Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Exercise 5

Name:

Student Number:

1. **Electricity market properties**

We consider a power system with 3 generators and 2 flexible consumers. The fixed marginal production production costs c_g^G (DKK/MWh) and maximum generation capacity $\overline{P}_g$ (MWh) of all generators $g \in \mathcal{G} = \{W_1, G_1, G_2\}$, the fixed marginal utility cost c_d^D (DKK/MWh), minimum $\underline{L}^d$ and maximum load capacity $\overline{L}^d$ of all flexible consumers $d \in \mathcal{D} = \{D_1, D_2\}$ (in MWh) are summarized in Table **??**.

	marginal cost (DKKMWh)	**Minimum capacity (MWh)**	**Maximum capacity (MWh)**
$\boldsymbol{W_1}$	0	0	80
$\boldsymbol{G_1}$	30	0	80
$\boldsymbol{G_2}$	35	0	80
$\boldsymbol{D_1}$	40	0	100
$\boldsymbol{D_2}$	20	0	50

The market operator wants to find the optimal dispatch of the generators and flexible consumers in order to maximize the social welfare of the system.

(a) Assuming that all market participants are price-takers, formulate this market-clearing as an optimization problem.

(b) Assuming that all market participants are price-takers, formulate the equilibrium among them.

(c) Show that the KKT conditions of the market-clearing problem formulated in Question a) and the equilibrium formulated in Question b) are identical. What does this mean for the properties of this market design, *under the assumption of perfect competition*?

(d) Using the formulation of the market-clearing problem and its KKT conditions, show that this market design ensures the following desirable properties:

   i. *budget balance* for the market operator, *(Tip: use the balance equation of the market-clearing problem.)*

   ii. *cost recovery* for all market participants. *(Tip: use the 1st order and complementarity conditions of the market-clearing problem.)*

2. **Optimal offering strategy of a price-maker**

(a) Assuming that producer G_1 acts as a price-maker and is strategic in quantity, and all other market participants are price-takers, formulate its optimal offering strategy as a bilevel optimization problem.

(b) Reformulate this bilevel problem as a single-level optimization problem, so-called Mixed Complementarity Problem (MCP), by replacing the lower-level optimization problem by its (necessary and sufficient) KKT conditions.

(c) Formulate the strong-duality condition for the lower-level problem. Then, linearize the bilinear terms in the upper-level objective function using the strong duality and the KKT conditions of the lower-level problem. *Please, check the book "Complementarity Modelling in Energy Markets" for guidance.*

(d) Solve the resulting optimization problem using a programming language of your choice. You can either reformulate the complementarity conditions of the lower-level problems usign the Fortuny-Amat linearization or introduce SOS1 variables. *Please, check the Python tutorial on complementarity modelling for guidance.*

(e) Compare the social welfare in the market when all market-participants are price-takers and when on market participant is strategic. What an you deduce with respect to the efficiency and incentive compatibility of this market design?