venerdì 13 dicembre 2019 09:17

$\exists f_x(x,y)$ $(f_x(x,y), f_y(x,y))$

$Df =$

$f(x,y) = \sqrt{x+y}$

$f(x,y) = (x_{-1})$

Calcolare le derivate

$f_x(x_{-1})$, $f_y(x_{-1})$

$\boxed{\exists\, Df = \frac{1}{2\sqrt{x}} \quad \forall x>0}$

$\exists f_x(x_{-1})$?

$f(x,y_{-1}) = \sqrt{x+1}$ ¿ derivata notevole

$\exists\, Df(x,y_{-1}) = \frac{1}{2\sqrt{x+1}}$, $\forall x = \frac{x+1 \geq 0}{x \geq -1}$ $\quad \frac{1}{2\sqrt{x+1}}$ $\quad \forall x \in R$

$\boxed{Df(x,y_{-1})}_{x=0} = \frac{1}{2\sqrt{1}}$

$\Rightarrow f_x(x_{-1}) = \frac{1}{2\sqrt{1}}$

$\exists f_y(x_{-1})$?

$\exists\, Df(x,y_{-1})_{y=-1}$?

$f(x,y) = \sqrt{x+y}$ $\quad \forall y \in R$

$\exists\, Df(x,y) = \frac{1}{2\sqrt{x+y}}$, $\forall y$ $\quad \forall y \in R$

$\boxed{Df(x,y)}_{y=-1} = \frac{1}{2\sqrt{x}}$

$\Rightarrow f_y(x_{-1}) = \frac{1}{2\sqrt{x}}$

- Vediamo (in quali (x,y)) esistono f_x, f_y

$f(x,y) = \sqrt{x+y}$

Se $x+y^2>0$ (ad (x,y) $f(x,y)$ esiste

$\exists\, f_x(x,y) = \frac{1}{2\sqrt{x+y^2}}$, $\forall x = \frac{1}{2\sqrt{x+y^2}}$, $\forall x = \frac{x}{2\sqrt{x+y^2}}$

$\exists\, f_y(x,y) = \frac{y}{\sqrt{x+y^2}}$

$x \to \boxed{x+y^2} \xrightarrow{\sqrt{\ }} \sqrt{x+y^2}$

Se $(x,y) \notin (x,y)$

$\nabla f(x,y) = \left(\frac{x}{\sqrt{x+y^2}}, \frac{y}{\sqrt{x+y^2}}\right)$

- Dove studiare la derivabilità per $(0,0)$

$\exists\, f_x(0,0)$?

$f(x,0) = \sqrt{x^2} = |x|$

$\exists\, (\nabla f(x,0))_{x=0} = ??$ $\boxed{NO}$

Analogamente

$\nexists\, f_y(0,0)$

- $f(x,y) = |x+y| = |y|$ (x)

- $f_x(x,y) = \boxed{|y| \cdot \frac{x}{|x|}}$ $\quad \forall (x,y) \in R^2$ $x \neq 0$.

Da $(0,0)$ non posso affacciarmi le formule di derivazione $\log$

$\exists\, f_x(0,0)$?

$\exists\, [\nabla f(x,y)]_{x=0} = ??$

$f(x,0) = 0$ $\quad \forall x \in R$

$\Rightarrow \exists\, (\nabla f(x,0))_{x=0} = 0 \xrightarrow{\text{cont}} \exists\, f_x(0,0) = 0$

$\exists\, f_x(0,1)$?

$\exists\, [\nabla f(x,1)]_{x=0}$?

$f(x,1) = |x|$ $\quad \forall x \in R$ $\quad \nexists\, [\nabla f(x,1)]_{x=0}$

$\nexists\, f_x(0,1)$

Se $f \xrightarrow{C^1} f$

$f(x,y) = \begin{cases} y \log(x+|y|) & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

$f(0,0) = 0$

f continua in $(0,0)$

$\Updownarrow$ def

$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0$

$\Updownarrow$

$\lim_{(x,y)\to(0,0)} \boxed{y} \cdot \log(x+|y|) = 0$

$\underset{\to 0}{y} \quad \underset{\to -\infty}{\log(x+|y|)}$

f.i.: $0 \cdot \infty$.

$\frac{\log(x+|y|)}{\frac{1}{y}}$ $\quad \frac{|y|}{y}$

$E_0 = \{(x,y) : y = 0 \quad x \neq 0\}$

$f_{|E_0}(x,0) = 0$

$E_1 = \{(x,y) : x = y \quad x \neq 0\}$

$f_{|E_1}(x,x) = x^2 \log(2x)$

$\lim_{(x,x)\to(0,0)} x^2 \log(2x) = 0$

$\lim_{t\to 0} t^2 \log(2|t|) = \lim_{t\to 0} \frac{\log(2|t|)}{\frac{1}{t^2}} = \infty$

$\boxed{\lim_{t\to 0} t^2 \log(2|t|) = 0 \quad x \neq 0}$

$\lim_{(x,y)\to(0,0)} y \log(x+|y|) =$

$= \lim_{(x,y)\to(0,0)} \underbrace{\frac{xy}{|x+y|}}_{\leq 1} \underbrace{(x+|y|) \log(x+|y|)}_{\to 0} = 0$

$0 \leq \frac{|xy|}{|x+|y||} = |x| \frac{|y|}{|x+|y||} \leq |x|$

$\Downarrow$

$0 \leq \left\{\frac{|xy|}{|x+|y||}\right\} = \frac{|x|}{|x+|y||} \frac{|y|}{|x+|y||} \leq 1$

$|x| \leq |x+|y|| \Rightarrow \boxed{\frac{|x|}{|x+|y||} \leq 1} \quad \forall x \in (x,y) \in (0,0)$

$x^2 \leq x^2+y^2 \Rightarrow \frac{x^2}{x^2+y^2} \leq 1$

$|x| = \sqrt{x^2} \leq \sqrt{x^2+y^2}$

$\Downarrow$

$\frac{|x|}{\sqrt{x^2+y^2}} \leq 1$

$|y| \leq \sqrt{(x^2+y^2)}$

$\boxed{|y| \leq x^2+y^2}$ $\quad |x| \cdot |y| \leq \sqrt{x^2+y^2} \cdot \sqrt{x^2+y^2} = (x^2+y^2)$

$\log(x+t) \leq t$ $\quad \forall t > -1$

- $|\operatorname{sen} t| \leq |t|$

$\lim_{t\to 0} \frac{\operatorname{sen} t}{t} = 1$

$\lim_{t\to 0} \frac{\operatorname{sen} t}{t} = \frac{\cos(0)}{1} = 1$

$\lim_{t\to 0} \frac{e^t - 1}{t} = 1$

$f(x,y) = \begin{cases} \frac{y \cdot \log(x+|y|)}{|y|} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

$\exists\, f_x(0,0)$?

$f(x,0) = \begin{cases} 0 & x \neq 0 \\ 0 & x = 0 \end{cases}$

$f(x,0)_{x=0} \quad \forall x \in R$

$\exists\, [f(x,0)]_{x=0} = 0 \Rightarrow f(0,0) = 0$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0}{h} = 0$

$\boxed{\exists\, f_x(0,0) = 0}$

Controllo se $f(x,y) \xrightarrow{C^1} f$ $\quad \boxed{\text{no}}$

$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-f(0,0)}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{\frac{y \log(x+|y|)}{|y|}}{\sqrt{x^2+y^2}} = 0$

$\lim_{(h,k)\to(0,0)} \frac{f(h,k)}{\sqrt{h^2+k^2}} =$

$= \lim_{(h,k)\to(0,0)} \boxed{\frac{k \cdot \log(1+|k|)}{\sqrt{h^2+k^2}}}$

f differenziabile (in (x,y))

$\Delta f(x,y)$

$\lim_{(h,k)\to(0,0)} \frac{f(x+h,y+k) - f(x,y) - f_x(x,y)h - f_y(x,y)k}{\sqrt{h^2+k^2}} = 0$

$df(x,y)[h] = f_x(x,y)h + f_y(x,y)k$

$\to 0$

$\frac{|k|}{\sqrt{h^2+k^2}} \leq 1$ $\quad \frac{|k|}{\sqrt{h^2+k^2}} \leq 1$