

# SOMMA GEOMETRICA

## Demonstrazione

$$\text{Sia } S_n = 1 + a + a^2 + \dots + a^n$$

portando 1 al primo membro

$$S_n - 1 = a + a^2 + \dots + a^n$$

$$S_n - 1 = a \underbrace{(1 + a + \dots + a^{n-1})}_{S_n - a^n}$$

Quindi

$$S_n - 1 = a (S_n - a^n)$$

$$S_n - 1 = a S_n - a^{n+1}$$

$$S_n - a S_n = 1 - a^{n+1}$$

$$S_n(1 - a) = 1 - a^{n+1}$$

$$S_n = \frac{1 - a^{n+1}}{1 - a} = \frac{a^{n+1} - 1}{a - 1}$$

## Funzione Logaritmica

$$d, b > 1, \quad a, c > 0$$

$$1. \log_b (a \cdot c) = \log_b a + \log_b c$$

### Dimostrazione

$$\text{Sia } x = \log_b a \quad \text{e} \quad y = \log_b c \Rightarrow$$
$$b^x = a \quad \text{e} \quad b^y = c$$

$$\text{Dunque } b^{x+y} = a \cdot c$$

Calcolando il log (in base b) di entrambi i membri

$$\log_b b^{(x+y)} = \log_b a \cdot c$$

$$(x+y) \underbrace{\log_b b}_1 = \log_b a \cdot c$$

$$\log_b a + \log_b c = \log_b a \cdot c$$

$$2. \log_b a^c = c \log_b a$$

$$y = \log_b a \Rightarrow b^y = a \Rightarrow b^{yc} = a^c$$

$$yc = \log_b a^c \Rightarrow c \cdot \log_b a = \log_b a^c$$

## SOMMATORIE

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Dimostrazione

The diagram shows two rows of terms in a summation. The top row is  $1 + 2 + 3 + \dots + n$  and the bottom row is  $n + (n-1) + (n-2) + \dots + 1$ . Each term in the top row is enclosed in an oval, and each term in the bottom row is also enclosed in an oval. Lines connect the ovals of the first row to the ovals of the second row, showing that  $1$  is paired with  $n$ ,  $2$  with  $n-1$ ,  $3$  with  $n-2$ , and so on, until  $n$  is paired with  $1$ . Below the bottom row, the expression  $n+1$  is written, indicating that each pair of terms sums to  $n+1$ .

Se calcoliamo  $(n+1)n$  otterremmo  
il doppio della somma richiesta  
Il valore che ci serve è dunque

$$\frac{(n+1)n}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proviamo per induzione

$$n=1 \quad \frac{1 \cdot (2) \cdot (3)}{6} = 1 \quad \text{ok}$$

Sia la tesi vera per  $n=k-1$

Proviamo il caso  $n=k$

$$\begin{aligned} \sum_{i=1}^k i^2 &= \sum_{i=1}^{k-1} i^2 + k^2 = \\ &= \frac{(k-1)k(2(k-1)+1)}{6} + k^2 = \\ &= \frac{(k-1)k(2(k-1)+1) + 6k^2}{6} = \\ &= \frac{k[(k-1)(2(k-1)+1) + 6k]}{6} = \end{aligned}$$

$$= \frac{k [(k-1)(2k-2+1) + 6k]}{6} =$$

$$= \frac{k [(k-1)(2k-1) + 6k]}{6} =$$

$$= \frac{k (2k^2 - 2k - k + 1 + 6k)}{6} =$$

$$= \frac{k (2k^2 + 6k + 1)}{6} = \frac{k(k+1)(2k+1)}{6}$$

## Esercizio

Determinare il numero di passi del seguente algoritmo

```
for (i=1; i ≤ n; i++)  
  for (j=1; j ≤ i; j++)  
    for (k=1; k ≤ j; k++)  
      { do something }
```

Assumiamo che  
il costo del  
corpo ( $t_k$ ) sia  
costante  
 $t_k = t$

$$\sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j t =$$

$$= \sum_{i=1}^n \sum_{j=1}^i \underbrace{j t}_{jt} =$$

perché l'ultima  
sommatrice costa  
 $jt$

$$= t \left( \sum_{i=1}^n \sum_{j=1}^i j \right) = t \sum_{i=1}^n \frac{i(i+1)}{2} =$$

$$= \frac{t}{2} \left( \sum_{i=1}^n i^2 + \sum_{i=1}^n i \right) =$$

$$= \frac{t}{2} \left( \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right) \approx n^3$$

# NOTAZIONE ASINTOTICA - ESEMPI

1.  $8n - 2 \in O(n)$

Prendendo  $C=8$   $n_0=1$

$$8n - 2 \leq 8n \quad \forall n \geq 1$$

————— x —————

2.  $5n^4 + 3n^3 + 2n^2 + 4n + 7 \in O(n^4)$

Prendendo  $C = 5 + 3 + 2 + 4 + 7 = 21$ ,  $n_0 = 1$

$$5n^4 + 3n^3 + 2n^2 + 4n + 7 \leq 21n^4 \quad \forall n \geq 1$$

————— x —————

3.  $6n^2 + 3n \log n + 2n + 5 \in O(n^2)$

$$C = 6 + 3 + 2 + 5 = 16 \quad n_0 = 1$$



$$4 \quad 6n^2 + 2^{\log n} + 4 \in O(n^2)$$

$\Downarrow$

$$6n^2 + n + 4 \in O(n^2)$$

$$5. \quad 3n^2 + 6n \in O(n^2)$$

$$C = 4 \quad n_0 = 6$$

## SEGMENTI DI SOMMA MASSIMA

Complessità della prima soluzione

Il ciclo for più interno viene eseguito al massimo  
 $j-i+1$  volte

Num. totale di passi dell'algoritmo:

$$\sum_{i=0}^{n-1} \sum_{j=i}^{n-1} \sum_{k=i}^j t = \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} j-i+1 =$$

$\uparrow$   
costante che possiamo porre 1

$$= \sum_{i=0}^{n-1} \sum_{j=1}^{n-i} j =$$

$$\boxed{\sum_{j=i}^{n-1} j-i+1 = \sum_{j=1}^{n-i} j}$$

$$= \sum_{i=0}^{n-1} \frac{(n-i)(n-i+1)}{2} \geq \sum_{i=0}^{n-1} \frac{(n-i)^2}{2} =$$

$$= \sum_{i=1}^n \frac{i^2}{2} = \frac{1}{2} \left( \sum_{i=1}^n i^2 \right) = \frac{1}{2} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$O(n^3)$$