

Assignment 3 (Turbulence spectra: from measurements to time-series synthesis).

For various applications in wind energy, time-series of wind are analyzed, treated, or simulated via spectra. In this assignment we work with measured time-series, as well as synthesizing turbulent time-series.

The assignment can be divided into four main categories, which are all somewhat related. The focus of each category can be described in terms of the data involved:

Data 1: Time-series of turbulent velocity measurements from a three-dimensional sonic anemometer at Høvsøre, at a height of 80 m. The file `hoevsoere1hr.uvw.csv` contains 1 hour of data sampled at 20 Hz. Its three columns are $\{u, v, w\}$, where u is in the mean horizontal wind direction and w is vertical.

Data 2: Time-series of 10-minute-mean wind speed measurements from Sprogø. Use the data in `sprogo_1.zip` (which you've also used in Assignment 1).

Data 3: Data you simulate, from simple linear Markov process. See below for details.

Data 4: Data you synthesize via Fourier Simulation. See below for details.

Do the following:

1. **Data 1:**

- (a) Calculate the mean value and variance for each of the (three) velocity components.
- (b) Calculate the friction velocity, u_* .
- (c) Calculate the temporal spectrum for the three wind components, both directly (unsmoothed) and via ensemble averaging: chop the time-series up into 1 and 6 equally long sub-series (corresponding to 1 hour and 10 minutes, respectively), and ensemble average the six 10-minute spectra. Plot both the 'raw' (unaveraged) and averaged spectra as $S(f)$ vs f in a log-log coordinate system.
- (d) Write a short code to make a smoothing filter that averages a spectrum over neighboring frequency bins with logarithmic separation, so that at most n spectral estimates appear per frequency decade. (Note: for practical use n should be between ~ 10 and 20 .) Apply the smoothing filter you just made to the 'raw' (unaveraged 1-hour) spectra calculated in the previous question, and plot the theoretical prediction for the inertial sub-range (the $5/3$ law) in the same plot; i.e., compare the power law exponent (slope in a log-log coordinate system). Also check quantitatively whether the $4/3$ law is observed.

2. **Data 2:** Calculate the frequency spectrum of the 21-year time-series of 10-minute averages from Sprogø, used in Assignment 1. For simplicity you can ignore any gaps in the data time-series, and just treat all wind speeds as one long time-series with constant sampling rate $f_s = 1/(10\text{min}) = \frac{1}{600}$ Hz.

- (a) Plot the spectrum as $fS(f)$ vs f , using appropriate axis scaling.
- (b) Applying your smoother to the spectrum, can you see a diurnal peak or an annual peak? I.e., what timescales have the highest peaks?

3. **Data 3** is made by you! It is the result of a simple auto-regressive process. In such a series the $(n+1)^{\text{th}}$ number is obtainable from the n^{th} value via

$$X_{n+1} = c_{ar}X_n + \xi_n, \quad (2)$$

where $0 < c_{ar} < 1$, and where $\xi_1, \xi_2, \xi_3 \dots$ are independent Gaussian random variables with zero mean and unit variance. By choosing an initial value, X_1 , you can thus generate all future values X_n . Make a computer program that simulates such a process, where the initial value X_1 is normally distributed with zero mean and variance $\sigma_X^2 = 1/(1 - c_{ar}^2)$.

- (a) Simulate three *long* time-series ($N \geq 10^4$, using $c_{ar} = 0.1, 0.4$ and 0.9 respectively (you can arbitrarily pick whatever Δt that each step corresponds to, e.g., 1 s or 0.1 s).
 - (b) Calculate the mean and variance of the three time-series just simulated in 3(a).
 - (c) For the auto-regressive process the autocorrelation function is given by $\rho(\tau) = c_{ar}^\tau$; plot $\rho(\tau)$ for each of the three time-series, and compare with this expression.
 - (d) Calculate the spectra for the three time-series and plot in log-log coordinates.
 - (e) What is the power-law exponent at the highest frequencies, and should it depend on c_{ar} ? [*Hint: deriving it is more reliable than trying to get from plots*]
 - (*) Bonus points: derive the integral timescale T_{int} ; then re-express $c_{ar}, \rho(\tau)$, and σ_X in terms of T_{int} .
4. You make **Data 4**, using the Kaimal spectrum. The latter is defined by

$$S_u(f) = u_*^2 \frac{52.5z/U}{(1 + 33fz/U)^{5/3}}. \quad (3)$$

The recipe for generating a time-series x_n , having a given spectrum (in this case the Kaimal spectrum) is given below.

- Choose a record length, N .
- Make a list X_ℓ of random complex numbers, of length $N/2$. Using a zero-mean Gaussian distribution with unit variance, make a random number for both the real part and the imaginary part. This means that you should generate a total of $2 * N/2 = N$ random numbers.
- Multiply each member (X_ℓ) with the correct Fourier amplitude σ_ℓ ; the latter is specified by the Kaimal spectrum.
- Expand X using the index rules given in the course notes:

$$\overline{X} = \left\{ \begin{array}{cccccccc} \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} & & \boxed{N/2+1} & \boxed{N-1} & \boxed{N} \\ a_0, a_1 + ib_1, a_2 + ib_2, a_3 + ib_3, \dots, a_{N/2} + 0, \dots, a_2 - ib_2, a_1 - ib_1 \end{array} \right\} \quad (4)$$

Use a built-in function for complex conjugation to make this easier (e.g. `conj(x)` in Matlab or `numpy.conjugate` in python).

- Inverse Fourier-transform X , making sure it is normalized correctly. If the resulting time-series is not real-valued (i.e., has a significant imaginary part), then you did not do the previous step correctly.

Now, do the following:

- (a) Make a program that generates $u(t)$ using this ‘recipe’ with the Kaimal spectrum (3) at a height of $z = 80$ m, under the conditions $U = 7.0 \text{ m s}^{-1}$ and $u_* = 0.25 \text{ m s}^{-1}$. Use it to generate a 10-minute $u(t)$, with $\Delta t = 0.05$ s; plot $u(t)$.
- (b) Calculate and plot the spectrum from the synthesized time-series to check that you get the Kaimal spectrum back.