Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Exercise 3

Name:

Student Number:

1. Energinet's optimal power flow problem

We consider a 3-bus power system comprising 3 generators: G_1 at node n_1, G_2 at node n_2, G_3 at node n_3, and 1 inflexible load: D_1 at node n_3, as illustrated in Figure 1.

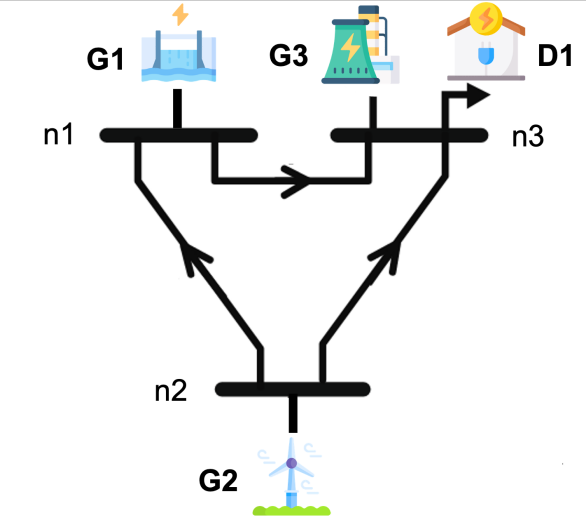

Figure 1: 3-bus power system

The production cost c_i^G (in DKK/MWh) and maximum generation capacity $\overline{P}_i^G$ of generator $i \in \mathcal{G} = \{1, ..., 3\}$ (in MWh), and inflexible demand P_i^D of load $i \in \mathcal{L} = \{1\}$ (in MWh) are summarized in Figure 2. The system operator wants to dispatch the generators in order to cover this load at the lowest possible cost.

The reactance x_{nm} and maximum flow in branches connecting nodes $n, m \in \mathcal{N} = \{1, ..., 3\}$ are summarized in Table 1. By convention $x_{nm} = x_{mn}$ and $\overline{P}_{nm}^F = \overline{P}_{mn}^F$.

branches	(n_1, n_2)	(n_1, n_3)	(n_2, n_3)
reactance (x_{nm})	0.4	0.4	0.4
max. flow $(\overline{P}_{nm}^F)$	150	150	150

Table 1: Branch parameters

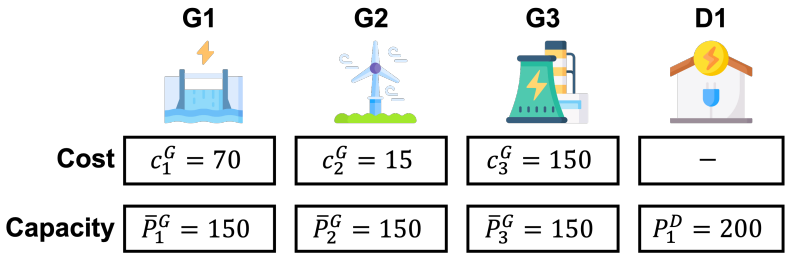

Figure 2: Generators and load parameters

(a) Formulate the optimal power flow problem in this system, using the DC linearization of the power flow equations. Specify the number of variables and constraints.

(b) Are the solutions of the Economic Dispatch (ED) problem from Exercise 2 feasible for this DC power flow equations? If not, which constraints are violated?

(c) Formulate the dual of the DC-OPF problem above. What do the dual variables associated with each constraint represent?

(d) Solve this optimization problem using python. Provide and analyze the values of the optimal primal variables, objective value, and dual variables associated with each constraint. In particular, disuss whether these solutions are feasible for the AC power flow equations, and interpret the values taken by the dual variables associated with each constraint.

(e) We now consider that the line capacities $\overline{P}_{12}^F$ and $\overline{P}_{13}^F$ are reduced to 75MWh. Answer questions a), b) and d) for these new values.