$\forall (x,y) \in \mathbb{R}^2: \quad xy \neq 0.$

$$\frac{1-\cos(xy)}{(x^2+y^2)^{3/2}} = \underbrace{\frac{1-\cos(xy)}{x^2y^2}}_{\downarrow\ (x,y)\to(0,0)\quad 1/2} \boxed{\frac{x^2y^2}{\sqrt{(x^2+y^2)^3}}}$$

perché

$$\lim_{(x,y)\to(0,0)} xy = 0$$

$$\lim_{t\to 0} \frac{1-\cos t}{t^2} = \frac{1}{2}$$

Vorrei che

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{(x^2+y^2)^3}} = 0$$

$y^2 = |y|\cdot|y|$

$$0 \leq \frac{x^2y^2}{\sqrt{(x^2+y^2)^3}} = \underbrace{\frac{x^2}{(x^2+y^2)}}_{\leq 1} \cdot \overbrace{\frac{|y|}{\sqrt{x^2+y^2}}}^{\leq 1} \cdot |y|$$

$$|y| = \sqrt{y^2} \leq \sqrt{x^2+y^2}$$

$$0 \leq \frac{x^2y^2}{\sqrt{(x^2+y^2)^3}} \leq |y| \qquad \forall (x,y) \neq (0,0)$$

$$\downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0.$$

STUDIAMO LA DIFF. in $(0,0)$

- Calcoliamo

$f_x(0,0) \qquad f_y(0,0)$

$$f(x,y) = \begin{cases} \dfrac{1-\cos(xy)}{\sqrt{(x^2+y^2)^3}} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

Vediamo se esiste $f_x(0,0)$?

$\left[D f(x,0)\right]_{x=0}$ ?

$f(x,0) = 0 \qquad \forall x \in \mathbb{R} \Rightarrow \exists \left[D f(x,0)\right]_{x=0} = 0$

$$\left.\begin{array}{l} f_x(0,0) = 0 \\ \text{Analogam.} \\ f_y(0,0) = 0 \end{array}\right\} \Rightarrow df(0,0) = 0$$

Bisogna vedere se!

$$\exists \lim_{(h,k)\to(0,0)} \frac{f(h,k) - \overbrace{f(0,0)}^{=0} - \overbrace{df(0,0)}^{=0}}{\sqrt{h^2+k^2}} = 0 \ ?$$

$$\lim_{(h,k)\to(0,0)} \frac{\dfrac{1-\cos(hk)}{\sqrt{(h^2+k^2)^3}}}{\sqrt{h^2+k^2}} =$$

$$= \lim_{(h,k)\to(0,0)} \underbrace{\frac{1-\cos(hk)}{(h^2+k^2)^2}}_{g(h,k)}$$

$E_0 = \{ (h,k) : \quad h=0 \quad k \neq 0 \}$

$g|_{E_0} = 0$

$E_m = \{ h,k) : \quad k = mh, \quad h \neq 0 \}$

$$g|_{E_m} = \frac{1-\cos(mh^2)}{[(1+m^2)h^2]^2} = \underbrace{\frac{1-\cos(mh^2)}{(1+m^2)^2\, m^2h^4}}_{\to 1/2} \cdot m^2$$

$m \neq 0$

$$\downarrow$$

$$\frac{m^2}{2(1+m^2)^2}$$