Расчетно графическая работа

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Задачи решили студенты НОЦ инфохимии:

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Вариант 8

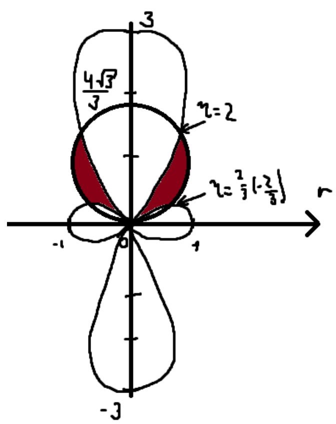
Задача 2

Пункт а

$$r = 1 - 2\cos 2\varphi,\ r = \frac{4\sqrt{3}}{3}\sin\varphi$$

$$\left(r \geq 1 - 2\cos 2\varphi,\ r \leq \frac{4\sqrt{3}}{3}\sin\varphi\right)$$

Решение:



$$r_1 = 1 - 2cos\varphi$$
$$r_2 = \frac{4\sqrt{3}}{3}\sin\varphi$$

$$1 - 2\cos 2\varphi = 1 - 2(1 - 2\sin^2\varphi)\frac{4\sqrt{3}}{3}\sin\varphi$$

$$4\sin^2\varphi - \frac{4\sqrt{3}}{3}\sin\varphi - 1 = 0$$

$$\sin\varphi\frac{\frac{4\sqrt{3}}{3} \pm \sqrt{\frac{16}{3} + 16}}{8} = \frac{4\frac{\sqrt{3}}{3} \pm 8\frac{\sqrt{3}}{3}}{8}$$

$$\varphi = \frac{\pi}{3} + 2\pi k \quad \varphi = \frac{2\pi}{3} + 2\pi k$$

$$\varphi = arcsin\left(-\frac{\sqrt{3}}{6}\right) + 2\pi k \quad \varphi = \pi - rcsin\left(-\frac{\sqrt{3}}{6}\right) + 2\pi k$$

Проинтегрируем:

$$S = 2 \cdot \frac{1}{2}\int_0^{\pi/3}\left(\frac{4\sqrt{3}}{3}\varphi\right)^2 - (1 - 2\cos 2\varphi)^2\,d\varphi = \int_0^{\pi/3}\frac{16}{3}\sin^2\varphi - (4\sin^2\varphi - 1)^2\,d\varphi =$$

$$= \int_0^{\pi/3} -16\sin^4\varphi + \frac{40}{3}\sin^2\varphi - 1\,d\varphi = \int_0^{\pi/3} -16\sin^2(1 - \cos^2\varphi) + \frac{40}{3}\sin^2 - 1\,d\varphi =$$

$$= \int_0^{\pi/3} (2\sin 2\varphi)^2 - \frac{8}{3}\sin^2\varphi - 1\, d\varphi = \left| \begin{array}{c} 1 - 2\sin^2 2\varphi = \cos 4\varphi \\ 4\sin^2 2\varpi = -2\cos 4\varphi + 2 \\ -\frac{8}{3}\sin^2\varphi = \frac{8}{6}(-2\sin^2\varphi) = \\ = \frac{8}{6}(1 - 2\sin^2\varphi) - \frac{8}{6} = \frac{8}{6}\cos 2\varphi - \frac{8}{6} \end{array} \right| =$$

$$= \int_0^{\pi/3} -2\cos 4\varphi + \frac{8}{6}\cos 2\varphi + 2 - \frac{4}{3} - 1\, d\varphi =$$

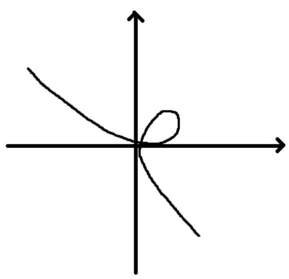
$$= \frac{-\sin 4\varphi}{2} + \frac{2\sin 2\varphi}{3} - \frac{1}{3}\varphi\Big|_0^{\frac{\pi}{3}} = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{3} - \frac{\pi}{9}\right) - 0 = \frac{7\sqrt{3}}{12} - \frac{\pi}{9}$$

Ответ: $\dfrac{7\sqrt{3}}{12} - \dfrac{\pi}{9}$

Пункт б

$$(x+y)^3 = tx$$

Решение:



$$(x+y)^3 = tx \quad y = tx$$

$$(x+tx)^3 = 2tx^2 \quad x^3(t+1)^3 = 2tx^2 \quad x(t+1)^3 = 2t$$

$$x = \frac{2t}{(t+1)^3};\ y = \frac{2t^2}{(t+1)^3}$$

$$x'_t = \frac{2(t+1)^3 - 6t(t+1)^2}{(t+1)^6} = \frac{2(t+1) - 6t}{(t+1)^4} = \frac{-4t+2}{(t+1)^4}$$

$$y'_t = \frac{4t(t+1)^3 - 6t^2(t+1)^2}{(t+1)^6} = \frac{4t(t+1) - 6t^2}{(t+1)^4} = \frac{-2t^2+4t}{(t+1)^4}$$

$$S = \frac{1}{2}\int_0^{+\infty} \left|y\cdot x' - xy'\right| dt = \frac{1}{2}\int_0^{+\infty} \frac{2t^2\cdot 2\cdot(-2t+1)}{(t+1)^7} - \frac{2t\cdot 2t\cdot(t+2)}{(t+1)^7}\ |\ dt =$$

$$= 2\int_0^{+\infty} \left(\frac{t^2}{(t+1)^7}(-2t+1+t-2)\ |\ dt = \int_0^{+\infty} \frac{t^2}{(t+1)^6} dt\right.$$

$$= \int_0^{+\infty} \frac{t^2+2t+1}{(t+1)^6} - \frac{2t+2}{(t+1)^6} + \frac{1}{(t+1)^6} dt = 2\int_0^{+\infty} (t+1)^{-4} - 2(t+1)^{-5} + \big(t+y^6 dt =$$

$$= \frac{2(t+1)^{-3}}{-3} - \frac{4(t+1)^{-4}}{-4} + \frac{2(t+1)^{-5}}{-5}\Big|_0^{+\infty} =$$

$$= -\frac{2}{3(t+1)^3} + \frac{1}{(t+1)^4} - \frac{2}{5(t+1)^5}\Big|_0^{+\infty} = \frac{-10(t+1)^2 + 15(t+1) - 6}{15(t+1)^5}\Big|_0^{+\infty}$$

$$= -\frac{-10+15-6}{15} = \frac{1}{15}$$

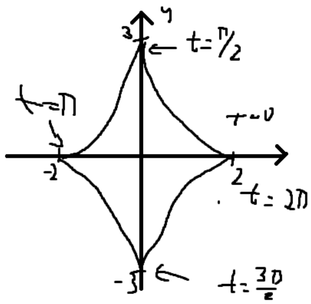
Ответ: $\dfrac{1}{15}$.

Задача 3

Пункт а

$$x = 2\cos^3 t,\ \ y = 3\sin^3 t$$

Решение:



$$x' = 2 \cdot 3 \cdot (-\sin t) \cdot \cos^2 t$$

$$y' = 9\cos t \cdot \sin^2 t$$

$$L = \int_0^{2\pi} \sqrt{(s')^2 + (y')^2}\, dt = 4\int_0^{\frac{\pi}{2}} 3 \cdot \sin t \cdot \cos t \sqrt{4\cos^2 t + 9\sin^2 t}\, dt =$$

$$= \left| \begin{array}{l} 4\cos^2 t + 9\sin^2 t = x \\ -8\sin t\cos t + 18\sin t\cos t\, dt = dx \\ 10\sin t\cos t\, dt = dx \\ t = \frac{\pi}{2} \quad x = 9 \\ t = 0 \quad x = 4 \end{array} \right| = \frac{6}{5}\int_4^9 \sqrt{x} dx =$$

$$= \frac{6}{5} \cdot \frac{2}{3} (\sqrt{x})^3 \bigg|_4^9 = \frac{4}{5}(27 - 8) = 15,2$$

Ответ: $15,2$

Пункт б

$$\delta] z^2 = 2x, xz = 3y \quad 0 \leqslant x \leqslant 8$$

Решение:
$z = 6t, x = 18t^2 \quad y = 36t^3$

$0 \leqslant 18t^2 \leqslant 8 \quad 0 \leqslant t^2 \leqslant \dfrac{4}{9} \quad t \in \left[-\dfrac{2}{3}; \dfrac{2}{3}\right]$

$\left(x'\right)^2 + \left(y'\right)^2 + \left(z'\right)^2 = (36t)^2 + \left(108t^2\right)^2 + 36 =$

$= 36^2 \cdot t^2 + 36^2 \cdot 9 \cdot t^4 + 36 = 36^2 \left(9t^4 + t^2 + \dfrac{1}{36}\right) = 36^2 \left(3t^2 + \dfrac{1}{6}\right)^2$

$L = \displaystyle\int_{-\frac{2}{3}}^{\frac{2}{3}} \sqrt{x'^2 + y^2 + z'^2} dt = \int_{-2/3}^{2/3} 36 \left(3t^2 + \frac{1}{6}\right) dt = 108 \int_{-2/3}^{2/3} t^2 + 6 \int_{-2/3}^{2/3} 1 dt =$

$= 72t^3\big|_0^{2/3} + 12t\big|_0^{2/3} = 72 \cdot \dfrac{8}{27} + 8 = \dfrac{64}{3} + 8$ **Ответ:** $\dfrac{64}{3} + 8$