Danmarks Tekniske Universitet

Introduction to Micrometeorology for Wind Energy

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Assignment 1

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Introduction and Data Cleaning

Two different data sets of wind measurements are provided, within the context of probability distributions and statistics. These data sets are:

- A couple decades of data of a mast on island of Sprogø (Great Belt Bridge), Denmark. This includes wind speed and two different vector of measurements of wind direction: at 70m and at 67.5m.
- A few hours data of turbulence measurements from DTU's Høvsøre turbine test center on the west coast of Denmark. With two columns: timestamp and stream wise wind velocity component .

First we convert all non-numerical values to NaN, as all those non-numerical values must correspond to error flags.

Wind Direction

Apart from the non-numerical values, the values for wind direction that are considered incorrect data are the ones with value over 360 or below 0. This could come from an error measurement (999 in this case) or the wind direction measurement not normalised at 0 - 360 and getting multiples of those values. so, we checked for values over 360 and below 0. As it turns out, the measurements are already normalised to a range of 0 to 360 degrees, measurement values of 999 were found which are classified as error. These values are converted to NaN.

There are two data column/vectors available measuring wind direction: one measuring at 67.5m and the other measuring at 70m. Many timestamps' data is found to be invalid, there are timestamps when none of the heights have a valid measurement, however there are no timestamps when both columns have a valid measurement. Based on this and on the fact that wind direction measurement should not change that much between the 70m and 67.5 layer, it is decided to merge those two columns creating a unique vector for wind direction. There is no conflict found as it is found either one proper measurement on the timestamp or both invalid measurements.

Wind Speed

As done for the wind direction, the wind speed data is checked to remove invalid wind speed data. The invalid wind speed measurements are defined as those which are either over 40 m/s or negative values, the upper limit of 40 m/s is set as after generic and simple inspection is seen that those would be outliers among the data. It is realised that the only invalid data is the one corresponding to 99.99 m/s values, which are also converted to NaN values.

The same filter is applied to both datasets.

1 Task 1: Mean and Standard Deviation of Wind Speed

Both Sprogø and Høvsøre datasets have available data on wind speed. The Høvsøre datasets has the stream-wise wind velocity component, but it is treated as wind speed measurement in this case.

The mean wind speed is computed using Equation 1

$$\mu = \frac{1}{N}\sum_{m=1}^{N} u_m \tag{1}$$

Where:

- N, number of data points
- u, value of indiviadual data point

The standard deviation is calculated using Equation 2

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \tag{2}$$

1.1 Sprogø

The mean wind speed and standard deviation for the sprog data is as follows:

Mean Wind Speed	8.24 m/s
Wind Speed Standard deviation	3.91 m/s

Table 1: Wind Speed Mean and standard deviation results for Sprogø

1.2 Høvsore 1

The mean wind speed and standard deviation for the hovsore data is as follows:

Mean Wind Speed	13.3 m/s
Wind Speed Standard deviation	1.54 m/s

Table 2: Wind Speed Mean and standard deviation results for Høvsore 1

2 Task 2: Mean and Standard Deviation of Wind Direction

It is needed to consider that the wind direction is cyclic, thus the mean can be calculated as it usually is, using Equation 1. According to [1], there are two ways to compute it:

- Taking the average of components using wind speed
- Taking the average of components disregarding wind speed

2.1 Mean Wind Direction - Averaging components using wind vectors

To calculate the mean wind direction of this first method Equation 3, Equation 6 and Equation 7 are used, where θ is the wind direction and U is the wind speed:

$$S_{va} = \frac{1}{N}\sum_{n=1}^{N} U_n sin\theta_n \tag{3}$$

$$C_{va} = \frac{1}{N}\sum_{n=1}^{N} U_n cos\theta_n \tag{4}$$

The average wind direction is then given by:

$$\bar{\theta} = mod\left[\frac{180^0}{\pi} arctan\left(\frac{S_a}{C_a}\right), 2\pi\right] \tag{5}$$

The results are shown in Table 3 :

S_{va}	-1.91
C_{va}	-1.30
Mean Wind Direction	235.63 deg

Table 3: Computed mean wind direction result by including wind speed

2.2 Mean Wind Direction - Averaging components Disregarding the wind speed

For this second method, Equation 6, Equation 7 and Equation 8 are used, where θ is the wind direction and U is the wind speed::

$$S_a = \frac{1}{N}\sum_{n=1}^{N} sin\theta_n \tag{6}$$

$$C_a = \frac{1}{N}\sum_{n=1}^{N} cos\theta_n \tag{7}$$

The average wind direction again given by the arc-tangent of the ratio of components as in

$$\bar{\theta} = mod\left[\frac{180^0}{\pi} arctan\left(\frac{S_a}{C_a}\right), 2\pi\right] \tag{8}$$

The computed values are as follows:

S_{va}	-0.19
C_{va}	-0.13
Mean Wind Direction	234.5 deg

Table 4: computed mean wind direction result by excluding wind speed

2.3 Standard deviation for wind direction

The Yamartino method which employs the classic vector component average is used to calculate the standard deviation, which is defined by Equation 9 and Equation 10.

$$\sigma_\theta = \sin\varepsilon \left[1 + \left(\frac{2}{\sqrt{3}} - 1\right)\varepsilon^3\right] \tag{9}$$

where:

$$\varepsilon = \sqrt{1 - {s_a}^2 - {c_a}^2} \tag{10}$$

And the result is in

Standard Deviation Wind Direction	1.53 deg

Table 5: computed mean wind direction result by excluding wind speed

3 Task 3: Probability Density Function of Wind Speed

After remove all those invalid data points in both datasets (section), now the probability density function of their wind speed will be plotted.

3.1 Sprogø

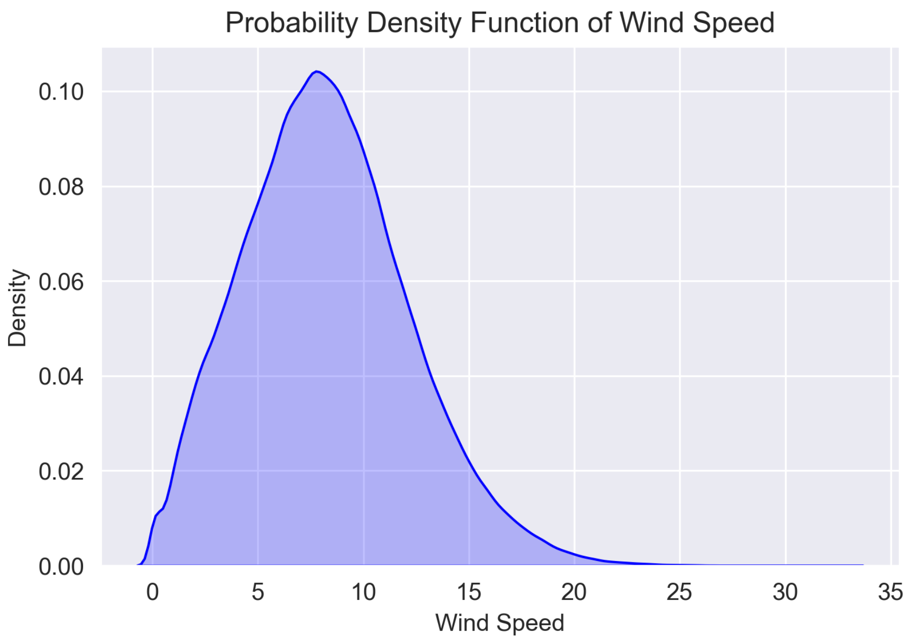

Figure 1: Probability Density Function of Wind Speed Sprogø

The Figure 1 shows the probability density funtion of the wind speed, it shows a Weibull function shape as expected. For this dataset the peak density, so the most repeated value is its already calculated mean: 8.24 m/s.

3.2 Høvsore 1

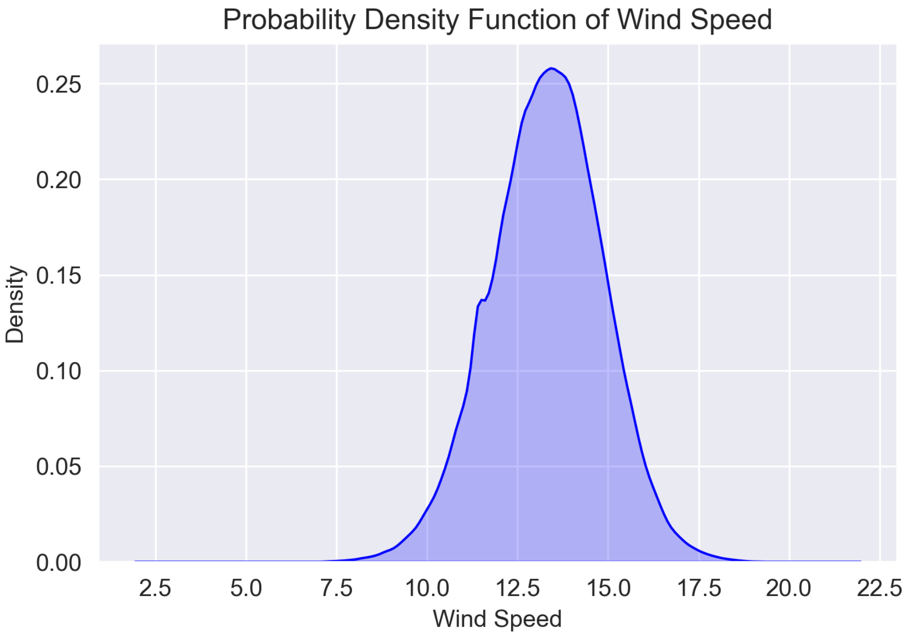

Figure 2: Probability Density Function of Wind Speed Høvsore 1

In this case of the measurements of Høvsore 1 Figure 2, the wind speed does not show a Weibull function shape. One of the reasons of this normal distribution instead of Weibull could be that it is stream-wise wind velocity component of turbulence data instead of raw wind speed. However, it can always be the case that at the site the wind speed distribution is as seen. For this dataset the peak density, so the most repeated value is its already calculated mean: 13.3 m/s.

4 Task 4: Cumulative Distribution Function of Wind Speed

Now the cumulative distribution function will be calculated and plotted for wind speed values of both datasets. The CDFs are calculated in an empirical way.

4.1 Sprogø

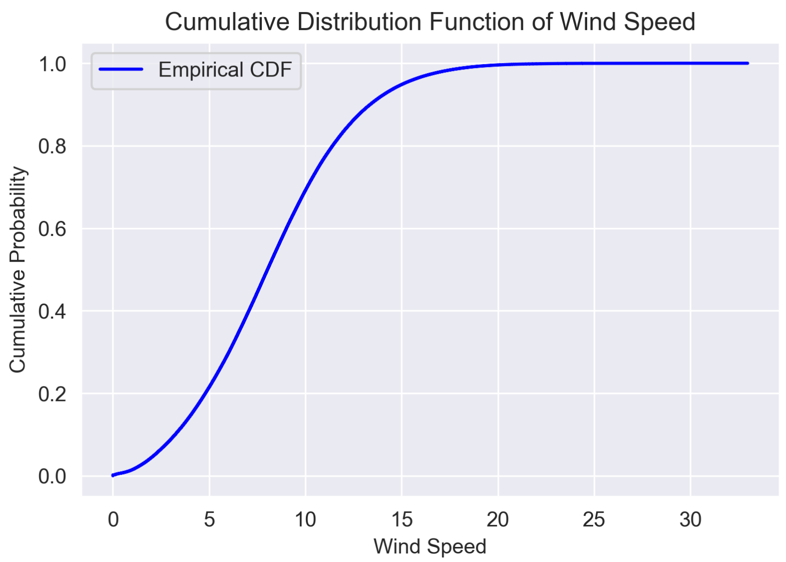

Figure 3: Probability Density Function of Wind Speed Sprogø

4.2 Høvsore 1

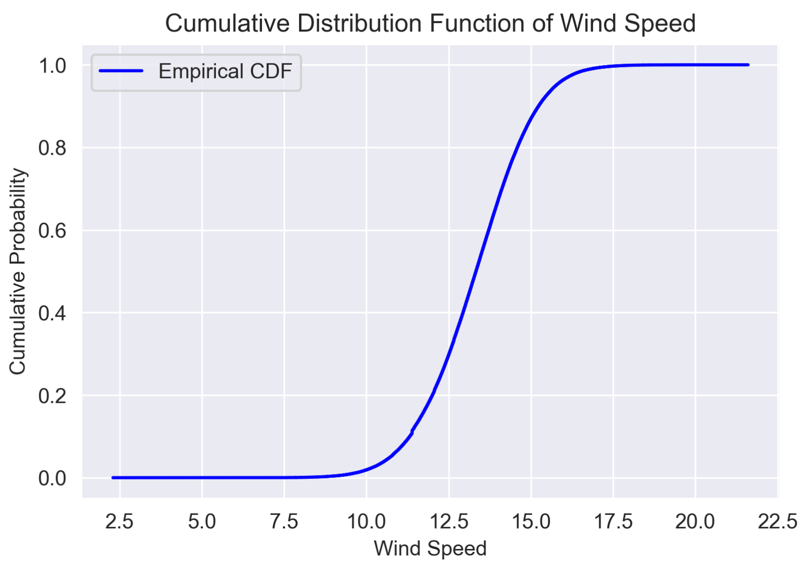

Figure 4: Cumulative Distribution Function of Wind Speed Høvsore 1

The Høvsore 1's CDF shows that this dataset has less spread in value data points, it has a steeper CDF.

5 Task 5: Probability Density Function of Wind Speed for normalised variables

Continuously, the Høvsore 1's CDF is re-plotted again, nevertheless, in this case the variables will be normalised. To normalise the variables the mean is subtracted from each data point and it is divided by the standard deviation, as Equation 11 shows.

$$\text{Normalized value} = \frac{X - \mu}{\sigma} \tag{11}$$

where:

- x, original value
- μ, mean of the data
- σ, standard deviation

Once the wind speed is normalised, the probability density funtion is computed again the same way as in section 4, si en empirical way.

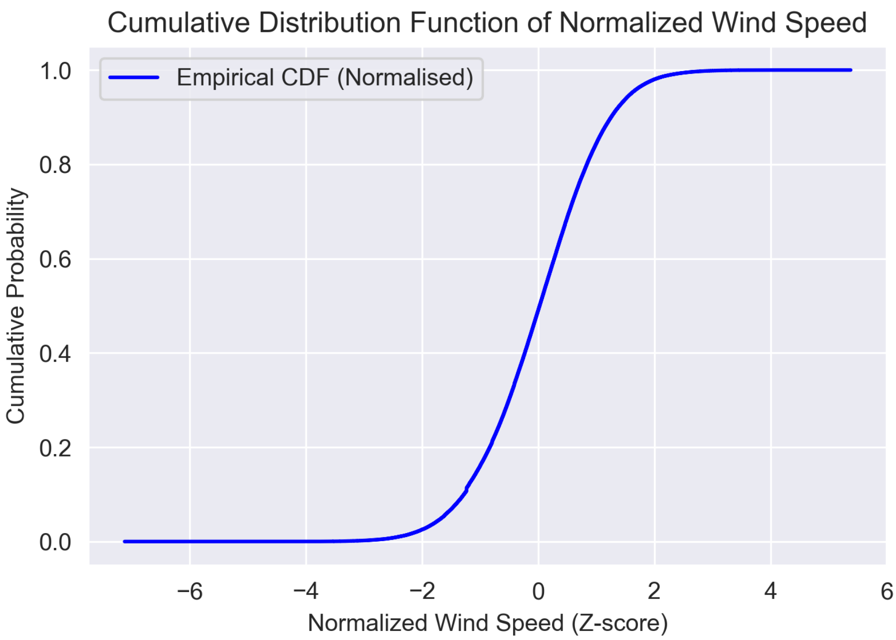

Figure 5: Cumulative Distribution Function of Normalised Wind Speed Høvsore 1

6 Task 6: Estimate Weibull parameters A and k

The Weibull A and k parameters will be estimated for the Sprogø dataset. These parameters will be estimated using two different methods:

- Based on the first and second non-central moments
- Based on the third moment and CDF-at-mean

6.1 Based on the first and second non-central moments

To use the non-central moments to estimate the Weibull parameters, the following relationship is used:

$$\mu_n = A^n \Gamma(1 + \frac{n}{k}) \tag{12}$$

Where:

- μ_n, non-central moment n
- A^n, Weibull parameter A scale parameter
- Γ, Gamma function
- n, nth non-central moment
- k, Weibull parameter k, shape parameter

The first non-crentral moment is equal to the mean of the Weibull distribution:

$$\mu_X = A\Gamma(1 + \frac{1}{k}) \tag{13}$$

And the variance is of the Weibull distribution is equal to central second moment:

$$\sigma_X^2 = A^2(\Gamma(1 + \frac{2}{k}) - \Gamma^2(1 + \frac{1}{k}) \tag{14}$$

As the variance (Equation 14) is the square of the standard deviation, the latest and the mean calculated in section 1 are used along with Equation 13 and Equation 14 to compute the Weibull parameters k and A.

To solve the presented system computationally, we give an initial guess and iterate till we get a fair result. The results are presented in Table 6.

Weibull A parameter	Weibull k parameter
9.299	2.227

Table 6: A and k Weibull parameters results

6.2 Based on the third moment and CDF-at-mean

First, the value of the third moment is defined as Equation 12:

$$\mu_3 = A^3 \Gamma(1 + \frac{3}{k}) \tag{15}$$

The cumulative distribution function of a Weibull distribution at mean is defined as Equation 16.

$$F(\mu) = 1 - e^{-(\frac{\mu}{A})^k} \tag{16}$$

Then using Equation 15 and Equation 16 as a system of equations and giving an initial guess, through interpolations the final results for the parameters are the ones on Table 7:

Weibull A parameter	Weibull k parameter
9.396	2.324

Table 7: A and k Weibull parameters results

7 Task 7: Theoretical Weibull-PDFs

In this seventh task, the two found Weibull distributions and the Sprogø's PDF are plotted together too spot the differences. The Weibull curves are ploted used the k and A parameters found using the two different methods, explained in subsection 6.1 and subsection 6.2. The PDF is the same as obtained in subsection 3.1.

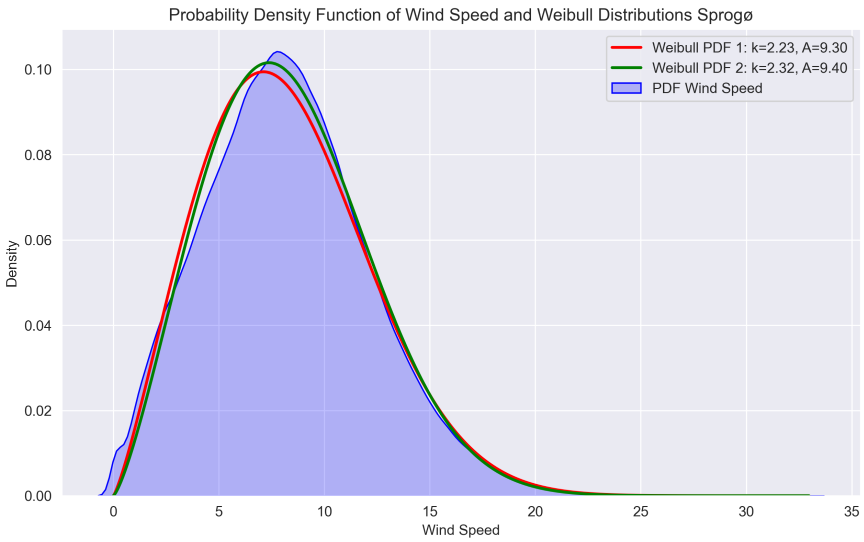

Figure 6: Probability Density Function of Wind Speed and Weibull Distributions Sprogø

Looking at the result Figure 6, it can be appreciated that both methods to find the Weibull PDF have resulted on reliable methods, as both are accurate with respect of the real distribution. Nonetheless, the method based on the third moment ans CDF-at-mean seems to give slightly better results.

8 Task 8: Investigation Data Conditioned on Wind Direction

As mention in section , the initial data offered 2 vectors of wind direction measurements: one at 67.5m and the other at 70m. As it has been found that most of the time when an invalid error was found in one of the vector the other had a proper value, It is decided to create a unique vector for wind direction merging both initial ones. Thus, this resulting vector has been the one that has been binned. The data is divided into 13 sectors, each sector is 30 deg wide and the vectors are centered starting at 0 deg and each following center bin increases 30 deg.

For each 12 bins, the classified as wind direction proper data is used to find the A and k Weibull parameters of the bin, to defining what is the Weibull cumulative distribution function of each 20 deg bin. The method use tho find those two parameters is the method based on the first and second moments, explained and used in subsection 6.1. Obtaining the following results presented in Table 8, the sectors are defined depending on the center degree value of the bin.

Sector	A parameter	k parameter
0	7.751	1.901
30	7.525	1.895
60	7.761	2.047
90	9.236	2.311
120	9.603	2.317
150	8.461	2.064
180	9.407	2.158
210	9.931	2.468
240	9.951	2.553
270	10.194	2.563
300	9.631	2.200
330	7.896	1.895

Table 8: A and k Weibull parameters results for each 30 deg sector

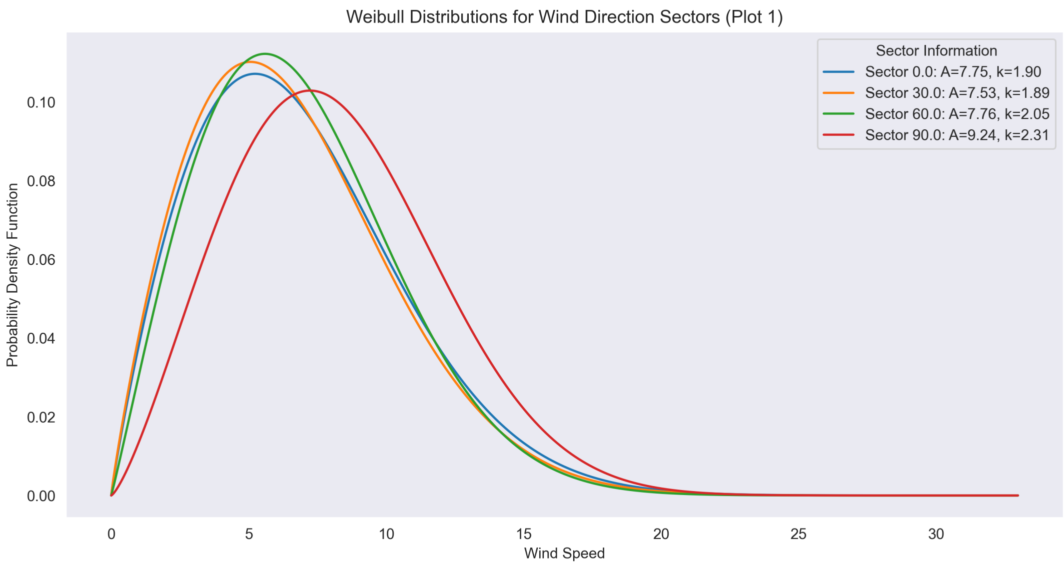

Figure 7: Weibull Distributions for Wind Direction Sectors 0, 30, 60 and 90

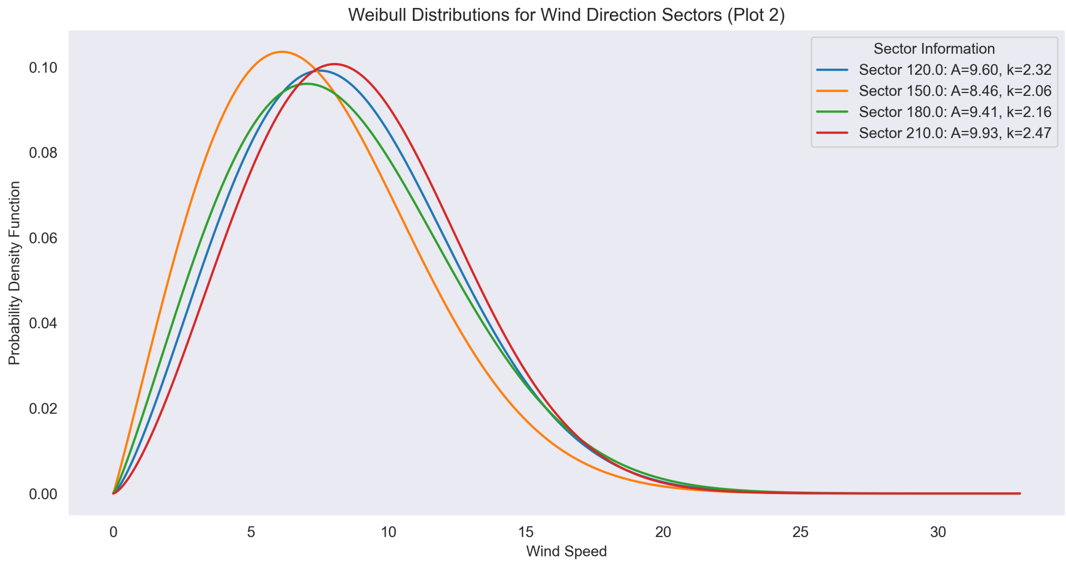

Figure 8: Weibull Distributions for Wind Direction Sectors 120, 150, 180 and 210

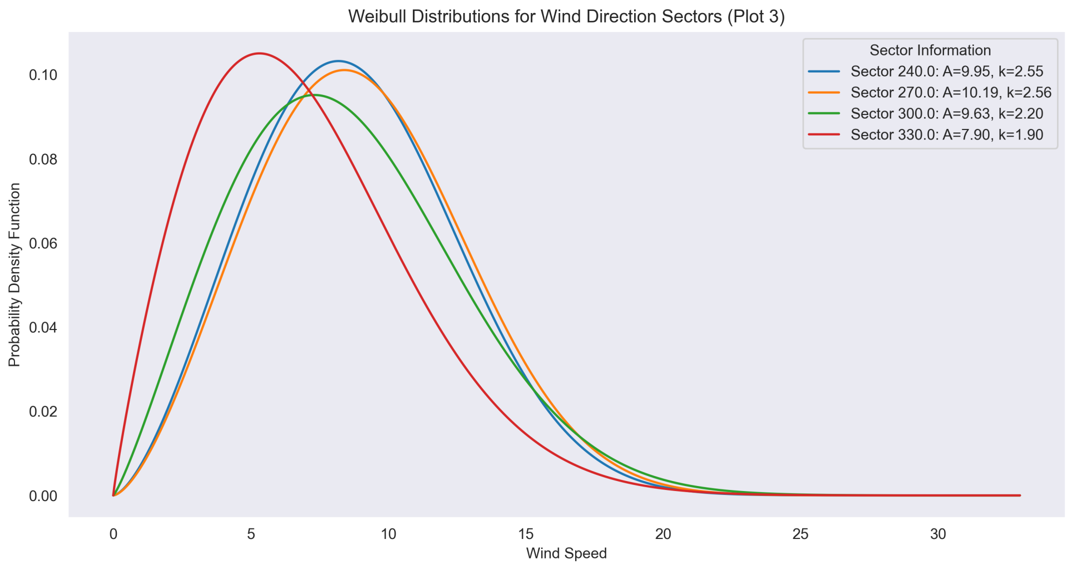

Figure 9: Weibull Distributions for Wind Direction Sectors 240, 270, 300 and 330

9 Task 9 - Seasonal and daily trends in the Sprogø data

This last task is about inspecting daily and seasonal trends for either the wind speed and the wind direction in the Sprogø's data.

9.1 Daily Trends

9.1.1 Wind Speed

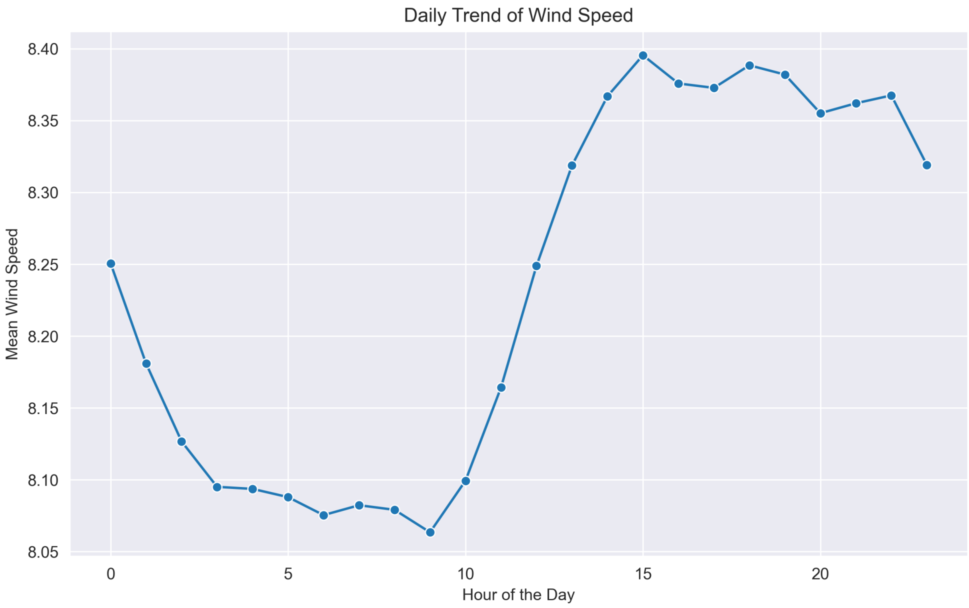

Figure 10: Daily Trend of Wind Speed

For the daily wind speed trend, Figure 10 shows the hourly mean wind speed for the entire set of days, what means the hourly mean taking as a set all the day measurements at that hour. It shows a clear trend of lower wind speed during night and early miring time, from 3am to 9 am. Then progressively increasing the hourly mean wind speed till the highest wind speed range of that that would be from 3pm to 10pm. Nevertheless, the hourly mean wind speed value throughout the day is quite stable, with a range between 8m/s and 8.4m/s. Therefore it can be concluded that the wind speed a long the day is stable in terms of velocity without major changes, always taking into account that no seasonal binning in the data has been done which can show different trends on the daily trend for season.

9.1.2 Wind Direction

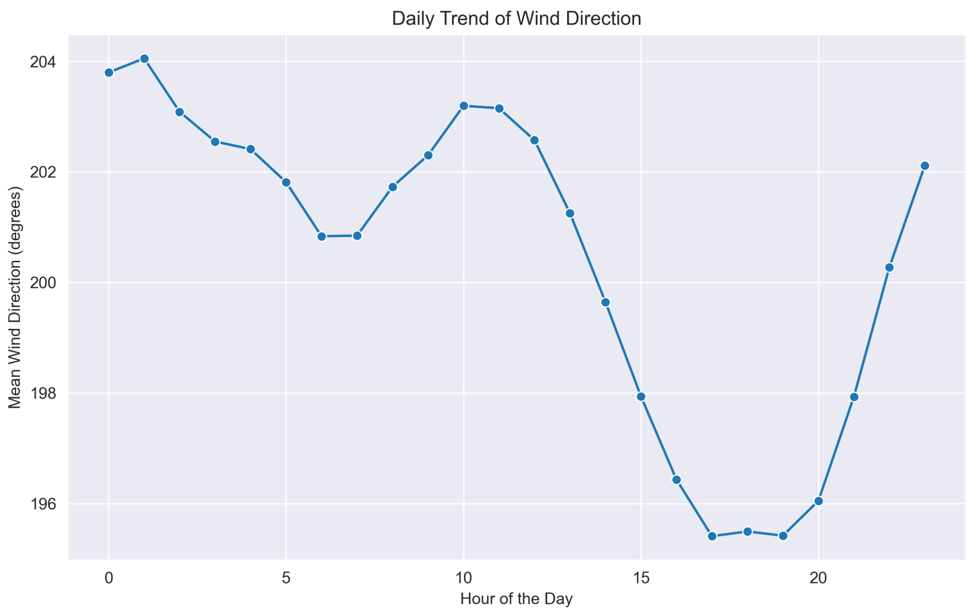

Figure 11: Daily Trend of Wind Direction

For the daily trend of wind direction, the result is shown in Figure 11. This plot shows hourly mean wind direction for all days of all years of the dataset. The trend shows that the site has strong main wind direction coming from the range of 195 deg to 204 deg, all hours mean wind direction fall in this range. With southern winds in the afternoon and northern winds during night.

9.2 Seasonal Trends

9.3 Wind Speed

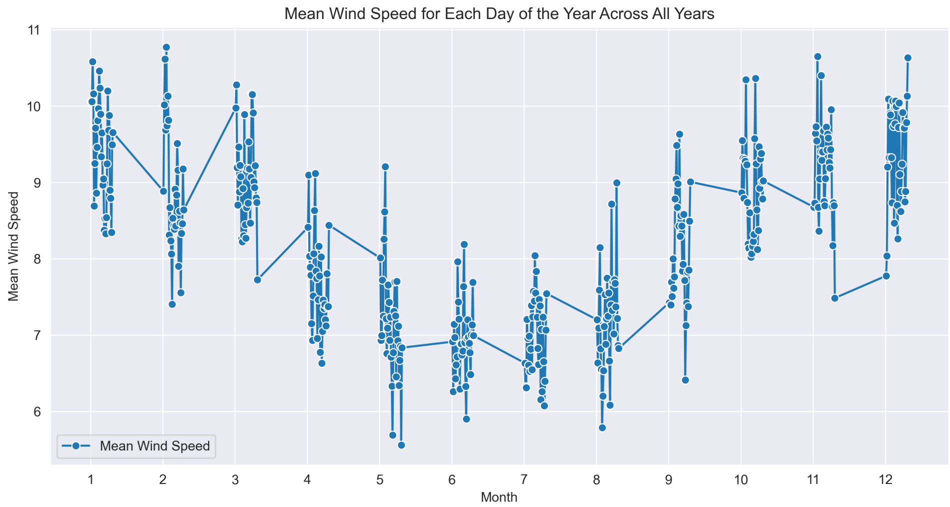

Figure 12: Seasonal Trend of Wind Speed

Figure 12 shows the mean wind speed for each day of the year across all years in the dataset. Here can be see the seasonal trend, the months that show higher mean wind speeds are January, February, November and December, thus winter time has higher wind speeds (around 9 m/s). From winter the mean wind speed seems to be getting lower till summer time when the wind speed reach its trough (around 7 m/s).

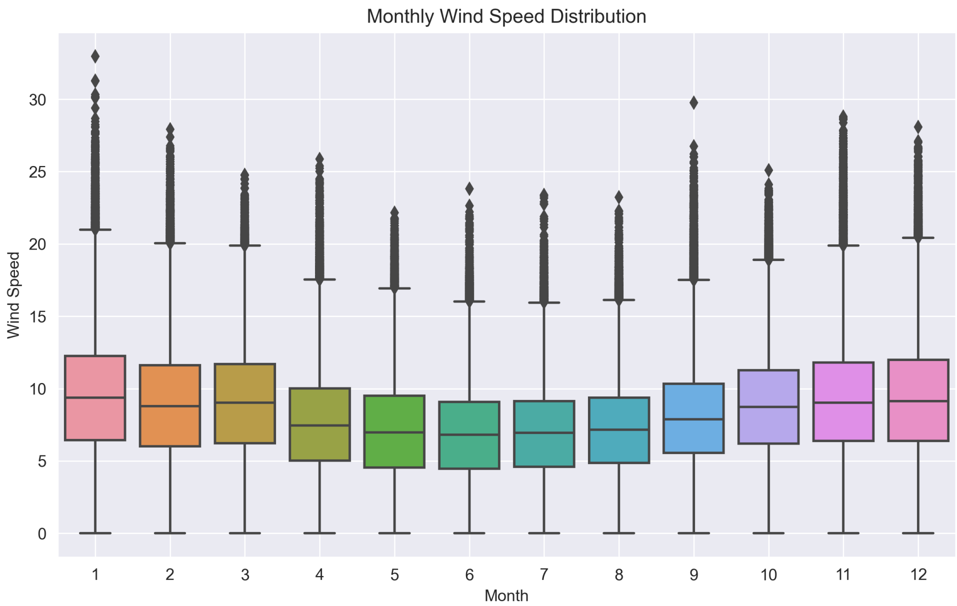

Figure 13: Monthly wind speed distribution

Figure 13 is a box plot of the monthly wind speed distribution. The coloured boxes show the IQR (from 25th percentile to the 75th percentile), the line inside represents the median. The whiskers is the range of data without outliers, whiskers extend from the Q1 (25th percentile) to the smallest value within 1.5 times the IQR and from Q3 to the largest value within 1.5 times the IQR. All data points outside whiskers are considered outliers. IT can be seen that all months have a pretty similar wind speed distribution, being those where the median is higher (winter) with the most spread distribution.

9.4 Wind Direction

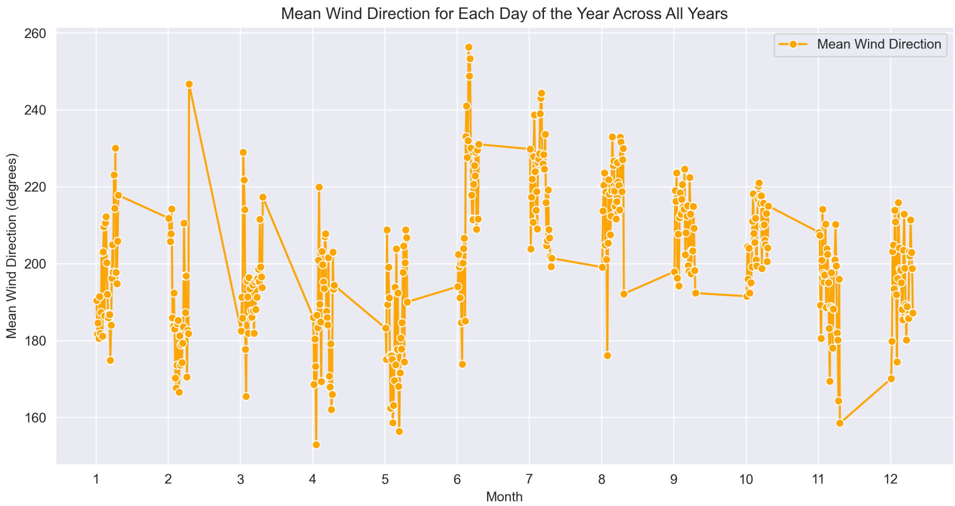

Figure 14: Seasonal Trend of Wind Direction

On the other hand, Figure 14 shows the mean wind direction of each day of the year across all years. It shows an interesting trend of the wind coming from southern directions as the spring approaches, however when summer starts the direction changes suddenly coming from northern direction reaching its peak and keep falling till the beginning of next summer. The mean wind direction does not change that much though, it moves in a range between 230 deg and 180 deg.

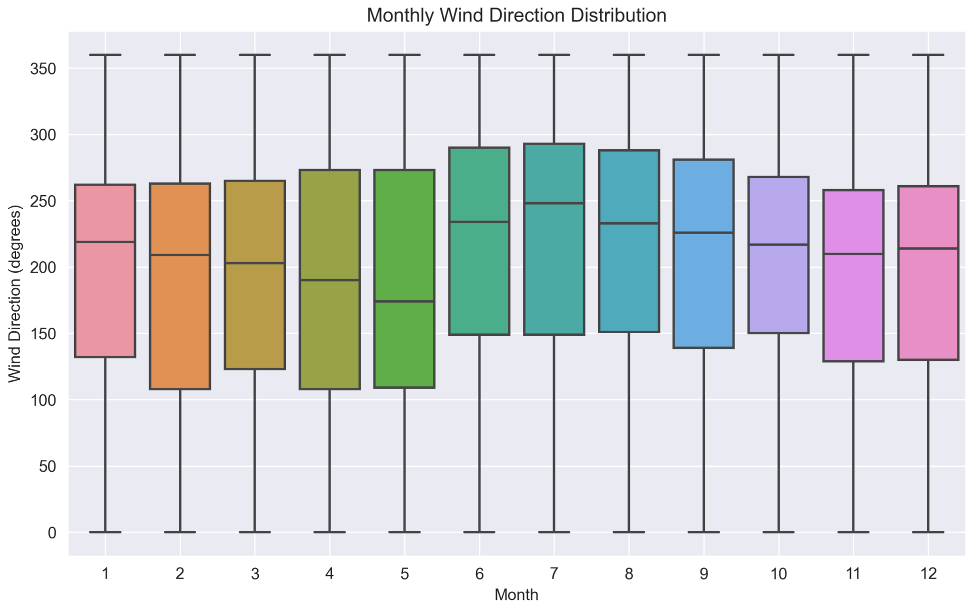

Figure 15: Seasonal Trend of Wond Direction

Figure 15 shows the box plots of monthly wind direction distribution, the plot shows the same statistic as the box plot above. Nevertheless, in this wind direction case it is seen less regularity in the monthly distributions than in the wind speed. It look like colder months (winter) and hotter months (summer) have a more tilted to right distribution and the milder season have a more normal distribution even having May with a distribution more tilted to the left. So hotter and colder months have less even distribution of wind direction, what means more sudden and abrupt changes in wind direction. It is also seen that all moths have a similar spread on their distributions.

References

[1] Jacob Berg; Mark Kelly; Jakob Mann; Morten Nielsen. “Micrometeorology for Wind Energy”. In: *DTU 46100* (2022).