Second order consensus

Distributed multi-vehicle coordinated control via local interaction

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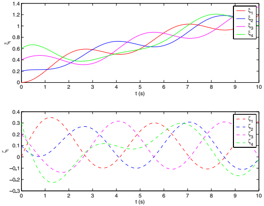

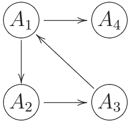

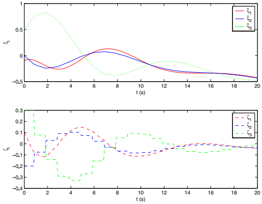

Overview

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Introduction

The paper proposes a **second order consensus** for mobile robots, such that some information states converge to a consistent value (e.g. position of the formation centre) while others converge to another consistent value (e.g. velocity of the formation centre)

The mobile robots communicate each others exchanging **direct information**. It is a general assumption because the robots in the formation can be heterogeneous.

The main aspect of the analysis are:

- Robot positions
- Robot velocities
- Dynamic graph of the information flow

First order consensus

The first order consensus so far proposed in the literature is the following:

$$\dot{\xi}_i = u_i$$

with:

$$u_i = -\sum_{j=1}^{n} g_{ij} k_{ij} (\xi_i - \xi_j), \quad i \in I$$

where $k_{ij} > 0$ and $g_{ij} = 1$ iff information flows from vehicle i to vehicle j, and zero otherwise.

The value of k_{ij} is defined by the topology and it is given by the relation: $a_{ij} = k_{ij} g_{ij}$, $\forall i \neq j$ where a_{ij} is an element of the adjacency matrix A.

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Second order consensus

Tanking into account the second order vehicle dynamics modelled by:

$$\dot{\xi}_i = \zeta_i$$
$$\dot{\zeta}_i = u_i$$

with:

$$u_i = -\sum_{j=1}^{n} g_{ij} k_{ij} [(\xi_i - \xi_j) + \gamma(\zeta_i - \zeta_j)], \quad i \in I$$

where k_{ij} and g_{ij} are defined as before and γ is a scaling factor.

The **positions** of the robots are described by ξ_is, while ζ_is describe the **velocities**. In this context u_is are the **accelerations**.

Second order consensus, matrix representation

Let $\xi = [\xi_1, \xi_2, ..., \xi_n]^T$ and $\zeta = [\zeta_1, \zeta_2, ..., \zeta_n]^T$. If we want to write in matrix notation the previous relations, we obtain:

$$\begin{bmatrix} \dot{\xi} \\ \dot{\zeta} \end{bmatrix} = \Gamma \begin{bmatrix} \xi \\ \zeta \end{bmatrix}$$

where:

$$\Gamma = \begin{bmatrix} 0_{n \times n} & I_n \\ -L & -\gamma L \end{bmatrix}$$

L is the Laplacian matrix, $0_{n \times n}$ is a matrix of all 0s and I_n is the identity matrix.

Convergence analysis, time invariant topology

The convergence is determined by the eigenvalues of Γ. So we solve the following equation:

$$det(\lambda I_{2n} - \Gamma) = det\left(\begin{bmatrix} \lambda I_n & -I_n \\ L & \lambda I_n + \gamma L \end{bmatrix}\right) = det(\lambda^2 I_n + (1 + \gamma\lambda)L)$$

With solution:

$$\lambda_{i\pm} = \frac{\gamma\mu_i \pm \sqrt{\gamma^2\mu_i^2 + 4\mu_i}}{2}$$

Where λ_{i+} and λ_{i-} are called eigenvalues of Γ associated with μ_i. We can note that if $\mu_i = 0$ then $\lambda_{i\pm} = 0$.

Convergence analysis, time invariant topology

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The proposed consensus protocol achieves **consensus asymptotically** if and only if matrix Γ has **exactly two zero eigenvalues** and all the **other eigenvalues have negative real parts**.

If all non-zero eigenvalues of $-L$ are real and therefore negative, it is straightforward to verify that all non-zero eigenvalues of Γ have negative real parts.

In the general case, some non-zero eigenvalues of Γ may have positive real parts even if all non-zero eigenvalues of $-L$ have negative real parts as shown in the following examples.

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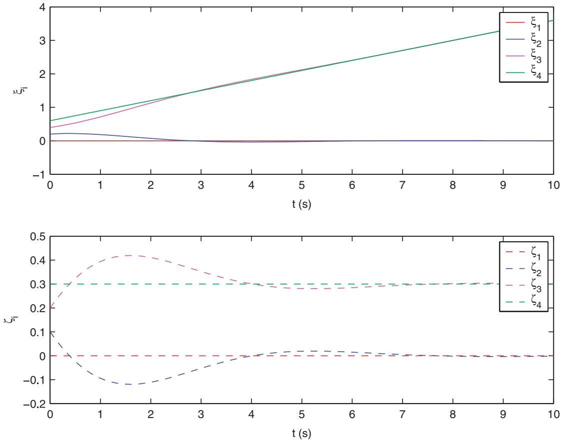
Case 1



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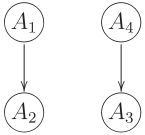
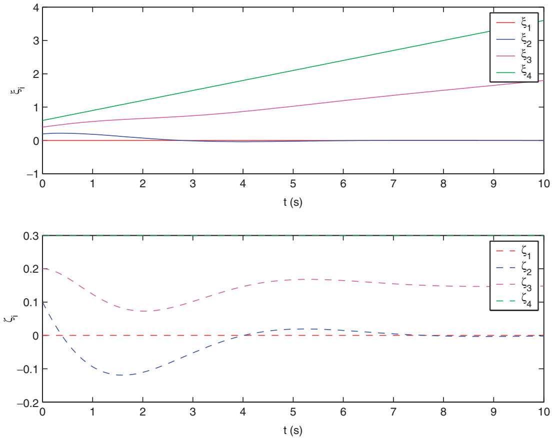
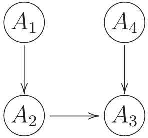


Consensus cannot be achieved since the information states from different groups do not affect one another.

We also know that L has at least two zero eigenvalues in this case, which in turn implies that Γ has at least **four zero eigenvalues**.

Case 2

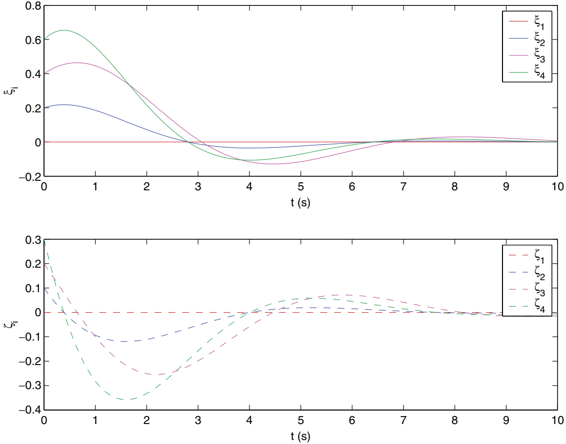




If a vehicle only has outgoing links without incoming ones, we call it a leader. Here we have **multiple leaders**.

Consensus cannot be achieved , L has at least two rows with all zero entries in this case, we know that L has at least two zero eigenvalues, which in turn implies that Γ has at least **four zero eigenvalues**.

Case 3

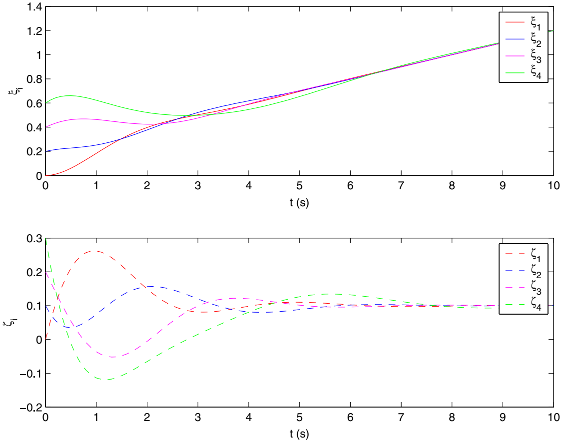


$$\begin{tikzcd} A_1 \arrow[r] \arrow[d] & A_4 \\ A_2 \arrow[r] & A_3 \end{tikzcd}$$

Here the information exchange topology has a **leader–follower structure** and L can be written as an upper diagonal matrix.

We know that zero is a simple eigenvalue of L and all non-zero eigenvalues are real. So, we know that **consensus is achieved asymptotically**.

Case 4A

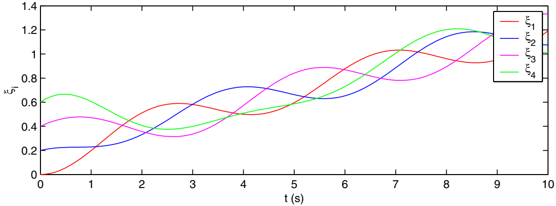


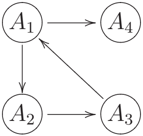
$$
\begin{tikzcd}
A_1 \arrow[r] \arrow[d] & A_4 \\
A_2 \arrow[r] & A_3 \arrow[ul]
\end{tikzcd}
$$

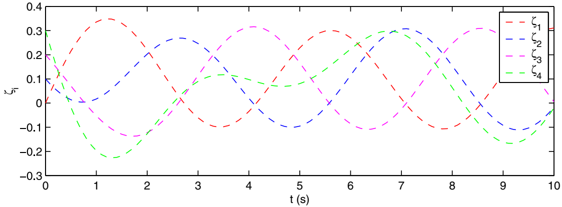
Having a (directed) spanning tree is a necessary condition for **consensus**, but it **may not be achieved**.

In this case **consensus can be reached for $\gamma = 1$**, but could not be reached for smaller γ.

Case 4B







Having a (directed) spanning tree is a necessary condition for **consensus**, but it **may not be achieved**.

In this case **consensus cannot be reached for** $\gamma = 0.4$.

Conclusions

If $-L$ has a simple zero eigenvalue and all the other eigenvalues are real, consensus protocol achieves consensus for any $\gamma > 0$:

Matrix L of a directed weighted graph has a simple zero eigenvalue and all the other eigenvalues have positive real parts if and only if the graph has a (directed) spanning tree.

Consensus protocol achieves consensus asymptotically if the information exchange topology has a (directed) spanning tree and:

$$\gamma > \max_{\mu_i \neq 0} \sqrt{\frac{2}{|\mu_i| \cos\left(\frac{\pi}{2} - \tan^{-1} \frac{-Re(\mu_i)}{Im(\mu_i)}\right)}}$$

References

- **Distributed multi-vehicle coordinated control via local information exchange** - Wei Ren and Ella Atkins
- **Consensus in networked multi-agent systems** - Carlo Piccardi - ftp://ftp.elet.polimi.it/outgoing/Carlo.Piccardi/VarieCsr/Lezioni/10_Consensus.pdf
- **Ros - Robotic operative system** http://www.ros.org
- **Gazebo - simulator** http://gazebosim.org
- **Kobra robot** http://www.nuzoo.it/it/

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Switching graph topology

In this case the analysis is more complex and depends on more factors. We can see some of them through an example. Let's consider the following graphs defined by the matrices:

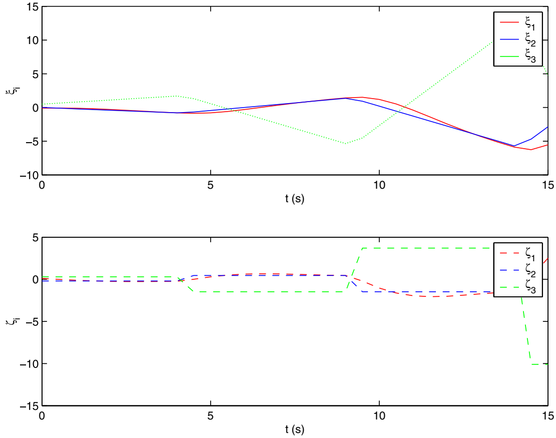
$$L_1 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -2 & 0 & 2 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Also let $\gamma_1 = \gamma_2 = \gamma_3 = 1$ and let Γ_i be defined as

$$\Gamma_i = \begin{bmatrix} 0_{n\times n} & I_n \\ -L_i & -\gamma_i L_i \end{bmatrix}$$

We note that Γ_2 **has two zero eigenvalues** and all the others have negative real parts, while both Γ_1 **and** Γ_3 **have four zero eigenvalues** and all the others have negative real parts.

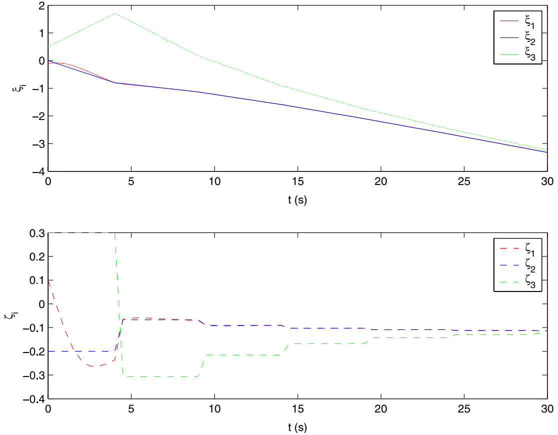
Switching graph topology



At each time interval of 5 s we let the information exchange topology be G_1 during 90% of the time and be G_2 during the rest of the time. Note that $G_1 \cup G_2$ has a (directed) spanning tree.

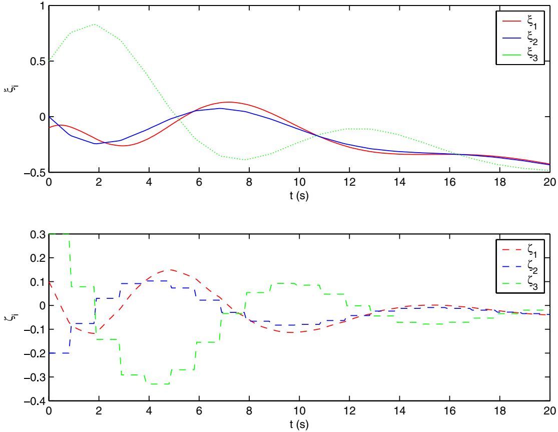
Using the **first-order consensus** protocol, **consensus can be achieved**, while **consensus cannot be achieved** using the **second-order consensus** protocol.

Switching graph topology



In contrast, if we increase the gain γ_2 to be 10, **consensus can be achieved asymptotically**.

Switching graph topology



Alternatively, if we reduce the length of each time interval to be 1 s, **consensus can be achieved asymptotically**.

Switching graph topology

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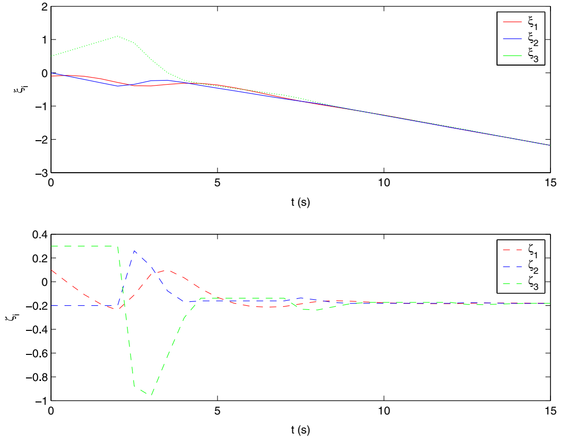
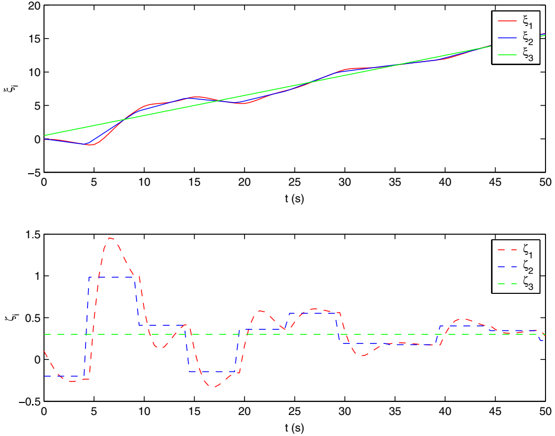


In addition, if we let the information exchange topology be G_1 during 50% of the time and be G_2 during the rest of the time with an interval of 5 s, **consensus can be achieved asymptotically**.

Switching graph topology



Next, at each time interval of 5 s, we let the exchange topology be G_1 during 90% of the time and be G_3 during the rest of the time. Note that $G_1 \cup G_3$ has a (directed) spanning tree and that graph G_3 is only a subset of graph G_2.

Consensus can be achieved asymptotically even if graph G_3 has less information exchange than graph G_2.