

**Assignment 2** (Wind profiles, shear, and stability).

In this assignment you will study vertical profiles of the mean wind speed, over horizontally homogeneous terrain (i.e., flat with a uniform surface).

Data from cup and sonic anemometers, mounted on the meteorological mast at the Danish Test Center for Large Wind Turbines at Høvsøre from the year 2008 are provided (under **Data+Files for Assignments/Files for Assignment 2**) in DTU-Learn and should be used in the analysis. Download the data set (**Hoevsore2008.csv**). It contains 10-minute averages of quantities measured from different heights, with 1 header-line to label the data columns; they are:

1. date [yyyymmddhhmm] ('name')
2.  $u_\star$  [ $\text{ms}^{-1}$ ] ('u\_star')
3.  $Q_0$  [ $\text{K ms}^{-1}$ ] ('wsc')
4. incoming wind direction [deg. clockwise from N] at  $z = 60$  m ('dir\_metmast\_60m')
5. surface-layer temperature [ $^\circ\text{C}$ ] ('T\_10m')
6. mean wind speed [ $\text{ms}^{-1}$ ] at 10m ('wsp\_metmast\_10m')
7. mean wind speed [ $\text{ms}^{-1}$ ] at 40m ('wsp\_metmast\_40m')
8. mean wind speed [ $\text{ms}^{-1}$ ] at 60m ('wsp\_metmast\_60m')
9. mean wind speed [ $\text{ms}^{-1}$ ] at 80m ('wsp\_metmast\_80m')
10. mean wind speed [ $\text{ms}^{-1}$ ] at 100m ('wsp\_metmast\_100m')

where the header-labels are given in parenthesis above.

**The following tasks should be addressed:**

1. Calculate the reciprocal of Obukhov length,  $L^{-1}$ , from the data (recall that since  $Q_0$  is often close to zero, i.e. neutral conditions, then  $1/L$  is more useful than  $L$ ). Show its PDF.
2. Choose easterly wind directions (wind *from* the East) between  $70$ – $110^\circ$ , and select wind speed data for *neutral* conditions from heights  $z = \{10, 40, 60, 80, 100\}$  m; *in this assignment we'll use the criterion  $|L^{-1}| < 0.0005 \text{ m}^{-1}$  for neutral stratification.*

(a.) Average the 10-min. mean wind speeds at each height, then plot the wind profile in a semi-logarithmic coordinate system (with  $U$  on the horizontal axis and  $z$  on the logarithmic vertical axis); include 'error bars' showing the long-term standard deviation  $\sigma_U$  at each  $z$ .

(b.) Fit a straight line to the average profile of mean wind speeds you just found in semi-log space, for the anemometers from 10–80 m; add this line to the plot you just made.

(c.) From the fit and log-law, what are the resulting (mean) values of  $u_\star$  and  $z_0$ ?

What type of vegetation does this  $z_0$  imply?

(d.) Vertical extrapolation via log-law: with the  $z_0$  just obtained under neutral conditions, use the mean wind speeds at 60m and 80m respectively to give two predictions for the mean wind under neutral conditions at 100m,  $U_{N,100}$ . What is the %-difference between each predicted  $U_{N,100}$  and the observed  $U_{N,100}$  calculated in task 2(a) earlier?

(e.) Calculate the long-term mean shear exponent  $\langle\alpha\rangle$  using a process analogous to 2(b) above, over 10–80m; hint: recall  $\alpha = d \ln U / d \ln z$ .

What  $z_0$  is implied by this neutral-condition  $\langle\alpha\rangle$ ?

(f.) Now calculate  $\langle\alpha\rangle$  from the means at  $z = 60$  m and 80 m. How does it compare to the shear exponent calculated over the range of observations from 10–80 m done in task 2(e)?

(g.) 'Shear extrapolation': from the two  $\alpha$  you calculated in tasks 2(e–f) above, employ the power-law profile to make predictions of the mean easterly wind speed at 100 m height in neutral conditions, with the 'reference' wind taken as the mean at  $z = 80$  m.

- (h.) How do the two  $\alpha$ -based predictions of  $U_{N,100}$  in 2(g) compare to the log-law predictions from 2(d), and to the actual mean measurements?
3. Again using data from the Easterly directions, now additionally select data for non-neutral stratification categories: use  $0.0005 < L^{-1} < 0.05 \text{ m}^{-1}$  for stable, and  $-0.05 < L^{-1} < -0.0005 \text{ m}^{-1}$  for unstable conditions, ignoring the more extreme stabilities.
- (a.) Plot vertical profiles of  $U$ , for stable and for convective conditions, in the same graph along with the neutral profile; also calculate and plot the mean wind profile over *all* conditions.
- (b.) Is it reasonable to neglect the extremely non-neutral conditions? Why?
- (\*) Bonus credit: make a second plot of the *non-dimensional* wind profiles ( $U/u_*$ ) for each condition; explain why the dimensional profiles in 3(a) can cross one another.
4. The Monin-Obukhov similarity function

$$\Phi_m(z/L) = \frac{dU/dz}{u_*/(\kappa z)} \quad (1)$$

is actually a measure of dimensionless shear, in terms of the stability.

- (a.) Now you will find profiles of  $\Phi_m$  (i.e.,  $z$  versus  $\Phi_m$ ): calculate  $\kappa z u_*^{-1} dU/dz$  averaged over stable, neutral, and unstable cases respectively, plotting them together on a single graph.
- (b.) The neutral value of  $\Phi_m$  should *ideally* be equal to 1. Why?
- (c.) How much do your neutral  $\Phi_m$  deviate from 1, and why?
5. (a.) Using the definition (1) for  $\Phi_m$ , derive the M-O wind profile

$$U(z) = \frac{u_*}{\kappa} \left[ \ln(z/z_0) - \Psi_m(z/L) \right];$$

i.e., write an (integral) expression for  $\Psi_m(z/L)$  in terms of  $\Phi_m(z/L)$ .

- (b.) Explain what use/effect  $\Psi(z/L)$  has on the wind profile (hint: consider what you plotted in task 3 above).
6. Which type of non-neutral conditions most strongly affect vertical extrapolation? How frequently do these occur?
7. Again as in task 3, plot the ‘total’ wind profile (all conditions) for the easterly winds considered. Now also over-plot the power-law profile for two cases: [a] using  $\alpha$  from 40-80m with the 40m winds for the reference, and [b] using  $\alpha$  from 60-80m and the 80m winds as reference. How do these compare?
8. For vertical extrapolation (‘VE’) from measurement heights to hub-heights, the industry often uses  $\alpha$  for each 10-minute period to ‘shear up’ and then calculate wind statistics.
- (a.) What happens to the Weibull- $k$  parameter in this process?
- (b.) What added info does M-O theory require, compared to the power-law?
- (c.) What are the assumptions behind using each type of profile for VE?