Dynamic PA power mode DPD on base of 2-dimensional Chebyshev polynomials

Alexander Degtyarev
MIPT
Moscow, Russia
degtyarev.aa@phystech.edu
0009-0009-5386-8034

Nikita Bakholdin
MIPT
Moscow, Russia
bakholdin.nikita@gmail.com
0000-0003-3341-3074

Anton Sushko
MIPT
Moscow, Russia
a.sushko2000@gmail.com
0009-0006-9243-2274

Sergei Bakhurin
MIPT
Moscow, Russia
bakhurin.sa@mipt.ru
0009-0004-5772-2554

***Abstract*—This paper addresses the challenge of power amplifier (PA) behavioral modeling in scenarios with non-stationary dynamically varying PA output power. Since different PA power modes correspond to distinct nonlinearities, conventional behavioral models, such as the Generalized Memory Polynomial (GMP) or Chebyshev polynomials, require extensions to account for the characteristics of different PA power modes. In this work, a two-dimensional Chebyshev polynomial-based model is employed to capture nonlinear distortion across a wide range of PA output power levels, from 0.069 W to 0.912 W, and to predict nonlinear behavior for intermediate power modes. The proposed model achieves high performance, Adjacent Channel Leakage Ratio (ACLR) less than -45 dB for the considered PA power modes. Furthermore, the model demonstrates up to a 14 dB improvement in performance compared to a model with the same number of parameters that does not consider PA power mode information.**

***Index Terms*—digital pre-distortion (DPD), Chebyshev polynomials, power amplifier dynamic mode, least squares method (LS), adjacent channel leakage ratio (ACLR)**

I. Introduction

Power amplifiers (PAs) generate nonlinearities that significantly impact the quality of radio frequency signals, reducing the performance of the transmitted signal. The distortions caused by PAs can be categorized as in-band and out-of-band interferences. Without a linearization method, these distortions result in an increased bit error rate (BER) at the receiver, generate interference signals and degrade the transmission quality of adjacent band signals. In order to prevent a distortions in modern base stations and cellular devices digital pre-distortion (DPD) techniques widely used [1]–[3].

The DPD device is introduced by a block with an inverse nonlinear characteristic, that changes the PA input signal in order to minimize non-linear distortion at the output of PA (fig. 1).

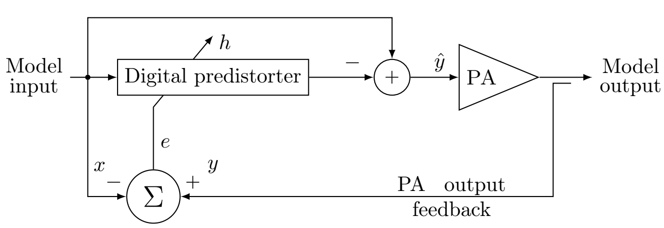


Fig. 1: DPD structure

Thus, proper implementation can increases PA usage efficiency, due to it can be exploited in highly non-linear modes [1].

Digital pre-distortion optimization task can be expressed with mathematical equation [4], which is in fact describes equalization of DPD input and PA output in block-wise mode:

$$||\mathrm{PA}(\boldsymbol{x} - \mathrm{DPD}(\boldsymbol{x}, \boldsymbol{h})) - \boldsymbol{x}||_2^2 \to \min_{\boldsymbol{h}}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$ – PA input signal vector, $\boldsymbol{h} \in \mathbb{C}^{P\times 1}$ – DPD model parameters vector, N – block length. Consider, that PA non-linear function $\mathrm{PA}(\boldsymbol{x})$ could be linearized within the vicinity of PA input:

$$\mathrm{PA}(\boldsymbol{x} - \mathrm{DPD}(\boldsymbol{x}, \boldsymbol{h})) \approx \mathrm{PA}(\boldsymbol{x}) - (D_{\boldsymbol{x}}\mathrm{PA}(\boldsymbol{x}))\mathrm{DPD}(\boldsymbol{x}, \boldsymbol{h}), \tag{2}$$

where $D_{\boldsymbol{x}}\mathrm{PA}(\boldsymbol{x}) \in \mathbb{C}^{\mathbb{N}\times\mathbb{N}}$ – PA output derivative w.r.t. the PA input. Since DPD must operate in such a mode of the PA that the level of nonlinear distortions remains significantly lower than the transmitter signal, then $D_{\boldsymbol{x}}\mathrm{PA}(\boldsymbol{x}) \approx \boldsymbol{I}$ – identity matrix. Thus, substituting (2) into (1) we derive expression:

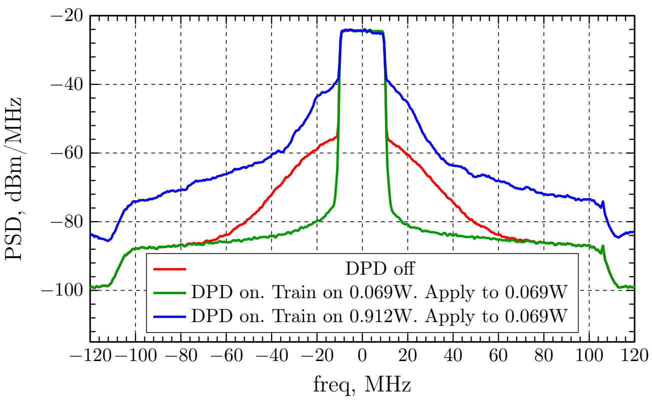
$$||\mathrm{DPD}(\boldsymbol{x}, \boldsymbol{h}) - \boldsymbol{e}||_2^2 \to \min_{\boldsymbol{h}}, \tag{3}$$

where $\boldsymbol{e} = \mathrm{PA}(\boldsymbol{x}) - \boldsymbol{x}$ – error vector. Current optimization problem (3) is commonly considered in papers dedicated to DPD task [2]–[7]. Mentioned task usually solved by LS, RLS, LMS algorithms [8], [9] etc. In the proposed letter we exploit LS method for single-layer models training, due to it achieves quadratic loss function global optimum in one optimization step.

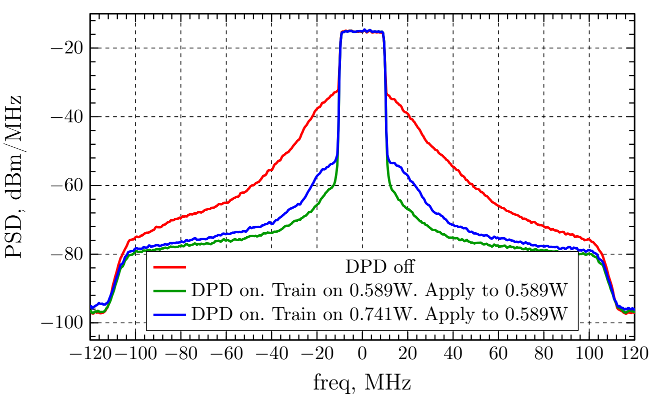
In the communication systems in order to meet certain requirements, resource blocks (RBs) in a data frame may be dynamically allocated according to real-time traffic, which can result in short time period power level changes thus reducing PA efficiency. This fact contradicts the assumption that DPD should operate in a stationary mode. Such problem was researched in [10].

On the other hand, it is necessary to significantly vary the input power levels of the PA in order to improve power efficiencies. To compensate the distortions caused by power changes DPD must be re-calibrated in real-time, which is not often economically justified in practice.

Moreover, transient processes during the switching between power modes of the PA result in distortions that are transmitted through the communication channel, causing brief communication protocol violations and degrading the spectral mask. This leads to a performance degradation when attempting to apply model parameters trained in one PA power mode to signals, corresponding to another mode, which is shown in fig. 2.



(a) Apply 1D model. Train on 0.912 W case, test on 0.069 W case



(b) Apply 1D model. Train on 0.741 W case, test on 0.589 W case

Fig. 2: PSD of PA output. Dynamic PA mode non-stationarity illustration

In fig. 2a parameters of single-dimensinal polynomial, trained on 0.069 W (green), 0.912 W (blue) cases and both applied to 0.069 W case. One can observe up to 32 dB ACLR degradation because of different PA behavior for chosen power modes. However, in case of model training on close PA output power modes 0.598 W (green) and 0.714 W (blue) (fig. 2b) performance degradation is 7.5 dB. Nevertheless, in real applications, this is still not permissible.

As a result, increased non-linear distortions may propagate through the communication channel while adjustment of the parameter. Consequently, a more favorable approach is the prediction of non-linear distortions across all operating modes of the amplifier, rather than tracking power modes.

In order to predict non-linear distortions caused by various PA power modes authors primarily propose different approaches of parameters re-calculation relatively to the those chosen as a reference.

For instance, the paper [11] suggests to divide trainable model parameters into static and dynamic part, where dynamic parameters are switched in accordance with PA power mode. The work [3] extends the concept suggested in [11] by application of power adaptive decomposed vector rotation (PDVR) model.

Another remarkable approach is introduced by AI-based methods application [6], [7]. As an example, the paper [6] suggest to use artificial neural network in order to gather information corresponding to different non-linearities and transform general memory polynomial (GMP) model parameters into those corresponding to the target power, i.e. predicting PA non-linearity related to the certain power mode.

In current article we introduce another dynamic PA power accounting method which is based on 2-dimensional Chebyshev polynomial offline training. In fact it represents traditional approach, which takes into consideration dynamic conditions within 2-nd dimension of non-linear model.

Note that general view of optimization problem, related to dynamic PA power scenario might be represented as the sum of loss functions corresponding to each power case (3):

$$\sum_{i=0}^{C-1} ||\mathrm{DPD}(\boldsymbol{x},\boldsymbol{p},\boldsymbol{h}) - \boldsymbol{e}_i||_2^2 \rightarrow \min_{\boldsymbol{h}}, \tag{4}$$

where C – number of PA power cases, $\boldsymbol{p} \in \mathbb{C}^{N\times 1}$ – vector of PA input power features. Thus, $(\boldsymbol{x},\boldsymbol{p})$ – 2-dimensional model input, $\boldsymbol{h} \in \mathbb{C}^{P\times 1}$ – DPD model parameters vector.

II. 2D Chebyshev Polynomial DPD Technique

Among non-linear digital signal processing tasks polynomials [1], [3], spline-based polynomials [12] are often used for the purpose of signal non-linearity description. Common single-dimensional DPD based on Chebyshev polynomial for one-dimensional input signal is represented as:

$$\begin{gathered} z_n = \sum_{i=0}^{P-1} h_i x_n T_i(|x_n|), \\ T_i(|x_n|) = \cos(i \arccos(|x_n|)), \end{gathered} \tag{5}$$

here P – Chebyshev polynomial order, $\{h_i\}_{i=0}^{P-1}$ – Chebyshev polynomial trainable parameters.

Non-linear model is required to take into account non-linear memory of the PA. For this reason proposed model (6) is expanded to the sum of non-linearities, fed by delayed signals $|x_{n-d}|$, where d – signal delay corresponding to the d-th branch. Thus, model output is shown as follows:

$$y_n = \sum_{d=-D}^{D} \sum_{i=0}^{P-1} h_{k,i} x_{n-d} T_i(|x_{n-d}|), \tag{6}$$

where $B = 2D + 1$ – number of model branches. Fig. 3 shows general scheme of chosen parallel non-linear Chebyshev polynomial-based model.

In current paper single-dimensional Chebyshev polynomial is tested on the case of dynamically changing PA output

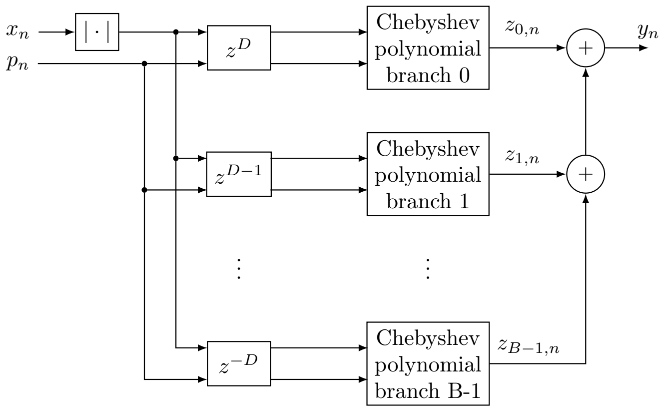


Fig. 3: B-branch non-linear Chebyshev polynomial-based model

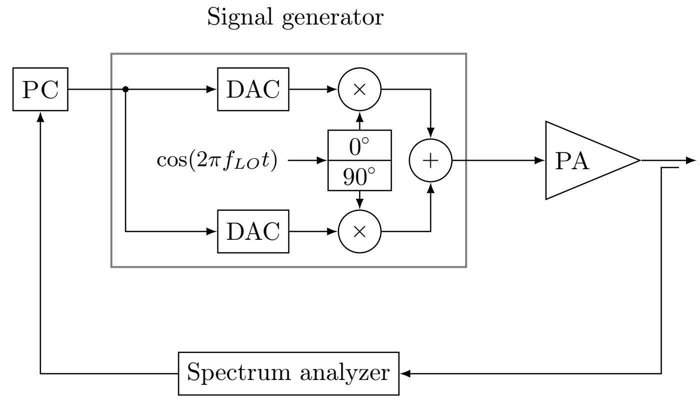


Fig. 4: The scheme of testbench

power. According to the results, represented in section IV, 1D non-linearity can't absorb dynamically changing PA power properties. Thus, current article introduces 2-dimensional non-linearity, which is fed by both features: signal magnitude $|x_n|$ and feature, which corresponds to current PA power mode p_n. Therefore, 2-dimensional polynomial output is as following:

$$\begin{aligned} z_n &= \sum_{i=0}^{P_1-1} \sum_{j=0}^{P_2-1} h_{i,j} x_n T_{i,j}(|x_n|, p_n), \\ T_{i,j}(|x_n|, p_n) &= T_i(|x_n|) T_j(p_n) = \\ &= \cos(i \arccos(|x_n|)) \cos(j \arccos(p_n)), \end{aligned} \tag{7}$$

here P_1, P_2 – 2-dimensional Chebsyhev polynomial orders related to magnitude and power mode feature dimensions correspondingly, $\{h_{i,j}\}_{i=0,j=0}^{P_1-1,P_2-1}$ – 2-dimensional Chebsyhev polynomial trainable parameters. 2D non-linearity basis functions $T_{i,j}(\cdot)$ are expressed through the multiplication of 1D non-linearity basis functions $T_i(\cdot)$, $T_j(\cdot)$, related to input signal magnitude and PA power correspondingly. Multi-branch 2-dimensional model output is represented as:

$$y_n = \sum_{d=-D}^{D} \sum_{i=0}^{P_1-1} \sum_{j=0}^{P_2-1} h_{i,j,k} x_{n-d} T_{i,j}(|x_{n-d}|, p_{n-d}), \tag{8}$$

where basis function $T_{i,j}$ is expressed similarly to (7).

Note, that input signal magnitude $|x_n|$ and PA mode featrure p_n are implied to be scaled into the same range $[0, 1]$ for the purpose of data standardization and from the perspective of satisfying Chebyshev polynomial orthogonality conditions.

III. Testbench description

Testbench structure is shown in fig. 4. The measurement setup consists of personal computer (PC), signal generator (SG) R&S SMW200A, spectrum analyzer (SA) R&S FSW85 and power amplifier ZKY66291-11 under testing.

PC loads baseband (BB) IQ data to SG which modulates BB to the carrier frequency and send to the PA input. PA output with nonlinear distortions transmitted to SA. After that IQ BB data send to the PC for further signal processing.

Power control was done by SG. Complex valued 20 MHz OFDM signal with 100 resource blocks, carrier $f_{LO} = 1.8$ GHz was used for experiments.

Totally, $C = 61$ power cases of the PA output powers are considered in dynamic range 11.2 dB in current simulations:

$$\begin{gathered} 0.069 \text{ W},\ 0.107 \text{ W},\ 0.143 \text{ W}, \cdots, 0.912 \text{ W} \\ -11.6 \text{ dBm}, -9.7 \text{ dBm}, -8.4 \text{ dBm}, \cdots, -0.4 \text{ dBm} \end{gathered} \tag{9}$$

Each PA power case consists of 147k complex samples. Even power cases are used for training:

$$\begin{gathered} 0.069 \text{ W},\ 0.143 \text{ W}, \cdots, 0.912 \text{ W} \\ -11.6 \text{ dBm}, -8.4 \text{ dBm}, \cdots, -0.4 \text{ dBm} \end{gathered} \tag{10}$$

Odd power cases are considered for model performance evaluation:

$$\begin{gathered} 0.107 \text{ W},\ 0.175 \text{ W}, \cdots, 0.906 \text{ W} \\ -9.7 \text{ dBm}, -7.6 \text{ dBm}, \cdots, -0.4 \text{ dBm}. \end{gathered} \tag{11}$$

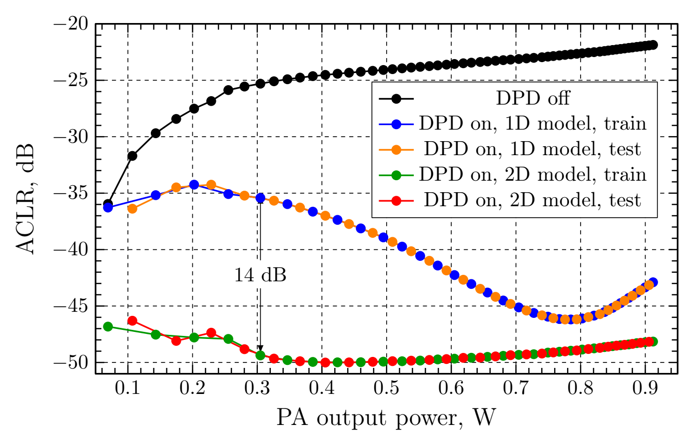
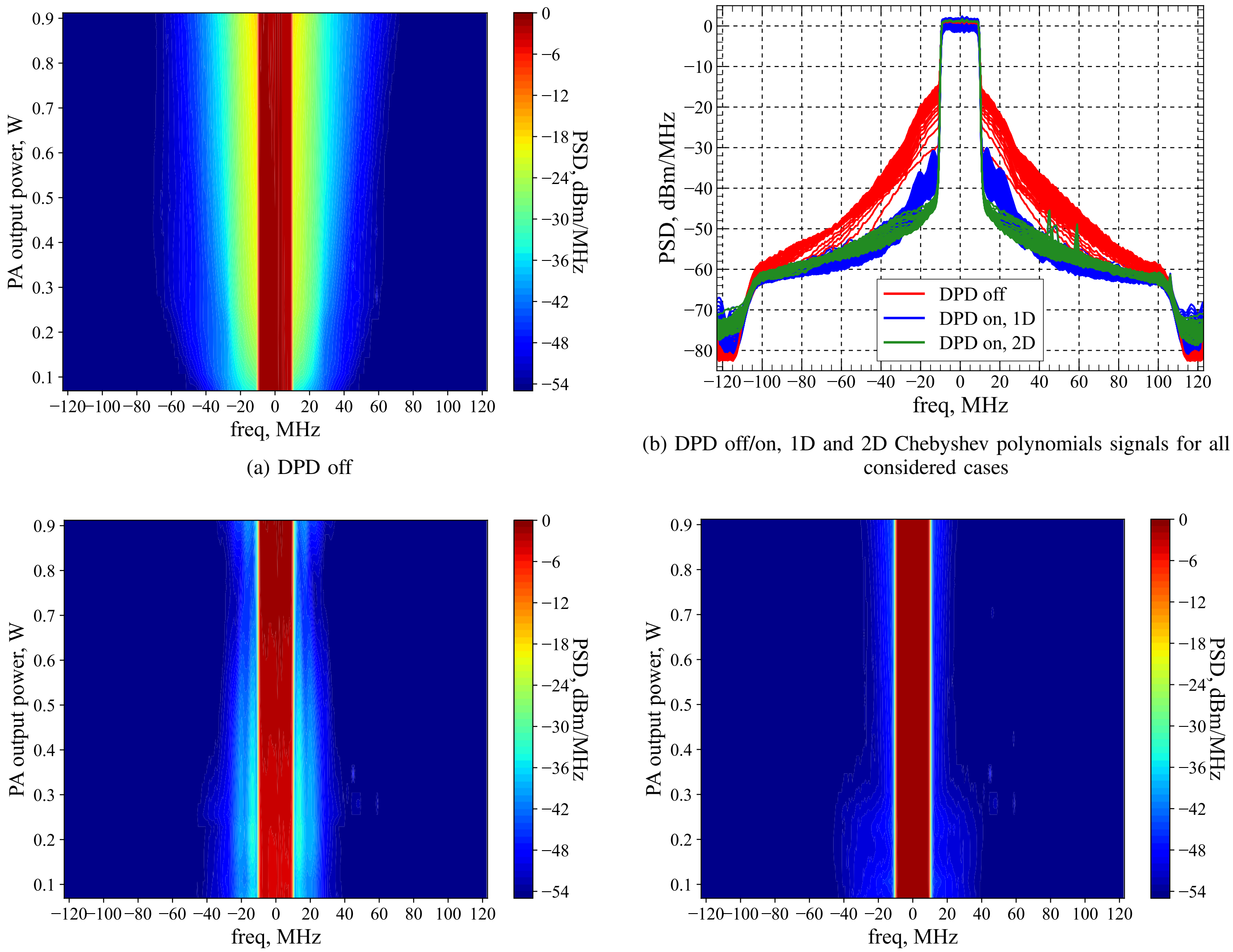


Fig. 5: Performance 1D- and 2D-dimensional Chebyshev polynomial on train and test datasets



(a) DPD off

(b) DPD off/on, 1D and 2D Chebyshev polynomials signals for all considered cases

(c) DPD on, 1D Chebyshev polynomial

(d) DPD on, 2D Chebyshev polynomial

Fig. 6: PSD of PA output. Application results of DPD on base of 1D and 2D Chebyshev polynomials

IV. Experimental Results

The number of branches in two-dimensional model $B = 9$. Thus, delays are in range $[-4, \cdots, 4]$, which is enough to describe PA memory effects according to simulations results. The orders of Chebyshev polynomial along signal magnitude $|x_n|$ and PA power feature p_n dimensions $P_1 = 22$ and $P_2 = 10$ correspondingly.

Chebyshev polynomial order P_2 is chosen to achieve acceptable ACLR values -45 dB in the whole range of powers

Total number of parameters for single-dimensional model is chosen to equalize total number of parameters for both considered models.

Thus, 1D and 2D polynomials-based models include the same number of trainable complex parameters equal 1980.

Performance of both mentioned models is shown in fig. 5. According to the simulations results 2-dimensional model provides ACLR < -45 dB in the whole range of PA output powers. Moreover, there is an improvement up to 14 dB comparing to 1-dimensional polynomial among all considered PA power modes.

Figure 6 shows power spectral densities of transmitted signal before DPD-based correction and after application of pretrained 1D and 2D Chebyshev polynomials. Model evaluation is made both on train and test data, resulting corrected PA output PSD plots are visualized in fig. 6.

Fig. 6a shows transmitted signal PSD color map before DPD application in correspondence to all PA output powers cases. Whereas fig. 6c, 6d provide PSD color map of transmitted signal after application of DPD model based on 1D and 2D Chebyshev polynomials correspondingly. In addition, fig. 6b represents PSD of TX before and after DPD application in the same plot with normalized output powers for different cases. Thus, according to fig. 5, 6c, 6d 2D-based DPD model provides significant performance improvement in comparison to the 1D-based model up to 14 dB among the whole chosen

PA output power range: 0.069 W, $\cdots$, 0.912 W.

V. Conclusion

Suggested offline DPD model, based on 2-dimensional Chebyshev polynomials, which provides PA output signal correction within the wide range of PA power modes: from 0.069 W to 0.912 W PA output power. The model if fed by PA input signal and the feature, which corresponds to PA input signal power. The model implemented and compared with DPD correction on base of 1-dimensional Chebyshev polynomials, fed by PA input signal.

2D-based model provides ACLR < -45 dB performance for the whole range of considered PA output power cases.

In addition, suggested model provides ACLR improvement up to 14 dB for different PA output power cases in comparison with 1D Chebyshev polynomial-based model with the same number of trainable complex parameters.

Taking the aforementioned into account, 2D Chebyshev polynomial-based model enables prediction of PA behavior in response to changes in power mode and addresses the issue of non-stationarity during DPD parameter extraction.

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