ZJU-UIUC Institute

Zhejiang University / University of Illinois at Urbana-Champaign Institute

ECE 470: Introduction to Robotics

Lecture 16

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Last Lecture

- We looked at
  - An overview of robot control
  - Feedback control
  - Joint control and the various components
  - 2nd Order Dynamics System (mechanical mass-spring-damper system)

Recall: Designing Control

- Design control system to achieve specific system behaviors
  - Steady state error
  - Rise time
  - Overshoot
  - Settling time
- Consider Stability

Recall: Designing Robot Control System

- The central question in designing a control system is the performance specification
  - System response
  - Stability analysis

Control of 2nd Order System

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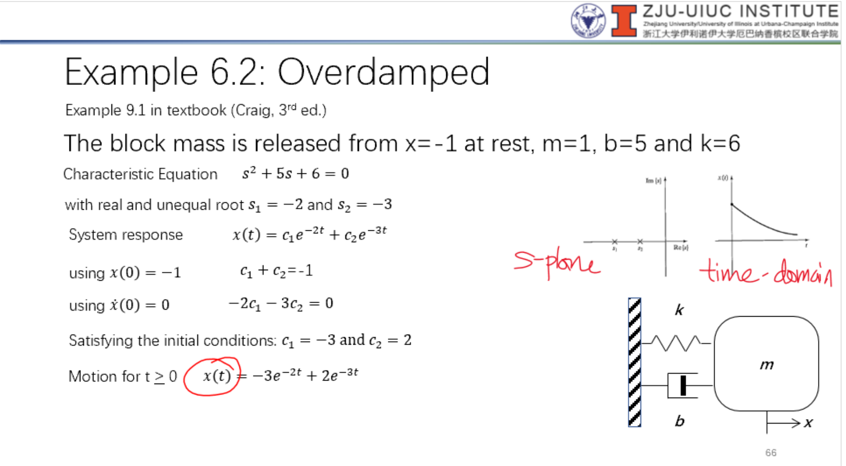
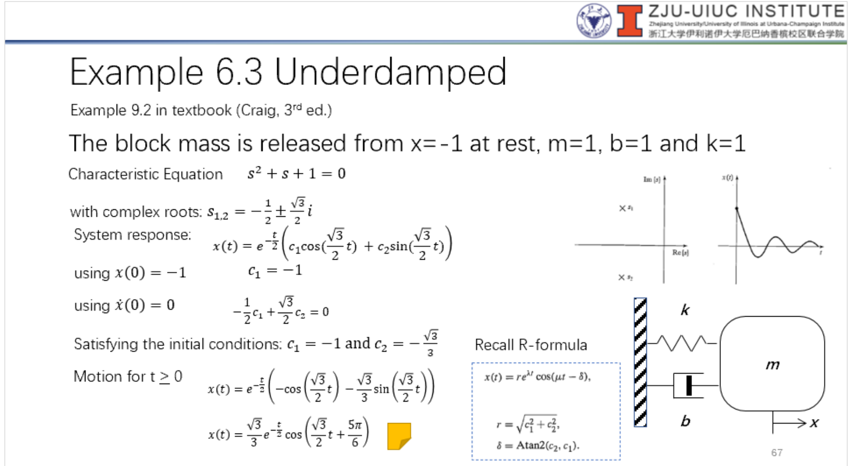
Second-Order System

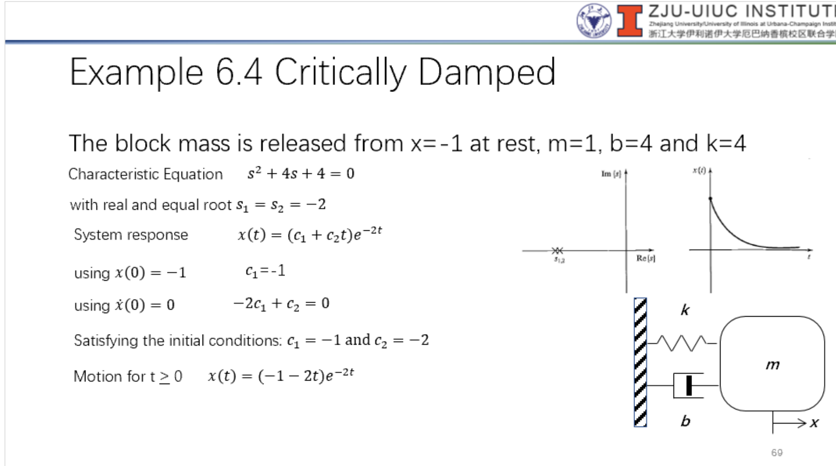
From Ex 6.2-4, the systems are all **stable** where *m, b and k* >0

In free oscillation, the system has natural response

In control applications, the system has forced response







Recall: Second-Order System

The Dynamics of the Mass-Spring-Damper System

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = 0$$

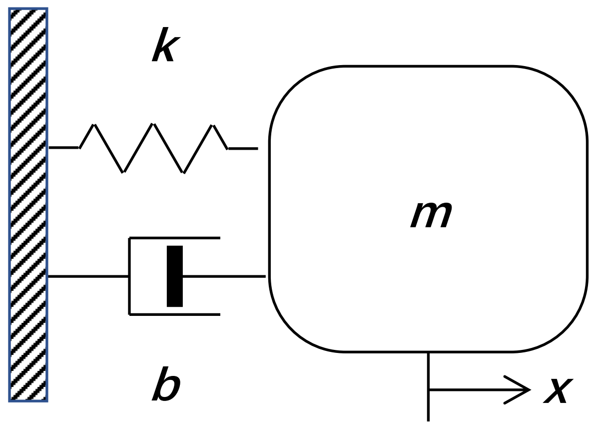
can be described as an oscillatory 2nd order system

$$\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = 0$$

characterized by

Natural frequency: $\omega_n = \sqrt{\frac{K}{m}}$

Damping Ratio: $\zeta = \frac{1}{2}\sqrt{\frac{b^2}{mk}}$



Recall: Second-Order System

Oscillatory 2nd order system with free response

Homogenous ODE $\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = 0$

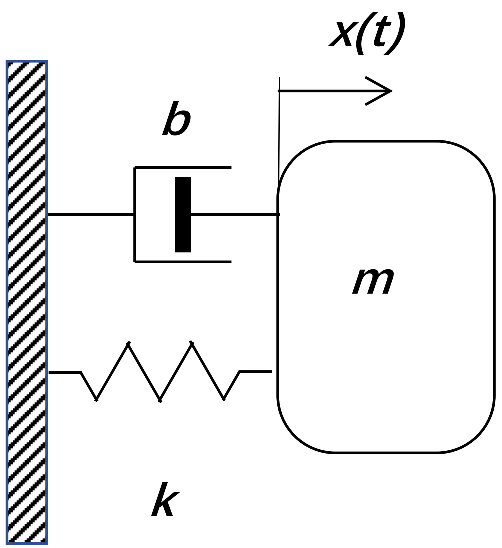
Free response

Overdamped $\zeta > 1$ Real and unequal roots: $s_1 = s_2$

Critically damped $\zeta = 1$ Real and Equal roots $s_1 \neq s_2$

Underdamped $0< \zeta <1$ Complex roots: $s_{1,2} = \lambda \pm \mu i$

No damping $\zeta = 0$ Imaginary roots: $s_{1,2} = \pm \mu i$



Forced Second-Order System

Newton's 2nd Law $$\sum F = m\ddot{x}$$

Net total Force= Drag force+ Spring force+ **External Force**

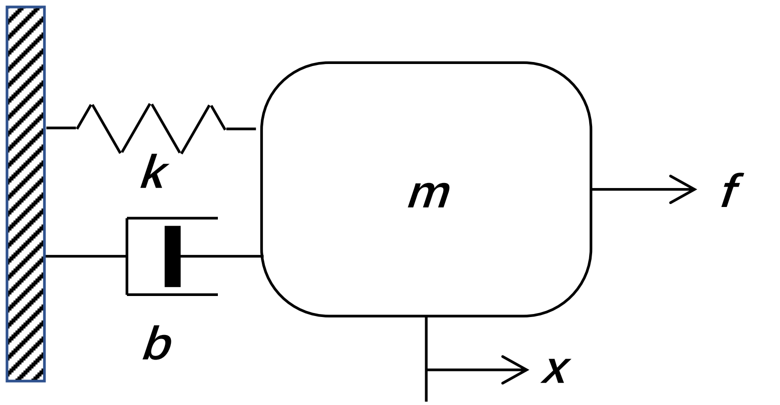
$F_{drag} = -b\dot{x}$

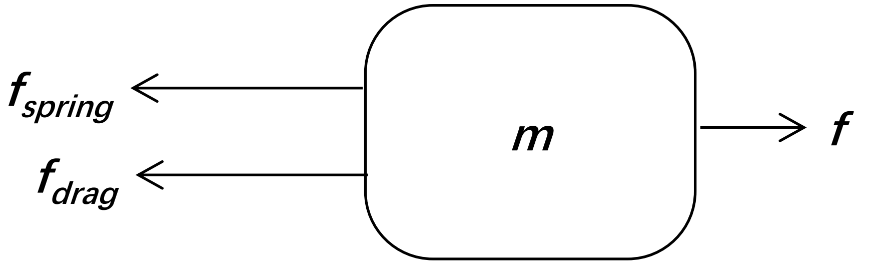
Stokes's law

$F_{Spring} = -kx$

Hooke's law

$$\boxed{m\ddot{x} = -b\dot{x} - kx + f(t)}$$





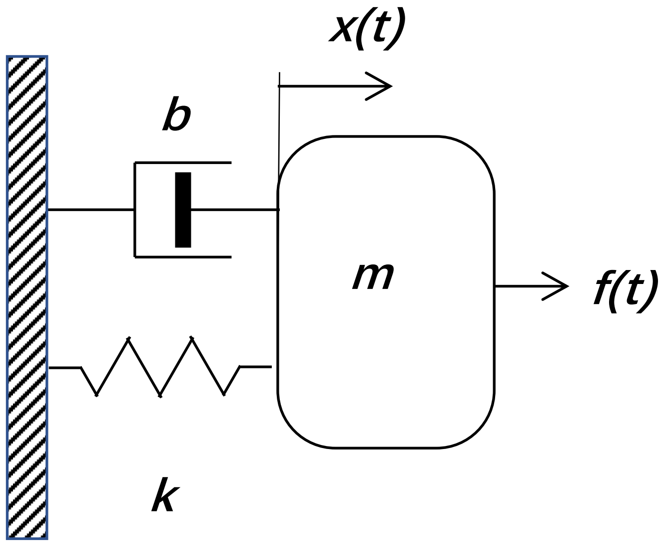
Free Body Diagram

Forced Second-Order System

Forced Oscillatory 2nd order system

Non-homogenous ODE $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = \boxed{}$

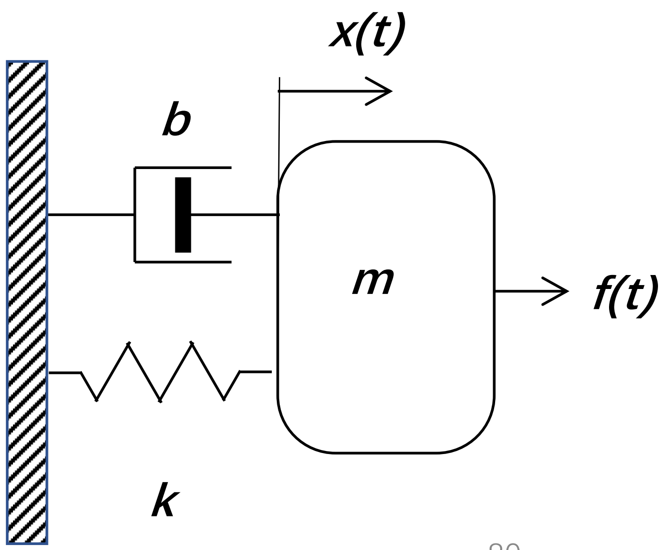
Forced response



Forced: Block Diagram

Block Diagram of the forced system

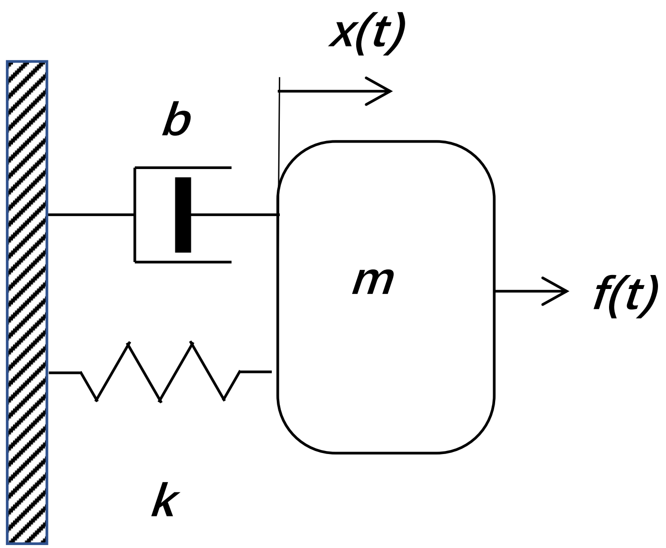
$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$



Block diagram in s-domain

Second order System can be represented as a double zero-initial-value integrator with negative feedback

$$ms^2X(s) + bsX(s) + kX(s) = F(s)$$



Control of a Second-Order System

Recap Open-loop example:

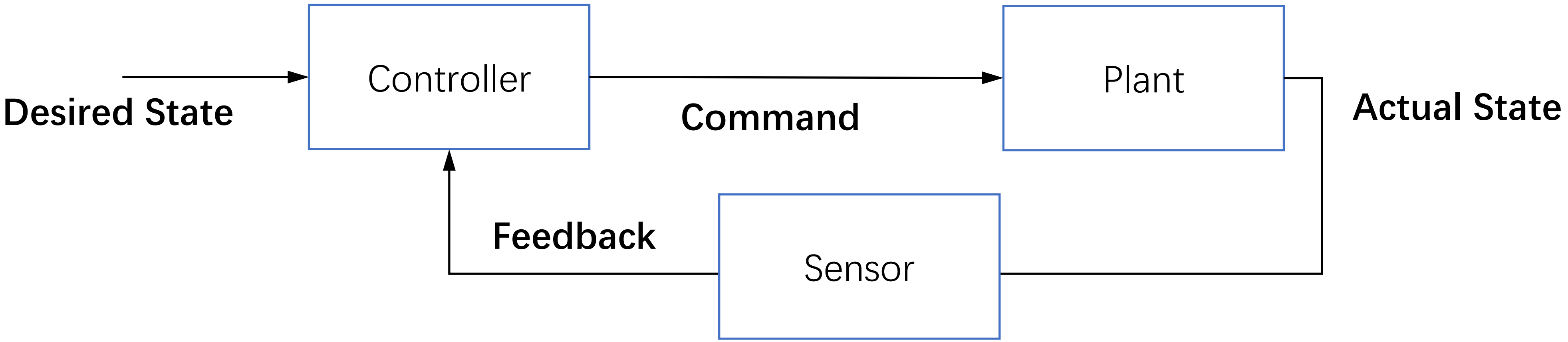
Control the robot to achieve a desired state

Through preestablished model, generate the command that will achieve the desired state. This is known as an **open-loop system**.

Control of a Second-Order System

Recap Closed-loop example:

Control the robot to maintain a desired state



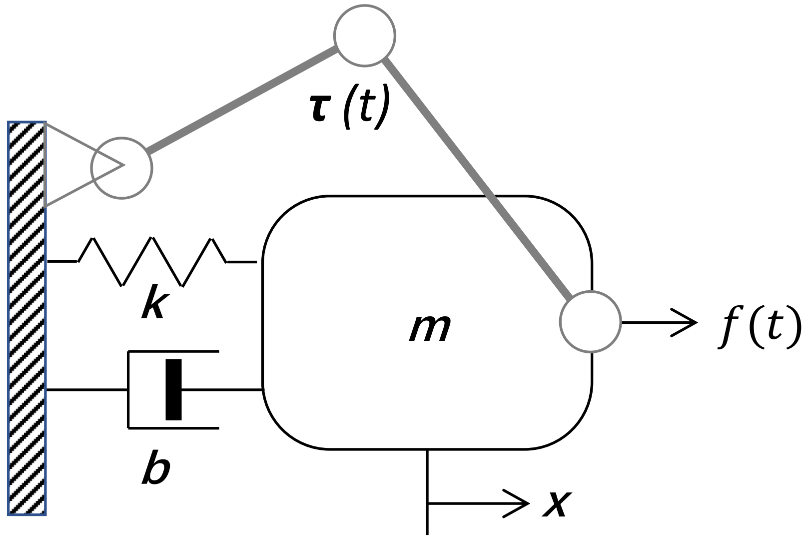
Through sensors, we are able to **feedback** the measurement to produce the command that will minimize the error between desired and actual targeted profile. This is known as a **closed-loop system**.

Control of a Second-Order System

In order to modify the system's behavior as desired, we can use closed-loop control, by using sensors, actuators and control system.

For example, if the position and velocity of the block can be measured, it is possible to apply a force f to the block as shown.

$$m\ddot{x} + b\dot{x} + kx = f(t)$$



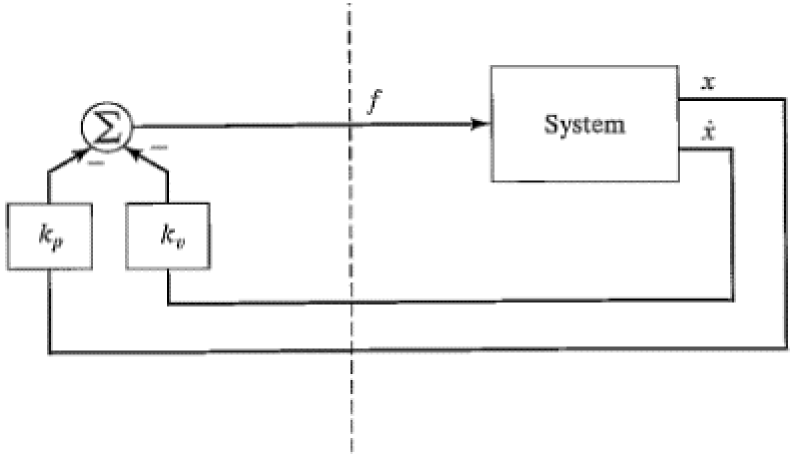
Control of a Second-Order System

Assume that we are able to sense $\dot{x}$ and x and feedback to the controller. We can propose a control law that determine the force that the actuator should apply as a function of the feedback as shown

$$f(t) = -k_p x - k_v \dot{x}$$

$$m\ddot{x} + b\dot{x} + kx = -k_p x - k_v \dot{x},$$

Control Gains



Closed-loop Stiffness

Example 6.5: Closed loop gain

Recall example 6.3: The block mass is released from x=-1 at rest, m=1, b=1 and k=1

Determine the gains K_p and K_v such that the system is being critically damped with a closed loop stiffness of 16.0

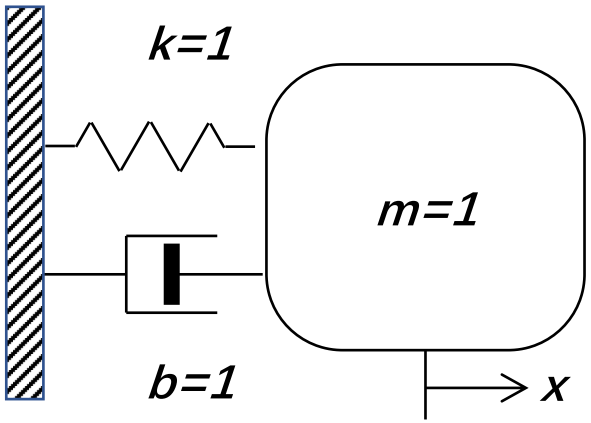
Control Law:

Closed Loop System:

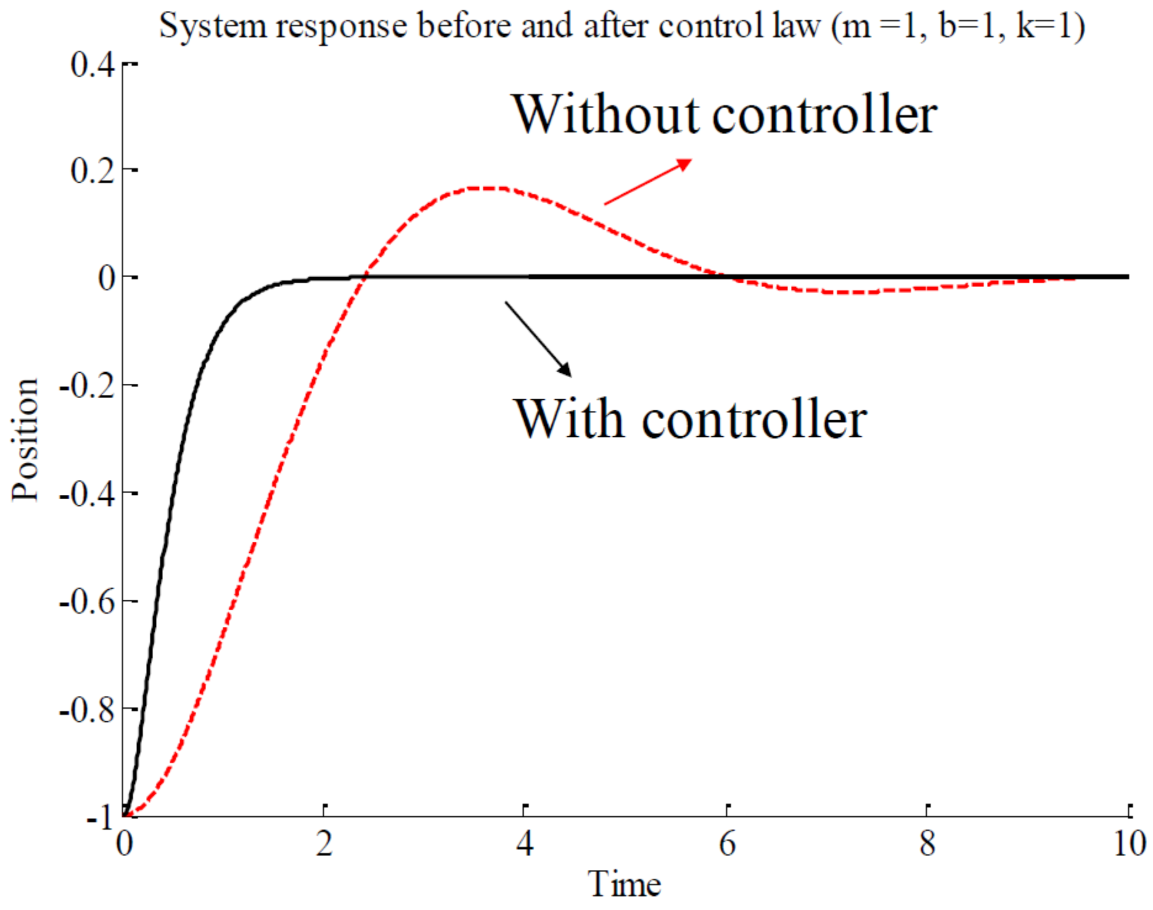
CL Stiffness, Damping:

Since $k' =$ _____ $= 16$, $k_p =$ _____
For Critically damped response, $b' =$ ______
Hence, $k_v =$ _________

Control Gains: $[k_p, k_v] = [\_\_, \_\_]$



Response


System response before and after control law (m =1, b=1, k=1)
Without controller
With controller
Position
Time
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10

Control-Law Partitioning

The control system consist of a Model-based portion and a Servo portion

Open loop equation: $m\ddot{x} + b\dot{x} + kx = f$

Model-based portion is a control law in the form

$$f = \alpha f' + \beta$$

Hence, the system equation is written as

$$m\ddot{x} + b\dot{x} + kx = \alpha f' + \beta$$

To make the system unit mass,

$$\alpha = m, \qquad \beta = b\dot{x} + kx$$

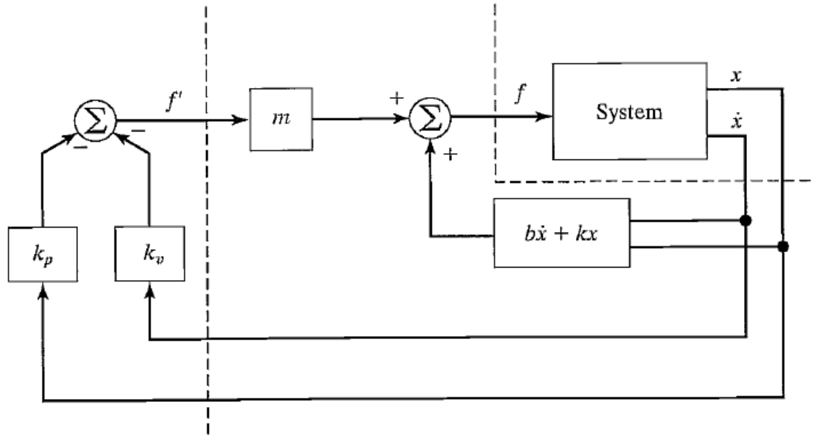
Therefore, $\ddot{x} = f'$

Proceed with servo portion of control law

$$f' = -kpx - kv\,\dot{x}$$

Since the model-based portion make $\ddot{x} = f'$,

$$\ddot{x} + k_p x + k_v\,\dot{x} = 0$$



Example 6.6: Control-Law Partitioning

Recall example 6.3: The block mass is released from x=-1 at rest, m=1, b=1 and k=1
Using control law partitioning, determine the α, β and gains k_p and k_v such that the system is being critically damped with a closed loop stiffness of 16.0

$$\alpha = m = 1$$
$$\beta = \dot{x} + x$$

Effective system equation

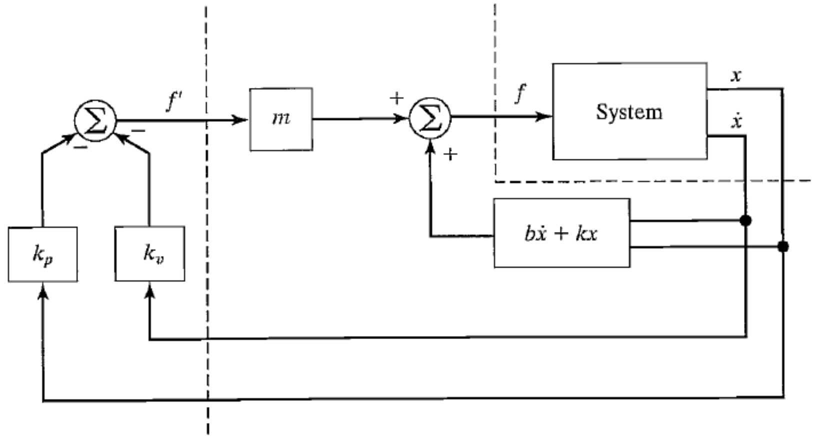
$$\ddot{x} + kv\,\dot{x} + kpx = 0$$

Hence

$$k_p = \_\_$$

For Critically damped response,

$$k_v = \_\_\_\_\_\_$$



Trajectory-Following

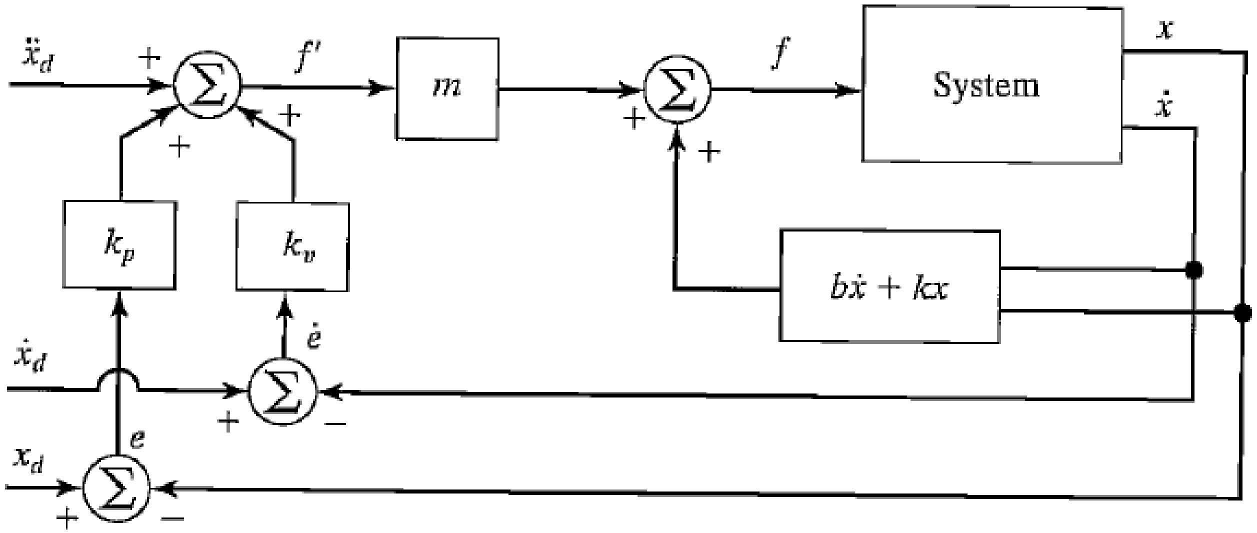
- In Example 6.5, we establish a control law that restore the mass to equilibrium (with forced response) according to our specified response behavior
- This is a form of position-regulation where we maintain the block at the desired position (equilibrium; x=0)
- More generally, we can specify desired motion trajectories to be followed

Trajectory-Following

Trajectory realized by a controller that minimizes **servo error**, $e = xd - x$ with a **servo-control law**, $f' = \ddot{x}_d + kv\,\dot{e} + kpe$

i.e. $\ddot{x} = \ddot{x}_d + kv\,\dot{e} + kpe$

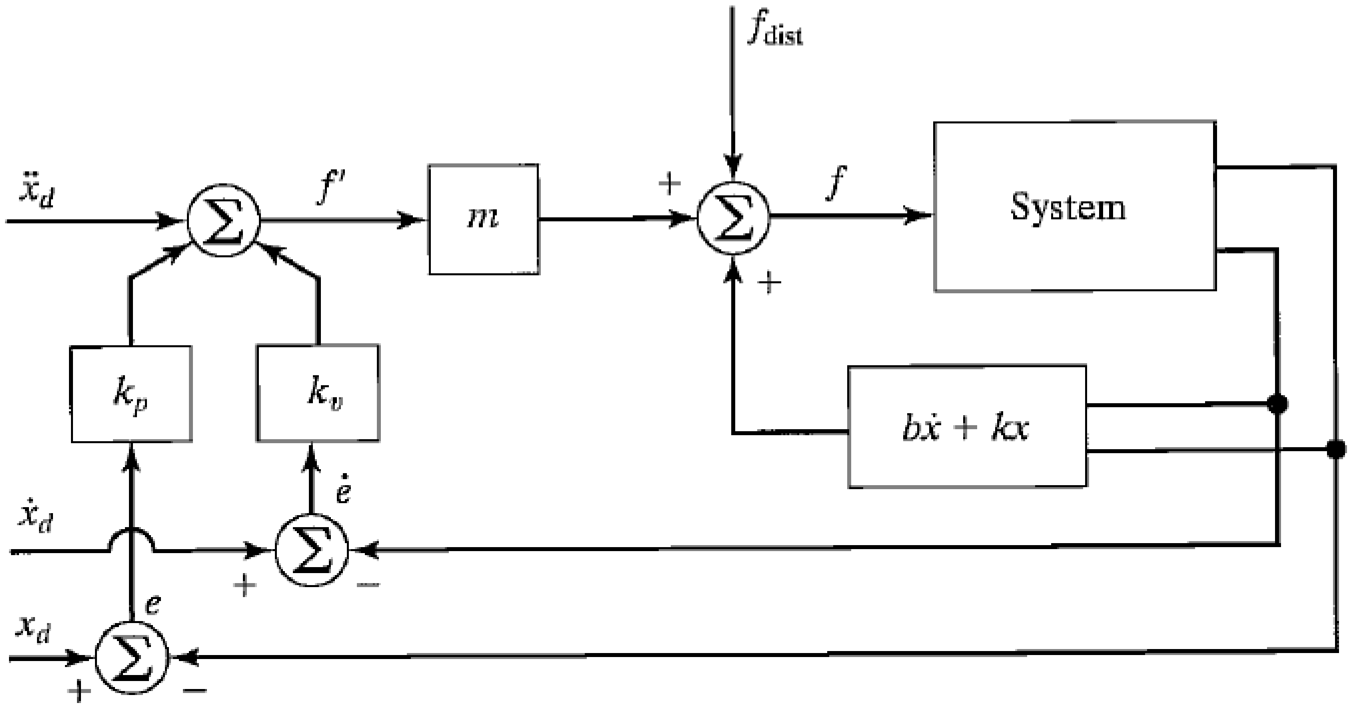
or $\ddot{e} + kv\,\dot{e} + kpe = 0$



Disturbance Rejection

There could be disturbance f_{dist} from the environment making the system equation to be

$$\ddot{e} + kv\,\dot{e} + kpe = f_{\text{dist}}$$

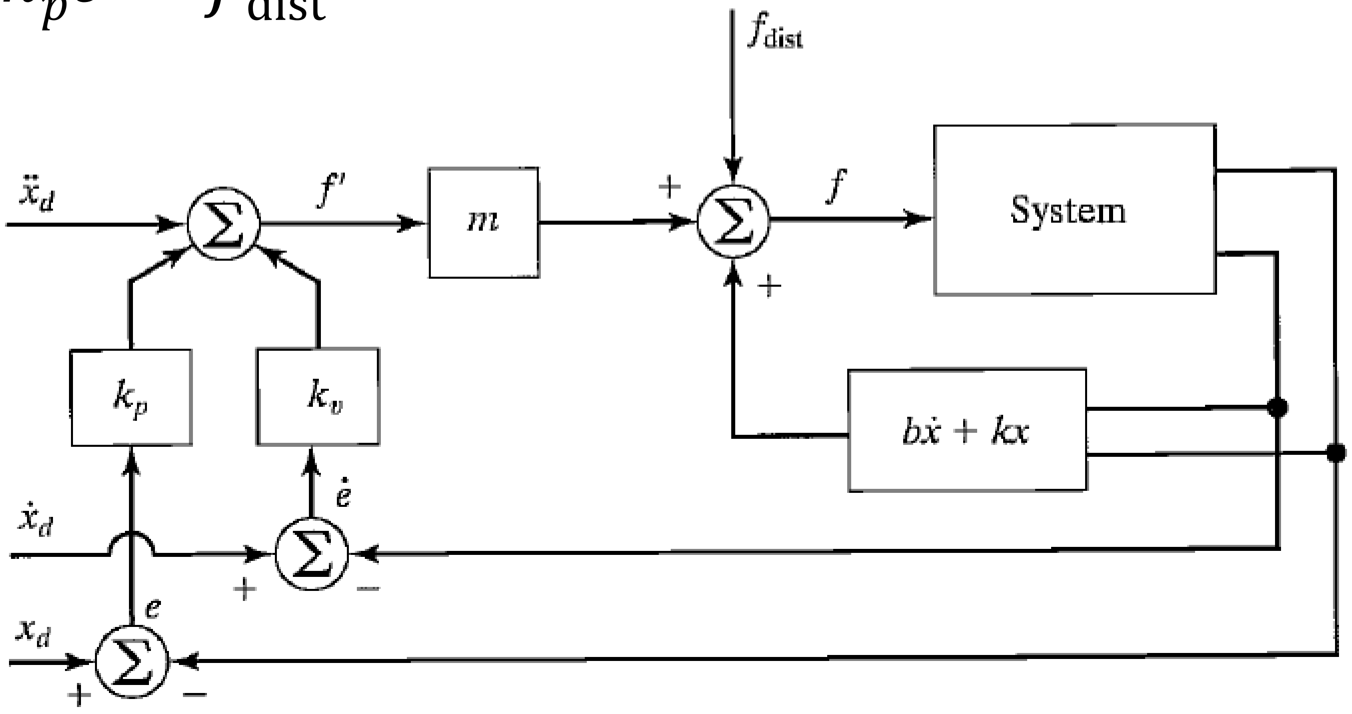


Disturbance Rejection

In case where f_{dist} is a constant, performing steady state analysis at rest, setting derivatives to be zero

$$\ddot{e} + kv\,\dot{e} + kpe = f_{\text{dist}}$$

$$k_p e = f_{\text{dist}}$$



Disturbance Rejection

In case where f_{dist} is a constant, performing steady state analysis at rest, setting derivatives to be zero

$$k_p e_{ss} = f_{\mathrm{dist}} \Rightarrow e_{ss} = f_{\mathrm{dist}}/k_p$$

Setting large k_p may reduce but cannot eliminate e_{ss}

Proportional-Integral-Derivative Control

To rectify the steady state error, an integral term can be added

Control law: $f' = \ddot{x}_d + kv\,\dot{e} + kpe + k_i \int edt$

Error equation: $\ddot{e} + kv\,\dot{e} + kpe + k_i \int e\,dt = f_{dist}$

Take time derivative, $\dddot{e} + kv\ddot{e} + kp\dot{e} + k_i e = \dot{f}_{dist}$

For constant disturbance, $\dot{f}_{dist} = 0$

$k_i e = 0$ so $e = 0$

Independent Joint Control

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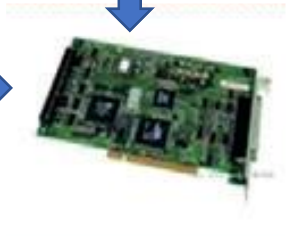
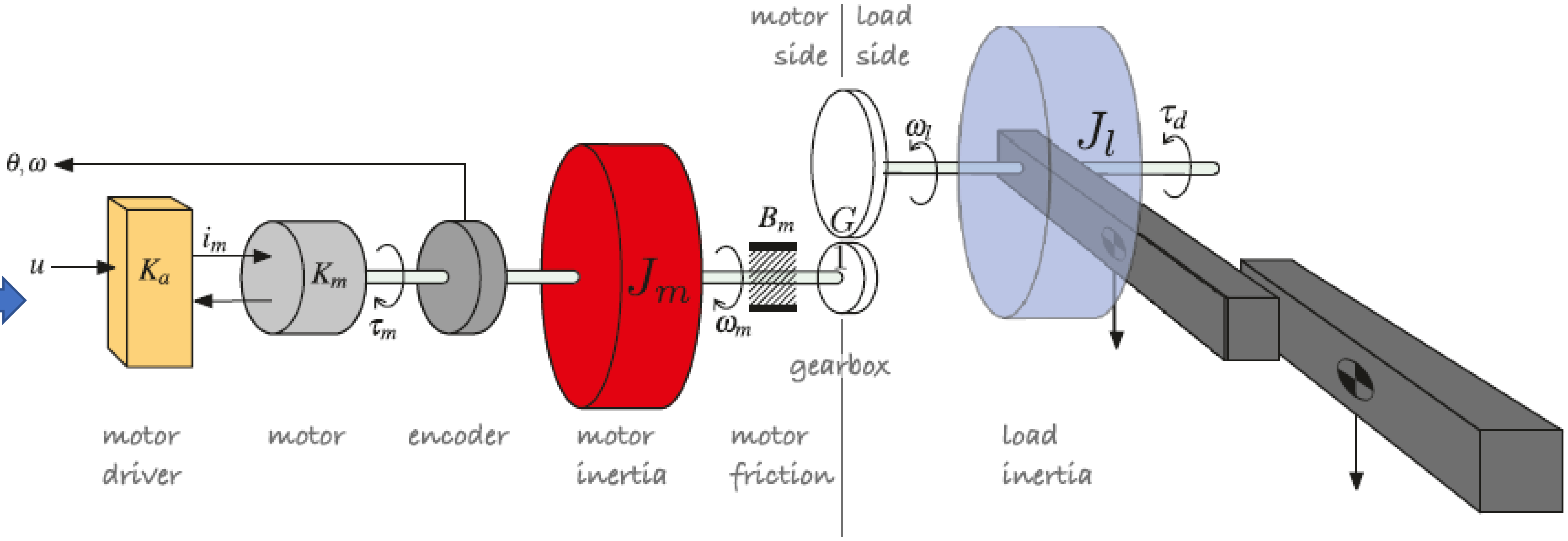
Independent Joint Control

- The 2nd order system dealt with so far has single DOF
- Ultimately, we are interested in multibody robotic systems that involve Multi-Input, Multi-Output (MIMO) control systems
- We shall first adopt an **independent joint control** approach with <u>N independent Single-Input Single-Output (SISO)</u> control systems

Joint Control in Robotic System

Trajectory Planning



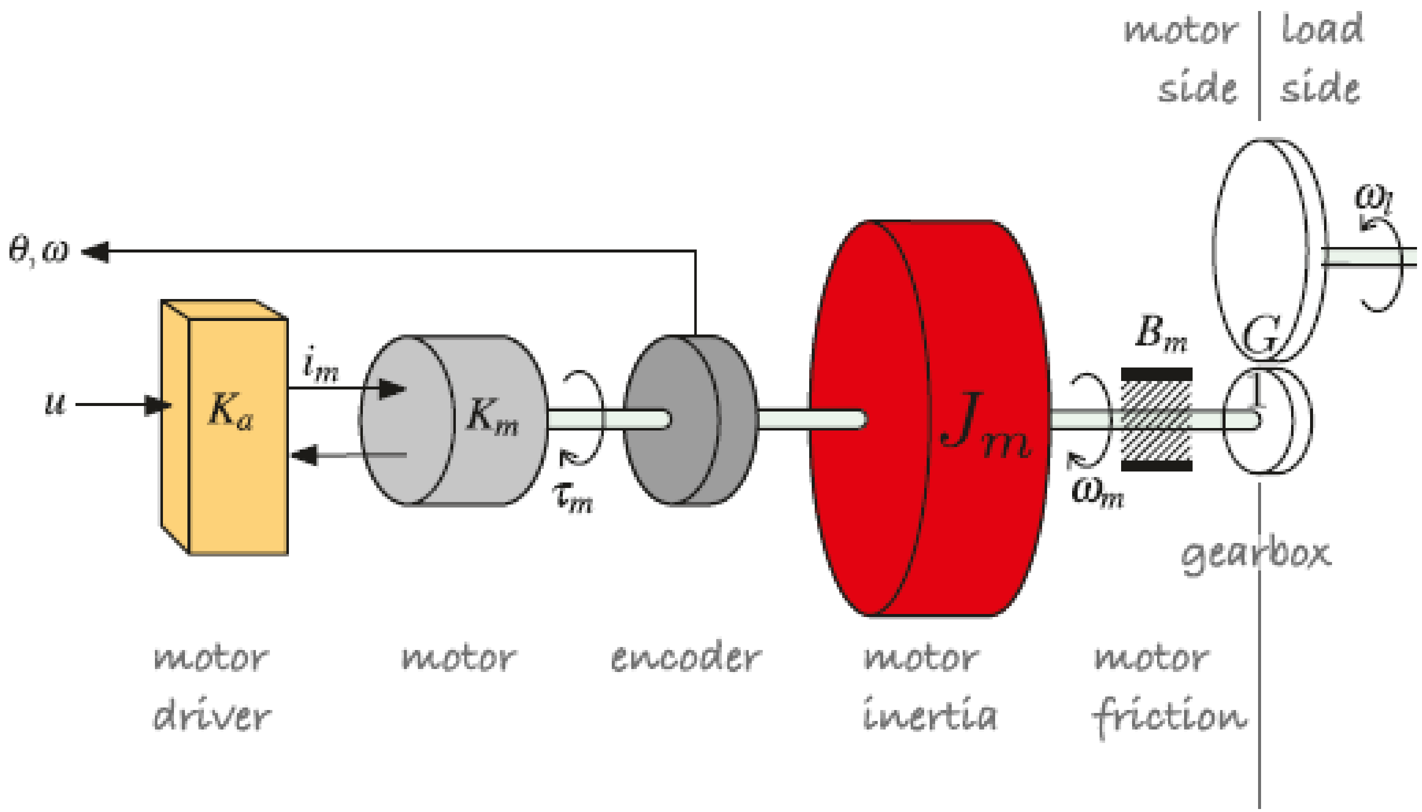


Human-Machine Interface

Controller

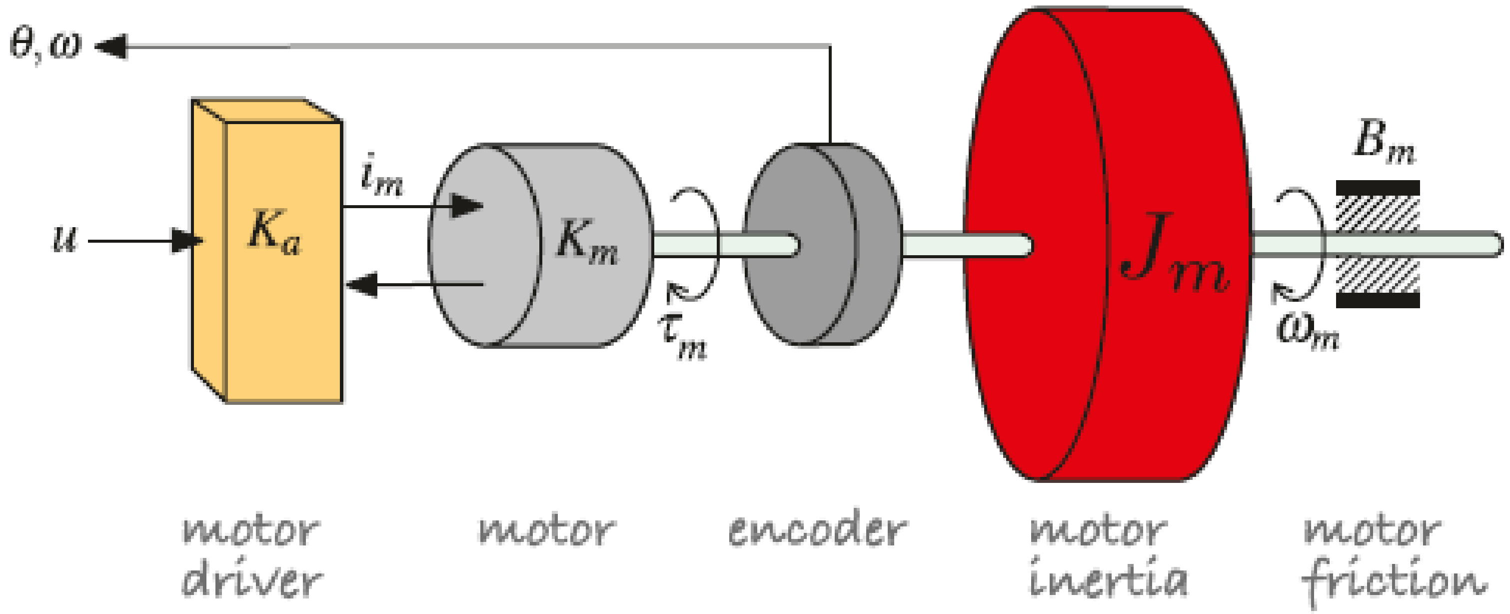
Modeling Single Joint Control

Integrated assembly



Modeling Single Joint Control

A demand voltage u controls the current i_m flowing into the **motor (Actuator)** which generates a torque τ_m that accelerates the rotational inertia J_m and is opposed by friction $B_m \omega_m$. The **encoder (Sensor)** measures rotational speed and angle



Modeling Single Joint Control

- How to model the resultant system as a 2nd order linear system?
    - Combining the Dynamics of the electromechanical system

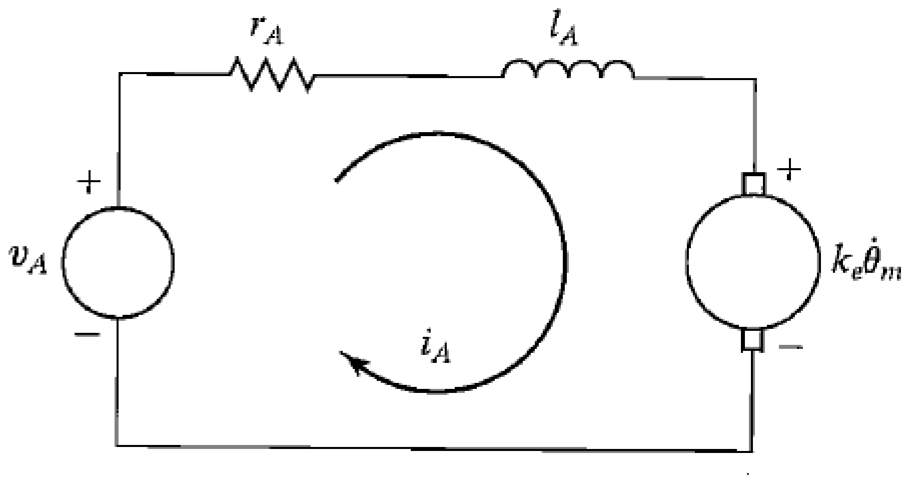
Modeling Single Joint Control

- How model the resultant system as a 2nd order linear system?

Physics Law:
Lenz Force, $F = qV \times B$

Motor Torque, $\tau_m = k_m i_a$

Back emf, $v = k_e \dot{\theta}_m$



Modeling Single Joint Control

- Model the Motor

Current

Physics Law:
Lenz Force, $F = \boxed{qV} \times \boxed{B}$

Magnetic Field

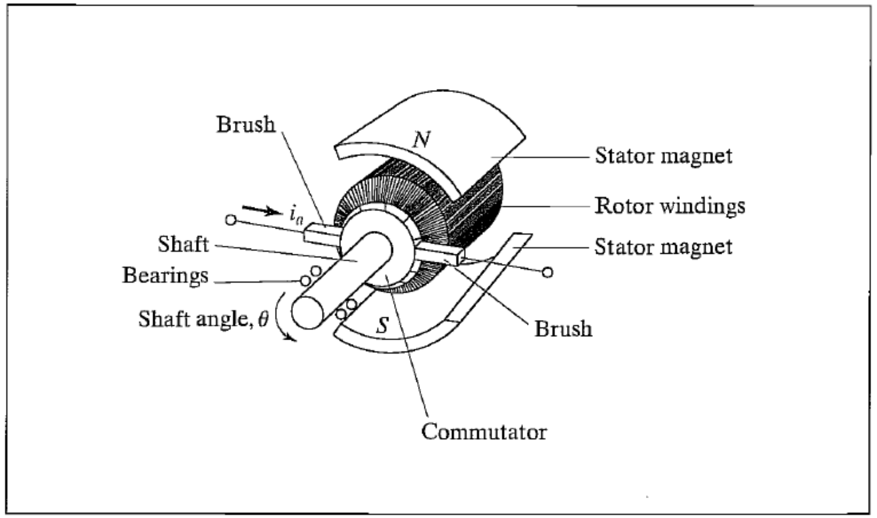
Motor Torque, $\tau_m = k_m i_A$
Back emf, $v = k_e \dot{\theta}_m$

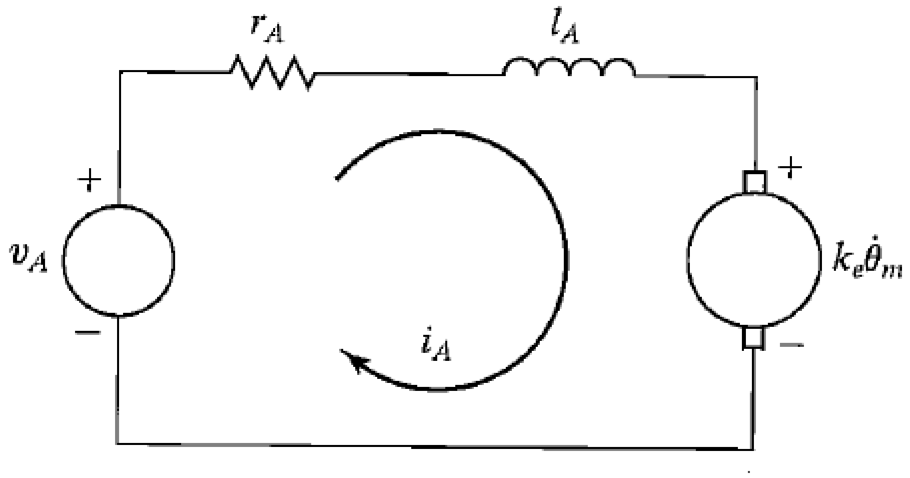
Torque Constant

Torque Constant

Kirchoff Law:

$$v_A = l_A \dot{i}_A + r_A i_A + k_e \dot{\theta}_m$$
$$v_A - k_e \dot{\theta}_m = l_A \dot{i}_A + r_A i_A$$





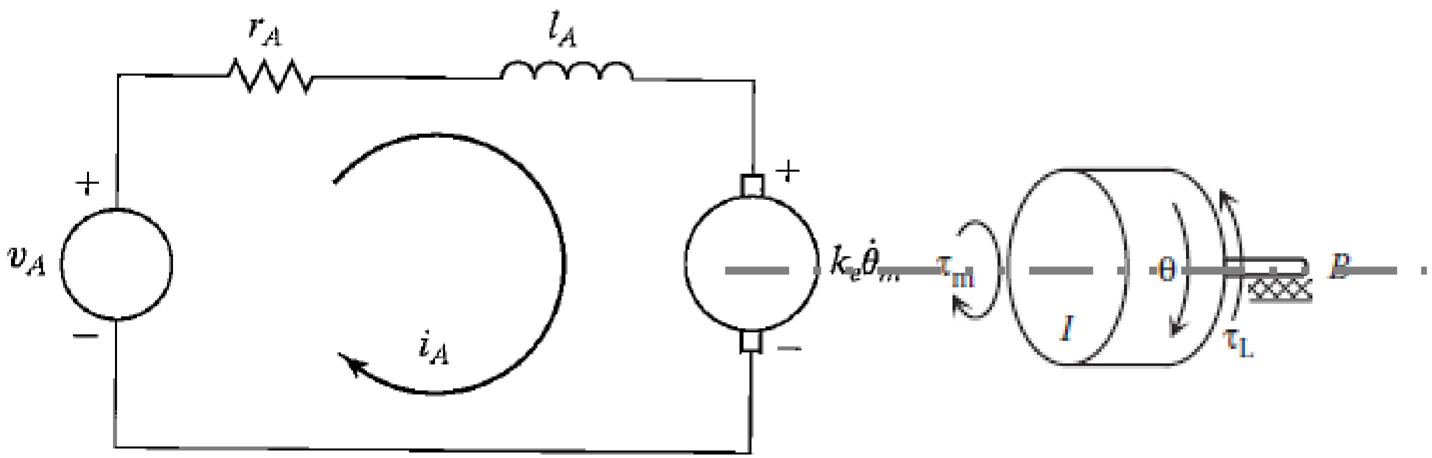
Modeling Single Joint Control

- Model the Motor

Motor Torque, $\tau_m = k_m i_A$
Back emf, $v = k_e \dot{\theta}_m$

Kirchoff Law:

$$v_A = l_A \dot{i}_A + r_A i_A + k_e \dot{\theta}_m$$
$$v_A - k_e \dot{\theta}_m = l_A \dot{i}_A + r_A i_A$$



$$\tau_m - \tau_L - b\dot{\theta} = I\ddot{\theta}$$

$$k_m i_A - b\dot{\theta}_m - I\ddot{\theta}_m = \tau_L$$

Modeling Single Joint Control

- Model the Motor-Gearing-Load

$$\tau_m = I_m\ddot{\theta}_m + bm\dot{\theta}_m + \left({}^{1}/_{\eta}\right)(I\ddot{\theta} + b\dot{\theta})$$

Gear Ratio, η:

$$\tau = \eta\tau_m,$$

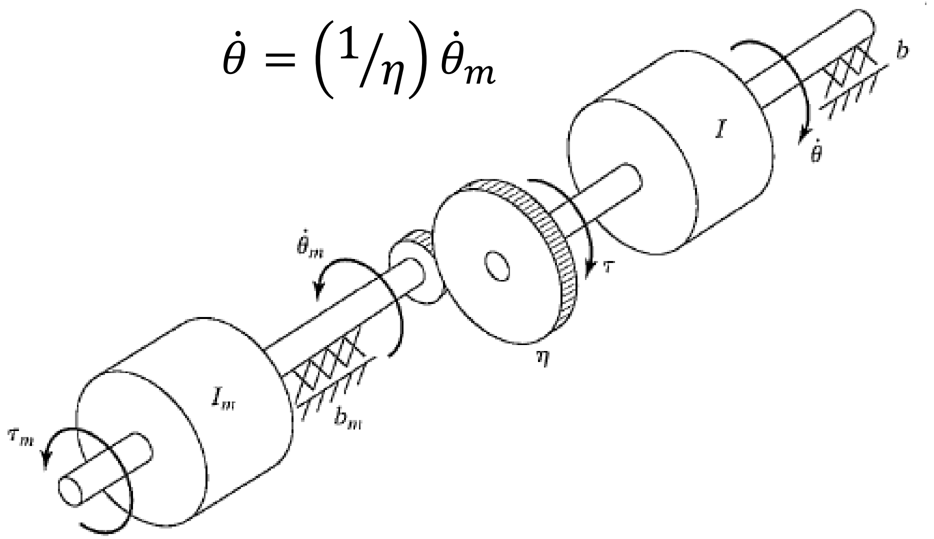
$$\dot{\theta} = \left({}^{1}/_{\eta}\right)\dot{\theta}_m$$

$$\tau_m = \left((Im + {}^{I}/_{\eta^2})\right)\ddot{\theta}_m + \left(b_m + \left({}^{b}/_{\eta^2}\right)\right)\dot{\theta}_m$$

$$\tau = \boxed{\left((\eta^2 I_m + I)\right)}\ddot{\theta} + \boxed{\left(\eta^2 b_m + b\right)}\ \dot{\theta}$$

Effective inertia

Effective damping



Robot Control Scheme

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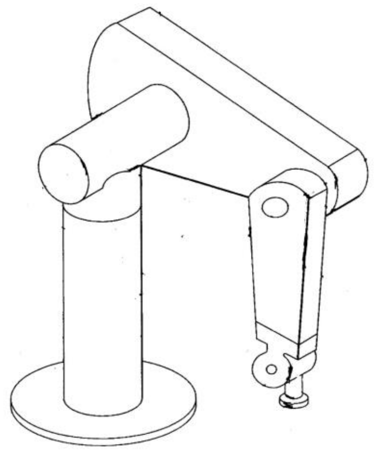
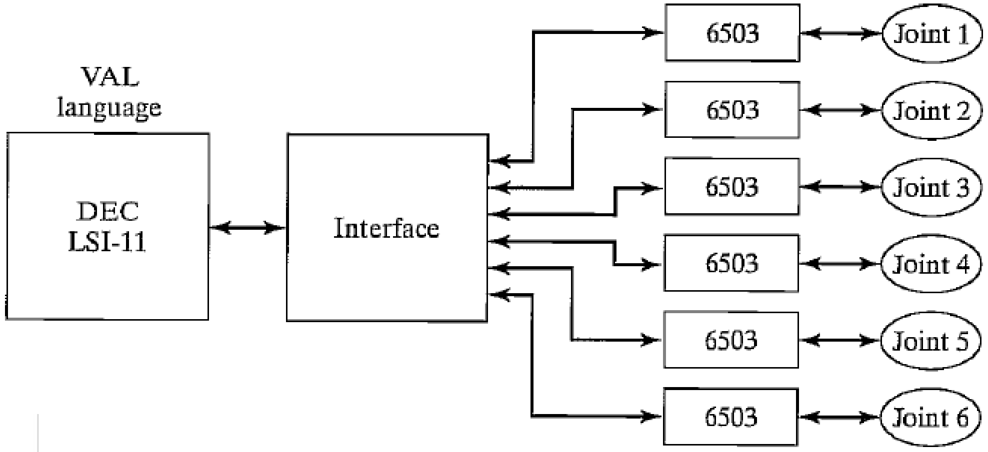
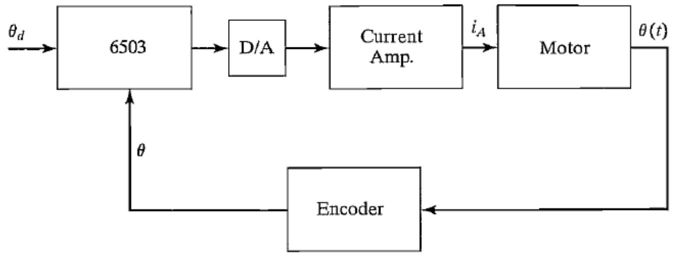
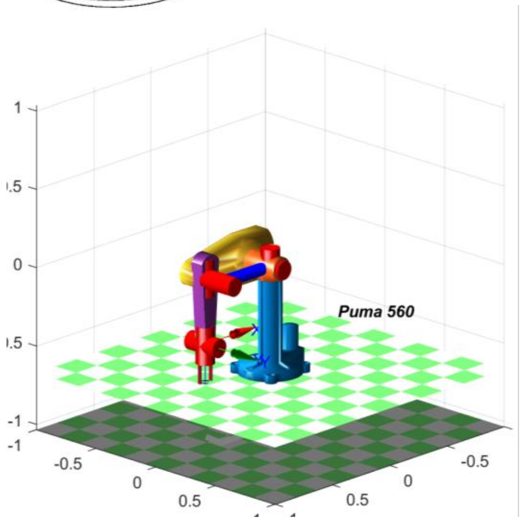
Joint Control for Robot

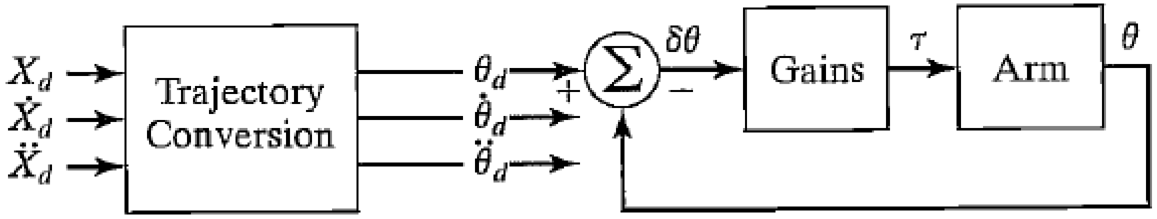
Example of industrial Robotic Arm:
PUMA 560











Joint based vs. Cartesian based

Textbok Chapter 10.8 (Craig 3rd Ed, 2005)

Computation before the control loop

Joint-based-based Control Scheme

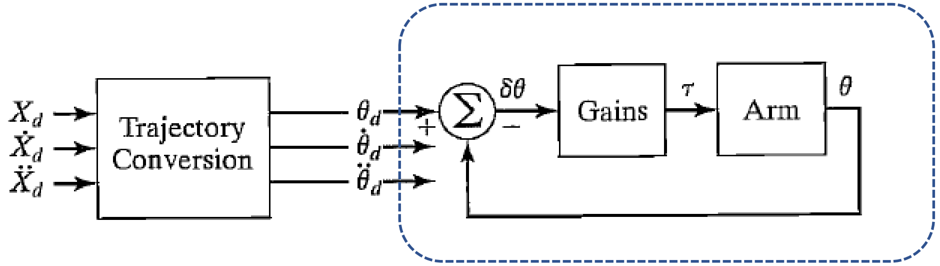


FIGURE 10.10: A joint-based control scheme with Cartesian-path input.

Computation within the control loop

Cartesian-based Control Scheme

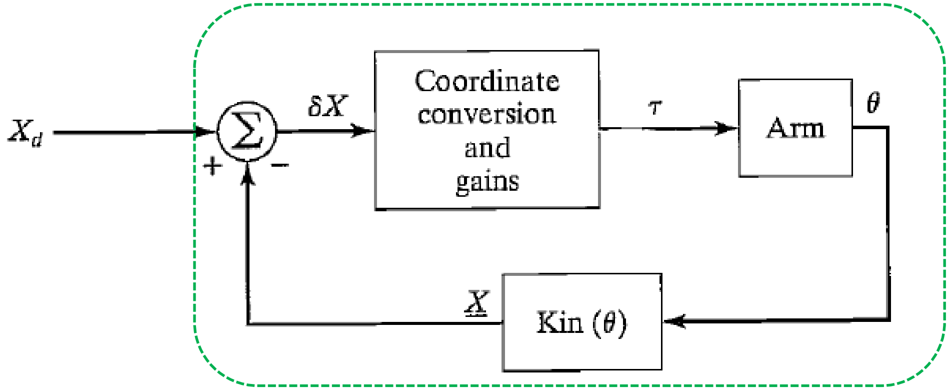


FIGURE 10.11: The concept of a Cartesian-based control scheme.

Joint based vs. Cartesian based

Question: As an engineering designing the control scheme, which ones will you choose for the following robots?

Multi-joint Snake Robot

https://www.gizbot.com/news/indian-scientists-developing-snake-robot-027613.html

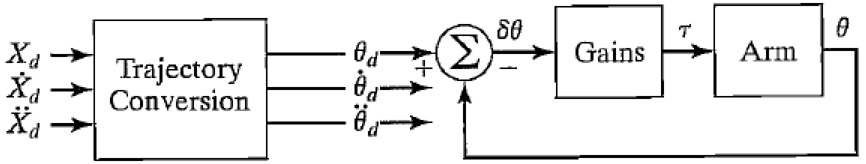


FIGURE 10.10: A joint-based control scheme with Cartesian-path input.

Serial Arm with 2 long links

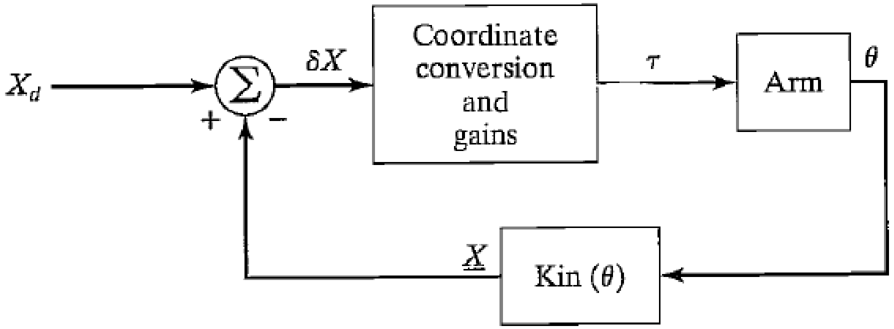


FIGURE 10.11: The concept of a Cartesian-based control scheme.

Cartesian Control Scheme

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Both are doing coordinate conversation. Which is doing in force domain?

Coordinate Conversion in Spatial Domain

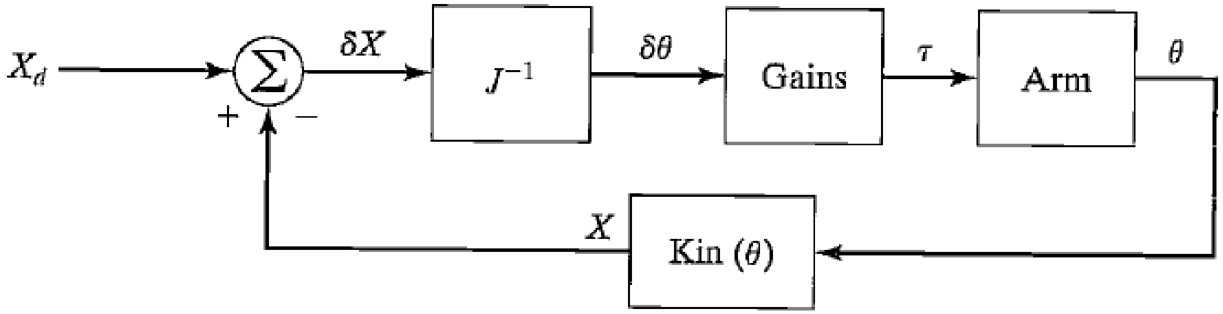

FIGURE 10.12: The inverse-Jacobian Cartesian-control scheme.

Coordinate Conversion in Force Domain

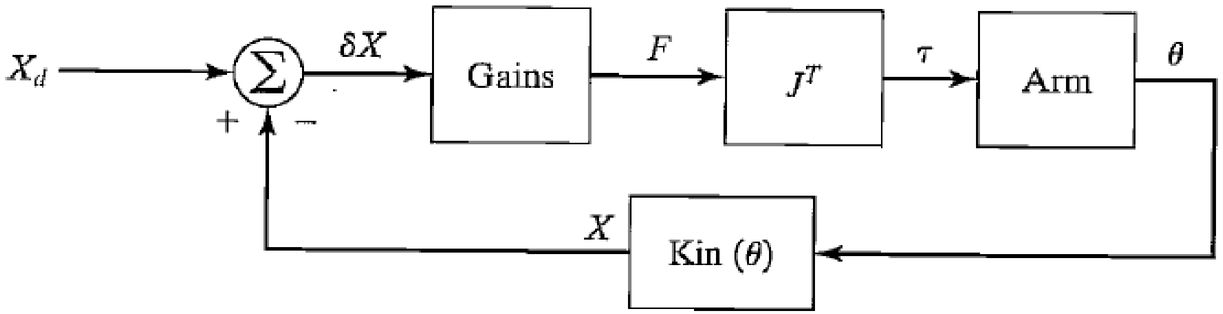

FIGURE 10.13: The transpose-Jacobian Cartesian-control scheme.