ECE 470: Introduction to Robotics Homework 5

Question 1.

Consider the single-link manipulator arm in Figure 1(a) as shown also in Figure 10.4 (Craig, Introduction to Robotics 3rd Ed.).

a) Given that the revolute joint moves the link over 2 cubic segments in **6s** from an initial angle $\boldsymbol{\theta_0 = 15\ \mathrm{deg}}$ to rest at a final position $\boldsymbol{\theta_f = 90\ \mathrm{deg}}$ through a via point $\boldsymbol{\theta_v = 30\ \mathrm{deg}}$ at $\boldsymbol{t_v = 3s}$ with a velocity of $\boldsymbol{\dot{\theta}_v = 15\ \mathrm{deg/s}}$, obtain the 8 parameters of the 2-segment cubic polynomial. (10 Points)

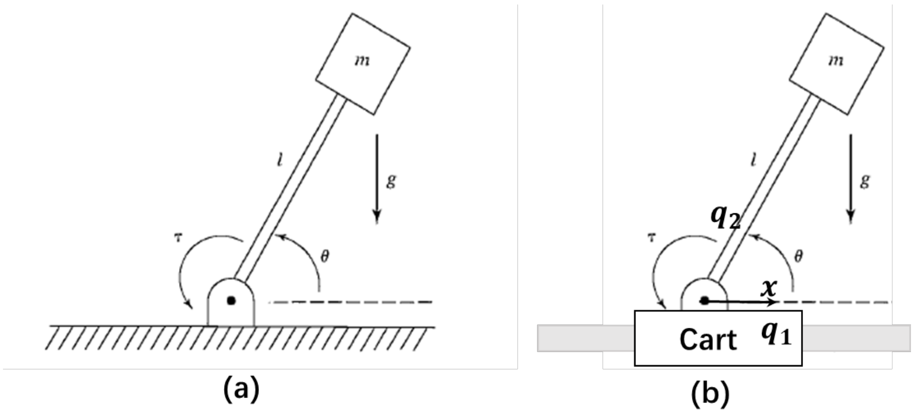


Figure 1

b) Imagine you decided to create a prismatic-revolute (PR) 2-dof robotic manipulator and mounted the single-link on a horizontally moving cart as shown in Figure 1(b),

I. Illustrate the configuration space of this PR manipulator with a sketch given that the joint limits are $x \subset [-d,\ d]$ and $\theta \subset [-\pi,\ \pi]$. (2 Points)

II. Describe a possible path in the configuration space if a vertical straight path is desired from point $(0, -\boldsymbol{l})$ to $(0, \boldsymbol{l})$ in the workspace (*Hint: circular motion projects to orthogonal axes as sinusoidal motion*) (3 Points)

III. Assuming the motor at $\boldsymbol{q_2}$ rotates at a constant speed of $\boldsymbol{\omega}$, suggest a trajectory for $\boldsymbol{q_1}$ (2 Points)

IV. Suggest a control scheme if the manipulator is tasked to performance ultrasound imaging over a region by sliding the probe along the $\boldsymbol{x}$ direction at a vertically downward controlled contact force with the surface. You may assume an additional joint $\boldsymbol{q_3}$ to orientate the ultrasound transducer as shown.in Figure 2 (3 Points)

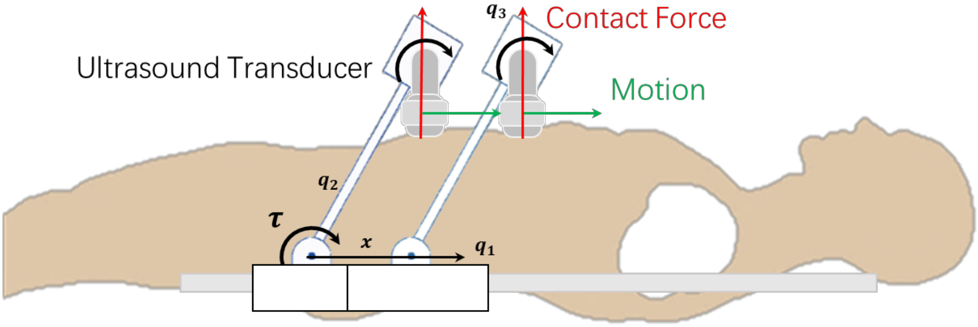


Figure 2

Question 2 (<u>Optional Bonus Question</u>)

Find the new equation of motion relating $\boldsymbol{f}$ and $\boldsymbol{\tau}$ to $\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}, \ddot{\boldsymbol{\theta}}, \dot{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}$ if the single-link manipulator is mounted on a horizontally moving cart as shown in Figure 1(b). (10 Points)

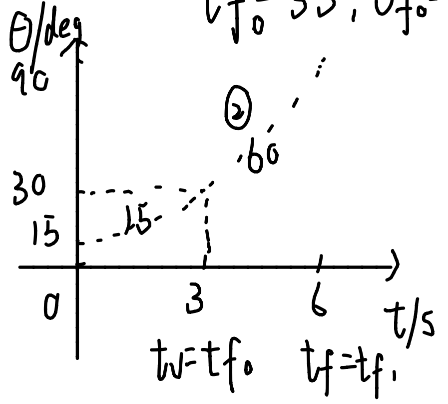
Q1. a)

① 1st. segment

$$\begin{cases} \theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ \dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 \\ \ddot{\theta}(t) = 2a_2 + 6a_3 t \end{cases} \Rightarrow \begin{cases} a_0 = \theta_0 = 15 \\ a_1 = \dot{\theta}_0 = 0 \\ a_2 = \frac{3}{t_{f_0}^2}(\theta_{f_0} - \theta_0) - \frac{2}{t_{f_0}}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_{f_0} \\ \quad = 20 \\ a_3 = -\frac{2}{t_f^3}(\theta_{f_0} - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_{f_0} - \dot{\theta}_0) \\ \quad = -3.9 \end{cases}$$

(unit: deg)

$t_{f_0} = 3s$, $\dot{\theta}_{f_0} = 15$

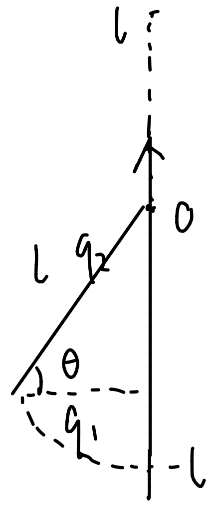


② Second segment

$$\begin{cases} \theta(t) = a_4 + a_5 t + a_6 t^2 + a_7 t^3 \\ \dot{\theta}(t) = a_5 + 2a_6 t + 3a_7 t^2 \\ \ddot{\theta}(t) = 2a_6 + 6a_7 t \end{cases} \Rightarrow \begin{cases} a_4 = \theta_1 = 30 \\ a_5 = \dot{\theta}_1 = 15 \\ a_6 = \frac{3}{t_{f_1}^2}(\theta_{f_1} - \theta_1) - \frac{2}{t_{f_1}}\dot{\theta}_1 - \frac{1}{t_f}\dot{\theta}_{f_1} \\ a_7 = -\frac{2}{t_f^3}(\theta_{f_1} - \theta_1) + \frac{1}{t_f^2}(\dot{\theta}_{f_1} - \dot{\theta}_1) \end{cases}$$

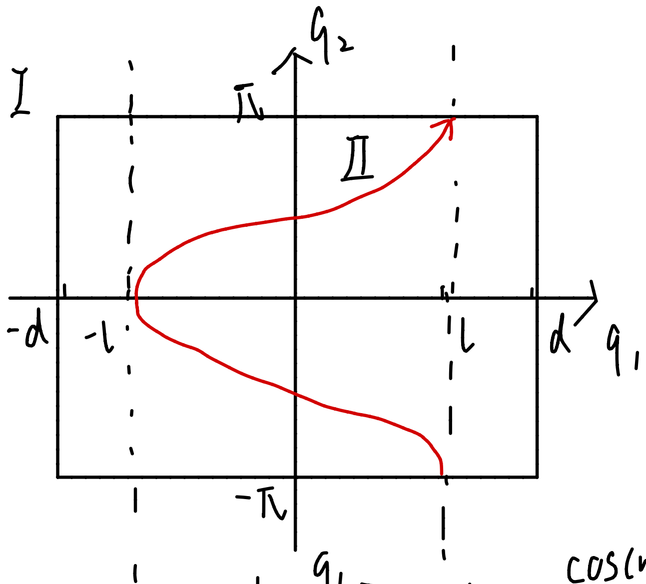
$t_{f_1} = 6s$, $\dot{\theta}_{f_1}$, $\theta_{f_1} = 90$

b)

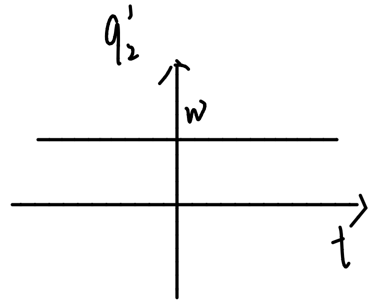


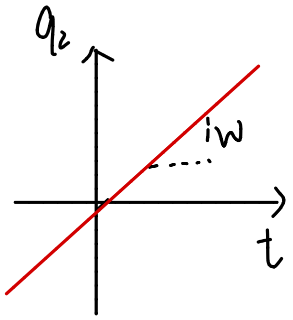
$q_1 = \cos q_2$

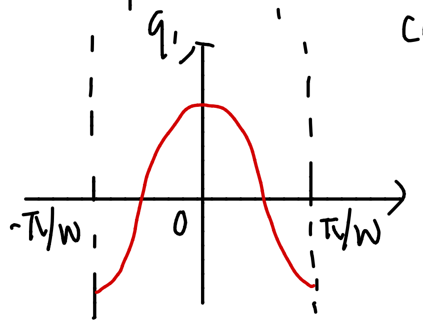
I



III



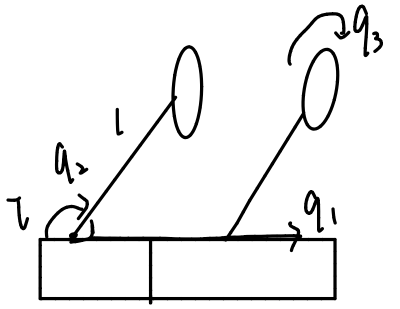




$\cos(wt)$

$T = \frac{2\pi}{w}$

IV



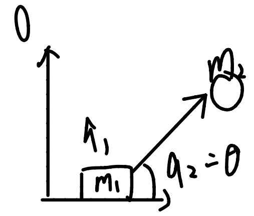
Hybrid Position-force Control

position control q_1 q_3

force control q_2

$$\tau = r \times F$$
$$= (l \cos q_2) F$$

Q2.



$$P_1 = \begin{pmatrix} q_1 \\ 0 \\ 0 \end{pmatrix} \quad {}^0V_1 = \begin{pmatrix} \dot{q}_1 \\ 0 \\ 0 \end{pmatrix} \; ; \quad P_2 = \begin{pmatrix} l\cos q_2 + q_1 \\ \sin q_2 \\ 0 \end{pmatrix}, \quad {}^0V_2 = \begin{pmatrix} -l\sin q_2 \dot{q}_2 + \dot{q}_1 \\ l\cos q_2 \dot{q}_2 \\ 0 \end{pmatrix}$$

$$K = \frac{1}{2} m_1 {}^0V_1^2 + \frac{1}{2} m_2 {}^0V_2^2$$
$$= \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 [(-l\sin q_2 \dot{q}_2 + \dot{q}_1)^2 + l^2 \cos^2 q_2 \, \dot{q}_2^2]$$

$$V = m_2 g l \sin q_2$$

$$L = K - V = \left(\frac{m_1 + m_2}{2}\right) \dot{q}_1^2 + \frac{m_2 l^2}{2} \dot{q}_2^2 - m_2 \dot{q}_1 \dot{q}_2 l \sin q_2 - m_2 g l \sin q_2$$

$$\frac{\partial L}{\partial q_2} = \begin{bmatrix} 0 \\ -m_2 \dot{q}_1 \dot{q}_2 l \cos q_2 - m_2 l g \cos q_2 \end{bmatrix}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_{i=1}} = (m_1 + m_2) \ddot{q}_1 - m_2 l \sin q_2 \ddot{q}_2 - m_2 \dot{q}_2^2 l \cos q_2$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_{i=2}} = m_2 l^2 \ddot{q}_2 - m_2 l \sin q_2 \ddot{q}_1 - m_2 l \cos q_2 \dot{q}_1 \dot{q}_2$$

$$F = \frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial q_i}$$

$$\therefore \begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} \frac{d}{dt}\frac{\partial L}{\partial q_1} - \frac{\partial L}{\partial q_1} \\ \frac{d}{dt}\frac{\partial L}{\partial q_2} - \frac{\partial L}{\partial q_2} \end{bmatrix} = \begin{bmatrix} (m_1+m_2)\ddot{q}_1 - m_2 l \sin q_2 \ddot{q}_2 - m_2 \dot{q}_2^2 l \cos q_2 \\ m_2 l^2 \ddot{q}_2 - m_2 l \sin q_2 \ddot{q}_1 - m_2 l g \cos q_2 \end{bmatrix}$$

where $q_2 = \theta$, $\dot{q}_2 = \dot{\theta}$, $\ddot{q}_2 = \ddot{\theta}$, $\ddot{q}_1 = \ddot{x}$

$$\therefore \begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} (m_1+m_2)\ddot{x} - m_2 l \sin\theta \ddot{\theta} - m_2 \dot{\theta}^2 l \cos\theta \\ m_2 l \ddot{\theta} - m_2 l \sin\theta \ddot{x} - m_2 l g \cos\theta \end{bmatrix}$$