Problem 39

We can use Euclid's formula to solve this one.

$$a = k(m^2 - n^2)b \qquad = k(2mn)c = k(m^2 + n^2) \tag{1}$$

This set of equations will generate all of the Pythagorean triples provided that $m > n$ and that m and n are coprime and not both odd. This makes the problem easy.

Further we can provide constrains on the values of m and n given the problem statement that $p \leq 1000$. From this we can put an upper bound on $c < 1000$ since if any of the three were 1000, this would exceed the perimeter by definition. Then we have a bound on k which is that $k(m^2 + n^2 \leq 1000$ and given that the smallest values of m and n are 2 and 1 respectively, the upper bound on k is:

$$k \leq 1000/5 = 200$$

Then we can make constraints on m and n similarly using the same relation. For n:

$$n \leq \sqrt{1000 - m^2} \rightarrow n \leq \sqrt{1000 - 4}$$

And using a floor function on the square root, we find $n \leq 31$. by the exact same principles, we find $m \leq 31$ as well.

```
for n in range(1,32):
    for m in range(n+1,32):
        if not gcd(m,n) == 1:
            continue
        if (m % 2 != 0) and (n % 2 != 0):
            continue
        if n >= m:
            continue
        for k in range(1,201):
            a = k * (m**2 - n**2)
            b = k * 2 * n * m
            c = k * (m**2 + n**2)

            p = a + b + c

            if p > 1000:
                continue

            triples[p].append((a,b,c))
```