Danmarks Tekniske Universitet

Wind Turbine Measurement Technique

Michael Courtney & Ida Nielsen

Assignment 1: Measurement Uncertainty

Alexandre del Barco (s232518)

February 23, 2025

Contents

List of Figures

List of Tables

Introduction

This report investigates the concept of measurement uncertainty in the context of determining the power output of a wind turbine. The primary objective is to analyze how uncertainties in measuring wind speed, air temperature, and barometric pressure affect the accuracy of power calculations. The study begins with the derivation of algebraic expressions for absolute and relative uncertainties under simplified conditions, assuming constant air density and power coefficient.

The analysis progresses to include more complex scenarios by incorporating the influence of temperature and pressure on air density, leading to a comprehensive expression for combined power uncertainty. A Monte Carlo simulation is conducted to validate the theoretical results, using statistical methods to model the variability in input measurements and assess their impact on the calculated power output.

The findings provide a detailed understanding of how measurement uncertainties propagate through calculations, highlighting their significance in practical applications such as wind energy assessment. The report presents the methodology, derivations, calculations, and results, contributing to a broader comprehension of measurement uncertainty in engineering contexts.

1 Power uncertainty expression - ρ and C_p constants

The classic formula for the power produced by a wind turbine is Equation 1:

$$P = \frac{1}{2}\rho A C_p V^3 \tag{1}$$

Where:

- P: power produced by a wind turbine
- ρ: air density
- A: swept area
- C_p: power coefficient
- V: wind speed

1.1 Algebraic expression for the power uncertainty

Firstly, it is assumed that both air density and power coefficient are constant. Thus, an algebraic expression for the power uncertainty (U_P) is derived, indeed it is associated with the wind speed uncertainty (U_v).

To do so, it is assumed that the uncertainties related to Equation 1 are independent of each other. Therefore, to compute the power uncertainty it is needed to combine uncertainties adding them in quadrature, as Equation 2 defines.

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u_i^2(x_i) \tag{2}$$

- y: measurand
- x_i: variables that define y
- f: function combination of x_i that define y
- u_c: combined standard uncertainty
- u_i: standard uncertainty of variable x_i

In this specific case then, f in Equation 2 is Equation 1; $u_c(P)$ is equal to U_P; and u_V is the same as U_V. Leaving then Equation 3

$$U_P = (\frac{\partial P}{\partial V})^2 U_V^2 = \frac{3}{2}\rho A c_p V^2 U_V \tag{3}$$

1.2 Expression for relative power uncertainty

If it is desired to get the expression for the relative power uncertainty, it is enough to just divide both sides of the resulted Equation 3 by P to then express $\frac{U_P}{P}$. This expression (relative uncertainty) shows the uncertainty as a percentage of the total result. Getting Equation 4 as result.

$$\frac{U_P}{P} = \frac{\frac{3}{2}\rho A c_p V^2 U_V}{\frac{1}{2}\rho A C_p V^3} = 3 \cdot \frac{U_V}{V} \tag{4}$$

1.3 Value of relative standard uncertainty of power

It is estimated that the relative standard uncertainty of the wind speed ($\frac{U_V}{V}$) to be 2%. Thus, by using Equation 4, the relative standard uncertainty of power is defined as:

$$U_P = 3 \cdot \frac{U_V}{V} = 3 \cdot 0.02 = 0.06 \tag{5}$$

2 Power uncertainty expression - ρ and C_p not constants

We now go one step closer to reality by recognizing that the air density (ρ) is calculated from measurements, specifically from the temperature and barometric pressure using the expression:

$$\rho = \frac{P_a}{R \cdot T} \tag{6}$$

Where:

- P_a: Barometric pressure
- T: Temperature
- R: Individual gas constant = 287.05 J/(kgK)

2.1 Algebraic Expression for combined power uncertainty

Assuming that all the uncertainty components in Equation 1 are uncorrelated and that air density is defined as Equation 6; an expression for the combined power uncertainty could be derived using again Equation 2. The difference with respect to subsection 1.1 is that in this case instead of having only wind speed (V) as the only uncertainty, temperature (T) and barometric pressure (P_a) are uncertainties now as well (x_i). Then leading to Equation 7

$$U_P = \left(\frac{\partial P}{\partial V}\right)^2 U_V^2 + \left(\frac{\partial P}{\partial P_a}\right)^2 U_{P_a}^2 + \left(\frac{\partial P}{\partial T}\right)^2 U_T^2 \tag{7}$$

Then solving each partial derivative:

$$U_P = \left(\frac{3P_aAC_PV^2}{2RT}\right)^2 U_V^2 + \left(\frac{1AC_PV^3}{2RT}\right)^2 U_{P_a}^2 + \left(\frac{-1P_aAC_PV^3T^{-2}}{2R}\right)^2 U_T^2 \tag{8}$$

2.2 Expression for combined relative power uncertainty

Following the same procedure as section 1, the expression for the combined relative power uncertainty ($\frac{U_P}{P}$) is derived, in this case in tems of the relative uncertainties of wind speed (U_V), temperature (U_T) and pressure (U_{P_a}). The result obtained is Equation 9:

$$\left(\frac{U_P}{P}\right)^2 = \left(\frac{3U_V}{V}\right)^2 + \left(\frac{U_{P_a}}{P_a}\right)^2 + \left(\frac{-U_T}{T}\right)^2 \tag{9}$$

It is observed a pattern when deriving the relative uncertainty expression, it seems to be the squared summation of each relative uncertainty times the power of the uncertainty.

2.3 Value of absolute uncertainty of the power

Assuming a wind speed uncertainty ($\frac{U_V}{V}$) of 4% at k=2, the accuracy of the temperature sensor to be ± 0.5°C, the standard uncertainty of the barometers 5hPa (U_{P_A}), wind speed to be 8 m/s (V), the temperature as 20°C ($T = 293.15K$), pressure as 980 hPa (P_a) and the power coefficient 0.45 (C_P). The absolute uncertainty of the power is calculated at these conditions.

But first, as the uncertainty for wind speed is given as relative, it is needed to covert it to standard:

$$U_V = \frac{8m/s \cdot 0.04}{2} = 0.16m/s \tag{10}$$

Then the accuracy of the thermometer is given as ± 0.5°C, this value will be converted have the temperature uncertainty in in Gaussian distribution:

$$U_T = \frac{0.5K}{\sqrt{3}} \tag{11}$$

Using Equation 9, both the relative ($\frac{U_P}{P}$) and absolute (U_P) uncertainties can be calculated.; and also the power (P).

$$\frac{U_P}{P} = \sqrt{\left(\frac{3U_V}{V}\right)^2 + \left(\frac{U_{P_a}}{P_a}\right)^2 + \left(\frac{-U_T}{T}\right)^2} = 6\% \tag{12}$$

$$U_P = \sqrt{\left(\frac{3U_V}{V}\right)^2 + \left(\frac{U_{P_a}}{P_a}\right)^2 + \left(\frac{-U_T}{T}\right)^2} \cdot \frac{1}{2}\frac{P_a}{RT} Ac_P V^3 = 17.16KW \tag{13}$$

$$P = \frac{1}{2}\frac{P_a}{RT} Ac_P V^3 = 284.942kW \tag{14}$$

Assigment1.ipynb file is also submitted where the calculations are shown

3 Monte-Carlo Solution

In this last section a Monte-Carlo solution is programmed in python to check the results obtained in section 2. The code could be found in *Assigment1.ipynb*

After defining the power as the combination of Equation 2 and Equation 6. Three populations of 10000 vales each are made, one for wind speed (V), another for temperature (T) and the last for barometric pressure (P_a). Both distributions for wind speed and pressure are defined as Gaussian distributions, however temperature is defined as uniform distribution as it is believed that each possible value has the same probability. The results of the populations can be seen summarised in the histograms showed in Figure 1

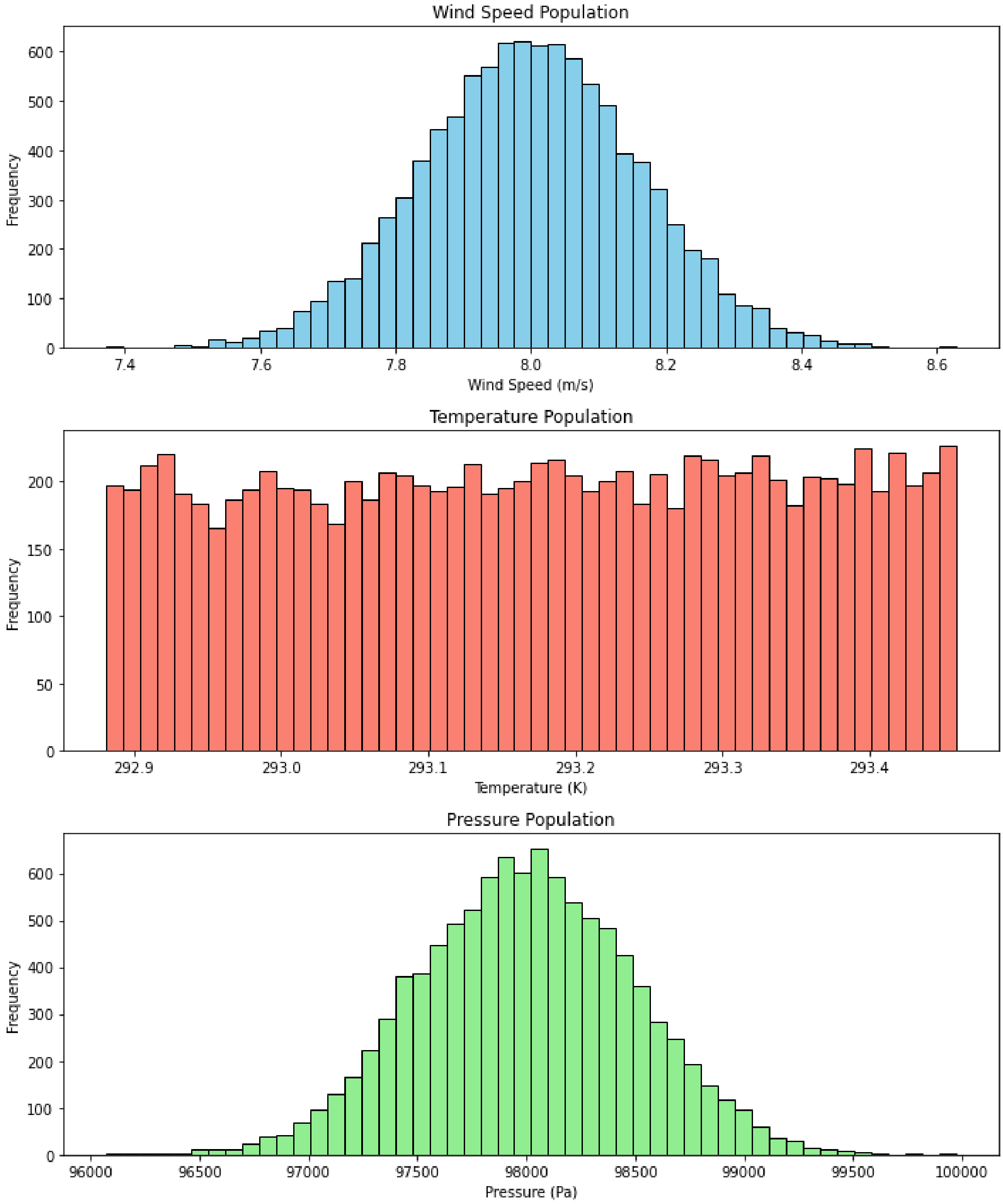

Figure 1: Populations wind speed, Temperature and Pressure

After using the created populations to compute 10000 possible values of power, the results are displayed in Table 1.

	Monte-Carlo 10000 values	Solution 2.3	Relative Error
Best Estimate [kW]	285.240	284.942	0.1%
Standard uncertainty [kW]	17.246	17.16	0.5%

Table 1: Comparison result Monte-Carlo solution

It can be concluded that Monte-Carlo solution is accurate with respect to the previous calculations, neither of best estimation and standard uncertainty calculation error exceeds 0.5%.

It has been tried to increase the number of samples for each population to see how this would impact the final result of the best power estimate and the standard uncertainty. The result are shown in Figure 2 and Figure 3, and it is seen that a larger number of samples does not mean getting a closer value neither for best estimate and the standard uncertainty, nevertheless the results do not get worse.

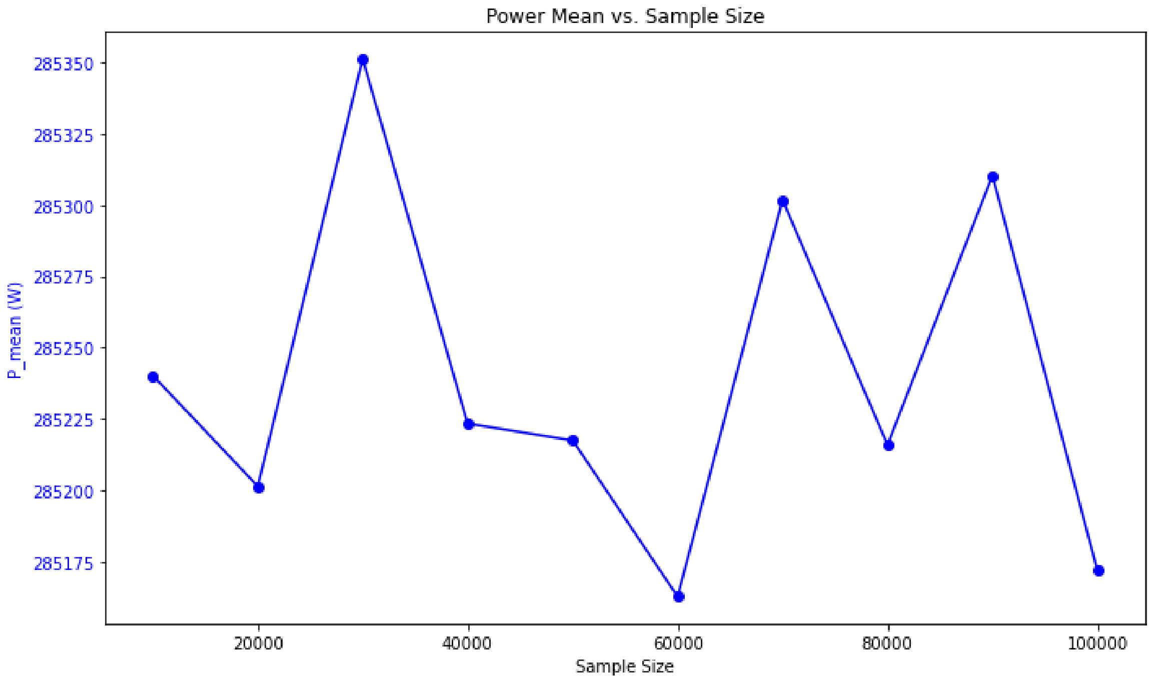

Figure 2: Best Power Estimate for different number of samples in the populations

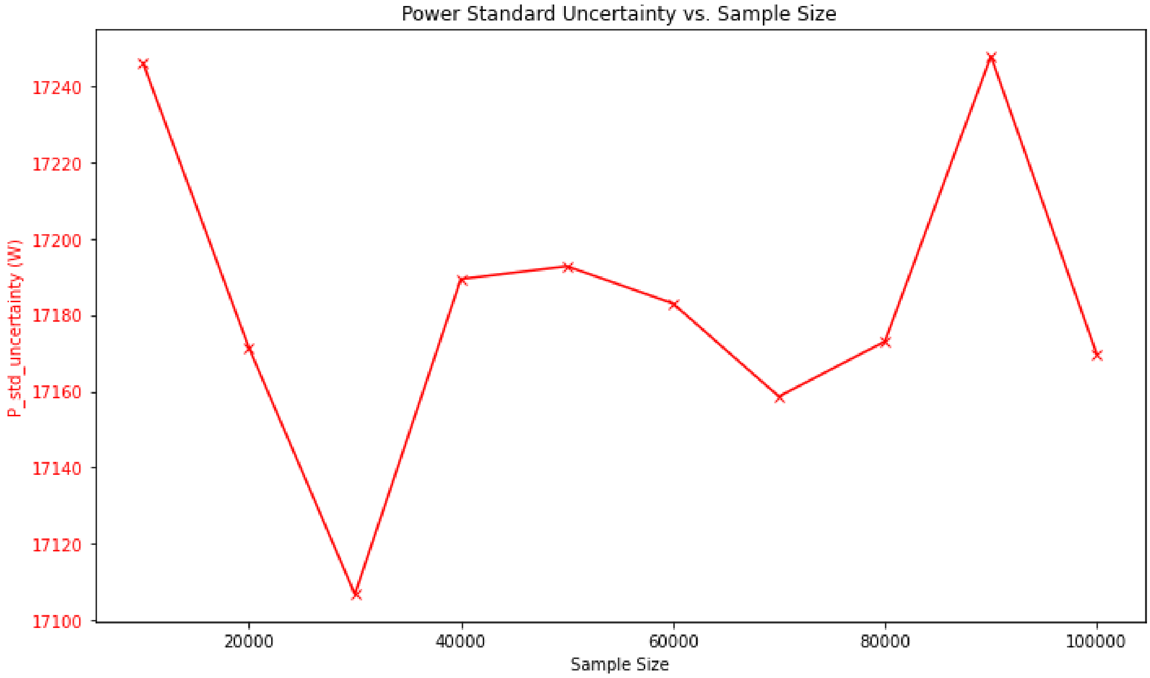

Figure 3: Standard uncertainty for different number of samples in the populations