Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Exercise 12

Name:

Student Number:

1. **Decomposition of stochastic economic dispatch**

We consider a power system with 1 wind farm (G_2), 2 thermal generators (G_1, G_3), and 1 inflexible load (D_1) equal to 200MWh. The generation of the thermal generators and the wind farm are controllable. In order to ensure that production meets demand, their generation must be dispatched in the day-ahead. All generators are remunerated for their day-ahead dispatch at a uniform (marginal) price.

However, the *available* wind power is uncertain and can take 4 distinct values in real-time. The normalized production for each scenario and their associated probabilities is provided in Table 1. Therefore, the generation of the thermal and wind generators may need to be

Table 1: Wind production scenarios

	1	**2**	**3**	**4**
Production	0.60	0.70	0.75	0.85
Probability	0.25	0.25	0.4	0.1

adjusted in real-time based on the realized *available* wind power, to ensure that the demand is met. Their day-ahead generation can be adjusted upward or downward by up to their adjustment capacity, as long as their final generation respects their generation capacity bounds. Based on the realized *available* wind power, all generators are remunerated for their real-time adjusted generation at a uniform (marginal) price, which may differ from the day-ahead price.

The day-ahead dispatch costs, upward and downward adjustment costs, generation capacity, upward and downward adjustment capacity of the generators, and the inflexible demand are summarized in Table 1. We observe that the upward adjustment costs of the generators

Table 2: Generators parameters

	G_1	G_2	G_3
Day-ahead cost (DKK/MWh)	75	5	80
Upward adjustment cost (DKK/MWh)	77	7	82
Downward adjustment cost (DKK/MWh)	74	4	79
Generation capacity (MW)	100	150	50
Upward adjustment capacity (MW)	10	150	50
Downward adjustment capacity (MW)	10	150	50

is greater than their day-ahead dispatch cost, and their downward adjustment cost is lower than their day-ahead dispatch cost. This entails that any upward adjustment to generator i's day-ahead dispatch will incur a positive penalty cost equal to $(c_i^{UP} - c_i^{G})$DKK/MWh, and any downward adjustment will incur a positive penalty cost equal to $(c_i^{G} - c_i^{DW})$DKK/MWh.

(a) The system operator aims at dispatching the generators to cover the inflexible load for all realizations of the *available* wind power, at the lowest *expected* cost. Formulate this ED as a stochastic optimization problem.

(b) Is this optimization problem decomposable? If so, specify the complicating variables/constraints and the number of subproblems into which it can be decomposed.

(c) Formulate the Benders decomposition algorithm to decompose this stochastic ED problem. Detail the steps of the algorithm, the formulation of the sub-problems and (uni- and multi-cut) master problem at each iteration, and the convergence criteria.

(d) Solve this stochastic ED problem using the Benders decomposition algorithm. Present and discuss the convergence of the algorithm with the uni- and multi-cut formulations of the master problem. Discuss the pros and cons of each approach.