AS01 – Measurement Uncertainty

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In this assignment, you will investigate some of the uncertainties involved in measuring the power produced by a wind turbine and use this to practice and reinforce your understanding of measurement uncertainty. Later, in the Power Performance module, you will learn how these uncertainties are really calculated in practice, but I hope you will be able to see the overlying concepts that we uncover here.

1) We all know the classic formula for the power produced by a wind turbine:

$$P = \tfrac{1}{2}\rho A C_p V^3 \qquad (1)$$

a) Firstly, assuming that both air density ρ and C_p are constant, derive an algebraic expression for the power uncertainty U_P associated with a wind speed uncertainty U_V.

b) Divide both sides of the expression for U_p that you obtained in a) by equation 1 to obtain an expression for the relative uncertainty $\frac{U_p}{P}$. Hint – since $\frac{U_p}{P}$ is dimensionless, you should get a dimensionless expression on the right-hand side as well.

c) If the relative standard uncertainty of the wind speed is estimated to be 2%, what will the relative standard uncertainty of the power be?

2) We now go one step closer to reality by recognizing that the air density ρ is calculated from measurements if the temperature T and barometric pressure p_a using the expression:

$$\rho = \frac{p_a}{RT} \qquad (2)$$

a) Assuming that all the uncertainty components are uncorrelated, derive an expression for the combined power uncertainty, including wind speed, temperature and pressure uncertainties.

b) If you can, derive an expression for the combined relative power uncertainty $\frac{U_p}{P}$ in terms of the relative uncertainties of wind speed $\frac{U_V}{V}$, temperature $\frac{U_T}{T}$ and pressure $\frac{U_{p_a}}{p_a}$. Can you see a pattern here?

c) We have a wind speed uncertainty of 4% at k=2 and are told that the accuracy of the temperature sensor is ± 0.5°C and that the standard uncertainty of the barometer is 5hPa. Calculate the absolute uncertainty of the power at 8 m/s, stating units and coverage factor. Take the temperature as 20°C and the barometric pressure as 980 hPa. The value of R is 287.05 J/(kgK). Choose an appropriate value for C_p.

3) Make a Monte-Carlo solution to check your result for 2 c). Use the programming tool you are most comfortable with. Follow these steps:

a) Start by making a function for the power with V , T and p_a as parameters.

b) Next make three populations (an array with, say 10000 values each) of the possible true values for each of V , T and p_a based on the mean values, uncertainties and distribution types indicated in question 2 c).

c) Finally, pick values from the populations, put them through the algorithm for power and make a population of the possible values of the power.

d) What is the best estimate and standard uncertainty for the power? Does this agree with your previous result from 2 c)? Does the result change if you use larger populations?

Final comments: You will see later that in a power performance test, the wind speed uncertainty dominates below rated power and then the uncertainty in the electrical power measurement (we didn't include it here) dominates above rated power. Hopefully you can already understand why this is!