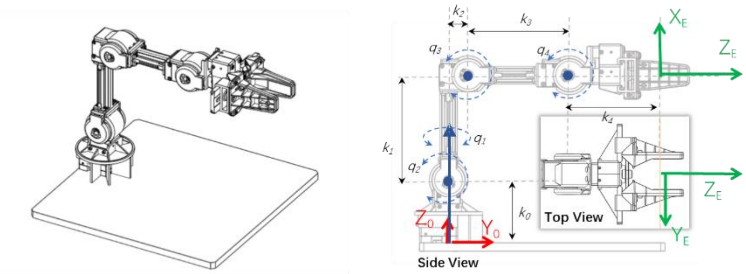
ECE 470: Introduction to Robotics Homework 2

Question 1. *(12 marks)*

A 4-DOF (excluding gripper) robotic serial manipulator arm is shown in Figure 1.



Using the D-H convention learned in class,

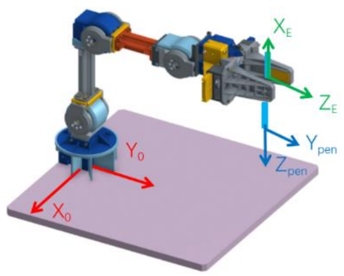
a) Assign frames to the links on a schematic diagram that represents the robot arm

b) Tabulate the D-H parameters

c) Obtain the forward kinematics representing the pose of end-effector frame {E} referenced from base frame {0}.

Question 2. *(8 marks)*

The serial manipulator arm is tasked to write on the board plane Z_o, with a pen attached to the gripper {E}. For the ink to flow, ${}^0Z_{pen}$ has to be $(0\ 0\ -1)^T$ i.e. vertically downwards. As shown in the diagram, axis X_E and Z_E are parallel to Z_{pen} and $Y_{pen.}$ respectively. The distance between Z_E and Y_{pen} is k_0.



State any assumption or condition while working on the following:

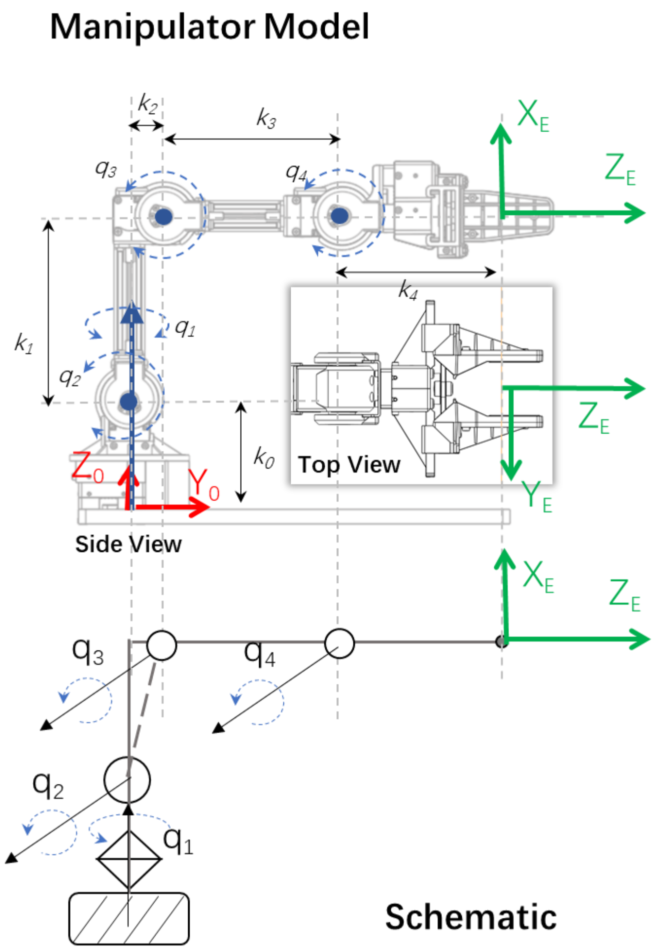
a) Write down the transformation matrix ${}^{E}_{pen}T$

b) If the pen tip is to be place on the board with coordinates ${}^0(u,v)$, find the expressions describing the joint variable $\boldsymbol{q}$ in terms of $k_{0\text{-}4}$, u and v.

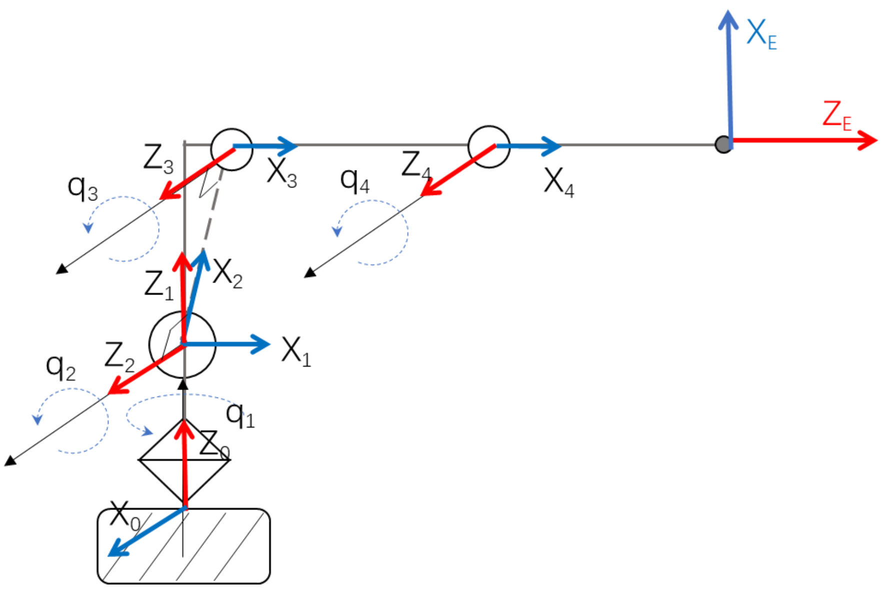
c) Describe the workspace of the writing task if the distance between Z_E and Y_{pen} is now change to $k_0/2$. Assume that q_2 can only move its link in a range of 0 to 180^o from the plane.

Solution

Question 1



(a) Frame Assignment



(b) D-H Parameters

	α	a	θ	d
${}^{0}T_1$	0	0	$q_1 = 90^o$	k_0
${}^{1}T_2$	90	0	$q_2 = \text{atan2}(k_1, k_2)$	0
${}^{2}T_3$	0	$\sqrt{{k_1}^2 + {k_2}^2} = K_{12}$	$q_3 = -\text{atan2}(k_1, k_2)$	0
${}^{3}T_4$	0	k_3	$q_4 = 0$	0

(c) Forward kinematics

(I) Obtain transformation between adjacent

${}^{i-1}_{i}\mathrm{T} = R_x(\alpha_{i-1})D_x(a_{i-1})\ R_z(\theta_i)D_z(d_i)$

$${}^{0}_{1}\mathrm{T} = [\mathrm{I}][\mathrm{I}]\begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}\begin{bmatrix} \mathbf{I} & \begin{matrix} 0 \\ 0 \\ k_0 \end{matrix} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & k_0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$${}^{1}_{2}\mathrm{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}[\mathrm{I}]\begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}[\mathbf{I}] = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$${}^{2}_{3}\mathrm{T} = [\mathbf{I}]\begin{bmatrix} \mathbf{I} & \begin{matrix} K_{12} \\ 0 \\ 0 \end{matrix} \\ \mathbf{0} & 1 \end{bmatrix}\begin{bmatrix} \cos q_3 & -\sin q_3 & 0 & 0 \\ \sin q_3 & \cos q_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}[\mathbf{I}] = \begin{bmatrix} c3 & -s3 & 0 & K_{12} \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$${}^{3}_{4}\mathrm{T} = [\mathrm{I}]\begin{bmatrix} \mathbf{I} & \begin{matrix} K_3 \\ 0 \\ 0 \end{matrix} \\ \mathbf{0} & 1 \end{bmatrix}\begin{bmatrix} \cos q_4 & -\sin q_4 & 0 & 0 \\ \sin q_4 & \cos q_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}[\mathbf{I}] = \begin{bmatrix} c4 & -s4 & 0 & K_3 \\ s4 & c4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix};\ {}^{4}_{E}\mathrm{T} = \begin{bmatrix} 0 & 0 & 1 & K_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

(II) ${}^{0}_{N}\mathrm{T} = {}^{0}_{1}\mathrm{T}\ {}^{1}_{2}\mathrm{T} \dots\dots {}^{i-1}_{i}\mathrm{T}\ {}^{i}_{i+1}\mathrm{T} \dots\dots {}^{N-2}_{N-1}\mathrm{T}\ {}^{N-1}_{N}\mathrm{T}$

$${}^{0}_{E}\mathrm{T}$$

$$= \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & k_0 \\ & \mathbf{0} & & 1 \end{bmatrix}\begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}\begin{bmatrix} c3 & -s3 & 0 & K_{12} \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}\begin{bmatrix} c4 & -s4 & 0 & k_3 \\ s4 & c4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & k_4 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$$
{}^{0}_{4}\mathrm{T} = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & k_0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c3 & -s3 & 0 & K_{12} \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c4 & -s4 & 0 & k_3 \\ s4 & c4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & k_0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c_{34} & -s_{34} & 0 & c_3k_3 + K_{12} \\ s_{34} & c_{34} & 0 & s_3k_3 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & k_0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c_{234} & -s_{234} & 0 & c_2(c_3k_3 + K_{12}) - s_2s_3k_3 \\ 0 & 0 & -1 & 0 \\ s_{234} & c_{234} & 0 & s_2(c_3k_3 + K_{12}) + c_2s_3k_3 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & k_0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} c_{234} & -s_{234} & 0 & c_2K_{12} + c_{23}k_3 \\ 0 & 0 & -1 & 0 \\ s_{234} & c_{234} & 0 & s_2K_{12} + s_{23}k_3 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_1c_{234} & -c_1s_{234} & s_1 & c_1(c_2K_{12} + c_{23}k_3) \\ s_1c_{234} & s_1s_{234} & -c_1 & s_1(c_2K_{12} + c_{23}k_3) \\ s_{234} & c_{234} & 0 & s_2K_{12} + s_{23}k_3 + k_0 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

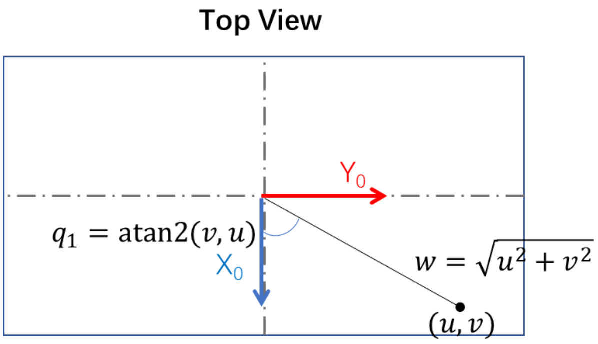
$$
{}^{0}_{E}\mathrm{T} = \begin{bmatrix} c_1c_{234} & -c_1s_{234} & s_1 & c_1(c_2K_{12} + c_{23}k_3) \\ s_1c_{234} & s_1s_{234} & -c_1 & s_1(c_2K_{12} + c_{23}k_3) \\ s_{234} & c_{234} & 0 & s_2K_{12} + s_{23}k_3 + k_0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & K_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

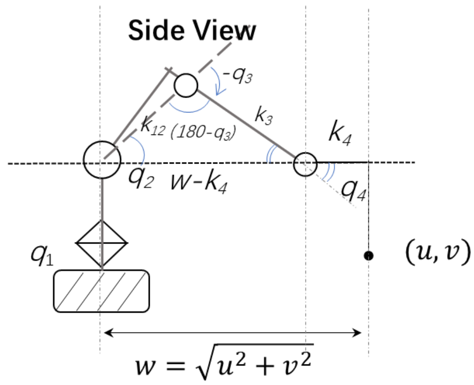
$$
= \begin{bmatrix} -c_1s_{234} & s_1 & c_1c_{234} & c_1(c_2K_{12} + c_{23}k_3) + c_1c_{234}k_4 \\ s_1s_{234} & -c_1 & s_1c_{234} & s_1(c_2K_{12} + c_{23}k_3) + s_1c_{234}k_4 \\ c_{234} & 0 & s_{234} & s_2K_{12} + s_{23}k_3 + k_0 + s_{234}k_4 \\ & \mathbf{0} & & 1 \end{bmatrix}
$$

Question 2

a) ${}^{E}_{pen}\mathrm{T} = \begin{bmatrix} 0 & 0 & -1 & -k_0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}$

b) Assuming pen is pointing vertically downward.





From top view, $q_1 = \mathrm{atan2}\,(v, u)$

From side view,

Cosine rule:

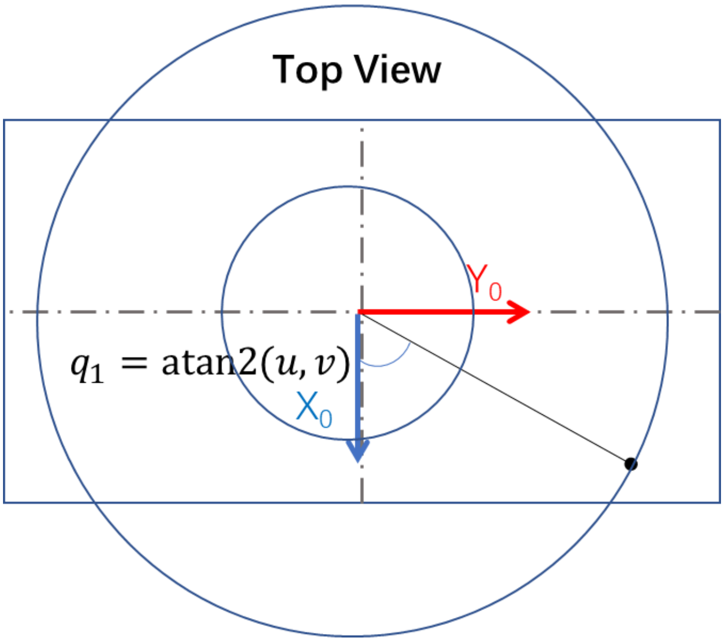
$$\cos q_4 = \frac{{k_3}^2+(w-k_4)^2-{k_{12}}^2}{2k_3(w-k_4)} \;\; ; \;\; q_4 = \mathrm{acos}\left(\frac{{k_3}^2+(w-k_4)^2-{k_{12}}^2}{2k_3(w-k_4)}\right)$$

Sine rule:

$$\sin(180 - q_3) = \sin q_3 = \frac{\sin q_4}{k_{12}}(w - k_4); \;\; q_3 = \mathrm{asin}\left(\frac{\sin q_4}{k_{12}}(w - k_4)\right)$$

$$\sin q_2 = \frac{\sin q_4}{k_{12}}k_3; \;\; q_2 = \mathrm{asin}\left(\frac{\sin q_4}{k_{12}}k_3\right)$$

c) The shape of the writing taskspace has the following shape.



Initially when the distance between Z_E and Y_{pen} is k_0 :

If $k_1 = k_3$, the internal envelop will have a radius of k_4

Shortening the distance will shorten the radius of reach while pen maintain vertically downwards. Hence, when the distance between Z_E and Y_{pen} is halved, the outer boundary shrinks in radius while the internal circular envelop radius increases.