lunedì 28 dicembre 2020 18:27

$$\lim_{(x,y)\to(0,0)} \frac{x}{y}\log(1+x)$$

$m \in \mathbb{R}$ $m \neq 0$

$E_m = \{(x,y)\in\mathbb{R}^2 : \ y = mx, \ x \neq 0\}$

$$f_{|E_m} = \frac{1}{m}\log(1+x) \xrightarrow[(x,y)\to(0,0)]{} 0$$

$$0 \le \left|\frac{x}{y}\log(1+x)\right| \qquad \frac{x}{\log y}\log(1+x)$$

$\log(1+y) \le y$

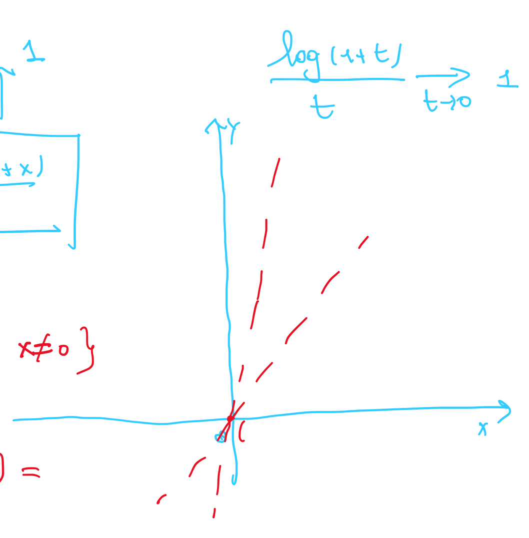
$y>0$ $\log y \le \log(1+y) \le y = |y|$

$|y| \geq |\log y|$?

$f \le g \overset{?}{\Rightarrow} |f| \overset{?}{\le} |g|$ NO!

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{y}\cdot\boxed{\frac{\log(1+x)}{x}} \qquad \frac{\log(1+x)}{x} \to 1$$

$$\frac{\log(1+t)}{t} \xrightarrow[t\to 0]{} 1$$



$F = \{(x,y) : \ y = x^2 \ \ x\neq 0\}$

$$f_{/F}(x,y) = \frac{x}{x^2}\log(1+x) =$$

$$= \frac{\log(1+x)}{x} \xrightarrow[(x,y)\to(0,0)]{} 1.$$

$$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x,y)$$

$f: \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} g(x,y) & \text{se } (x,y)\neq(x_0,y_0) \\ \ell & \text{se } (x,y) = (x_0,y_0) \end{cases}$$

Studio la diff. in (x_0,y_0)

f è continua in (x_0,y_0) ?

NO $\Rightarrow$ f non è diff. in (x_0,y_0)

SI

definizione

$\Downarrow$

$$\lim_{(h,k)\to(0,0)} \frac{\Delta f - df(x_0,y_0)}{\sqrt{h^2+k^2}} = 0$$

$$df(x_0,y_0) = \underline{f_x(x_0,y_0)}\,h + \underline{f_y(x_0,y_0)}\,k$$

condizione sufficiente per la diff.

$\Downarrow$

$\exists f_x, f_y$ in un intorno di (x_0,y_0) e sono ivi continue

$\exists f_x, f_y$ in un intorno di (x_0,y_0)

$+$

$$\lim_{(x,y)\to(x_0,y_0)} f_x(x,y) = f_x(x_0,y_0)$$

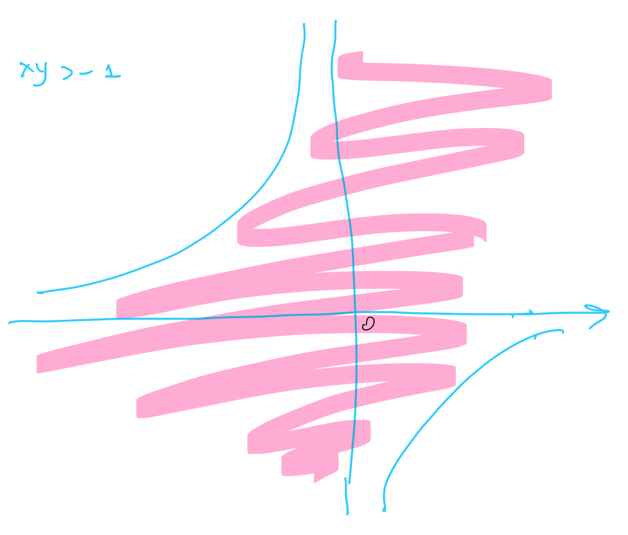
$$\lim_{(x,y)\to(x_0,y_0)} f_y(x,y) = f_y(x_0,y_0)$$

$$f(x,y) = \begin{cases} \boxed{\dfrac{xy}{x^2+y^2}} & \text{se } (x,y)\neq(0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

$$f(x,y) = \log(1+xy)$$

$\text{dom} f = \{(x,y)\in\mathbb{R}^2 : \ 1+xy>0\}$

$xy+1>0 \iff xy>-1$



$$f(x,y) = \begin{cases} \dfrac{xy^4}{x^2+y^2} & \text{se } (x,y)\neq(0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

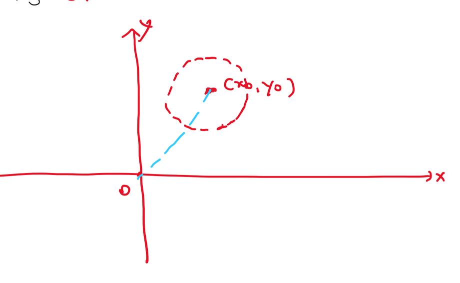
$$0 \le |f(x,y)| = \frac{|x|\,|y^4|}{x^2+y^2} = |x|\,y^2 \underbrace{\boxed{\frac{y^2}{x^2+y^2}}}_{\le 1} \le |x|y^2 \xrightarrow[(x,y)\to(0,0)]{} 0$$

f è continua in $(0,0)$

Studio la differenziabilità in $\mathbb{R}^2$

se $(x_0,y_0)\neq(0,0)$

$\exists\, I_\delta(x_0,y_0) \subseteq \mathbb{R}^2\setminus\{(0,0)\}$



In $I_\delta(x_0,y_0)$

$$f(x,y) = \frac{xy^4}{x^2+y^2} \qquad \forall (x,y)\in I_\delta(x_0,y_0) \subseteq \mathbb{R}^2\setminus\{(0,0)\}$$

$\exists f_x, f_y$ in $I_\delta(x_0,y_0)$ e sono continue in ogni punto di $I_\delta(x_0,y_0)$

$\Downarrow$

f è differenziabile in ogni punto di $I_\delta(x_0,y_0)$

$\Downarrow$

f è diff in (x_0,y_0)

- Studio la diff. in $(0,0)$

1° modo) USO LA DEF.

$\exists f_x(0,0)$ $\exists f_y(0,0)$?

$f(x,0) = 0$ $\forall x\in\mathbb{R} \Rightarrow [D f(x,0)]_{x=0} = 0$

$\Rightarrow \exists f_x(0,0) = 0$

$f(0,y) = 0$ $\forall y\in\mathbb{R} \Rightarrow [D f(0,y)]_{y=0} = 0.$

$\Rightarrow \exists f_y(0,0) = 0$

$df(0,0) = 0$

Controllo se

$$\lim_{(h,k)\to(0,0)} \frac{f(h,k)-f(0,0)}{\sqrt{h^2+k^2}} = 0.$$

$$\lim_{(h,k)\to(0,0)} \frac{hk^4}{\sqrt{h^2+k^2}\,(h^2+k^2)} \overset{?}{=} 0$$

$\forall (x,y)\neq(0,0)$

$$0 \le \frac{|h|\,k^4}{\sqrt{h^2+k^2}\,(h^2+k^2)} \le \frac{\cancel{\sqrt{h^2+k^2}}\,k^4}{\cancel{\sqrt{h^2+k^2}}\,(h^2+k^2)} \le \frac{k^2\,\cancel{(k^2+h^2)}}{\cancel{h^2+k^2}} \to 0$$

2° modo) Calcolo

$$f_x(x,y) = y^4\,\frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = y^4\,\frac{y^2-x^2}{(x^2+y^2)^2}$$

f_x è continua in $(0,0)$ se:

$$\lim_{(x,y)\to(0,0)} f_x(x,y) = \underbrace{f_x(0,0)}_{=0}$$

$$\boxed{\lim_{(x,y)\to(0,0)} y^4\,\frac{y^2-x^2}{(x^2+y^2)^2} = 0.}$$

$f: A \to \mathbb{R}$ $A \subseteq \mathbb{R}^2$

$\underline{v} = (v_1, v_2)$ direzione

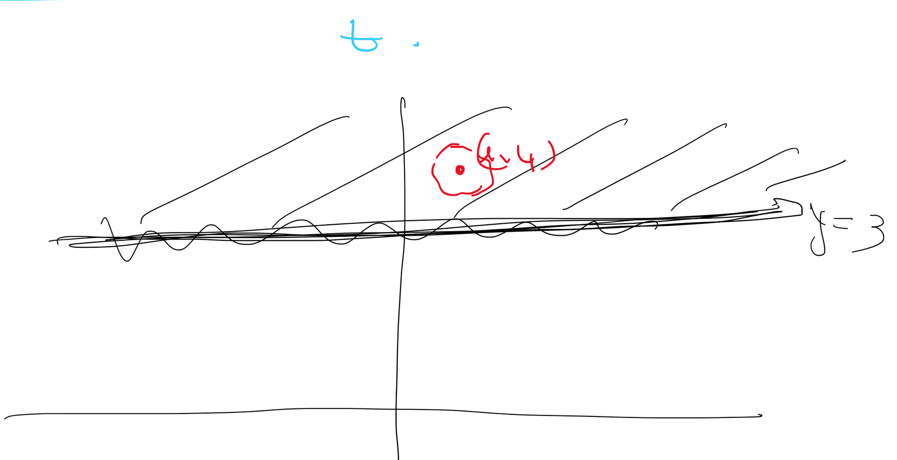
$(x_0,y_0) \in \mathring{A}$

$$\frac{\partial f}{\partial \underline{v}}(x_0,y_0) \overset{\text{def}}{=} \lim_{t\to 0} \frac{F(t)-F(0)}{t} =$$

$$= \lim_{t\to 0} \frac{f(x_0+tv_1, y_0+tv_2) - f(x_0,y_0)}{t}.$$

- $f(x,y) = \boxed{x}\sqrt{y-3}$

$$\frac{\partial f}{\partial \underline{v}}(1,4)$$



f diff. in $(1,4)$?

$f_x(x,y) = \sqrt{y-3}$ $\forall (x,y) \in \text{dom} f$

$f_y(x,y) = \dfrac{x}{2\sqrt{y-3}}$ $\forall (x,y)\in\mathbb{R}^2$ $\underline{y>3}$

f_x, f_y sono continue in $(1,4)$

$\Downarrow$

f è differenziabile in $(1,4)$