Расчетно графическая работа

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Задачи решили студенты НОЦ инфохимии:

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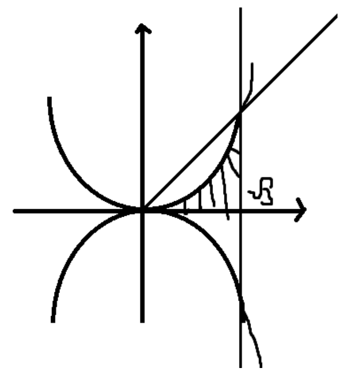
Вариант 3

Задача 2

Пункт а

$$r = 2|\operatorname{tg}\varphi|, r = \frac{\sqrt{3}}{\cos\varphi}$$

Решение:



$$2|\operatorname{tg}\varphi| = \frac{\sqrt{3}}{\cos\varphi}$$

$$2\cos\varphi|\operatorname{tg}\varphi| = \sqrt{3}$$

$$2\sin\varphi \cdot \sin\varphi = \sqrt{3}$$

$$\sin\varphi \cdot \sin\varphi = \frac{\sqrt{1}}{2} \Rightarrow \varphi = \pm\frac{\pi}{2}$$

$$s_{1/2} = \frac{1}{2}\int_0^{\pi/3} (2\operatorname{tg}\varphi)^2\, d\varphi = \frac{1}{2}\cdot\varphi\int_0^{\pi/3} \operatorname{tg}^2\varphi\, d\varphi = 2\int_0^{\pi/3} \frac{\sin^2\varphi}{\cos^2\varphi}\, d\varphi =$$

$$= 2\int_0^{\pi/3} \frac{1-\cos^2\varphi}{\cos^2\varphi}\, d\varphi = 2\int_6^{\pi/3} \frac{1}{\cos^2\varphi}\, d\varphi - 2\int_0^{\pi/3} d\varphi =$$

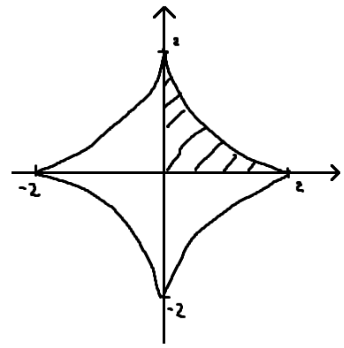
$$= (2\operatorname{tg}\varphi - 2\varphi)\Big|_0^{\pi/3} = 2\sqrt{3} - \frac{2\pi}{3} = s_{1/2} \qquad S = 4\sqrt{3} - \frac{4\pi}{3}$$

Ответ: $S = 4\sqrt{3} - \frac{4\pi}{3}$

Пункт б

$$\left(\frac{x}{2}\right)^{2/3} + \left(\frac{y}{2}\right)^{2/3} = 1$$

Решение:



$$\left(\frac{x}{2}\right)^{2/3} + \left(\frac{y}{2}\right)^{2/3} = 1$$

$$\left(\frac{x}{2}\right)^{1/3} = \cos t$$

$$\left(\frac{y}{2}\right)^{1/5} = \sin t$$

$$\left(\cos^2 t + \sin^2 t = 1\right)$$

$$\begin{cases} x = 2\cos^3 t \\ y = 2\sin^3 t \end{cases}, \ t \in [0, 2\pi] \quad \begin{cases} y' = 6\sin^2 t \cos x \\ x' = -6\cos^2 x \sin x \end{cases}$$

$$\frac{1}{4}s = \frac{1}{2}\int_0^{\pi/2} \left(2\cos^9 t \cdot 6\sin^2 t \cos t + 6\cos^2 t \sin t 2\sin^3 t\right) dt =$$

$$= \frac{1}{2} \cdot 12 \int_0^{\pi/2} \left(\cos^4 t \sin^2 t + \sin^4 t \cos^2 t\right) dt = 6\int_0^{\pi/2} \left(\sin^2 t \cos^2 t \left(\cos^2 t + 1\sin^2 t\right) dt =$$

$$= 6\int_0^{\pi/2} \sin^2 t \operatorname{cs}^2 t dt = 6 \cdot \frac{1}{4}\int_0^{\pi/2} \sin^2 2t dt = \frac{3}{2}\int_0^{\pi/2} \sin^2 2t dt =$$

$$= \frac{3}{2}\int_0^{\pi/2} \frac{1-\cos 4t}{2} dt = \frac{3}{4}\int_0^{\pi/2} \frac{d6}{3} - \frac{3}{4}\int_0^{\pi/2} \cos 4t dt =$$

$$= \frac{3}{4} \cdot \frac{\pi}{2} - \left.\frac{\sin 4t}{4}\right|_0^{\pi/2} = \frac{3}{32}\pi - \frac{\sin 2\pi}{4} + 0 = \frac{3\pi}{8}$$

$$\frac{1}{4}s = \frac{3n}{7} >= \frac{9\pi}{2}$$

Ответ:

$$\text{S} = \frac{9\pi}{2}$$

Задача 3

Пункт а

$$x = 6 - 3t^2, \quad y = 4t^3, x \geqslant 0$$

Решение:

$$\begin{aligned}
&6 - 3t^2 \geq 0 \quad t^2 \leq 2, t \in [-\sqrt{2}; \sqrt{2}] \\
&x' = -6t; y' = 12t^2 \quad 1 \quad i(x)^2 + \left(y'\right)^2 = 36t^2 + 144t^4 = 36t^2\left(1 + 4t^2\right) \\
&L = \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{x^2 + y'^2}dt = \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{36t^2\left(1 + 4t^2\right)}dt = \\
&= 2\int_0^{\sqrt{2}} 6t\sqrt{1 + 4t^2}dt = 12\int_0^{\sqrt{2}} t\sqrt{1 + 4t^2}dt = 6\int_0^{\sqrt{2}} \sqrt{1 + 4t^2}dt^2 = \\
&= \frac{3}{2}\int_0^{\sqrt{2}} \sqrt{1 + 4t^2}d\left(1 + 4t^2\right) = \left(1 + 4t^2\right)^{\frac{3}{2}}\Big|_0^{\sqrt{2}} = 27 - 1 = 26
\end{aligned}$$

Ответ: 26

Пункт б

$$y^3 = 3z, \ 2yx = 1, \ 1 \leq y \leq 3$$

Решение:

$$\begin{aligned}
&y = t, \quad z = \frac{t^3}{3}, \quad x = \frac{1}{2t}, \quad 1 \leq t \leq 3 \\
&x' = -\frac{1}{2t^2}, \quad y' = 1, \quad z' = t^2 \\
&\left(x'\right)^2 + \left(y'\right)^2 + \left(z'\right)^2 = \frac{1}{4t^4} + 1 + t^4 = \left(\frac{1}{2t^2} + t^2\right)^2 \\
&L = \int_1^3 \sqrt{x'^2 + y'^2 + z'^2}dt = \int_1^3 \frac{1}{2t^2} + t^2dt = -\frac{1}{2t} + \frac{t^3}{3}\Big|_1^3 = \\
&= \left(-\frac{1}{6} + 9\right) - \left(-\frac{1}{2} + \frac{1}{3}\right) = 9 + \frac{1}{2} - \frac{1}{6} - \frac{1}{3} = 9
\end{aligned}$$

Ответ: 9

Вариант 6

Задача 2

Пункт а

Пункт б

Задача 3

Пункт а

Пункт б

Вариант 8

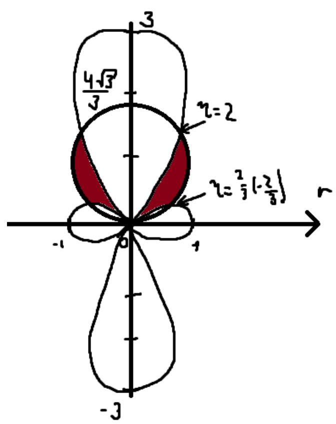
Задача 2

Пункт а

$$r = 1 - 2\cos 2\varphi,\ r = \frac{4\sqrt{3}}{3}\sin\varphi$$

$$\left(r \geq 1 - 2\cos 2\varphi,\ r \leq \frac{4\sqrt{3}}{3}\sin\varphi\right)$$

Решение:



$$r_1 = 1 - 2cos\varphi$$
$$r_2 = \frac{4\sqrt{3}}{3}\sin\varphi$$

$$1 - 2\cos 2\varphi = 1 - 2(1 - 2\sin^2\varphi)\frac{4\sqrt{3}}{3}\sin\varphi$$

$$4\sin^2\varphi - \frac{4\sqrt{3}}{3}\sin\varphi - 1 = 0$$

$$\sin\varphi\frac{\frac{4\sqrt{3}}{3} \pm \sqrt{\frac{16}{3} + 16}}{8} = \frac{4\frac{\sqrt{3}}{3} \pm 8\frac{\sqrt{3}}{3}}{8}$$

$$\varphi = \frac{\pi}{3} + 2\pi k \quad \varphi = \frac{2\pi}{3} + 2\pi k$$

$$\varphi = arcsin\left(-\frac{\sqrt{3}}{6}\right) + 2\pi k \quad \varphi = \pi - rcsin\left(-\frac{\sqrt{3}}{6}\right) + 2\pi k$$

Проинтегрируем:

$$S = 2\cdot\frac{1}{2}\int_0^{\pi/3}\left(\frac{4\sqrt{3}}{3}\varphi\right)^2 - (1 - 2\cos 2\varphi)^2\,d\varphi = \int_0^{\pi/3}\frac{16}{3}\sin^2\varphi - (4\sin^2\varphi - 1)^2\,d\varphi =$$

$$= \int_0^{\pi/3} -16\sin^4\varphi + \frac{40}{3}\sin^2\varphi - 1\,d\varphi = \int_0^{\pi/3} -16\sin^2(1 - \cos^2\varphi) + \frac{40}{3}\sin^2 - 1\,d\varphi =$$

$$= \int_0^{\pi/3} (2\sin 2\varphi)^2 - \frac{8}{3}\sin^2\varphi - 1\, d\varphi = \left| \begin{array}{c} 1-2\sin^2 2\varphi = \cos 4\varphi \\ 4\sin^2 2\varpi = -2\cos 4\varphi + 2 \\ -\frac{8}{3}\sin^2\varphi = \frac{8}{6}(-2\sin^2\varphi) = \\ = \frac{8}{6}(1-2\sin^2\varphi) - \frac{8}{6} = \frac{8}{6}\cos 2\varphi - \frac{8}{6} \end{array} \right| =$$

$$= \int_0^{\pi/3} -2\cos 4\varphi + \frac{8}{6}\cos 2\varphi + 2 - \frac{4}{3} - 1\, d\varphi =$$

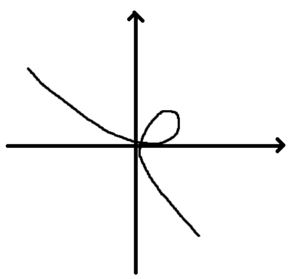
$$= \frac{-\sin 4\varphi}{2} + \frac{2\sin 2\varphi}{3} - \frac{1}{3}\varphi \Big|_0^{\frac{\pi}{3}} = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{3} - \frac{\pi}{9}\right) - 0 = \frac{7\sqrt{3}}{12} - \frac{\pi}{9}$$

Ответ: $\dfrac{7\sqrt{3}}{12} - \dfrac{\pi}{9}$

Пункт б

$$(x+y)^3 = tx$$

Решение:



$$(x+y)^3 = tx \quad y = tx$$

$$(x+tx)^3 = 2tx^2 \quad x^3(t+1)^3 = 2tx^2 \quad x(t+1)^3 = 2t$$

$$x = \frac{2t}{(t+1)^3};\ y = \frac{2t^2}{(t+1)^3}$$

$x'_t = \dfrac{2(t+1)^3 - 6t(t+1)^2}{(t+1)^6} = \dfrac{2(t+1) - 6t}{(t+1)^4} = \dfrac{-4t+2}{(t+1)^4}$

$y'_t = \dfrac{4t(t+1)^3 - 6t^2(t+1)^2}{(t+1)^6} = \dfrac{4t(t+1) - 6t^2}{(t+1)^4} = \dfrac{-2t^2+4t}{(t+1)^4}$

$S = \dfrac{1}{2}\displaystyle\int_0^{+\infty} \left| y\cdot x' - xy' \right| dt = \dfrac{1}{2}\displaystyle\int_0^{+\infty} \dfrac{2t^2\cdot 2\cdot(-2t+1)}{(t+1)^7} - \dfrac{2t\cdot 2t\cdot(t+2)}{(t+1)^7}\ |\ dt =$

$= 2\displaystyle\int_0^{+\infty} \Big(\dfrac{t^2}{(t+1)^7}(-2t+1+t-2)\ |\ dt = \displaystyle\int_0^{+\infty} \dfrac{t^2}{(t+1)^6}dt$

$= \displaystyle\int_0^{+\infty} \dfrac{t^2+2t+1}{(t+1)^6} - \dfrac{2t+2}{(t+1)^6} + \dfrac{1}{(t+1)^6}dt = 2\displaystyle\int_0^{+\infty} (t+1)^{-4} - 2(t+1)^{-5} + \big(t+y^6 dt =$

$= \dfrac{2(t+1)^{-3}}{-3} - \dfrac{4(t+1)^{-4}}{-4} + \dfrac{2(t+1)^{-5}}{-5}\Big|_0^{+\infty} =$

$= -\dfrac{2}{3(t+1)^3} + \dfrac{1}{(t+1)^4} - \dfrac{2}{5(t+1)^5}\Big|_0^{+\infty} = \dfrac{-10(t+1)^2 + 15(t+1) - 6}{15(t+1)^5}\Big|_0^{+\infty}$

$= -\dfrac{-10+15-6}{15} = \dfrac{1}{15}$

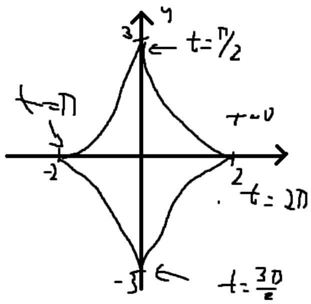
Ответ: $\dfrac{1}{15}$.

Задача 3

Пункт а

$$x = 2\cos^3 t,\ \ y = 3\sin^3 t$$

Решение:



$$x' = 2\cdot 3\cdot(-\sin t)\cdot\cos^2 t$$

$$y' = 9\cos t\cdot\sin^2 t$$

$$L = \int_0^{2\pi}\sqrt{(s')^2+(y')^2}\,dt = 4\int_0^{\frac{\pi}{2}} 3\cdot\sin t\cdot\cos t\sqrt{4\cos^2 t+9\sin^2 t}\,dt =$$

$$= \left| \begin{array}{l} 4\cos^2 t + 9\sin^2 t = x \\ -8\sin t\cos t + 18\sin t\cos t\,dt = dx \\ 10\sin t\cos t\,dt = dx \\ t = \frac{\pi}{2} \quad x = 9 \\ t = 0 \quad x = 4 \end{array} \right| = \frac{6}{5}\int_4^9 \sqrt{x}dx =$$

$$= \frac{6}{5}\cdot\frac{2}{3}(\sqrt{x})^3\Big|_4^9 = \frac{4}{5}(27-8) = 15,2$$

Ответ: $15,2$

Пункт б

$$\delta]z^2 = 2x, xz = 3y \quad 0 \leqslant x \leqslant 8$$

Решение:

$z = 6t, x = 18t^2 \quad y = 36t^3$

$0 \leqslant 18t^2 \leqslant 8 \quad 0 \leqslant t^2 \leqslant \dfrac{4}{9} \quad t \in \left[-\dfrac{2}{3}; \dfrac{2}{3}\right]$

$\left(x'\right)^2 + \left(y'\right)^2 + \left(z'\right)^2 = (36t)^2 + \left(108t^2\right)^2 + 36 =$

$= 36^2\cdot t^2 + 36^2\cdot 9\cdot t^4 + 36 = 36^2\left(9t^4 + t^2 + \dfrac{1}{36}\right) = 36^2\left(3t^2 + \dfrac{1}{6}\right)^2$

$L = \displaystyle\int_{-\frac{2}{3}}^{\frac{2}{3}} \sqrt{x'^2 + y^2 + z'^2}dt = \int_{-2/3}^{2/3} 36\left(3t^2 + \frac{1}{6}\right)dt = 108\int_{-2/3}^{2/3} t^2 + 6\int_{-2/3}^{2/3} 1dt =$

$= 72t^3\big|_0^{2/3} + 12t\big|_0^{2/3} = 72\cdot\dfrac{8}{27} + 8 = \dfrac{64}{3} + 8$ **Ответ:** $\dfrac{64}{3} + 8$