ECE 470: Introduction to Robotics Homework 1

Question 1.

In Figure 1, Frame {A} and {B} are not connected.

a) Determine the transformation matrix ${}^{A}_{B1}T$ after {B} rotates 45° about its axis X_B to become {B1}.

b) Determine the inverse matrix ${}^{A}_{B1}T^{-1}$ in (a)

c) Determine the transformation matrix ${}^{A}_{B2}T$ if {B1} revolves 45° about Y_A to become {B2}.

d) Determine the transformation matrix ${}^{A1}_{B2}T$ if {A} rotates -90° about its X_A to become {A1}.

(10 Points)

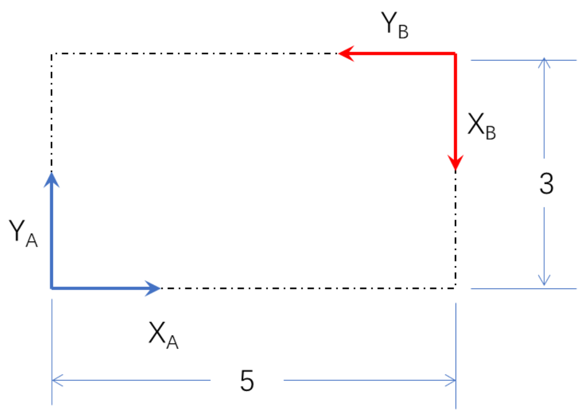

Figure 1

Question 2.

A cuboid with Frame {M} and Frame {C} attached rigidly is shown in Figure 2. The universe frame of reference {U} serves as an absolute frame that is always fixed. The cuboid motion is described by the series of transformation operations.

1> Rotation about the z axis of Frame C by 30°, then
2> Translation of (1, 2, 3) along Frame C, then
3> Rotation about the x axis of Frame M by 45°, and then
4> Rotation about the y axis of Frame U by 60°.

Let ${}^{U}T_{C_i}$ and ${}^{U}T_{M_i}$ be the 4 × 4 homogeneous transformation matrices that describes the position and orientation of Frames C and M, respectively, in U after motion i.

Find

i. ${}^{U}T_{C_1}$

ii. ${}^{U}T_{C_2}$

iii. ${}^{U}T_{C_3}$

iv. ${}^{U}T_{C_4}$

v. ${}^{U}T_{M_4}$

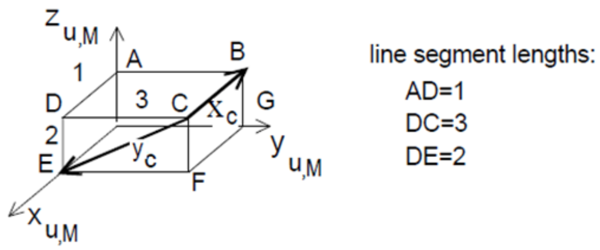

Figure 2

(10 Points)

Solution

Question 1

a)

$$ {}^{A}_{B1}T = {}^{A}_{B}T\,{}^{B}_{B1}T; \qquad {}^{A}_{B}T = \left(\begin{array}{ccc|c} 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \qquad {}^{B}_{B1}T = rot_x(45) = \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & c45 & -s45 & 0 \\ 0 & s45 & c45 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) $$

$$ {}^{A}_{B1}T = \left(\begin{array}{ccc|c} 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & c45 & -s45 & 0 \\ 0 & s45 & c45 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} 0 & -c45 & s45 & 5 \\ -1 & 0 & 0 & 3 \\ 0 & -s45 & -c45 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 5 \\ -1 & 0 & 0 & 3 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) $$

(b)

$$ {}^{A}_{B1}T^{-1} = \begin{pmatrix} {}^{A}_{B1}R' & -{}^{A}_{B1}R'\,{}^{A}_{B1}P \\ 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 0 & -1 & 0 & 3 \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & \frac{5}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{5}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{array}\right) $$

(c)

$$ {}^{A}_{B2}T = {}^{A}_{A2}T\,{}^{A2}_{B2}T; \qquad {}^{A}_{A2}T = rot_y(45) = \left(\begin{array}{ccc|c} c(45) & 0 & s(45) & 0 \\ 0 & 1 & 0 & 0 \\ -s(45) & 0 & c(45) & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \qquad {}^{A2}_{B2}T = {}^{A}_{B1}T = \left(\begin{array}{ccc|c} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 5 \\ -1 & 0 & 0 & 3 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) $$

$$ {}^{A}_{B2}T = \left(\begin{array}{ccc|c} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 5 \\ -1 & 0 & 0 & 3 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} 0 & -1 & 0 & \frac{5}{\sqrt{2}} \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & \frac{-5}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

(d)

$$ {}^{A1}_{B2}T = {}^{A1}_{A}T\,{}^{A}_{B2}T; $$

$$ {}^{A1}_{B2}T = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)\begin{pmatrix} 0 & -1 & 0 & \frac{5}{\sqrt{2}} \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & \frac{-5}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & \frac{5}{\sqrt{2}} \\ 0 & 0 & 1 & \frac{5}{\sqrt{2}} \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Question 2

$${}^{U}_{C0}T = \left[{}^{\widetilde{U}}_{C0}R \quad {}^{\widetilde{U}}_{C0}P\right] \text{ where } {}^{\widetilde{U}}_{C0}R = \left(\begin{pmatrix}-1\\0\\0\end{pmatrix} \quad \underset{\mathbf{0}}{\widetilde{\begin{pmatrix}0\\-3\\-2\end{pmatrix}}} \quad \widetilde{\begin{pmatrix}0\\-2\\3\end{pmatrix}} \right) \text{ and } {}^{\widetilde{U}}_{C0}P = \begin{pmatrix}1\\3\\2\\1\end{pmatrix}$$

$${}^{U}_{C0}T = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -0.8321 & -0.5507 & 3 \\ 0 & -0.5507 & 0.8321 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(i)

$${}^{U}_{C1}T = {}^{U}_{C0}T\,{}^{C0}_{C1}T = {}^{U}_{C0}T\ rot_z(30)$$

$${}^{U}_{C1}T = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -0.8321 & -0.5507 & 3 \\ 0 & -0.5507 & 0.8321 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -0.5 & 0 & 0 \\ 0.5 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.866 & 0.5 & 0 & 1 \\ -0.416 & -0.721 & -0.555 & 3 \\ -0.277 & -0.48 & 0.832 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)

$${}^{U}_{C2}T = {}^{U}_{C1}T\,{}^{C1}_{C2}T = {}^{U}_{C0}T\ trans(1,2,3)$$

$${}^{U}_{C2}T = \begin{bmatrix} -0.866 & 0.5 & 0 & 1 \\ -0.416 & -0.721 & -0.555 & 3 \\ -0.277 & -0.48 & 0.832 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.866 & 0.5 & 0 & 1.134 \\ -0.416 & -0.721 & -0.555 & -0.521 \\ -0.277 & -0.48 & 0.832 & 3.258 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii)

$${}^{U}_{C3}T = {}^{U}_{C2}T\,{}^{C2}_{M2}T\,{}^{M2}_{M3}T\,{}^{M3}_{C3}T = {}^{U}_{C2}T\,{}^{C}_{M}T\ rot_x(45)\,{}^{M}_{C}T$$

$$= \begin{bmatrix} -0.866 & 0.5 & 0 & 1.134 \\ -0.416 & -0.721 & -0.555 & -0.521 \\ -0.277 & -0.48 & 0.832 & 3.258 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -0.832 & -0.551 & 3 \\ 0 & -0.551 & 0.832 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c45 & -s45 & 0 \\ 0 & s45 & c45 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -0.832 & -0.551 & 3 \\ 0 & -0.551 & 0.832 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.866 & 0.354 & 0.354 & 1.662 \\ -0.416 & -0.117 & -0.902 & -2.696 \\ -0.277 & -0.928 & 0.249 & 4.872 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv)

$${}^{U}_{C4}T = {}^{U}_{U4}T\,{}^{U4}_{C4}T = rot_y(60)\,{}^{U}_{C3}T$$

$${}^{U}_{C4}T = \begin{bmatrix} c60 & 0 & s60 & 0 \\ 0 & 1 & 0 & 0 \\ -s60 & 0 & c60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.866 & 0.354 & 0.354 & 1.662 \\ -0.416 & -0.117 & -0.902 & -2.696 \\ -0.277 & -0.928 & 0.249 & 4.872 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.673 & -0.627 & 0.392 & 5.05 \\ -0.416 & -0.117 & -0.902 & -2.696 \\ 0.611 & -0.77 & -0.182 & 0.997 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(v)

$${}^{U}_{M4}T = {}^{U}_{C4}T\,{}^{C4}_{M4}T = {}^{U}_{C4}T\,{}^{C0}_{U}T = \begin{bmatrix} 0.673 & 0.304 & 0.674 & 2.117 \\ 0.416 & 0.589 & -0.685 & -3.535 \\ -0.611 & 0.742 & 0.276 & -1.169 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$