Расчетно графическая работа

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Задачи решили студенты НОЦ инфохимии:

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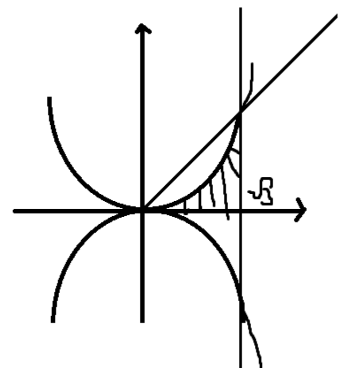
Вариант 3

Задача 2

Пункт а

$$r = 2|\operatorname{tg}\varphi|, r = \frac{\sqrt{3}}{\cos\varphi}$$

Решение:



$$2|\operatorname{tg}\varphi| = \frac{\sqrt{3}}{\cos\varphi}$$

$$2\cos\varphi|\operatorname{tg}\varphi| = \sqrt{3}$$

$$2\sin\varphi \cdot \sin\varphi = \sqrt{3}$$

$$\sin\varphi \cdot \sin\varphi = \frac{\sqrt{1}}{2} \Rightarrow \varphi = \pm\frac{\pi}{2}$$

$$s_{1/2} = \frac{1}{2}\int_0^{\pi/3}(2\operatorname{tg}\varphi)^2\,d\varphi = \frac{1}{2}\cdot\varphi\int_0^{\pi/3}\operatorname{tg}^2\varphi\,d\varphi = 2\int_0^{\pi/3}\frac{\sin^2\varphi}{\cos^2\varphi}\,d\varphi =$$

$$= 2\int_0^{\pi/3}\frac{1-\cos^2\varphi}{\cos^2\varphi}\,d\varphi = 2\int_6^{\pi/3}\frac{1}{\cos^2\varphi}\,d\varphi - 2\int_0^{\pi/3}d\varphi =$$

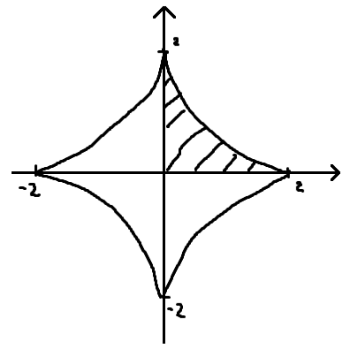
$$= (2\operatorname{tg}\varphi - 2\varphi)\Big|_0^{\pi/3} = 2\sqrt{3} - \frac{2\pi}{3} = s_{1/2} \qquad S = 4\sqrt{3} - \frac{4\pi}{3}$$

Ответ: $S = 4\sqrt{3} - \frac{4\pi}{3}$

Пункт б

$$\left(\frac{x}{2}\right)^{2/3} + \left(\frac{y}{2}\right)^{2/3} = 1$$

Решение:



$$\left(\frac{x}{2}\right)^{2/3} + \left(\frac{y}{2}\right)^{2/3} = 1$$

$$\left(\tfrac{x}{2}\right)^{1/3} = \cos t \qquad \left(\cos^2 t + \sin^2 t = 1\right)$$
$$\left(\tfrac{y}{2}\right)^{1/3} = \sin t$$

$$\begin{cases} x = 2\cos^3 t \\ y = 2\sin^3 t \end{cases}, \ t \in [0, 2\pi] \qquad \begin{cases} y' = 6\sin^3 t \cos x \\ x' = -6\cos^2 x \sin x \end{cases}$$

$$\frac{1}{4}s = \frac{1}{2}\int_0^{\pi/2} \left(2\cos^3 t \cdot 6\sin^2 t \cos t + 6\cos^3 t \sin t 2\sin^3 t\right)\, dt =$$

$$= \frac{1}{2} \cdot 12 \int_0^{\pi/2} \left(\cos^4 t \sin^2 t + \sin^4 t \cos^2 t\right)\, dt = 6\int_0^{\pi/2} \left(\sin^2 t \cos^2 t \left(\cos^2 t + 1\sin^2 t\right)\right)\, dt =$$

$$= 6\int_0^{\pi/2} \sin^2 t \cos^2 t\, dt = 6 \cdot \frac{1}{4}\int_0^{\pi/2} \sin^2 2t\, dt = \frac{3}{2}\int_0^{\pi/2} \sin^2 2t\, dt =$$

$$= \frac{3}{2}\int_0^{\pi/2} \frac{1-\cos 4t}{2}\, dt = \frac{3}{4}\int_0^{\pi/2} dt - \frac{3}{4}\int_0^{\pi/2} \cos 4t\, dt = \frac{3}{4}\cdot\frac{\pi}{2} - \left.\frac{\sin 4t}{4}\right|_0^{\pi/2} =$$

$$= \frac{3}{32}\pi - \frac{\sin 2\pi}{4} + 0 = \frac{3\pi}{8} \qquad \frac{1}{4}s = \frac{3\pi}{8} \quad \Rightarrow \quad s = \frac{9\pi}{2}$$

Ответ: $S = \dfrac{9\pi}{2}$

Задача 3

Пункт а

$$x = 6 - 3t^2, \quad y = 4t^3, x \geqslant 0$$

Решение:

$$
\begin{aligned}
&6 - 3t^2 \geq 0 \quad t^2 \leq 2, t \in [-\sqrt{2}; \sqrt{2}] \\
&x' = -6t; y' = 12t^2 \quad 1 \quad i(x)^2 + \left(y'\right)^2 = 36t^2 + 144t^4 = 36t^2\left(1 + 4t^2\right) \\
&L = \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{x^2 + y'^2} dt = \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{36t^2\left(1 + 4t^2\right)} dt = \\
&= 2\int_0^{\sqrt{2}} 6t\sqrt{1 + 4t^2} dt = 12\int_0^{\sqrt{2}} t\sqrt{1 + 4t^2} dt = 6\int_0^{\sqrt{2}} \sqrt{1 + 4t^2} dt^2 = \\
&= \frac{3}{2}\int_0^{\sqrt{2}} \sqrt{1 + 4t^2} d\left(1 + 4t^2\right) = \left(1 + 4t^2\right)^{\frac{3}{2}}\Big|_0^{\sqrt{2}} = 27 - 1 = 26
\end{aligned}
$$

Ответ: 26

Пункт б

$$y^3 = 3z, \ 2yx = 1, \ 1 \leq y \leq 3$$

Решение:

$$
\begin{aligned}
&y = t, \quad z = \frac{t^3}{3}, \quad x = \frac{1}{2t}, \quad 1 \leq t \leq 3 \\
&x' = -\frac{1}{2t^2}, \quad y' = 1, \quad z' = t^2 \\
&\left(x'\right)^2 + \left(y'\right)^2 + \left(z'\right)^2 = \frac{1}{4t^4} + 1 + t^4 = \left(\frac{1}{2t^2} + t^2\right)^2 \\
&L = \int_1^3 \sqrt{x'^2 + y'^2 + z'^2} dt = \int_1^3 \frac{1}{2t^2} + t^2 dt = -\frac{1}{2t} + \frac{t^3}{3}\Big|_1^3 = \\
&= \left(-\frac{1}{6} + 9\right) - \left(-\frac{1}{2} + \frac{1}{3}\right) = 9 + \frac{1}{2} - \frac{1}{6} - \frac{1}{3} = 9
\end{aligned}
$$

Ответ: 9

Вариант 6

Задача 2

Пункт а

$$r = \sqrt{\frac{\sin 3\varphi}{\sin\varphi}}, r = 4\cos\varphi$$

Решение:

$$S_1 = \frac{1}{2}\int_0^{\pi}\left(\sqrt{\frac{\sin 3\varphi}{\sin\varphi}}\right)^2 d\alpha = \frac{1}{2}\int_0^{\pi}\frac{\sin 3\alpha}{\sin\alpha}\,d\alpha = \frac{1}{2}\int_0^{\pi}\frac{3\cos^2\alpha\sin\alpha - \sin^3\alpha}{\sin\alpha}d\alpha =$$
$$= \frac{1}{2}\int_0^{\pi} 3\cos^3\alpha\,d\alpha - \frac{1}{2}\int_0^{\pi}\sin^2\alpha\,d\alpha = \frac{3}{2}\int_0^{\pi}\frac{\cos 2\alpha + 1}{2}\,d\alpha - \frac{1}{2}\int_0^{\pi}\frac{1-\cos 2\alpha}{2}\,d\alpha =$$
$$= \left|\begin{array}{c} u = 2\alpha \\ d\alpha = \frac{d\varphi}{2}\end{array}\right| \quad \Rightarrow \quad \begin{cases} \frac{3}{2}\int_0^{\pi}\frac{\cos 2\alpha+1}{2}\,d\alpha = \frac{1}{2}\int\frac{\cos\varphi}{2}\,d\varphi + \frac{1}{2}\int\frac{d\varphi}{2} = \frac{\sin 2\alpha}{4} + \frac{\alpha}{2} \\ \frac{1}{2}\int_0^{\pi}\frac{1-\cos 2\alpha}{2}\,d\alpha = \frac{\alpha}{2} - \frac{\sin 2\alpha}{4}\end{cases}$$
$$S_1 = \left(\frac{3}{2}\left(\frac{\sin 2\alpha}{4} + \frac{\alpha}{2}\right) - \frac{1}{2}\frac{\alpha}{2} - \frac{\sin 2\alpha}{\alpha}\right)\Bigg)^{\pi} = \frac{\pi}{2}$$
$$2S_2 = \frac{1}{4}\int_0^{2\pi} 16\cos^2\alpha d\alpha = 4\int_0^{2\pi}\frac{1+\cos 2\alpha}{2}d\alpha = 2\cdot\left(\int_0^{0\pi} d\alpha + \int_0^{2\pi}\cos 2\alpha d\alpha\right) \Rightarrow$$
$$\Rightarrow S_2 = 2\pi$$
$$S_{\text{между ккривыми}} = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

Ответ $\frac{3\pi}{2}$

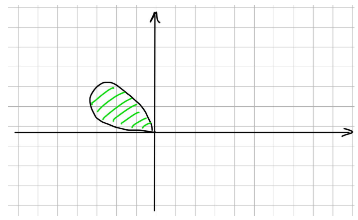
Пункт б

$$x^3 = ay^4 - x^2y$$

Решение:

$$x^3 = ay^4 - x^2y \quad \begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$
$$r^3\cos^3\varphi = ar^4\sin^4\varphi - r^2\cos^2\varphi r\sin\varphi$$
$$r^3\cos^3\varphi = r^4 a\sin^4\varphi - r^3\cos^2\varphi\sin\varphi \,|: 2^3$$
$$\cos^3\varphi = r\,\mathrm{asin}^4\varphi - \cos^2\varphi\sin\varphi$$
$$\cos^2\varphi\sin\varphi + \cos^3\varphi = r\cdot a\sin^4\varphi$$

$$r = \frac{\cos^2\varphi\sin\varphi + \cos^3\varphi}{a\sin^4\varphi}$$



$$S = \int_{\varphi_1}^{\varphi_2} r^2(\varphi)d\varphi = \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{\cos^2\varphi\sin\varphi + \cos^3\varphi}{a\sin^4\varphi}\right)^2 d\varphi =$$

$$= \quad \frac{1}{a^2}\int_{\frac{\pi}{2}}^{3\frac{\pi}{4}} \frac{\operatorname{tg}^2\varphi + 2\operatorname{tg}\varphi + 1}{\cos^2\varphi + \operatorname{tg}^8\varphi} d\varphi = \begin{cases} u = 2\operatorname{tg}\varphi \\ du = \frac{1}{\cos^2\varphi}d\varphi \end{cases}$$

$$= \frac{1}{a^2}\int \frac{32u^2 + 128u + 128}{u^8}du = \frac{1}{a^2}\int \left(\frac{32}{u^6} + \frac{128}{u^7} + \frac{128}{u^8}\right)du =$$

$$= -\frac{672u^2 + 2240u + 1920}{105a^2u^7} \text{ обратная замена}$$

$$\left(-\frac{\operatorname{ctg}^7\varphi\operatorname{tg}^2\varphi}{5\mathrm{a}^2} - \frac{\operatorname{ctg}^7\varphi\operatorname{tg}\varphi}{3a^2} - \frac{\operatorname{ctg}^7\varphi}{7a^2}\right)\Bigg|_{\frac{\pi}{2}}^{\frac{3\pi}{4}} = \frac{1}{105a^2}$$

Ответ: $\frac{1}{105a^2}$

Задача 3

Пункт а

$$x = 6 - 3t^3, y = \frac{9\left(2t^2 - t^4\right)}{8}, y \geqslant 0$$

Решение:

$$\begin{aligned}
&(\gamma) = \int_{T_0}^{T_1} \sqrt{(x'(t))^2 + (y'(t))^2} dt \\
&x'(t) = -9t^2 \\
&y'(t) = \frac{9}{2}t - \frac{9}{2}t^3
\end{aligned}$$

Найдем промежутки:

$$\begin{aligned}
&\frac{9\left(2t^2 - t^4\right)}{8} \geqslant 0 \Rightarrow \\
&\Rightarrow t \in [-\sqrt{2}; \sqrt{2}]
\end{aligned}$$

$$\begin{aligned}
x'(t)^2 &= Mt^2 \\
y'\left(t^2\right) &= \frac{11}{t^2} - \frac{81}{2}t^4 + \frac{81}{4}t^6 \\
|\gamma| &= \int_{-\sqrt{2}}^{\sqrt{2}} \left(\frac{81}{2}t^4 + \frac{81}{4}t^2 + \frac{81}{4}t^6\right) dt = \left(\frac{81t^5}{10} + \frac{21t^3}{4} + \frac{81t^7}{28}\right)\Bigg|_{-\sqrt{2}}^{2} = \frac{4883\sqrt{2}}{35}
\end{aligned}$$

Ответ: $\dfrac{4883\sqrt{2}}{35}$

Пункт б

$$z^3 = 12x, \quad 2zy = 4, \quad \frac{2}{3} \leqslant x \leqslant 18$$

Решение:

$$\text{Пусть } x = t \ \Rightarrow \ z = \sqrt[3]{12t}, \quad y = \frac{4}{2\sqrt[3]{12t}}$$

$$|y| = \int_{t_0}^{t_1} \sqrt{(x')^2 + (y')^2 + (z')^2\, dt}$$

Тогда $x' = 1; \quad z' = \dfrac{\sqrt[3]{12}}{3\sqrt[3]{t^2}}; \quad y' = -\dfrac{2\sqrt[3]{18t^2}}{9t^2}$

$$\int_{\frac{2}{3}}^{18} \left(1 + \frac{\sqrt[3]{12}}{3\sqrt[3]{t^2}} - \frac{2\sqrt[3]{18t^2}}{9t^2}\right) dt = \int_{\frac{2}{3}}^{18} 1\, dt + \int_{\frac{2}{3}}^{18} \frac{12^{1/3}}{3t^{2/3}} dt + \int_{\frac{2}{3}}^{18} \frac{2 \cdot 18^{1/3}}{9t^{4/3}} dt =$$

$$\left(t+\sqrt[3]{12t}+\frac{2\sqrt[3]{18}}{3\sqrt[3]{t}}\right)\Bigg|_{\frac{2}{3}}^{18}=22\frac{2}{3}$$

Ответ: $22\dfrac{2}{3}$

Вариант 8

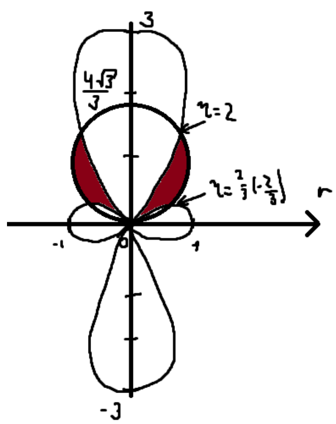
Задача 2

Пункт а

$$r = 1 - 2\cos 2\varphi,\ r = \frac{4\sqrt{3}}{3}\sin\varphi$$

$$\left(r \geq 1 - 2\cos 2\varphi,\ r \leq \frac{4\sqrt{3}}{3}\sin\varphi\right)$$

Решение:



$$r_1 = 1 - 2cos\varphi$$
$$r_2 = \frac{4\sqrt{3}}{3}\sin\varphi$$

$$1 - 2\cos 2\varphi = 1 - 2(1 - 2\sin^2\varphi)\frac{4\sqrt{3}}{3}\sin\varphi$$

$$4\sin^2\varphi - \frac{4\sqrt{3}}{3}\sin\varphi - 1 = 0$$

$$\sin\varphi\frac{\frac{4\sqrt{3}}{3} \pm \sqrt{\frac{16}{3} + 16}}{8} = \frac{4\frac{\sqrt{3}}{3} \pm 8\frac{\sqrt{3}}{3}}{8}$$

$$\varphi = \frac{\pi}{3} + 2\pi k \quad \varphi = \frac{2\pi}{3} + 2\pi k$$

$$\varphi = arcsin\left(-\frac{\sqrt{3}}{6}\right) + 2\pi k \quad \varphi = \pi - rcsin\left(-\frac{\sqrt{3}}{6}\right) + 2\pi k$$

Проинтегрируем:

$$S = 2 \cdot \frac{1}{2}\int_0^{\pi/3}\left(\frac{4\sqrt{3}}{3}\varphi\right)^2 - (1 - 2\cos 2\varphi)^2\,d\varphi = \int_0^{\pi/3}\frac{16}{3}\sin^2\varphi - (4\sin^2\varphi - 1)^2\,d\varphi =$$

$$= \int_0^{\pi/3} -16\sin^4\varphi + \frac{40}{3}\sin^2\varphi - 1\,d\varphi = \int_0^{\pi/3} -16\sin^2(1 - \cos^2\varphi) + \frac{40}{3}\sin^2 -1\,d\varphi =$$

$$= \int_0^{\pi/3} (2\sin 2\varphi)^2 - \frac{8}{3}\sin^2\varphi - 1\,d\varphi = \left| \begin{array}{c} 1-2\sin^2 2\varphi = \cos 4\varphi \\ 4\sin^2 2\varpi = -2\cos 4\varphi + 2 \\ -\frac{8}{3}\sin^2\varphi = \frac{8}{6}(-2\sin^2\varphi) = \\ = \frac{8}{6}(1-2\sin^2\varphi) - \frac{8}{6} = \frac{8}{6}\cos 2\varphi - \frac{8}{6} \end{array} \right| =$$

$$= \int_0^{\pi/3} -2\cos 4\varphi + \frac{8}{6}\cos 2\varphi + 2 - \frac{4}{3} - 1\,d\varphi =$$

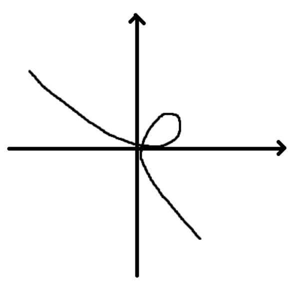
$$= \frac{-\sin 4\varphi}{2} + \frac{2\sin 2\varphi}{3} - \frac{1}{3}\varphi\Big|_0^{\frac{\pi}{3}} = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{3} - \frac{\pi}{9}\right) - 0 = \frac{7\sqrt{3}}{12} - \frac{\pi}{9}$$

Ответ: $\frac{7\sqrt{3}}{12} - \frac{\pi}{9}$

Пункт б

$$(x+y)^3 = tx$$

Решение:



$$(x+y)^3 = tx \quad y = tx$$

$$(x+tx)^3 = 2tx^2 \quad x^3(t+1)^3 = 2tx^2 \quad x(t+1)^3 = 2t$$

$$x = \frac{2t}{(t+1)^3};\ y = \frac{2t^2}{(t+1)^3}$$

$$x'_t = \frac{2(t+1)^3 - 6t(t+1)^2}{(t+1)^6} = \frac{2(t+1) - 6t}{(t+1)^4} = \frac{-4t+2}{(t+1)^4}$$

$$y'_t = \frac{4t(t+1)^3 - 6t^2(t+1)^2}{(t+1)^6} = \frac{4t(t+1) - 6t^2}{(t+1)^4} = \frac{-2t^2+4t}{(t+1)^4}$$

$$S = \frac{1}{2}\int_0^{+\infty} \left|y\cdot x' - xy'\right| dt = \frac{1}{2}\int_0^{+\infty} \frac{2t^2\cdot 2\cdot(-2t+1)}{(t+1)^7} - \frac{2t\cdot 2t\cdot(t+2)}{(t+1)^7} \mid dt =$$

$$= 2\int_0^{+\infty} \Big(\frac{t^2}{(t+1)^7}(-2t+1+t-2) \mid dt = \int_0^{+\infty} \frac{t^2}{(t+1)^6} dt$$

$$= \int_0^{+\infty} \frac{t^2+2t+1}{(t+1)^6} - \frac{2t+2}{(t+1)^6} + \frac{1}{(t+1)^6} dt = 2\int_0^{+\infty} (t+1)^{-4} - 2(t+1)^{-5} + \big(t+y^6 dt =$$

$$= \frac{2(t+1)^{-3}}{-3} - \frac{4(t+1)^{-4}}{-4} + \frac{2(t+1)^{-5}}{-5}\Big|_0^{+\infty} =$$

$$= -\frac{2}{3(t+1)^3} + \frac{1}{(t+1)^4} - \frac{2}{5(t+1)^5}\Big|_0^{+\infty} = \frac{-10(t+1)^2 + 15(t+1) - 6}{15(t+1)^5}\Big|_0^{+\infty}$$

$$= -\frac{-10+15-6}{15} = \frac{1}{15}$$

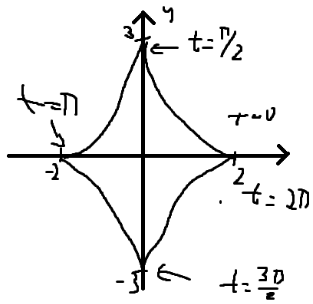
Ответ: $\frac{1}{15}$.

Задача 3

Пункт а

$$x = 2\cos^3 t,\ \ y = 3\sin^3 t$$

Решение:



$$x' = 2 \cdot 3 \cdot (-\sin t) \cdot \cos^2 t$$

$$y' = 9\cos t \cdot \sin^2 t$$

$$L = \int_0^{2\pi} \sqrt{(s')^2 + (y')^2}\, dt = 4\int_0^{\frac{\pi}{2}} 3 \cdot \sin t \cdot \cos t \sqrt{4\cos^2 t + 9\sin^2 t}\, dt =$$

$$= \left| \begin{array}{l} 4\cos^2 t + 9\sin^2 t = x \\ -8\sin t\cos t + 18\sin t\cos t\, dt = dx \\ 10\sin t\cos t\, dt = dx \\ t = \frac{\pi}{2} \quad x = 9 \\ t = 0 \quad x = 4 \end{array} \right| = \frac{6}{5}\int_4^9 \sqrt{x} dx =$$

$$= \frac{6}{5} \cdot \frac{2}{3}(\sqrt{x})^3 \Big|_4^9 = \frac{4}{5}(27 - 8) = 15,2$$

Ответ: $15,2$

Пункт б

$$\delta] z^2 = 2x, xz = 3y \quad 0 \leqslant x \leqslant 8$$

Решение:
$z = 6t, x = 18t^2 \quad y = 36t^3$

$0 \leqslant 18t^2 \leqslant 8 \quad 0 \leqslant t^2 \leqslant \frac{4}{9} \quad t \in \left[-\frac{2}{3}; \frac{2}{3}\right]$

$\left(x'\right)^2 + \left(y'\right)^2 + \left(z'\right)^2 = (36t)^2 + \left(108t^2\right)^2 + 36 =$

$= 36^2 \cdot t^2 + 36^2 \cdot 9 \cdot t^4 + 36 = 36^2 \left(9t^4 + t^2 + \frac{1}{36}\right) = 36^2 \left(3t^2 + \frac{1}{6}\right)^2$

$L = \int_{-\frac{2}{3}}^{\frac{2}{3}} \sqrt{x'^2 + y^2 + z'^2} dt = \int_{-2/3}^{2/3} 36\left(3t^2 + \frac{1}{6}\right) dt = 108\int_{-2/3}^{2/3} t^2 + 6\int_{-2/3}^{2/3} 1 dt =$

$= 72t^3\big|_0^{2/3} + 12t\big|_0^{2/3} = 72 \cdot \frac{8}{27} + 8 = \frac{64}{3} + 8$ **Ответ:** $\frac{64}{3} + 8$