Technical University
of Denmark

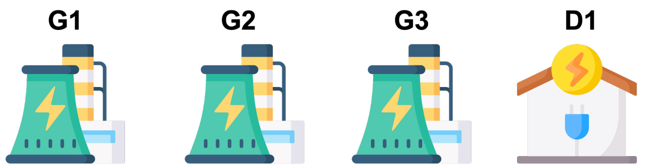
46750 - Optimization in Modern Energy Systems
Exercise 11

Name:

Student Number:

1. **Single-period economic dispatch (ED) problem**

We consider a power system with 3 generators, with quadratic cost functions $\beta_g^G \boldsymbol{p_g} + \alpha_g^G \boldsymbol{p_g}^2$ for $g \in \{G_1, G_2, G_3\}$, and 1 inflexible load D_1. The cost parameters and capacities of these units are summarized in Figure 1.



	G1	G2	G3	D1
Cost parameters	$\alpha_1^G = 0.1$, $\beta_1^G = 70$	$\alpha_2^G = 0.4$, $\beta_2^G = 15$	$\alpha_3^G = 0.2$, $\beta_3^G = 150$	–
Capacities	$\overline{P}_1^G = 150$	$\overline{P}_2^G = 150$	$\overline{P}_3^G = 150$	$\overline{P}_1^D = 200$

Figure 1: Generators and load parameters

(a) Formulate and solve the ED problem for this power system, and provide the optimal solutions, i.e., primal and dual variables and objective value. *Bonus question: observe the value of the electricity market price (dual variable of the balance equation). What is it equal to?*

(b) Is this optimization problem decomposable? And, if so, how? Define the complicating variables/constraints and the number of subproblems.

(c) Solve this optimization problem using the LR method.

   i. Provide the formulation of the equivalent optimization problem in which the complicating constraint is relaxed (based on Lagrangian duality).

   ii. Provide a detailed outline of the LR algorithm.

   iii. Provide the formulation of the subproblems and master problem at each iteration.

   iv. Implement the LR algorithm in the programming language of your choice and provide the optimal solutions of the original problem. You may try different values of the subgradient parameters a and b, the sensitivity parameter ϵ, the initial value of the Lagrange multiplier $\lambda^{(0)}$, and the maximum number of iterations N, and compare the convergence performances of the LR algorithm.

(d) Solve this optimization problem using the ALR method and ADMM algorithm.

i. Provide the formulation of the equivalent *augmented* optimization problem in which the complicating constraint is relaxed (based on Lagrangian duality).

ii. Provide a detailed outline of the ADMM algorithm.

iii. Provide the formulation of the subproblems and master problem at each iteration.

iv. Implement the ADMM algorithm in the programming language of your choice and provide the optimal solutions of the original problem. You may try different values of the penalty parameter γ, the sensitivity parameter ϵ, the initial value of the Lagrange multiplier $\lambda^{(0)}$, and the maximum number of iterations N, and compare the convergence performances of the ADMM algorithm.

2. **Multi-period ED problem (if you have time and want to go further)**

We consider the power system consisting of 3 generators and 1 inflexible load described in Question 1, with the parameters described in Figure 1. In addition, we consider the maximum hourly ramping rate of each generator $\overline{R}_1^{up/down} = \overline{R}_2^{up/down} = \overline{R}_3^{up/down} = 40$ and their initial production $p_{1,init} = p_{2,init} = p_{3,init} = 0$, and we introduce a Battery Energy Storage System (BESS) B_1 with a maximum state of charge (SOC) $\overline{SOC}_1 = 600$, maximum charging and discharging powers $\overline{P}_1^{ch} = \overline{P}_1^{dis} = 200$, and charging and discharging efficiencies $\rho_1^{ch} = 0.95$ and $\rho_1^{dis} = \dfrac{1}{0.95}$, and initial state of charge $SOC_{1,init} = 300$. We consider the following inflexible load $\overline{P}_{1,t}^D$ over 24 hours:

$$\overline{P}_{1,t}^D \in \{40, 55, 60, 65, 70, 200, 210, 225, 200, 180, 175, 170,\\ 150, 145, 150, 155, 170, 200, 230, 250, 200, 140, 75, 55\}.$$

(a) Formulate the multi-period ED problem in this power system over 24 hours, while considering that the final state of charge of the BESS at the end of the optimization period should be greater or equal to its initial state of charge.

(b) Is this optimization problem decomposable and is it suitable for the LR and/or ALR methods? Justify your answer.

(c) Provide the formulation of the equivalent *augmented* optimization problem in which the complicating constraints of this problem are relaxed (based on Lagrangian duality), the outline of the ADMM algorithm, and the formulation of the subproblems, and master problems at each iteration.

(d) Solve this optimization problem directly and using the ADMM algorithm.

3. **Single-period optimal power flow (DC-OPF) problem (if you have time and want to go further)**

We consider the power system consisting of 3 generators and 1 inflexible load described in Question 1, with the parameters described in Figure 1. In addition, we now consider the underlying 3-bus network, such that: G_1 is at node n_1, G_2 at node n_2, G_3 at node n_3, and D_1 at node n_3, as illustrated in Figure 2.

The reactance x_{nm} and maximum flow in branches connecting nodes $n, m \in \mathcal{N} = \{1, ..., 3\}$ are summarized in Table 1. By convention $x_{nm} = x_{mn}$ and $\overline{P}_{nm}^F = \overline{P}_{mn}^F$.

(a) Formulate the single-period DC-OPF problem in this power system.

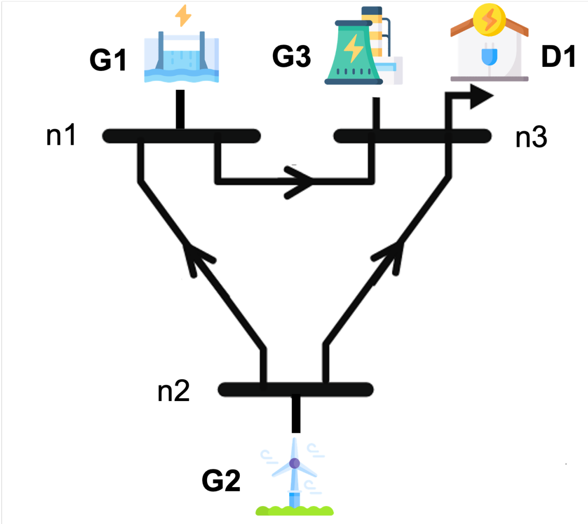


Figure 2: 3-bus power system

branches	(n_1, n_2)	(n_1, n_3)	(n_2, n_3)
reactance (x_{nm})	0.4	0.4	0.4
max. flow $(\overline{P}^F_{nm})$	150	150	150

Table 1: Branch parameters

(b) Is this optimization problem decomposable and is it suitable for the LR and/or ALR methods? Justify your answer. *Please see the lecture related to additional applications and extensions for guidance.*

(c) Provide the formulation of the equivalent *augmented* optimization problem in which the complicating constraints of this problem are relaxed (based on Lagrangian duality), the outline of the *consensus ADMM algorithm*, and the formulation of the subproblems, and master problems at each iteration.

(d) Solve this optimization problem directly and using the consensus ADMM algorithm.