Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Exercise 4

Name:

Student Number:

1. **Lagrangian Duality and Optimality Conditions**

We consider a power system with 3 generators (G_1, G_2, G_3) and 1 inflexible load (D_1). Each generator has a production cost $C_i^G = \alpha_i^G(\boldsymbol{p_i})^2 + \beta_i^G \boldsymbol{p_i}$ (in DKK). The production cost parameters α_i^G and β_i^G (in DKK/MWh) and maximum generation capacity $\overline{P}_i^G$ of all generators $i \in \mathcal{G} = \{1, ..., 3\}$ (in MWh), and the inflexible demand $\overline{P}_i^D$ of all loads $i \in \mathcal{L} = \{1\}$ (in MWh) are summarized in Figure 1. The system operator wants to dispatch

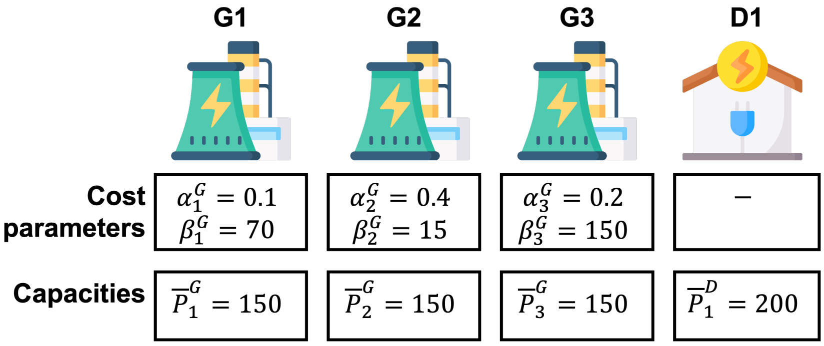

Figure 1: Generators and load parameters

the generators in order to cover this load at the lowest possible cost.

(a) Formulate the economic dispatch problem in this system.

(b) Solve this optimization problem and provide the values of its primal and dual variables, and objective value.

(c) Is this optimization problem convex? Justify your answer.

(d) Formulate the Lagrangian function, and the KKT conditions of this optimization problem. Are these KKT conditions necessary and sufficient optimality conditions? Justify your answer.

(e) Compare the KKT conditions of this problem for $\alpha_i^G = 0$ (i.e. linear production cost case) to the primal-dual optimality conditions of the linear ED problem in Exercise 3. Discuss how Lagrangian and LP duality relate to each others.

(f) Based on the KKT conditions, express the uniform price λ^* as a function of the marginal cost of the generators, and derive a relationship between the marginal production costs of the producers, at optimality.