4.4 Scheduling Multipurpose Batch Processes using State-Task Networks

References

Floudas, C. A., & Lin, X. (2005). Mixed integer linear programming in process scheduling: Modeling, algorithms, and applications. Annals of Operations Research, 139(1), 131-162.

Harjunkoski, I., Maravelias, C. T., Bongers, P., Castro, P. M., Engell, S., Grossmann, I. E., ... & Wassick, J. (2014). Scope for industrial applications of production scheduling models and solution methods. Computers & Chemical Engineering, 62, 161-193.

Kondili, E., Pantelides, C. C., & Sargent, R. W. H. (1993). A general algorithm for short-term scheduling of batch operations—I. MILP formulation. Computers & Chemical Engineering, 17(2), 211-227.

Méndez, C. A., Cerdá, J., Grossmann, I. E., Harjunkoski, I., & Fahl, M. (2006). State-of-the-art review of optimization methods for short-term scheduling of batch processes. Computers & Chemical Engineering, 30(6), 913-946.

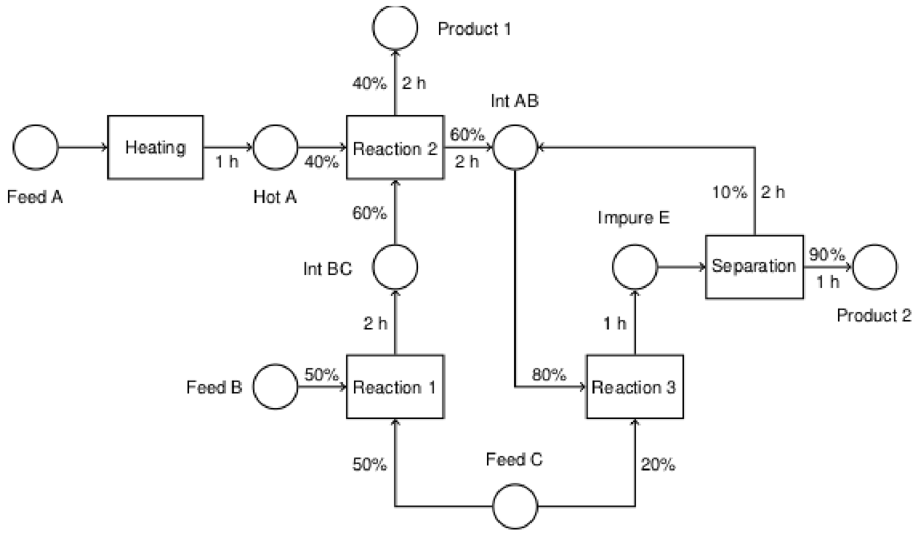
Shah, N., Pantelides, C. C., & Sargent, R. W. H. (1993). A general algorithm for short-term scheduling of batch operations—II. Computational issues. Computers & Chemical Engineering, 17(2), 229-244.

Wassick, J. M., & Ferrio, J. (2011). Extending the resource task network for industrial applications. Computers & chemical engineering, 35(10), 2124-2140.

Example (Kondili, et al., 1993)

A state-task network is a graphical representation of the activities in a multiproduct batch process. The representation includes the minimum details needed for short term scheduling of batch operations.

A well-studied example due to Kondili (1993) is shown below. Other examples are available in the references cited above.



Each circular node in the diagram designates material in a particular state. The materials are generally held in suitable vessels with a known capacity. The relevant information for each state is the initial inventory, storage capacity, and the unit price of the material in each state. The price of materials in intermediate states may be assigned penalities in order to minimize the amount of work in progress.

The rectangular nodes denote process tasks. When scheduled for execution, each task is assigned an appropriate piece of equipment, and assigned a batch of material according to the incoming arcs. Each incoming arc begins at a state where the associated label indicates the mass fraction of the batch coming from that particular state. Outgoing arcs indicate the disposition of the batch to product states. The outgoing are labels indicate the fraction of the batch assigned to each product state, and the time necessary to produce that product.

Not shown in the diagram is the process equipment used to execute the tasks. A separate list of process units is available, each characterized by a capacity and list of tasks which can be performed in that unit.

Exercise

Read this reciped for Hollandaise Sauce: http://www.foodnetwork.com/recipes/tyler-florence/hollandaise-sauce-recipe-1910043. Assume the available equipment consists of one sauce pan and a double-boiler on a stove. Draw a state-task network outlining the basic steps in the recipe.

Encoding the STN data

The basic data structure specifies the states, tasks, and units comprising a state-task network. The intention is for all relevant problem data to be contained in a single JSON-like structure.

In [1]:

```
H = 20

Kondili = {
    'TIME':  range(0,H+1),
    'STATES': {
        'Feed_A'   : {'capacity': 500, 'initial': 500, 'price':  0},
        'Feed_B'   : {'capacity': 500, 'initial': 500, 'price':  0},
        'Feed_C'   : {'capacity': 500, 'initial': 500, 'price':  0},
        'Hot_A'    : {'capacity': 100, 'initial':   0, 'price': -1},
        'Int_AB'   : {'capacity': 200, 'initial':   0, 'price': -1},
        'Int_BC'   : {'capacity': 150, 'initial':   0, 'price': -1},
        'Impure_E' : {'capacity': 100, 'initial':   0, 'price': -1},
        'Product_1': {'capacity': 500, 'initial':   0, 'price': 10},
        'Product_2': {'capacity': 500, 'initial':   0, 'price': 10},
    },
    'ST_ARCS': {
        ('Feed_A',   'Heating')   : {'rho': 1.0},
        ('Feed_B',   'Reaction_1'): {'rho': 0.5},
        ('Feed_C',   'Reaction_1'): {'rho': 0.5},
        ('Feed_C',   'Reaction_3'): {'rho': 0.2},
        ('Hot_A',    'Reaction_2'): {'rho': 0.4},
        ('Int_AB',   'Reaction_3'): {'rho': 0.8},
        ('Int_BC',   'Reaction_2'): {'rho': 0.6},
        ('Impure_E', 'Separation'): {'rho': 1.0},
    },
    'TS_ARCS': {
        ('Heating',    'Hot_A')    : {'dur': 1, 'rho': 1.0},
        ('Reaction_2', 'Product_1'): {'dur': 1.5, 'rho': 0.4},
        ('Reaction_2', 'Int_AB')   : {'dur': 1.5, 'rho': 0.6},
        ('Reaction_1', 'Int_BC')   : {'dur': 2, 'rho': 1.0},
        ('Reaction_3', 'Impure_E') : {'dur': 1, 'rho': 1.0},
        ('Separation', 'Int_AB')   : {'dur': 2, 'rho': 0.1},
        ('Separation', 'Product_2'): {'dur': 1, 'rho': 0.9},
    },
    'UNIT_TASKS': {
        ('Heater',    'Heating')   : {'Bmin': 0, 'Bmax': 100, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Reactor_1', 'Reaction_1'): {'Bmin': 0, 'Bmax':  80, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Reactor_1', 'Reaction_2'): {'Bmin': 0, 'Bmax':  80, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Reactor_1', 'Reaction_3'): {'Bmin': 0, 'Bmax':  80, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Reactor_2', 'Reaction_1'): {'Bmin': 0, 'Bmax':  80, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Reactor_2', 'Reaction_2'): {'Bmin': 0, 'Bmax':  80, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Reactor_2', 'Reaction_3'): {'Bmin': 0, 'Bmax':  80, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
        ('Still',     'Separation'): {'Bmin': 0, 'Bmax': 200, 'Cost': 1, 'vCost': 0, 'Tc
lean': 0},
    },
}
```

Setting a Time Grid

The following computations can be done on any time grid, including real-valued time points. TIME is a list of time points commencing at 0.

Creating a Pyomo Model

The following Pyomo model closely follows the development in Kondili, et al. (1993). In particular, the first step in the model is to process the STN data to create sets as given in Kondili.

One important difference from Kondili is the adoption of a more natural time scale that starts at $t = 0$ and extends to $t = H$ (rather than from 1 to H+1).

A second difference is the introduction of an additional decision variable denoted by $Q_{j,t}$ indicating the amount of material in unit j at time t. A material balance then reads

$$Q_{jt} = Q_{j(t-1)} + \sum_{i\in I_j} B_{ijt} - \sum_{i\in I_j} \sum_{\substack{s\in\bar{S}_i \\ s\ni t-P_{is}\geq 0}} \bar{\rho}_{is} B_{ij(t-P_{is})} \qquad \forall j,t$$

Following Kondili's notation, I_j is the set of tasks that can be performed in unit j, and $\bar{S}_i$ is the set of states fed by task j. We assume the units are empty at the beginning and end of production period, i.e.,

$$\begin{aligned} Q_{j(-1)} &= 0 \qquad \forall j \\ Q_{j,H} &= 0 \qquad \forall j \end{aligned}$$

The unit allocation constraints are written the full backward aggregation method described by Shah (1993). The allocation constraint reads

$$\sum_{i\in I_j} \sum_{t'=t}^{t-p_i+1} W_{ijt'} \leq 1 \qquad \forall j,t$$

Each processing unit j is tagged with a minimum and maximum capacity, B_{ij}^{min} and B_{ij}^{max}, respectively, denoting the minimum and maximum batch sizes for each task i. A minimum capacity may be needed to cover heat exchange coils in a reactor or mixing blades in a blender, for example. The capacity may depend on the nature of the task being performed. These constraints are written

$$B_{ij}^{min} W_{ijt} \leq B_{ijt} \leq B_{ij}^{max} W_{ijt} \qquad \forall j, \forall i \in I_j, \forall t$$

Characterization of Tasks

In [2]:

```
STN = Kondili

STATES = STN['STATES']
ST_ARCS = STN['ST_ARCS']
TS_ARCS = STN['TS_ARCS']
UNIT_TASKS = STN['UNIT_TASKS']
TIME = STN['TIME']
H = max(TIME)
```

In [3]:

```
TASKS = set([i for (j,i) in UNIT_TASKS])                                # set of all tasks

S = {i: set() for i in TASKS}                                           # S[i] input set of st
es which feed task i
for (s,i) in ST_ARCS:
    S[i].add(s)

S_ = {i: set() for i in TASKS}                                          # S_[i] output set of
states fed by task i
for (i,s) in TS_ARCS:
    S_[i].add(s)

rho = {(i,s): ST_ARCS[(s,i)]['rho'] for (s,i) in ST_ARCS}              # rho[(i,s)] input frac
tion of task i from state s

rho_ = {(i,s): TS_ARCS[(i,s)]['rho'] for (i,s) in TS_ARCS}             # rho_[(i,s)] output f
raction of task i to state s

P = {(i,s): TS_ARCS[(i,s)]['dur'] for (i,s) in TS_ARCS}                # P[(i,s)] time for
task i output to state s

p = {i: max([P[(i,s)] for s in S_[i]]) for i in TASKS}                 # p[i] completion time
for task i

K = {i: set() for i in TASKS}                                           # K[i] set of units ca
ble of task i
for (j,i) in UNIT_TASKS:
    K[i].add(j)
```

Characterization of States

In [4]:

```
T = {s: set() for s in STATES}                                          # T[s] set of tasks re
iving material from state s
for (s,i) in ST_ARCS:
    T[s].add(i)

T_ = {s: set() for s in STATES}                                         # set of tasks
producing material for state s
for (i,s) in TS_ARCS:
    T_[s].add(i)

C = {s: STATES[s]['capacity'] for s in STATES}                          # C[s] storage capacity
for state s
```

Characterization of Units

In [5]:

```
UNITS = set([j for (j,i) in UNIT_TASKS])

I = {j: set() for j in UNITS}                                    # I[j] set of tasks pe
formed with unit j
for (j,i) in UNIT_TASKS:
    I[j].add(i)

Bmax = {(i,j):UNIT_TASKS[(j,i)]['Bmax'] for (j,i) in UNIT_TASKS}  # Bmax[(i,j)] maximum
capacity of unit j for task i
Bmin = {(i,j):UNIT_TASKS[(j,i)]['Bmin'] for (j,i) in UNIT_TASKS}  # Bmin[(i,j)] minimum
capacity of unit j for task i
```

Pyomo Model

In [6]:

```
from pyomo.environ import *
import numpy as np

TIME = np.array(TIME)

model = ConcreteModel()

model.W = Var(TASKS, UNITS, TIME, domain=Boolean)              # W[i,j,t] 1 if task i
starts in unit j at time t
model.B = Var(TASKS, UNITS, TIME, domain=NonNegativeReals)     # B[i,j,t,] size of batch
assigned to task i in unit j at time t
model.S = Var(STATES.keys(), TIME, domain=NonNegativeReals)    # S[s,t] inventory of sta
te s at time t
model.Q = Var(UNITS, TIME, domain=NonNegativeReals)            # Q[j,t] inventory of
unit j at time t
model.Cost = Var(domain=NonNegativeReals)
model.Value = Var(domain=NonNegativeReals)

# Objective is to maximize the value of the final state (see Kondili, Sec. 5)
model.Obj = Objective(expr = model.Value - model.Cost, sense = maximize)

# Constraints
model.cons = ConstraintList()
model.cons.add(model.Value == sum([STATES[s]['price']*model.S[s,H] for s in STATES]))
model.cons.add(model.Cost == sum([UNIT_TASKS[(j,i)]['Cost']*model.W[i,j,t] +
        UNIT_TASKS[(j,i)]['vCost']*model.B[i,j,t] for i in TASKS for j in K[i] for t in
TIME]))

# unit constraints
for j in UNITS:
    rhs = 0
    for t in TIME:
        # a unit can only be allocated to one task
        lhs = 0
        for i in I[j]:
            for tprime in TIME:
                if tprime >= (t-p[i]+1-UNIT_TASKS[(j,i)]['Tclean']) and tprime <= t:
                    lhs += model.W[i,j,tprime]
        model.cons.add(lhs <= 1)

        # capacity constraints (see Konkili, Sec. 3.1.2)
        for i in I[j]:
            model.cons.add(model.W[i,j,t]*Bmin[i,j] <= model.B[i,j,t])
            model.cons.add(model.B[i,j,t] <= model.W[i,j,t]*Bmax[i,j])

        # unit mass balance
        rhs += sum([model.B[i,j,t] for i in I[j]])
```

```
        for i in I[j]:
            for s in S_[i]:
                if t >= P[(i,s)]:
                    rhs -= rho_[(i,s)]*model.B[i,j,max(TIME[TIME <= t-P[(i,s)]])]
        model.cons.add(model.Q[j,t] == rhs)
        rhs = model.Q[j,t]

    # terminal condition
    model.cons.add(model.Q[j,H] == 0)

# state constraints
for s in STATES.keys():
    rhs = STATES[s]['initial']
    for t in TIME:
        # state capacity constraint
        model.cons.add(model.S[s,t] <= C[s])

        # state mass balanace
        for i in T_[s]:
            for j in K[i]:
                if t >= P[(i,s)]:
                    rhs += rho_[(i,s)]*model.B[i,j,max(TIME[TIME <= t-P[(i,s)]])]

        for i in T[s]:
            rhs -= rho[(i,s)]*sum([model.B[i,j,t] for j in K[i]])
        model.cons.add(model.S[s,t] == rhs)
        rhs = model.S[s,t]

# additional production constraints
model.cons.add(model.S['Product_2',H] >= 250)

SolverFactory('glpk').solve(model).write()
```

```
  Signal handler called from  /Users/jeff/anaconda3/lib/python3.6/subprocess.py
_try_wait 1404
  Waiting...
  Signaled process 30126 with signal 2
ERROR: Solver (glpk) returned non-zero return code (-1)
ERROR: Solver log: GLPSOL: GLPK LP/MIP Solver, v4.65 Parameter(s) specified in
    the command line:
     --write /Users/jeff/Dropbox/Git/ND-Pyomo-
        Cookbook/notebooks/scheduling/tmp0uvmbh6h.glpk.raw --wglp
        /Users/jeff/Dropbox/Git/ND-Pyomo-
        Cookbook/notebooks/scheduling/tmpr8n6to0a.glpk.glp --cpxlp
        /Users/jeff/Dropbox/Git/ND-Pyomo-
        Cookbook/notebooks/scheduling/tmpjuds8idv.pyomo.lp
    Reading problem data from '/Users/jeff/Dropbox/Git/ND-Pyomo-
    Cookbook/notebooks/scheduling/tmpjuds8idv.pyomo.lp'...
    /Users/jeff/Dropbox/Git/ND-Pyomo-
    Cookbook/notebooks/scheduling/tmpjuds8idv.pyomo.lp:5778: warning: lower
    bound of variable 'x1' redefined /Users/jeff/Dropbox/Git/ND-Pyomo-
    Cookbook/notebooks/scheduling/tmpjuds8idv.pyomo.lp:5778: warning: upper
    bound of variable 'x1' redefined 890 rows, 612 columns, 2486 non-zeros 168
    integer variables, all of which are binary 5946 lines were read Writing
    problem data to '/Users/jeff/Dropbox/Git/ND-Pyomo-
    Cookbook/notebooks/scheduling/tmpr8n6to0a.glpk.glp'... 5051 lines were
    written GLPK Integer Optimizer, v4.65 890 rows, 612 columns, 2486 non-
    zeros 168 integer variables, all of which are binary Preprocessing... 459
    rows, 552 columns, 1730 non-zeros 168 integer variables, all of which are
    binary Scaling...
     A: min|aij| =  1.000e-01  max|aij| =  2.000e+02  ratio =  2.000e+03
    GM: min|aij| =  3.162e-01  max|aij| =  3.162e+00  ratio =  1.000e+01 EQ:
    min|aij| =  1.000e-01  max|aij| =  1.000e+00  ratio =  1.000e+01 2N:
    min|aij| =  5.000e-02  max|aij| =  1.600e+00  ratio =  3.200e+01
    Constructing initial basis... Size of triangular part is 459 Solving LP
    relaxation... GLPK Simplex Optimizer, v4.65 459 rows, 552 columns, 1730
    non-zeros
          0: obj =  -0.000000000e+00 inf =   1.600e+02 (2)
        161: obj =   3.783856096e+03 inf =   9.934e-14 (0)
```

```
*   257: obj =   9.121531481e+03 inf =   6.361e-14 (0) 1 OPTIMAL LP
    SOLUTION FOUND Integer optimization begins... Long-step dual simplex
    will be used +   257: mip =     not found yet <=              +inf
    (1; 0) +   818: >>>>>   6.429520833e+03 <=   9.121531481e+03  41.9%
    (67; 0) +  1069: >>>>>   7.386666667e+03 <=   9.121531481e+03  23.5%
    (127; 3) +  1309: >>>>>   8.203750000e+03 <=   9.121531481e+03  11.2%
    (173; 31) +  1592: >>>>>   8.526222222e+03 <=   9.121531481e+03   7.0%
    (191; 100) +  4660: >>>>>   8.635833333e+03 <=   9.121531481e+03
    5.6% (576; 135) + 13019: >>>>>   8.706185185e+03 <=   9.121531481e+03
    4.8% (1608; 313) + 16046: >>>>>   8.725500000e+03 <=   9.121531481e+03
    4.5% (1946; 484) + 33655: >>>>>   8.728851852e+03 <=   9.121531481e+03
    4.5% (4044; 837) + 41870: >>>>>   8.910611111e+03 <=   9.121531481e+03
    2.4% (5037; 978) + 68385: mip =   8.910611111e+03 <=   9.121531481e+03
    2.4% (5793; 4912) + 80975: >>>>>   8.941111111e+03 <=
    9.121531481e+03   2.0% (7227; 5015) + 84651: >>>>>   8.961777778e+03
    <=   9.121531481e+03   1.8% (6533; 7110) +111699: mip =
    8.961777778e+03 <=   9.121531481e+03   1.8% (8670; 8139) +137551: mip
    =   8.961777778e+03 <=   9.121531481e+03   1.8% (11053; 8461) +163408:
    mip =   8.961777778e+03 <=   9.121531481e+03   1.8% (13423; 8769)
    +190751: mip =   8.961777778e+03 <=   9.121531481e+03   1.8% (15740;
    9142) +194516: >>>>>   8.962777778e+03 <=   9.121531481e+03   1.8%
    (16105; 9182) +214583: mip =   8.962777778e+03 <=   9.121531481e+03
    1.8% (17629; 10003) +239892: mip =   8.962777778e+03 <=
    9.121531481e+03   1.8% (19842; 10343) +262539: mip =   8.962777778e+03
    <=   9.121531481e+03   1.8% (21931; 10572) +274398: >>>>>
    8.963777778e+03 <=   9.121531481e+03   1.8% (23019; 10705) Time used:
    60.0 secs.  Memory used: 32.0 Mb. +297658: mip =   8.963777778e+03 <=
    9.121531481e+03   1.8% (24790; 11490) +323501: mip =   8.963777778e+03
    <=   9.121531481e+03   1.8% (26950; 11807) +346369: mip =
    8.963777778e+03 <=   9.121531481e+03   1.8% (29222; 12116) +369526:
    mip =   8.963777778e+03 <=   9.121531481e+03   1.8% (31202; 12437)
    +391286: mip =   8.963777778e+03 <=   9.121531481e+03   1.8% (33135;
    12760) +412573: mip =   8.963777778e+03 <=   9.121531481e+03   1.8%
    (35027; 13074) +433462: mip =   8.963777778e+03 <=   9.121531481e+03
    1.8% (36860; 13360) +451949: mip =   8.963777778e+03 <=
    9.121531481e+03   1.8% (38718; 13624) +471024: mip =   8.963777778e+03
    <=   9.121531481e+03   1.8% (40679; 13856)
```

```
---------------------------------------------------------------------------
ApplicationError                          Traceback (most recent call last)
<ipython-input-6-046334ce38aa> in <module>()
     71 model.cons.add(model.S['Product_2',H] >= 250)
     72
---> 73 SolverFactory('glpk').solve(model).write()

~/anaconda3/lib/python3.6/site-packages/pyomo/opt/base/solvers.py in solve(self, *args,
**kwds)
    624                     logger.error("Solver log:\n" + str(_status.log))
    625                 raise pyutilib.common.ApplicationError(
--> 626                     "Solver (%s) did not exit normally" % self.name)
    627             solve_completion_time = time.time()
    628             if self._report_timing:

ApplicationError: Solver (glpk) did not exit normally
```

Analysis

Profitability

In [ ]:

```
print("Value of State Inventories = {0:12.2f}".format(model.Value()))
print("  Cost of Unit Assignments = {0:12.2f}".format(model.Cost()))
print("             Net Objective = {0:12.2f}".format(model.Value() - model.Cost()))
```

State Inventories

In [8]:

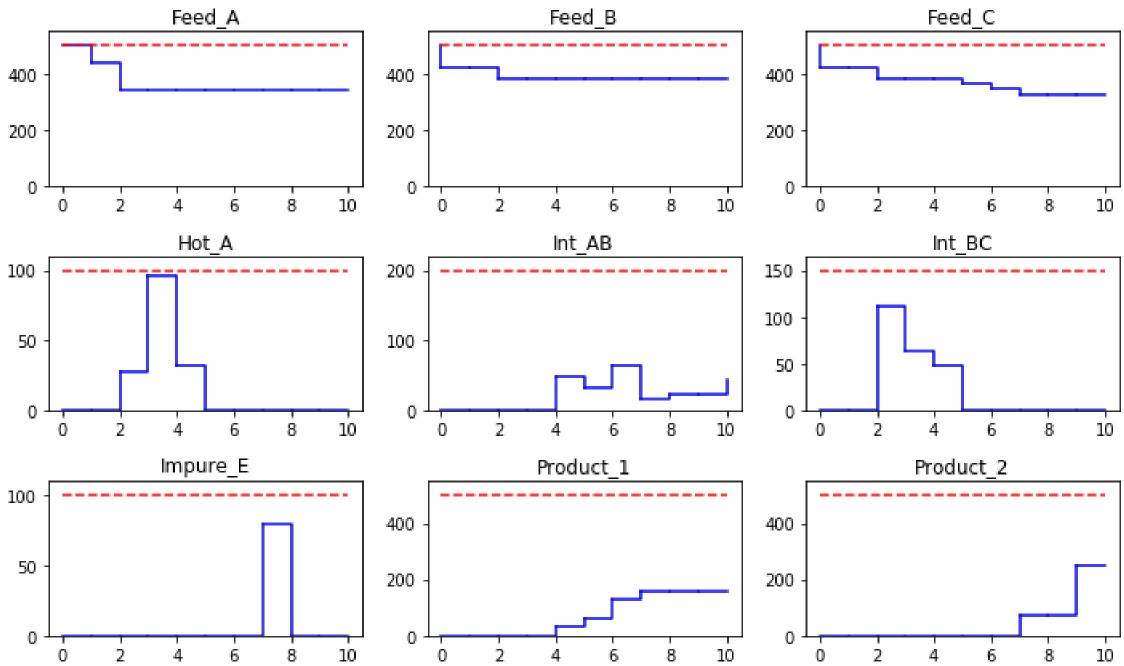
```
%matplotlib inline
import matplotlib.pyplot as plt
import pandas as pd
from IPython.display import display, HTML

pd.DataFrame([[model.S[s,t]() for s in STATES.keys()] for t in TIME], columns = STATES.
keys(), index = TIME)
```

Out[8]:

	Feed_A	Feed_B	Feed_C	Hot_A	Int_AB	Int_BC	Impure_E	Product_1	Product_2
0	500.0	420.0	420.0	0.0	0.0	0.0	0.0	0.0	0.0
1	440.0	420.0	420.0	0.0	0.0	0.0	0.0	0.0	0.0
2	340.0	380.0	380.0	28.0	0.0	112.0	0.0	0.0	0.0
3	340.0	380.0	380.0	96.0	0.0	64.0	0.0	0.0	0.0
4	340.0	380.0	380.0	32.0	48.0	48.0	0.0	32.0	0.0
5	340.0	380.0	364.0	0.0	32.0	0.0	0.0	64.0	0.0
6	340.0	380.0	348.0	0.0	64.0	0.0	0.0	128.0	0.0
7	340.0	380.0	324.0	0.0	16.0	0.0	80.0	160.0	72.0
8	340.0	380.0	324.0	0.0	24.0	0.0	0.0	160.0	72.0
9	340.0	380.0	324.0	0.0	24.0	0.0	0.0	160.0	252.0
10	340.0	380.0	324.0	0.0	44.0	0.0	0.0	160.0	252.0

In [9]:

```
plt.figure(figsize=(10,6))
for (s,idx) in zip(STATES.keys(),range(0,len(STATES.keys()))):
    plt.subplot(ceil(len(STATES.keys())/3),3,idx+1)
    tlast,ylast = 0,STATES[s]['initial']
    for (t,y) in zip(list(TIME),[model.S[s,t]() for t in TIME]):
        plt.plot([tlast,t,t],[ylast,ylast,y],'b')
        #plt.plot([tlast,t],[ylast,y],'b.',ms=10)
        tlast,ylast = t,y
    plt.ylim(0,1.1*C[s])
    plt.plot([0,H],[C[s],C[s]],'r--')
    plt.title(s)
plt.tight_layout()
```



Unit Assignment

In [10]:

```
UnitAssignment = pd.DataFrame({j:[None for t in TIME] for j in UNITS},index=TIME)

for t in TIME:
    for j in UNITS:
        for i in I[j]:
            for s in S_[i]:
                if t-p[i] >= 0:
                    if model.W[i,j,max(TIME[TIME <= t-p[i]])]() > 0:
                        UnitAssignment.loc[t,j] = None
        for i in I[j]:
            if model.W[i,j,t]() > 0:
                UnitAssignment.loc[t,j] = (i,model.B[i,j,t]())

UnitAssignment
```

Out[10]:

	Heater	Reactor_1	Reactor_2	Still
0	None	(Reaction_1, 80.0)	(Reaction_1, 80.0)	None
1	(Heating, 60.0)	None	None	None
2	(Heating, 100.0)	(Reaction_1, 80.0)	(Reaction_2, 80.0)	None
3	None	None	(Reaction_2, 80.0)	None
4	None	(Reaction_2, 80.0)	(Reaction_2, 80.0)	None
5	None	(Reaction_3, 80.0)	(Reaction_2, 80.0)	None
6	None	None	(Reaction_3, 80.0)	(Separation, 80.0)
7	None	(Reaction_3, 40.0)	(Reaction_3, 80.0)	None
8	None	None	None	(Separation, 200.0)
9	None	None	None	None
10	None	None	None	None

Unit Batch Inventories

In [11]:

```
pd.DataFrame([[model.Q[j,t]() for j in UNITS] for t in TIME], columns = UNITS, index =
TIME)
```

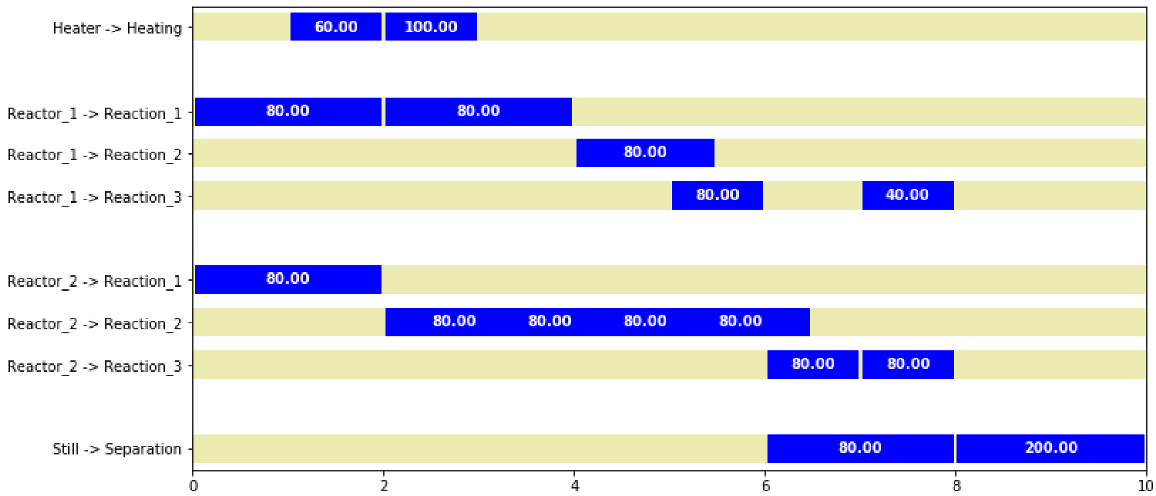
Out[11]:

	Still	Reactor_1	Reactor_2	Heater
0	0.0	80.0	80.0	0.0
1	0.0	80.0	80.0	60.0
2	0.0	80.0	80.0	100.0
3	0.0	80.0	160.0	0.0
4	0.0	80.0	160.0	0.0
5	0.0	160.0	160.0	0.0
6	80.0	0.0	160.0	0.0
7	8.0	40.0	80.0	0.0
8	200.0	0.0	0.0	0.0
9	20.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0

Gannt Chart

In [12]:

```
%matplotlib inline
import matplotlib.pyplot as plt

plt.figure(figsize=(12,6))

gap = H/500
idx = 1
lbls = []
ticks = []
for j in sorted(UNITS):
    idx -= 1
    for i in sorted(I[j]):
        idx -= 1
        ticks.append(idx)
        lbls.append("{0:s} -> {1:s}".format(j,i))
        plt.plot([0,H],[idx,idx],lw=20,alpha=.3,color='y')
        for t in TIME:
            if model.W[i,j,t]() > 0:
                plt.plot([t+gap,t+p[i]-gap], [idx,idx],'b', lw=20, solid_capstyle='butt'
)
                txt = "{0:.2f}".format(model.B[i,j,t]())
                plt.text(t+p[i]/2, idx, txt, color='white', weight='bold', ha='center',
va='center')
plt.xlim(0,H)
plt.gca().set_yticks(ticks)
plt.gca().set_yticklabels(lbls);
```



Trace of Events and States

In [13]:

```
sep = '\n--------------------------------------------------------------------------------
-----------\n'
print(sep)
print("Starting Conditions")
print("    Initial Inventories:")
for s in STATES.keys():
        print("        {0:10s}  {1:6.1f} kg".format(s,STATES[s]['initial']))

units = {j:{'assignment':'None', 't':0} for j in UNITS}

for t in TIME:
    print(sep)
    print("Time =",t,"hr")
    print("    Instructions:")
    for j in UNITS:
        units[j]['t'] += 1
        # transfer from unit to states
        for i in I[j]:
            for s in S_[i]:
                if t-P[(i,s)] >= 0:
                    amt = rho_[(i,s)]*model.B[i,j,max(TIME[TIME <= t - P[(i,s)]])]()
                    if amt > 0:
                        print("        Transfer", amt, "kg from", j, "to", s)
    for j in UNITS:
        # release units from tasks
        for i in I[j]:
            if t-p[i] >= 0:
                if model.W[i,j,max(TIME[TIME <= t-p[i]])]() > 0:
                    print("        Release", j, "from", i)
                    units[j]['assignment'] = 'None'
                    units[j]['t'] = 0
        # assign units to tasks
        for i in I[j]:
            if model.W[i,j,t]() > 0:
                print("        Assign", j, "with capacity", Bmax[(i,j)], "kg to task",i
"for",p[i],"hours")
                units[j]['assignment'] = i
                units[j]['t'] = 1
        # transfer from states to starting tasks
        for i in I[j]:
            for s in S[i]:
                amt = rho[(i,s)]*model.B[i,j,t]()
                if amt > 0:
                    print("        Transfer", amt,"kg from", s, "to", j)
    print("\n    Inventories are now:")
    for s in STATES.keys():
        print("        {0:10s}  {1:6.1f} kg".format(s,model.S[s,t]()))
    print("\n    Unit Assignments are now:")
    for j in UNITS:
        if units[j]['assignment'] != 'None':
            fmt = "        {0:s} performs the {1:s} task with a {2:.2f} kg batch for ho
ur {3:f} of {4:f}"
            i = units[j]['assignment']
            print(fmt.format(j,i,model.Q[j,t](),units[j]['t'],p[i]))

print(sep)
print('Final Conditions')
print("    Final Inventories:")
for s in STATES.keys():
        print("        {0:10s}  {1:6.1f} kg".format(s,model.S[s,H]()))
```

```
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Starting Conditions
    Initial Inventories:
        Feed_A       500.0 kg
        Feed_B       500.0 kg
```

```
        Feed_B       500.0 kg
        Feed_C       500.0 kg
        Hot_A          0.0 kg
        Int_AB         0.0 kg
        Int_BC         0.0 kg
        Impure_E       0.0 kg
        Product_1      0.0 kg
        Product_2      0.0 kg

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Time = 0 hr
    Instructions:
        Assign Reactor_1 with capacity 80 kg to task Reaction_1 for 2 hours
        Transfer 40.0 kg from Feed_C to Reactor_1
        Transfer 40.0 kg from Feed_B to Reactor_1
        Assign Reactor_2 with capacity 80 kg to task Reaction_1 for 2 hours
        Transfer 40.0 kg from Feed_C to Reactor_2
        Transfer 40.0 kg from Feed_B to Reactor_2

    Inventories are now:
        Feed_A       500.0 kg
        Feed_B       420.0 kg
        Feed_C       420.0 kg
        Hot_A          0.0 kg
        Int_AB         0.0 kg
        Int_BC         0.0 kg
        Impure_E       0.0 kg
        Product_1      0.0 kg
        Product_2      0.0 kg

    Unit Assignments are now:
        Reactor_1 performs the Reaction_1 task with a 80.00 kg batch for hour 1.000000
of 2.000000
        Reactor_2 performs the Reaction_1 task with a 80.00 kg batch for hour 1.000000
of 2.000000

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Time = 1 hr
    Instructions:
        Assign Heater with capacity 100 kg to task Heating for 1 hours
        Transfer 60.0 kg from Feed_A to Heater

    Inventories are now:
        Feed_A       440.0 kg
        Feed_B       420.0 kg
        Feed_C       420.0 kg
        Hot_A          0.0 kg
        Int_AB         0.0 kg
        Int_BC         0.0 kg
        Impure_E       0.0 kg
        Product_1      0.0 kg
        Product_2      0.0 kg

    Unit Assignments are now:
        Reactor_1 performs the Reaction_1 task with a 80.00 kg batch for hour 2.000000
of 2.000000
        Reactor_2 performs the Reaction_1 task with a 80.00 kg batch for hour 2.000000
of 2.000000
        Heater performs the Heating task with a 60.00 kg batch for hour 1.000000 of 1.0
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Time = 2 hr
```

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    Instructions:
        Transfer 80.0 kg from Reactor_1 to Int_BC
        Transfer 80.0 kg from Reactor_2 to Int_BC
        Transfer 60.0 kg from Heater to Hot_A
        Release Reactor_1 from Reaction_1
        Assign Reactor_1 with capacity 80 kg to task Reaction_1 for 2 hours
        Transfer 40.0 kg from Feed_C to Reactor_1
        Transfer 40.0 kg from Feed_B to Reactor_1
        Release Reactor_2 from Reaction_1
        Assign Reactor_2 with capacity 80 kg to task Reaction_2 for 1.5 hours
        Transfer 32.0 kg from Hot_A to Reactor_2
        Transfer 48.0 kg from Int_BC to Reactor_2
        Release Heater from Heating
        Assign Heater with capacity 100 kg to task Heating for 1 hours
        Transfer 100.0 kg from Feed_A to Heater

    Inventories are now:
        Feed_A       340.0 kg
        Feed_B       380.0 kg
        Feed_C       380.0 kg
        Hot_A         28.0 kg
        Int_AB         0.0 kg
        Int_BC       112.0 kg
        Impure_E       0.0 kg
        Product_1      0.0 kg
        Product_2      0.0 kg

    Unit Assignments are now:
        Reactor_1 performs the Reaction_1 task with a 80.00 kg batch for hour 1.000000
of 2.000000
        Reactor_2 performs the Reaction_2 task with a 80.00 kg batch for hour 1.000000
of 1.500000
        Heater performs the Heating task with a 100.00 kg batch for hour 1.000000 of 1.
000000

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Time = 3 hr
    Instructions:
        Transfer 100.0 kg from Heater to Hot_A
        Assign Reactor_2 with capacity 80 kg to task Reaction_2 for 1.5 hours
        Transfer 32.0 kg from Hot_A to Reactor_2
        Transfer 48.0 kg from Int_BC to Reactor_2
        Release Heater from Heating

    Inventories are now:
        Feed_A       340.0 kg
        Feed_B       380.0 kg
        Feed_C       380.0 kg
        Hot_A         96.0 kg
        Int_AB         0.0 kg
        Int_BC        64.0 kg
        Impure_E       0.0 kg
        Product_1      0.0 kg
        Product_2      0.0 kg

    Unit Assignments are now:
        Reactor_1 performs the Reaction_1 task with a 80.00 kg batch for hour 2.000000
of 2.000000
        Reactor_2 performs the Reaction_2 task with a 160.00 kg batch for hour 1.000000
of 1.500000

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Time = 4 hr
    Instructions:
```

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        Transfer 80.0 kg from Reactor_1 to Int_BC
        Transfer 32.0 kg from Reactor_2 to Product_1
        Transfer 48.0 kg from Reactor_2 to Int_AB
        Release Reactor_1 from Reaction_1
        Assign Reactor_1 with capacity 80 kg to task Reaction_2 for 1.5 hours
        Transfer 32.0 kg from Hot_A to Reactor_1
        Transfer 48.0 kg from Int_BC to Reactor_1
        Release Reactor_2 from Reaction_2
        Assign Reactor_2 with capacity 80 kg to task Reaction_2 for 1.5 hours
        Transfer 32.0 kg from Hot_A to Reactor_2
        Transfer 48.0 kg from Int_BC to Reactor_2

    Inventories are now:
        Feed_A      340.0 kg
        Feed_B      380.0 kg
        Feed_C      380.0 kg
        Hot_A        32.0 kg
        Int_AB       48.0 kg
        Int_BC       48.0 kg
        Impure_E      0.0 kg
        Product_1    32.0 kg
        Product_2     0.0 kg

    Unit Assignments are now:
        Reactor_1 performs the Reaction_2 task with a 80.00 kg batch for hour 1.000000
of 1.500000
        Reactor_2 performs the Reaction_2 task with a 160.00 kg batch for hour 1.000000
of 1.500000

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Time = 5 hr
    Instructions:
        Transfer 32.0 kg from Reactor_2 to Product_1
        Transfer 48.0 kg from Reactor_2 to Int_AB
        Assign Reactor_1 with capacity 80 kg to task Reaction_3 for 1 hours
        Transfer 16.0 kg from Feed_C to Reactor_1
        Transfer 64.0 kg from Int_AB to Reactor_1
        Release Reactor_2 from Reaction_2
        Assign Reactor_2 with capacity 80 kg to task Reaction_2 for 1.5 hours
        Transfer 32.0 kg from Hot_A to Reactor_2
        Transfer 48.0 kg from Int_BC to Reactor_2

    Inventories are now:
        Feed_A      340.0 kg
        Feed_B      380.0 kg
        Feed_C      364.0 kg
        Hot_A         0.0 kg
        Int_AB       32.0 kg
        Int_BC        0.0 kg
        Impure_E      0.0 kg
        Product_1    64.0 kg
        Product_2     0.0 kg

    Unit Assignments are now:
        Reactor_1 performs the Reaction_3 task with a 160.00 kg batch for hour 1.000000
of 1.000000
        Reactor_2 performs the Reaction_2 task with a 160.00 kg batch for hour 1.000000
of 1.500000

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Time = 6 hr
    Instructions:
        Transfer 32.0 kg from Reactor_1 to Product_1
        Transfer 48.0 kg from Reactor_1 to Int_AB
        Transfer 80.0 kg from Reactor_1 to Impure_E
```

```
        Transfer 32.0 kg from Reactor_2 to Product_1
        Transfer 48.0 kg from Reactor_2 to Int_AB
        Assign Still with capacity 200 kg to task Separation for 2 hours
        Transfer 80.0 kg from Impure_E to Still
        Release Reactor_1 from Reaction_2
        Release Reactor_1 from Reaction_3
        Release Reactor_2 from Reaction_2
        Assign Reactor_2 with capacity 80 kg to task Reaction_3 for 1 hours
        Transfer 16.0 kg from Feed_C to Reactor_2
        Transfer 64.0 kg from Int_AB to Reactor_2

    Inventories are now:
        Feed_A       340.0 kg
        Feed_B       380.0 kg
        Feed_C       348.0 kg
        Hot_A          0.0 kg
        Int_AB        64.0 kg
        Int_BC         0.0 kg
        Impure_E       0.0 kg
        Product_1    128.0 kg
        Product_2      0.0 kg

    Unit Assignments are now:
        Still performs the Separation task with a 80.00 kg batch for hour 1.000000 of 2
.000000
        Reactor_2 performs the Reaction_3 task with a 160.00 kg batch for hour 1.000000
of 1.000000

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Time = 7 hr
    Instructions:
        Transfer 72.0 kg from Still to Product_2
        Transfer 32.0 kg from Reactor_2 to Product_1
        Transfer 48.0 kg from Reactor_2 to Int_AB
        Transfer 80.0 kg from Reactor_2 to Impure_E
        Assign Reactor_1 with capacity 80 kg to task Reaction_3 for 1 hours
        Transfer 8.0 kg from Feed_C to Reactor_1
        Transfer 32.0 kg from Int_AB to Reactor_1
        Release Reactor_2 from Reaction_2
        Release Reactor_2 from Reaction_3
        Assign Reactor_2 with capacity 80 kg to task Reaction_3 for 1 hours
        Transfer 16.0 kg from Feed_C to Reactor_2
        Transfer 64.0 kg from Int_AB to Reactor_2

    Inventories are now:
        Feed_A       340.0 kg
        Feed_B       380.0 kg
        Feed_C       324.0 kg
        Hot_A          0.0 kg
        Int_AB        16.0 kg
        Int_BC         0.0 kg
        Impure_E      80.0 kg
        Product_1    160.0 kg
        Product_2     72.0 kg

    Unit Assignments are now:
        Still performs the Separation task with a 8.00 kg batch for hour 2.000000 of 2.
000000
        Reactor_1 performs the Reaction_3 task with a 40.00 kg batch for hour 1.000000
of 1.000000
        Reactor_2 performs the Reaction_3 task with a 80.00 kg batch for hour 1.000000
of 1.000000

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```

```
Time = 8 hr
    Instructions:
        Transfer 8.0 kg from Still to Int_AB
        Transfer 40.0 kg from Reactor_1 to Impure_E
        Transfer 80.0 kg from Reactor_2 to Impure_E
        Release Still from Separation
        Assign Still with capacity 200 kg to task Separation for 2 hours
        Transfer 200.0 kg from Impure_E to Still
        Release Reactor_1 from Reaction_3
        Release Reactor_2 from Reaction_3

    Inventories are now:
        Feed_A      340.0 kg
        Feed_B      380.0 kg
        Feed_C      324.0 kg
        Hot_A         0.0 kg
        Int_AB       24.0 kg
        Int_BC        0.0 kg
        Impure_E      0.0 kg
        Product_1   160.0 kg
        Product_2    72.0 kg

    Unit Assignments are now:
        Still performs the Separation task with a 200.00 kg batch for hour 1.000000 of
2.000000

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-

Time = 9 hr
    Instructions:
        Transfer 180.0 kg from Still to Product_2

    Inventories are now:
        Feed_A      340.0 kg
        Feed_B      380.0 kg
        Feed_C      324.0 kg
        Hot_A         0.0 kg
        Int_AB       24.0 kg
        Int_BC        0.0 kg
        Impure_E      0.0 kg
        Product_1   160.0 kg
        Product_2   252.0 kg

    Unit Assignments are now:
        Still performs the Separation task with a 20.00 kg batch for hour 2.000000 of 2
.000000

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-

Time = 10 hr
    Instructions:
        Transfer 20.0 kg from Still to Int_AB
        Release Still from Separation

    Inventories are now:
        Feed_A      340.0 kg
        Feed_B      380.0 kg
        Feed_C      324.0 kg
        Hot_A         0.0 kg
        Int_AB       44.0 kg
        Int_BC        0.0 kg
        Impure_E      0.0 kg
        Product_1   160.0 kg
        Product_2   252.0 kg

    Unit Assignments are now:
```

```
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-

Final Conditions
    Final Inventories:
        Feed_A       340.0 kg
        Feed_B       380.0 kg
        Feed_C       324.0 kg
        Hot_A          0.0 kg
        Int_AB        44.0 kg
        Int_BC         0.0 kg
        Impure_E       0.0 kg
        Product_1    160.0 kg
        Product_2    252.0 kg
```

In [ ]: