ECE 6560 Final Project - Image Smoothing

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1 Problem Description

Although image processing techniques began to appear around the 1960s, the proliferation of inexpensive digital computing power has greatly widened the application pool. Image processing techniques can now be found in areas such as digital photography, medical imaging, and object detection/tracking.

All these application areas can be affected by a very common problem: image noise. Noise can render further processing ineffective, thus there exist a variety of approaches by which we can attempt to smooth and denoise our images. Traditional image processing techniques may rely on fixed kernels that can be swept over an image in order to smooth it. However, this type of approach will result in uniform smoothing can blur the edges of an image.

We can instead leverage PDEs to describe how an image should be updated depending on its local characteristics. More specifically, we will examine Ansiotropic Diffusion. This technique can be used to reduce image noise while lessening the blurring that is done to edges. This project will attempt to develop PDEs that can be used to reduce noise in high/low contrast images.

We need more information about the actual high/low contrast problem!

2 Mathematical Modeling

Before explicitly developing our PDEs, we must first understand what behavior we want our system to have. The calculus of variations can be used to minimize an energy functional. The choice of setup for the energy functional will determine the system behavior.

Let's begin by defining our image as

$$I(x, y)$$

Note that the derivations in sections (2) and (3) are being performed exclusively in continuous space: $x, y \in \mathbb{R}$.

3 Derivation of PDE

Now, let's introduce the Euler-Lagrange equation:

$$L_f - \frac{\partial}{\partial x} L_{f'} = 0 \text{ (1-D)}$$
$$L_I - \frac{\partial}{\partial x} L_{I_x} - \frac{\partial}{\partial y} L_{I_y} = 0 \text{ (2-D)}$$

We can begin working towards obtaining our PDE by setting up a gradient descent:

$$I_t = -\nabla_I E$$
$$I_t = -L_I + \frac{\partial}{\partial x} L_{I_x} + \frac{\partial}{\partial y} L_{I_y}$$

We will now have to compute terms L_I, L_{I_x}, L_{I_y} using the previously obtained energy functional:

$$L(I, I_x, I_y, x, y) = \frac{\lambda}{1+e^{\alpha}}, \text{ where } \alpha = -\frac{1}{\beta}(\|\nabla_I\|)$$

$$L_I = \frac{\partial}{\partial I}(L)$$
$$L_I = 0$$
$$L_{I_x} = \frac{\partial}{\partial I_x}(L)$$
$$L_{I_x} = \frac{\lambda}{\beta} \frac{e^{\alpha}}{(1+e^{\alpha})^2} \frac{I_x}{\sqrt{I_x^2 + I_y^2}}$$
$$L_{I_y} = \frac{\partial}{\partial I_y}(L)$$
$$L_{I_y} = \frac{\lambda}{\beta} \frac{e^{\alpha}}{(1+e^{\alpha})^2} \frac{I_y}{\sqrt{I_x^2 + I_y^2}}$$

Now that we have obtained our expressions for L_{I_x} and L_{I_y}, we must compute their partial derivatives as shown by the Euler-Lagrange equation.

4 Discretization and Implementation

5 Experimental Results

6 Summary