RF Amplifier Design
Lecture Notes

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Contents

Chapter 1

Introduction

1.1 Active Circuits in RF Front-Ends

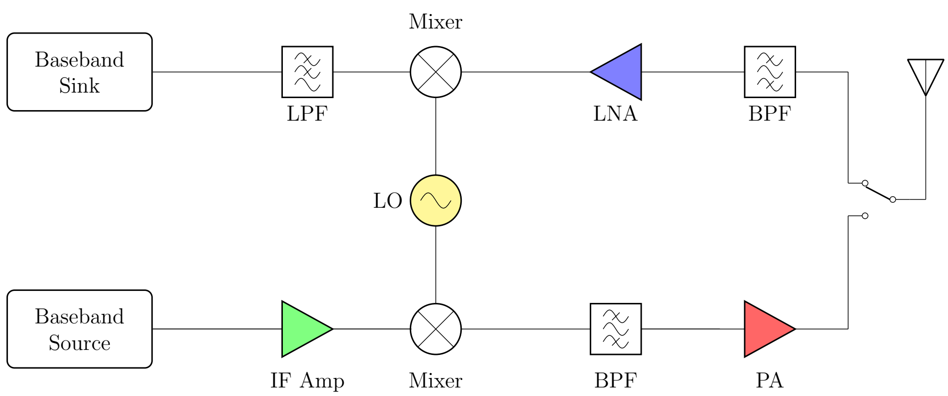


Figure 1.1: Heterodyne radio highlighting common active components

Active circuit design is considered the most challenging in RF and microwave engineering, although it is present in all radios. Some applications of amplifiers in the telecommunications industry include base stations along with mobile devices (Common 5G bands: 700 MHz - 900 MHz, 1.8 GHz - 2.5 GHz, 3.3 GHz - 4.2 GHz, 24 GHz - 30 GHz, etc.), Wi-Fi (2.4 GHz ISM, 5.8 GHz, 6 GHz), and Bluetooth (2.4 GHz ISM). In addition, some wireless backhaul links use higher frequency bands (60 GHz - 80 GHz). Radar and radio location require high power wideband amplifiers for air traffic, automotive (24 GHz, 77 GHz), weather (8 GHz

- 12 GHz), and defense applications. Satellite communications are also becoming more common with access points like Starlink and Amazon Kuiper (10 GHz - 14 GHz). These frequencies are examples and not 'gospel', most of these examples use specific subsets of these bands that are authorized in a given country.

Amplifier design cannot be reduced to a 'cookbook recipe' or a simple algorithmic process. For the inexperienced, understanding the core theory, reviewing examples, and illustrative case studies may be the best way to learn this craft. These notes will follow a 'learn-by-example' style to reinforce the theory.

1.2 Design Space

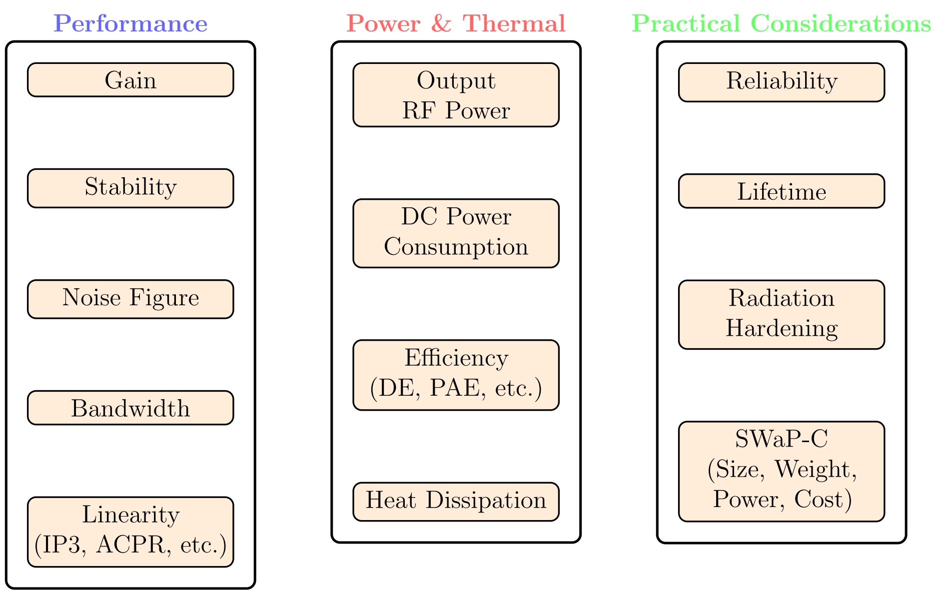


Figure 1.2: Active Microwave Circuits Design Space

A microwave circuit is considered 'active' when a device that requires an external power source is present. Amplifiers, oscillators, and some mixers are examples of active microwave circuits. They typically use transistors (FETs, BJTs, HEMTs, etc.) or vacuum tubes as the active device which provides signal gain. There are special considerations for these circuits to behave as expected. Fig. 1.2 presents the multi-dimensional design space of many active microwave circuits.

Although it is impossible to optimize for all the dimensions in this space simultaneously, the amplifier community has developed strategies to satisfy certain subsets of the design space for key purposes. These main categories are:

- **Linear Amplifiers**: The power will be considered a small-signal, providing high gain and low distortion
- **Wideband Amplifiers**: Matching over half-octaves with flat gain
- **Low Noise Amplifiers (LNA)**: Target low noise figure for receiver front-ends.
- **Power Amplifiers (PA)**: Target high output power and high efficiency for transmitter front-ends.

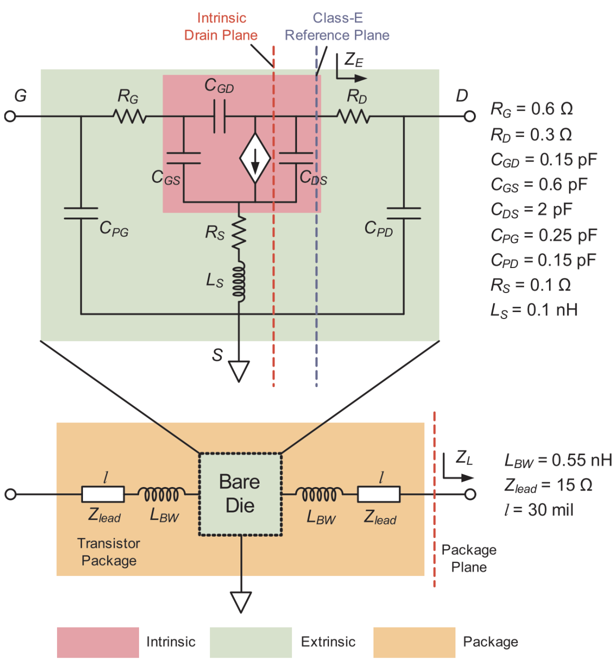


Figure 1.3: Typical transistor parasitics. Taken from [1]

1.3 Transistors and the need for matching

Fig. 1.3 shows an equivalent circuit for packaged GaN HEMT transistor, including typical intrinsic, extrinsic, and package parasitics. Note that the transistor does not present an impedance Z_0 in either the input or output. For that reason, carefully designed matching networks are needed to extract the desired performance within a given bandwidth. These parasitics are the ultimate limit to the device performance (gain, bandwidth, noise, etc.). In addition, some large-signal designs will carefully manipulate the waveforms at the intrinsic plane.

1.4 Amplifier Topology

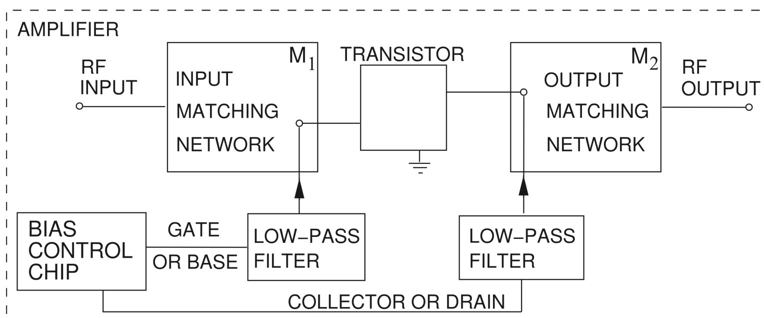


Figure 1.4: RF Amplifier Block Diagram. Taken from [2]

Fig. 1.4 shows the block diagram of a typical RF amplifier. The input and output matching networks provide good power transfer, have minimal loss, and present the impedances that target the desired figures of merit (FoM). These networks are not necessarily targeting conjugate matching for maximum power transfer. They balance the desired metrics (i.e. gain, noise, efficiency, etc.) with carefully chosen target impedances.

The DC biasing networks should provide DC power to the active device while isolating the RF signal from the supply. The DC network has an RF low-pass response (DC short, RF open) and can optionally be integrated with the matching networks. Synthesizing these circuits is the primary goal of amplifier design. Advanced topologies may include other circuitry such as feedback networks and harmonic terminations for wider bandwidths or higher efficiency.

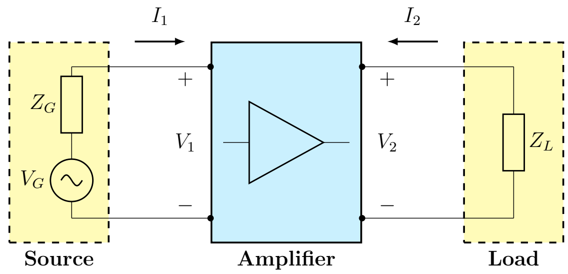
Chapter 2

Linear Amplifier Design

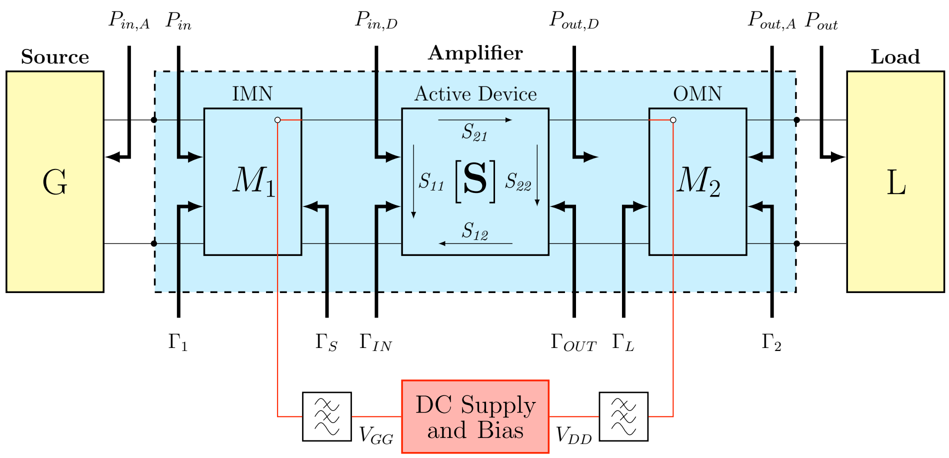
The ideas of linear amplifier design are the foundation for all types of amplifiers. In this chapter, the assumption is that the amplifier will work in the small-signal domain at the operating point and the device behavior can be approximated to the Scattering (S)-parameters. Note that the device behavior is dependent on the bias. Hence, the S-parameters will only be valid for the given bias point, and it is important to consider this when designing, simulating, and measuring amplifiers. Foundries and manufacturers can provide device models that can include effects of bias and non-linear response for circuit simulators such as Keysight Advanced Design System (ADS). The objectives of linear amplifier design are to bias the selected device and synthesize the matching networks for maximum gain while maintaining stability.

2.1 Gain Definitions

As the device is decomposed, we can consider the incident and transmitted power between each stage. This naturally begs the question of how the gain is defined when there are different power levels? For example, is the input power the one available at the source or available to the active device? There are different definitions of gain that will help during the design and analysis of linear amplifiers. Fig. 2.1 and Table 2.1 defined the incident and transmitted power at each stage. There are some special cases between these powers presented in Table 2.2 when the matching networks are conjugately matched or lossless.



(a) Amplifier circuit excluding DC supply and bias.



(b) Topology including DC supply (V_{DD}), DC bias (V_{GG}), input matching network (IMN) and output matching network (OMN).

Figure 2.1: Generic RF amplifier topology.

Table 2.1: Power parameters used in gain definitions

Power	Description
$P_{in,A}$	Available input power from the source. $P_{in} \leq P_{in,A}$.
P_{in}	Actual input power delivered to the amplifier.
$P_{in,D}$	Actual input power delivered to the device. $P_{in,D} \leq P_{in}$.
$P_{out,D}$	Available device output power of the device.
$P_{out,A}$	Available amplifier output power. $P_{out,A} \leq P_{out,D}$.
P_{out}	Actual output power delivered to load. Amplifier output power. $P_{out} \leq P_{out,A}$.

Table 2.2: Special relations for power parameters.

	Conjugate Matching	**Lossless Network**
Input	$P_{in} = P_{in,A}$	$P_{in} = P_{in,D}$
Output	$P_L = P_{out,A}$	$P_{out,A} = P_{out,D}$

> **Note.** The matching networks are generally assumed to be passive ($\Gamma \leq 1$) in these notes. This is not necessarily the case, for example, in oscillators.

2.1.1 Power Gain

Before defining the gain in terms of S-parameters, recall the definition of reflection coefficients for two-port networks in (2.1). The input and output reflection coefficients Eqs. (2.1a) and (2.1b) of a device are calculated from the device S matrix along with the source and load reflection coefficients Γ_S and Γ_L. The source and load reflection coefficients are the normalized impedances presented to the device by the matching networks. These are the source and load impedances transformed by the respective matching network.

A common misconception is that matching the device is achieved by presenting it with the system impedance $Z_S = Z_L = Z_0$. This results in $\Gamma_S = \Gamma_L = 0$ which leads to $G_T = |S_{21}|^2$. Although it seems like the best gain, it generally much lower than what can be achieved with matching networks since the input and output impedances of the device are not Z_0. The matching networks have to be chosen so that they present the right impedance to match the device. For output conjugate matching, $\Gamma_L = \Gamma^*_{OUT}$ while for the input conjugate matching, $\Gamma_S = \Gamma^*_{IN}$.

$$\Gamma_{IN} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \tag{2.1a}$$

$$\Gamma_{OUT} = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} \tag{2.1b}$$

The following gain definitions help characterize the performance of an amplifier and guide the design.

Definition 1. System Gain (Actual Power Gain)

Power delivered to the load relative to the power delivered by the source.

$$G = \frac{P_L}{P_{in}} = |S_{21}|^2 \frac{\left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{IN}|^2\right) |1 - S_{22}\Gamma_L|^2} \tag{2.2}$$

Definition 2. Transducer Gain

Power delivered to the load relative to the available power from the source.

$$G_T = \frac{P_L}{P_{in,A}} = |S_{21}|^2 \frac{\left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_{IN}\Gamma_S|^2 |1 - S_{22}\Gamma_L|^2} \tag{2.3}$$

This can be considered the actual gain of the amplifier and is a key metric as it will be discussed in the following sections. It takes into account how much of the input power was amplified. In the ideal case, needs optimum and lossless input & output matching.

Definition 3. Power Gain

Power delivered to the load relative to the power delivered to the device. This is with optimum (conjugate) input matching.

$$G_P = \frac{P_L}{P_{in,D}} = |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2 \left(1 - |\Gamma_{IN}|^2\right)} \tag{2.4}$$

Definition 4. Available Power Gain

Power available to the load relative to the input power available from the source. This is with optimum (conjugate) output matching.

$$G_A = \frac{P_{out,A}}{P_{in,A}} = |S_{21}|^2 \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{OUT}|^2\right)} \tag{2.5}$$

Since active devices such as transistors are bilateral ($S_{21} > 0$, $S_{12} \neq 0$), the input reflection coefficient is dependent on the load reflection coefficient and the output reflection coefficient is dependent on the source reflection coefficient. For example, if $\Gamma_S = \Gamma^*_{OUT}$ is selected and an input matching network is synthesized, Γ_{OUT} will be shifted. If then, the output matching network is synthesized such that $\Gamma_L = \Gamma^*_{OUT}$, the selected Γ_L will shift the target Γ_{IN}.

Synthesizing these networks is like tracking a moving target as Γ_{IN} is dependent on Γ_L and Γ_{OUT} is dependent on Γ_S. Another reason that makes active microwave circuit design difficult is that both input and output matching networks have to be designed simultaneously.

2.1.2 Unilateral Gain

A unilateral device ($S_{21} > 0$, $S_{12} = 0$), would greatly simplify the design as it decouples the relation between input-load and output-source reflection coefficients. In this case, the input and output target reflection coefficients are reduced to $\Gamma_S = S^*_{11}$ and $\Gamma_L = S^*_{22}$ becoming a 'cookbook' recipe. Although zero reverse gain might not be possible in practice, we can generally approximate a transistor as a unilateral device when the loop gain $S_{21}S_{12}$ is small and the relation between the input and output becomes negligible. The following gain definitions are helpful when designing an amplifier, including device comparison and selection.

Definition 5. Unilateral Transducer Gain
Transducer gain for a unilateral device.

$$G_{TU} = \left(\frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S S_{11}|^2}\right) |S_{21}|^2 \left(\frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L S_{22}|^2}\right) \tag{2.6}$$

The unilateral transducer gain can also be decomposed as three independent gain terms in the form

$$G_{TU} = G_S G_o G_L \tag{2.7}$$

where,

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S S_{11}|^2} \tag{2.8a}$$

$$G_o = |S_{21}|^2 \tag{2.8b}$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L S_{22}|^2} \tag{2.8c}$$

Definition 6. Maximum Unilateral Transducer Gain

Maximum transducer gain for a unilateral device if presented with conjugate matching such that $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$.

$$G_{TU,max} = |S_{21}|^2 \left(\frac{1}{1 - |S_{11}|^2}\right)\left(\frac{1}{1 - |S_{22}|^2}\right) \tag{2.9}$$

This is a FoM to compare the maximum power gain that can be achieved by a device.

Definition 7. Unilateral Power Gain

Power gain for a unilateral device.

$$U = \frac{\left|\dfrac{S_{21}}{S_{12}} - 1\right|^2}{2k\left|\dfrac{S_{21}}{S_{12}}\right| - 2\Re\left(\dfrac{S_{21}}{S_{12}}\right)} = \frac{|S_{12}S_{21}S_{11}S_{22}|}{\left(1 - |S_{11}|^2\right)\left(1 - |S_{22}|^2\right)} \tag{2.10}$$

where, k is Rollet's stability factor.

This is a useful FoM to consider as it is difficult to get useful gain at frequencies beyond $U \leq 1$.

$U = G_{MA}$ for $S_{12} = 0$ (see bilateral case for the definition of G_{MA}). A unilateral device can be achieved with feedback networks.

2.1.3 Bilateral Gain

In the bilateral case ($S_{21} > 0$, $S_{12} \neq 0$), a device is conjugately matching to obtain maximum power transfer as $\Gamma_S = \Gamma_{IN}^*$ and $\Gamma_L = \Gamma_{OUT}^*$. By solving the system of equations and calculating the transducer gain, we can obtain the following gain definitions [3].

Definition 8. Maximum Available Power Gain

Maximum transducer gain in a bilateral device with input and output conjugately matched.

$$G_{MA} = G_{T,max} = \left|\frac{S_{21}}{S_{12}}\right| \left(k - \sqrt{k^2 - 1}\right), \quad k \geq 1 \tag{2.11}$$

Corollary. Rollet's Stability Factor

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \tag{2.12a}$$

$$\text{where, } \Delta = S_{11}S_{22} - S_{21}S_{12} \tag{2.12b}$$

A necessary and sufficient condition for unconditional stability is $k > 1$ and $|\Delta| < 1$. This will be discussed further in Section 2.2.2.

To achieve G_{MA}, the input and output matching networks need to be conjugately matched and the stability condition needs to be met. Note that Rollet's stability factor is frequency dependent. It can be stable at some frequencies and unstable at others. It is important that both conditions need to be met simultaneously to satisfy the stability condition.

Note that if the Rollet's stability criterion is not met ($k < 1$), stable gain can be achieved, but the input and output networks cannot be simultaneously optimal. When the amplifier is potentially unstable, stability can be ensured by adding loss in the form of shunt admittance or a series resistance to the device so that the augmented device has $k = 1$. This is considered the maximum stable gain. An alternative is to purposely mismatch one of the matching networks to achieve maximum stable gain instead of maximum available gain.

Definition 9. Maximum Stable Gain
Maximum stable transducer gain achieved with $G_{MA}\,|_{k=1}$.

$$G_{MS} = \left|\frac{S_{21}}{S_{12}}\right| \tag{2.13}$$

This is often the practical limit for unconditionally stable gain with moderate design effort.

Maximum Gain in RF Circuit Simulation

Simulation software such as Keysight ADS often combines G_{MA} and G_{MS} as

Definition 10.

$$G_{MAX} = \begin{cases} G_{MA}, & \text{if } k \geq 1 \\ G_{MS}, & \text{if } k < 1 \end{cases} \tag{2.14}$$

2.1.4 Gain Circles

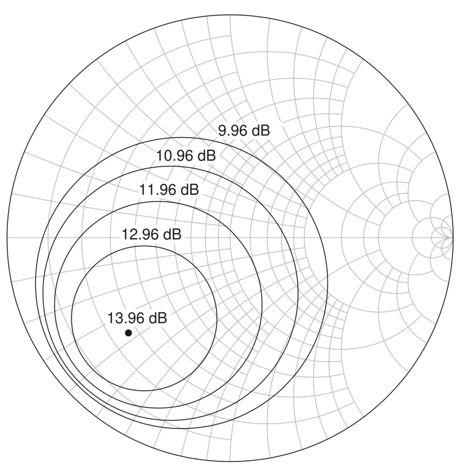


Figure 2.2: Example of available gain (G_A) circles with 1 dB step.

The definitions of gain are obtained from the magnitude of the reflection coefficients and S-parameters, which means they are complex valued by nature and can be plotted in the Γ plane (derivations are detailed in the literature). For example, the available gain G_A from (2.5) can be plotted in the Γ_S plane to obtain the contours that result in a constant gain. Fig. 2.2 shows an example of these input gain circles. The center of this family of circles defines the reflection coefficient Γ_S which will result in the maximum gain G_{MAX}. Then, the contours of Γ_S with decreasing gain are plotted. The same concept can be applied to the output Γ_L plane and power gain G_P in (2.4). There are dedicated tools to obtain these contours in simulation software such as Keysight ADS. Other techniques to obtain contours for different FoM will be explored later (source and load pulls).

TODO: ADD DEFINITIONS FOR SOURCE AND LOAD GAIN MISMATCH CIRCLES

TODO: MORE DETAILS ON LOAD/SOURCE PULLS

TODO: NOISE FIGURE CIRCLES

2.2 Stability Analysis

Any active circuit can potentially have a chaotic response in which the signal amplitude grows independently of the input. Stability is one of the most important considerations in amplifier design as it needs to be ensured for all frequencies, load impedances, and source impedances. Oscillator design is not as trivial as designing an unstable amplifier because of this chaotic behavior, as they intend to generate a stable sinusoidal signal.

Stability analysis is tedious and difficult, discouraging rigorous analysis, promoting assumptions and shortcuts which lead to costly redesigns. When the analysis results in potential instability, one of two techniques follow. (1) Circuit techniques are used to stabilize the circuit by adding loss, which sacrifices performance. (2) The load and source impedance presented to the device are carefully chosen and controlled so that the device operates within the stable limits [4]. Even smart, talented, and experienced engineers have suffered from instability [5].

Gain monotonically decreases with frequency, causing lower frequencies to be unstable even with high frequency stability. The maximum stable gain indicates whether the target gain will be difficult or impossible to realize with the selected device. For example, broadband amplifiers may need to intentionally suppress lower frequency gain by introducing losses at these frequencies to ensure stability while maintaining the high frequency gain. This can be achieved, for example, with a parallel RC network in series with the amplifier input where the resistor introduces losses at low frequencies but is shorted by the capacitor at higher frequencies.

Oscillation can initiate when the signals reflected between the source and input (2.15a) or alternatively between the output and load (2.15b) increase in amplitude.

$$|\Gamma_S \Gamma_{IN}| > 1 \tag{2.15a}$$

$$|\Gamma_L \Gamma_{OUT}| > 1 \tag{2.15b}$$

Unconditional stability can be achieved for any source and load impedance when both oscillatory conditions (2.15b) are not met. By substituting (2.1) into (2.15b) we get the conditions for unconditional stability Eqs. (2.16a) and (2.16b) at the input and output, respectively [2].

$$|\Gamma_S \Gamma_{IN}| = \left| \Gamma_S S_{11} + \frac{S_{21} S_{12} \Gamma_S \Gamma_L}{1 - S_{22} \Gamma_L} \right| < 1 \tag{2.16a}$$

$$|\Gamma_L \Gamma_{OUT}| = \left| \Gamma_L S_{22} + \frac{S_{21} S_{12} \Gamma_S \Gamma_L}{1 - S_{11} \Gamma_S} \right| < 1 \tag{2.16b}$$

For a passive source and load, $\Gamma_S \leq 1$ and $\Gamma_L \leq 1$, so the stability conditions can be reduced to $\Gamma_{IN} \leq 1$ and $\Gamma_{OUT} \leq 1$. However, this is not always the case as many amplifiers have multiple stages. In this case, stability analysis is difficult as multiple feedback paths exist. Ensuring each stage is stable using the methods presented in these notes are only an approximation of stability as often feedback paths are ignored and some source or load impedances could have $|\Gamma| > 1$ between stages. As the S parameters are fixed by the device and particular frequency, the remaining factors Γ_S and Γ_L presented by the matching networks determine stability.

> **Note.** The following subsections present different perspectives to analyze stability. Each metric tells a story, although each one can guarantee unconditional stability on its own, they are stronger together. Operating closer to the limits of stability of a device will result in better performance but require smarter engineers like you.

2.2.1 Stability Circles and Regions

The result of solving the stability conditions for the source and load reflection coefficients assuming a passive source and load are Eqs. (2.17a) and (2.17b). These equations are written in the form of circles of reflection coefficients.

$$\left|\Gamma_S - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| \tag{2.17a}$$

$$\left|\Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \tag{2.17b}$$

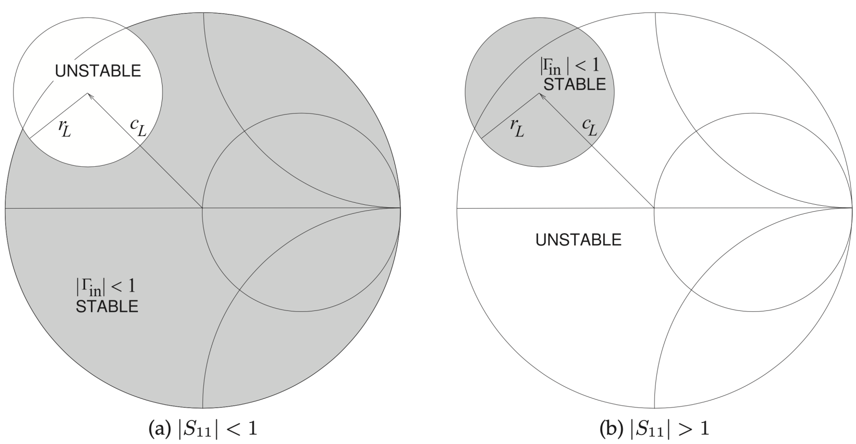


Figure 2.3: Stability circles on the Γ plane where the shaded region indicates the values that result in unconditional stability. Taken from [2].

Definition 11. Stability Circles

Stability circles are the potentially unstable boundaries that divide the stable and unstable regions of the source and load.

Source stability circles are defined as

$$|\Gamma_S - c_S| = r_S \tag{2.18}$$

where, the circle center c_S and radius r_S are defined as

$$c_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \tag{2.19a}$$

$$r_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|. \tag{2.19b}$$

Similarly, load stability circles are defined as

$$|\Gamma_L - c_L| = r_L \tag{2.20}$$

where, the circle center c_L and radius r_L are defined as

$$\text{Load Center: } c_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \tag{2.21a}$$

$$\text{Load Radius: } r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \tag{2.21b}$$

Recall (2.12b), $\Delta = S_{11}S_{22} - S_{21}S_{12}$.

Corollary. Stability Regions

The region resulting in unconditional stability will be defined depending on the magnitude of the input and output return losses, while the complementary region will be unstable.

$$\text{Source stable region: } \begin{cases} |\Gamma_S - c_S| > r_S, & \text{if } |S_{11}| < 1 \\ |\Gamma_S - c_S| < r_S, & \text{if } |S_{11}| > 1 \end{cases} \tag{2.22a}$$

$$\text{Load stable region: } \begin{cases} |\Gamma_L - c_L| > r_L, & \text{if } |S_{22}| < 1 \\ |\Gamma_L - c_L| < r_L, & \text{if } |S_{22}| > 1 \end{cases} \tag{2.22b}$$

To add some perspective, recall the Smith chart plots all passive impedances for $|\Gamma| \leq 1$ (i.e., positive resistances only: $\Re(Z) \geq 0$). When the stability circles and regions are overlaid on the Smith chart, they indicate which source Γ_S (i.e., normalized source impedance) or load Γ_L (i.e., normalized load impedance) causes potential instability at the input or output, respectively. Instability can still be caused with feedback networks as those could potentially have negative resistances (i.e., active circuits).

2.2.2 Rollet's Stability Criterion: k-Δ factors

Rollet's stability factor is derived by applying the stability conditions to the generic amplifier topology with scattering matrix S' in Fig. 2.1. Unconditional stability is achieved when $|S'_{11}| < 1$ for all passive source impedances (i.e., $|\Gamma_S| \leq 1$) and $|S'_{22}| < 1$ for all passive load impedances (i.e., $|\Gamma_L| \leq 1$). This conditions are interpreted as the source and load impedance having a positive resistance.

Definition 12. Rollet's Stability Criterion

The Rollet's stability factor for an amplifier with passive source and load is defined as

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} > 1. \tag{2.23}$$

$k > 1$ is a necessary but not sufficient condition for unconditional stability as an auxiliary condition needs to be satisfied simultaneously. A common auxiliary condition is (2.24), where Δ is the determinant of S'.

$$|\Delta| = |S_{11}S_{22} - S_{21}S_{12}| < 1 \tag{2.24}$$

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Corollary. Rollet's Alternate Auxiliary Conditions
One of the following auxiliary conditions can be used in lieu of $|\Delta| < 1$.

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \tag{2.25a}$$

$$B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\Delta|^2 > 0 \tag{2.25b}$$

$$C_1 = 1 - |S_{11}|^2 - |S_{12}S_{21}|^2 > 0 \tag{2.25c}$$

$$C_2 = 1 - |S_{22}|^2 - |S_{12}S_{21}|^2 > 0 \tag{2.25d}$$

The magnitude of k indicates the difficulty of achieving an unconditionally stable design. If $k \gg 1$, the design is straightforward. Achieving a stable design will become more difficult as $k \to 1$. If $k \leq 1$, the design is difficult and potentially unstable. In this last case, $k < 1$ does not indicate that the amplifier will be unstable, but rather that the source and load impedance presened by the matching networks have to be carefully selected considering the stability regions. This will however, limit the source and load impedances that could be presented to the amplifier as some could lead to the unstable region. When these impedances can be controlled to stay in the stable region, a potentially unstable amplifier can operate closer to the device ultimate performance limits, beyond G_{MS} but below $G_{TU,max}$.

2.2.3 Edwards–Sinsky Stability Criterion: μ factors

Edwards and Sinsky developed a measure of the relative stability that can be used to compare different designs. This test measures the distance of the nearest potentially unstable point from the stability circles to the origin on the Γ plane (i.e., center of the Smith chart). Larger values of the μ stability factor, also called geometric stability factor, indicate greater stability for a given frequency. This criterion also assumes a passive source and load termination (i.e., $|\Gamma_S| \leq 1$ and $|\Gamma_L| \leq 1$).

Definition 13. Edwards-Sinsky Stability Criterion

The μ-factor stability criterion and the dual parameter μ' are defined with unconditional stability as

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta| + |S_{12} S_{21}|} > 1 \tag{2.26a}$$

$$\mu' = \frac{1 - |S_{22}|^2}{|S_{11} - S_{22}^* \Delta| + |S_{12} S_{21}|} > 1 \tag{2.26b}$$

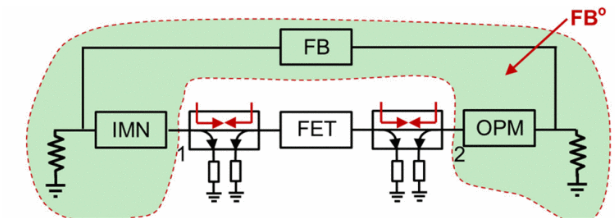
μ and μ' measure the input and output stability, respectively.

In contrast to Rollet's k-Δ test, the μ factor does not need an auxiliary condition to guarantee unconditional stability — a single metric indicates the grade of stability. Additionally, the μ and μ' factors measure the relative stability of the input and output, which helps to isolate the network causing potential instability.

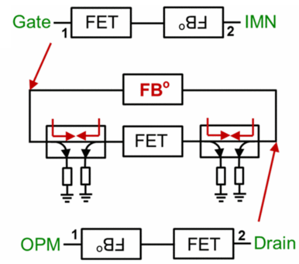
2.2.4 Next-Gen Analysis: Winslow S-Probe

Circuits used to be stabilized in the lab empirically by adding loss until the device became stable. Modern designs are increasingly complex and this is difficult to achieve. The classical Rollet's stability criterion is based on some assumptions that may not always hold true and is limited to two-port devices treated as black boxes. Modern radios cascade multiple amplifiers which could lead to the terminations with $|\Gamma| > 1$. To solve this, a generalized approach was recently developed with the Winslow S-Probe (WS-Probe) being the cornerstone of this new perspective [6]. This non-invasive probe quickly and efficiently derives stability measures in small- and large-signal circuits.

Fig. 2.4 shows the proposed simulation setup, where two bi-directional couplers are inserted on the active device source and load. The ideal couplers sample and inject signals between the nodes they are placed. This topology divides the amplifier into two major sections, (1) the device



(a) Proposed topology.



(b) Reduced topology.

Figure 2.4: Winslow S-Probes. Taken from [6].

itself and (2) the augmented feedback network FB° which encompasses the feedback network (FB), input matching network (IMN), output matching network (OMN), source, and load. The feedback network can include any intentional and unintentional feedback paths, which may include other active devices or have their own instabilities.

The probes obtain the in-situ bidirectional admittances at the gate and drain nodes by mathematically opening and closing the feedback loop using the flipped S-parameters of FB°. From these, the transfer functions can be derived and analyzed following the Kurokawa approach. The mathematical derivations, integration, and use in Keysight ADS can be found in the literature [5, 6, 7].

2.3 Noise Figure Circles

2.4 Efficiency

Amplifier efficiency measures the amount of DC power P_{DC} that is converted into useful RF power $P_{RF,out}$. This metric is important for high-power amplifiers, as increasing the gain leads to lower power consumption to achieve a given output power. In cascaded systems, the stage with the highest power dominates the efficiency of the entire cascade. Some metrics consider the input RF power $P_{RF,in}$ or the corresponding gain as this portion of the output RF power was not generated but rather provided to the device.

Definition 14. **Drain Efficiency (DE)**

Ratio of output RF power to consumed DC power.

$$\eta_D = \frac{P_{RF,out}}{P_{DC}} \tag{2.27}$$

The term 'drain' can be replaced by 'collector' in BJTs and HBTs.

Definition 15. **Power Added Efficiency (PAE)**

Ratio of generated RF power to consumed DC power.

$$\eta_{PAE} = \frac{P_{RF,out} - P_{RF,in}}{P_{DC}} \tag{2.28}$$

Definition 16. **Total Efficiency**

Ratio of output to input power, considered an alternate version of PAE. Also referred to as overall efficiency.

$$\eta_{total} = \frac{P_{RF,out}}{P_{DC} + P_{RF,in}} \tag{2.29}$$

Remark. **Average efficiency** considers the average of the time-varying levels of modulated signals when calculating the FoM.

When the amplifier has sufficient power gain, all the efficiencies will approach the same value η. Table 2.3 compares different efficiencies assuming fixed output RF and input DC powers.

Waste factor was recently introduced as a generalized efficiency metric for cascaded systems analogous to noise factor [8]. This is a FoM that measures the added power that is wasted by the cascade given a signal with input power P_{in}.

Definition 17. **Waste Factor**

The waste factor (or waste figure WF in dB) is defined as

$$W = \frac{1}{\eta_W} = \frac{P_{consumed}}{P_{out}} = \frac{P_{non-signal} + P_{out}}{P_{out}} \tag{2.30}$$

Table 2.3: Efficiency in relation to gain for ideal (Class-A) linear amplifier.

Gain (dB)	η_D	η_{PAE}	η_{total}
3	40%	25%	50%
6	44%	37%	50%
10	48%	45%	50%
15	49%	48%	50%
20	50%	50%	50%

where, $P_{consumed}$ is the total power consumed, P_{out} is the output signal power, and $P_{non-signal}$ is the total power consumed minus the output signal power (i.e., the wasted power).

The waste figure in dB is defined as

$$WF = 10\log_{10}(W) \tag{2.31}$$

The amplifier metrics can be converted to W as

$$W = \frac{1}{\eta_{total}} \tag{2.32}$$

$$W = \frac{1}{\eta_{PAE}}\left(1 + \frac{P_{RF,in}}{P_{DC}}\right)\left(1 - \frac{1}{G}\right) \tag{2.33}$$

where, G is the device gain.

The waste factor of a cascade with N stages with corresponding waste factors W_n and gain G_n $(n = 1, 2, ..., N)$ can be calculated as

$$W_{casc} = W_N + \frac{W_{N-1} - 1}{G_N} + \frac{W_{N-2} - 1}{G_N G_{N-1}} + ... + \frac{W_1 - 1}{\prod_{n=2}^{N} G_n} \tag{2.34}$$

An amplifier with 100% efficiency will result in a waste factor $W = 1$ ($WF = 0$dB) and will increase as the amplifier efficiency drops. If the power is consumed by the device (or cascade), the waste factor will be ∞. The wasted power for a device (or cascade) can be calculated as

$$P_{wasted} = (W - 1)P_{out}. \tag{2.35}$$

2.5 Matching and Biasing Networks

Different techniques can be used to bias the device. Two important considerations are that there is DC continuity between the DC feed and the transistor ports and that there is no DC short circuit for the supply. Some valid techniques are shown below.

$\lambda/4$ Transformer

TODO: ADD SCHEMATIC FOR COMMON BIASING NEWORKS

The capacitor and DC supply are effectively an RF short circuit (given that the capacitance has a reactance $> Z_0/10$). Hence, a $\lambda/4$ (90° electrical length) short-circuited stub will look like an open circuit at the transistor pin and should not change the impedance of the device at the center frequency. The capacitor filters out supply noise and provides an alternate RF short to prevent leaked RF from going to the supply. Alternatively, if you have a short-circuited stub in your matching network, you can terminate it like this and use the length you need to provide both matching and DC bias. The disadvantage of transmission lines is that they are limited to small fractional bandwidths. But that is okay for this project since we have a narrowband design. Be careful if you have an additional shorted stub, as it will short-circuit the supply to ground. In real life, the supply will drop the voltage since it cannot provide the infinite current needed to maintain the node voltage. You could add a DC block capacitor with a large reactance (similar to the next example) to prevent a DC short. Essentially, don't use a shorted stub like TL7 (which is disabled in this diagram):

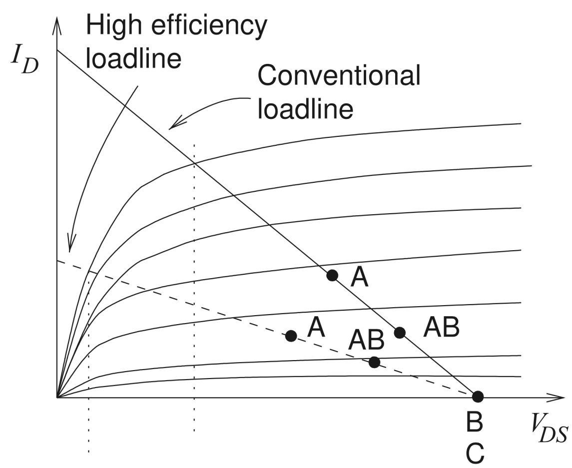
LC Bias Tee

You could use an inductor instead of a $\lambda/4$ transformer if the reactance is large ($> 10Z_0$). Keep in mind, at high frequencies, inductors have low Q and low self-resonance frequency, which will be hard to design. These are more common in sub-3 GHz designs. The series capacitor that follows will block any external DC. More importantly, if you have a shorted stub, this capacitor will provide a DC open circuit, so it doesn't short out the supply. If the matching network has this topology, you could use the values for matching as your elements and provide both matching and biasing through the same network.

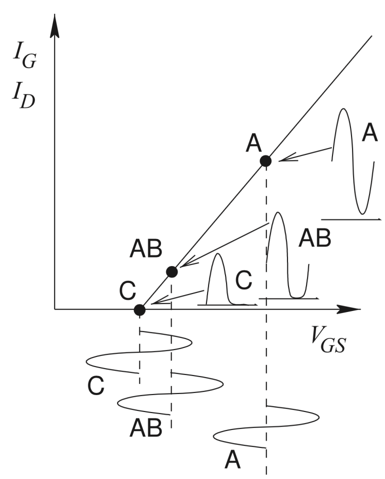
Power delivery network (PDN)

Decoupling capacitors are used to remove noise from the power supply and provide a stable DC voltage by shorting the high-frequency noise. Due to the parasitic resistance and inductance of lumped capacitors, a network of shunt capacitors with increasing capacitance and different packages is used between the DC supply connector and the DC feed point. The decoupling network can be thought of as a low-pass filter (LPF), and multiple shunt capacitors provide a broadband short circuit that could not be achieved from the parasitics of a single capacitor. A small current loop (ground vias placed close to the capacitor and the device) is desired.

2.6 Classes of Operation



(a) Output transfer characteristics



(b) Input transfer characteristics

Figure 2.5: Current-Voltage (I-V) characteristics of a transistor showing the quiescent point for conduction mode operation.

After an appropriate active device has been chosen, selecting the DC bias is the first step in the amplifier design workflow. It will fix the small signal behavior (i.e., S-parameters) of the device and mode of operation. The input and output transfer current-voltage (I-V) characteristics are obtained by plotting the output current I_D against V_{GS} for the input and against V_{DS} for the output transfer characteristics. Depending on the selection of the supply voltage V_{DS} and the bias voltage V_{GS}, the device will produce a DC current output known as quiescent current $I_{DD,Q}$. The point on the IV characteristics that corresponds to the combination of parameters is called the quiescent point, which determines the class of operation. In practice, biasing to achieve the desired quiescent current is more accurate than specifying a fixed gate voltage (i.e., use V_{GG} to get the desired $I_{DD,Q}$).

In linear conduction mode classes of operation, the higher order harmonics are shorted while the device amplifies a certain portion (conduction angle) of the signal. In Class-A, the bias is set to have a quiescent current in the midpoint of the linear input characteristics. Both the negative and positive parts of the signal will receive the same amplification before clipping. In Class-B, the quiescent current is at the 'knee' or start of the linear region. Here, the positive half of the signal is amplifier while the negative half is clipped, this is a non-linear behavior. In Class-AB, the bias is selected between Class-A and B as a tradeoff between linearity and power consumption. Any point between the mid-point and start of the linear region is categorized as Class AB. In Class-C, the bias is lower than the knee, the signal needs a large amplitude to be amplified, as a sort of peaking behavior.

Class	Description	Conduction Angle
A	Full cycle conduction	$\theta = 360°$
AB	More than half-cycle conduction	$180° < \theta < 360°$
B	Half cycle conduction	$\theta = 180°$
C	Less than half cycle conduction	$\theta < 180°$

Table 2.4: Conduction mode classes of operation

Linear behavior is desired in low noise amplifiers (LNA) hence, the conduction mode Class-A and -AB are preferred. Another category used in power amplifiers (PAs) are the switching mode classes of operation, which strategically terminate the harmonics and treat the device as a switch. As a result, they have worse linearity and better efficiency.

Chapter 3

Power Amplifiers

3.1 Load Pull

A load pull (or source pull) consists of a simulation or test where the termination impedance is swept at a set of points to determine the FoM of a particular device. In power amplifiers (PAs), output power and efficiency are the most important FoM. The target reflection coefficients Γ_S or Γ_L depend on the amplifier type and design goals. It is important to understand there is not a single impedance to match the amplifier but rather a continuous domain with performance tradeoffs.

3.2 Examples

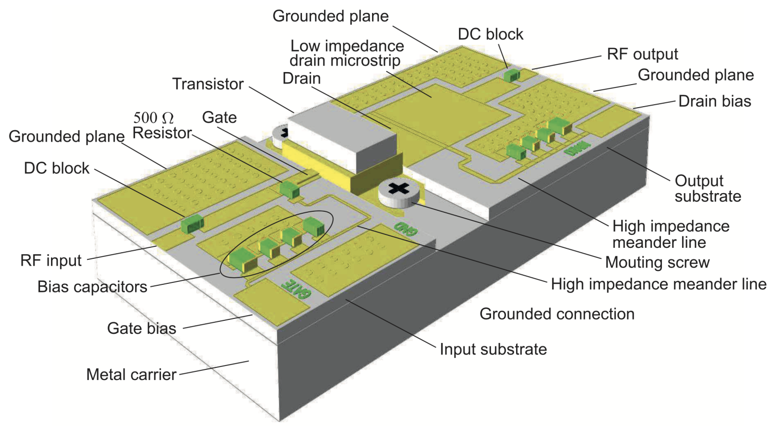
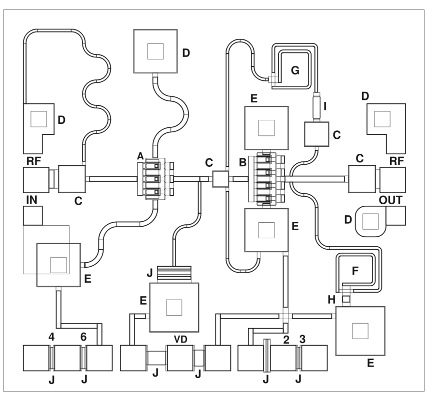
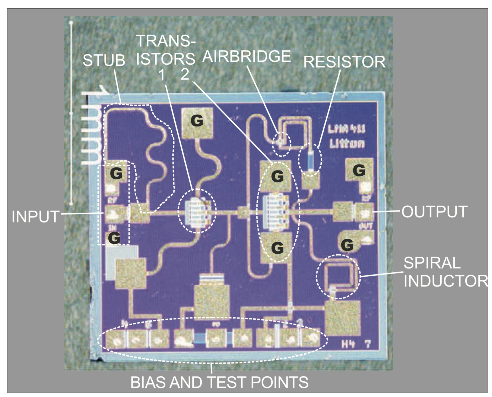


Figure 3.1: WiMax power amplifier for 3.4 GHz to 3.8 GHz. Taken from [2].



(a) Layout



(b) Micrograph

Figure 3.2: X-Band (8 - 12 GHz) MMIC power amplifier with (A,B) pHEMT devices, (C) DC blocking capacitors, (E) RF short capacitor, (F) RF choke inductor, (G) feedback inductor, (I,J) feedback and bias resistors. Taken from [2].

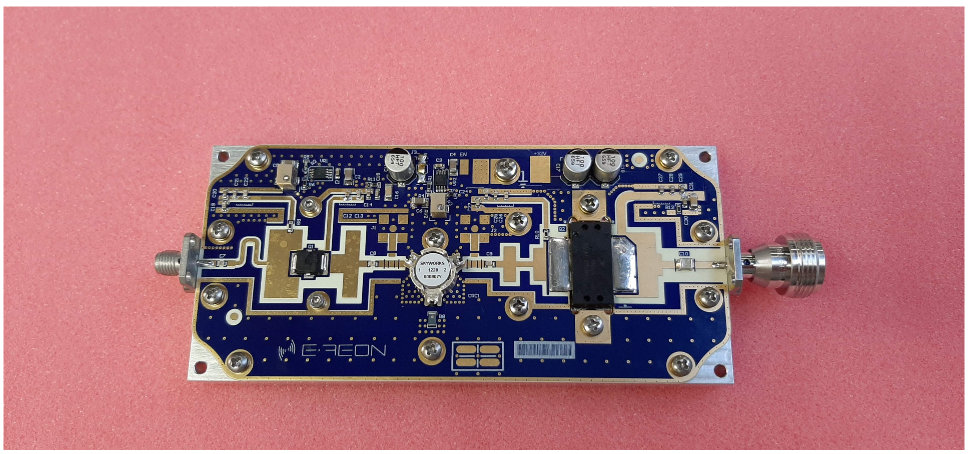
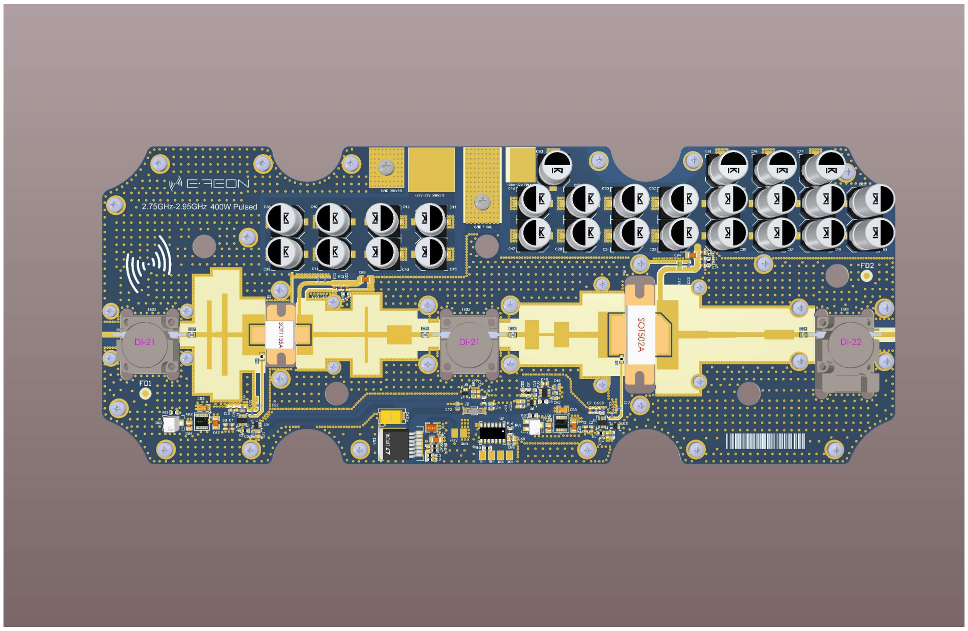
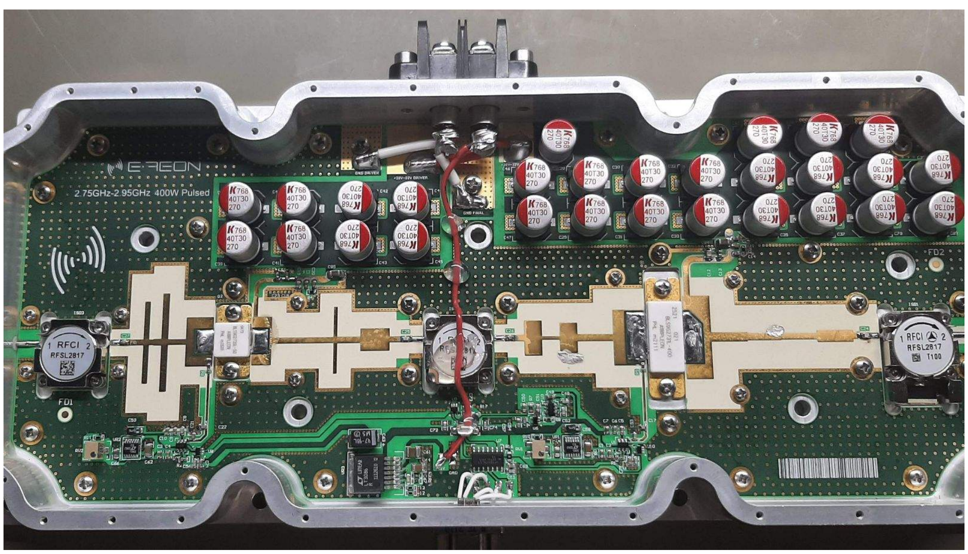


Figure 3.3: E-REON PowerBlast 300: 2.45 GHz ISM 250 W LDMOS amplifier with 33 dB gain and 60% efficiency [9].



(a) Design and Layout



(b) Implementation

Figure 3.4: E-REON 2.856 GHz 400 W solid state power amplifier unit used as to drive a Klystron tube in a medical MRI [9].

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