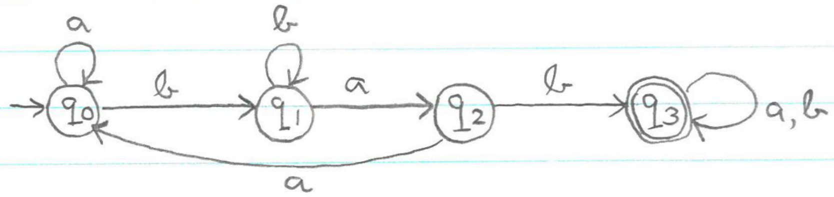
solution exam 2IT71/2IT17 automata and process theory
23/1/2015

① DFA D:



state	pathset
q_0	$\{w \in \{a,b\}^* \mid w = \varepsilon \lor w = a \lor$ (w does not contain bab and ends on aa) $\}$
q_1	$\{w \in \{a,b\}^* \mid w$ does not contain bab and ends on $b\}$
q_2	$\{w \in \{a,b\}^* \mid w$ does not contain bab and ends on $ba\}$
q_3	$\{w \in \{a,b\}^* \mid w$ contains $bab\}$

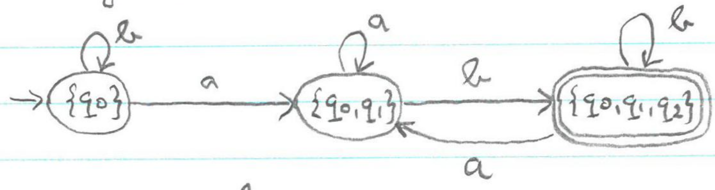
② ⓐ DFA D is derived using the so-called "subset construction" from the proof of Theorem 2.13 but only the reachable states are calculated

... indicates state added by the epsilon closure

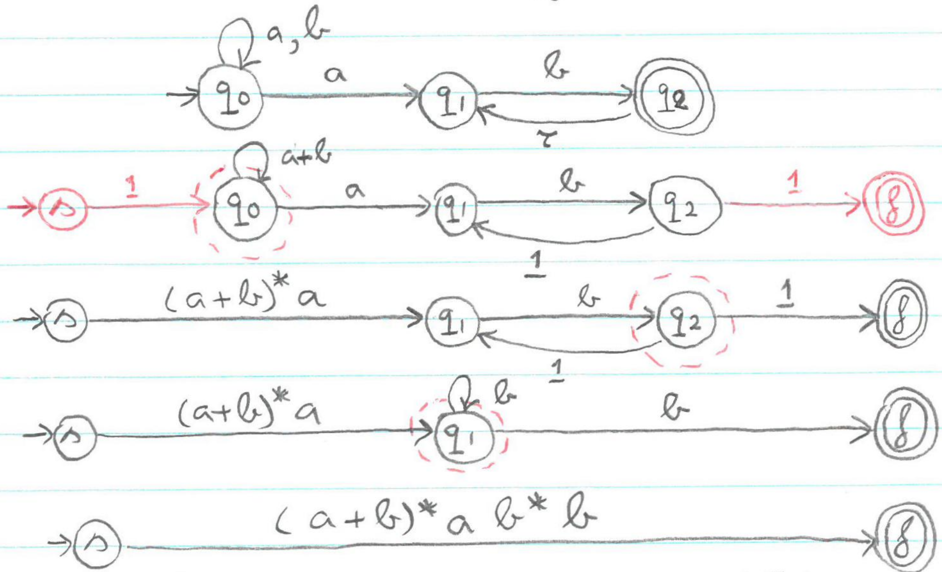
transition table

	DFA state	a	b
initial state →	$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
	$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_1\}$
final state	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_1\}$

transition diagram



ⓑ



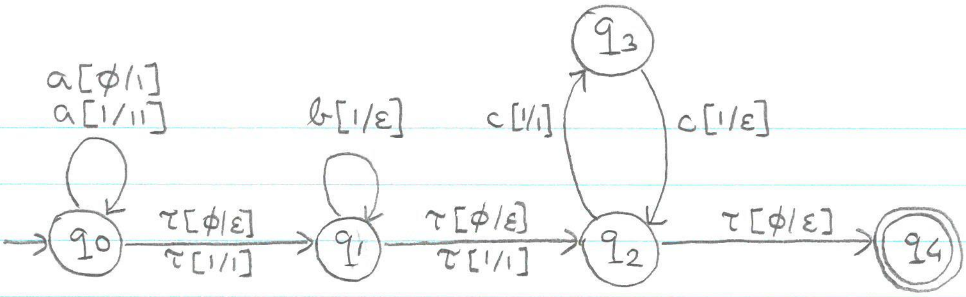
regular expression: $(a+b)^*ab^*b$

3 Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA with $\mathcal{L}(D) = L$.
Define DFA $D' = (Q, \Sigma, \delta, q_0, F')$
where $F' = \{ q \in Q \mid \delta(q, a) \in F \}$.
We will show that $\mathcal{L}(D') = L'$:
for all $u \in \Sigma^*$ we derive

$$u \in \mathcal{L}(D')$$
$$\overset{\text{val}}{=} \quad \{ \text{definition } \mathcal{L}(D') \}$$
$$\exists_q [\, q \in F' : (q_0, u) \vdash^*_{D'} (q, \varepsilon) \,]$$
$$\overset{\text{val}}{=} \quad \{ \text{def. } F' ;\ \vdash^*_{D'} = \vdash^*_D \text{ since } \vdash_{D'} = \vdash_D \}$$
$$\exists_q [\, \delta(q, a) \in F : (q_0, u) \vdash^*_D (q, \varepsilon) \,]$$
$$\overset{\text{val}}{=} \quad \{ \delta(q,a) \in F \overset{\text{val}}{=} \exists_f [f \in F : \delta(q,a) = f] ;\ \text{lemma } (2.3) \}$$
$$\exists_{q,f} [\, f \in F : (q_0, ua) \vdash^*_D (q, a) \wedge \delta(q, a) = f \,]$$
$$\overset{\text{val}}{=} \quad \{ \text{def. } \vdash_D \}$$
$$\exists_{q,f} [\, f \in F : (q_0, ua) \vdash^*_D (q, a) \wedge (q, a) \vdash_D (f, \varepsilon) \,]$$
$$\overset{\text{val}}{=} \quad \{ \text{def. } \vdash^*_D \}$$
$$\exists_f [\, f \in F : (q_0, ua) \vdash^*_D (f, \varepsilon) \,]$$
$$\overset{\text{val}}{=} \quad \{ \text{def. } \mathcal{L}(D) \}$$
$$ua \in \mathcal{L}(D)$$
$$\overset{\text{val}}{=} \quad \{ \mathcal{L}(D) = L \}$$
$$ua \in L$$
$$\overset{\text{val}}{=} \quad \{ \text{def. } L' \}$$
$$u \in L'$$

Since D' is a DFA and $\mathcal{L}(D') = L'$,
it follows that L' is regular.

4



interpretation: $(q_0, w, \varepsilon) \vdash^* (q, \varepsilon, x)$

state q	input w	stack x	constraints
q_0	a^k	1^k	$k \geq 0$
q_1	$a^k b^l$	1^{k-l}	$0 \leq l \leq k$
q_2	$a^k b^l c^{2m}$	$1^{k-(l+m)}$	$0 \leq l+m \leq k$
q_3	$a^k b^l c^{2m+1}$	$1^{k-(l+m)}$	$0 \leq l+m < k$
q_4	$a^k b^l c^{2m}$	ε	$k = l+m$

5 a $L = \{ a^k b^p c^l \mid k \geq p \wedge l \geq p \}$ is not context-free

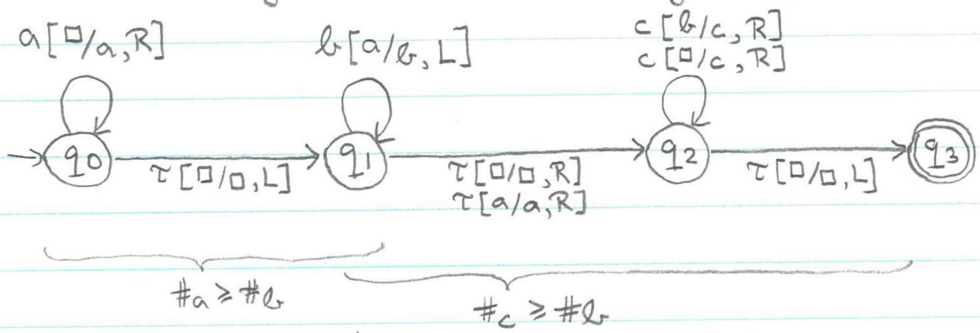
proof:

- Let $m > 0$.
  - Choose $w = a^m b^m c^m$, then $w \in L$ and $|w| = 3m \geq m$.
    - Let u, v, x, y, z be strings such that $w = uvxyz$, $vy \neq \varepsilon$, and $|vxy| \leq m$.

      Case distinction:
      - vy contains only a's: $uv^0xy^0z = a^{m-|vy|} b^m c^m \notin L$ since $|vy| > 0$.
      - vy contains only c's: $uv^0xy^0z = a^m b^m c^{m-|vy|} \notin L$ since $|vy| > 0$
      - vy contains b's and no c's: $\#_b(uv^2xy^2z) > m = \#_c(uv^2xy^2z)$ so $uv^2xy^2z \notin L$
      - vy contains b's and no a's: $\#_b(uv^2xy^2z) > m = \#_a(uv^2xy^2z)$ so $uv^2xy^2z \notin L$

Using (the contraposition of) the Pumping Lemma it follows that L is not context-free.

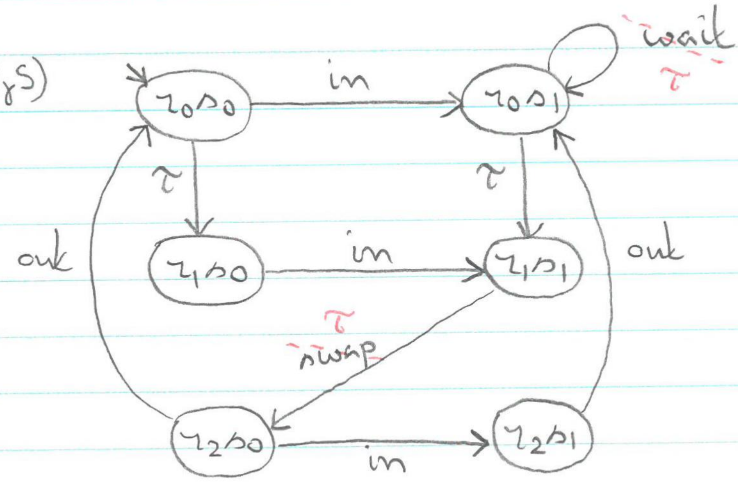
(b) reactive Turing machine accepting $L = \{a^k b^p c^\ell \mid k \geq p \wedge \ell \geq p\}$



accepting computation sequence for aabcc:

$$
\begin{aligned}
& (q_0,\ aabcc,\ \langle\square\rangle) \\
\vdash\ & (q_0,\ abcc,\ a\langle\square\rangle) \\
\vdash\ & (q_0,\ bcc,\ aa\langle\square\rangle) \\
\vdash\ & (q_1,\ bcc,\ a\langle a\rangle) \\
\vdash\ & (q_1,\ cc,\ \langle a\rangle b) \\
\vdash\ & (q_2,\ cc,\ a\langle b\rangle) \\
\vdash\ & (q_2,\ c,\ ac\langle\square\rangle) \\
\vdash\ & (q_2,\ \varepsilon,\ acc\langle\square\rangle) \\
\vdash\ & (q_3,\ \varepsilon,\ ac\langle\square\rangle)
\end{aligned}
$$

⑥ $\partial_H(R \|_\gamma S)$



$\tau_I(\partial_H(R \|_\gamma S))$