Validation of Rocket code

Alex Fausett

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1 Introduction

This short document highlights the key steps in validating rocket code written in c++ to simulate thrust levels of hybrid rockets.

2 First Validation Method AMROC Aquilla Launch Service

My first approach to validate the code involved taking the known performance values from the U-75 hybrid rocket motor on the AMROC Aquillia launch service vehicle shown in Fig [1].

> The fourth stage U-75 hybrid motor has the following performance specifications: an average vacuum thrust of 9,000 lbs, a vacuum total impulse of 7.7 x 10^5 lbs-sec, a vacuum specific impulse of 288 secs, a nominal burn time of 85 secs, an average chamber pressure of 349 psia, a nozzle expansion ratio of 75:1, a propellant weight of 2,670 lbs, a diameter of 22 inches, and a motor length of 69 inches.

Figure 1: Known performance values Page 8 The commercial Aquila Launch Vehicle

this paper [1] was written in imperial units and so the first step was to convert from Imperial SI units to metric SI units. This can be seen below in table[1] and in the code as a commented section at the beginning of the main function as well as the conversion of these into variables followed by the call for the thrust equation.

Table 1: Rocket Motor Parameters

Parameter	**Value**
Thrust	40 kN
Total Impulse	288 secs
Chamber Pressure	2.406e6 pascals
Expansion Ratio	75
Diameter	0.5588 m
Motor Length	1000 mm

The first validation code "HybridRocketCodeAquillaValidation" runs the simulation code with the expected and initial parameters stated above in table[1] and some additional parameter values shown as variables such as Mol, Gammas, OFtemps, and oxidizer flow rate.

```
int main() {
    ///AMROC values to aim for thrust = 9000 lbs or 40033.99 N, vaccum Isp = 288 secs
    /// burn time = 85s, chamber pressure = 349 psia or 2.406e6 pascals, Expansion = 75, propellent weight = 2670 lbs or 1211 kg
    /// diameter = 22 inches or 0.5588 m, motor length = 69 inches or 1.756 m

    // amroc validation numbers
    double Pc = 2.406e6; // pascals N/m^2 average chamber pressure (conversion 2.406e+6 pascals from 349 psia)
    double l = 1.756;   // length of rocket in meters currently 1.756 m, 69 inches to validate with amroc rocket
    double RI = 0.5588/2;  // amroc validation number 22 inches or 0.5588 m diameter so half for radius
    double E = 75; // expansion ratio
    double gammas = 1.2;
    double Mol = 23;
    double OFtemps = 2800;
    double OxidizerFlowRate = 13.665; //13.665 to 14.25 kg/sec as per belows calculations

    double thrust = hybrid_rocket_thrust(RI, l, T: OFtemps, Mol, kappa: gammas, Pc, E, mDot_o: OxidizerFlowRate);
    printf("Thrust : %f \n", thrust);
```

Figure 2: Expected values and parameters in main

It's important to note how some of these variables were defined. The molar ratio, OFtemps, and Gammas are chosen from NASA CEA output. Solving for the mass flow rate of the oxidizer and velocity is as follows:

- Thrust is 40033.99 newtons
- Burn time is 85 seconds
- Propellant mass is 1211.092 kg

First, we calculate the flow rate:

$$\text{Flow rate} = \frac{\text{Propellant mass}}{\text{Burn time}}$$

$$= \frac{1211.092\,\text{kg}}{85\,\text{s}}$$
$$= 14.248\,\text{kg/s}$$

Next, using the thrust equation $F = m \cdot v$:

$$40033.99\,\text{N} = 14.248\,\text{kg/s} \cdot v$$

Solving for v:

$$v = \frac{40033.99\,\text{N}}{14.248\,\text{kg/s}}$$
$$= 2809.797\,\text{m/s}$$

This can be further validated by approximating the mass flow rate of fuel using the volume of fuel multiplied by the density of HTPB at 920 kg/m$\hat{3}$ to solve for the mass of fuel in kg and then approximating using the same method as before. This is less accurate though as the whole rocket is not fuel, hence why the fuel value is 300kg above the true value Fig[3].

```
/// we can also appproximate fuel mass as
/// volume of fuel = pi * r^2 * L * density of fuel = pi * 0.5588^2 * 1.756 * 920 (HTPB) = 1584.801 kg of fuel
/// unit check = m^3 * kg/m^3
```

Figure 3: Fuel mass approximation via volume

Next was to solve the individual mass flow rate of oxygen. Since we know the size of the whole motor we can approximate the size of the solid fuel tank by looking at the diagram in the Aquilla launch service paper Fig [5], we can approximate the solid fuel tank as the grey area highlighted by the green lines. Then we can take the volume of the cylinder and multiply it by the density of HTPB to get the mass of fuel, then we know mass flow rate = mdotfuel + mdotoxidizer)/time we know three variables so rearranging for a mass flow rate of oxidizer we get 13.6655 kg/s and we know total flow rate is 14.25 so the flow rate of solid fuel is the difference between total flow rate and oxidizer flow rate Fig [6].

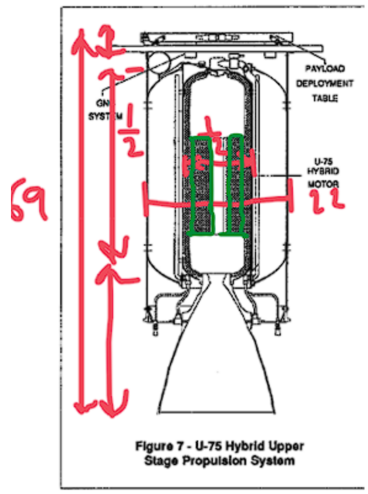

Figure 4: Fuel size approximation

```
/// validating fuel
/// fuel tank is about 0.1397 m (1/2 of motor width) and length is half the 69 inches so 34.5 inches to m which is 0.8763 m
/// Volume in si = pi * 0.1397^2 * 0.8763 = 0.05372 m^3
/// multiply by density 49.429 kg
/// solving mDot ox = (1211 kg - 49.429 kg)/85secs = 13.6655 kg/s
/// so if flow rate is 14.25 then fuel flow rate = 0.5845 kg/s
```

Figure 5: Fuel size approximation-math

Now with the variables in place, we can see the method for solving mass flow rate produces values close to what we expect. The fuel flow rate is higher than expected but this is reasonable as we are neglecting thermodynamic properties which would reduce the flow rate.

```
G_o = (mDot_o*1000) / (M_PI * pow( X: (RI*100),  Y: 2));  //  gm/cm s   RI: 0.27939999999999998
rDot = a * pow( X: G_o,  Y: n); // millimeters per second   G_o: 5.5719467817343284    a: 0.30399999999999999    n: 0.52700000000000002
mDot_f = (Pf * A_b * rDot/1000);/// kg/m^3 * m^2 * m/s   A_b: 3.0826965877944144    rDot: 0.75165621270684702    Pf: 920
mDot = mDot_o + mDot_f;   mDot_o: 13.664999999999999    mDot_f: 2.1317577987374001    mDot: 15.796757798737399
```

Figure 6: Mass flow rate calculation

It should be noted the values of a = 0.304 and n = 0.527 are both from the paper Hybrid Rocket Fuel Regression Rate Data and Modeling [2].

Following this we can confirm velocity is calculated correctly as shown below. We can see line 4 in the photo v_e = 2888.33 m/s. This is acceptably close to the value of velocity we expect and gives the correct value of thrust at 45626.405 newtons. This is slightly higher than the expected value but that is to be expected as we are estimating the value. It should be noted the formula used is from the Aquilla launch service paper [3].

```
// ------------------------------Calculating Ve Exit velocity----------------------------------//
double R = 8314.41; // # Gas constant J / kmol * K   R: 8314.4099999999999
double Pe = 0;  // # exit pressure from hybrid-rocket/Reference_code/program_2.c   Pe: 2269.4372017041023
float v_e = 0; // # Calculate the exit velocity   v_e: 2888.33984
double Thrust = 0; // # Calculate the rocket thrust from Space-Propulsion-Analysis-and-Design-McGraw-Hill-(1995)   Thrust: 45626.404952161771

Pe = Pc*pratio(kappa,E);   E: 75

v_e = sqrt( X: (2 * kappa*R * T) / ((kappa - 1)*MOL) * (1.0 - pow( X: (Pe / Pc), Y: ((kappa - 1) / (kappa)))));   T: 2800   MOL: 23   kappa: 1.2

Thrust = mDot * v_e;   mDot: 15.796757798737399

double Ip = v_e/9.81;   Ip: 294.42811862895002

printf( "velocity %f\n",v_e) ;   v_e: 2888.33984
printf("Specific impulse V_e: %f \n", Ip);   Ip: 294.42811862895002
```

Figure 7: Velocity and Thrust Calculation

2.1 Second Validation Method U75 Hybrid Rocket Motor

I also attempted to validate the code using values from the paper titled "Optimization design in single wagon-wheel fuel grain of hybrid rocket motor" to investigate different fuel grains for hybrid motors. Still, I had benchmark values for the U75 hybrid Rocket, the same Rocket used in the Aquilla launch service so I created a second validation program with the values shown in the table below.

Table 2: Rocket Motor Parameters

Parameter	**Value**
Thrust	11 kN
Total Impulse	1100 kN sec
Chamber Pressure	5 MPa
Expansion Ratio	100
Diameter	380 mm
Motor Length	1000 mm

this was also successful as the calculated values closely resembled the sought-after values but since they were using a wheel grain and we had just assumed a solid cylindrical grain the error in approximations was larger for example thrust is shown below in table[3].

3 Conclusion

In conclusion, the code has been validated through two different example values for testing the U-75 rocket. The first example from the Aquilla launch service paper is a more accurate validation as the second example uses values from a Rocket that had a wagon wheel grain. Overall the Thrust values were calculated correctly indicating the code is preforming as expected.

Table 3: Wagon Wheel Comparison

Parameter	**Expected**	**Code value**	**Error**
Thrust	11000	10358.535811	641.464189

References

[1] Kirk J. Flittie and Scott McFarlane. The commercial aquila launch vehicle. In *27th Joint Propulsion Conference*, Sacramento, CA, June 24-26 1991. AIAA/SAE/ASME, American Institute of Aeronautics and Astronautics. Paper AIAA-91-2046.

[2] Wei Tian, Guobiao Cai, Hui Tian, Hao Zhu, Yuanjun Zhang, and Xingtong Li. Optimization design in single wagon-wheel fuel grain of hybrid rocket motor. *Journal of Astronautics*, 1(4):212–221, 2024. Key Laboratory of Spacecraft Design Optimization & Dynamic Simulation Technologies, Ministry of Education, China.

[3] Greg Zilliac and M. Arif Karabeyoglu. Hybrid rocket fuel regression rate data and modeling. In *42nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit*, number AIAA 2006-4504, Sacramento, California, July 2006. American Institute of Aeronautics and Astronautics.

[2] [3] [1]