Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Exercise 8

Name:

Student Number:

1. **Chance-Constrained Economic Dispatch**

We consider a power system with 1 wind farm (G_2), 2 thermal generators (G_1, G_3), and 1 inflexible load (D_1) equal to 200MWh. The real-time (normalized) *available* wind power of the wind farm is modeled as a random variable $\overline{p}_{G_2} = \mu_{G_2} + \overline{W}_{G_2}\xi_{G_2}$, where $\mu_{G_2} = 110$ (MWh) is the mean predicted value, $\overline{W}_{G_2} = 150$ (MW) is the total installed wind capacity and ξ_{G_2} is the random variable representing the normalized wind prediction error, following a *Gaussian distribution* of mean $\mu_\xi = 0$ and standard deviation $\sigma_\xi = 0.09$. The uncertainty on this parameter is represented by the following scenarios: In order to ensure that production

Table 1: Scenarios of (normalized) wind prediction error ξ_{G_2}

Scenario (s)	1	2	3	4	5	6	7	8	9	10
Error $\xi_{G_2}^{(s)}$	-0.09	-0.04	-0.10	0.08	0.07	0.04	0.06	-0.01	0.02	0.07

meets demand, the generation of the thermal and wind generators must be dispatched in the day-ahead, and adjusted in real-time. The day-ahead dispatch of all generators is remunerated at the day-ahead market-clearing price.

To ensure that the demand is met in real-time, *for all realizations* of the uncertain available wind power, the generation of the thermal and wind generators may need to be adjusted upward/downward in real-time based on the realized *available* wind power. Their day-ahead generation can be adjusted upward or downward by up to their real-time adjustment capacity. However, their upward/downward adjustment and their total production (after adjustment) should not exceed their real-time adjustment capacity and their total generation capacity, respectively. Based on the realized *available* wind power, all generators are remunerated for their real-time adjusted generation using a single balancing market price, which may differ from the day-ahead price.

The day-ahead dispatch costs, upward and downward adjustment costs, generation capacity, upward and downward adjustment capacity of the generators, and the inflexible demand are summarized in Table 3. We observe that the upward adjustment costs of the generators is greater than their day-ahead dispatch cost, and their downward adjustment cost is lower than their day-ahead dispatch cost. This entails that any upward adjustment to generator i's day-ahead dispatch will incur a positive penalty cost equal to $(c_i^\uparrow - c_i^G)$EUR/MWh, and any downward adjustment will incur a positive penalty cost equal to $(c_i^G - c_i^\downarrow)$EUR/MWh.

(a) The system operator aims at dispatching the generators to cover the inflexible load at

Table 2: Generators parameters

	G_1	G_2	G_3
Day-ahead cost (EUR/MWh)	75	6	80
Upward adjustment cost (EUR/MWh)	77	8	82
Downward adjustment cost (EUR/MWh)	74	5	79
Generation capacity (MW)	100	150	50
Upward adjustment capacity (MW)	10	150	50
Downward adjustment capacity (MW)	10	150	50

the lowest possible cost. Formulate this economic dispatch problem as a multi-stage stochastic optimization problem.

(b) Solve this optimization problem, and analyze the results including the system cost, load shedding and generators profits in expectation and for each scenario.

(c) Now consider the following out-of-sample realizations of the available wind power: For

Table 3: Out-of-sample scenarios of (normalized) wind prediction error ξ_{G_2}

Scenario (s)	1	2	3	4	5	6	7	8	9	10
Error $\xi_{G_2}^{(s)}$	0.09	-0.09	0.12	-0.07	0.04	0.04	-0.15	-0.02	0.07	-0.01

each one of these out-of-sample realizations, and for the fixed optimal values of the day-ahead dispatch form Question 1.(b), compute the optimal real-time adjustment of the generators and evaluate the rate of out-of-sample constraint violations and the expected system cost.

(d) Assuming that each 2nd stage constraint should only be satisfied with a high probability of $1-\epsilon$, formulate the system operator's problem as a chance-constrained optimization problem, using a sample avergae approximation (SAA).

(e) Solve this chance-constrained economic dispatch problem for different values of $\epsilon \in \{0.1, 0.2, 0.3\}$ and analyze the results (compare them to the ones in Question 1.(a).

(f) For each one of the out-of-sample realizations in Question 1.(c), and for the fixed optimal values of the day-ahead dispatch form Question 1.(e), compute the optimal real-time adjustment of the generators and evaluate the rate of out-of-sample constraint violations and the expected system cost.

2. **Moment-Based Reformulation of Chance-Constrained Optimization Problems**

If you have time, attempt the follow questions (no coding required):

(a) Formulate the real-time adjustment decision variables as linear decision rules of the continuous uncertain input parameters representing the real-time (normalized) *available* wind power of the wind farm ξ_{G_2}.

(b) Formulate the system operator's problem as a chance constrained optimization problem, with continuous uncertain input parameters.

(c) Using the assumption that the (normalized) wind prediction error ξ_{G_2} follows a *Gaussian distribution* of mean $\mu_\xi = 0$ and standard deviation $\sigma_\xi = 0.09$, reformulate the chance constraints of the system operator's problem as convex constraints.