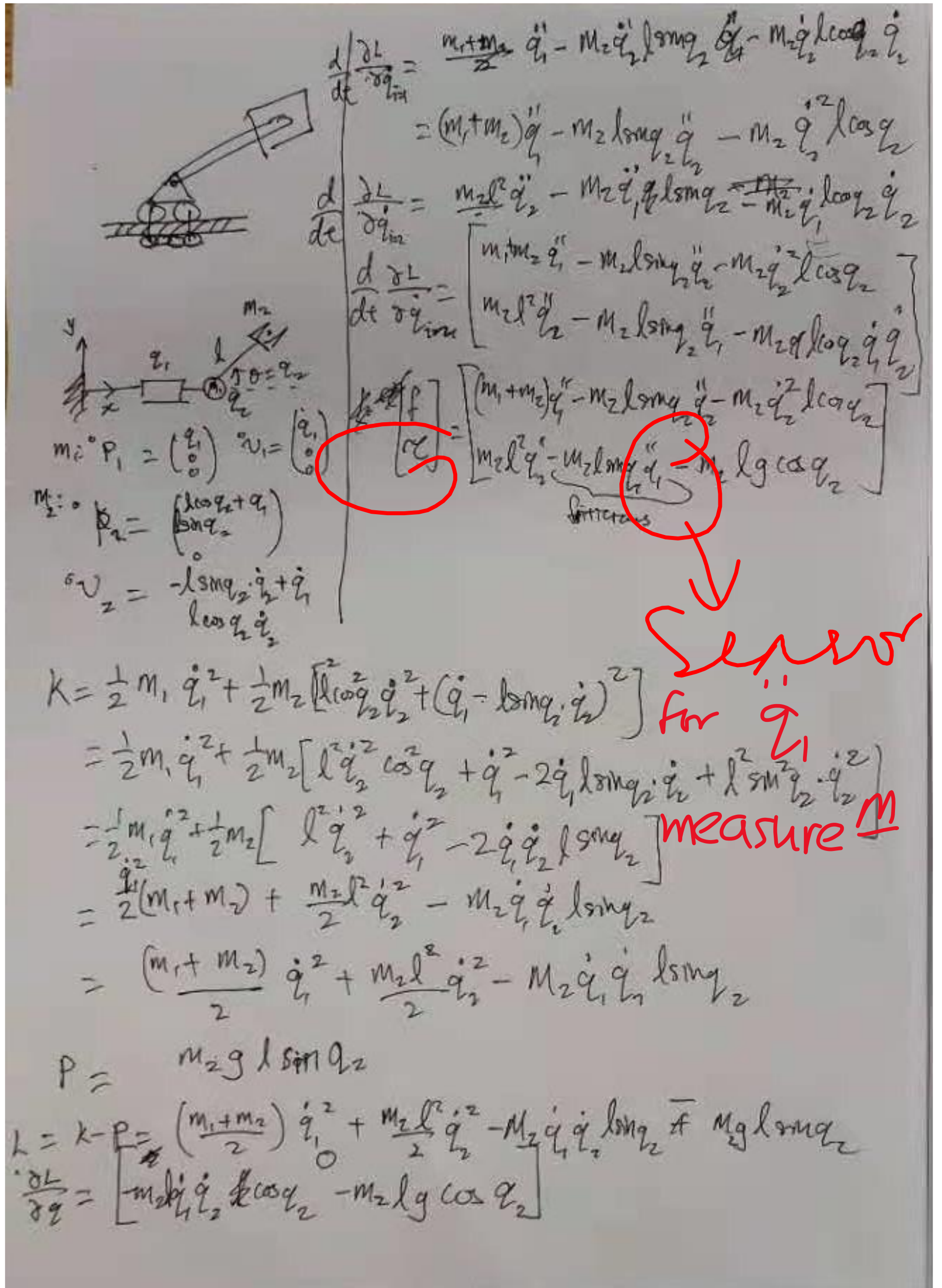
Review: Dynamics

- Example HW5

Find the new equation of motion relating $\boldsymbol{f}$ and $\boldsymbol{\tau}$ to $\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}, \ddot{\boldsymbol{\theta}}, \dot{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}$ if the single-link manipulator is mounted on a horizontally moving cart as shown in the Figure.

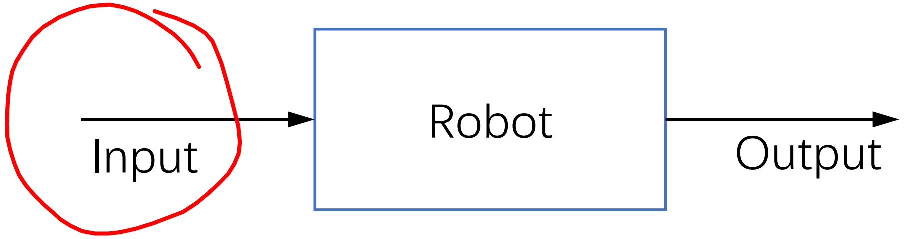






Impt Take Away: : Control

- **Control**: Generate the command to input to the robot to achieve a desired outcome
- In this course, we are interested in designing control systems that generate the appropriate inputs for the robotic system to achieve a desired outcome in a dynamical environment with a specified performance



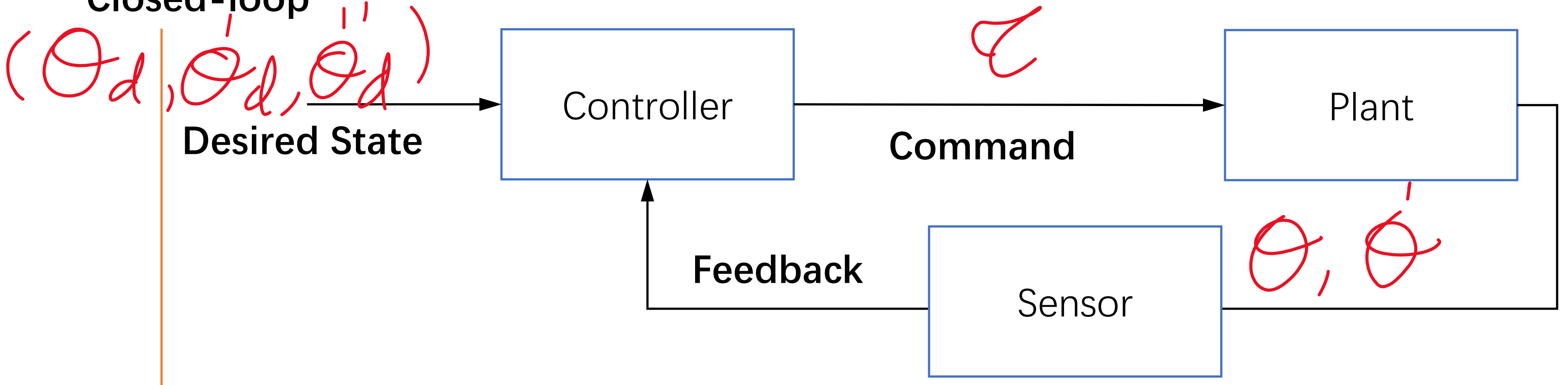
Impt Take Away: Open vs. Closed

$\tau = M\ddot{\theta}_d + V(\theta, \dot{\theta}) + G(\theta_d)$

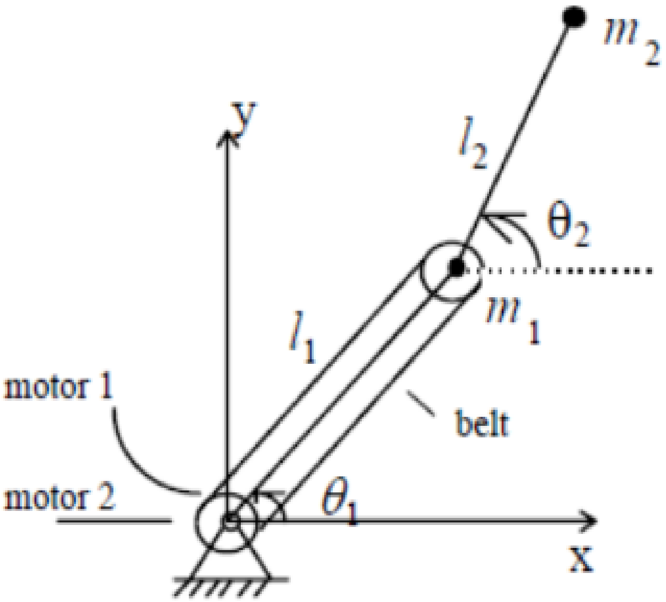
Open-loop

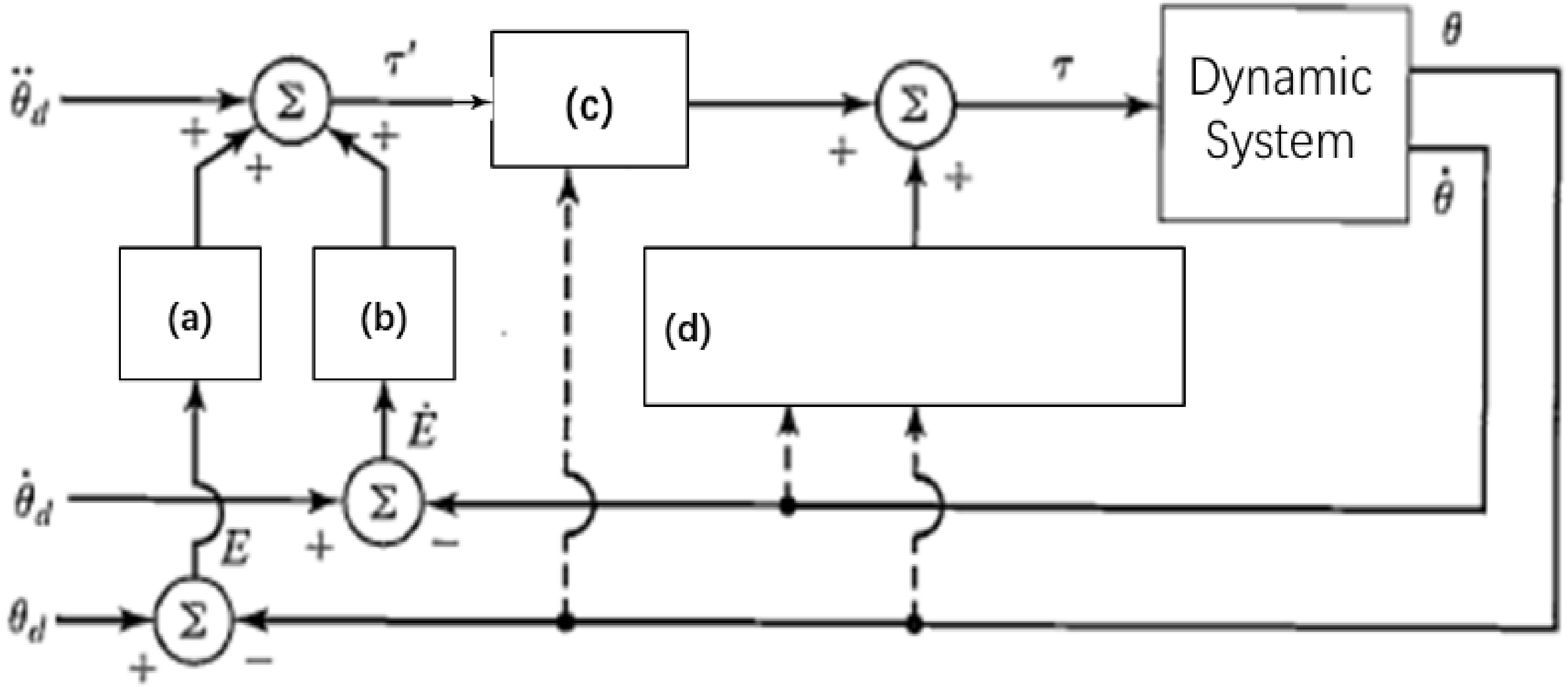
Through preestablished model, generate the command that will achieve the desired state.

Closed-loop

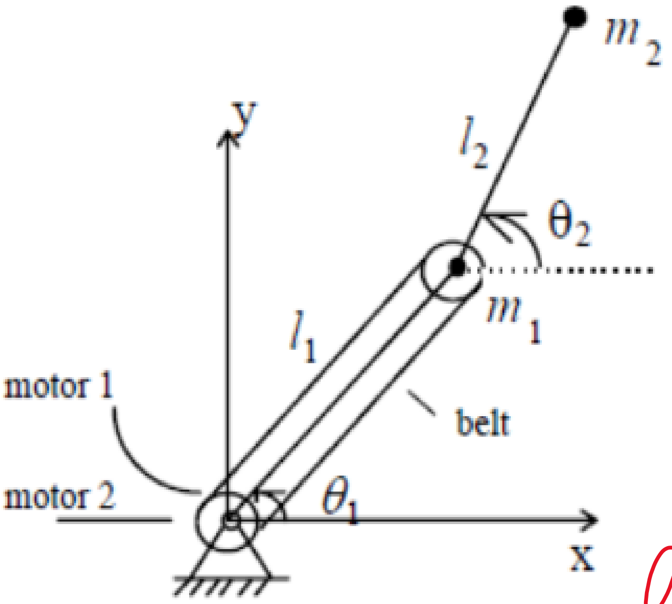


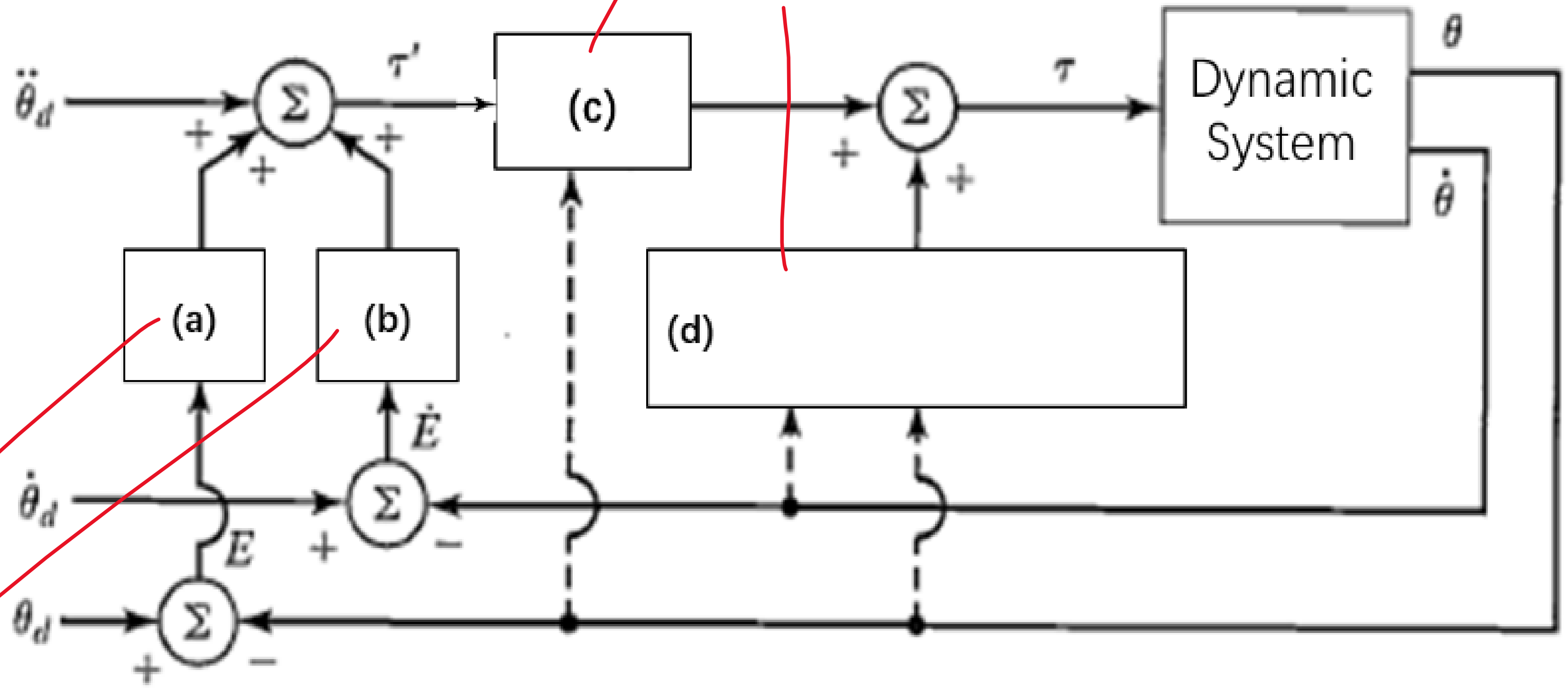
Through sensors, we are able to **feedback** the measurement to produce the command that will minimize the error between desired and actual targeted profile.


y
x
m_2
l_2
θ_2
m_1
l_1
motor 1
belt
motor 2
θ_1

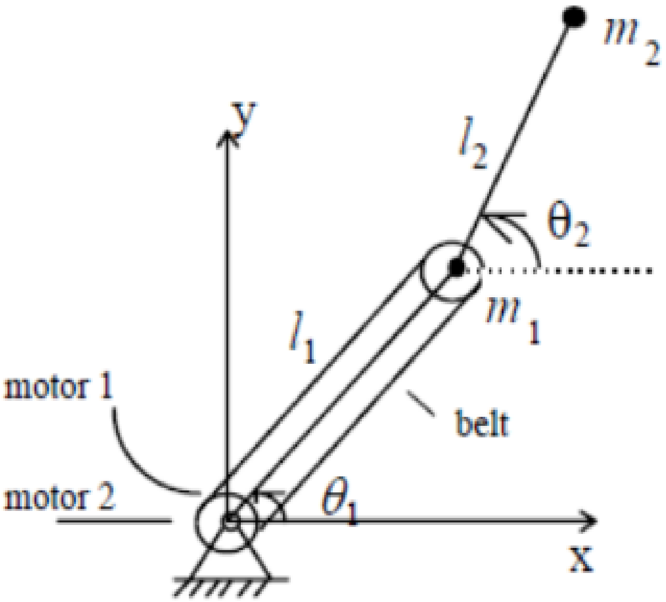

$\ddot{\theta}_d$
τ'
(c)
τ
Dynamic System
θ
$\dot{\theta}$
(a)
(b)
(d)
$\dot{E}$
$\dot{\theta}_d$
E
θ_d

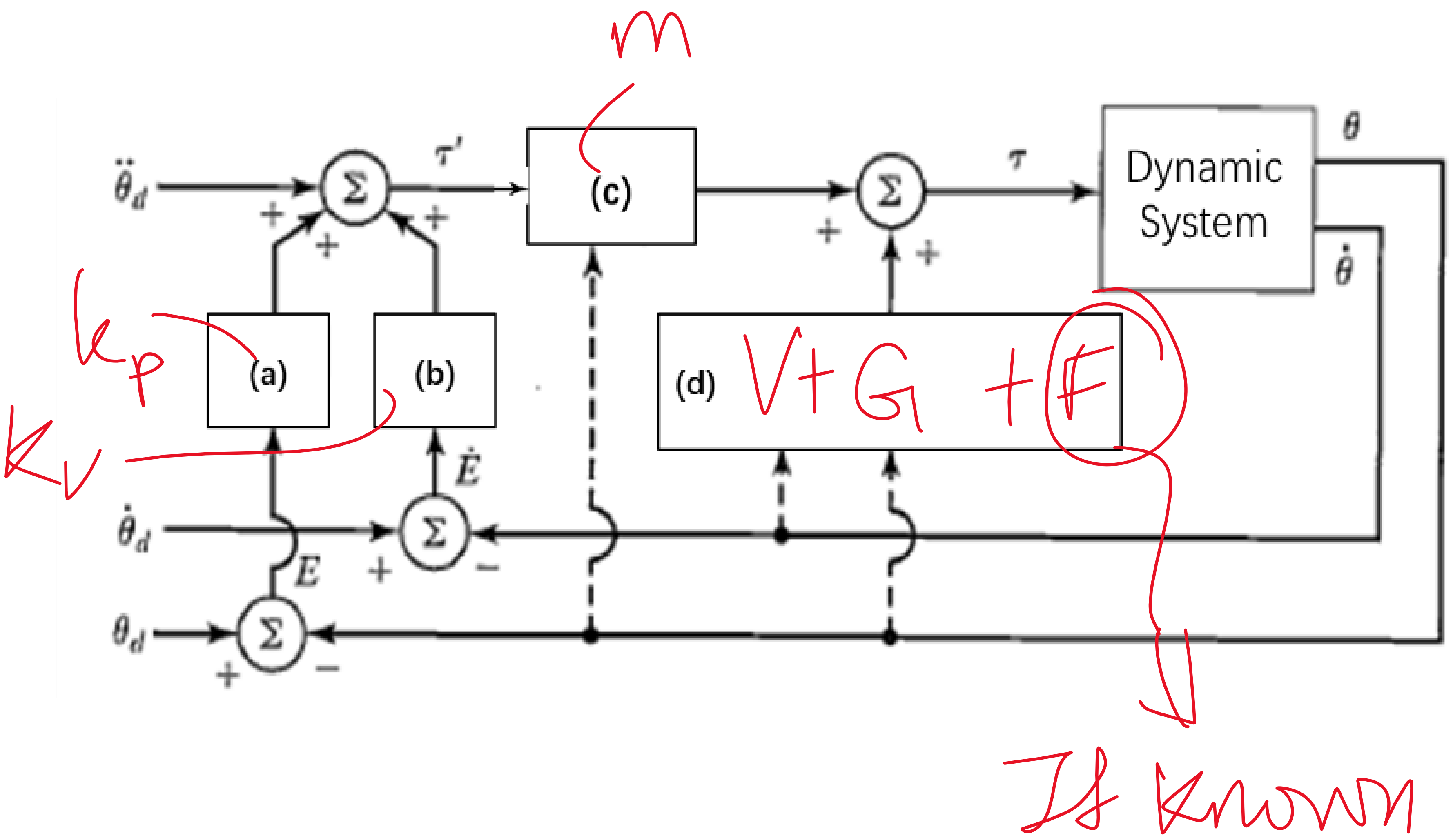
Information of Model

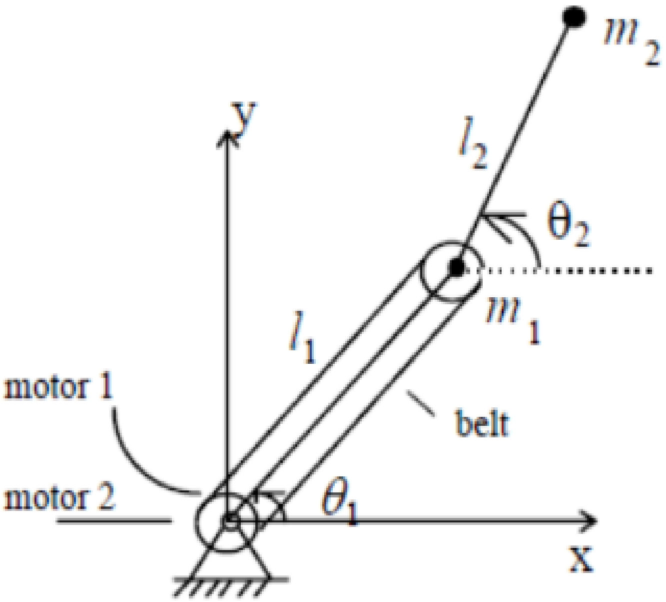


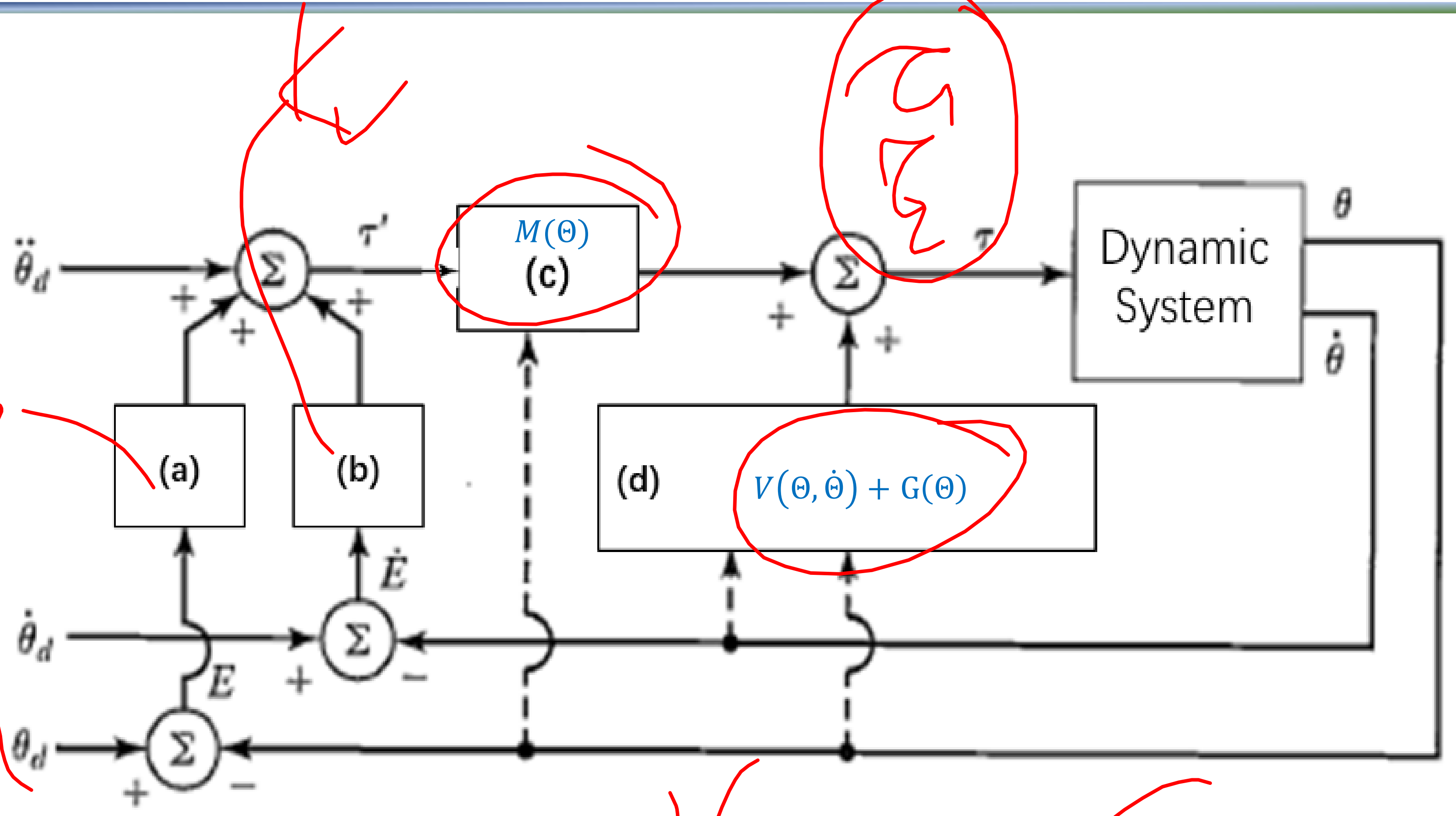


Gain

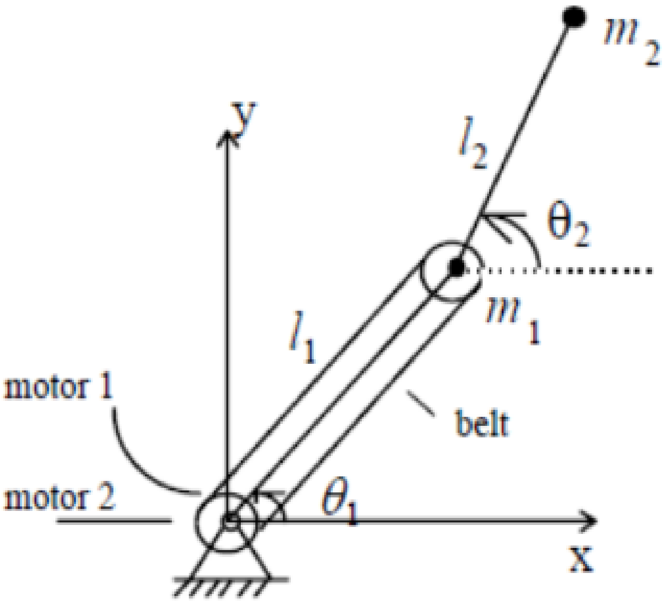

y
x
m_2
l_2
θ_2
m_1
l_1
motor 1
motor 2
belt
θ_1

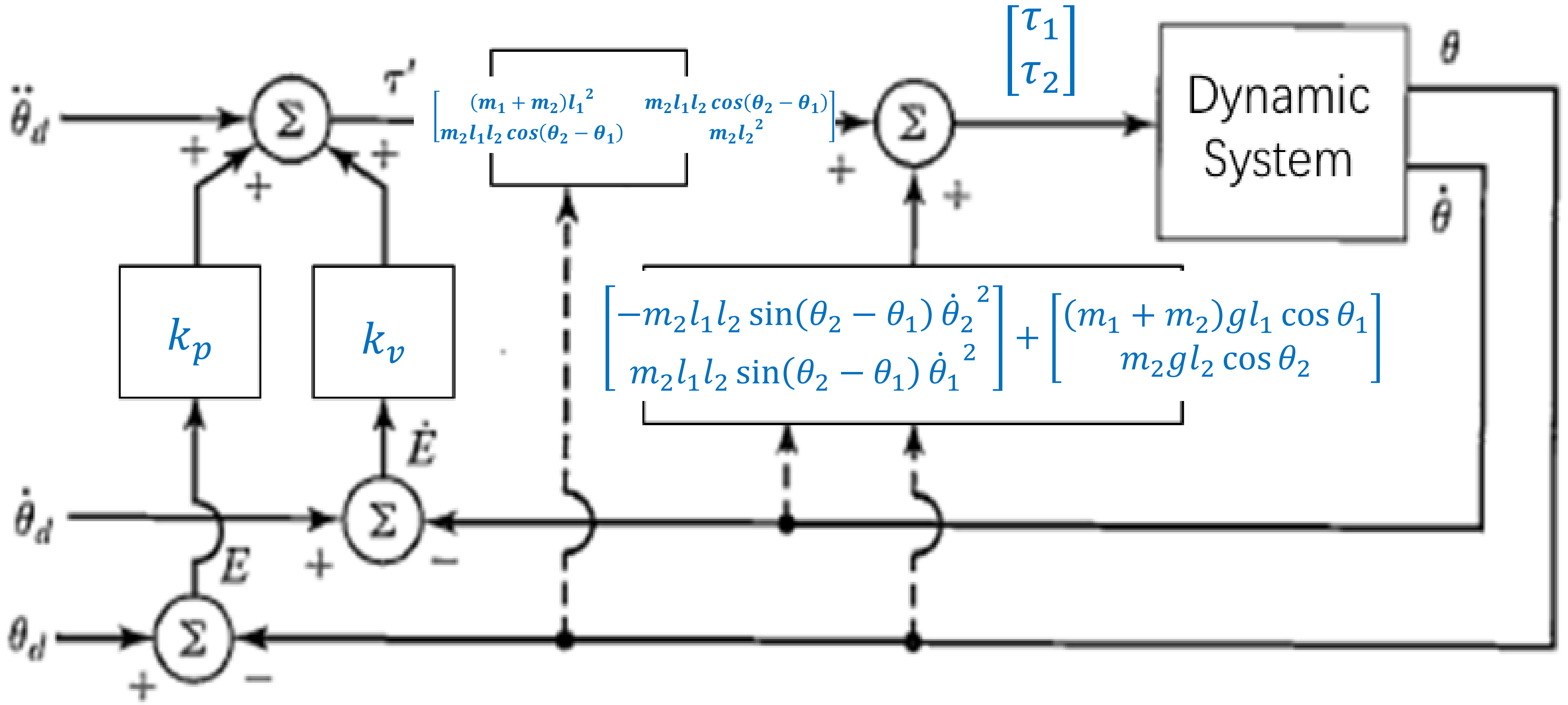

m
k_p
K_v
$\ddot{\theta}_d$
τ'
(c)
τ
Dynamic System
θ
$\dot{\theta}$
(a)
(b)
(d) V+G +F
$\dot{E}$
$\dot{\theta}_d$
E
θ_d
If known



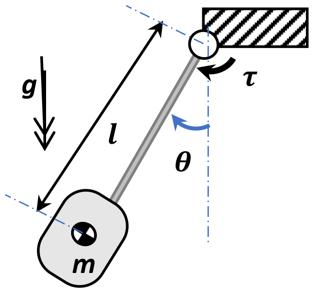


$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \underbrace{\begin{bmatrix}(m_1+m_2){l_1}^2 & m_2 l_1 l_2\cos(\theta_2-\theta_1)\\ m_2 l_1 l_2\cos(\theta_2-\theta_1) & m_2{l_2}^2\end{bmatrix}}_{M}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + \underbrace{\begin{bmatrix}-m_2 l_1 l_2\sin(\theta_2-\theta_1)\,{\dot{\theta}_2}^{\,2}\\ m_2 l_1 l_2\sin(\theta_2-\theta_1)\,{\dot{\theta}_1}^{\,2}\end{bmatrix}}_{V} + \underbrace{\begin{bmatrix}(m_1+m_2)g l_1\cos\theta_1\\ m_2 g l_2\cos\theta_2\end{bmatrix}}_{G}$$

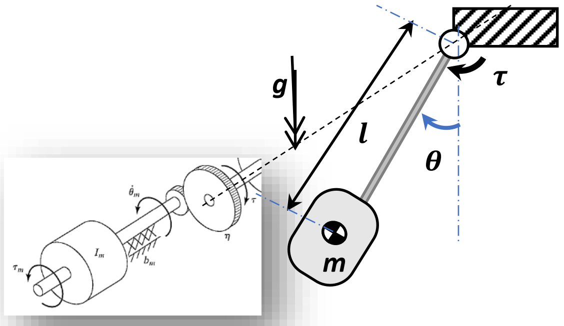




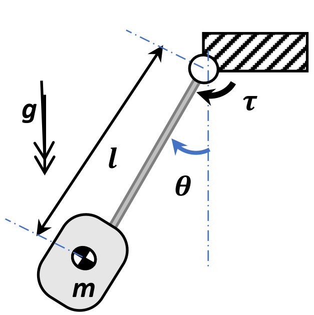
$$\begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix} = \begin{bmatrix} (m_1+m_2)l_1^2 & m_2 l_1 l_2 \cos(\theta_2-\theta_1) \\ m_2 l_1 l_2 \cos(\theta_2-\theta_1) & m_2 l_2^2 \end{bmatrix} \begin{bmatrix}\ddot{\theta}_1 \\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix} -m_2 l_1 l_2 \sin(\theta_2-\theta_1)\,\dot{\theta}_2^2 \\ m_2 l_1 l_2 \sin(\theta_2-\theta_1)\,\dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} (m_1+m_2) g l_1 \cos\theta_1 \\ m_2 g l_2 \cos\theta_2 \end{bmatrix}$$



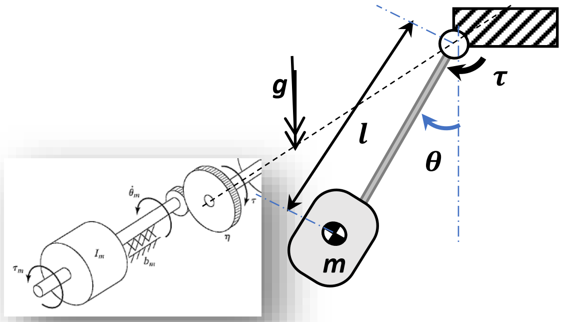
(a)



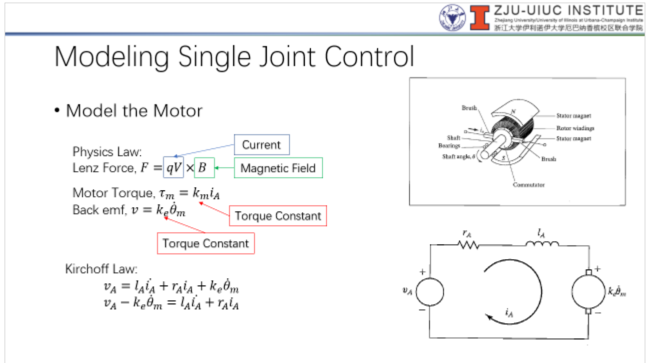
(b)

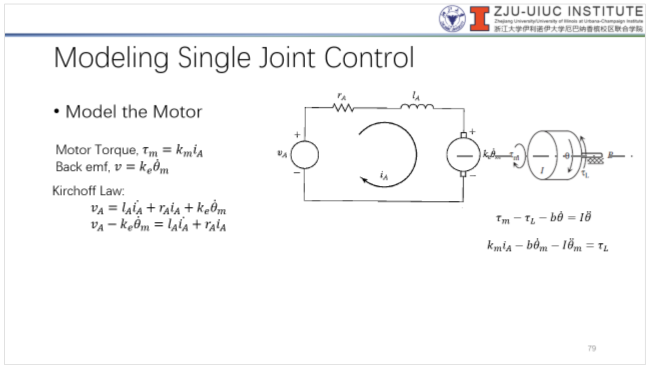

g
l
θ
τ
m

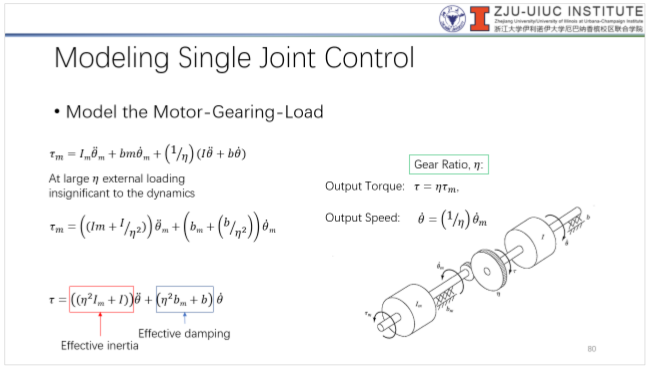
(a)


g
l
θ
τ
m

(b)


Modeling Single Joint Control
• Model the Motor
Physics Law:
Lenz Force, F = qV × B
Current
Magnetic Field
Motor Torque, τm = km iA
Back emf, v = ke θ̇m
Torque Constant
Torque Constant
Kirchoff Law:
vA = lA i̇A + rA iA + ke θ̇m
vA − ke θ̇m = lA i̇A + rA iA


Modeling Single Joint Control
• Model the Motor
Motor Torque, τm = km iA
Back emf, v = ke θ̇m
Kirchoff Law:
vA = lA i̇A + rA iA + ke θ̇m
vA − ke θ̇m = lA i̇A + rA iA
τm − τL − bθ̇ = Iθ̈
km iA − bθ̇m − Iθ̈m = τL


Modeling Single Joint Control
• Model the Motor-Gearing-Load
At large η external loading insignificant to the dynamics
Gear Ratio, η:
Output Torque: τ = ητm
Output Speed: θ̇ = (1/η) θ̇m
Effective inertia
Effective damping

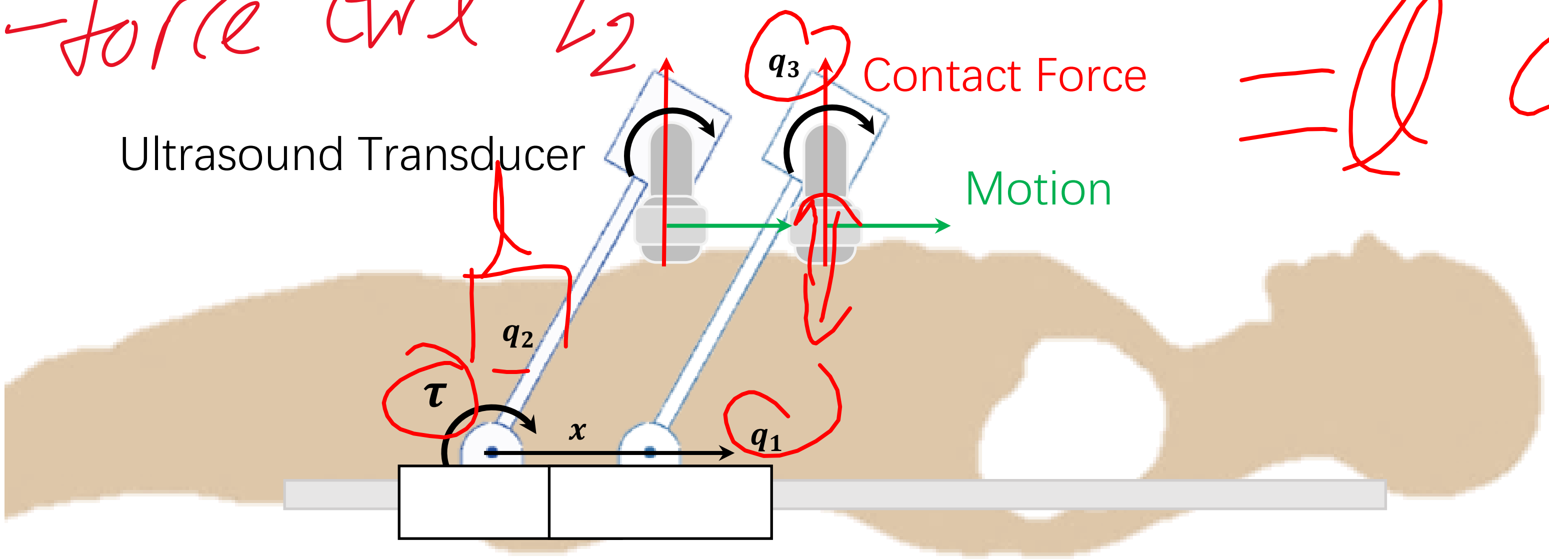
Imagine you decided to create a prismatic-revolute (PR) 2-dof robotic manipulator and mounted the single-link on a horizontally moving cart as shown in Figure 1(b),

IV. Suggest a control scheme if the manipulator is tasked to performance ultrasound imaging over a region by sliding the probe along the $\boldsymbol{x}$ direction at a vertically downward controlled contact force with the surface. You may assume an additional joint $\boldsymbol{q_3}$ to orientate the ultrasound transducer as shown.

Hybrid position-force control

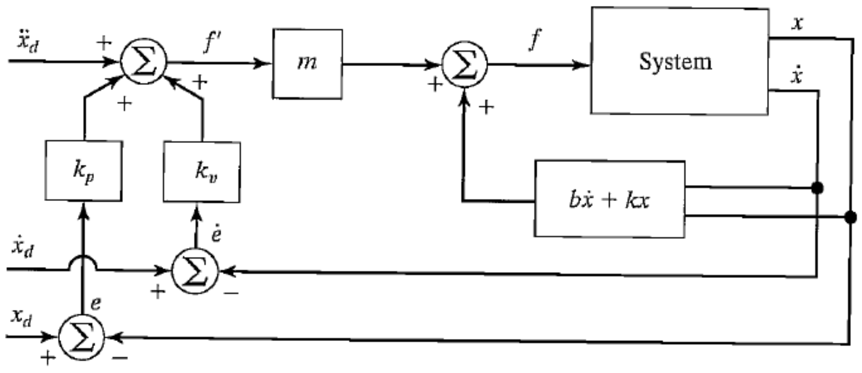
position ctrl q_1, q_3 $\quad \tau = \underline{r} \times F$

force ctrl q_2 $\quad = (l \cos q_2) F$

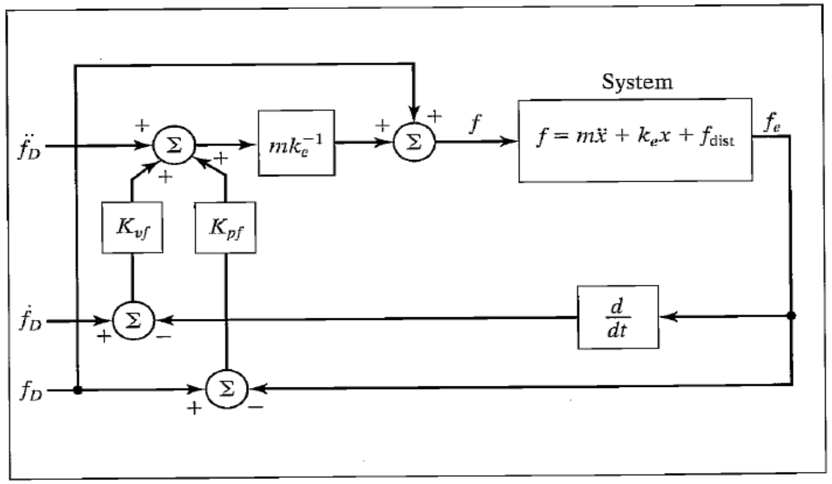


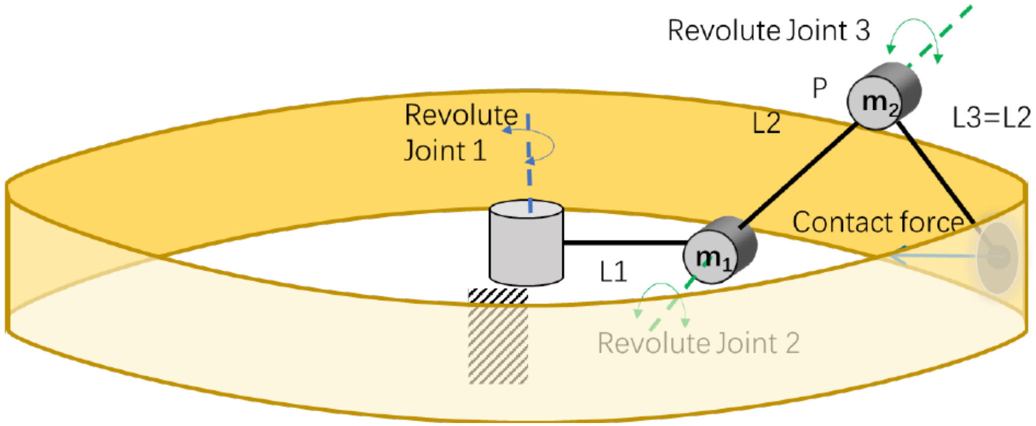
Impt Take Away: : Motion & Force Control

Motion Control



Force Control

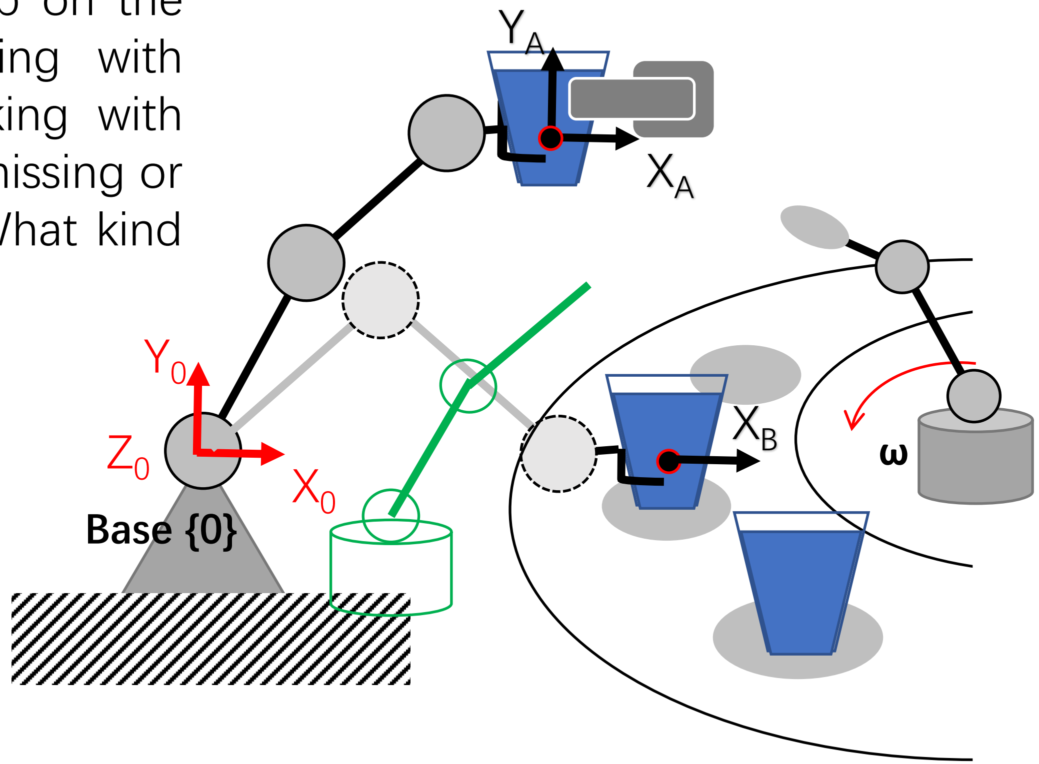




Example in Past Yr
Homework 4 2020SP

Impt Take Away : Types of Planning

A robot is tasked to bring a cup from A to B. Not to spill the water, place the cup on the saucers on a conveyer belt moving with angular with velocity **ω** while working with other robots that replace and clean missing or dirty in the same operation space. What kind of planning is required?



Path Planning

Trajectory Planning

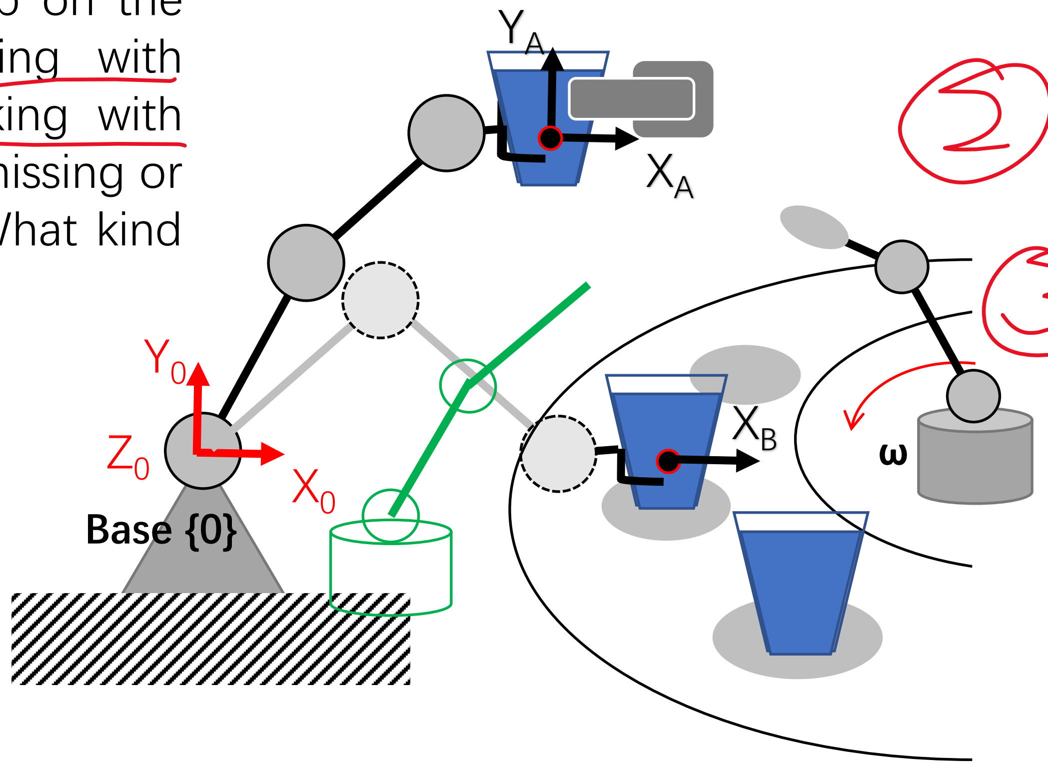
Motion Planning

Impt Take Away : Types of Planning

1 A robot is tasked to bring a cup from A to B. Not to spill the water, place the cup on the saucers on a conveyer belt moving with

2 angular with velocity **ω** while working with

3 other robots that replace and clean missing or dirty in the same operation space. What kind of planning is required?



1 **Path Planning**

2 **Trajectory Planning**

3 **Motion Planning**