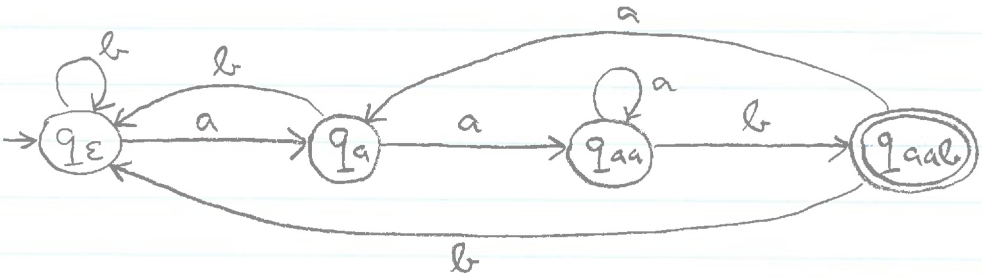
solution exam 2IT71 / 2IT17 Automata and process theory
10/4/2015

(1)



state	pathset
q_ε	$\{w \in \{a,b\}^* \mid w = \varepsilon \vee (w \text{ ends in } b, \text{ but not in } aab)\}$
q_a	$\{w \in \{a,b\}^* \mid w = a \vee (w \text{ ends in } ba)\}$
q_{aa}	$\{w \in \{a,b\}^* \mid w \text{ ends in } aa\}$
q_{aab}	$\{w \in \{a,b\}^* \mid w \text{ ends in } aab\}$

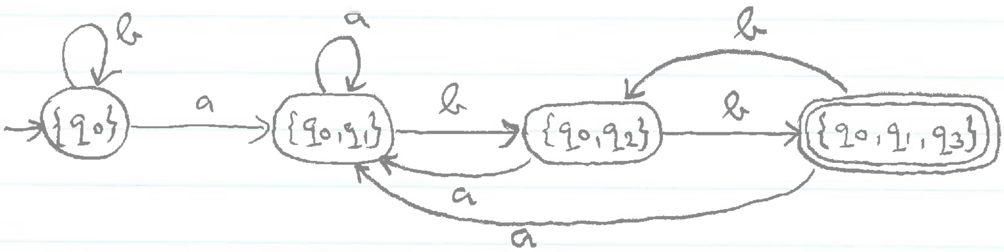
(2) (a) DFA D is derived using the so-called "subset construction" from the proof of Theorem 2.13, but only the reachable states are calculated

- transition table

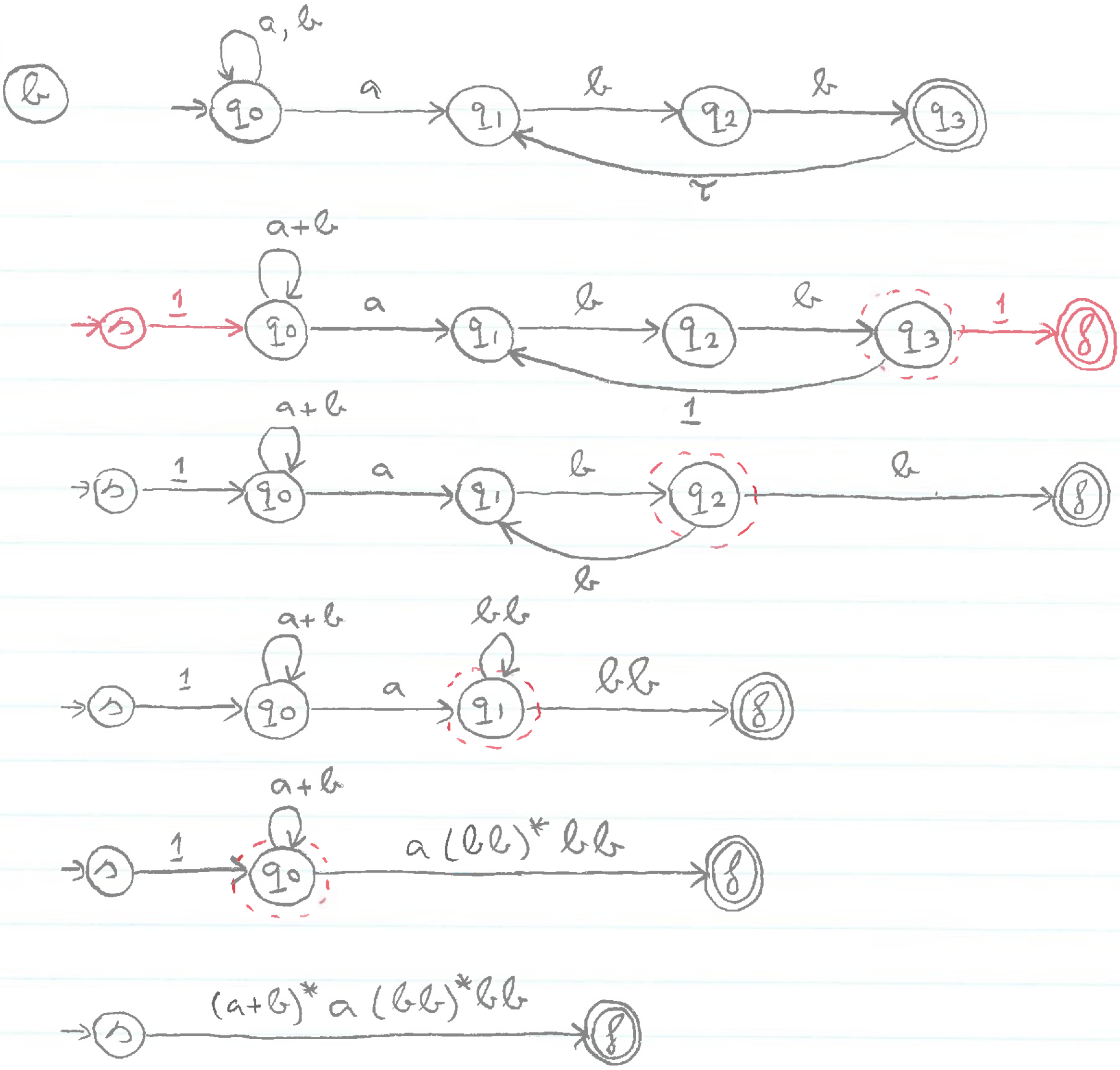
	DFA state	a	b
initial state →	$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
	$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
	$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_1\}$
final state →	$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

... indicates state added by the epsilon closure

- transition diagram

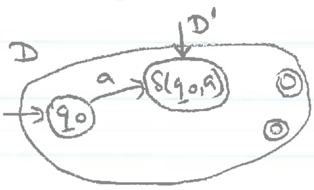


(b)



regular expression : $(a+b)^* a (bb)^* bb$

(3) Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting L.
Define DFA $D' = (Q, \Sigma, \delta, \delta(q_0, a), F)$



We have that L' is regular, since $\mathcal{L}(D') = L'$:
for $u \in \Sigma^*$ we have

$u \in \mathcal{L}(D')$

$\overset{\text{val}}{\equiv}$ { def. $\mathcal{L}(D')$ }

$\exists f\,[f \in F : (\delta(q_0,a), u) \vdash^*_{D'} (f, \varepsilon)]$

$\overset{\text{val}}{\equiv}$ { $(q_0, au) \vdash_D (\delta(q_0,a), u)$, $\vdash_{D'} = \vdash_D$ }

$\exists f\,[f \in F : (q_0, au) \vdash^*_D (f, \varepsilon)]$

$\overset{\text{val}}{\equiv}$ { def. $\mathcal{L}(D)$ }

$au \in \mathcal{L}(D)$

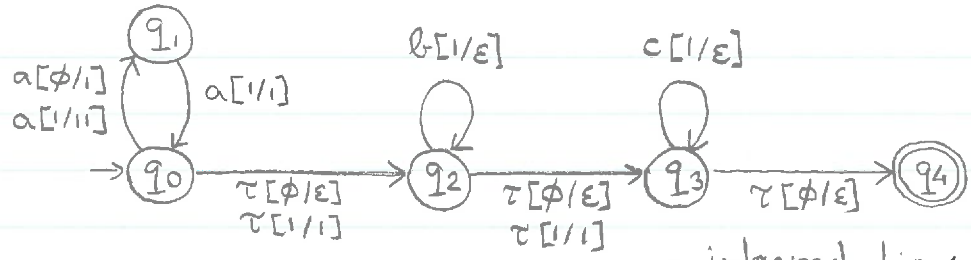
$\overset{\text{val}}{\equiv}$ { $\mathcal{L}(D) = L$ }

$au \in L$

$\overset{\text{val}}{\equiv}$ { def. L' }

$u \in L'$

④



interpretation: $(q_0, w, \varepsilon) \vdash^* (q, \varepsilon, x)$

state q	input w	stack x	constraints
q_0	a^{2k}	1^k	$k \geq 0$
q_1	a^{2k+1}	1^{k+1}	$k \geq 0$
q_2	$a^{2k} b^l$	1^{k-l}	$0 \leq l \leq k$
q_3	$a^{2k} b^l c^m$	$1^{k-(l+m)}$	$0 \leq l+m \leq k$
q_4	$a^{2k} b^l c^m$	ε	$k = l+m$

⑤ ⓐ $L = \{ a^k b^{2k} c^l \mid l \geq k \}$ is not context-free

proof

$\forall_m$ .... Let $m > 0$.

$\exists_w$ .... Choose $w = a^m b^{2m} c^m$, then $w \in L$ and $|w| = 4m \geq m$

$\forall_{u,v,x,y,z}$ .... Let $u, v, x, y,$ and z be strings such that $w = uvxyz$, $vy \neq \varepsilon$, and $|vxy| \leq m$.

case distinction:

$\exists_i$ .... – vy contains only a's: choose $i = 0$:
$uv^0xy^0z = a^{m-|vy|} b^{2m} c^m \notin L$, since $|vy| > 0$.

– vy contains only c's: choose $i = 0$:
$uv^0xy^0z = a^m b^{2m} c^{m-|vy|} \notin L$, since $|vy| > 0$.

– vy contains b's and no c's: choose $i = 2$:
$\#_b(uv^2xy^2z) > 2m = \#_c(uv^2xy^2z)$
vy contains b's ... no c's in vy

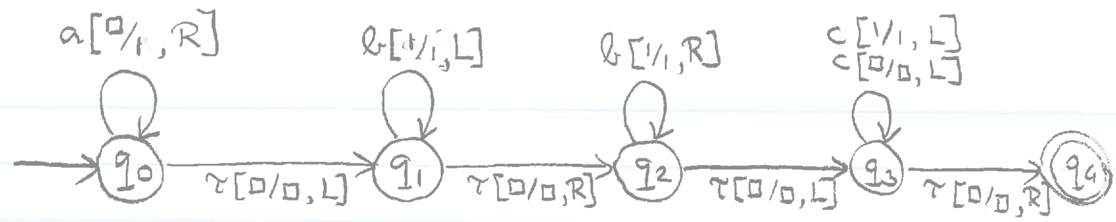
so $uv^2xy^2z \notin L$

– vy contains b's and no a's: choose $i = 0$:
$\#_b(uv^0xy^0z) < 2m = 2\#_a(uv^0xy^0z)$
vy contains b's ... no a's in vy

so $uv^0xy^0z \notin L$

Using (the contraposition of) the Pumping Lemma it follows that L is not context-free.

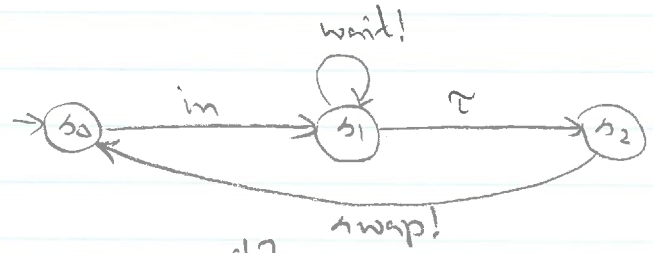
(a)



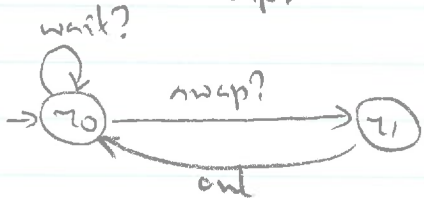
$(q_0, abbcc, \langle \square \rangle)$
$\vdash (q_0, bbcc, 1\langle \square \rangle)$
$\vdash (q_1, bbcc, \langle 1 \rangle)$
$\vdash (q_1, bcc, \langle \square \rangle 1)$
$\vdash (q_2, bcc, \langle 1 \rangle)$
$\vdash (q_2, cc, 1\langle \square \rangle)$
$\vdash (q_3, cc, \langle 1 \rangle)$
$\vdash (q_3, c, \langle \square \rangle 1)$
$\vdash (q_3, \varepsilon, \langle \square \rangle \square 1)$
$\vdash (q_4, \varepsilon, \langle \square \rangle 1)$

(b)

S



R:



$\partial_H(S \|_\gamma R)$

$\tau_I(\partial_H(S \|_\gamma R))$