Danmarks Tekniske Universitet

Introduction to Micrometeorology for Wind Energy

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Assignment 2: Wind Profiles, Shear and Stability

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October 7, 2024

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Introduction and Data Cleaning

This assignment is focused on studying the vertical profiles of the mean wind speed, over horizontally homogeneous terrain. To do so, Data from cup and sonic anemometers, mounted on the meteorological mast at the Danish Test Center for Large Wind Turbines at Høvsøre from the year 2008 is used.

The provided data contains 10-minute averages of quantities measured from different heights: 10m, 40m, 60m, 80m and 100m.

After a general check, it is decided that the data does not need further treatment as it looks reliable.

1 Task 1: Reciprocal of Obukhov length

The first step is calculating the Obukhov lenght (L) from the dataset mentioned in section . However, as the Q_0 is close to zero, the system is in natural constitutions, then $1/L$ is more useful to be computed than L. Thus, for each timestamp ($1/L$) will be calculated using Equation 1

$$\frac{1}{L} = (\frac{kg}{\theta_0})\frac{-Q_0}{u_\star^3} \tag{1}$$

Where:

- $k = 0.4$, is the Karman constant
- k, is the gravity
- θ_0, is the mean near surface temperature
- Q_0, is the surface heat flux
- $u_\star^3$, is the friction velocity

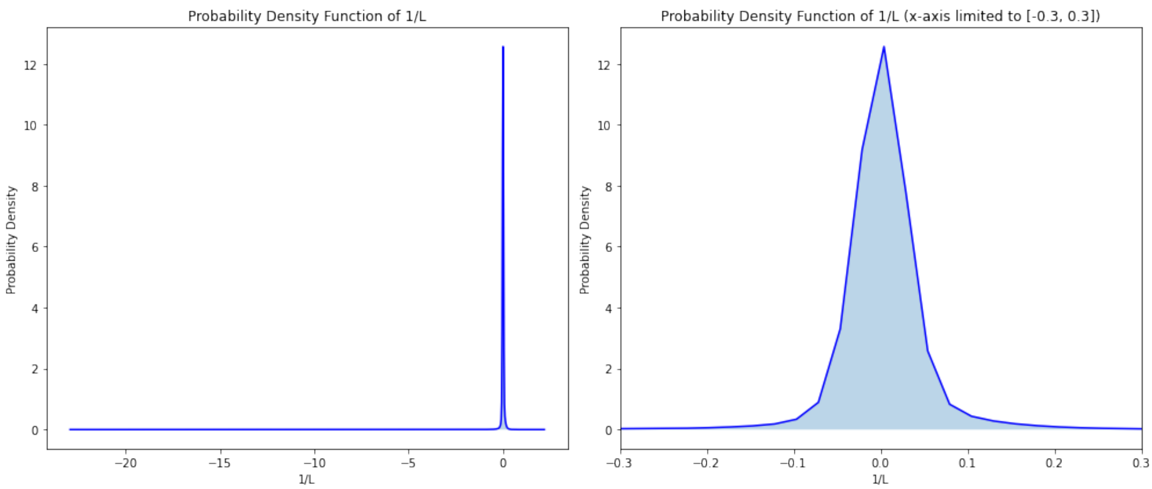

Figure 1: Probability density function $1/L$ for Høvsøre dataset, right figure 1/L limited to [-0.3, 0.3]

To visualise the results of the $1/L$ computation for each timestamp, the probability density function of it is show in Figure 1. The PDF appears to be highly skewed with a very sharp spike near zero and virtually no probability density for negative values far from zero. The sharp spike at or near zero indicates that the probability density is very high when the value of $1/L$ approaches zero from the right. This suggests that the variable L (which would be the inverse of $1/L$) takes on very large values with high probability. In simpler terms L is likely to be a very large positive number.

Knowing that:

- $L^{-1} > 0$, means stable atmosphere
- $L^{-1} < 0$, means unstable atmosphere
- $L^{-1} = 0$, means neutral conditions

Therefore, a sharp spike at zero suggests that neutral conditions are most likely to occur. In other words, the atmospheric state is most frequently in a condition where turbulence production by mechanical shear is roughly balanced by buoyancy effects. The very little probability density for negative values of L^{-1} indicates that unstable conditions are relative rare. For positive values of L^{-1}, the probability is still low.

2 Task 2: Velocities from easterly directions and neutral conditions

a) and b) Plotting of mean velocities with height

The mean wind speeds at each height (10m, 40m, 60m, 80m, and 100m) have been calculated using the filtered data for easterly wind directions and neutral conditions. The resulting average wind speeds are:

Height (m)	**Wind Speed (m/s)**
10	6.37
40	7.71
60	8.22
80	8.65
100	9.02

Table 1: Wind Speed at Different Heights

These wind speeds were then plotted in a semi-logarithmic coordinate system, with wind speed U on the horizontal axis and height z on the logarithmic vertical axis. Error bars were included in the plot to show the long-term standard deviation of the wind speeds at each height, providing an indication of the variability in the measurements. In addition, a straight line was fitted to the wind speed profile for the anemometers located between 10 m and 80 m, using a linear regression in semi-log space.

In figure 2, an exponential increase in mean velocity with height can be observed, which appears as an almost linear growth in the semi-logarithmic plot. Additionally, the size of the error bars, represented by the standard deviation, is noticeably large.

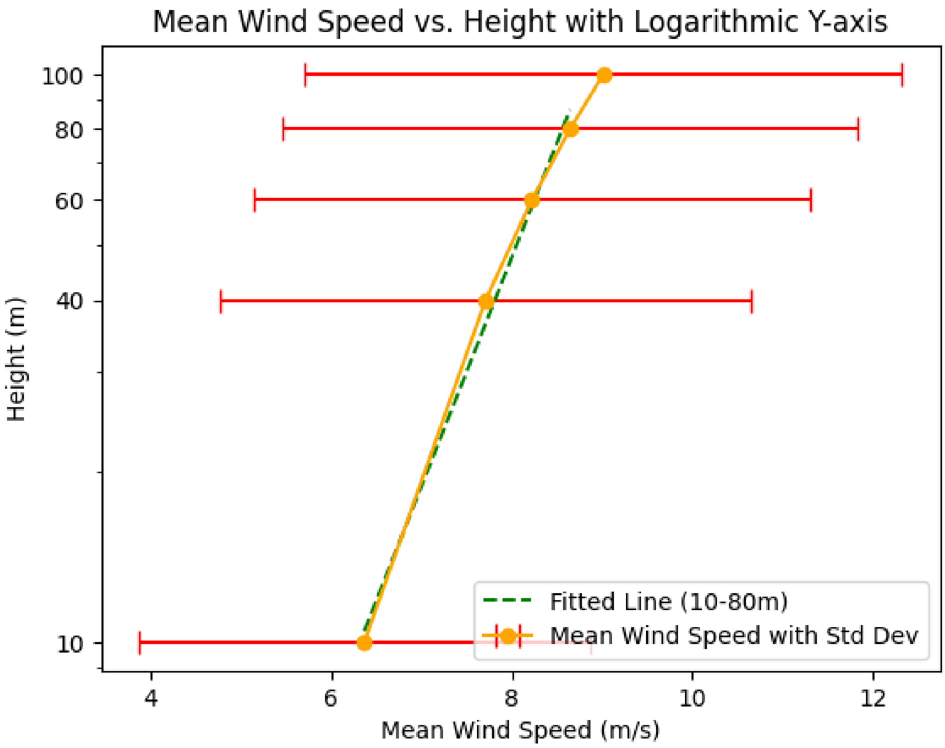

Figure 2: Semi-logarithmic representation of the wind profile for different heights

c) Calculation of z_0 and u_*

The fitted line can also be used to infer parameters such as the surface roughness length z_0 and friction velocity u_*, which are derived from the slope and intercept of the line:

$$U(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right)$$

$$U(z) = \frac{u_*}{\kappa} \left(\ln(z) - \ln(z_0)\right)$$

$$\ln(z) = \frac{\kappa}{u_*} U(z) + \ln(z_0)$$

$$y = m \cdot U(z) + b$$

$$m = \frac{\kappa}{u_*}, \quad b = \ln(z_0)$$

$$u_* = \frac{\kappa}{m}, \quad z_0 = e^b$$

The resulting parameters are $u_* = 0.43$ and $z_0 = 0.028$. According to [1], this roughness length suggests the terrain is a "farmland with very few buildings or trees".

d) Vertical extrapolation via log-law:

Using the log-law and the calculated z_0, the wind speeds at 100 m were predicted based on the data at 60 m and 80 m. The predicted wind speeds at 100 m are:

$$u_{60\to100} = 8.77\,\mathrm{m/s}$$

$$u_{80\to100} = 8.89\,\mathrm{m/s}$$

The percentage differences between these predictions and the observed wind speed at 100 m are:

$$\text{Difference using 60 m:}\,2.73\%$$

$$\text{Difference using 80 m:}\,1.40\%$$

These small percentage differences indicate that the vertical extrapolation using the log-law provides reasonably accurate predictions of the wind speed at 100 m, especially when using data from 80 m. The lower error from 80 m suggests that extrapolation from heights closer to the target height yields more accurate predictions under neutral conditions.

e) Calculation of the long-term mean shear exponent α

The long-term mean shear exponent α is a parameter that describes how wind speed varies with height in the surface layer. It is defined as:

$$\alpha = \frac{d \ln U}{d \ln z}$$

By performing a linear regression on the logarithmic transformation of the heights and wind speeds, the slope α is obtained:

$$\alpha = \text{slope}$$

The value of the long-term mean shear exponent α is:

$$\alpha = 0.145$$

Next, we calculate the roughness length z_0 using the reference height z_{ref}, which is the geometric mean of the heights:

$$z_{\text{ref}} = (10 \times 40 \times 60 \times 80)^{1/4}$$

The roughness length z_0 is then computed as:

$$z_0 = z_{\text{ref}} \times \exp\left(-\frac{1}{\alpha}\right) = 0.038$$

This roughness length z_0 differs from the one previously calculated but still falls within the terrain classification described as "farmland with very few buildings or trees".

The plot generated below shows the relationship between mean wind speed and height on a logarithmic scale, visually representing the wind profile under neutral conditions.

The fitted line, based on the logarithmic transformation of both the wind speeds and heights, provides a clear visual representation of the relationship. As shown, the fitted line closely follows the wind speed measurements between 10 m and 80 m, confirming the logarithmic behavior of the wind profile. Any deviations observed at higher heights, such as at 100 m, may indicate that the assumptions of the log-law become less valid as we move farther from the reference measurements.

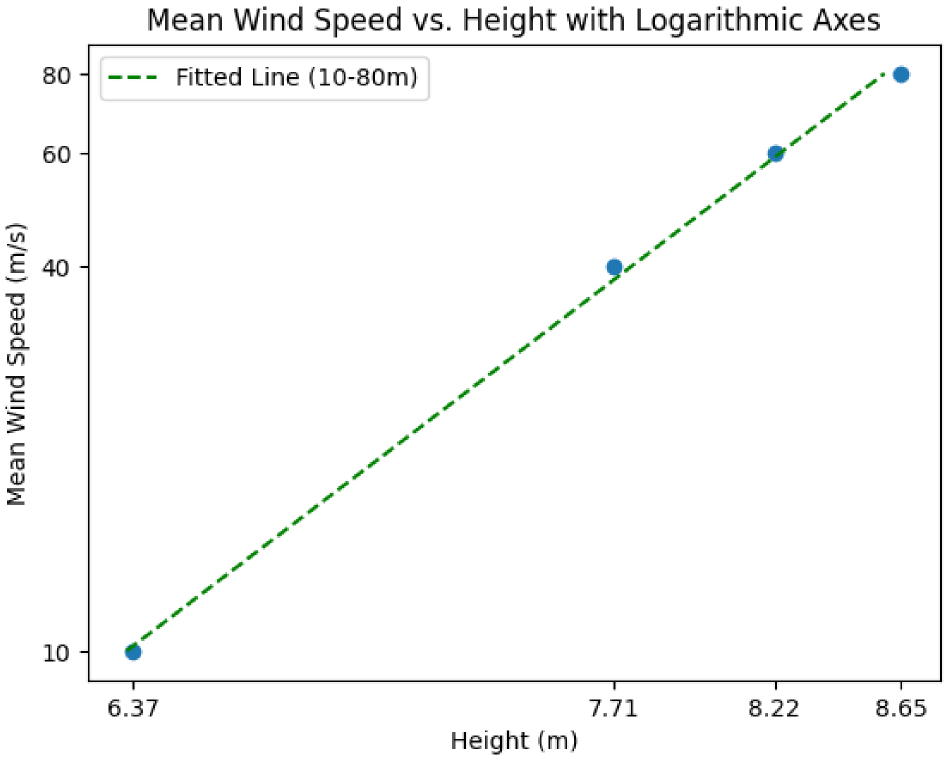

Figure 3: Logarithmic representation of the wind profile for different heights

(f) Shear Exponent Calculation using 60 m and 80 m Data

The shear exponent α was calculated using the wind speeds at heights 60 m and 80 m, following the formula:

$$\alpha = \frac{\ln\left(\frac{U_{80m}}{U_{60m}}\right)}{\ln\left(\frac{80}{60}\right)}$$

Substituting the values for the wind speeds at 60 m and 80 m, the calculated shear exponent is:

$$\alpha = 0.1753$$

This shear exponent represents the rate of change in wind speed with height between 60 m and 80 m. When compared to the shear exponent calculated over the range from 10 m to 80 m in part (e), the exponent calculated here is 20.78% larger. This indicates that the change in wind speed between 60 m and 80 m is more pronounced than over the broader range of 10 m to 80 m, suggesting stronger wind shear at higher altitudes.

(g) and (h) Shear Extrapolation and Comparison

Using the shear exponents calculated in parts (e) and (f), the wind speed at 100 m was predicted under neutral conditions. The predictions were made using the wind speed at 80 m and the shear exponents as follows:

$$U_{100,\alpha} = U_{80} \left(\frac{100}{80}\right)^{\alpha_{\text{slope}}}$$

$$U_{100,\alpha_{\text{means}}} = U_{80} \left(\frac{100}{80}\right)^{\alpha_{\text{means}}}$$

The percentage differences between the predicted wind speeds and the observed wind speed at 100 m were calculated, as well as the differences in the predictions using the wind speeds at 60 m and 80 m. The results are summarized in the following table:

Reference Height	Error using α_{slope}	Error using α_{means}
60 m	1.84%	2.53%
80 m	0.47%	1.15%
Actual wind speed	0.94%	0.27%

Table 2: Percentage differences between predicted and observed wind speeds at 100 m.

The results indicate that the predictions using the shear exponents are fairly accurate, with small percentage differences from the observed values. The error is slightly lower when using the wind speed at 80 m, which is expected as this height is closer to 100 m.

3 Task 3: Analysis of Wind Profiles under Different Stability Conditions

In this task, the vertical profiles of wind speed (U) as a function of height for different atmospheric stability conditions are analyzed. Data from the Easterly directions are used, and the analysis is limited to non-neutral stratification categories defined as $0.0005 < 1/L < 0.05$ for stable conditions, and $-0.05 < 1/L < -0.0005$ for unstable conditions, while neutral conditions are based on near-zero values of $1/L$. For this study, three datasets corresponding to different standard deviations (0.05, 1 and 3 times) of the $1/L$ probability density function (PDF) are displayed and used to assess the effect of extreme stratifications on wind profiles. These datasets were chosen with the definition of outliers in mind. In this case, outliers are data points that fall beyond 3 times the standard deviation of the 1/L distribution, usually representing rare or extreme atmospheric conditions.

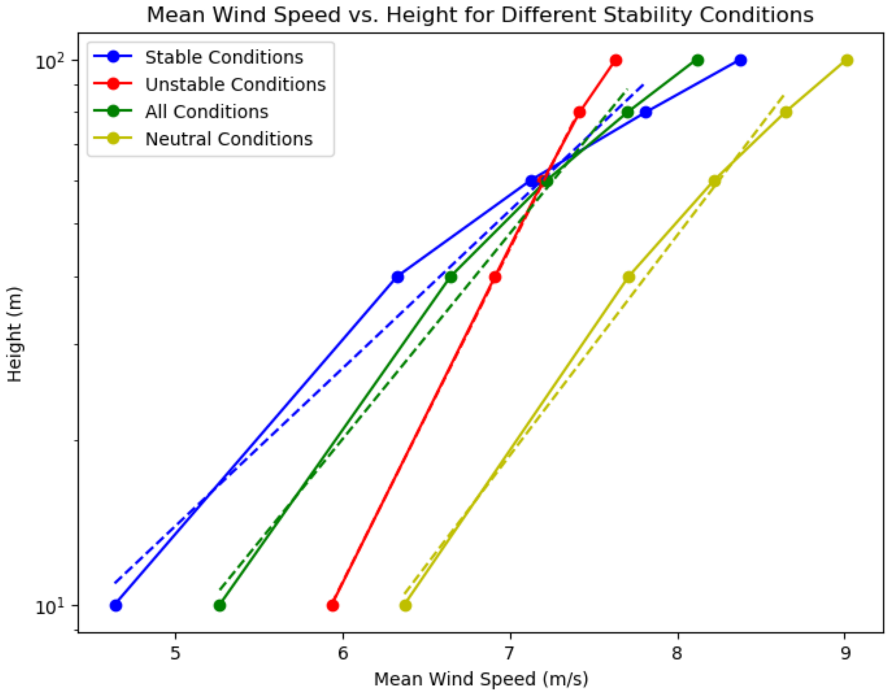

Figure 4: Wind speed profiles for stable, unstable, neutral and all conditions using data within 0.05 standard deviation of the| $1/L$ |PDF.

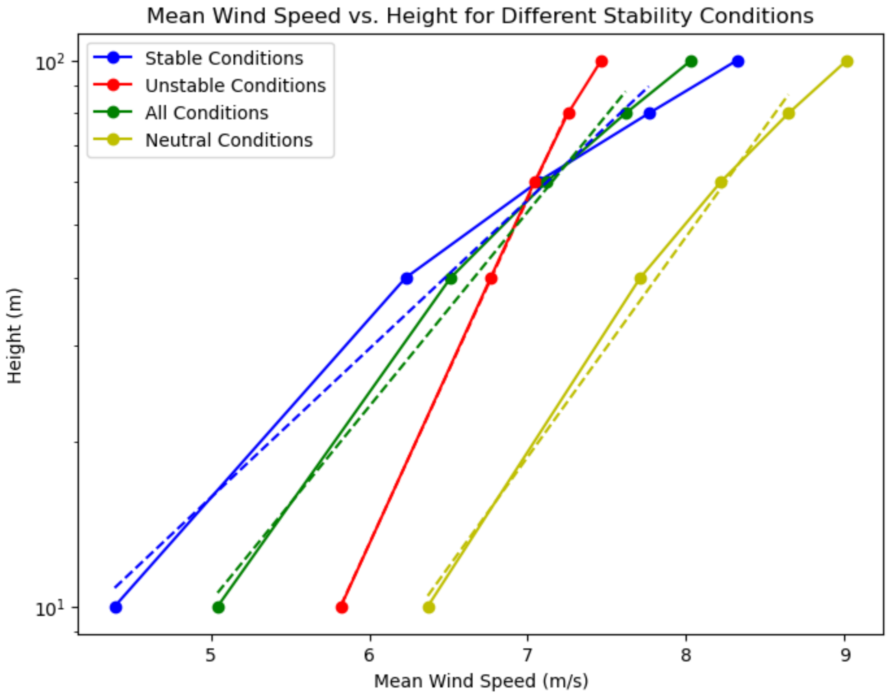

Figure 5: Wind speed profiles for stable, unstable, neutral and all conditions using data within 1 standard deviation of the| $1/L$ |PDF.

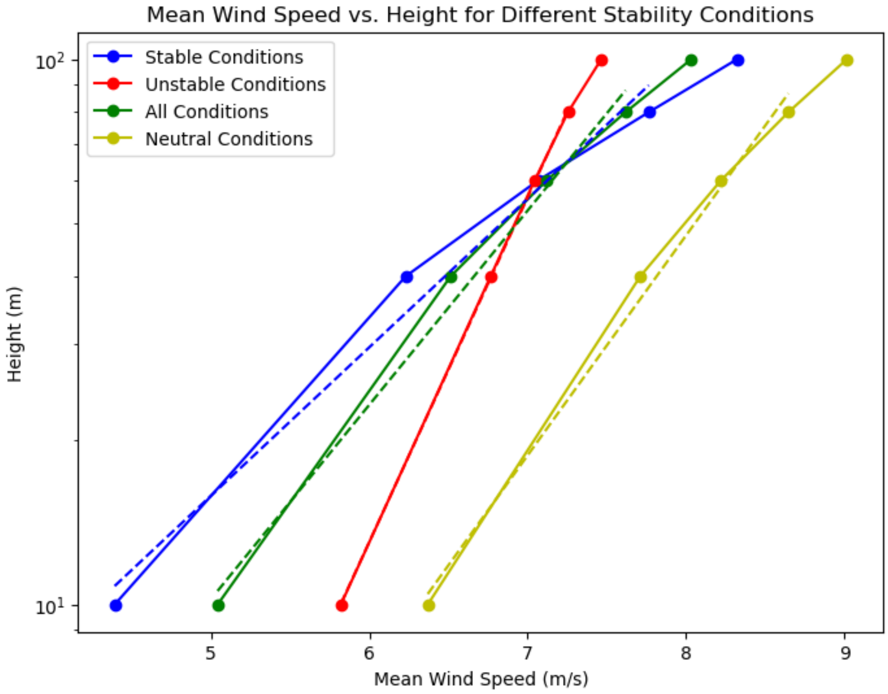

Figure 6: Wind speed profiles for stable, unstable, neutral and all conditions using data within 3 standard deviation of the| $1/L$ |PDF.

In the wind speed profiles displayed, distinct behaviors due to the nature of atmospheric mixing under different stability regimes are observed:

- **Stable Conditions**: The wind speed under stable conditions shows a gradual increase with height. This is because a stable atmosphere resists vertical mixing, leading to a slower growth in wind speed with altitude.
- **Unstable Conditions**: In contrast, the unstable profile shows a steeper increase in wind speed. The unstable atmosphere promotes vertical mixing, causing more rapid changes in wind speed as the height increases.
- **Neutral Conditions** : The neutral profile lies between the stable and unstable profiles. It reflects a state where vertical mixing is moderate, and the wind speed increases at a rate between the other two conditions.
- **All Conditions** : The profile using all conditions combines data from all stratification regimes, averaging the effects of stability and instability.

3.1 Neglecting Extremely Non-neutral Conditions

Neglecting extremely non-neutral conditions (those beyond the range of $0.0005 < \left|\frac{1}{L}\right| < 1$ standard deviations) is considered reasonable for this dataset. This is evident from the minimal changes observed between figures 4 and 5, 6, as well as from the near-identical values in table 3 when comparing the datasets limited to 1 and 3 standard deviations. The extreme non-neutral conditions are infrequent and do not meaningfully alter the overall wind profile trends. Including them in the analysis could introduce biases or outliers, which do not reflect the common wind profiles encountered in most atmospheric situations. Moreover, extreme non-neutral conditions are often outliers resulting from rare meteorological events or measurement noise, which may disproportionately affect the results.

Table 3: Wind Speed Data at Different Heights and Standard Deviations

		10m	40m	60m	80m	100m
0.05 std	**Stable**	4.63	6.32	7.12	7.81	8.37
	Unstable	5.93	6.91	7.19	7.41	7.63
	Neutral	6.36	7.71	8.22	8.64	9.01
	All	5.26	6.64	7.22	7.70	8.11
1 std (% difference)	**Stable**	5.26%	1.53%	0.80%	0.62%	0.70%
	Unstable	1.93%	2.11%	2.16%	2.19%	2.22%
	Neutral	0.00%	0.00%	0.00%	0.00%	0.00%
	All	4.22%	1.99%	1.39%	1.11%	1.04%
3 std (% difference)	**Stable**	5.26%	1.53%	0.80%	0.62%	0.70%
	Unstable	1.93%	2.11%	2.16%	2.19%	2.22%
	Neutral	0.00%	0.00%	0.00%	0.00%	0.00%
	All	4.22%	1.99%	1.39%	1.11%	1.04%

4 Task 4: Monin-Obukhov similarity function

The Monin-Obukhov similarity function (Equation 2), describes the structure of turbulence in the atmospheric boundary layer, by defining the dimensionless shear $\Phi_m\left(\frac{z}{L}\right)$.

$$\Phi_m\left(\frac{z}{L}\right) = \frac{\frac{dU}{dz}}{\frac{u_*}{\kappa z}} \tag{2}$$

4.1 Profiles of Φ_m

For each height (10m, 40m, 60m, 80m) wind speed measurement, the mean is computed, creating a profile of four dots. These four dots create four different segments on the profile. Then, the slope of each segment (representing how quickly the wind speed changes with altitude) is calculated, obtaining then the dU/dz part form Equation 2. This result along with the height (z) are the variable parts of Equation 2. This procedure is done fore stable, unstable and neutral cases. Getting the result shown in Figure 7.

$\Phi_m(\frac{z}{L})$ is a measure of dimensionless shear, which adjusts the relationship between wind speed gradient and height to account for atmospheric stability effects. It is central to describing how wind speed varies with height in different thermal stratification regimes (neutral, stable, and unstable), and is crucial for understanding and modeling turbulence in the atmospheric boundary layer. On Figure 7, a vertical lines is plotted on $\Phi = 1$ this line shows the limit between stable (right-hand side of $\Phi = 1$) and unstable conditions (left-hand side of $\Phi = 1$. The result obtained match with the latest description, by showing that the data filtered by unstable stable and neutral conditions by using L value matins in their condition when calculating Φ.

It is observed in results shown in Figure 7, that when having stable conditions the dimensionless shear increases way quicken over height than when having unstable and neutral conditions. Whereas unstable when unstable conditions, the dimensionless shear is the one that increases the less by increasing height. It is a similar profile that the one obtained in neutral conditions.

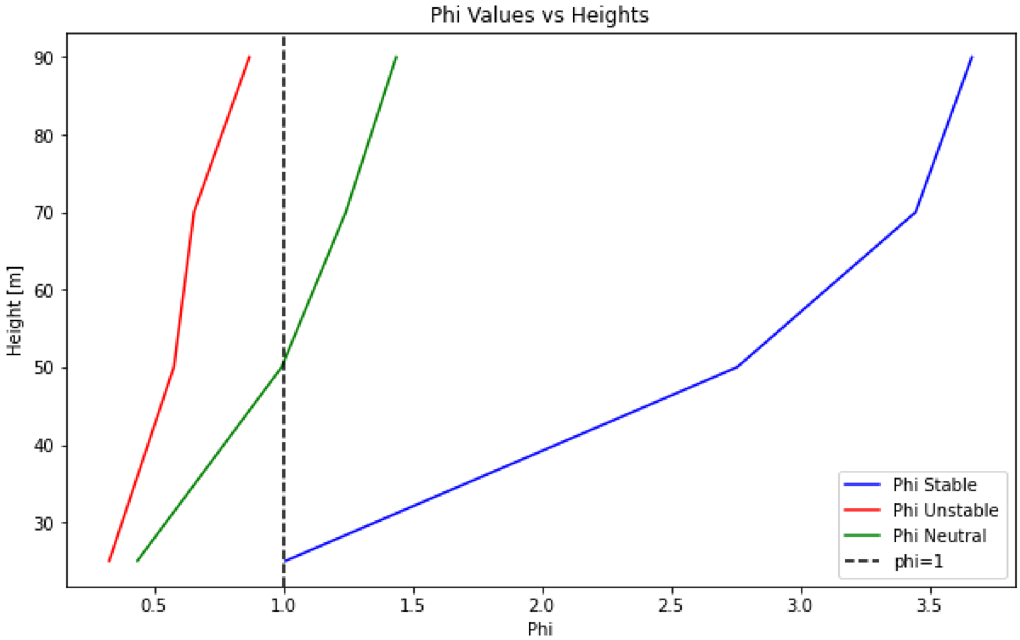

Figure 7: Φ_m stable, unstable and neutral profiles vs height

4.2 Natural value of Φ_m

When $z/L = 0$ the value of $\Phi_m(\frac{z}{L})$ is close to 1, which means the wind speed profile follows the logarithmic law. When $z/L > 0$, (stable conditions), $\Phi_m(\frac{z}{L})$ is usually greater than 1, indicating that the wind speed gradient is stronger due to suppression of turbulence by stable stratification. When $z/L < 0$, (unstable conditions) is typically less than 1, indicating enhanced mixing and a weaker wind speed gradient due to convective turbulence. The neutral value of Φ_m should ideally be equal to 1 because, under neutral atmospheric stability conditions, the effects of buoyancy are negligible, and turbulence is driven purely by mechanical shear.

4.3 Deviation of Φ_m from 1

The neutral Φ_m profile deviates from $\Phi_m = 0.44$ to $\Phi_m = 1.44$, thus clearly not staying on a vertical line with all values of Φ_m for neutral conditions being equal to 1, which theoretically it is what is supposed to happen. The deviations of Φ_m from the neutral value of 1 (ranging from 0.44 to 1.44) are likely due to a combination of factors like: measurement errors or localized influences on the wind profile; or it could simply be a place with non-stationary conditions and with intermittency in turbulence, creating a heterogeneous and variable surface roughness.

5 Task 5: Integral Expression for $\Phi_m(z/L)$

5.1 Derivation M-O Wind Profile

The Monin-Obukhov (M-O) wind profile equation is used to describe the variation of wind speed with height in the atmospheric surface layer, accounting for the effects of atmospheric stability. This equation builds upon the logarithmic wind profile and includes a correction factor that depends on stability conditions.

Using the definition of the dimensionless shear $(\Phi_m(z/L))$ given by the Monin-Obukhov similarity function (Equation 2) the M-O wind profile is derived (Equation 9). The in-between step from the later two equations mentions are the ones showed by: Equation 3, Equation 4, Equation 5, Equation 6, Equation 7, Equation 8, as follows:

$$\frac{dU}{dz} = \Phi_m(z/L)\frac{u_*}{kz} \tag{3}$$

$$dU = \Phi_m(z/L)\frac{u_*}{kz}dz \tag{4}$$

$$\int_{u(z)=0}^{u(z)} du = \int_{z_0}^{z} \Phi_m(z/L)\frac{u_*}{kz}dz \tag{5}$$

$$U(z) = \frac{u_*}{k}\int_{z_0}^{z} \frac{1}{z}\Phi_m(z/L)dzz \tag{6}$$

$$= \frac{u_*}{k}\int_{z_0}^{z} \frac{1}{z}\left(1 - (1 - \Phi_m(z/L))\right)dz \tag{7}$$

$$\text{Where Where } \psi_m(z/L) = \int_{z_0}^{z} \frac{1}{z}\left(1 - \Phi_m(z/L)\right)dz \tag{8}$$

$$U(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_0}\right) - \psi_m(z/L)\right] \tag{9}$$

5.2 Effect of Ψ on wind profile

The $\psi_m(z/L)$ is the stability correction function for momentum, which accounts for the influence of atmospheric stability. The $\psi_m(z/L)$ is a key component in the Monin-Obukhov wind profile equation that modifies the classic logarithmic wind profile to account for the effects of atmospheric instability. It adjusts the wind speed profile based on whether the atmosphere is neutral, stable, or unstable. In neutral conditions ($z/l = 0$, $\psi_m(z/L) = 0$), leading to a pure logarithmic profile. In stable conditions ($z/l > 0$, $\psi_m(z/L) > 0$), resulting in a steeper wind gradient due to turbulence. In unstable conditions (($z/l < 0$, $\psi_m(z/L) < 0$)), creating a shallower gradient with enhanced mixing. Thus, $\psi_m(z/L)$ adjusts the wind profile for non-neutral effects.

6 Task 6: Non-Neutral conditions Affecting Vertical Extrapolation

If the easterly wind conditions are analyzed, it can be observed that 94.7% of the samples correspond to non-neutral atmospheric conditions, with 56.6% being stable conditions and 38.1% unstable conditions. On the other hand, 5.4% of the samples represent neutral conditions 8.

From a theoretical perspective, vertical extrapolation is most strongly affected by stable conditions, as the thermal stratification limits vertical air mixing, leading to a more conservative (lower) wind speed at higher altitudes. In contrast, unstable conditions, which represent 38.1%, enhance vertical mixing, often resulting in a steeper wind speed gradient and less conservative extrapolation. Since stable conditions occur in more than half of the measurements, these are the dominant factor influencing vertical wind speed extrapolation. Therefore, the frequency of stable conditions suggests that the bias towards conservative wind speeds will be present in more than one out of every two measurements, making them the most impactful non-neutral condition.

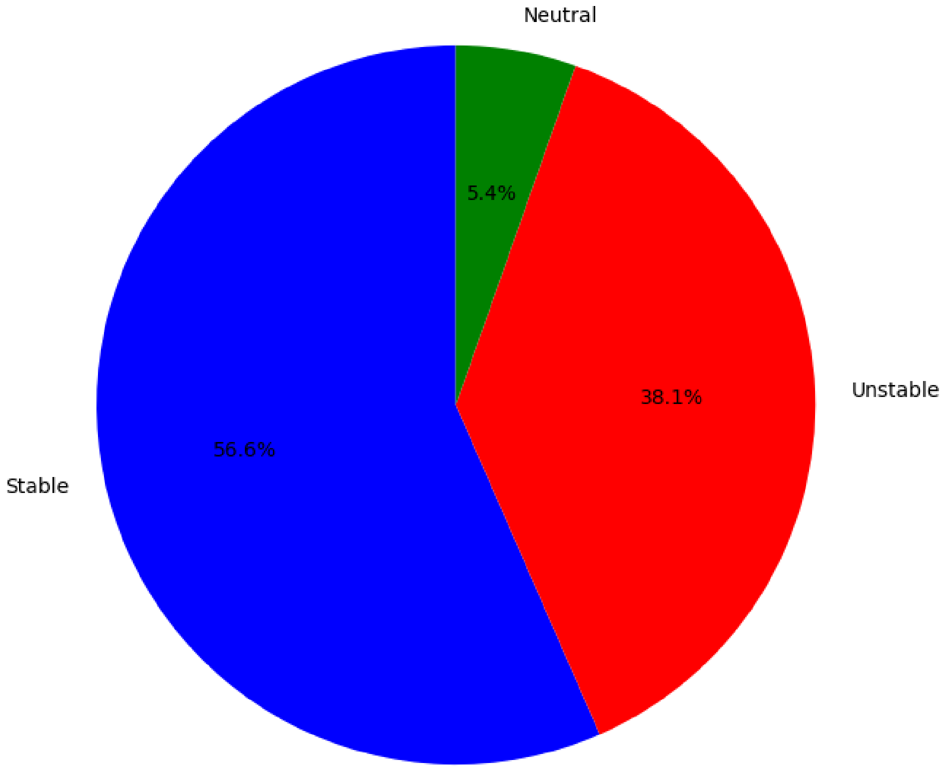

Figure 8: Distribution of neutral and non-neutral conditions for easterly winds

7 Task 7: Wind Profiles and Power-law Profiles

In this task, the total wind profile (represented by the green line) shows how wind speed changes with height across all conditions. It is important to note that in this plot, the y-axis (height) is non-logarithmic. This allows for a clearer comparison of the wind shear, as the changes in wind speed with height are more easily observable on a linear scale.

The red line corresponds to the power-law profile using α from the 40-80 m range, with the 40 m wind speed as the reference, while the blue line uses α from the 60-80 m range, with the 80 m wind speed as the reference.

From the comparison, it can be observed that the power-law profile using α for the 40-80 m range better predicts wind speeds at lower heights, closer to the surface. On the other hand, the power-law profile based on the 60-80 m range provides a more accurate prediction of wind speeds closer to the 100 meters mark.

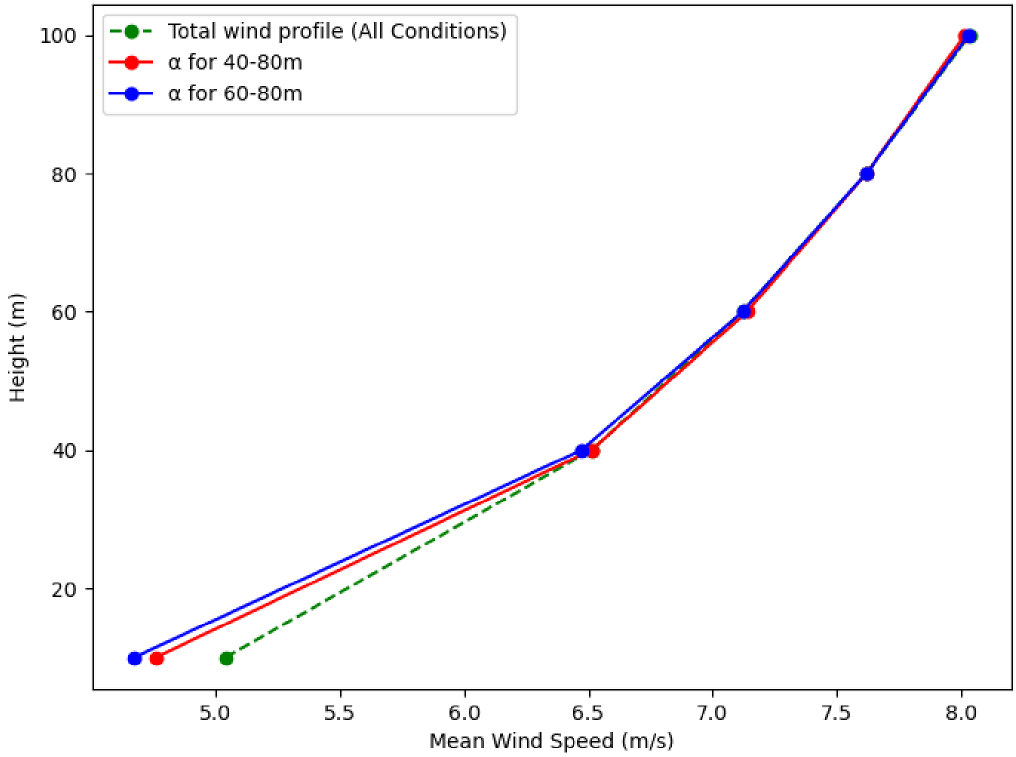

Figure 9: Wind profiles obtained from task 3 and the calculated α for different reference heights.

An important observation is that both profiles tend to closely match the total wind profile in the mid-range heights (40-80 m), showing good agreement with the overall trend. However, the small divergence at the lower and upper heights suggests that the choice of height range for calculating α affects accuracy at different altitudes. This highlights the importance of selecting an appropriate reference range for power-law approximations depending on the height range of interest. Overall, the closer the reference measurement is to the target height, the more accurate the results, while accuracy decreases as the reference point moves further from the height of interest.

8 Task 8: Vertical Extrapolation

8.1 How "shearing-up" affects the Weibull shape parameter.

The Weibull-k parameter is not invariant to vertical extrapolation. As the wind speed is "sheared up" to higher altitudes, the wind speed distribution can change, particularly under different atmospheric stability conditions.

Stable atmosphere: Under stable conditions (higher α), the wind profile tends to show more variability with altitude, which can decrease the k-value, indicating a wider spread in wind speeds.

Unstable atmosphere: Under unstable conditions (lower α), the wind profile is more uniform, potentially resulting in a higher k-value, reflecting less variability in wind speeds across heights.

8.2 Additional info required for M-O theory

For vertical extrapolation using Monin-Obukhov (M-O) theory, more information is needed compared to the simpler power-law method. M-O theory relies on parameters like the Obukhov length, friction velocity, and surface roughness to account for atmospheric stability, especially in non-neutral conditions.

These parameters can be challenging to measure accurately. For instance, the Obukhov length is difficult to measure directly because it depends on temperature gradients and turbulence, both of which fluctuate throughout the day. Similarly, friction velocity and surface roughness are affected by local terrain and environmental factors, which may vary with time or season, making them harder to measure reliably under certain conditions.

While the power-law relies solely on the wind shear exponent α, which is calculated based on observed wind speeds at different heights.

8.3 Assumptions behind the 3 different vertical extrapolation profiles.

1. Log-Law Assumptions

- **Spatial and Temporal Homogeneity:** The log-law assumes that the wind profile is homogeneous over space and time.
- **Neutral conditions:** It assumes neutral atmospheric conditions, meaning there's little impact from temperature or buoyancy. In unstable or very stable conditions, it won't work well.
- **Stable surface:** It assumes that the terrain (surface roughness) doesn't change much over time or space.
- **Results:** The log-law is most accurate close to the ground (within the ASL).

2. Power-Law Assumptions

- **Constant wind shear:** It assumes the wind shear stays the same (constant α) at all levels and can be used in higher altitudes. This can lead to inaccuracies if conditions change with altitude.
- **More adaptable:** It works under a wider range of atmospheric conditions compared to the log-law, but struggles with extreme stability or instability in the atmosphere due to the lack of the buoyancy term.
- **Results:** Can be used more easily based on measured data but for changing α with height it can fail.

3. Monin-Obukhov (M-O) Theory Assumptions

- **Handles complex conditions:** M-O theory is best for situations where the atmosphere is not neutral, like in stable or unstable conditions.
- **Accounts for temperature effects:** It takes into account how temperature and turbulence affect wind, which makes it more accurate in non-neutral conditions.
- **Results:** Its harder to use because it requires more detailed information, like temperature gradients and turbulence, which can be tricky to measure accurately but can be more efficient in higher altitudes.

References

[1] Jacob Berg; Mark Kelly; Jakob Mann; Morten Nielsen. “Micrometeorology for Wind Energy”. In: *DTU 46100* (2022).