- 1) Please forgive the formatting of the results page. Everything is still readable, but I couldn't present them in RMarkdown directly without exceeding the 8-page limit for the paper.
 - 2) Please email me if there are any problems accessing my code or data through the google drive: alexff@stanford.edu

Are Developed Markets More Efficient than Developing Markets? -Testing the Efficient Market Hypothesis through measures of Serial Dependence and Out-Of-Sample Prediction

Alexandre Farman-Farma

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Abstract

I perform a series of statistical tests and cross validations on stock index data from 11 developed and 19 developing countries to test the weak form of the Efficient Market Hypothesis (EMH) proposed by Eugene Fama. I find strong evidence that developing markets exhibit behavior that is less in keeping with this hypothesis than developed markets. I also find a SARIMA model predicts price movements better than the EMH in these markets. I find no strong evidence of a trend showing either class of markets becoming more or less efficient over time.

Introduction

The Efficient Market Hypothesis (EMH) is one of the most powerful ideas in Economics. Developed in large part by Euguene Fama in the 1970s, this theory states "[Any] market in which prices always fully reflect available information is called *efficient*", and posits that markets with freely determined prices are efficient. It's an astonishing proposal that all public information about an asset could be baked into a single, market-determined price, and that this process could update continuously and without planning.

Importantly, the EMH (even in it's weak form) can be tested. A consequence of the hypothesis is that changes in prices should be independent of their historical values, as they update only in response to new information, which cannot have also been reflected in historical prices. In theory then, it should be impossible to consistently predict price changes based off their previous values. "In other words, [in efficient markets] asset returns should exhibit no long-term memory of the price series that precedes it." At a high level:

EMH \implies Price Changes Exhibit No Serial Dependence \implies No Predictability from Price History

This paper puts this theory to the test, asking whether financial markets really are efficient through two related questions:

- 1. Are developed markets more efficient than developing markets?
- 2. Do we see evidence of these markets become more efficient over time?

Initially, the answer to both questions could conceivably be "yes", considering that market efficiency should increase with better public access to information and greater transaction volumes, both of which track with both development and technological advances over time (e.g. modern trading algorithms which process huge amounts of public data). These questions are not only important for the obvious reason of identifying

¹Fama, E. Efficient Capital Markets: A Review of Theory and Empirical Work. 1970.

²Hull, M & McGroarty, F. Do emerging markets become more efficient as they develop? Long memory persistence in equity indices. 2013.

market inefficiencies which might be profitably arbitraged. If a relationship between development and market efficiency is discovered, measures of market efficiency could be used as a powerful empirical tool to quantify how the emerging financial sectors of developing economies are evolving over time.

In this paper, I investigate these questions through two avenues:

- 1. **Statistical Measures of Serial Dependence** (Hurst Exponent, Ljung-Box, Lo-MacKinley Variance-Ratio, Dicky-Fuller).
- 2. Cross Validation Scores of Prediction Models (SARIMA Model)

I used data from the London Stock Exchange Group's DataStream database and find the following results:

- 1. Developing markets exhibit less efficient behavior than developed markets, across a wide range of measures.
- 2. The performance of developed markets themselves are inefficient, across some measures.
- 3. This translates to predictability. A simple SARIMA model predicts prices in developing markets better than the EMH.
- 4. No time trend for increasing efficiency was discovered. However inference here is limited by sparse data.

Related Work

Given the clear financial incentive, as well as the centrality of the Efficient Market Hypothesis to the field of Price Theory in economics, the question of exploring market efficiency is not a new one. Matthew Hull and Frank McGroarty's Do emerging markets become more efficient as they developed served as a key inspiration for this research. Our research builds on theirs by incorporating more tools to measure inefficiency than the Hurst Exponent, cross-validating basic predictions from SARIMA models and extending the range of analysis out from 1995-2011 to 1960-2024, where possible. M. Bęben and A. Orłowski's paper applying the Hurst Exponent to developed vs. developing markets was also an inspiration. Other work in the field of economics helps to build the theory for why the Efficient Market Hypothesis breaks down in certain contexts. For proposed economic explanations of why we observe the types inefficiencies found in this analysis, see Grossman & Stiglitz's On the Impossibility of Informationally Efficient Markets and Dacorogna et al.'s An Introduction to High-Frequency Finance. Finance. Finance Finance, the foundational theory for this paper, but also guides their readers towards how to test hypotheses like this one using their theory.

Data

I collected data from 30 stock market index funds (11 developed, 29 developing) to use as indicators of stock market performance in different regions. This data was collected using DataStream access to the London Stock Exchange Group's database, courtesy of terminals at the Stanford GSB's Bass Library. Classification of markets into "Developed" or "Developing" was conducted in accordance with the March 2024 FTSE Equity Country Classification Review.⁹ This data is summarized in the appendix.

There was relatively little need to pre-process this data. After each series begins, there are no missing values. Some background pre-processing occurred between importing all series starting from 01/01/1960 from DataStream, and adjusting the start of the time series to correspond with when these indexes began. (Only the Nikkei 225 was active at 01/01/1960.)

³Hull, M & McGroarty, F. Do emerging markets become more efficient as they develop? Long memory persistence in equity indices. 2013.

⁴M, Beben & A, Orłowski. correlations in financial time series: established versus emerging markets. 2000.

⁵S, Grossman & J. Stiglitz. On the Impossibility of Informationally Efficient Markets. 1980.

⁶M, Dacorogna et al. An Introduction to High-Frequency Finance. 2001

⁷Fama, E. Efficient Capital Markets: A Review of Theory and Empirical Work. 1970.

⁸Lo, A & MacKinley, A. Stock Prices do not Follow Random Walks: Evidence from a Simple Specification Test. 1951.

⁹FTSE Equity Country Classification Interim Announcement March 2024. 2024.

Methods

I propose a two pronged approach to quantifying to what extent a market is efficient. First, calculate statistical measures of serial dependence for each market. Second, cross validate prediction models on out-of-sample data from each market, and compare scores. In this section, I define the version of the EMH that I will test and I demonstrate why my statistical and cross-validation approaches test this hypothesis.

Hypothesis Definition - What does is mean to test the Efficient Market Hypothesis?

The Efficient Market Hypothesis, actually reflects a family of hypotheses. These are:

Weak Form EMH: Asset prices p_t fully reflect all historical trading information, including previous prices.

Semi-Strong Form EMH: Asset prices p_t fully reflect all publicly available data, including previous prices.

Strong Form EMH: Asset prices p_t fully reflect all publicly and privately available data, including previous prices.

So, while I test the Weak Form EMH because it relates most directly to time-series price data, in testing the Weak Form EMH I implicitly also test the Semi-Strong Form EMH and Strong Form EMH.¹⁰

Most famously, the EMH states:

$$E[r_{t+1}|\Phi_t] = 0$$

where $r_{t+1} = p_{t+1} - p_t$ is absolute returns at time t and $E[x|\Phi_t]$ is the conditional expectation of a return on the basis of all historical trading information at time t (under weak form EMH).

As such we have:

$$E[p_{t+1}|\Phi_t] = p_t$$

We all know, however, that prices do not remain steady for ever. Under the EMH, price changes are driven by new information shocks which we model through a 0-mean noise term with constant variance Z_t . In general, proving properties of the distribution of how any and all new information shocks affect prices is a tricky endeavor. The weak form EMH process specified by Fama in his original paper is that $Z_t \sim Gaussian(0, \sigma^2)$ with constant variance, and iid. That this should be the case is not entirely obvious: for example, markets are commonly known to enter periods of higher and lower volatility. Indeed there is an exciting literature testing the constant variance assumption, and the role of derivative markets in stabilizing this variance. Nevertheless, taking the weak-form EMH, as described by Fama, at face value we have the following unit-root relationship to test:

$$p_{t+1} = p_t + Z_t, Z_t \overset{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2)$$

Statistical Technique 1. Hurst Exponent - Measuring Long Term $Memory^{14}$ 15

We calculate the Hurst Exponent, H, defined as:

$$\mathbb{E}\left[\frac{R(n)}{S(n)}\right] = Cn^H \text{ as } n \to \infty,$$

Where: R(n) is the range of the first n cumulative deviations from the mean.

S(n) is the standard deviation of the series over time n.

n is the number of observations in the time series.

C is a constant.

¹⁰Fama, E. "Efficient Capital Markets". 1970.

¹¹A full discussion of distributional assumptions can be found in Efficient Capital Markets, section 4.

¹²Bentes, S et al. Long memory and volatility clustering. 2008.

¹³Singh, S & Tripathi, L. The Impact of Derivatives on Stock Market Volatility. 2016.

¹⁴Technique originally developed in: Hurst, H. Long-Term Storage Capacity of Resevoirs. 1951.

¹⁵Useful description of relation of Hurst to financial markets in: Kroha, P & Škoula, M. Hurst Exponent and Trading Signals Derived from Market Time Series. 2018.

A H-value = 0.5 indicates a unit root process. H in [0.5,1] indicates a process with long term positive autocorrelation (sometimes called "momentum"). H in [0,0.5] indicates a process with long term mean reversion.

As such, the extent to which H differs from 0.5 evidences against the unit root behavior predicted for efficient markets. Unlike the unit-root tests discussed below, the Hurst exponent is a statistic which is less sensitive to short term noise and focuses more on long-range dependencies. As such, it's a welcome tool for this analysis, despite the fact that it lacks a significance test. In keeping with simulations conducted by Bo Qian and Khaled Rasheed at the University of Georgia, we use [0.45,0.65] as a good proxy for a 95% confidence interval.¹⁶

Statistical Technique 2. Ljung-Box-Pierce Test - Testing Multiple Autocorrelation

We calculate the Ljung-Box-Pierce test statistic, Q, defined as:

$$Q = n(n+2) \sum_{i=1}^{k} \frac{\rho_k^2}{n-i}, Q \sim \chi_{k-1}^2$$
 asymptotically

Where: ρ_i is the sample autocorrelation for lag i

k is the range of lags we sum over. We experiment over a range of k: 5, 20, 50.

Because we are testing if an AR(1) model generated this data, we have k-1 df. ¹⁷

If we apply the Ljung-Box-Pierce test to the differenced time series $r_t = p_t - p_{t-1}$, which is just the returns series, we expect to observe that there is no significant autocorrelation in returns as returns should be Gaussian Noise under the Weak form EMH. $(r_t = p_t - p_{t-1} = p_{t-1} + Z_t - p_{t-1} = Z_t)$ As such, the p-value of this test can be used as a tool to quantify to what degree each index exhibits autocorrelation in returns, which violates the weak form EMH (where returns should be iid).

Statistical Technique 3. Lo MacKinley Test - Testing Random Walk via Variance of Multi-Period Returns

We calculate the Variance-Ratio test statistic, Ω_k , for a lag k, on the returns series:

$$\Omega_k = \frac{VR(k) - 1}{\sqrt{\text{Var}(VR(k))}}, \Omega_k \sim Gaussian(0, 1) \text{ asymptotically }$$

$$\text{Where:} VR(k) = \frac{\sigma_k^2}{k \cdot \sigma_1^2}, \text{and:} \sigma_k^2 = \frac{1}{k} \text{Var}\left(\sum_{j=0}^{k-1} r_{t-j}\right)$$

Because the total return from time t to time k+t periods is simply the sum of intermediate, one-period returns:

$$\left(\text{K Period Return} \right)_t = \sum_{i=0}^{k-1} r_{t-j}$$

So, this test compares the variance of k-th period returns to the variance of one-period returns. For an efficient market, we would expect: $\operatorname{Var}((K \operatorname{Period} \operatorname{Return})_t) = \operatorname{Var}\left(\sum_{j=0}^{k-1} r_{t-j}\right) = k \cdot \sigma^2$ as returns are iid with variance σ^2 . So, we can use this test to identify if the variance of long-returns in each index corresponds

¹⁶Qian, B & Rasheed, K. Hurst exponent and financial market predictability. 2004.

 $^{^{17}}$ See lecture notes Def. 5.55.

¹⁸Technique originally developed in: Lo, A & MacKinley, A. Stock Prices do not Follow Random Walks: Evidence from a Simple Specification Test. 1951.

¹⁹Useful overview of Variance Ratio Tests and implementations: Charles, A & Darné, O. Variance ratio tests of random walk: An overview. 1988.

with what the EMH predicts, and so test the hypothesis. We apply this test using both an asymptotic and empirically adjusted R implementation. We also do so for both the M1 statistic, where iid noise is assumed (which tracks best with the weak form EMH) and the M2 statistic, where some heteroskedasticity is allowed for (which is more permissive). We do so for multiple lags k : 5, 20, 50.

Statistical Technique 4. Dicky-Fuller Test - Testing Unit Root Relationship²⁰

For each series, we train the following regression:

$$r_t = \gamma \cdot p_{t-1} + \epsilon_t$$

And calculate:

DF Test Statistic =
$$\frac{\hat{\gamma}}{\text{S.E.}(\hat{\gamma})}$$

We chose to implement this Dicky-Fuller test, and not a Dicky Fuller test with a constant term or t-polynomial term because our expected EMH relationship:

$$p_{t+1} = p_t + Z_t, Z_t \stackrel{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2)$$

Directly implies:

$$r_t = \gamma \cdot p_{t-1} + \epsilon_t$$
, where: $\epsilon \stackrel{\text{i.i.d.}}{\sim} Gaussian(0, \sigma^2)$ and: $\gamma = 0$

So this regression exactly tests our expected relationship. For the same reason, I did not implement the (more popular) Augmented Dicky Fuller test, which trains:

$$r_t = \alpha + \beta t + \gamma \cdot p_{t-1} + \sum_{i=1}^k \delta_i r_{t-i} + \epsilon_t$$

The Ljung-Box-Pierce test is a substitute for capturing if there are significant auto correlations in returns, denoted δ_i above.

Because we expect $\gamma = 0$ under the EMH, we can use the DF test statistic as a measure of how much each index exhibits the unit-root relationship characteristic of an efficient market, and test against the critical values of the Dickey-Fuller distribution for this statistic.

Cross Validation 1. SARIMA models - Do we observe better predictability in less efficient markets?

Finally, we fit a SARIMA model to our returns series training data and compute cross validation scores on test data. This is specifically done by testing on 1 unseen data point at a time, as opposed to a series of unseen points. This decision is informed by how trading prediction algorithms are used in practice: just predicting stock movements one point out in time, and doing so continuously. As our series differ in length, we train on the first 90% of total observations for each series. We then compare the SARIMA model score to the score of a $E[r_{t+1}|\Phi_t]=0$ model, which is what the EMH predicts.

While looking at the magnitude of the EMH cross validation scores alone is a measure of how much a market's behaviour is in keeping with the EMH, we train the SARIMA model and compare scores to explore whether any inefficiencies discovered translate into better predictability with a simple time series model.

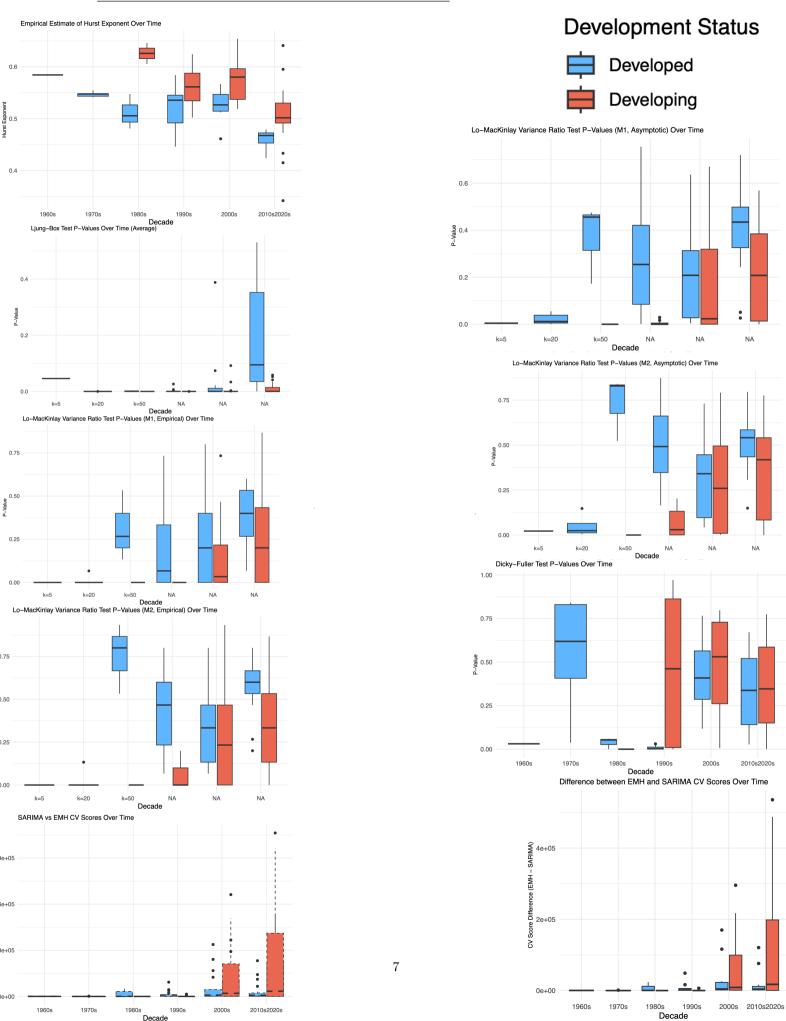
Results

Developed vs Developing Markets, aggregated:

²⁰Technique originally developed in: Dicky, D & Fuller, W. Distribution of the Estimators for Autoregressive Time Series With a Unit Root. 1979.

Empirical Estimate of Hurst Exponent Development Status Developed Developing Ljung-Box p-values Developed Developing **Development Status** Dicky-Fuller Test P-values 1.00 p value 0.50 0.50 0.25 0.00 0.00 k=50 Developing k=5 k=20 Development Status Lo-MacKinley p-values (iid assumption) Lo-MacKinley p-values (w. heteroskedasticity) 1.00 1.00 0.75 p-value 0.50 p-value 05.0 0.25 0.25 0.00 k=5_asy k=20_asy k=50_asy k=20_emp k=50_emp k=20 asv k=5_emp k=20_emp k=50_emp Lag SARIMA vs EMH Predicted Scores EMH - SARIMA CV score 1200000 6e+05 800000 4e+05 400000 6 0e+00 SARIMA EMH Development Status Model

Developed vs Developing Markets, over time:



Conclusions

1. We observe strong evidence that developing markets exhibit less efficient behavior than developed markets.

We observe that the majority of developing markets studied have Hurst exponents greater than 0.5, as opposed to developed markets whose Hurst exponents cluster around 0.5. This indicates a higher degree of momentum, over long periods, is present in developing markets than in developed ones, although almost all Hurst exponents for both series fall within the 95% confidence interval proposed by Qian and Rasheed [0.45, 0.65]. ²¹

The difference in ranges for the p-values of the Ljung-Box test and the variations of the Lo-MacKinley Variance Ratio (iid vs. heteroskedastic assumption, asymptotic vs. empirically adjusted p-values) test similarly confirms the hypothesis that developing markets exhibit less efficient behavior than developed markets. We also observe that the cross-validation score for EMH predictions in developing markets is significantly worse than in developed markets.

The only evidence contrary to our hypothesis comes from the Dicky-Fuller test, wherein developing markets have less extreme p-values when tested for unit root behavior. However, it is worth noting that neither developed nor developing markets consistently fail the Dicky-Fuller test. One possible interpretation for this is that the relative inefficiency of developing markets comes not from strong violation of the unit-root hypothesis over range-one, but via long range autocorrelation, as detected by the Hurst Exponent, and the increasingly extreme p-values for the Box-Ljung test and Lo-Mackinley test variations as k, the lag range explored, increases.

2. Developed markets themselves sometimes exhibit inefficient behavior.

While the Hurst exponent, Lo-MacKinley Variance Ratio test variations, Dicky Fuller test, and (with some outliers) cross-validations for developed markets all exhibit behavior predicted by the EMH, there are some measures in which even developing markets fail. This is most notably the case in the Ljung-Box test, as we test over progressively greater lags. This evidences that there is some joint-autocorrelation in prices in developed markets, over longer ranges. And indeed, in some instances this is reflected in a SARIMA model that fits far better to new data than the EMH prediction. In the main, developed markets largely exhibit efficient behavior, but we identify the question of inefficient behavior in joint autocorrelations for future study.

3. A simple SARIMA model predicts prices in developing markets better than the EMH.

Having demonstrated that the behavior of developing markets is less efficient across these metrics, we then show that even a simple SARIMA model performs far better than the EMH in developing markets. For practical applications, this is among the most significant findings of our paper as it translates theoretical measures of market inefficiency to a more powerful predictive rule for how prices evolve in these markets. The fact that the EMH fails in these conditions doesn't imply a better prediction rule of this sort must be found, for example, the EMH could fail only in its assumption of homoskedastic variance; as such the fact that the SARIMA model performs so well is a separate discovery of our paper, and is a promising sign that more sophisticated models could potentially model prices in developing markets better.

4. No time trend for increasing efficiency was discovered.

Looking across the decade plots, no clear relationship between time and market efficiency is visible across any metric. However, does not necessarily indicate that there is no true relationship to be found, as our decade data pre-2000, especially for developing markets is very sparce, as many of these indices did not yet exist. This is a key limitation with the data collected for this paper, and warrants future research.

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²¹Qian, B & Rasheed, K. Hurst exponent and financial market predictability. 2004.

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Appendix 1. Data

Table 1: Time Series Index Data

Region	Index	Start	End	n	Frequency	Source	Classification
Australia	S&P/ASX 200	2000-11-16	2024-05-31	6142	Daily	LSEG	Developed
Chile	IPSA CLP	1990-01-02	2024-05-31	8979	Daily	LSEG	Developing
China	SE COMP	1990-12-19	2024-05-31	8728	Daily	LSEG	Developing
Columbia	COLCAP	2011-07-06	2024-05-31	3368	Daily	LSEG	Developing
Egypt	EGX 30	1998-01-01	2024-05-31	6892	Daily	LSEG	Developing
France	CAC 40	1987-07-09	2024-05-31	9627	Daily	LSEG	Developed
Germany	DAX	1964-12-31	2024-05-31	15502	Daily	LSEG	Developed
Hungary	BUX	1991-01-02	2024-05-31	8718	Daily	LSEG	Developing
India	BSE	2016-10-18	2024-05-31	1989	Daily	LSEG	Developing
Indonesia	IDX COMP	1983-04-04	2024-05-31	10740	Daily	LSEG	Developing
Italy	FTSE MIB	1997-12-31	2024-05-31	6893	Daily	LSEG	Developed
Japan	NIKKEI 225	1960-01-01	2024-05-31	16806	Daily	LSEG	Developed
Malaysia	FTSE KLCI	1980-01-02	2024-05-31	11588	Daily	LSEG	Developing
Mexico	MEXICO IPC	1988-01-04	2024-05-31	9500	Daily	LSEG	Developing
Netherlands	AEX	1983-01-03	2024-05-31	10805	Daily	LSEG	Developed
Nigeria	NIGERIA ALL SHARE	2000-01-14	2024-05-31	6361	Daily	LSEG	Developing
Pakistan	KARACHI SE 100	1988-12-30	2024-05-31	9241	Daily	LSEG	Developing
Peru	IGBVL	1991-01-02	2024-05-31	8718	Daily	LSEG	Developing
Poland	WARSAW GENERAL	1991-04-16	2024-05-31	8644	Daily	LSEG	Developing
Russia	MOEX RUSSIA	1997-09-22	2024-05-31	6965	Daily	LSEG	Developing

Saudi Arabia	TASI	1998-10-19	2024-05-31	6685	Daily	LSEG	Developing
South Africa	FTSE/JSE ALL SHARE	1995-06-30	2024-05-31	7546	Daily	LSEG	Developing
Spain	IBEX 35	1987-01-05	2024-05-31	9760	Daily	LSEG	Developed
Sweden	OMX STOCK- HOLM 30	1986-01-02	2024-05-31	10022	Daily	LSEG	Developed
Switzerland	SMI	1988-06-30	2024-05-31	9372	Daily	LSEG	Developed
Thailand	BANGKOK S.E.T	1975-04-30	2024-05-31	12808	Daily	LSEG	Developing
Turkey	BIST NATIONAL 100	1988-01-04	2024-05-31	9500	Daily	LSEG	Developing
United Kingdom	FTSE 100	1983-12-30	2024-05-31	10546	Daily	LSEG	Developed
United States	S&P 500 COMP	1963-12-31	2024-05-31	15764	Daily	LSEG	Developed
Vietnam	HOCHIMINH SE	2000-07-28	2024-05-31	6221	Daily	LSEG	Developing

Appendix 2. Code

Code:

https://drive.google.com/file/d/1N5Kgz3H80ajWKgAhdcstBGio2rVM-6rx/view?usp=sharing

Data:

https://docs.google.com/spreadsheets/d/1LhX9fzRlxhJq7ey6qXtZq9F4sEUCAhgS/edit?usp=sharing&ouid=106419429795166406559&rtpof=true&sd=true