ZJU-UIUC Institute

Zhejiang University / University of Illinois at Urbana-Champaign Institute

ECE 470: Introduction to Robotics

Lecture 27

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Camera-Based Pose Estimation

ECE 470 Introduction to Robotics

Schedule Check

- **Lecture**

O. Overview
- Science & Engineering in Robotics

I. Spatial Representation & Transformation
- Coordinate Systems; Pose Representations; Homogeneous Transformations

II. Kinematics
- Multi-body frame assignment; D-H Convention; Joint-space; Work-space; Forward/Inverse Kinematics

III. Velocity Kinematics and Static Forces
- Translational/Rotational Velocity; Joint torque; Generalized Force Coordinates; Jacobian; Singularity

Fundamentals

Week 1-4

Revision/ ~~Quiz on Week 5~~

IV. Dynamics
- Acceleration of Body; Newton-Euler Equations of Motion; Lagrangian Formulation

V. Control
- Closed-Loop Control and Feedback, Control of 2nd order system, Independent Joint Control, Force Control

VI. Planning
- Joint-Based Scheme; Cartesian-Based Scheme; Collision Free Path Planning

Essentials

Week 6-9

Revision/ ~~Quiz on Week 10~~

VII. Robot Vision (Perception)
- Image Formation; Image Processing; Visual Tracking & Pose Estimation; Vision-based Control & Image-guided robotics

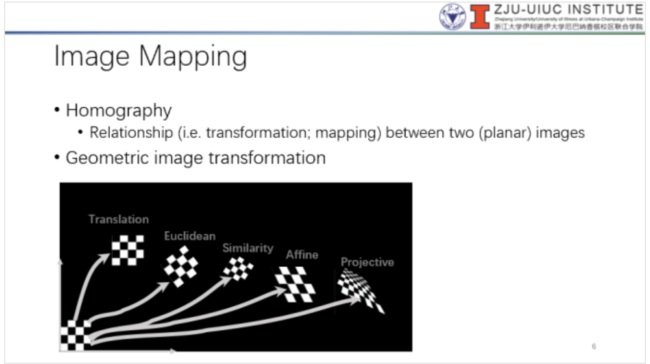
Applied

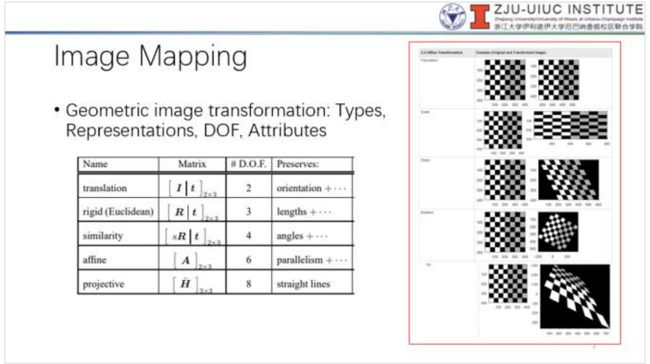
Week 11-14

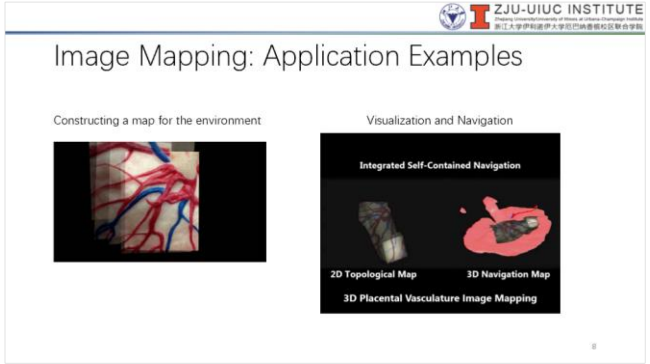
Reading Wk/ Exam on Week 15-16

Image Mapping

- Relating different viewpoints of on a common scene
- Image registration and geometric transformation
- How?
  - Recall the techniques learn so far: detect, describe, match···
  - Then, solve for <u>transformation (homography) </u>based on a specific model: translation, rigid, similarity, affine, projective ······







Visual Sensing and Perception

- The Physics
  - Principles of image projection (camera model)
- The Mathematics
  - Projective geometry and spatial representation
- The Techniques
  - Camera-based pose estimation, camera calibration and robot-camera calibration

Camera Model

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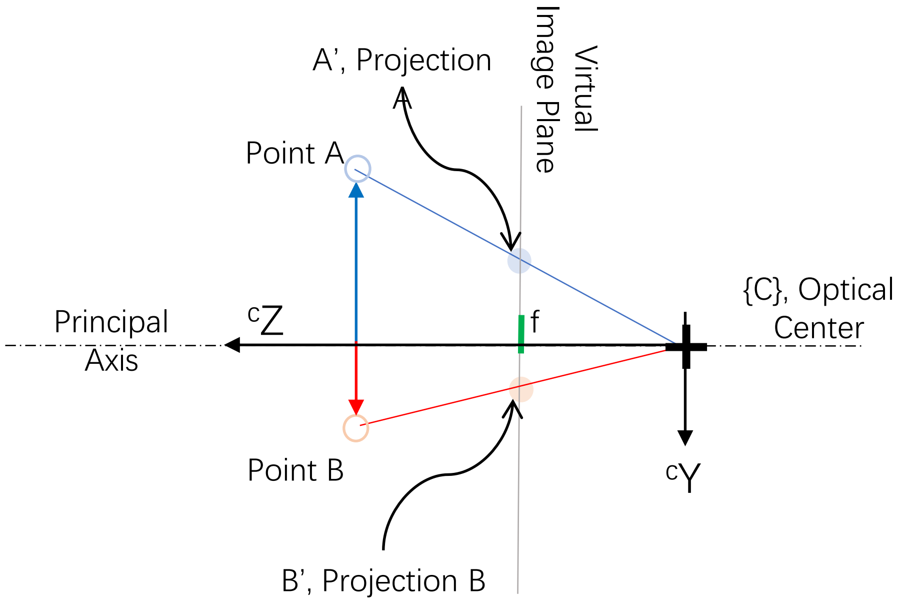
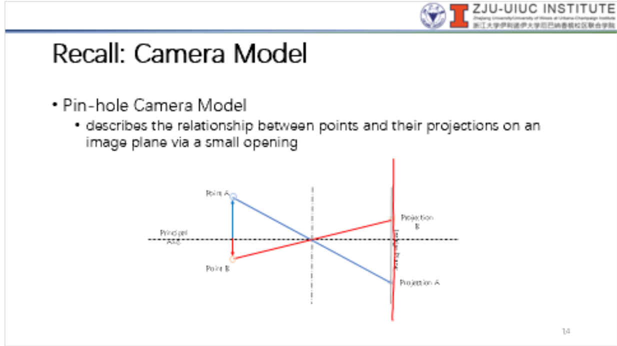
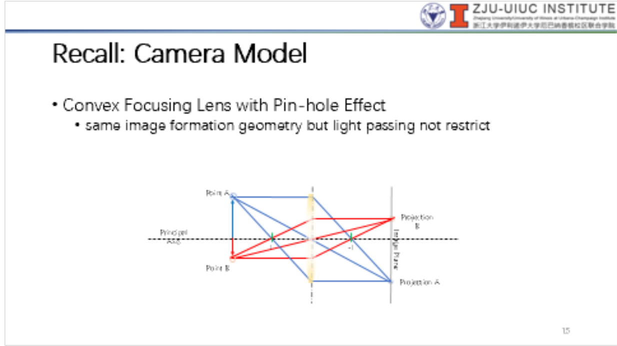
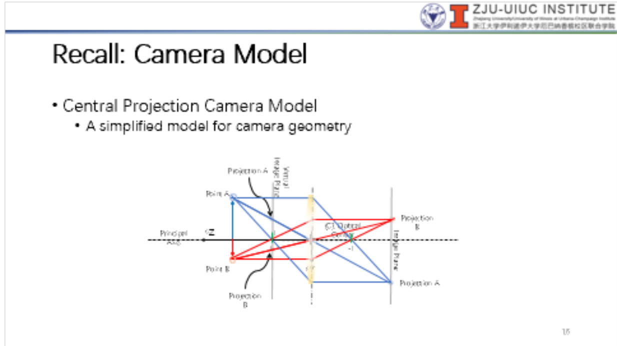
Recall: Camera Model

- Central Projection Camera Model
  - A simplified model for camera geometry

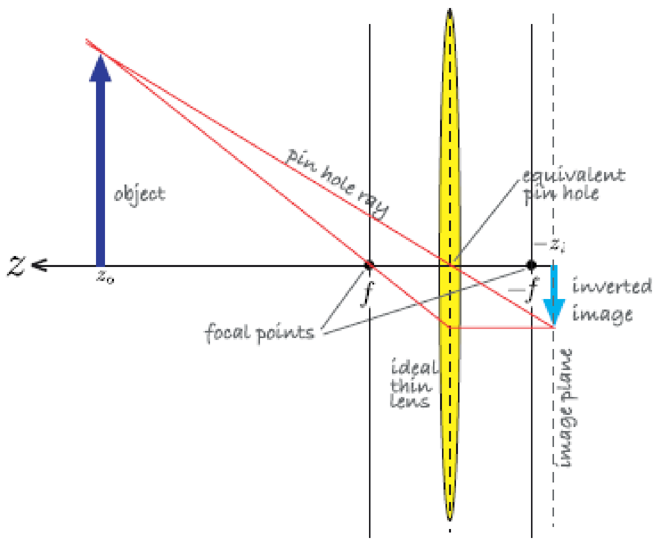


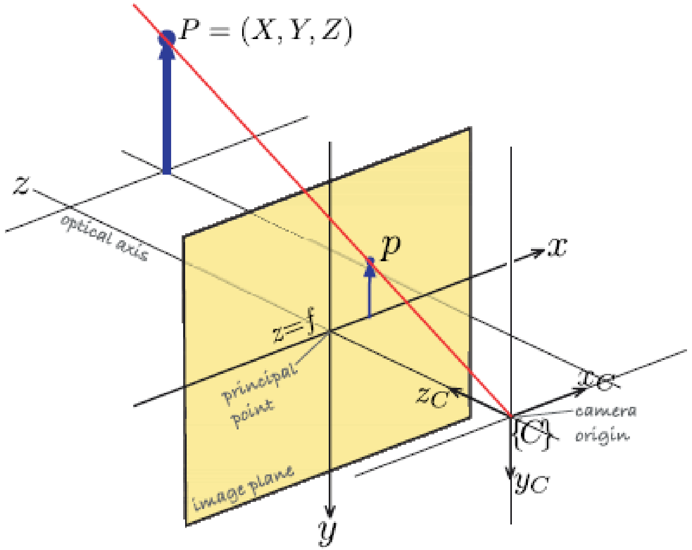






Camera Model

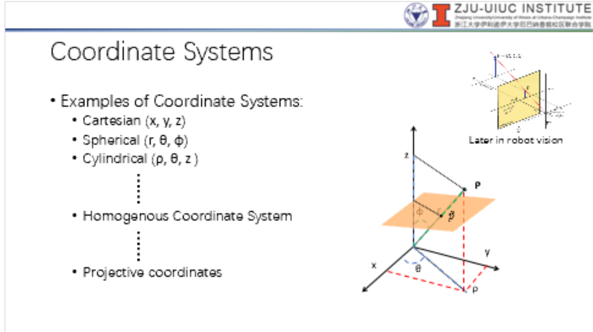




$$x = f_x \frac{X}{Z}$$

$$y = f_y \frac{Y}{Z}$$

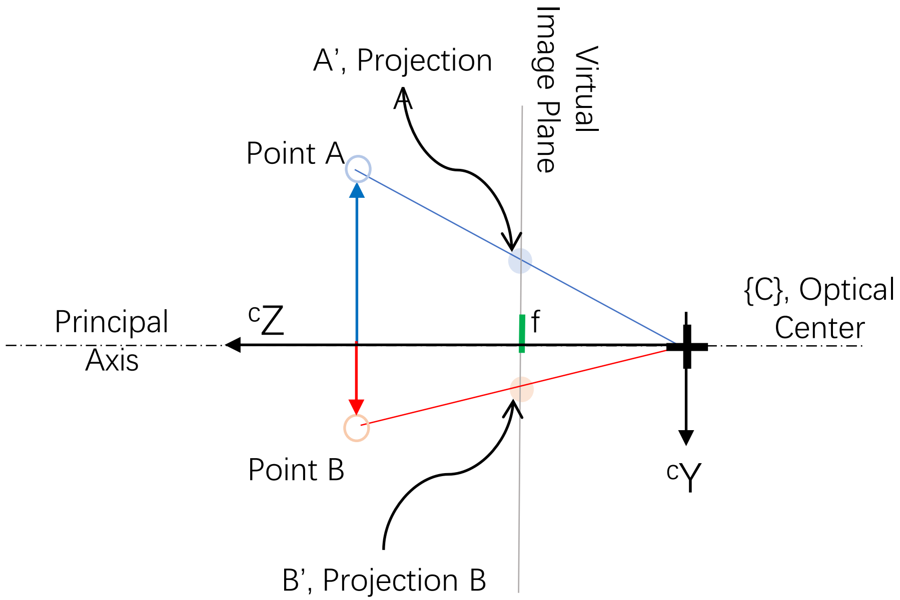
Corke, Peter. *Robotics, vision and control: fundamental algorithms in MATLAB*.



Matrix Representation: Intrinsic Matrix

- Intrinsic Matrix, $[K]$
  - focal length: $(f_x, f_y)^T$
  - principal point: $(i_c, j_c)^T$
  - skew coefficient: a

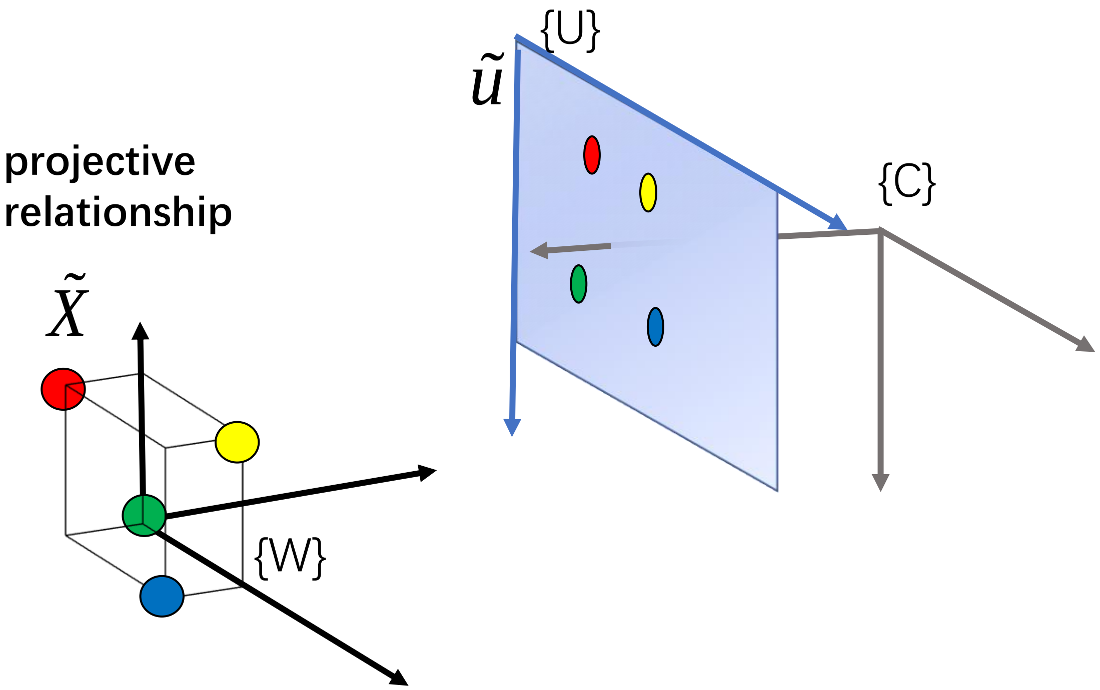
$$K = \begin{bmatrix} f_x & a & i_c \\ 0 & f_y & j_c \\ 0 & 0 & 1 \end{bmatrix}$$



What about a moving camera?

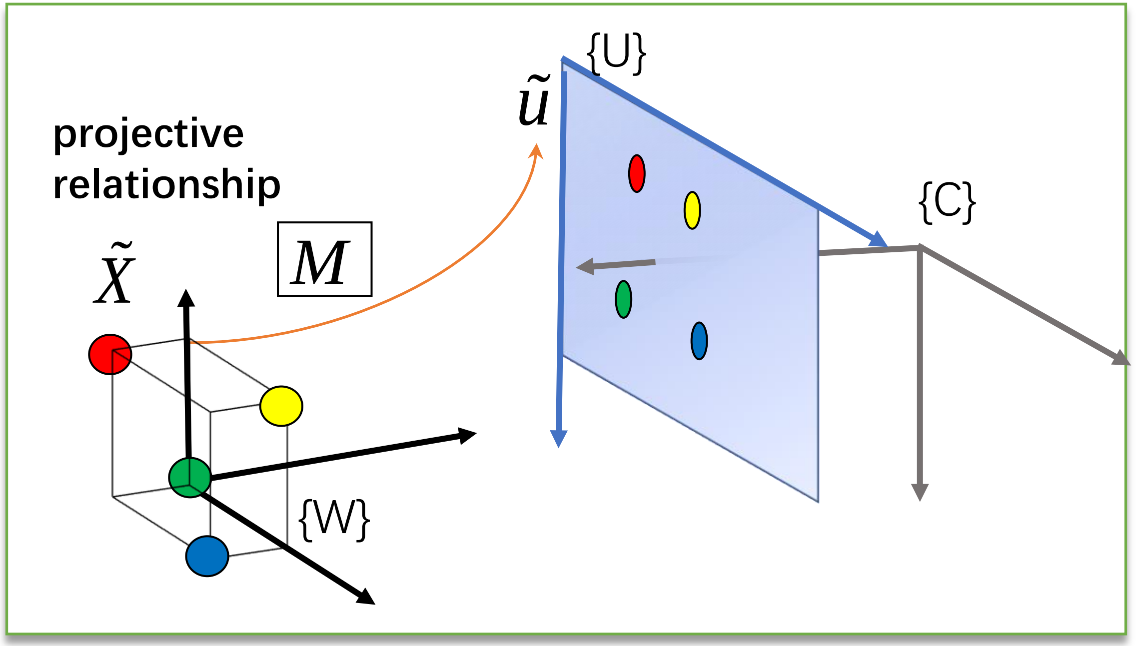
Matrix Representation: Extrinsic Matrix

- Extrinsic Matrix, $^{c}[\ R \mid t\ ]$
  - R: Orientation of world reference frame w.r.t. camera coord.
  - t: Position offset of world reference frame w.r.t. camera coord.



Camera Matrix

- Camera Matrix, *M*
  - Relates world with image coord. System
  - 2 Components:
    - Extrinsic Matrix
    - Intrinsic Matrix



Camera Matrix, *M*

For a given set of points

${}^{W}\tilde{X}$ in 3D,

the projected set of points can be expressed as

$s\tilde{u} = M\,{}^{W}\tilde{X}.$

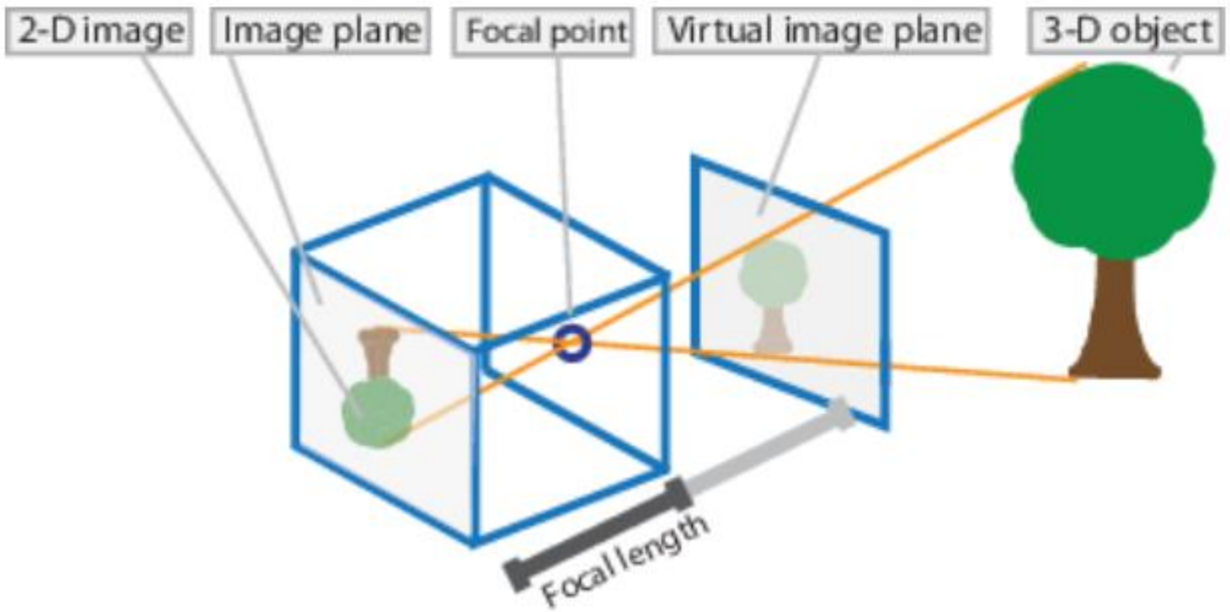
where $M = K\,{}^{C}[R\,|\,t]_{W}$,

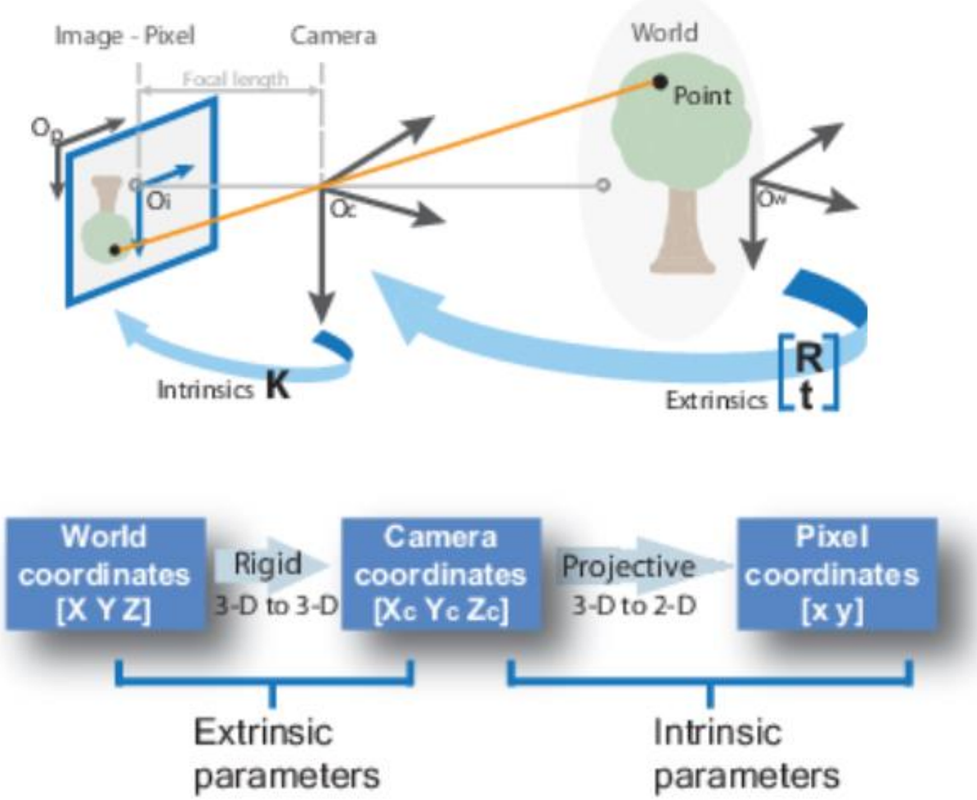
$K =$ intrinsic matrix

$[R\,|\,t] =$ extrinsic matrix

Camera Model

- Pin-hole Projective Camera Model

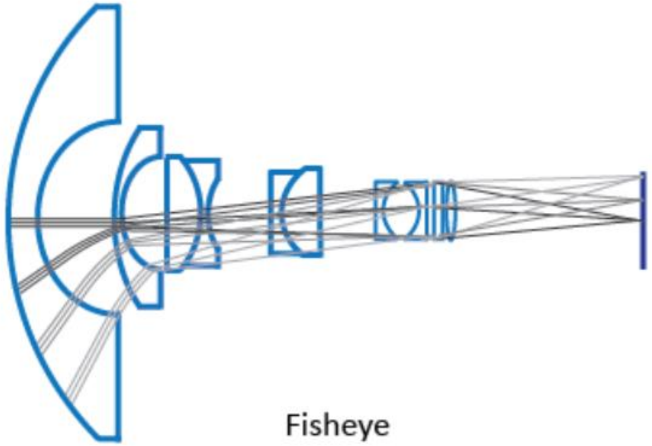




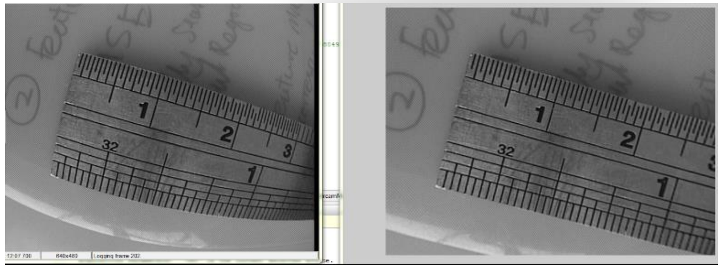
Camera Model

- Pin-hole Projective Camera Model
- Lens Distortion Model

Pinhole



Fisheye



Camera Calibration

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Camera Calibration

- Aim
  - To obtain a mapping between 3D pts in the world to their pixel coord. on the image plane
- Application
  - Robot & automation; object measurement; vision-based control; navigation; 3D reconstruction of scene…

Camera Calibration

- Two approach to solve for Camera Matrix, P
  1. Direct Linear Transformation (DLT)
     - Obtain several corresponding 3D world pts. $\tilde{X}$ and 2D image pts. $\tilde{u}$
       - (Qn: how many pairs needed?)
       - (Qn: what happen if all points of $\tilde{X}$ lie on a plane?)
     - Estimate a matrix $\hat{P}$ that minimized the square error

  2. Solve K &$[R|t]$ separately (aka camera resection)
     - Perform geometric calibration to obtain K
     - Use perspective-n-point method to solve $[R|t]$
       - (Qn: how many points needed?)

Camera Calibration (DLT)

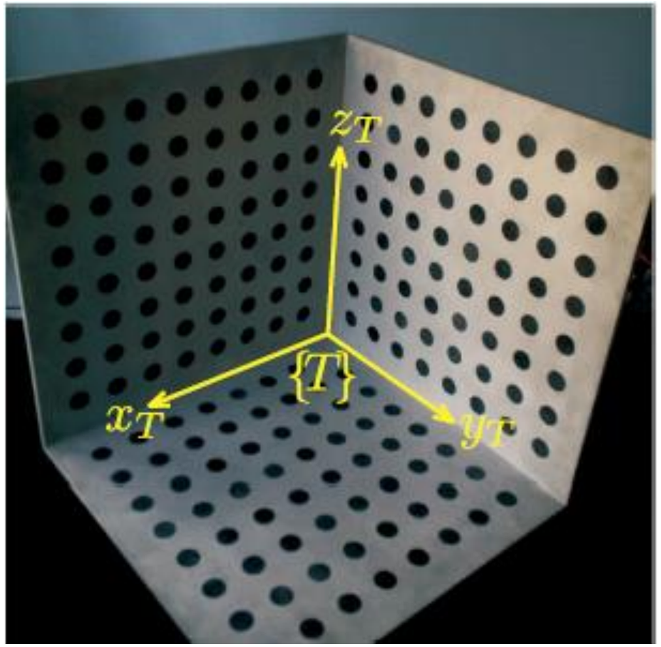
- Solve entire camera matrix

For a given point ${}^{w}\tilde{X}$ in 3D world, its projection on the image plane can be expressed as

$$s\tilde{u} = M\,{}^{w}\tilde{X}$$

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} M(1,1:4) \\ M(2,1:4) \\ M(3,1:4) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Qn: What is the last row of equation for?



Corke, Peter. *Robotics, vision and control: fundamental algorithms in MATLAB*.

Camera Calibration (DLT)

$$\begin{bmatrix} M(1,1:4) \\ M(2,1:4) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} - s \begin{bmatrix} u \\ v \end{bmatrix} = \tilde{0}$$

where $s = [M(3,1:4)] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

$$M(1,1)X + M(1,2)Y + M(1,3)Z + M(1,4) - s\tilde{u} = 0$$
$$M(2,1)X + M(2,2)Y + M(2,3)Z + M(2,4) - s\tilde{v} = 0$$

Substitute *s*,

$$M(1,1)X + M(1,2)Y + M(1,3)Z + M(1,4) - M(3,1)Xu - M(3,2)Yu - M(3,3)Zu - M(3,4)u = 0$$
$$M(2,1)X + M(2,2)Y + M(2,3)Z + M(2,4) - M(3,1)Xv - M(3,2)Yv - M(3,3)Zv - M(3,4)v = 0$$

Solve for M in the form of $\boldsymbol{AM} = \boldsymbol{0}$

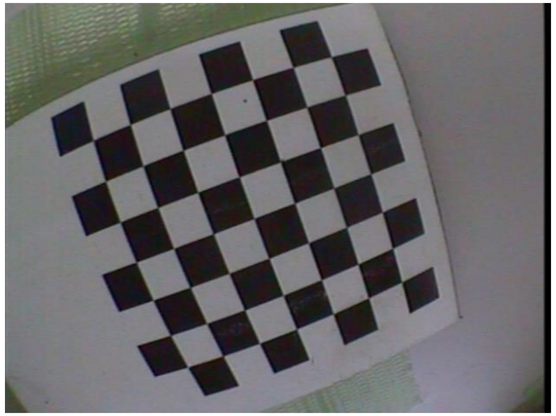
Don’t worry, matlab can deal with it, not so impt…

What if they are planar?

Camera Calibration (Geometric Calibration)

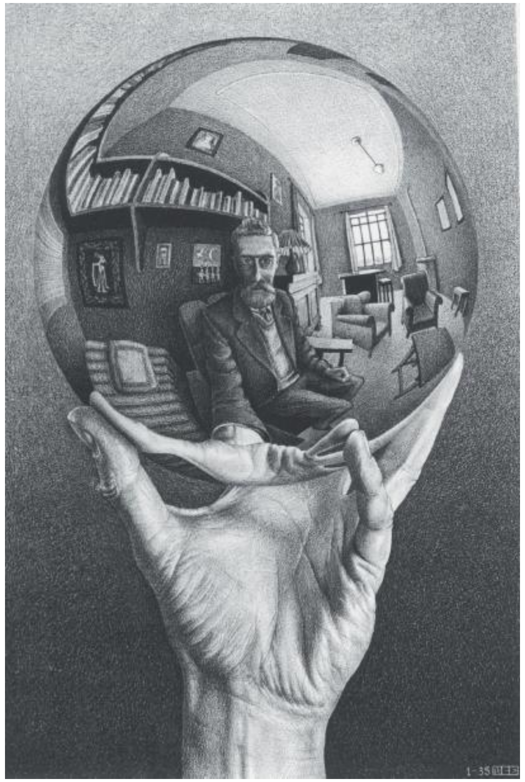
- Intrinsic parameters
  - (i_o, j_o): Principal Point (optical center)
  - (f_x, f_y): Focal Length
  - a: Skew Coefficient

$$\mathbf{K} = \begin{pmatrix} f_x & a & i_o \\ 0 & f_y & j_o \\ 0 & 0 & 1 \end{pmatrix}$$

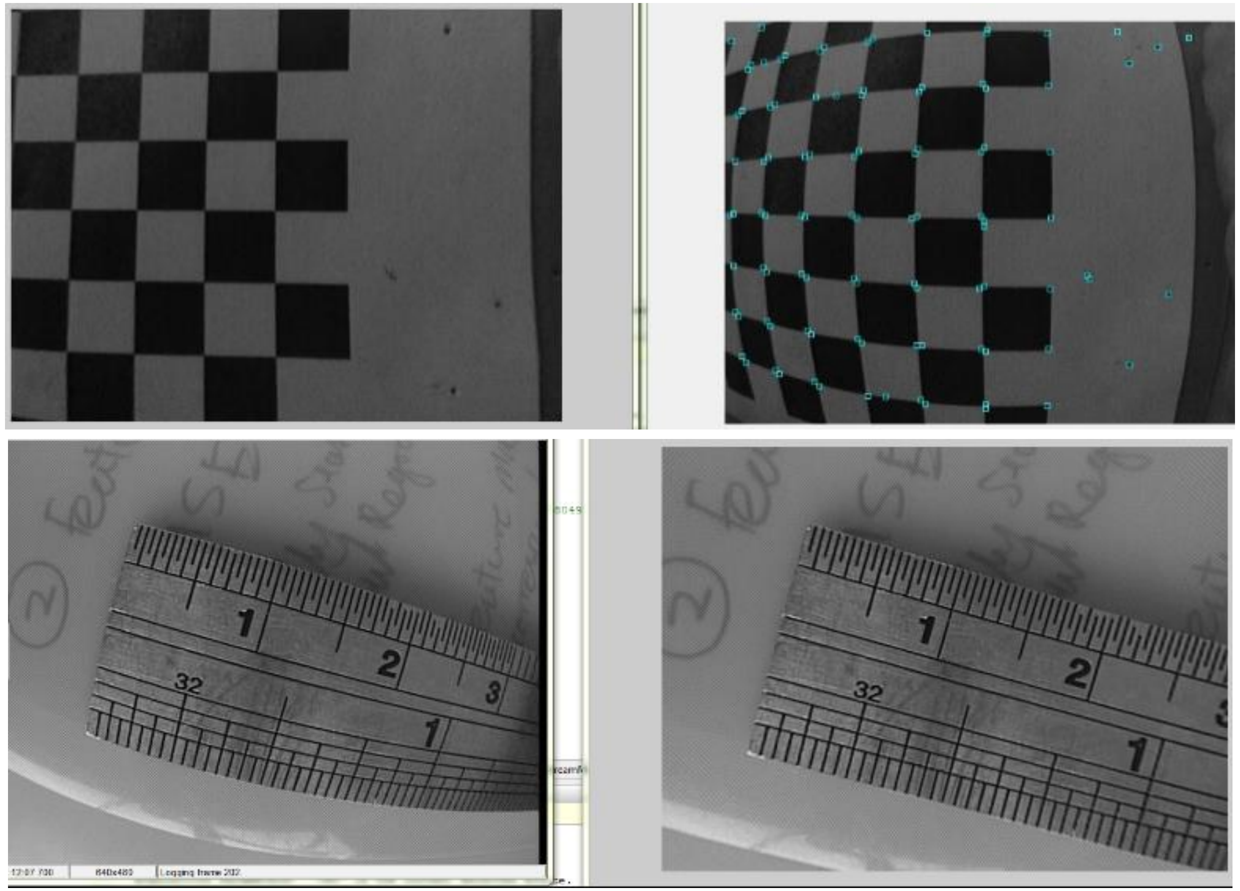


Camera Calibration (Geometric Calibration)

- Lens Distortion

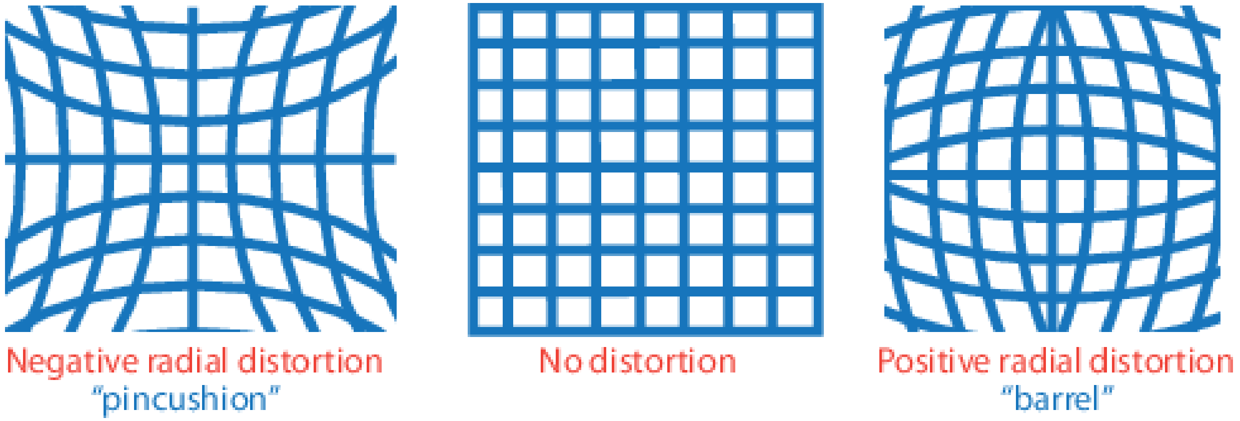


Hand with Reflecting Sphere, M. C. Escher,1935



Camera Calibration (Geometric Calibration)

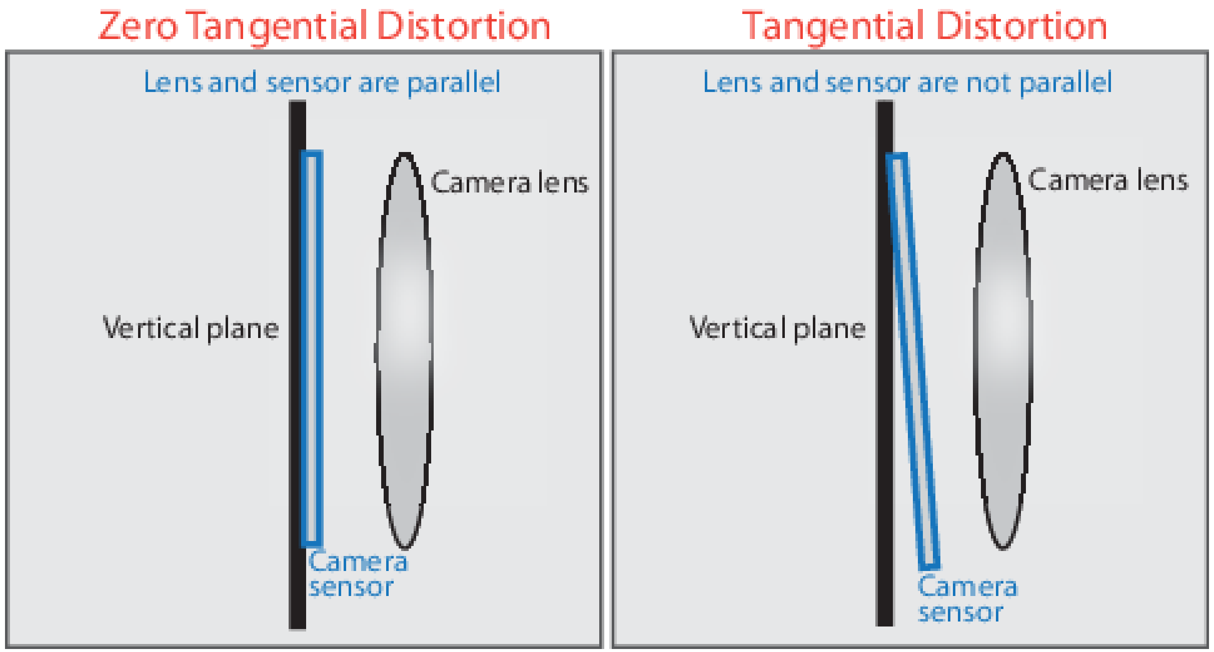
- Lens Distortion
  - Radial
  - Tangential



"What Is Camera Calibration?", Mathworks Inc., http://www.mathworks.com/help/vision/ug/camera-calibration.html

Camera Calibration (Geometric Calibration)

- Lens Distortion
  - Radial
  - Tangential



"What Is Camera Calibration?", Mathworks Inc., http://www.mathworks.com/help/vision/ug/camera-calibration.html

Camera Model

- Lens Distortion

Pinhole Model

$$(\bar{x}\quad \bar{y})^{\mathrm{T}} = \mathbf{C}\left({}^{\mathrm{u}}\mathbf{X}\right)$$

Intrinsic Parameter

$$\mathbf{C} = \begin{pmatrix} f_x & a & i_o \\ 0 & f_y & j_o \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{k}\mathrm{c} = \left(k_1 \quad k_2 \quad l_1 \quad l_2\right)$$

$$\bar{\mathbf{X}}_{distort} = (1 + k_1 r^2 + k_2 r^4)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + (l_1 \quad l_2)\begin{pmatrix} 2\bar{x}\bar{y} & r^2 + 2\bar{y}^2 \\ r^2 + 2\bar{x}^2 & 2\bar{x}\bar{y} \end{pmatrix}$$

Brown-Conrady (Plum-Blob) Model

$${}^{\mathrm{u}}\mathbf{X} \rightarrow \text{Pinhole Projection} \xrightarrow{(\bar{x}\quad \bar{y})^{\mathrm{T}}} \text{Lens Distortion} \xrightarrow{\bar{\mathbf{X}}_{distort}}$$

Camera Calibration (Geometric Calibration)

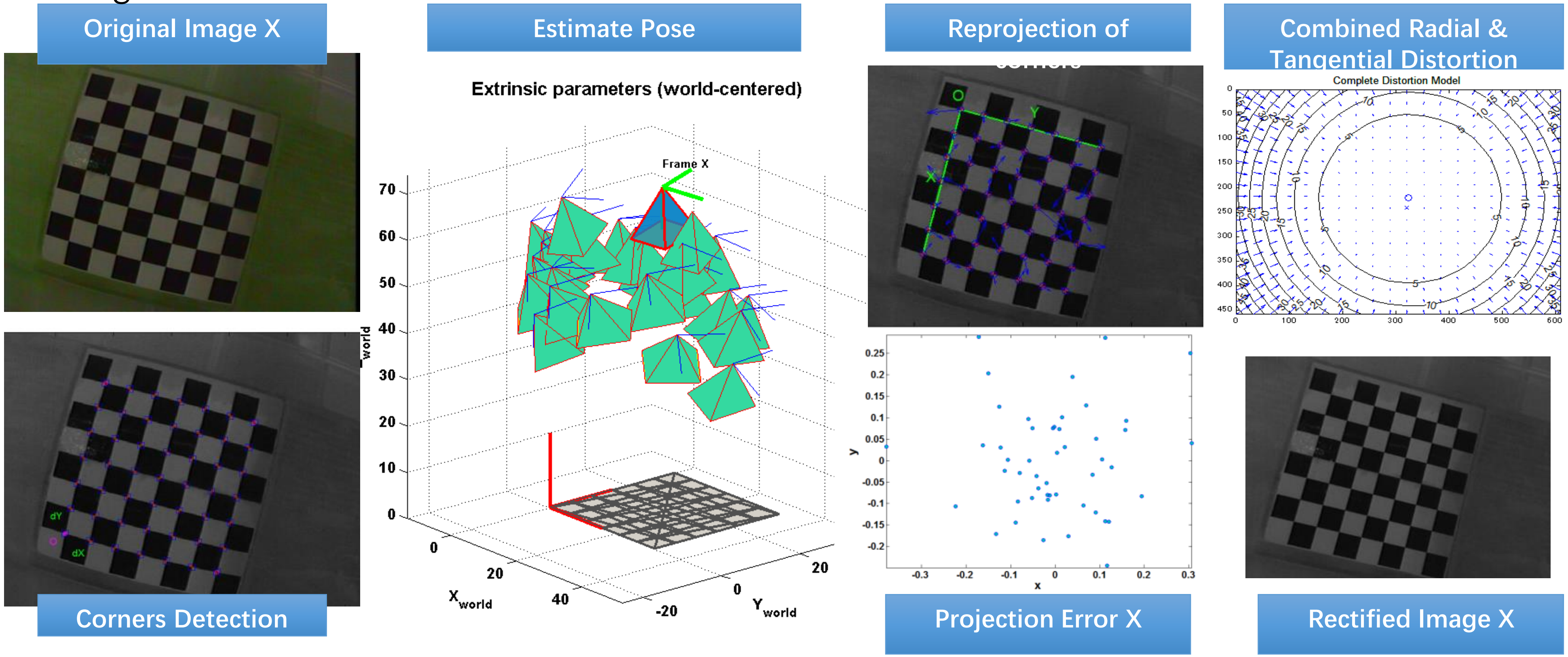
- Lens Distortion
  - Brown-Conrady Model (FYI)

Brown-Conrady (Plum-Blob) Model

$$\overline{\mathbf{X}}_{distort} = \underbrace{(1 + k_1 r^2 + k_2 r^4)\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix}}^{\text{Radial}} + \overbrace{(l_1 \quad l_2)\begin{pmatrix} 2\overline{xy} & r^2 + 2\bar{y}^2 \\ r^2 + 2\bar{x}^2 & 2\overline{xy}\end{pmatrix}}^{\text{Tangential}}$$

Camera Calibration (Geometric Calibration)

Example of calibration process using Matlab Camera Calibration Toolbox developed by Bouget



Calibration Process

Camera Calibration (Geometric Calibration)

- Intrinsic Properties

Camera Geometry

$$(\bar{x}\quad \bar{y})^{\mathrm{T}} = \mathbf{K}\left({}^{\mathrm{C}}\mathbf{X}\right)$$

Intrinsic Parameter

$$\mathbf{K} = \begin{pmatrix} f_x & a & i_o \\ 0 & f_y & j_o \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{k}\mathrm{c} = \left(k_1 \quad k_2 \quad l_1 \quad l_2\right)$$

$$\overline{\mathbf{X}}_{distort} = (1 + k_1 r^2 + k_2 r^4)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + (l_1 \quad l_2)\begin{pmatrix} 2\bar{x}\bar{y} & r^2 + 2\bar{y}^2 \\ r^2 + 2\bar{x}^2 & 2\bar{x}\bar{y} \end{pmatrix}$$

Brown-Conrady (Plum-Blob) Model

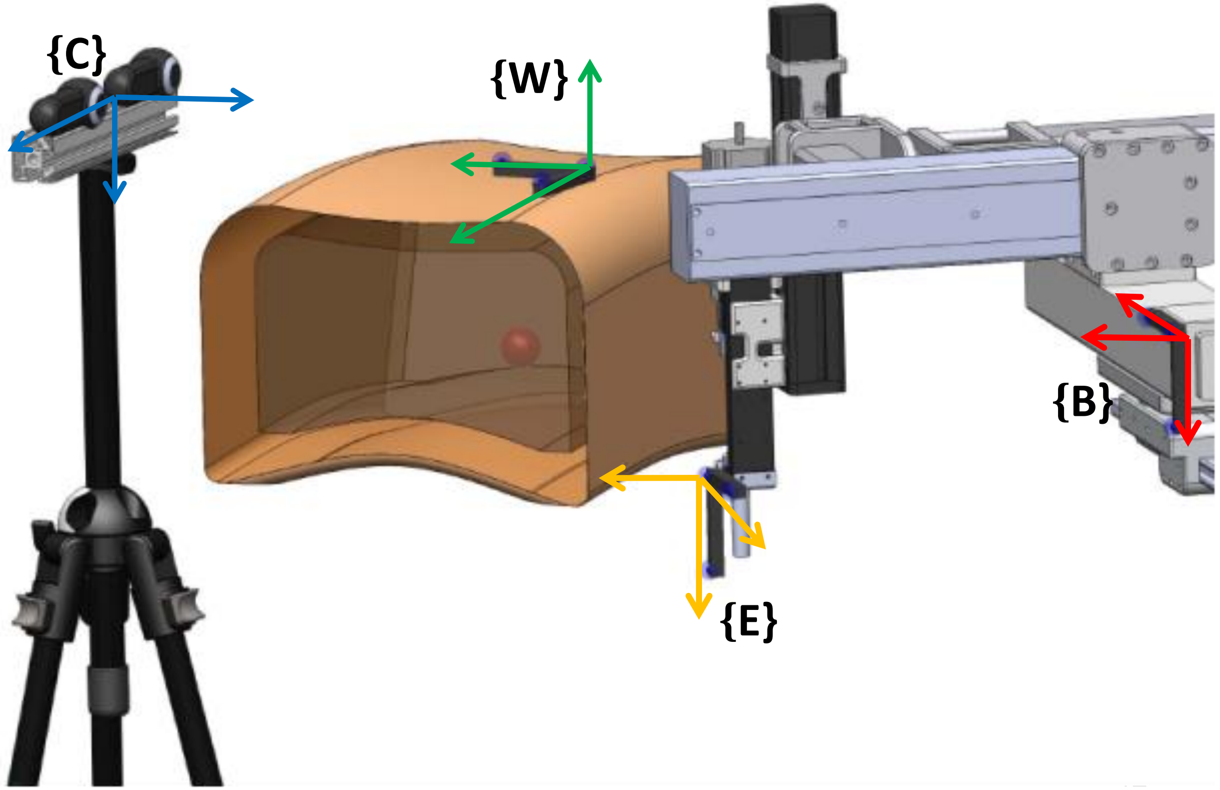
$${}^{\mathrm{C}}\mathbf{X} \longrightarrow \boxed{\text{Pinhole Projection}} \xrightarrow{(\bar{x}\quad \bar{y})^{\mathrm{T}}} \boxed{\text{Lens Distortion}} \xrightarrow{\overline{\mathbf{X}}_{distort}}$$

Camera Calibration Summary

- Transformations from world (Cartesian) to image (pixel) coordinates

Case Problem 1

- Given K, ${}^{c}[R \mid t]_{W}$, ${}^{W}T_{B}$, ${}^{B}P_{E}$
- Find the image coordinates of point E

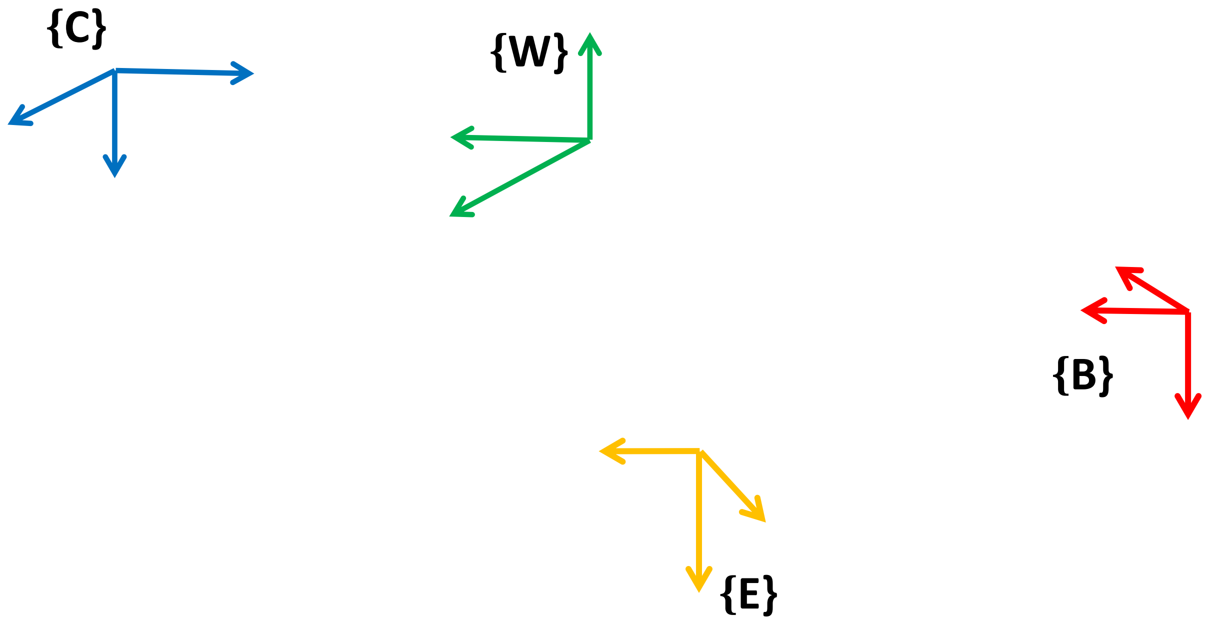


Case Problem 1

Using relationship, $s\mathbf{u} = \mathbf{K}\ {}^{c}[\mathbf{R} \mid \mathbf{t}]_{W}\ X$

$s\mathbf{u} = \mathbf{K}\ {}^{c}[\mathbf{R} \mid \mathbf{t}]_{W}\ {}^{W}T_{B}\ {}^{B}P_{E}$

$s[u\ v\ 1]^{T} = \mathbf{K}\ {}^{c}[\mathbf{R} \mid \mathbf{t}]_{W}\ {}^{W}T_{B}\ {}^{B}P_{E}$



There will be 3 rows
using the last row, solve for s
Obtain image coord, **u**

After calibration, how do we estimate the pose of the camera?

Robot-Camera Calibration

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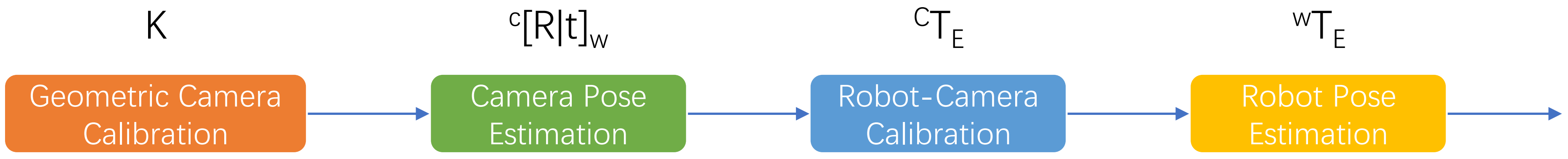
Robot-Camera Calibration

- For robotics, we are interested in **pose of robot** end-effector
- **Camera pose** and camera calibration is not enough
- **Robot-camera calibration** is required

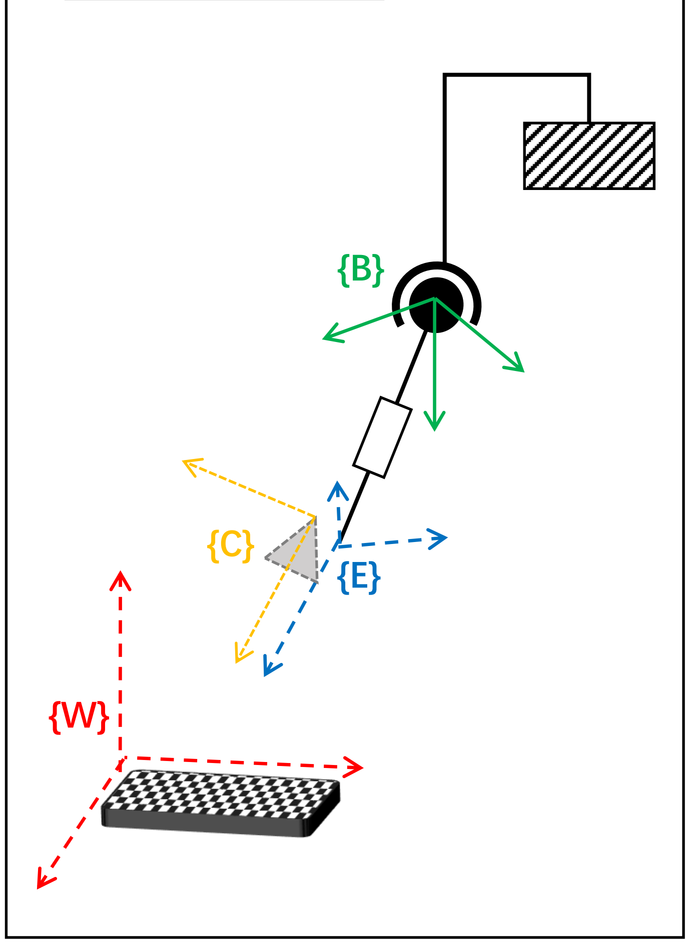
Robot-Camera Calibration

- Also known as Tracker-camera; Hand-eye calibration
- To obtain the end-effector pose w.r.t. the camera, ${}^{E}T_{C}$
- Use camera-acquired data for robot pose estimation

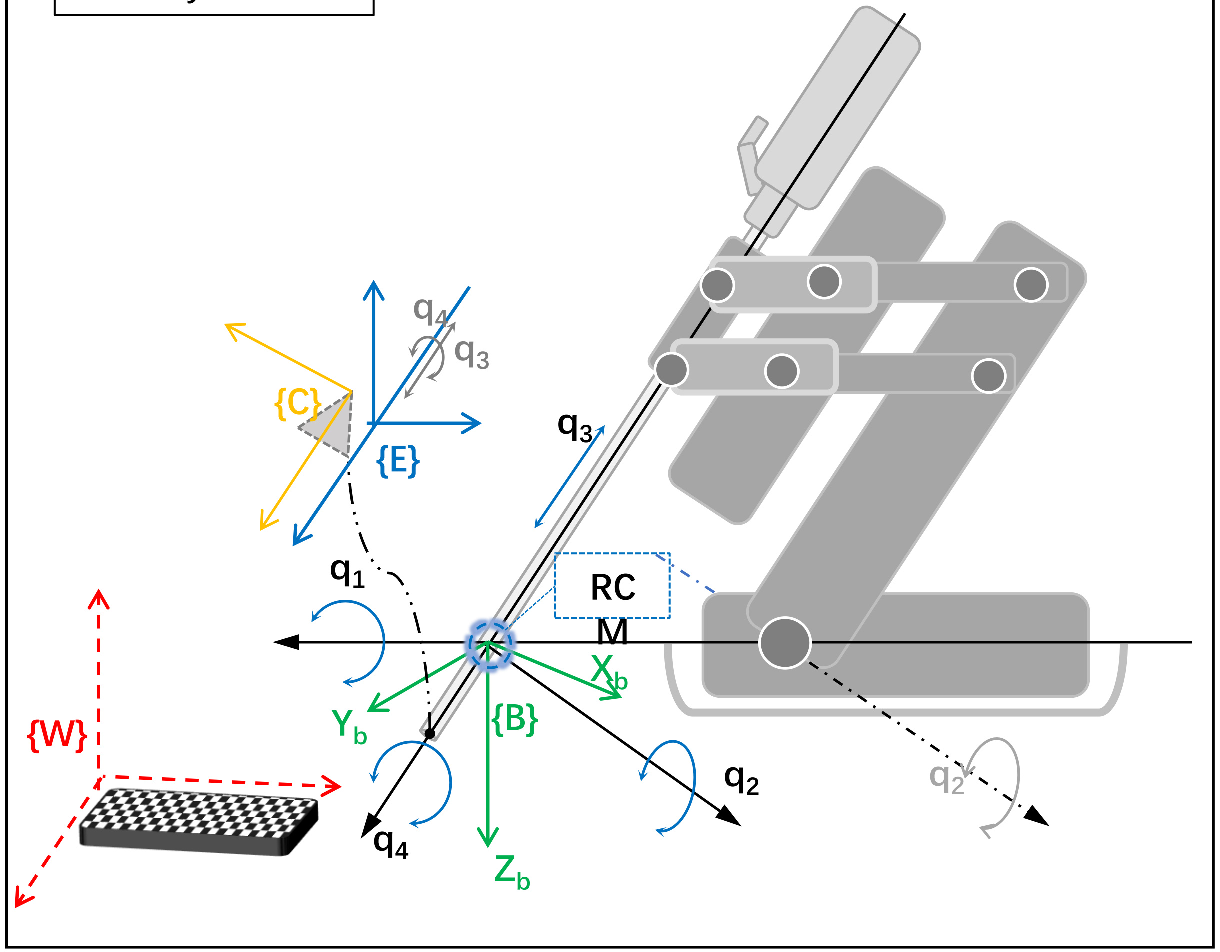


Robot-Camera Calibration: Illustration

Schematic



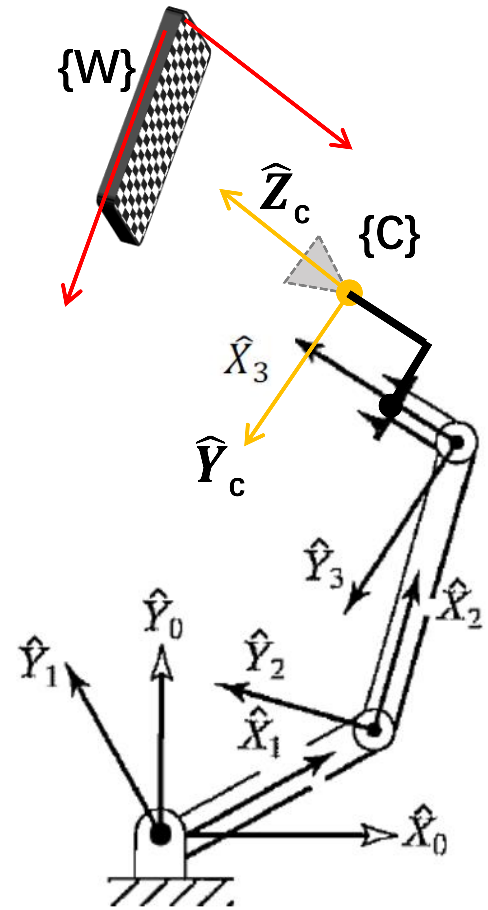
Physical



Case Problem 2

Find the camera matrix M that maps a 3D point (x,y,z) in world ref. frame {W} to its projection on image coordinates (u,v) given the following:

${}^{W}\mathbf{T}_{0}$, : Robot base in world ref. frame
${}^{3}\mathbf{T}_{c}$: Camera in Link 3 ref. frame
K: Intrinsic matrix of camera
$(\theta_1, \theta_2, \theta_3)$: Joint Variables



Case Problem 2

Using relationship, $\mathbf{M}=\mathbf{K}\ {}^{c}[\mathbf{R}|\mathbf{t}]_{w}$

where ${}^{c}[\mathbf{R}|\mathbf{t}]_{w}= {}^{c}\mathbf{T}_{w}$

We can express M as $\mathbf{M}=\mathbf{K}\ [{}^{w}\mathbf{T}_{0}\,{}^{0}\mathbf{T}_{3}\,{}^{3}\mathbf{T}_{c}]^{-1}$

by substituting ${}^{c}\mathbf{T}_{w} = [{}^{w}\mathbf{T}_{c}]^{-1}$

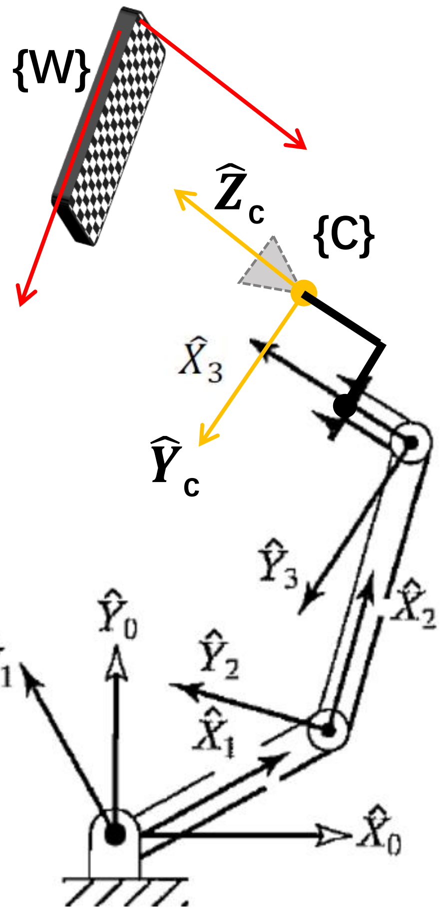
(Qn: what is the physical meaning of ${}^{w}\mathbf{T}_{c}$ or ${}^{c}\mathbf{T}_{w}$?)

Do forward kinematics,

$${}^{0}\mathbf{T}_{3}={}^{0}\mathbf{T}_{1}\,{}^{1}\mathbf{T}_{2}\,{}^{2}\mathbf{T}_{3}$$

$$\boxed{\mathbf{M}=\mathbf{K}\ [{}^{w}\mathbf{T}_{0}\,{}^{0}\mathbf{T}_{1}\,{}^{1}\mathbf{T}_{2}\,{}^{2}\mathbf{T}_{3}\,{}^{3}\mathbf{T}_{c}]^{-1}}$$

Qn: Can we do camera calibration with this setup?
The robot is planar, you need many views in 3D for calibration



Camera Pose Estimation

- Solve extrinsic parameters
  - Perspective-n-Point Problem
    - Given point correspondence and known $\boldsymbol{K}$ (calibrated), obtain $[\boldsymbol{R}|\boldsymbol{t}]$
  - To localized the camera with 6 dof in 3D space
    - $\geq$ 3 non-collinear points needed

In Matlab,

Calibration Toolbox: compute_extrinsic()

Robotics Toolbox: estpose

In OpenCV,

solvePnP

What if there is no known structure/model (like the checkerboard) in the scene?

Camera Pose Estimation

- EPnP Algorithm (FYI)

$$ {}^{k}\mathbf{z}_{l} \cdot \left({}^{k}\begin{pmatrix} i_l & j_l & 1 \end{pmatrix}^{\mathrm{T}} \right) = K \begin{pmatrix} R & | \, t \end{pmatrix} $$

For M virtual control points $\mathbf{q}=\{q_1 \ldots q_m \cdots q_M\}$,

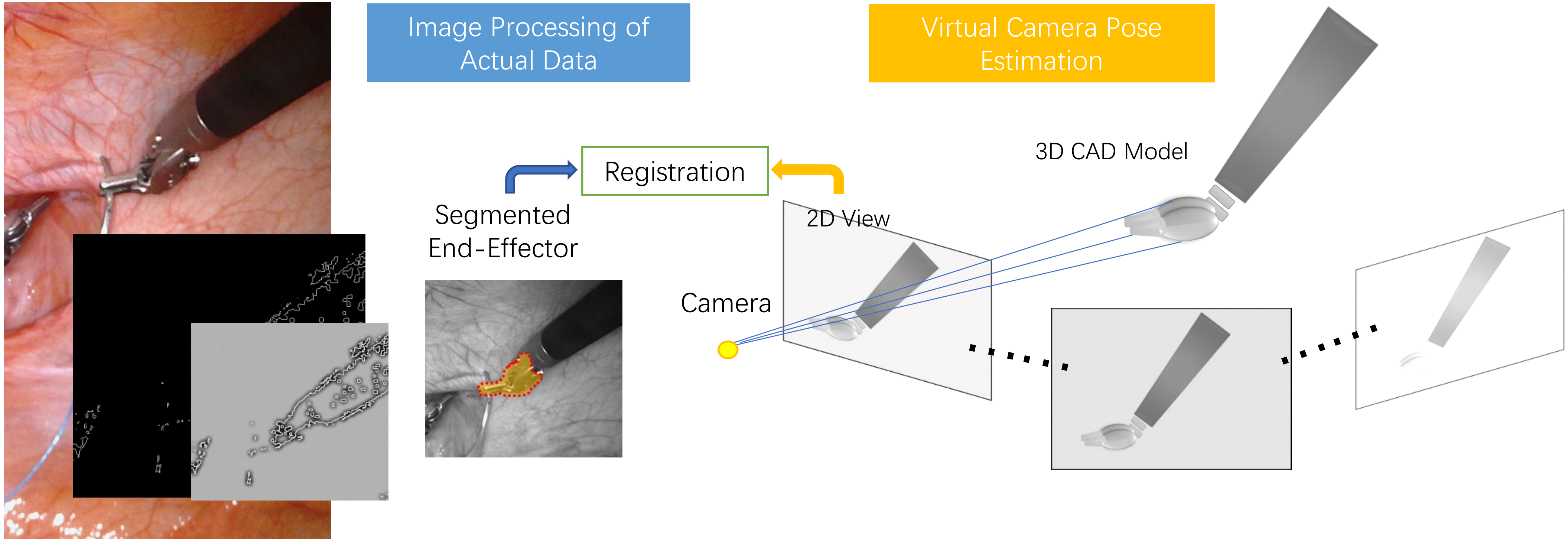
$$ {}^{k}\mathbf{z}_{l} \cdot \left({}^{k}\begin{pmatrix} i_l & j_l & 1 \end{pmatrix}^{\mathrm{T}} \right) = K \sum_{m}^{M} \lambda_{lm} \, {}^{c}q_m $$

λ_{lm} : homogeneous Barycentric coordinates

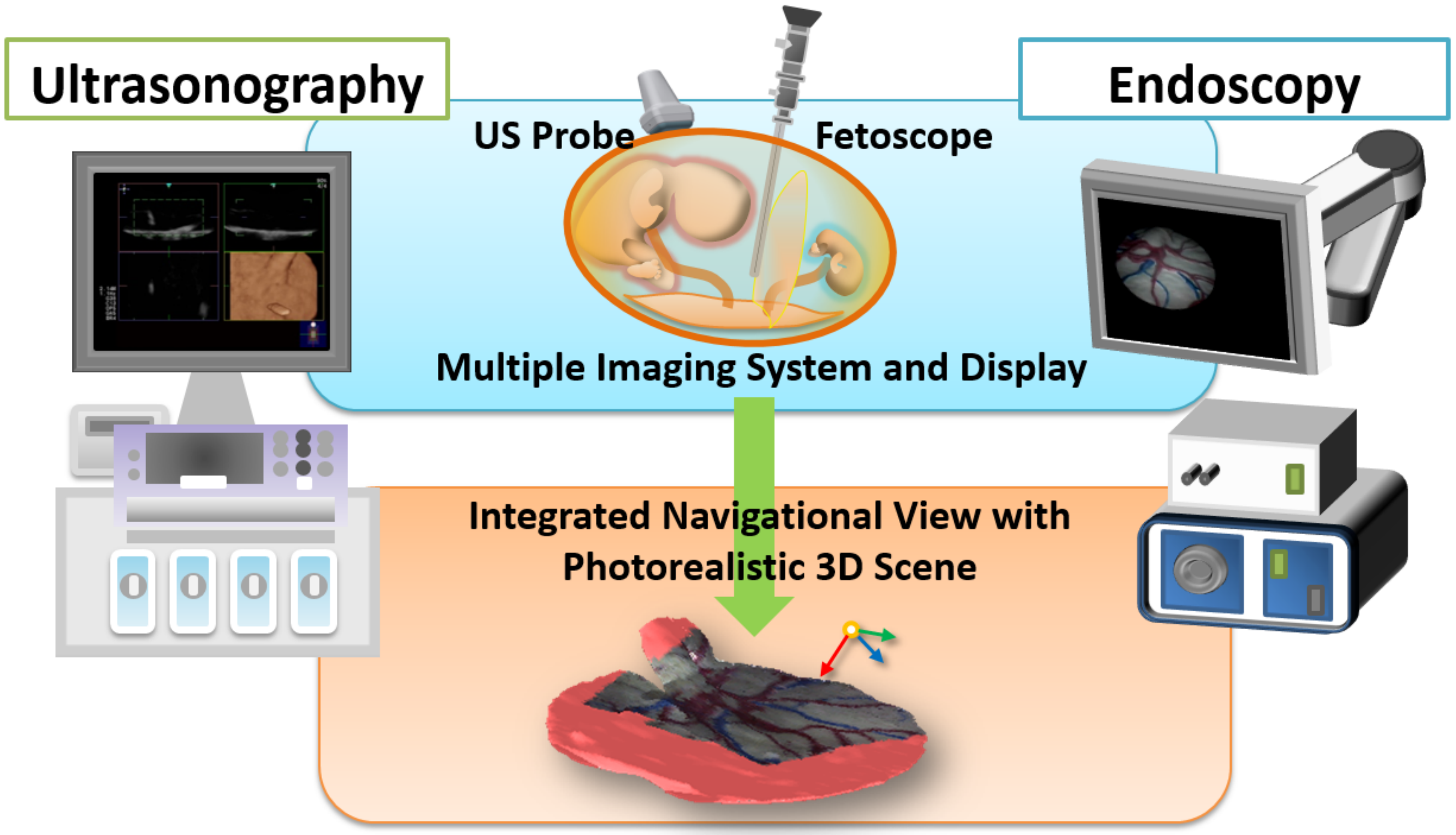
V. Lepetit, F. Moreno-Noguer and P. Fua. EPnP: An Accurate O(n) Solution to the PnP Problem, in International Journal Of Computer Vision, vol. 81, p. 155-166, 2009.

Camera Pose Estimation

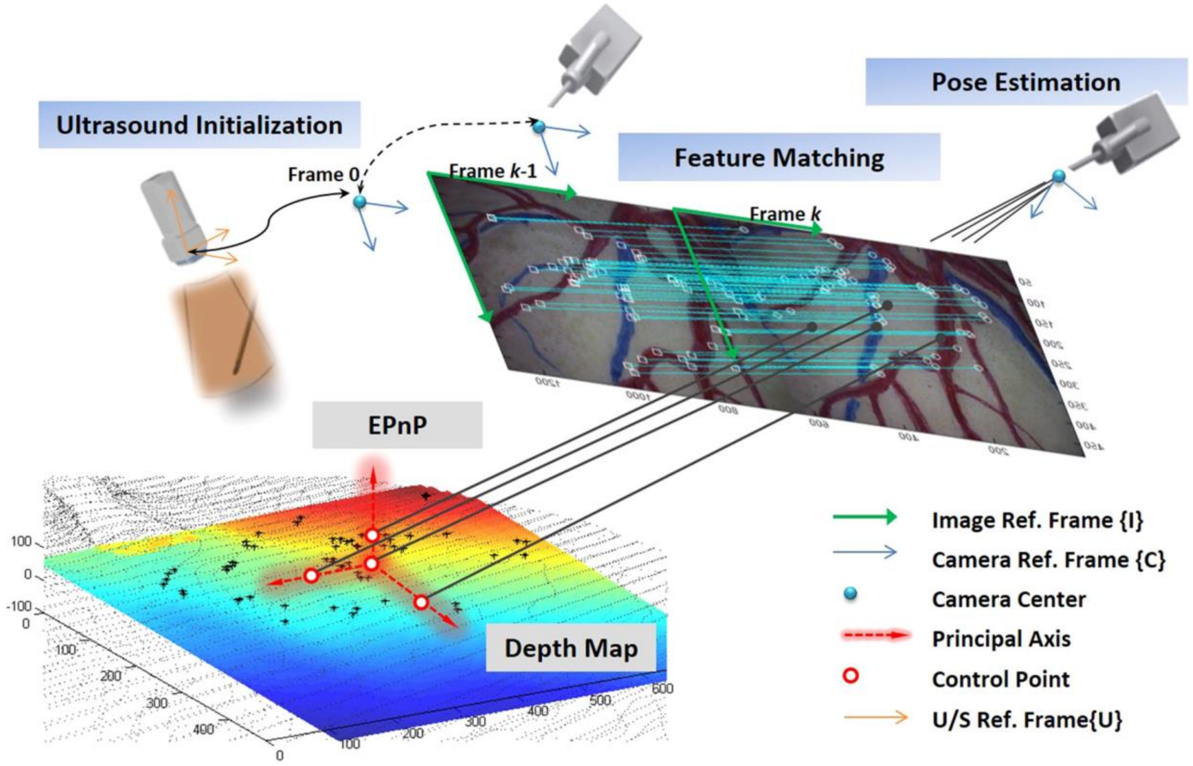
- Solve extrinsic parameters
  - Other model based approach

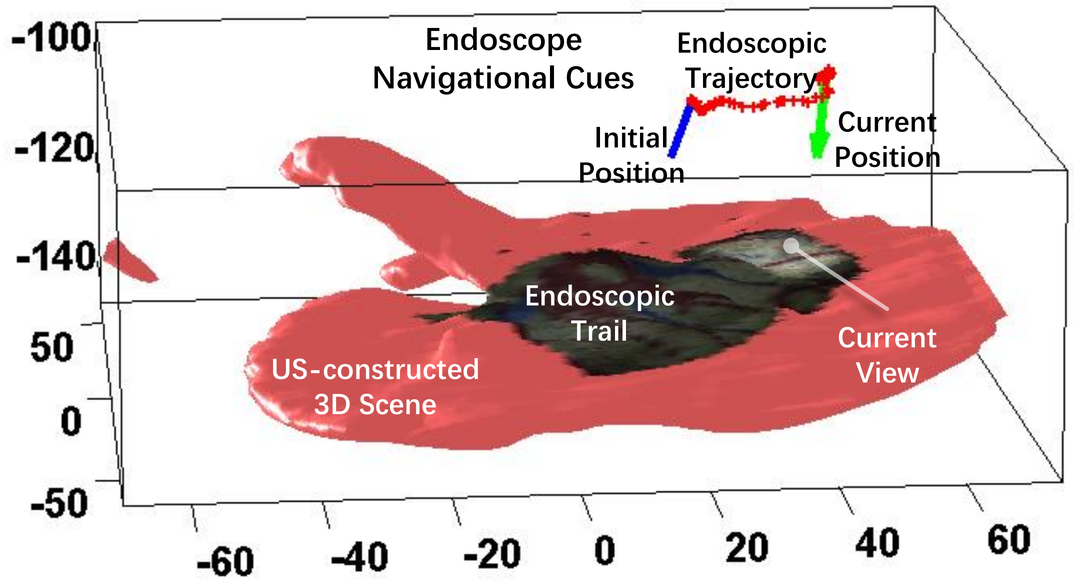


Pose Estimation: Fusion of Data



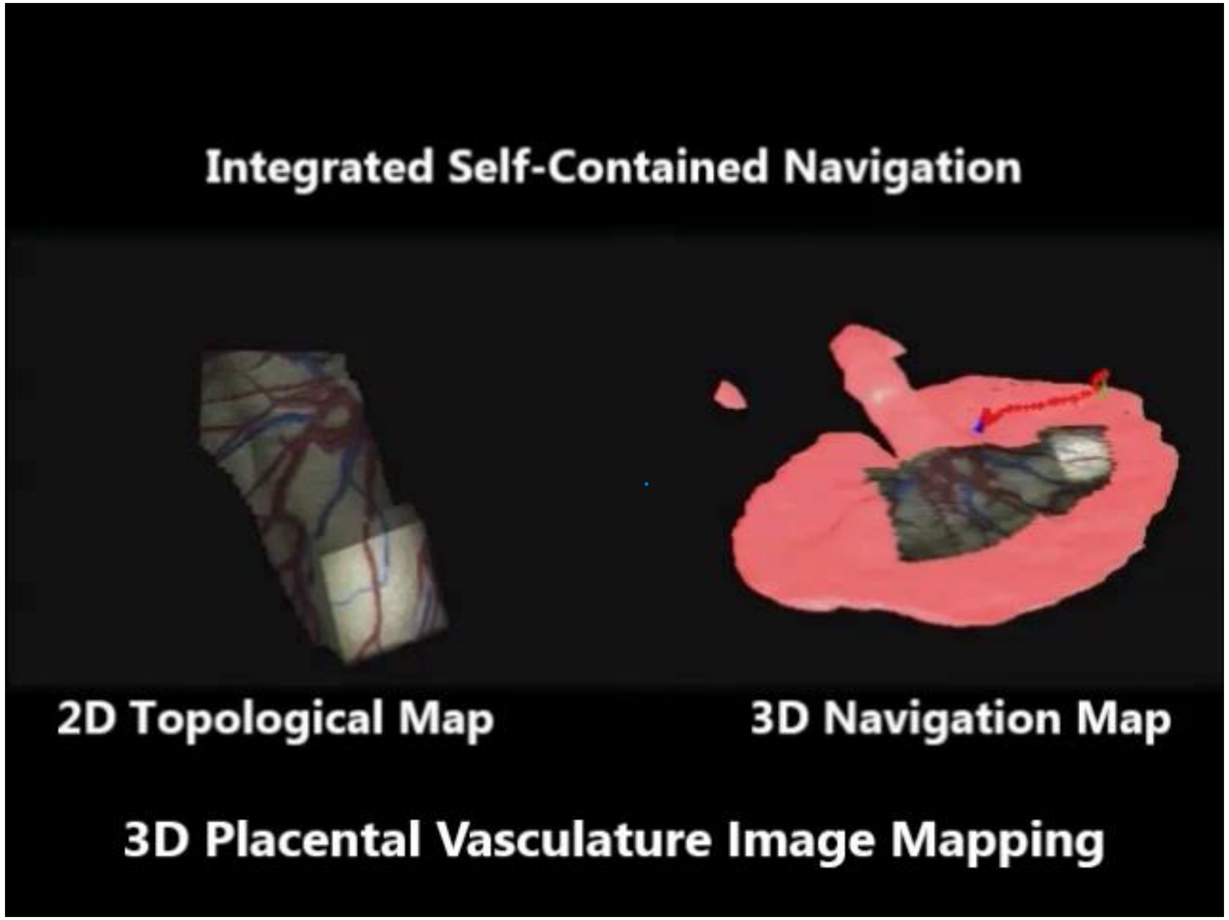
Pose Estimation: Fusion of Data





“Self-contained image mapping of placental vasculature in 3D ultrasound-guided fetoscopy”, Yang et al., 2015

Pose Estimation: Fusion of Data



After calibration, how do we estimate the pose of the camera?
Or the pose of our robot?
How about spatial representation of the surrounding?

Describing the pose of camera is mathematically equivalent to

describing the surrounding spatial information

A Problem of Spatial Representation and 3D Vision

3D Vision

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