Investigating Efficiency in Developing Markets

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formation. A market in which prices always "fully reflect" available information is called "efficient."

Most famously, the EMH states:

$$E[r_{t+1}|\Phi_t] = 0$$

where $r_{t+1} = p_{t+1} - p_t$ is absolute returns at time t and $E[x|\Phi_t]$ is the conditional expectation of a return on the basis of all historical trading information at time t (under weak form EMH).

As such we have :

$$E[p_{t+1}|\Phi_t] = p_t$$

Our Questions

1. Are developed markets more efficient than developing markets?

2. Do we see evidence of these markets become more efficient over time?

Jane Street

Contents

Data

Table 1: Time Series Index Data

Data	
australia_data	6142 obs. of 4 variables
australia_data_2000s2020s	8750 obs. of 4 variables
australia_data_2010s2020s	3761 obs. of 4 variables
chile_data	8979 obs. of 4 variables
chile_data_1990s2000s	5218 obs. of 4 variables
chile_data_2000s	2609 obs. of 4 variables
chile_data_2010s2020s	3761 obs. of 4 variables
china_data	8728 obs. of 4 variables

```
$ date    : Date[1:8728], format:  ...
$ china   : num [1:8728] 100 104 109 115 120 ...
$ t       : int [1:8728] 1 2 3 4 5 6 7 8 9 10 ...
$ returns: num [1:8728] NA 4.41 4.74 5.42 5.7 ...
```

Methods

- **EMH Statistically**
- **Hurst Exponent**
- **Ljung-Box Test**
- **Lo-Mackinley Test**
- **Dicky-Fuller Test**
- **Cross Validations**

The Efficient Market Hypothesis, actually reflects a family of hypotheses. These are:

Weak Form EMH: Asset prices p_t fully reflect all historical trading information, including previous prices.

Semi-Strong Form EMH: Asset prices p_t fully reflect all publicly available data, including previous prices.

Strong Form EMH: Asset prices p_t fully reflect all publicly and privately available data, including previous prices.

So, while I test the Weak Form EMH because it relates most directly to time-series price data, in testing the Weak Form EMH I implicitly also test the Semi-Strong Form EMH and Strong Form EMH.

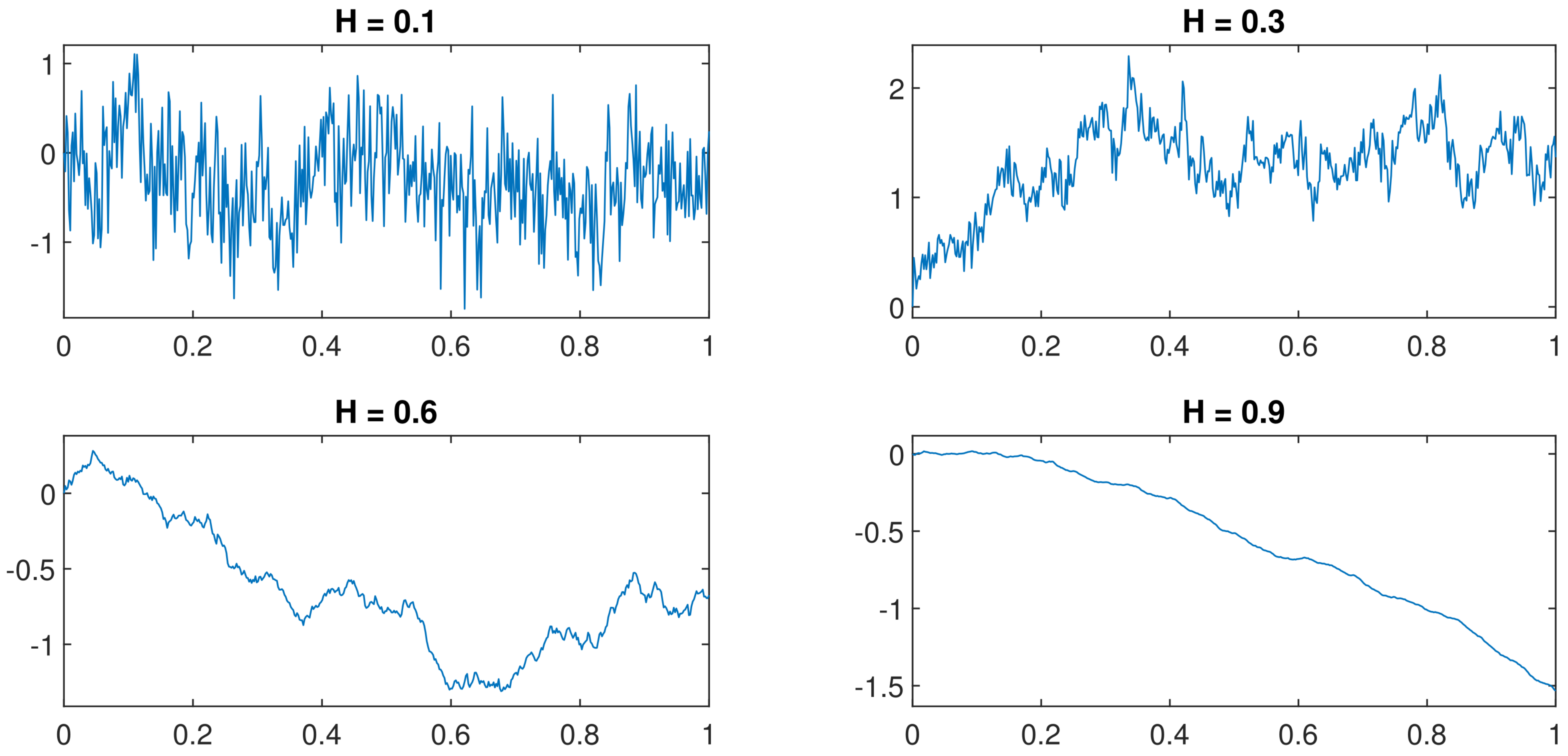
$$p_{t+1} = p_t + Z_t, Z_t \overset{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2)$$

We all know, however, that prices do not remain steady for ever. Under the EMH, price changes are driven by new information shocks which we model through a 0-mean noise term with constant variance Z_t. In general, proving properties of the distribution of how any and all new information shocks affect prices is a tricky endeavor. The weak form EMH process specified by Fama in his original paper is that $Z_t \sim Gaussian(0, \sigma^2)$ with constant variance, and iid. That this should be the case is not entirely obvious: for example, markets are commonly known to enter periods of higher and lower volatility. Indeed there is an exciting literature testing the constant variance assumption, and the role of derivative markets in stabilizing this variance. Nevertheless, taking the weak-form EMH, as described by Fama, at face value we have the following unit-root relationship to test:

$$p_{t+1} = p_t + Z_t, Z_t \overset{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2)$$

Methods

- EMH Statistically
- **Hurst Exponent**



Methods

- EMH Statistically
- Hurst Exponent
- **Ljung-Box Test**

We calculate the Ljung-Box-Pierce test statistic, Q, defined as:

$$Q = n(n+2)\sum_{i=1}^{k}\frac{\rho_k^2}{n-i}, Q \sim \chi^2_{k-1} \text{ asymptotically}$$

Where: ρ_i is the sample autocorrelation for lag i

k is the range of lags we sum over. We experiment over a range of k (k= XXXX , XXXX , XXXX)

Because we are testing if an AR(1) model generated this data, we have $k-1$ df. See lecture notes Def. 5.55.

If we apply the Ljung-Box-Pierce test to the differenced time series $r_t = p_t - p_{t-1}$, which is just the returns series, we expect to observe that there is no significant autocorrelation in returns as returns should be Gaussian Noise under the Weak form EMH. ($r_t = p_t - p_{t-1} = p_{t-1} + Z_t - p_{t-1} = Z_t$.) As such, the p-value of this test can be used as a tool to quantify to what degree each index exhibits autocorrelation in returns, which violates the weak form EMH (where returns should be iid).

Methods

- EMH Statistically
- Hurst Exponent
- Ljung-Box Test
- **Lo-Mackinley Test**

We calculate the Variance-Ratio test statistic, Ω_k, for a lag k, on the returns series:

$$\Omega_k = \frac{VR(k) - 1}{\sqrt{\mathrm{Var}(VR(k))}}, \Omega_k \sim Gaussian(0,1) \text{ asymptotically}$$

$$\text{Where:} VR(k) = \frac{\sigma_k^2}{k \cdot \sigma_1^2}, \text{and:} \sigma_k^2 = \frac{1}{k} \mathrm{Var}\left(\sum_{j=0}^{k-1} r_{t-j}\right)$$

Because the total return from time t to time k+t periods is simply the sum of intermediate, one-period returns:

$$(\text{K Period Return})_t = \sum_{j=0}^{k-1} r_{t-j}$$

So, this test compares the variance of k-th period returns to the variance of one-period returns. For an efficient market, we would expect: $\mathrm{Var}((\text{K Period Return})_t) = \mathrm{Var}\left(\sum_{j=0}^{k-1} r_{t-j}\right) = k \cdot \sigma^2$ as returns are iid with variance σ^2. So, we can use this test to identify if the variance of long-returns in each index corresponds with what the EMH predicts, and so test the hypothesis. We do so for multiple lags k : XXXX , XXXX , XXXX

Methods

- EMH Statistically
- Hurst Exponent
- Ljung-Box Test
- Lo-Mackinley Test
- **Dicky-Fuller Test**

For each series, we train the following regression:

$$r_t = \gamma \cdot p_{t-1} + \epsilon_t$$

And calculate:

$$\text{DF Test Statistic} = \frac{\hat{\gamma}}{\text{S.E.}\,(\hat{\gamma})}$$

We chose to implement this Dicky-Fuller test, and not a Dicky Fuller test with a constant term or t-polynomial term because our expected EMH relationship:

$$p_{t+1} = p_t + Z_t, Z_t \overset{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2)$$

Directly implies:

$$r_t = \gamma \cdot p_{t-1} + \epsilon_t, \text{where: } \epsilon \overset{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2) \text{ and: } \gamma = 0$$

So this regression exactly tests our expected relationship. For the same reason, I did not implement the (more popular) Augmented Dicky Fuller test, which trains:

$$r_t = \alpha + \beta t + \gamma \cdot p_{t-1} + \sum_{i=1}^{k} \delta_i r_{t-i} + \epsilon_t$$

The Box-Ljung-Pierce test is a substitute for capturing if there are significant auto correlations in returns, denoted δ_i above.

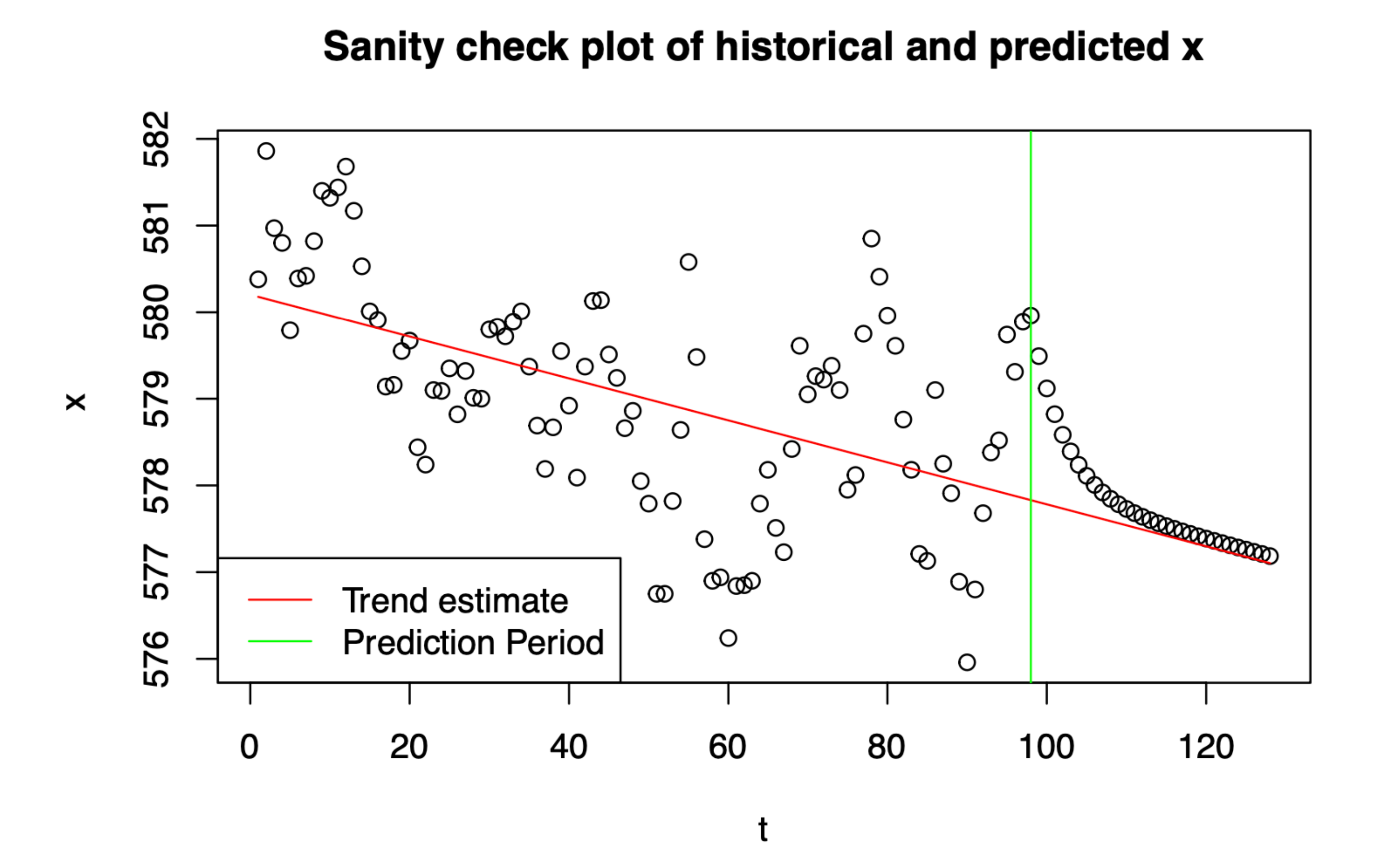
Because we expect $\gamma = 0$ under the EMH, we can use the DF test statistic as a measure of how much each index exhibits the unit-root relationship characteristic of an efficient market, and test against the critical values of the Dickey-Fuller distribution for this statistic.

Methods

- EMH Statistically
- Hurst Exponent
- Ljung-Box Test
- Lo-Mackinley Test
- Dicky-Fuller Test
- **Cross Validations**

Cross Validation 1. SARIMA models - Do we observe better predictability in less efficient markets?

Finally, we fit a SARIMA model to our returns series training data and compute cross validation scores on test data. This is specifically done by testing on 1 unseen data point at a time, as opposed to a series of unseen points. This decision is informed by how trading prediction algorithms are used in practice: just predicting stock movements one point out in time, and doing so continuously. As our series differ in length, we train on the first 90% of total observations for each series. We then compare the SARIMA model score to the score of a $E[r_{t+1}|\Phi_t] = 0$ model, which is what the EMH predicts.



Results

Methods Applied

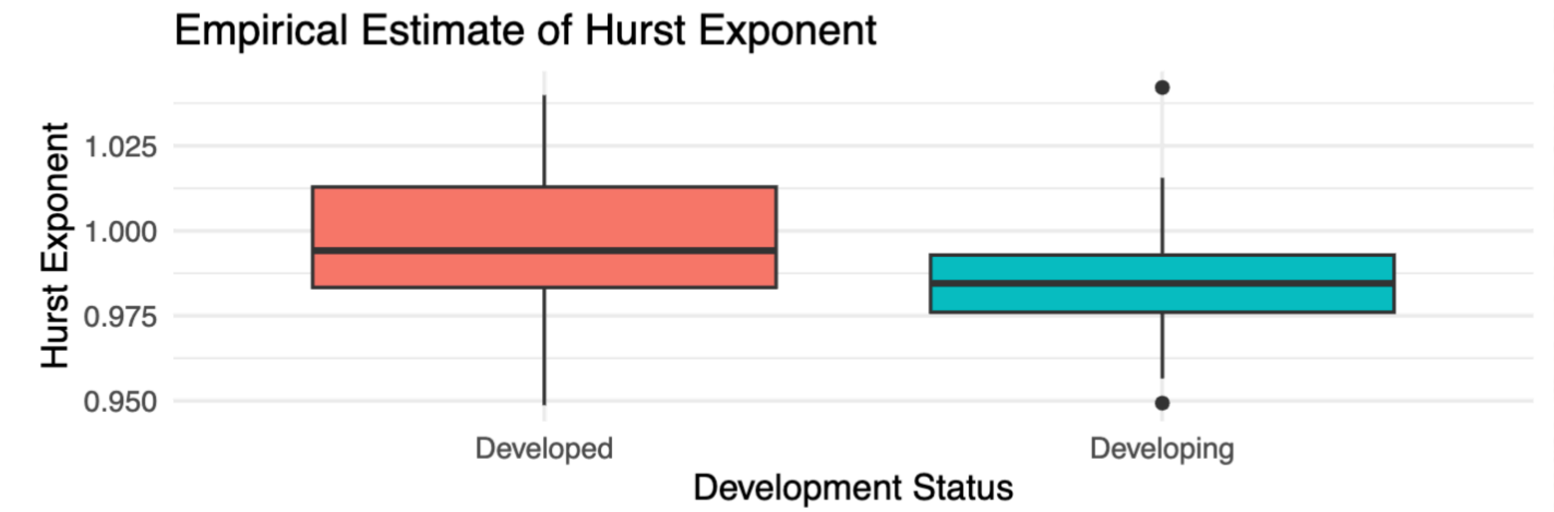
- Hurst Exponent
- Ljung-Box Test
- Lo-Mackinley Test
- Dicky-Fuller Test
- Cross Validations

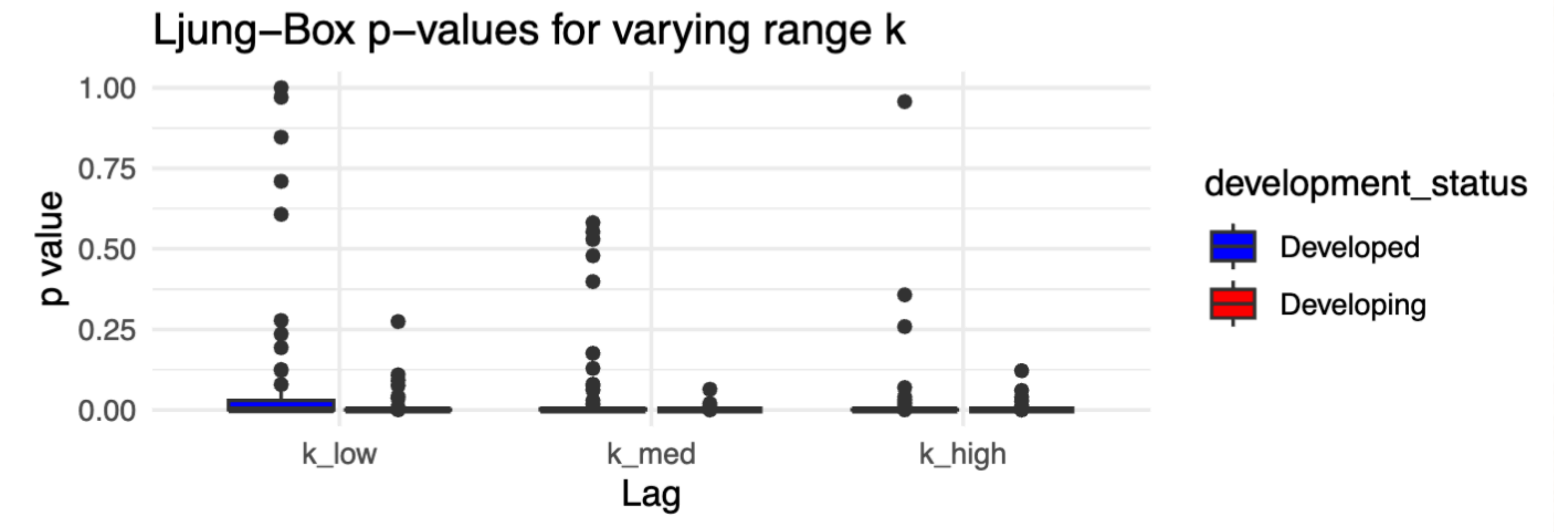
Datasets

- 19 Developing vs. 11 Developed
- Decade data: 1960s -> 2010-2024s

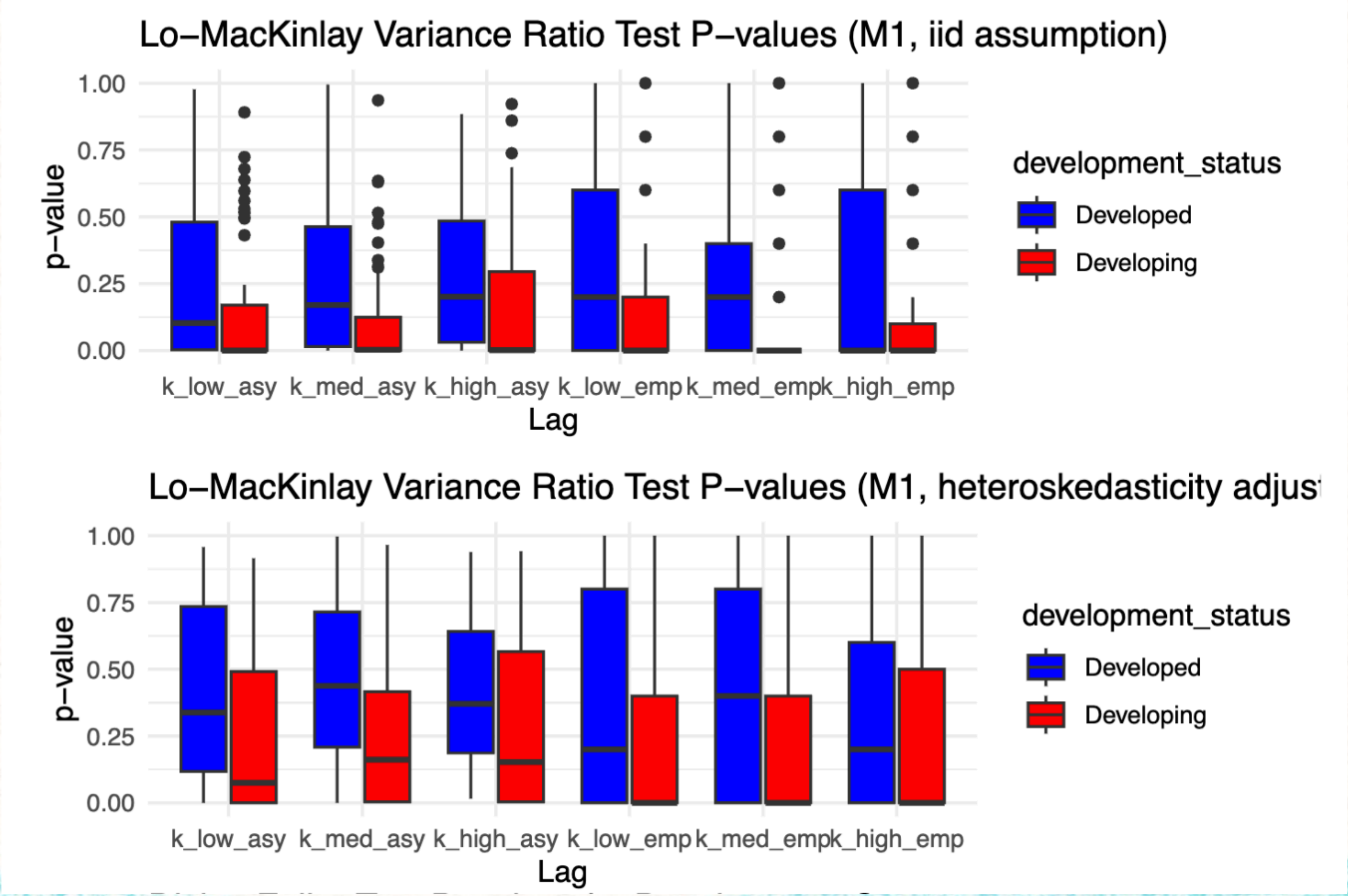
(Switch tabs to RStudio)

Results – Developed vs. Developing

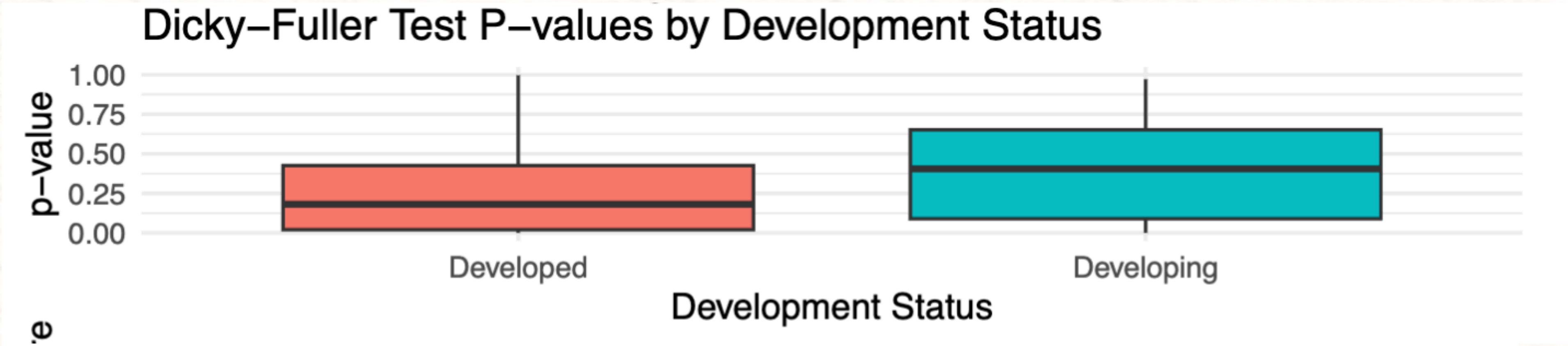


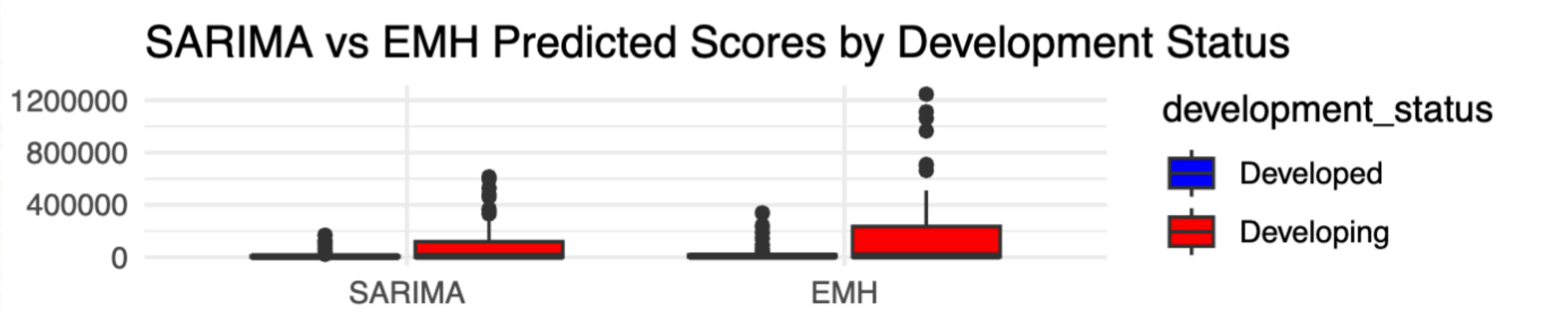


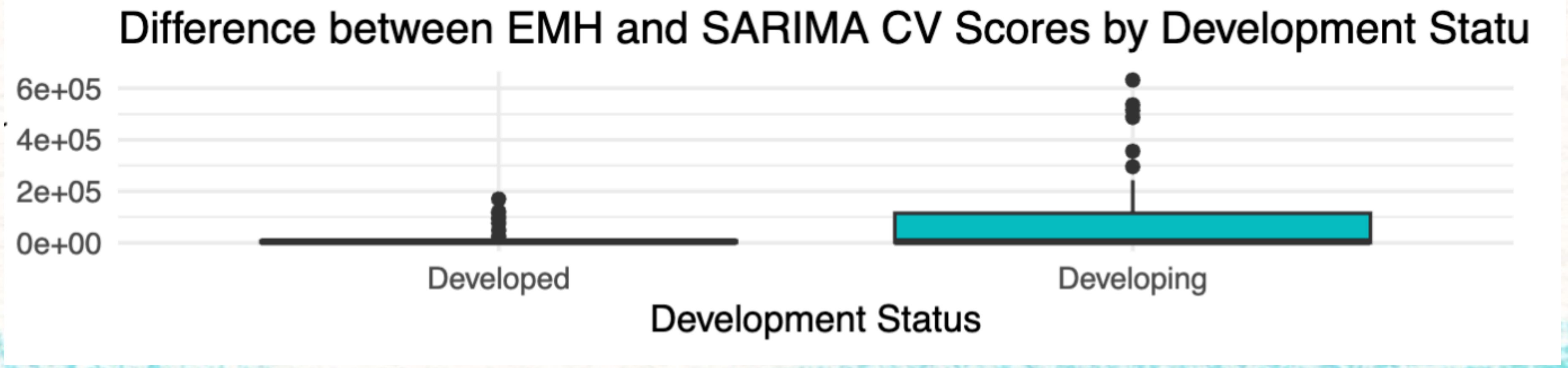
Results – Developed vs. Developing



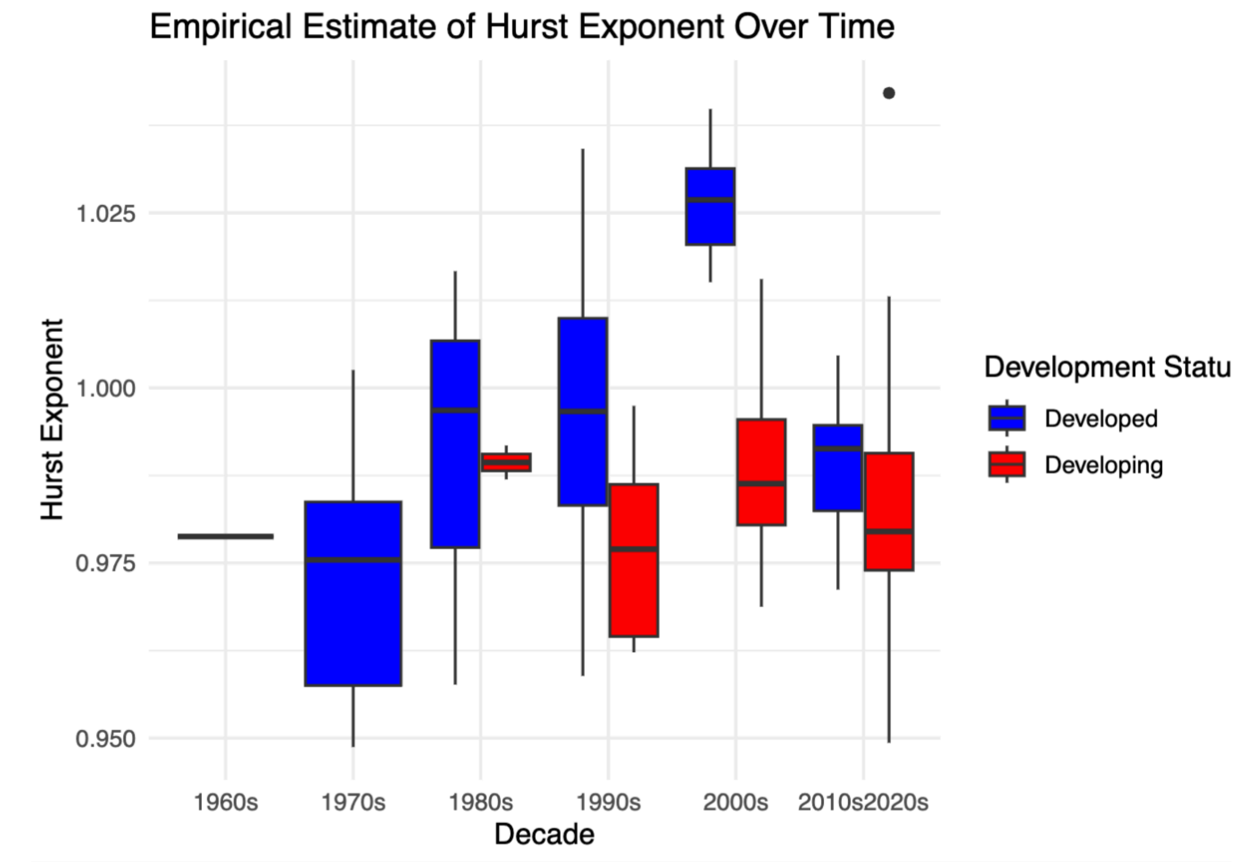
Results – Developed vs. Developing

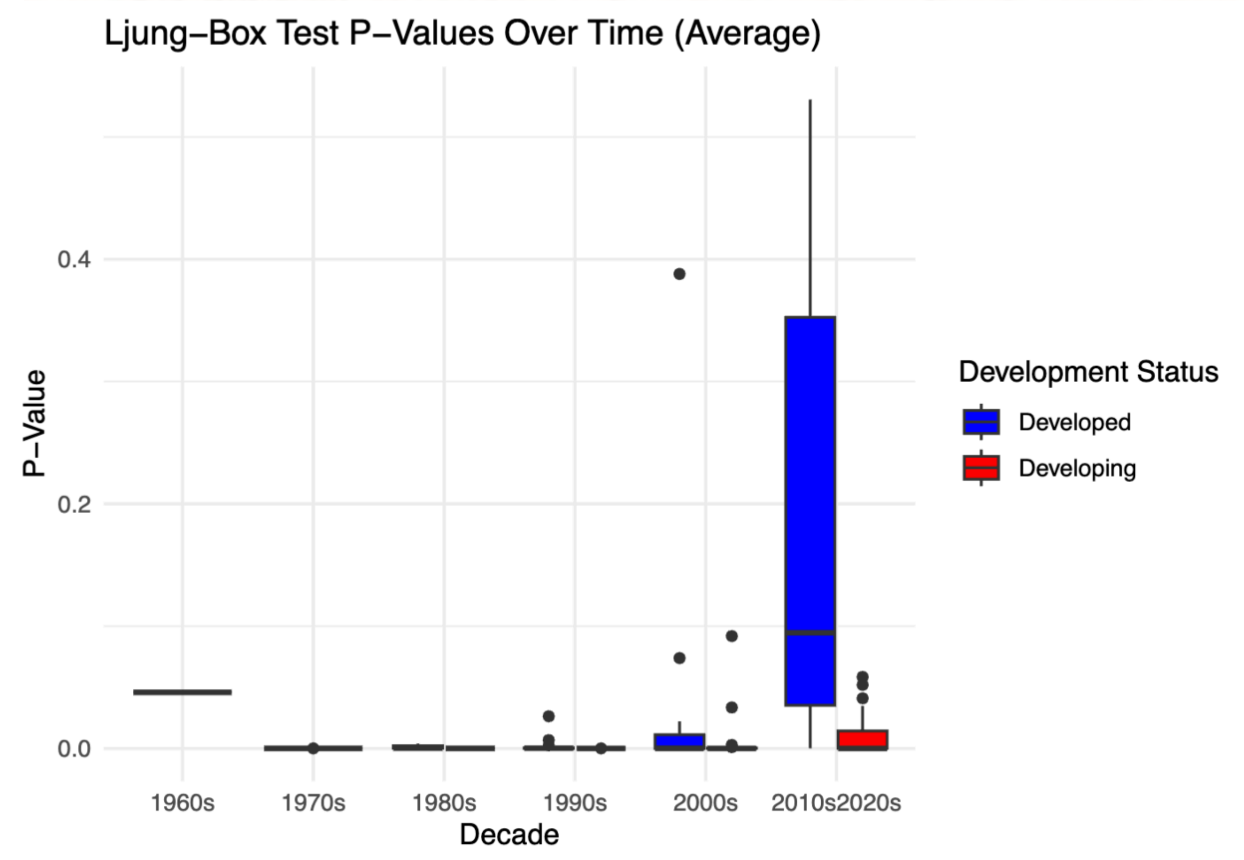






Results – Decades





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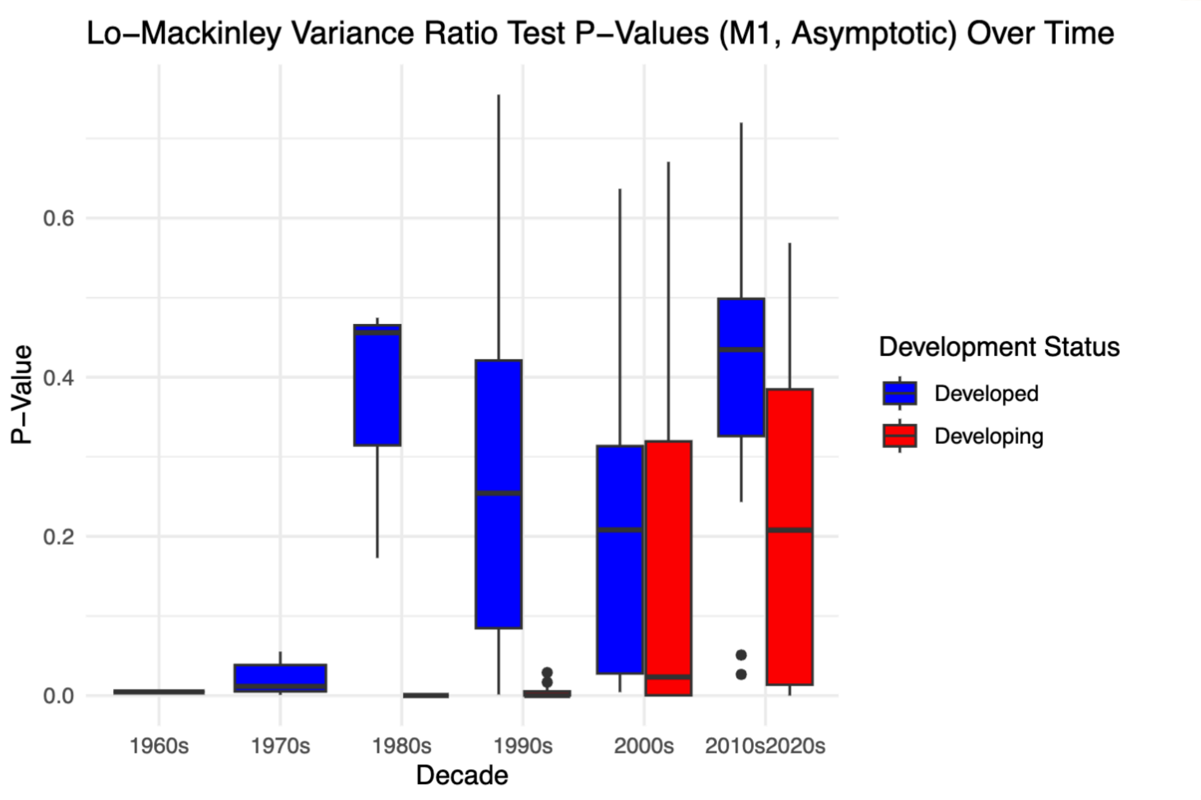

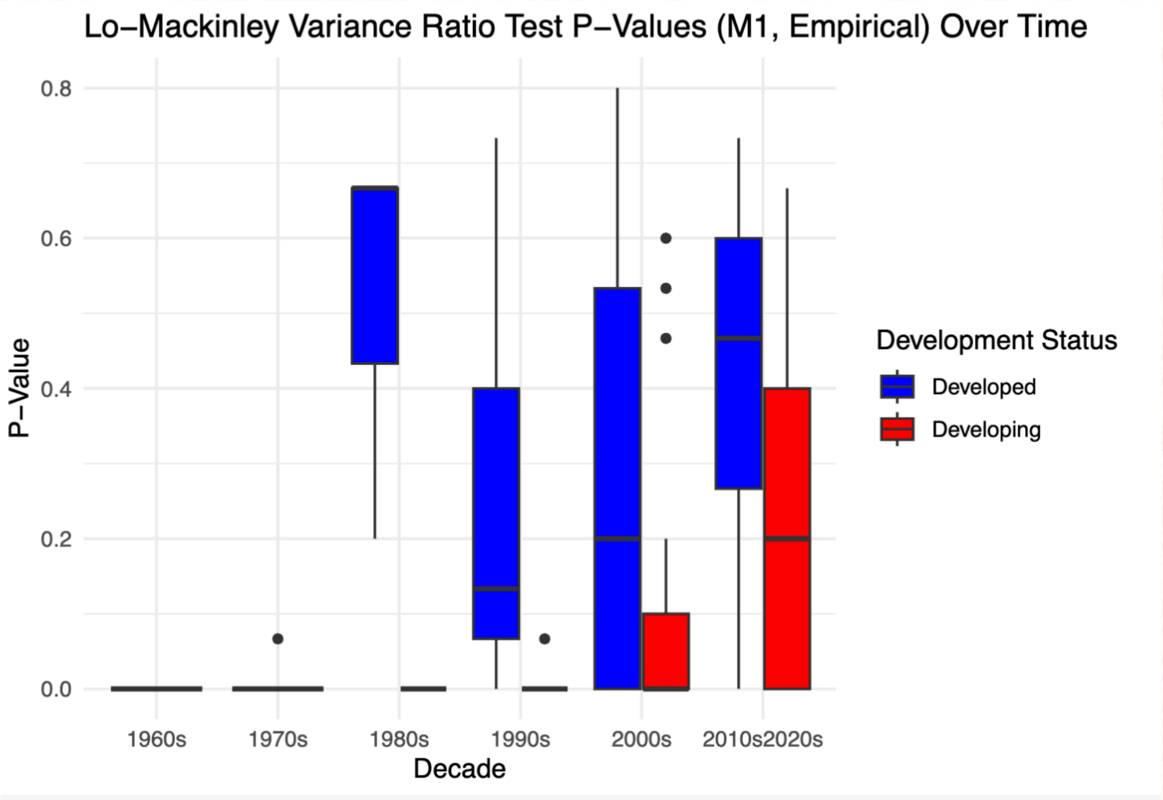

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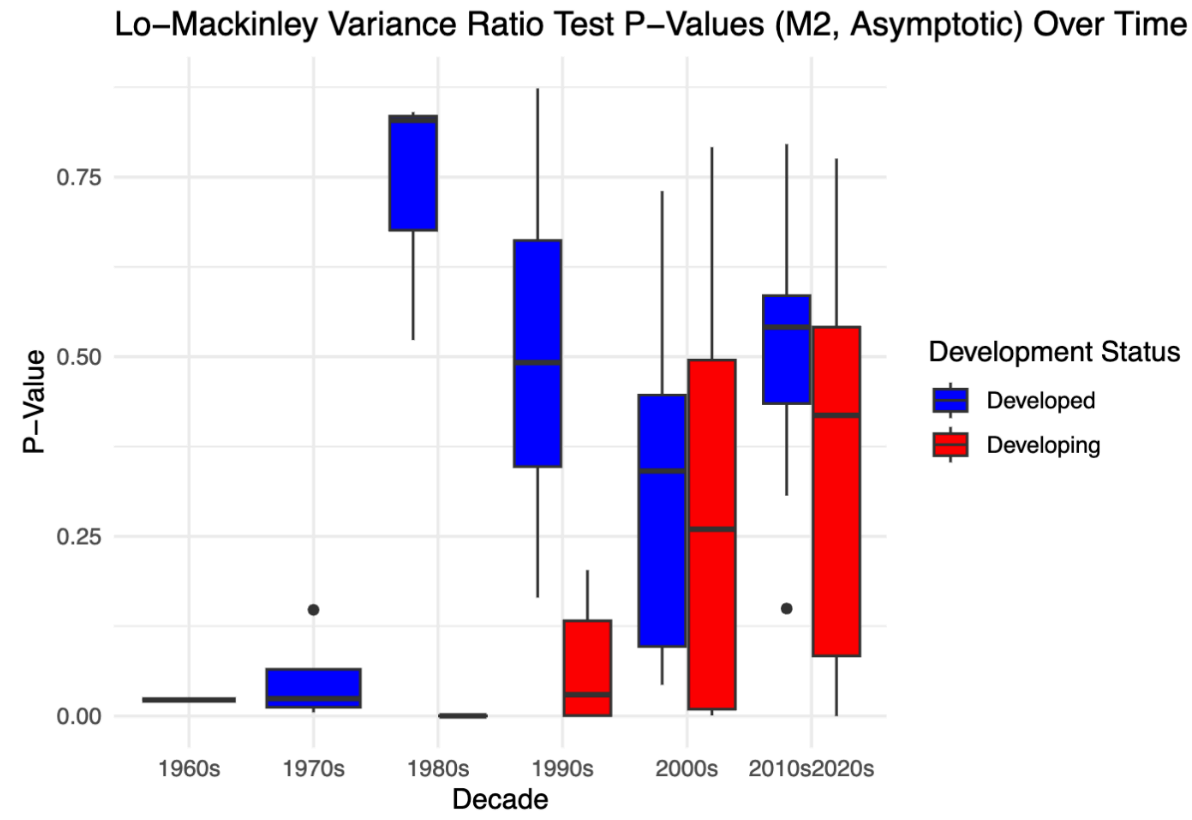

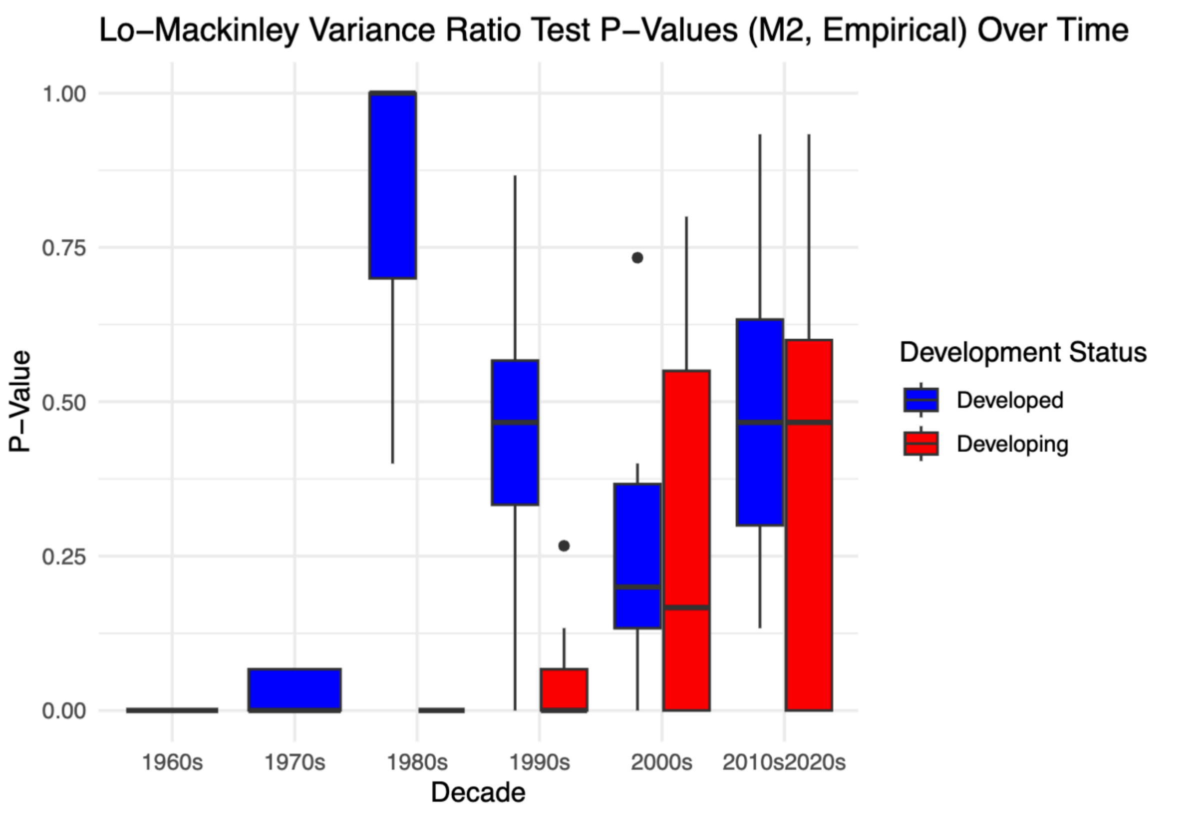

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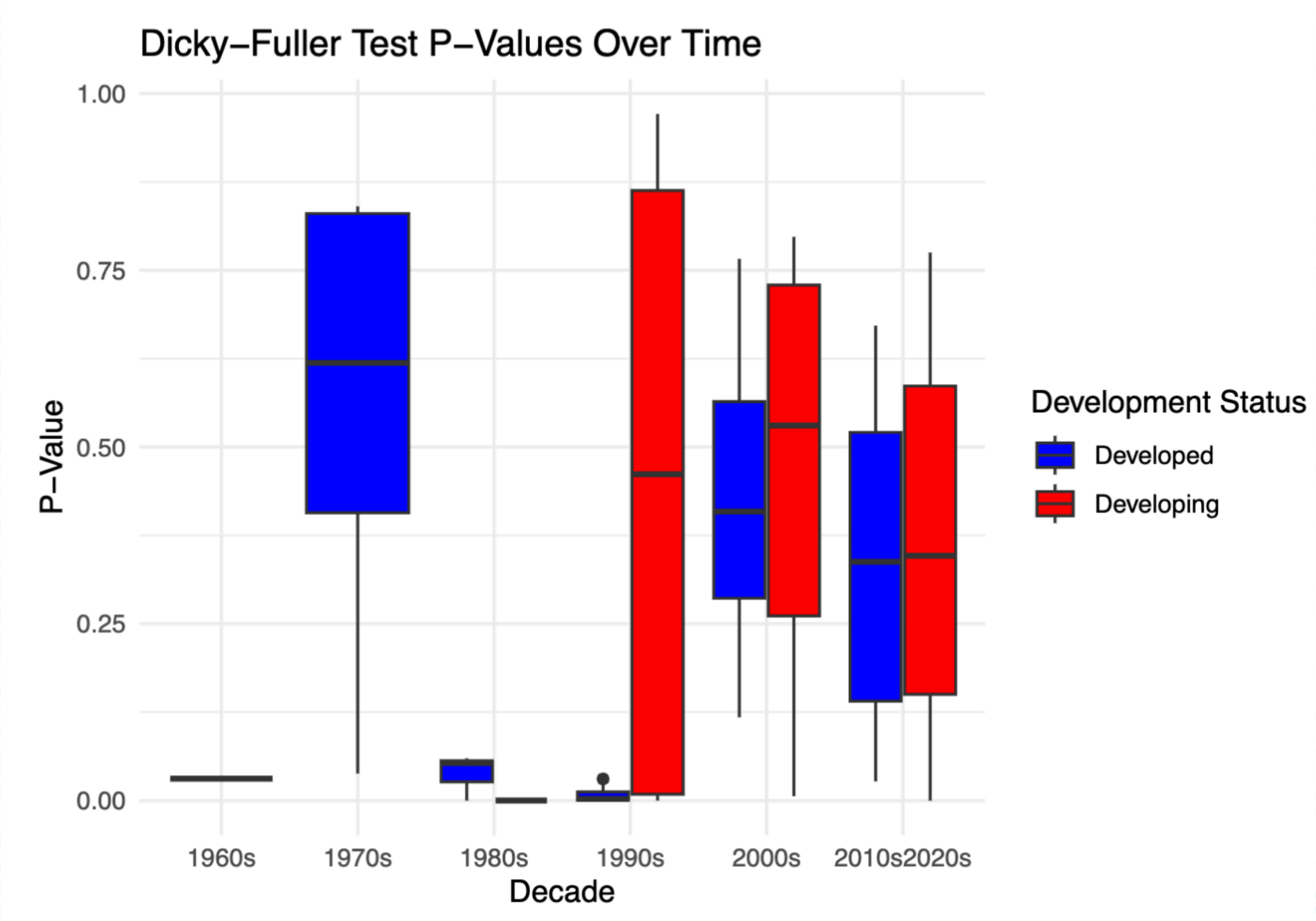

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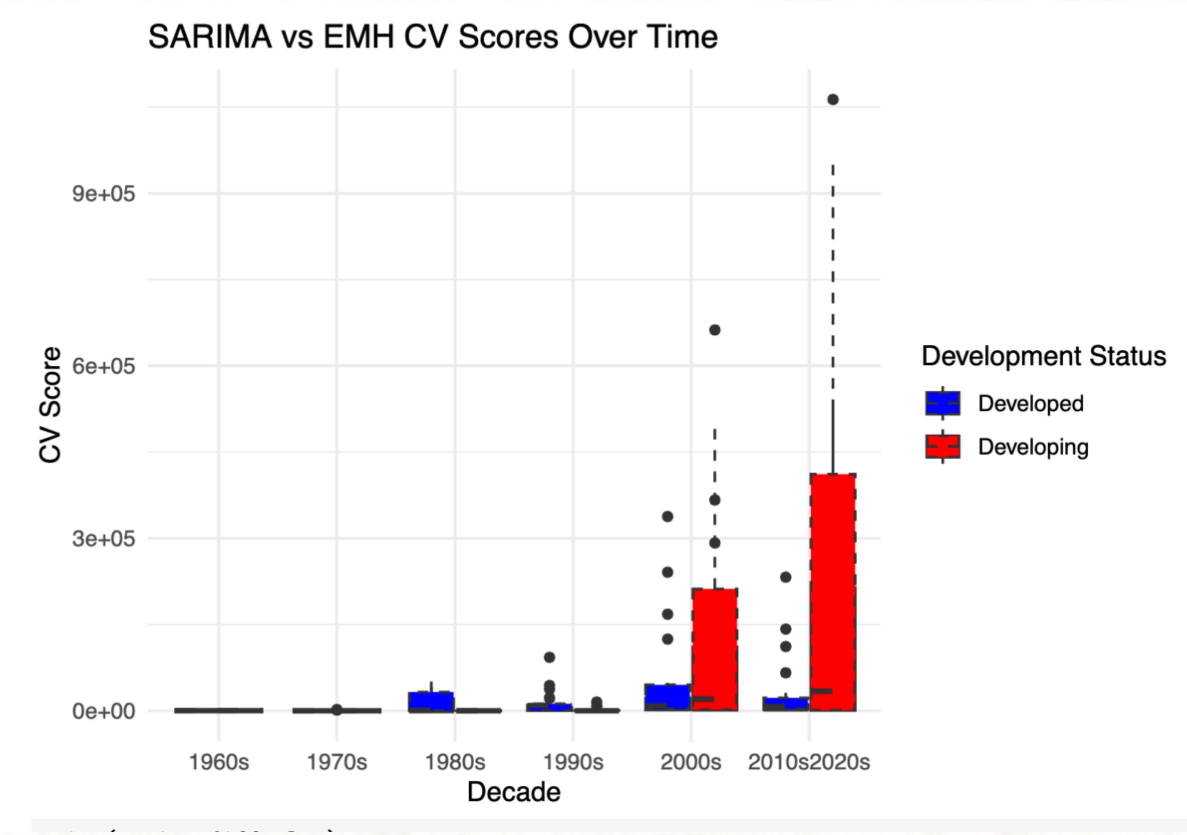

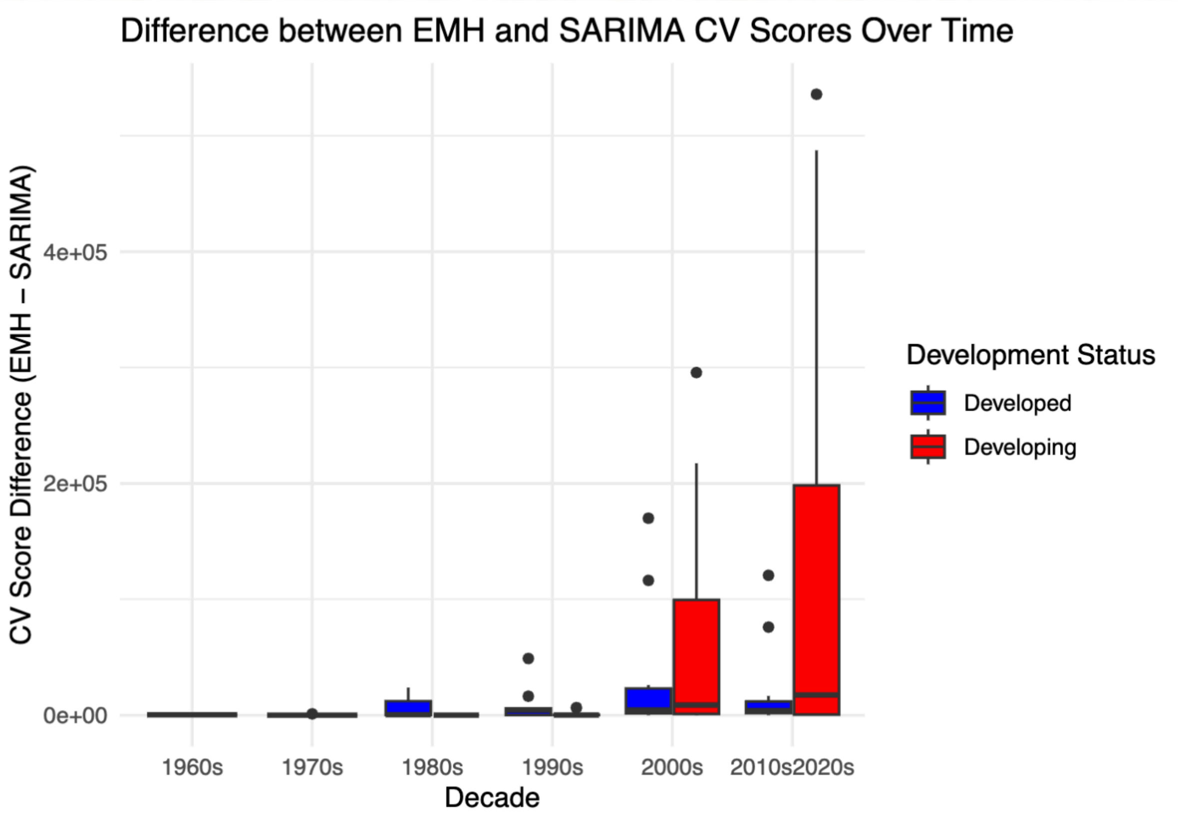

Discussion

<u>Methods</u>

- Weak EMH vs Heteroskedastic
- Power of DF test relative to Ljung-Box, Lo-Mackinley
- Interpretation of CV scores

<u>Limitations</u>

- No economic reasoning for *why* EMH failing
- No discussion of relative power of tests in this context
- Decade data too sparse to infer lack of time-relationship conclusively.

<u>Data</u>

- Reliability
- Weekend Adjustments
- Volume for Decade Analysis

<u>Takeaways</u>

- We **do** observe stronger evidence for EMH behavior in Developed markets.
- We observe even a simple SARIMA model has better OOS prediction than the EMH in developing markets
- Time relationship inconclusive.

Thank You!