

# Definition

Given data about a user, his environment, and some items of interest (*training data*), determine items to recommend.

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We don't have to find the  $\max k$ .

It's enough to find  $k$  within some  $\max n$ .

# Examples

- Amazon
- Google News (or Le Monde)
- Facebook
- Medical testing
- App Store / Google Play
- Youtube
- Advertising
- Netflix, last.fm, Spotify, Pandora, . . .
- Browser (URL recommendations)
- Search

# Client Value Proposition

- Find opportunities
- Reduce choice
- Explore options
- Discover long tails
- Recreation

# Provider Value Proposition

- Offer a unique or additional service (beyond competitors)
- Customer trust and loyalty
- Increase sales, CTR, conversions
- Better understand customers

# Recommendation

Content-based filtering <i>(filtrage basée sur le contenu)</i>	More things similar to what I like
Collaborative filtering <i>(filtrage collaboratif)</i>	More of what other people who like what I like like
Knowledge-based filtering <i>(filtrage basée sur connaissance)</i>	More of what I need.

# Content-based filtering

*More things similar to what I like*

*Plus de ce qui ressemble à ce que j'aime*

## Advantages

yes! No need for community

yes! Possible to compare items

## Disadvantages

no Understand content

yes Cold start problem

no Serendipity

# Collaborative filtering

*More of what other people who like what I like like*  
*Plus de ce que d'autres qui aiment ce que j'aime aiment*

## Advantages

yes! No need to understand content

yes! Serendipity

yes! Learn market

## Disadvantages

no User feedback

yes Cold start problem (users)

yes Cold start problem (items)



# Knowledge-based filtering

*More of what I need*

*Plus de ce qu'il faut*

## Advantages

yes! Deterministic

yes! Certainty

no! Cold start problem

yes! Market knowledge

## Disadvantages

yes Studies to bootstrap

yes Static model, doesn't learn from trends

# Utility Matrix

- Users (utilisateurs)
- Items (objets)

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The goal is to fill in the blanks.

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$
$U_1$	1				
$U_2$			1	1	1
$U_3$		1		1	1

Example: books sales at Amazon.

*But thousands or millions of columns and rows.*

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$U_2$			5	1	4
$U_3$		2		5	1

Example: film advice at Netflix.

*But thousands or millions of columns and rows.*

# Utility Matrix

How do we make the matrix?

- Ask users
- Observe users

That's usually expensive. . .

# Item Profiles

Examples:

- Films  $\Rightarrow ?$
- Books  $\Rightarrow ?$
- News  $\Rightarrow ?$
- Images  $\Rightarrow ?$

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Films :

Content: actors, directors, year (decade, etc.), length

Collaborative: seen, opinion (1–5), when seen relative to release

# Item Profiles

Examples:

- Films  $\Rightarrow ?$
- Books  $\Rightarrow ?$
- News  $\Rightarrow ?$
- Images  $\Rightarrow ?$

Books:

Content : authors, genre, year (decade, etc.), number of pages, content (very difficult)

Collaborative: read, opinion (1–5), how read



# Item Profiles

Examples:

- Films  $\Rightarrow ?$
- Books  $\Rightarrow ?$
- News  $\Rightarrow ?$
- Images  $\Rightarrow ?$

News:

Content : source, section, TF-IDF word vectors

Collaborative:

# Item Profiles

Examples:

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- Books  $\Rightarrow ?$
- News  $\Rightarrow ?$
- Images  $\Rightarrow ?$

Images :

Content:

Collaborative:

# Item Profiles

Examples:

- Films  $\Rightarrow ?$
- Books  $\Rightarrow ?$
- News  $\Rightarrow ?$
- Images  $\Rightarrow ?$

Also: user profile, user behavior

Vectors

Similarity

# Similarity : Jaccard Index

or: *Indice de Jaccard, Jaccard similarity coefficient*

Similarity:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

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$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

Distance:

$$J_{\delta}(A, B) = 1 - J(A, B)$$

# cosine similarity

or: *mesure cosinus*, *Similarité cosinus*

Similarity:

$$\cos \theta = \frac{A \cdot B}{\|A\| \|B\|}$$

# cosine similarity

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We only consider non-empty components in the vector.

# Texts: TF-IDF

- Vectors of word frequencies
- Frequency  $n \rightarrow$  significance

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- Frequency  $\rightarrow$  significance
- Term Frequency - Inverse Document Frequency

## Texts: TF-IDF

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}} \qquad IDF_i = \log_2 \left( \frac{N}{n_i} \right)$$

$$TF-IDF_{ij} = TF_{ij} \cdot IDF_i$$

with :

$f_{ij}$  = frequency of word  $i$  in document  $j$

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Variation: boolean, log, stop word filtering

# Content-Based Filtering

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Then, we can cluster (*regroupement, partitionnement de données*), etc.

# Content-Based Filtering

Based on item profiles

- More stable (in principle)
- $O(n^2)$  (but often less, items often aren't categorised together)
- Can reduce to threshold
- Can pre-calculate, queries become faster

# Collaborative Filtering

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$
$U_1$	3		4	2	
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User profile

# Collaborative Filtering

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Item profile

# Utility Matrix Symmetry

- Propose items based on users
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But remember: 2 items being similar  $\nRightarrow$  2 users similar.

Thought experiment: consider comparing people vs comparing objects.



# Utility Matrix Symmetry

- Propose items based on users
- Propose users based on items

To estimate  $m_{u,i}$ ,

- Find  $k$  users like  $U_u$
- Find  $k$  items like  $I_i$

## Utility Matrix : Estimate $m_{u,i}$

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- Find  $k$  users like  $U_u$ , take  $\frac{1}{k} \sum_{j=1}^k m_{u_j,i}$
- Find  $k$  items like  $I_i$ , take  $\frac{1}{k} \sum_{j=1}^k m_{u,i_j}$

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We have to compute the entire line (or the part which is likely to be important)

## Utility Matrix : Estimate $m_{u,i}$

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Once we've computed  $U_u$ , the other  $k$  users lets us take a shortcut.

## Utility Matrix : Estimate $m_{u,i}$

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For  $I_i$ , we have to compute most of the  $I_j$  before we can fill in a single line. But item-item filters are often more reliable.

## Utility Matrix : Estimate $m_{u,i}$

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$
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In any case, we can mostly precompute in advance.

# Utility Matrix

The matrix is sparse.

$\implies$  clustering  $\implies$  reduced matrix

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Estimate on the reduced matrix, then take items and users as representative for the cluster.



# Amazon : Item-to-Item Collaborative Filtering

Observations :

Clustering is expensive, reduces quality

# Amazon : Item-to-Item Collaborative Filtering

Observations :

Dimension reduction reduces quality

# Amazon : Item-to-Item Collaborative Filtering

Observations :

Users interact with very few items

# Amazon : Item-to-Item Collaborative Filtering

Observations :

Rapid response desirable

# Amazon : Item-to-Item Collaborative Filtering

Scales independent of the number of users or of items

- Online
- Offline

G. Linden, B. Smith, J. York, *Amazon.com Recommendations: Item-to-Item Collaborative Filtering*, Internet Computing (7, 1), 22 Jan 2003.

# Amazon : Item-to-Item Collaborative Filtering

Offline (Precomputation)

```
for each item  $l_1$  to sell do  
  | for each user  $C$  who has purchased  $l_1$  do  
    | for each item  $l_2$  bought by  $C$  do  
      |  $(l_1, l_2)++$   
    end  
  end  
  for each item  $l_2$  do  
    |  $S_{l_1, l_2} \leftarrow S(l_1, l_2)$   
  end  
end
```

# Slope One

Linear regression on user opinions (ratings)

Daniel Lemire and Anna Maclachlan, *Slope One Predictors for Online Rating-Based Collaborative Filtering*, Proceedings of SIAM Data Mining (SDM) 2005.

# Slope One : algorithm

Offline :

**for** *chaque*  $I_i, I_j$  **do**

$\mathcal{U} \leftarrow \{\text{users who have expressed an opinion on } I_i, I_j\}$

$\text{dev}_{i,j} \leftarrow \frac{1}{\|\mathcal{U}\|} \sum_{u \in \mathcal{U}} (r_u(i) - r_u(j))$

**end**

Online (for  $u$ ) :

$\mathcal{V} \leftarrow \{j \mid u \text{ has expressed an opinion on } I_j\}$

$r_u(i) \leftarrow \frac{1}{\|\mathcal{V}\|} \sum_{u \in \mathcal{V}} (\text{dev}_{i,j} - r_u(j))$



# Dimensionality reduction

SVD, typically  $k = 20 \dots 100$

$$M = U\Sigma V^*$$

# Dimensionality reduction

SVD, typically  $k = 20 \dots 100$

$$(a_1 \quad \dots \quad a_m) \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \text{scalar}$$

# Dimensionality reduction

SVD, typically  $k = 20 \dots 100$

$$\begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix} (b_1 \quad \cdots \quad b_n) = \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & & \vdots \\ c_{m,1} & \cdots & c_{m,n} \end{pmatrix}$$

# Dimensionality reduction

SVD, typically  $k = 20 \dots 100$

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ \vdots & \vdots & \vdots \\ a_{m,1} & a_{m,2} & a_{m,3} \end{pmatrix} \begin{pmatrix} b_{1,1} & \cdots & b_{1,n} \\ b_{2,1} & \cdots & b_{2,n} \\ b_{3,1} & \cdots & b_{3,n} \end{pmatrix} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & & \vdots \\ c_{m,1} & \cdots & c_{m,n} \end{pmatrix}$$

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SVD, typically  $k = 20 \dots 100$

$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,k} \\ \vdots & & \vdots \\ a_{m,1} & \cdots & a_{m,k} \end{pmatrix} \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & & \vdots \\ c_{k,1} & \cdots & c_{k,n} \end{pmatrix} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & & \vdots \\ c_{m,1} & \cdots & c_{m,n} \end{pmatrix}$$

# Challenges

- How do we measure success?
- What are our features?

# Clustering

- kNN
- Curse of Dimensionality
- Scalability

# Clustering

- kNN *k*-Nearest Neighbor
- Curse of Dimensionality
- Scalability  $10^7$  clients,  $10^6$  objets













