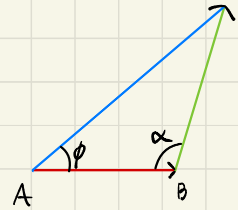
$\vec{A} = 200\ Km$ →

$\vec{B} = 300\ Km$ ↗

$\alpha = 60°\ E$



Determinare la direzione (lunghezza $\vec{C}$ e di ϕ).

$$C = \sqrt{A^2 + B^2 - 2AB \cdot \cos(\pi - \alpha)}$$ Teorema del coseno

$$\frac{\sin(\phi)}{B} = \frac{\sin(\pi - \alpha)}{C}$$ Teorema dei seni

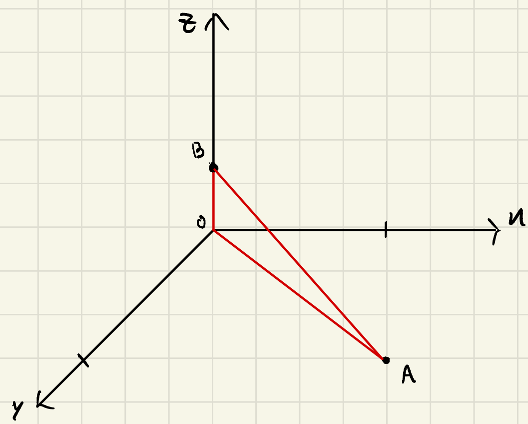
---

$F_1 = \hat{x} + 2\hat{y} + 3\hat{z}$ (in N)

$F_2 = 4\hat{x} - 5\hat{y} - 2\hat{z}$ (in N)

$A(20, 15, 0)$

$B(0, 0, 7)$



$r_a = 20\hat{x} + 15\hat{y}$

$r_b = 7\hat{z}$

$\Rightarrow \Delta r = r_b - r_a = (-20\hat{x} - 15\hat{y} + 7\hat{z})\ m$

$$F = F_1 + F_2 = (\hat{x} + 2\hat{y} + 3\hat{z}) + (4\hat{x} - 5\hat{y} - 2\hat{z}) \Rightarrow (5\hat{x} - 3\hat{y} - \hat{z})\ N$$

$$F \cdot \Delta r = 5 \cdot (-20)\hat{x} + (-3)(-15)\hat{y} + 7(-1)\hat{z} \Rightarrow -48\ J$$

$v_0(t) = 0$ $\qquad\qquad h = ?$

$t = 4{,}8\,s$

velocità suono $= 340\,m/s$

$$y_{sasso}(t) = h - \frac{1}{2} g t^2$$

t_1 Tempo caduta sasso $\quad y(t_1) = 0 \qquad t_1 = \sqrt{\frac{2h}{g}}$

t_2 tempo del suono a percorrere $h \qquad t_2 = \frac{h}{v_s}$

$t = t_1 + t_2 = 4{,}8\ m/s$

$$\sqrt{\frac{2h}{g}} + \frac{h}{v_s} = t \Rightarrow \sqrt{\frac{2h}{g}} = t - \frac{h}{v_s} \Rightarrow \frac{2h}{g} = \left(t - \frac{h}{v_s}\right)^2 =$$

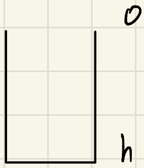
$$\Rightarrow \left[\left(t - \frac{h}{v_s}\right) > 0 \Rightarrow h < v_s t,\ h < 1632\right.$$

$$\frac{2h}{g} = t^2 - \left(\frac{h}{v_s}\right)^2 - 2t\frac{h}{v_s} \Rightarrow$$

$$\Rightarrow h = \left[v_s \left(t + \frac{v_s}{g} \right) \pm \sqrt{\left(\frac{v_s}{g} \right)^2 \ldots} \right.$$

$h_1 = 99{,}5 \qquad h_2 = 26756$

h_2 vidé $\quad h < 1632$



$v_s = 340\,m/s$ $\qquad h = ?$

$t = 4,8\,s$

$\underline{t_1 + t_2 = t} \Rightarrow \tau_2 = \tau_0 - \tau_1$

$$\underbrace{h(t_1)}_{y} = \underbrace{h_0}_{0} + \underbrace{v_0}_{0} t_1 + \frac{1}{2} g t_1^2 \Rightarrow h = \underline{\frac{1}{2} g t_1^2} \Rightarrow t_1 = \sqrt{\frac{2h}{g}}$$

$$t_2 = \frac{h}{v_s}$$

$$\underbrace{x(\tau_2)}_{0} = \underbrace{x_0}_{h} - v_s \cdot \tau_2 \Rightarrow h = \underline{v_s \cdot \tau_2}$$

$$\frac{1}{2} g t_1^2 = v_s \cdot t_2 \Rightarrow$$

$$\frac{\frac{1}{2} g t_1^2}{v_s} = t - t_1 \Rightarrow$$

$$\frac{1}{2} \frac{g}{v_s} \cdot t_1^2 + t_1 - t = 0 \Rightarrow 0,0144 \cdot t_1^2 + t_1 - 4,8\,s = 0$$

Δ

13

$t_0 = 0$

$x_1 = ?$

$t_1 = 3,0\ s$

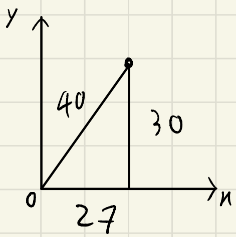
$v_0 = 8,0\,\hat{i} + 15\,\hat{j}\ (m/s)$

$a = 1,5\,\hat{i} - 4,0\,\hat{j}\ (m/s^2)$

$x_0 = 0$

$x(t) = x_0 + v_0 t + \frac{1}{2} a_x t^2$

$v_x(t) = v_{x_0} + a_x t$

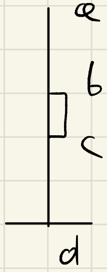


$x = 0 + (8,0\,\hat{i} + 15\,\hat{j}) \cdot 3,0 + \frac{1}{2}(1,5\,\hat{i} - 4,0\,\hat{j}) \cdot 9$

$x = 24\,\hat{i} + 45\,\hat{j} + \frac{1}{2}(13,5\,\hat{i} - 36\,\hat{j})$

$x = 24\,\hat{i} + 45\,\hat{j} + 6,75\,\hat{i} - 18\,\hat{j} \Rightarrow (30,75\,\hat{i} - 27\,\hat{j})\,m$

$d(x_0, x) = \sqrt{27^2 + 30^2} = 40,36\ m$



$h_{BC} = 1{,}2\ m$

$\Delta t_f = 0{,}125\ s$

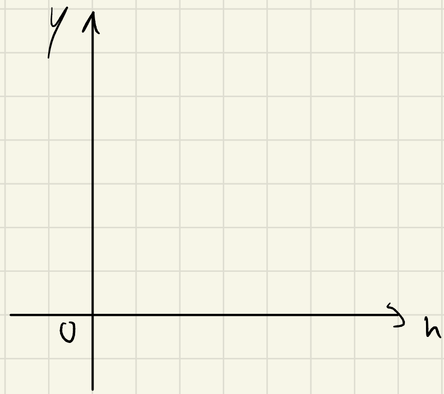
$\Delta t'_f = 2{,}00\ s$

$t_0 = 0$

$v_0 = 7{,}2\ m/s$

$a = 3{,}0\hat{i} - 2{,}0\hat{j}\ (m/s)$

$d(x_0, x)$



$$x(t) = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$$

$$v(t) = v_{x_0} + a_x t$$

$$12 = 0 + 7{,}2 \cdot t + \frac{1}{2}(3{,}0\hat{i} - 2{,}0\hat{j}) \cdot t^2 \Rightarrow$$

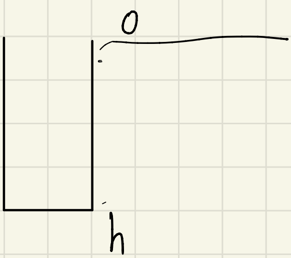
$$(3{,}0\hat{i} - 2{,}0\hat{j})t^2 + 7{,}2t - 12 = 0 \Rightarrow$$

$$\Delta = 51{,}84 + (144\hat{i} - 96\hat{j}) \Rightarrow$$

$$h = \frac{-7,2 \pm (51,84 + (144\hat{\imath} - 96\hat{\jmath}))}{2(3,0\hat{\imath} - 2,0\hat{\jmath})}$$

$t = 4,8\,s$ $\qquad$ $h = ?$

$V_s = 340\,m/s$



$t = t_1 + t_2$

$t_2 = t - t_1$

$$h = h_0 + V_{h0}\,t + \frac{1}{2} a_h t^2$$

$$h_{0\text{-}h} = 0 + 0 + \frac{1}{2} g t^2 \Rightarrow H = \frac{1}{2} g t_2^2 \Rightarrow t_1 = \sqrt{\frac{2H}{g}}$$

$$h_{h\text{-}0} = h = 340 \cdot t_2 \qquad t_2 = \frac{h}{340}$$

$\downarrow$

0

$$h = 340 \cdot t_2$$

17

$v_0 = 500\ Km/h$

$h = 5\ Km$

$\alpha = ?$

$\tan\alpha = \frac{x}{h}$

$$\begin{cases} x(t) = v_{x_0}\, t \\ y(t) = V_{y_0}\, t - \frac{1}{2} g t^2 \end{cases}$$

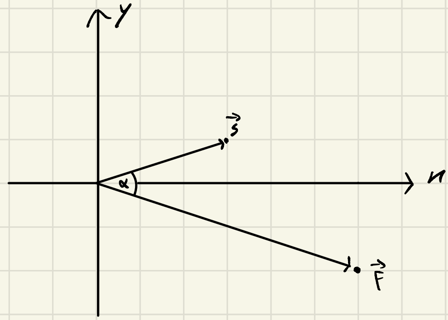
$$y_{CADUTA} = h - \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$x = V_0 \cdot t_{CAD} \Rightarrow V_0 \cdot \sqrt{\frac{2h}{g}}$$

$$\tan\alpha = \frac{V_0 \cdot \sqrt{\frac{2h}{g}}}{h} \Rightarrow 88$$

$F = (6\hat{i} - 2\hat{j})\,N$ $\quad W = ?$

$s = (3\hat{i} + \hat{j})\,m$ $\quad \alpha = ?$



$$W = \vec{F} \cdot \vec{s} = |\bar{F}|\,|\vec{s}| \cdot \cos\alpha$$

$$= (3 \cdot 6) + ((-2) \cdot 1) = 16\,J$$

$$\cos\alpha = \frac{\overbrace{F \cdot s}^{W}}{|F| \cdot |s|} = \frac{16}{\sqrt{40} \cdot \sqrt{10}} \Rightarrow$$

$$|F| = \sqrt{F_i^2 + F_j^2} = \sqrt{40}$$

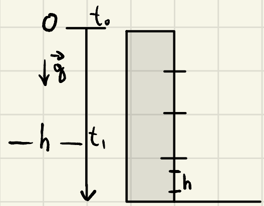
$$|s| = \sqrt{s_i^2 + s_j^2} = \sqrt{10}$$

$$\Rightarrow \alpha = \arccos \frac{16}{\sqrt{40} \cdot \sqrt{10}} = 36.9°$$

12

$h = 1{,}8\,m$

$\Delta t = 0{,}1\,s$



$$y(t) = y_0 + v_0 t + \frac{1}{2} a t^2$$

RIVEDERE

$$y(t = t_1) = h = v_0 t_1 + \frac{1}{2} g t_1^2 \Rightarrow v_0 = \frac{h}{t_1} - \frac{1}{2} g t_1 \Rightarrow 17{,}5\,m/s$$

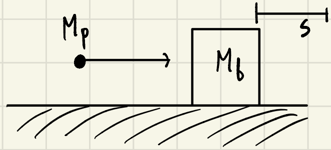
$$\boxed{v_F^2 = v_i^2 + 2a\,(x_F - x_i)}$$

$$v_0^2 = 0 + 2g \cdot h \Rightarrow h = \frac{v_0^2}{2g} \Rightarrow \frac{(17{,}5)^2}{2g} = 15{,}6\,m$$

$h/3 = 5 + 1 = 6$ portineria

ogni piano è alto 3 m

$M_b = 1\,Kg$ $\qquad$ $v_p = ?$



$\mu = 0.7$

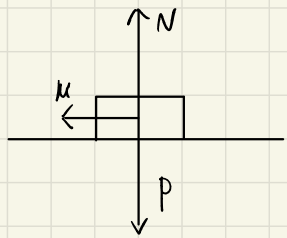
$M_p = 40\,g = 0{,}04\,Kg = 4 \cdot 10^{-2}\,Kg$

$s = 34\,cm = 0{,}34\,m = 3{,}4 \cdot 10^{-1}\,m$

Urto completamente anelastico → conservazione quantità di moto

$p_i = M_p v_p$ $\qquad$ $M v_p = (M_b + M_p) \cdot v'$

$p_F = (M_p + M_b) \cdot v'$ $\qquad$ $v_p = \dfrac{M_p + M_b}{M_p} \cdot v'$



$W_{NC} = \Delta E_M = K + U$

$\Delta K = K_s - K_0 = -\frac{1}{2} m v'^2$

$N = P = (M_p + M_b)\,g$

$W_{NC} = -\mu\,(M_b + M_p)\,g \cdot s$

$\vec{F} = -\mu N$

$= \mu\,(M_b + M_p)\,g$

$\mu\,(M_p + M_b)\,g\,s = \frac{1}{2}\,(M_b + M_p)\,v'^2$

$v'^2 = 2\mu g s$

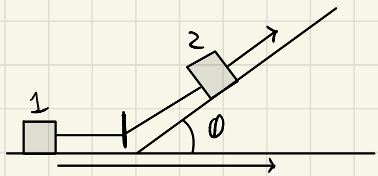
$v_p = \dfrac{M_p + M_b}{M_p} \cdot \sqrt{2\mu g s}$.

- $F = 12\,N$ Tensione cavo = ?

$m_1 = 3\,Kg$

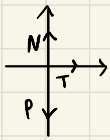
$m_2 = 1\,Kg$

$\phi = 37°$



$F = m \cdot a$

Primo corpo



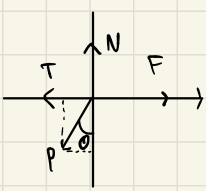
$\vec{N} + \vec{T} + \vec{P} = m_1 \vec{a}$

$T = m \vec{a}$

$N - m_1 g = 0$

$N = m_1 g$

Secondo corpo:



$\vec{F} + \vec{T} + \vec{P} + \vec{N} = m_2 \vec{a}$

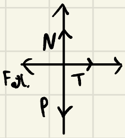
$\vec{F} - \vec{T} - \vec{P} - mg \cdot sen\,\theta = m_2 a$

$N - mg \cdot \cos\theta = 0 \Rightarrow N = mg \sin\theta$

$$\begin{cases} T = m_1 a \\ F - T - m_2 g \sin\theta = m_2 a \end{cases} \Rightarrow \begin{cases} T = m_1 a \\ F - m_1 a - m_2 g \sin\theta = m_2 a \end{cases} \Rightarrow$$

$$a = \frac{F - m_2 g \cdot \sin\theta}{m_1 + m_2} \quad ; \quad T = m_1 \frac{F - m_2 g \cdot \sin\theta}{m_1 + m_2} = 4{,}57$$

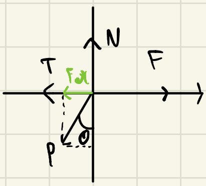
Con attrito



$$T - F_{att} = m_1 a$$

$$N = m_1 g$$

$$T - \mu m_1 g = m_1 a$$



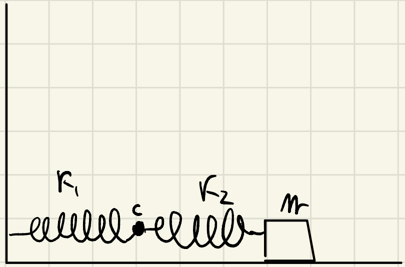
$$N - m_2 g \cdot \cos\theta = 0 \Rightarrow N = m_2 g \cdot \cos\theta$$

$$\begin{cases} F - T - m_2 g \cdot \sin\theta - \mu m_2 g \cdot \cos\theta = m_2 a \\ T - \mu m_1 g = m_1 a \Rightarrow T = m_1 (a + \mu g) \end{cases}$$

$$F - m_1 (a + \mu g) - m_2 g \sin\theta - \mu m_2 g \cdot \cos\theta = m_2 a$$

$$a (m_1 + m_2) = F - m_1 \mu g - m_2 g \sin\theta - \mu m_2 g \cdot \cos\theta$$

$$a = \frac{F - g \left[\mu (m_1 + m_2 \cos\theta) + m_2 \cdot \sin\theta \right]}{m_1 + m_2}$$



$$f = \frac{\omega}{2\pi} \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\vec{F} = -kx$$

$$\begin{cases} +k_1 x_1 = +k_2 x_2 \\ x = x_1 + x_2 \end{cases} \Rightarrow \begin{cases} x_1 = \dfrac{k_2 x_2}{k_1} \\ x = \left(\dfrac{k_2}{k_1} + 1\right) x_2 \Rightarrow \left(\dfrac{k_2 + k_1}{k_1}\right) x_2 \end{cases}$$

$$x_2 = \frac{k_1}{k_2 + k_1}$$

$$-k_2 x_2 = ma \Rightarrow -m \frac{d^2 x}{dt^2}$$

$$m \frac{d^2 x}{dt^2} + k_2 \frac{k_1}{k_2 + k_1} x = 0$$

$$\frac{d^2 x}{dt^2} + \frac{1}{m} \frac{k_1 k_2}{k_2 + k_2} x = 0 \qquad k_{eq} = \frac{k_2 k_1}{k_2 + k_1}$$

$$\omega = \sqrt{\frac{k_{eq}}{m}}$$

$$\frac{1}{\omega^2} = \frac{m}{k_{eq}} = m \frac{k_2 + k_1}{k_2 k_1}$$

$$\frac{\cancel{K_2}}{\cancel{K_2} K_1} + \frac{\cancel{K_1}}{K_2 \cancel{K_1}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{\omega^2} = \frac{m}{K_{eq}} = m\left(\frac{1}{K_1} + \frac{1}{K_2}\right) = \frac{m}{K_1} + \frac{m}{K_2} = \frac{1}{\omega^2} + \frac{1}{\omega^2}$$

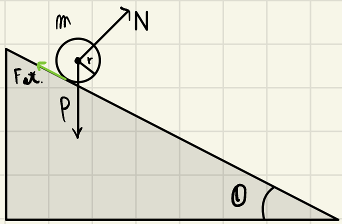
$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} \Rightarrow f^2 = \frac{1}{\frac{1}{f_1^2} + \frac{1}{f_2^2}} \Rightarrow f = \frac{1}{\sqrt{\frac{1}{f_1^2} + \frac{1}{f_2^2}}}$$

$m = 6\,Kg$ $\qquad \theta = ?$

$r = 0,12\,m$ $\qquad \omega = ?$

$\mu_s = 0,22$

$h = 0,98\,m$



$a_{CM} = \alpha \cdot r$

$$\vec{P} + \vec{F}_{at.} = m\vec{a}$$

$$mg \cdot \sin\theta - F_{at.} = m\,a_{CM}$$

$$\tau^{(E)} = I\alpha \qquad \text{(Per il Teorema del momento angolare)}$$

$$\tau = \bar{r} \times \bar{F}_{at.} = r \times F_{at.} \cdot \sin\frac{\pi}{2} = r F_{at}$$

$$r F = I \frac{a_{CM}}{r}$$

$$\begin{cases} F_{att} = I \dfrac{a_{CM}}{r} \\ mg\sin\theta - F = m a_{CM} \end{cases} \qquad a_{CM} = g\sin\theta - \frac{F}{m} = g\sin\theta - \frac{I a_{CM}}{m r^2}$$

$$a_{CM} = \frac{g\sin\theta}{1 + \frac{I}{mr^2}}$$

$$F_{att} = \frac{I}{r^2} \quad \frac{g\sin\theta}{1 + \frac{I}{mr^2}}$$

$$F_{att} \le \mu_s N = \mu_s \, mg \cdot \cos\theta$$

$$\frac{mg\cdot\cos\theta}{1 + \frac{mr^2}{I}} \le \mu_s \, mg\cos\theta$$

$$tg\,\theta \le \mu_s \left(1 + \frac{mr^2}{I}\right) \le \mu_s \cdot 2$$

$w = ?$ $\qquad\qquad w = \dfrac{v_{CM}}{r}$

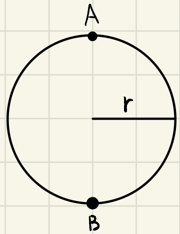
$$mg(h-r) = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I w^2$$

$$mg(h-r) = \frac{1}{2} m w^2 r + \frac{1}{2} m r^2 w^2$$

$$\cancel{m} g(h-r) = \cancel{m} w^2 r \Rightarrow g(h-r) = w^2 r \Rightarrow w = \sqrt{\frac{g(h-r)}{r^2}}$$

$$w = \frac{\sqrt{g(h-r)}}{r}$$

19



$R = 40\,m$ $\quad v_1 = ?$ $\quad t = ?$

$v_B = 40\,Km/h = 11{,}11\,m/s$

$P_{CIRCONF} = 2\pi r = 251{,}3\,m$

$\alpha = 2{,}5 \cdot 2\pi = 15{,}7\,rad$

$\alpha_A - \alpha_B = \pi$ $\qquad \alpha_A = \omega_A t$

$\omega_A t - \omega_B t = \pi$

$t(\omega_A - \omega_B) = \pi$

$t = \dfrac{\pi}{\omega_A - \omega_B} \Rightarrow \omega = \dfrac{V}{R} \Rightarrow \dfrac{\pi R}{V_A - V_B} = t$

$\alpha_B = \omega_B t \Rightarrow \dfrac{V_B}{\cancel{R}} \cdot \dfrac{\pi \cancel{R}}{V_A - V_B} \Rightarrow \dfrac{11{,}11}{\cancel{40}} \cdot \dfrac{\pi \cdot \cancel{40}}{V_A - 11{,}11} = 15{,}7$

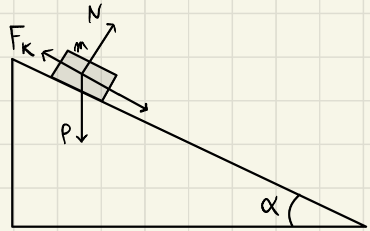
$V_A = 13{,}3\,m/s \Rightarrow 47{,}9\,Km/h$

$t = \dfrac{\pi \cdot 40}{13{,}3 - 11{,}11} = 57{,}3\,s$

22

$m = 3\,Kg$ $\qquad a = ?$

$\alpha = 30°$ $\qquad \mu_K = ?$



$d = 2\,m$

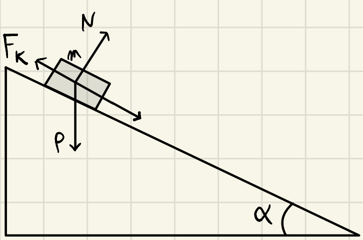
$t = 1{,}50\,s$ $\qquad v = 1{,}33\,m/s$

$$\vec{F}_K + \vec{N} + \vec{P} = m\vec{a}$$

$$\begin{cases} \vec{F}_K = \mu_K \vec{N} \\ -\vec{F}_K + \cancel{m}g\sin\alpha = \cancel{m}\vec{a} \\ N - mg\cos\alpha = 0 \end{cases} \Rightarrow \begin{array}{l} \mu_K \cdot \cancel{mg}\cdot\cos\alpha = \cancel{mg}\sin\alpha \Rightarrow \frac{\sqrt{3}}{3} \\ F_K = 14{,}7 \\ N = mg\cdot\cos\alpha \end{array}$$

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2$$

$$2 = 0 + 0 + \frac{1}{2}at^2 \Rightarrow a = 1{,}78\,m/s^2$$



$$ma = \cancel{\vec{N}} + \vec{F}_{//} + \cancel{\vec{F}_\perp} + \vec{F}_K$$

$$ma = \vec{F}_{//} + \vec{F}_K \Rightarrow$$

$$ma = \vec{F}_{//} + \mu_K \cdot \vec{N} \Rightarrow \underbrace{ma = P\sin\alpha + \mu_K(-P\cos\alpha)}$$

$$\vec{F}_\perp = \vec{P}\cdot\cos\alpha \Rightarrow$$

$$m a = P \cdot \operatorname{sen}\alpha + \mu_k (-P \cos\alpha)$$

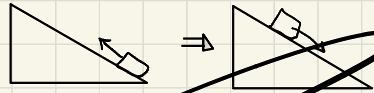
$a - g \sin\alpha$

$$\cancel{m} a = \cancel{m} g \cdot \operatorname{sen}\alpha + \underline{\underline{\mu_k}} \cdot (-mg \cdot \cos\alpha) \Rightarrow$$

$$\mu_k = \frac{a - g \operatorname{sen}\alpha}{(-mg \cos\alpha)} \Rightarrow 0{,}368$$

27 VEDERE SIMULAZIONE COMPITO!

29 $m = 1\ Kg$ $x = ?$



$\mu_k = 0{,}2$ $t = ?$

$V_0 = 3\ m/s$

$\alpha = 30°$

$F = m g \mu_k$

$$\begin{cases} x(t) = x_0 + V_0 t + \frac{1}{2} g t^2 \\ v(t) = V_0 + a t \end{cases}$$

$a = g \sin\alpha - \mu_k g \cos\alpha \Rightarrow -3{,}20\ m/s^2$

$0 = 3 + (-3{,}20) \cdot t \Rightarrow t = 0{,}93\ s$

$x(t) = 0 + 3 \cdot 0{,}93 + \frac{1}{2} g \cdot (0{,}93)^2 \Rightarrow 7\ m$

29

$m = 1\,Kg$ $\qquad l = ?$

$\mu_k = 0{,}2$ $\qquad t = ?$

$V_0 = 3\,m/s$

$\alpha = 30°$

$$\begin{cases} \frac{1}{2} m v^2 - F_e \cdot l = mgh \\ h = \sin\alpha \cdot l \end{cases} \Rightarrow \frac{1}{2} m v^2 - (F_e \cdot l) = m g \sin\alpha\, l$$

$$\frac{1}{2} m v^2 = m g \sin\alpha\, l + F_e \cdot l \Rightarrow$$

$$\frac{1}{2} m v^2 = l\,(m g \sin\alpha + F_e)$$

$F_e = \mu_k \cdot mg \cdot \cos\alpha$

$F_e = 0{,}98$

$$\frac{\frac{1}{2} m v^2}{m g \sin\alpha + F_e} = l \Rightarrow 0{,}76$$

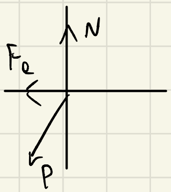
$ma = -F_k - P\sin\alpha$

$a =$

$$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2 \Rightarrow 0{,}7 = 0 + 3t + \frac{1}{2} \cdot 0{,}98 \cdot t^2 \Rightarrow$$

$$\Rightarrow 0{,}49 t^2 + 3t - 0{,}7 = 0 \Rightarrow \frac{-3 \pm \sqrt{9 + 4 \cdot 0{,}49 \cdot 0{,}7}}{0{,}98} \Rightarrow$$

$v(t) = V_0 + at$



$$\vec{N}+\vec{P}+\vec{F}_e=ma$$

$$\cancel{N}+\cancel{P}_\perp+P_{\parallel}+F_e=ma \Rightarrow a=\frac{F}{m}=0,98$$

$$+mg\sin\alpha-mg\cos\alpha\,\mu_r=ma$$

$$g(\sin\alpha-\cos\alpha\,\mu)=a$$

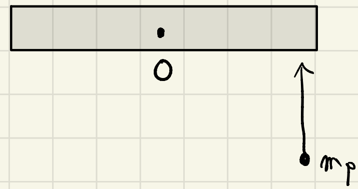
$$x(t)=x_0+v_0t+\frac{1}{2}at^2$$

$$0=0,76+0+\frac{1}{2}at^2\Rightarrow$$

$$\Rightarrow \frac{1}{2}at^2=-0,76\Rightarrow\frac{(-0,76)\cdot 2}{a}\Rightarrow$$

$M = 4\,kg$ $\qquad \omega = ?$

$l = 1\,m$ $\qquad \tau = ?$



$m_p = 1\,kg$

$v_p = 5\,m/s$

$N = \text{Giri} = 20$

$$L = r \wedge m v \qquad L_i = \frac{L}{2} m v$$

$$L_f = I \omega$$

$$I = I_{\substack{\text{asta}\\\text{centro}}} + I_p = \frac{1}{12} M \cdot L^2 + m \left(\frac{L}{2}\right)^2 = \left(\frac{1}{12} M + \frac{1}{4} m\right) L^2$$

Conservazione momento angolare

$$\frac{L}{2} m v = \underbrace{\left(\frac{1}{12} M + \frac{1}{4} m\right)}_{I_s} L^2 \omega \Rightarrow \omega = \frac{m v I_s}{2 L^2} \Rightarrow 4,3 \text{ rad/s}$$

$$W = \int_0^{\Delta\theta} \tau^{(E)} \, d\theta \qquad \Delta\theta = N 2\pi$$

$$\Delta V = \tau^{(E)} \cdot N 2\pi \Rightarrow \frac{1}{2} I_s \omega^2 = \tau^{(E)} N \cdot 2\pi$$

$$\tau^{(E)} = \frac{I_s \omega^2}{N 4\pi} = \frac{I_s \omega^2}{80\pi} =$$

$L_i = I_A \omega$

$L_F = I' \omega'$

$I_A \omega = I' \omega' \Rightarrow$

$I_A \to \frac{1}{3} m e^2$

$I' \Rightarrow I_A + I_m$

$I_m \Rightarrow m e^2$

$mg L = \frac{1}{2} I_A \omega^2 + mg \frac{e}{2} \to \omega$

$mg \frac{e}{\cancel{2}} = \frac{1}{\cancel{2}} I_A \omega^2 \Rightarrow$

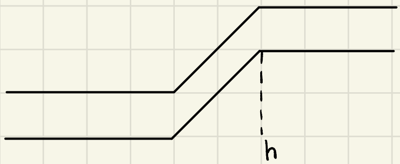
$$\omega = \sqrt{\frac{mge}{I_A}} = \sqrt{\frac{2mge}{me^2}} = \sqrt{\frac{3g}{e}}$$

$I_A \omega = I' \omega' \Rightarrow \omega' \ldots$

0 $\quad s = 10^{-2}\ m^2$ $\qquad P = ?$

$h = 10\ m$

$v = 3\ m/s$



$$P = \frac{dW}{dt} \qquad dW = dm\, g h$$

$$\frac{dm}{dt} = s \cdot v \rho \qquad \frac{dW}{dt} = s v g h \rho$$

40

$\omega = 5 \text{ rad/s}$ $\qquad$ $A = ?$

$t_0 = 0 \text{ s}$ $\qquad$ $\phi = ?$

$\Delta x = 25 \text{ cm} = 0{,}25 \text{ m}$

$v_0 = -40 \text{ cm/s} = -0{,}40 \text{ m/s}$

$$\begin{cases} x(t) = A \cdot \sin(\omega t + \phi) \\ v(t) = \omega A \cdot \cos(\omega t + \phi) \end{cases} \Rightarrow$$

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} \Rightarrow \sqrt{0{,}25^2 + \left(\frac{-0{,}40 \text{ m/s}}{5 \text{ rad/s}}\right)^2} = 0{,}26 \text{ m}$$

$$\operatorname{tg} \phi = \frac{x_0 \omega}{v_0} \Rightarrow \phi = \operatorname{arctg} \frac{x_0 \omega}{v_0} \Rightarrow -72^\circ$$

41

$l = 2,23\ m$ $T = ?$

$m = 6,74\ kg$ $E = ?$

$v_0 = 2,06\ m/s$ $\alpha = ?$

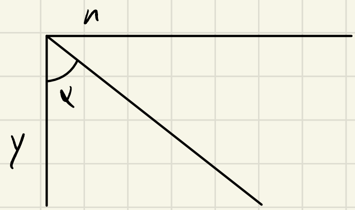
$$\omega = \sqrt{\frac{g}{l}} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow 2,99\ s$$

(17)

$y = 5\ Km = 5000\ m$

$v_0 = 500\ Km/h \Rightarrow 138,88\ m/s$

$\alpha = ?$



$$\begin{cases} x(t) = x_0 + vt \\ y = y_0 + v_y t + \frac{1}{2} g t^2 \end{cases} \Rightarrow \quad t = \sqrt{\frac{2h}{g}} = 31,94\ s$$

$x = 0 + 138,88 \cdot 31,94 = 4435,82\ m$

$$tg\,\alpha = \frac{x}{y} \Rightarrow \alpha = arctg\,\frac{x}{y} = 41,57^0$$

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(24)

$F = 50N$

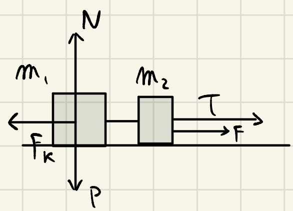
$m_1 = 10\ Kg$

$m_2 = 20\ Kg$

$\mu_k = 0,1$

$T = ?$

$a = ?$



29

$m = 1\,kg$

$\mu_k = 0{,}2$

$v_0 = 3\,m/s$

$\alpha = 30^{\circ}$

$s = ?$

$t_0, t_1 = ?$

56

$$\rho_A = 1 \ g/cm^3 \Rightarrow 1000 \ g/m^3$$

$$F_{AR} = -\rho_c \cdot g \cdot \cdot V$$

$$F_A = F_p \qquad\qquad V \cdot \rho = M$$

$$F + F_p = -g \, \rho_c \cdot V_T \qquad\qquad V_T = \frac{M}{\rho} =$$

$$F = -g \, \rho_c \cdot \frac{M}{\rho_A} - Mg \Rightarrow 245N \qquad\qquad F + F_p = F_{AR}$$

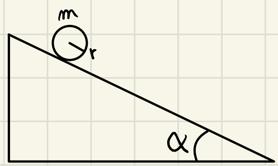
53

$m = 6\ Kg$

$r = 0{,}12\ m$

$\mu_s = 0{,}22$

$\alpha = ?$



$$A_{cM} = \frac{g \operatorname{sen} \alpha}{1 + \dfrac{I_{cM}}{mR^2}}$$

10

$v_0 = 0$ $\qquad h = ?$

$t = 4{,}8s$

$v_s = 340 m/s$

$$\begin{cases} y(t) = y_0 + \overset{0}{v_0} t + \frac{1}{2} g t^2 \\ v(t) = v + at \end{cases} \qquad \begin{cases} y(t) = y_0 + vt \\ v(t) = v_0 \end{cases}$$

$$y(t) = h - \frac{1}{2} g t^2 \qquad \overset{h}{y}(t) = \overset{0}{y_0} + v_s t_2$$

$$y(t_1) = 0 \Rightarrow t_1 = \sqrt{\frac{2h}{g}} \Rightarrow \qquad t_2 = \frac{h}{v_s}$$

$$t = t_1 + t_2 = 4{,}8s \Rightarrow t_1 = t - t_2 > 0 \Rightarrow t > t_2 \Rightarrow t > \frac{h}{v_s} \Rightarrow h < 1632$$

$$\sqrt{\frac{2h}{g}} + \frac{h}{v_s} - t = 0 \Rightarrow \sqrt{\frac{2h}{g}} = t - \frac{h}{v_s} \Rightarrow \left(t - \frac{h}{v_s}\right)^2 = \frac{2h}{g}$$

$$t^2 + \frac{h^2}{v_s^2} - 2t\frac{h}{v_s} - \frac{2h}{g} = 0 \Rightarrow$$

$$23{,}04 + \frac{h^2}{115600} - 0{,}028h - 0{,}20h = 0 \Rightarrow$$

$$\frac{h^2}{115600} - 0{,}228h + 23{,}04 = 0 \Rightarrow$$

$$h = \frac{0{,}228 \pm \sqrt{0{,}051 - 0{,}00079}}{0{,}000017} \Rightarrow$$

$$h_1 = 26\,756 \text{ m}$$

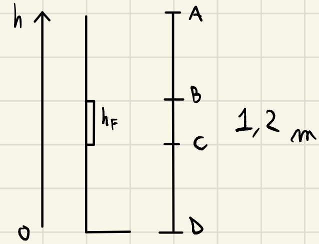
$$h_2 = 99 \text{ m}$$

11

$h_F = 1{,}2\ m$ $\qquad\qquad h = ?$

$\Delta t_{BC} = 0{,}125\ s$

$\Delta t'_{CD} = 2{,}00/2 = 1\ s$



$$\begin{cases} y(t) = \underset{h}{y_0} + v_0 t + \frac{1}{2} g t^2 \\ v(t) = v_0 + g t \end{cases}$$

$$y(t_D) = h - \frac{1}{2} g t_D^2 = 0 \Rightarrow \boxed{h = \frac{1}{2} g t_D^2}$$

$y(t = -\frac{1}{2} g t^2$

$t_D = \Delta t_{AB} + \Delta t_{BC} + \Delta t_{CD}$

$\Delta t_{AB} = t_B - t_A = t_B - 0 = t_B$

$y(t_B) = h - \frac{1}{2} g t_B^2$

$y(t_C) = h - \frac{1}{2} g t_C^2$

$$h = y_B - y_C = -\frac{1}{2} g t_B^2 + \frac{1}{2} g t_C^2 \Rightarrow \frac{1}{2} g (t_C^2 - t_B^2) \Rightarrow$$

$$\frac{1}{2} g \underbrace{(t_C - t_B)}_{\Delta t_{BC}} (t_C + t_B)$$

$\Delta t_{BC} = t_C - t_B \Rightarrow t_C = \Delta t + t_B$

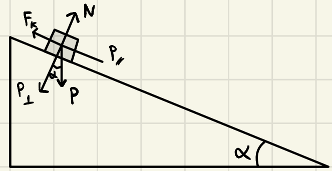
$$h = \frac{1}{2} g\, \Delta t\, (\Delta t + 2 t_B) \Rightarrow 2 t_B = \frac{2h}{g \cdot \Delta t} - \Delta t \Rightarrow 0{,}917\ s$$

$$t_D = \overset{\Delta t_{AB}}{0{,}917} + \overset{\Delta t_{BC}}{0{,}125} + \overset{\Delta t_{CD}}{1} \approx 2{,}042$$

$$h = \frac{1}{2} g t_D^2 = 20{,}43\ m$$

22 $m = 3\,kg$ $\qquad a = ?$

$\alpha = 30°$ $\qquad \mu_k = ?$



$d = 2{,}00\,m$

$t = 1{,}50\,s$

$$x(t) = \cancelto{0}{x_0} + \cancelto{0}{v_0}t + \frac{1}{2}at^2 \Rightarrow \frac{1}{2}at^2 \Rightarrow a = \frac{2x(t)}{t^2} \Rightarrow 1{,}7\,m/s^2$$

$$\vec{P} + \vec{N} + F_k = m\vec{a}$$

$$(\cancel{\vec{P}_{\perp}} + \vec{P}_{\parallel}) + \cancel{\vec{N}} + \vec{F}_k = m\vec{a} \Rightarrow P_{\parallel} + F_k = ma \Rightarrow$$

$$\cancel{m}g\sin\alpha - \mu_k \cancel{m}g\cos\alpha = \cancel{m}a \Rightarrow a = g(\sin\alpha - \mu_k\cos\alpha)$$

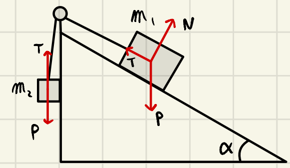
$$\mu_k = \frac{g\sin\alpha - a}{g\cos\alpha} \Rightarrow 0{,}37$$

23

$\alpha = 30°$ $\qquad a_1 = ?$

$m_1 = 40 \, Kg$ $\qquad a_2 = ?$

$m_2 = 30 \, Kg$ $\qquad T = ?$



(1)

$$\cancel{\vec{N}} + (\cancel{\vec{P}_\perp} + \vec{P}_{//}) + T = m\vec{a}$$

$$| m g \sin\alpha + T = m\vec{a} |$$

$$m_1 g \sin\alpha - T_1 = -m_1 a$$

(2)

$$\vec{P} + \vec{T} = m_2 \vec{a}$$

$$m_2 g - T_2 = m_2 a$$

$a = a_1 = a_2 \qquad T = T_1 = T_2$ perchè fune ideale.

$$\begin{cases} m_1 g \sin\alpha - T + m_1 a = 0 \\ m_2 g - T - m_2 a = 0 \end{cases}$$

$$\begin{cases} \cdot\cdot \; T = m_1 g \sin\alpha + m_1 a \\ \cdot \; m_2 g - (m_1 g \sin\alpha + m_1 a) - m_2 a = 0 \end{cases}$$

• $m_2 g - m_1 g \sin\alpha - m_1 a - m_2 a = 0 \Rightarrow$

$$g(m_2 - m_1 \sin\alpha) + a(-m_1 - m_2) = 0$$

$$a = \frac{-g(m_2 - m_1 \sin\alpha)}{-m_1 - m_2} \Rightarrow 1,4 \, m/s^2$$

•• $T = 40 \cdot g \sin 30° + 40 \cdot 1,4 \Rightarrow 252 \, N$

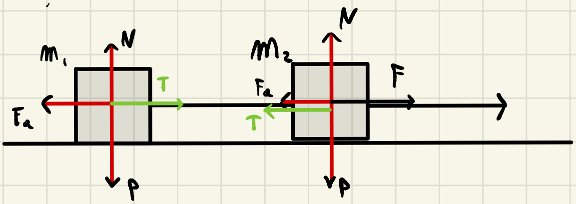
(24)

$F = 50N$ $\quad T = ?$

$m_1 = 10 kg$ $\quad a = ?$

$m_2 = 20 kg$

$\mu = 0,1$



$$\vec{P} + \vec{N} + \vec{F}_k + \vec{T} = m\vec{a} \qquad \vec{P} + \vec{N} + \vec{F}_k + \vec{T} + \vec{F} = m_2\vec{a}$$

$$\begin{cases} \vec{T} - \vec{F}_k = m_1\vec{a} \\ \vec{N} = m_1 g \\ \vec{F}_{k_1} = \mu_k \vec{N} \end{cases} \qquad \begin{cases} \vec{F} - \vec{F}_k - \vec{T} = m_2\vec{a} \\ \vec{N} = m_2 g \\ \vec{F}_{k_2} = \mu_k \cdot \vec{N} \end{cases}$$

fune ideale : $T = T_1 = T_2$, $a = a_1 = a_2$

$F_{k_1} = \mu_k m_1 g = 9,8 \qquad F_{k_2} = 19,6$

$$\begin{cases} \vec{T} - \vec{F}_{k_1} = m_1\vec{a} \\ \vec{F} - \vec{F}_{k_2} - \vec{T} = m_2\vec{a} \end{cases} \Rightarrow \begin{cases} ^{\bullet\bullet} T = m_1 a + F_{k_1} \\ ^{\bullet} F - F_{k_2} - m_1 a - F_{k_1} = m_2\vec{a} \end{cases}$$

$$\bullet \; a(m_1 + m_2) = F - F_{k_2} - F_{k_1} \Rightarrow a = \frac{F - F_{k_2} - F_{k_1}}{m_1 + m_2} \Rightarrow 0,68$$

$\bullet\bullet \; T = 16,66 \text{ N}$

27

$m = 1\ Kg$ $\mu_k = ?$

$v_0 = 4\ m/s$

$s = 136\ cm \Rightarrow 1,36\ m$

$$F_k = \mu_k\, m g \quad = \mu_k = \frac{F_k}{mg} \Rightarrow \frac{a}{g}$$

$$\begin{cases} x(t) = x_0 + v_0 t + \frac{1}{2} g t^2 \\ v(t) = v_0 + at \end{cases} \Rightarrow \begin{cases} 1,36 = 4t + \frac{1}{2} g t^2 \\ 0 = 4 + at \end{cases} \Rightarrow$$

$$\frac{1}{2} g t^2 + 4t - 1,36 = 0 \quad \Rightarrow t = \frac{-4 \pm \sqrt{16 + 4\left(\frac{1}{2}g\right)(1,36)}}{g} \Rightarrow$$

$$a = \frac{4}{t} = 12\ m/s^2$$

$t_1 = 0,25\ s$

$t_2 = -1,07\ s$

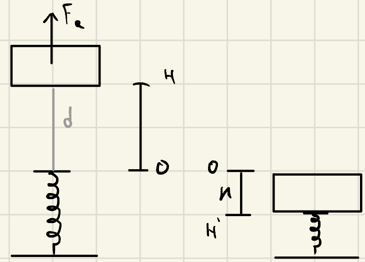
30

$m = 2000\,Kg$ $\qquad v_f = ?$

$d = 4{,}0\,m$ $\qquad s_M = ?$

$K_M = 1{,}5 \cdot 10^5\,N/m$

$F_e = 4900\,N$



Energia potenziale e quella cinetica aumentano.

$$W = E_0 - E_H \Rightarrow (\underset{0}{U_0} + K_0) - (\underset{mgh}{U_H} + \underset{0}{K_H}) \Rightarrow -K_0 - mgh \Rightarrow$$

$$\Rightarrow \frac{1}{2} m v_0^2 - mgh = W$$

$$W = F \cdot \Delta s \Rightarrow F_K \cdot (0 - \overset{d}{4}) \Rightarrow W = -F_K \cdot d$$

$$\frac{1}{2} m v_0^2 = mgd - F_K \cdot d \Rightarrow v_0 = \sqrt{\frac{2(mgd - F_K d)}{m}} = 7{,}66\,m/s$$

$$\bullet\bullet \quad W = (U_{H'} + \overset{0}{K_{H'}}) - (\overset{0}{U_0} + \overset{\frac{1}{2}mv^2}{K_0}) \Rightarrow -mgn + \frac{1}{2} k n^2 - \frac{1}{2} m v^2$$

$$W = -F_K \cdot n$$

$$-mgn + \frac{1}{2} k n^2 - \frac{1}{2} m v^2 = -F_K n$$

$$\omega - mgh + \frac{1}{2}kh^2 - mgd - F_k d + F_n h = 0 \Rightarrow$$

$$\frac{1}{2}kh^2 + (F_k - mg)h + (F_k - mg)d = 0$$

$$h = \frac{-F_k + mg \pm \sqrt{(F_k - mg)^2 - 4\left(\frac{1}{2}k\right)(F_k - mg)}}{k} \Rightarrow$$

$$h = \frac{14700 \pm 68015{,}365}{k}$$

1,4 DA RIVEDERE

$m = 2000\,kg$

$d = 4{,}0\,m$

$k_M = 1{,}5 \cdot 10^5\,N/m$

$F_a = 4900\,N$

(28)

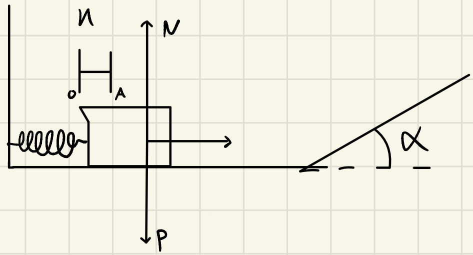
$m = 100\ g = 0{,}1\ Kg$

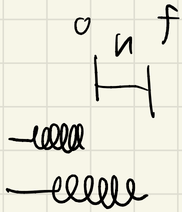
$K = 100\ N/m$

$n = 10\ cm = 0{,}1\ m$

$\alpha = 30°$

$\mu_K = 0{,}100$





$$\Delta E = \left(\overset{0}{U_A} + \overset{\frac{1}{2}mv^2}{K_A}\right) - \left(\overset{\frac{1}{2}kn^2}{U_o} + \overset{0}{K_o}\right)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}kn^2 \Rightarrow \qquad v = \sqrt{\frac{kn^2}{m}} \Rightarrow 3{,}16\ m/s$$

DA VEDERE

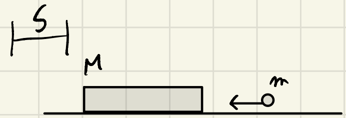
32

$M = 1\,kg$ $v_m = ?$

$\mu = 0{,}7$

$m = 40\,g = 0{,}04\,kg$

$s = 34\,cm = 0{,}34\,m$



40

$\omega = 5 \text{ rad/s}$ $\qquad$ $A = ?$

$t = 0$ $\qquad$ $\phi = ?$

$s = 25 \text{ cm} = 0,25 \text{ m}$

$v_0 = -40 \text{ cm/s} = -0,40 \text{ m/s}$

$$A = \sqrt{r_0^2 + \left(\frac{v_0}{\omega}\right)^2} \Rightarrow \sqrt{0,25^2 + \left(\frac{0,40}{5}\right)^2} \Rightarrow 0,26 \text{ m}$$

$$tg(\phi) = \frac{r_0 \omega}{v_0} \Rightarrow -72,25$$

41

$l = 2,23 \text{ m}$ $\qquad$ $T = ?$

$m = 6,74 \text{ kg}$ $\qquad$ $E = ?$

$v_0 = 2,06 \text{ m/s}$ $\qquad$ $\alpha = ?$

$$T = \frac{2\pi}{\omega} \Rightarrow 2\pi\sqrt{\frac{l}{g}} \Rightarrow 2,99 \text{ s}$$

$E = U + K$ forze conservative $\Rightarrow U = E$; $K = E$

$$E = \frac{1}{2} m v^2 \Rightarrow 14,30 \text{ J}$$
