Física Computacional - Prática 9

Questão 1

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The program `laplace.py` aims to solve the *Laplace's Equation*, $\nabla^2\phi = 0$, for an electrostatic potential ϕ on a square plate, subject to the boundary conditions

$$\begin{cases} \phi = 1.0V, & x = 0, \\ \phi = 0.0V, & x = L,\ y = 0, \quad \text{or} \quad y = L. \end{cases}$$

The program makes use of *divided differences method* to solve the equation. In two dimensions, the equation takes the form

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0. \tag{1}$$

Our method consists of dividing the domain of ϕ (the square plate) into a grid of small squares[1] and using the approximation for the derivative via Taylor expansion, truncaded at first order:

$$\frac{\partial^2\phi}{\partial x^2}(x,y) = \frac{\phi(x-a,y) + \phi(x+a,y) - 2\phi(x,y)}{a^2},$$

and the same for the y variable. Using this and putting into the equation (1), we have

$$\frac{1}{a^2}[\phi(x-a,y) + \phi(x+a,y) + \phi(x,y-a) + \phi(x,y+a) - 4\phi(x,y)] = 0,$$

and, rearranging,

$$\phi(x,y) = \frac{1}{4}[\phi(x-a,y) + \phi(x+a,y) + \phi(x,y-a) + \phi(x,y+a)]. \tag{2}$$

Because we divided the domain into small squares, the (x,y) point can be interpreted as a center of a square with side length equal to a. So, $(x+a,y)$, $(x-a,y)$, $(x,y-a)$ and $(x,y+a)$ are the first (squares) neighbors.

[1] It could be another form insted of squares.

Equation (2) enables us to solve the problem. We fix the potential at the boundaries and guess the values[2] of ϕ as a first to calculate new values for the potential ϕ' via (2), where ϕ' is the left hand side and the right hand side we use the values of phi. After the calculations, we set ϕ`[][]` $= \phi'$`[][]` and iterate over and over. This method is known as *Jacobi Method*.

We can chose between several stop conditions for the program. The used in `laplace.py` is that the maximum diference between ϕ and ϕ' be lass than a target value (the target accuracy defined at line 7).

The flowchart on the next page shows more explicitly the process explained above.

Running the program, we get a graph. The picture produced is displayed below.

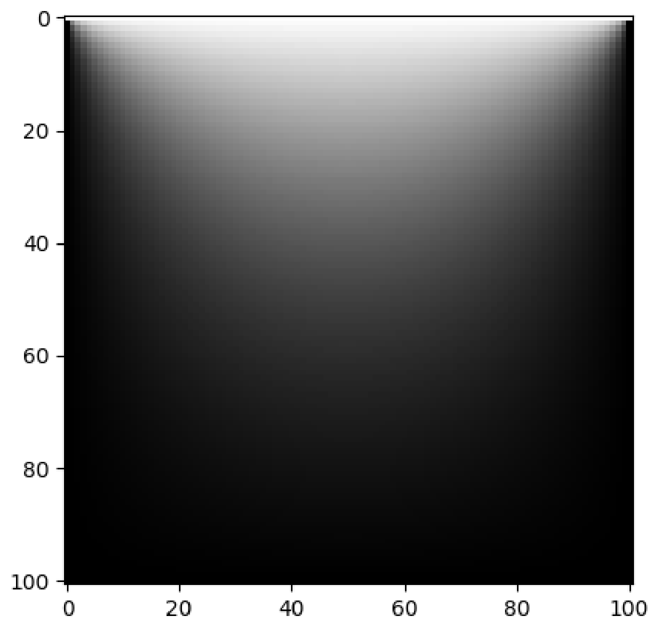

Figura 1: Solution for equation (1)

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[2]It does not need to be acurated.

Cont.

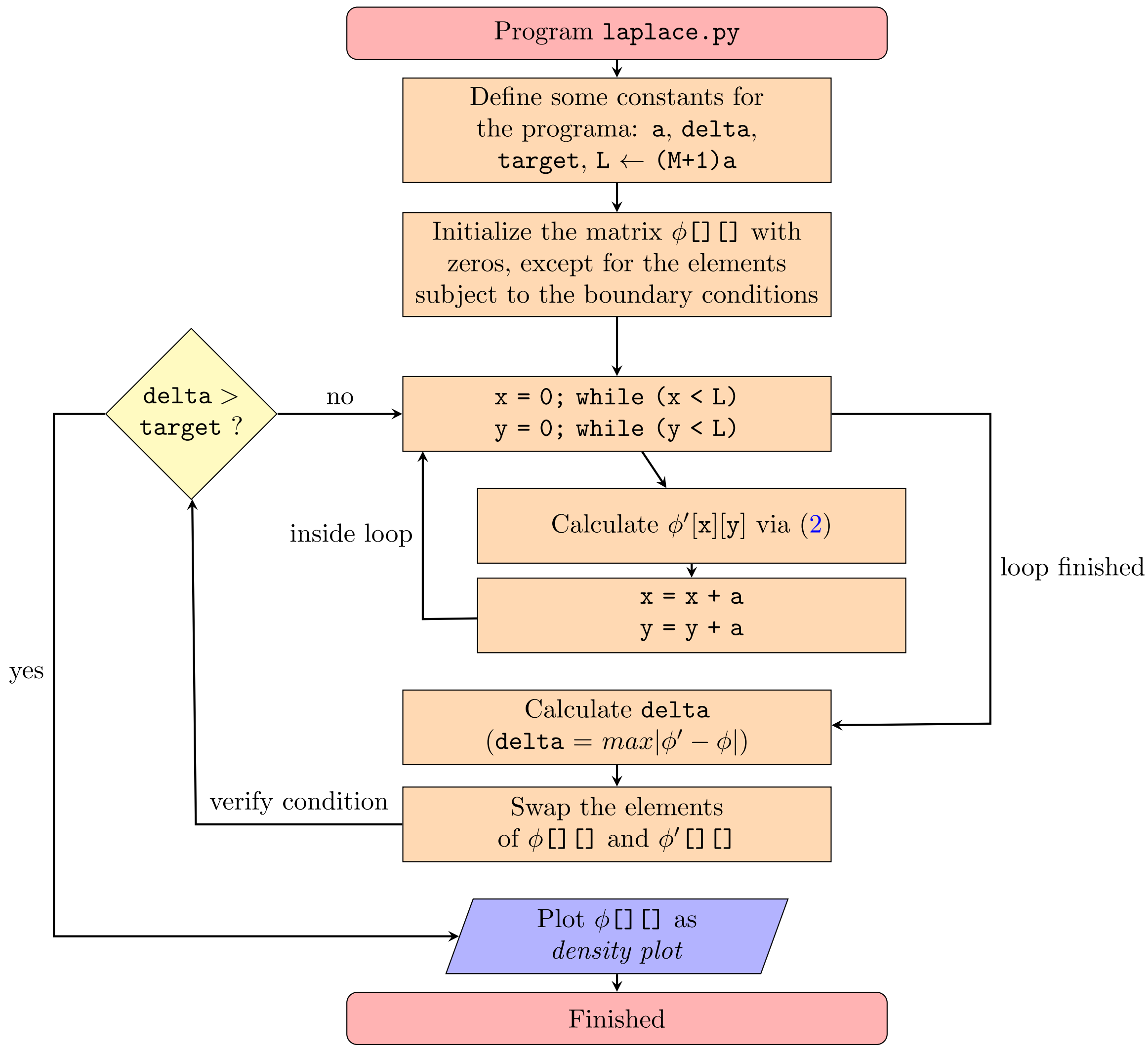

Figura 2: Flowchart of the program `laplace.py`

The End.