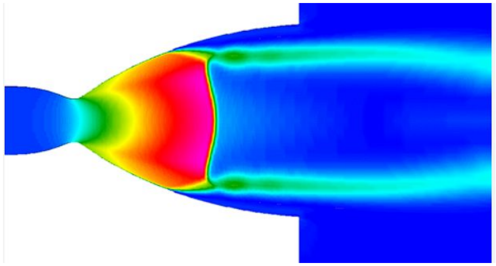
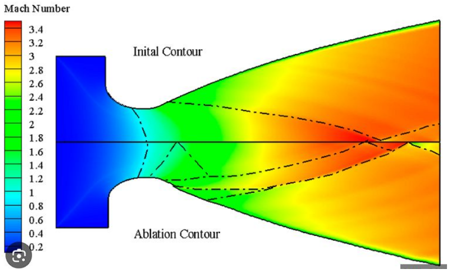
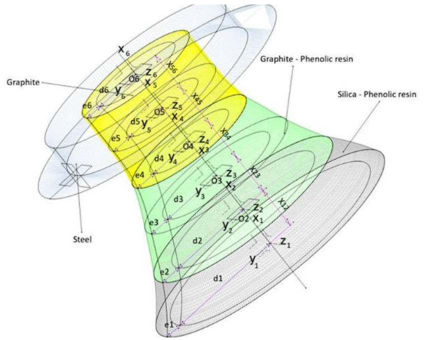


Thrust vectored Rocket Project

Nozzle design process





Initial Research

https://youtu.be/PD6XpFfR68E?si=1Hmg1o76Y_aFVljb

Awesome lecture on how Rocket fuel grains can be made and Rocket nozzles.

Grain development notes

- Can use **anything that burns** but **paraffin** is cheap, non toxic, and reusable
- Make **individual segments** (fuel grains) as its unreliable to make a long tube filled with fuel grain
- Easier to fabricate a **cylindrical grain** then **drill out the centre** so it burns outwards towards the walls (this prevents it burning downwards meaning the top of the tube would be exposed to extreme temperatures)
- Solid paraffin is denser than liquid paraffin so it will shrink as it cools and the outer layers and bottom will cool first as heat is emitted but it means the left over void has to be filled by the next pour resulting in a convex mating and a divot in the top of the grain

- Solution artificially **extend tube** by wrapping a piece of tape along the bottom which can be easily removed later to **cut** the **defective grain**.

- Anything removed from **grains can be remelted and reused**
- Grain will heat up within seconds and radiant energy will travel through the grain with a pure wax grain it will disperse through the grain but we want it to radiate in the top micron thick layer as it burns to cause it to super heat and melt and blend with nitrous oxide to produce exhaust.

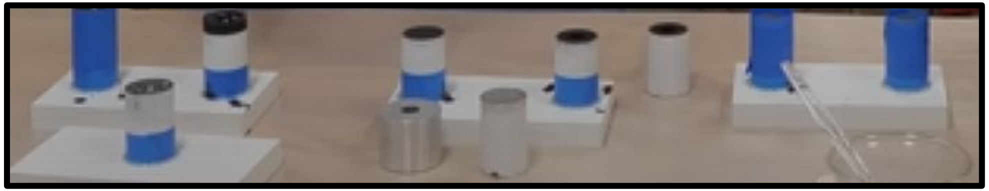
- Solution **add carbon black** mixed in at **1% by weight ratio.** This will absorb light and strengthen our grain as any fracture in the wax will stopped by the carbon therefore avoiding any fatal fractures instead limited to microns thick.

- The carbon is more dense than the paraffin so before make sure to mix the carbon black up and then pour and only pour **10-15% each pour** to allow each pour to cool quickly meaning the carbon black won't have time to sink.

Step by step

Fuel grain step by step

- Heat Mixing pot
- Place 100g wax in pot
- Add one gram carbon black
- Mix till melted (about 5mins)
- Pour 10 - 15% in fuel cell
- Let cool (approx 5 minutes)
- Repeat till fill
- Peel tape and remove defective wax
- Drill cylindrical centre using drill bit
- Place excess material in jar



Initial reasons for why a Nozzle

Step by step for why

Throw composite into open flame - large ball of fire no thrust

Put composite in tube - big fountain of flame and tiny thrust in opposite direction no pressure slow velocity

Add a plate with a small circle hole in the centre - increases pressure increases thrust

The gas has to accelerate to its maximum velocity almost instantly as it leaves the whole therefore low maximum velocity (lots of turbulent eddies at that circle reducing efficiency) so we add conversion area to allow more time for the gas to accelerate to normally 90 degrees in total. For gas to go forward it needs to go from a higher to lower pressure (bernoulli's principle)

Materials

Bentonite clay (kitty litter) - fragile will only last one test - force it into the shape or cast it from concrete

Commercial plastic nozzle - urea formaldehyde resin doesn’t melt instead chars/oblets so when reinforced will glass but not good with hybrid rocket

Ultimately a high temperature metal like iconal, tungsten or graphite.

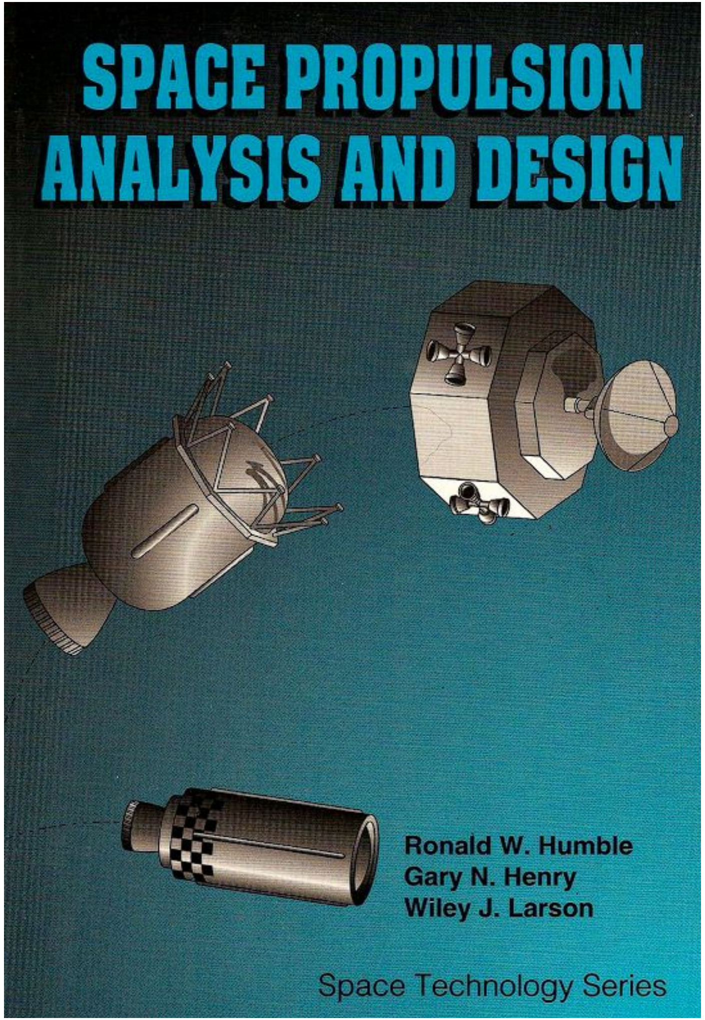
Isostatically molded microfine particle graphite cylinders can be whole bored to create a nozzle. 4300 degrees melting point, higher than tungsten. Sublation rate at one atmosphere is about 3600 degrees so they are very Temperature resistant and some What tolerant to the oxidizing Environment inside the chamber

Bernoulli's principle explanation in terms of Nozzle designs

In a room of stagnant air molecules rebounding of the walls creates pressure at the speed of sound. If you open two walls to allow the air to flow, due to the conservation of momentum (as long as heat is not added to the system) the force is in the axis of motion and so the pressure orthogonal to that must decrease hence why the gas moving through a tube decreases pressure, acceleration is caused from the rebounding of particles inside the tube. As the gas continues to accelerate it gets harder for particles to 'overtake' and transfer energy they cannot go faster than sound therefore the max speed at the convergent zone is always mach 1 so if you continue to add gas the pressure must increase throughout the entire chamber. Now the gas leaving the chamber goes from higher pressure to lower pressure. The potential energy stored in the gas wil convert to kinetic energy and add to the thrust pushing the velocity past mach 1. The gas will expand in a hemispherical zone and not be efficient so the thrust will be in the desired direction but also laterally which is undesired and wasted energy. So we add a divergent section (gradually increasing cone behind the throat) results in a gradual increase in pressure and velocity which is more efficient and results in the expansion of thrust that you want.
This expansion is determined from the pressure inside the chamber and outside the chamber. In space since outside has no pressure particles will expand forever so by making the nozzle very very big you can tweak out very small amounts of additional efficiency or thrust basically forever only limited by the bulk of the nozzle. For example sea level raptor engines have a smaller nozzle than the vacuum operation nozzles. At sea level it is the ratio between the inside and one atmosphere and as a good rule of thumb for every twelve atmospheres of pressure you want to add again the area of the throat of your nozzle.
So at 12 atmosphere you would want the divergent area to have twice the area of the throat of the nozzle or the square root of 2 or 1.4 times the diameter. At 60 atmospheres you'd want a area increase of five to six times the area of the throat of the nozzle.2.2 to 2.4 times the diameter.

Finally the expanding angle needs to be slightly more gentle then the convergent angle for example close efficiency has been found to 30 degrees in total for the divergent angle. So 90 degrees for the convergent and 30 for the divergent with a minimal diameter throat

Space review on hybrid rocket systems



Chapter **Page**

Considerations

7.7 Case Study

In Chap. 10, we derive propulsion-system requirements for a liquid-rocket system assuming that the propellants are RP-1 (kerosene) and liquid oxygen (LOx). The performance of these propellants is very close to the performance of a polybutadiene / LOx hybrid system, so we believe we can design a hybrid propulsion system based on these similar propellants.

7.7.1 Summarize Requirements

We intend to design the Option 4 system outlined in Chap. 10. As in the design case study for liquid rockets (Sec. 5.5), we need to add margin to the numbers to increase the likelihood that our design is successful. Remember that this option is a two-stage system in which the first stage is a high-thrust chemical rocket providing a transfer from low-Earth orbit to a 5000-km circular orbit. From there, a low-thrust ion rocket is used to spiral up to geostationary altitude. Table 7.11 lists the key requirements and suggests reasonable design margins.

<u>Summary</u>

The performance of propellants are largely based on performance from LOX liquid rocket systems and therefore much has been used from the previous simulations in Chap.10

It should also be considered through the design process that they designing a hybrid system for second stage low thrust spiral stated in mission option number 4 so there is no atmosphere externally.

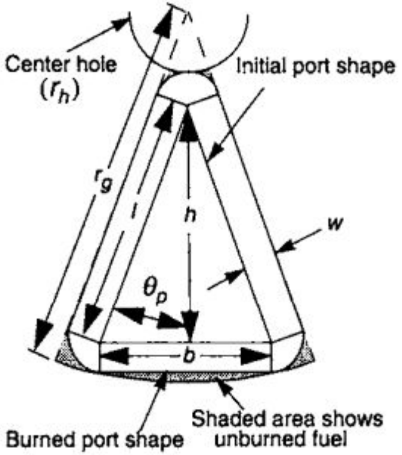
Option 4 -pg 606

4. A two-stage transfer. The first stage is a high-thrust Hohmann transfer, and the second stage is a low-thrust spiral. Staging occurs at a 5000-km altitude. All of the plane change is performed after reaching GSO altitude.

Pg 420 design labelling

Given:

N	= number of ports (decision)
m_{fuel}	= mass of fuel required
ρ	= density of fuel
G_{oi}	= allowable initial oxidizer flux
$\dot{m}_{oi}$	= initial required oxidizer flow rate
$\dot{m}_{fi}$	= initial required fuel flow rate
a, n, m	= regression-rate parameters



Calculate:

$A_{pi} = \dfrac{\dot{m}_{oi}}{NG_{oi}}$	initial port area
$\theta_p = \dfrac{\pi}{N}$	port half-angle
$h = \sqrt{\dfrac{A_{pi}}{\tan\theta_p}}$	triangle height $\left(A = \frac{1}{2}bh\right)$
$b = 2h\tan\theta_p$	length of triangle base
$l = \dfrac{h}{\cos\theta_p}$	length of the side
$P_{pi} = 2l + b$	initial port perimeter
$G_{fi} = \dfrac{\dot{m}_f}{NA_{pi}}$	initial fuel flux
$L_p = \left(\dfrac{\dot{m}_{fi}}{N\rho_{fuel}a(G_{oi}+G_{fi})^n P_{pi}}\right)^{\frac{1}{m+1}}$	port length [see Eq. (7.88)]
$w^2 + \dfrac{2l+b}{\pi}w - \dfrac{m_{fuel}}{N\rho L_p \pi} = 0$	solve quadratic for thickness of fuel web (w)
$r_h = \dfrac{w}{\sin\theta_p} - w$	radius of center hole
$r_g = h + 2w + r_h$	radius of grain

Fig. 7.23. Geometry Relations for Triangular Ports. This geometry applies to wagon-wheel ports in which the total number of ports is greater than seven. For fewer ports, we must take into account the curvature of the triangle base.

Summary

Although not part of the nozzle design highlights the given variables and variable needed to be calculated.

Steps of Simulation

Table 7.9. Simulation Algorithm. This algorithm estimates the performance of a hybrid rocket over time. We assume that regression rate is constant down the port length.

Action	Inputs	Outputs
1. Initialize simulation $t = 0$	• Grain geometry • History of oxidizer flow • Nozzle configuration • Regression-rate parameters • Throat-erosion parameters • History of ambient pressure • Simulation parameters (time step size - Δt) • Thermochemistry data	
2. Determine fuel-regression rate	• $\dot{m}_{ox}$ - oxidizer mass flow rate(kg/s) • A_p - cross-section area of port (m^2) • ρ - fuel density (kg/m^3) • N - number of ports	• G_{ox} - oxidizer mass flux in each port (kg/m^2·s) • G - total mass flux (kg/m^2·s) • $\dot{r}$ - fuel regression rate (m/s)
3. Determine total mass flow rate	• $\dot{r}$ • S_p - port surface area (m^2) • ρ • N • $\dot{m}_{ox}$	• $\dot{m}_{fuel} = \dot{r}\rho S_p N$ - fuel flow rate (kg/s) • $\dot{m}_{prop} = \dot{m}_{fuel} + \dot{m}_{ox}$ - total flow rate (kg/s)
4. Determine thermo-chemistry	• $\dot{m}_{ox}$ • $\dot{m}_{fuel}$ • Thermochemistry data (tables or curve-fit expressions, Appendix B)	• $\frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}}$ - oxidizer-to-fuel ratio • T_c - flame temperature (K) • γ - ratio of specific heats • $\mathcal{M}$ - molecular mass (kg/kmol) • c^* - characteristic velocity [Eq. (3.133)]
5. Determine chamber, nozzle, and engine performance	• $\dot{m}_{prop}$ • A_t - nozzle throat area (m^2) • ε - nozzle expansion ratio • p_a - ambient pressure (Pa) • λ - nozzle efficiency • c^* • γ	• $p_c = \frac{\dot{m}_{prop} c^*}{A_t}$ - chamber pressure (Pa) • M_e - exit Mach number [Eq. (3.100)] • p_e - nozzle exit pressure [Eq. (3.95)] • v_e - nozzle exit velocity [Eq. (3.112)] • $A_e = \varepsilon A_t$ [Eq. (3.100)] • $F = \lambda[\dot{m}_{prop} v_e + (p_e - p_a) A_e]$ [Eq. (1.6)] • $I_{sp} = \frac{F}{\dot{m}_{prop} g_0}$

__Summary__

Step by step on simulation to estimate the performance of a hybrid rocket.

Mass Estimates

7.6.6 Summary of Relationships between Mass Estimates for Components

Because there is virtually no published database concerning mass estimates for hybrid rockets, we estimate system masses using numbers for liquid and solid rockets, as appropriate. Table 7.10 summarizes the equation or section in which the component is discussed.

Of course, these relationships derive from empirical data on existing systems, so they may or may not apply to hybrids, depending on vehicle design. For example, we typically do not use separate internal insulation in hybrid rockets. Instead, we add a bit of excess fuel. Another example is nozzle design, which is very specific for solid rockets. It must account for particulate erosion and other solid-rocket problems. However, without a good database for hybrids, we rely on these numbers for estimates.

Summary

Due to lack of published knowledge concerning mass estimates for hybrid rocket systems we estimate them using solid and liquid rockets. On the right is a table of estimates and where they are from in the paper.

Table 7.10. Summary of Relationships between Mass Estimates for Components.

Component	Relationship
Motor case	• Section 6.3.3 (m_{mc})
Thrust skirt and polar boss	• Section 6.3.4 • $m_{tspb} = 0.1\, m_{mc}$ (kg)
Nozzle	• Section 7.6.4 [Eq. (7.103)] • $m_{noz} = 125\left(\frac{m_{prop}}{5400}\right)^{\frac{2}{3}}\left(\frac{\varepsilon}{10}\right)^{\frac{1}{4}}$ (kg) • m_{prop} = total propellant mass (kg) • ε = nozzle expansion ratio
Thrust-vector control	• Section 6.3.8 (m_{tvc})
Injector	• Section 5.4.1 (m_{inj})
Oxidizer tank	• Section 5.4.4 (m_{tank})
Pressurant system	• Section 5.4.5 (m_{press})
Turbo-pumps	• Section 5.4.3 (m_{pump})
Feed system	• Section 5.4.2 (m_{fs})
Structural supports	• $m_{ss} = 0.1\,[m_{mc} + m_{noz} + m_{tvc} + m_{inj} + m_{tank} + m_{press} + m_{pump} + m_{fs}]$ • 10% of the other inert masses

Nozzle design

Constants

5. Determine chamber, nozzle, and engine performance	• $\dot{m}_{prop}$ • A_t - nozzle throat area (m^2) • ε - nozzle expansion ratio • p_a - ambient pressure (Pa) • λ - nozzle efficiency • c^* • γ	• $p_c = \frac{\dot{m}_{prop} c^*}{A_t}$ - chamber pressure (Pa) • M_e - exit Mach number [Eq. (3.100)] • p_e - nozzle exit pressure [Eq. (3.95)] • v_e - nozzle exit velocity [Eq. (3.112)] • $A_e = \varepsilon A_t$ [Eq. (3.100)] • $F = \lambda[\dot{m}_{prop} v_e + (p_e - p_a) A_e]$ [Eq. (1.6)] • $I_{sp} = \frac{F}{\dot{m}_{prop} g_0}$

Summary

O/F ratio varies with time and the parameters can be seen above. Cooled nozzles can be more efficient but are more complicated.

Side note TVC

7.6.4 Nozzle Design

Nozzle design closely parallels that of the solid rocket, with one major difference. In the solid rocket, the oxygen-to fuel ratio (*O/F*) is constant. As discussed in previous sections, for hybrid rockets, *O/F* typically increases with time and becomes oxygen-rich in the latter stages of the burn. Hence, the nozzle material must withstand an oxygen-rich atmosphere. Graphite, good for solids, is not good for hybrids that go oxygen-rich, because graphite oxidizes easily. Phenolic-based materials are better in this case.

Equation (7.103) gives an empirically derived expression for nozzle mass:

$$m_{noz} = 125\left(\frac{m_{prop}}{5400}\right)^{\frac{2}{3}}\left(\frac{\varepsilon}{10}\right)^{\frac{1}{4}} \tag{7.103}$$

where m_{noz} = nozzle mass (kg)
m_{prop} = total mass of propellant (kg)
ε = nozzle expansion ratio

Regeneratively cooled nozzles (Chap. 5) are possible for hybrids but are more complex than the standard ablative nozzles discussed in Chap. 6. If we use the oxidizer as the coolant, the nozzle may oxidize (burn), so we must carefully analyze this approach before using it.

7.6.5 Thrust-Vector Control

Techniques for controlling the thrust vector are the same as for solids with or exception. Liquid Injection Thrust Vector Control (LITVC) is easier to use i hybrids because the system already has a liquid. The mass penalty for addition liquid is small and is easily accounted for in the design.

Design steps

Table 7.11. Summary of Propulsion-System Requirements. The middle column gives the requirement from Chap. 10, and the far right column discusses the approach we use to ensure the requirement is met.

Name	Requirement	Modification
Maximum initial vehicle mass	12,000 kg	We iteratively select the structural mass fraction to get this number. Hopefully, the resulting mass fraction is conservative.
Payload mass	4914 kg	Includes 3387-kg payload and an electric stage
Δv	1721 m/s	1893 m/s (10% margin)
Minimum thrust-to-weight ratio	0.3	We choose 1.5 to decrease the burn duration.
Envelope limits	3 m long x 3 m diameter	Diameter is not a problem, but length may be difficult to meet.

In any design, some iterating and optimizing are necessary. Usually, we can iterate to develop a "better" design until the end of time. In the following design, we make several decisions necessary to the design so we can illustrate the process. These decisions give us a workable design (ignoring the length restriction) but are not "optimal." We could then vary the decision parameters to derive a better design.

Summary

Identifying a few key parameters in the design which you can later change to iterate the design.

Other constants of importance

Table 7.10. Summary of Relationships between Mass Estimates for Components.

Component	Relationship
Motor case	• Section 6.3.3 (m_{mc})
Thrust skirt and polar boss	• Section 6.3.4 • $m_{tspb} = 0.1 m_{mc}$ (kg)
Nozzle	• Section 7.6.4 [Eq. (7.103)] • $m_{noz} = 125\left(\frac{m_{prop}}{5400}\right)^{\frac{2}{3}}\left(\frac{\varepsilon}{10}\right)^{\frac{1}{4}}$ (kg) • m_{prop} = total propellant mass (kg) • ε = nozzle expansion ratio
Thrust-vector control	• Section 6.3.8 (m_{tvc})
Injector	• Section 5.4.1 (m_{inj})
Oxidizer tank	• Section 5.4.4 (m_{tank})
Pressurant system	• Section 5.4.5 (m_{press})
Turbo-pumps	• Section 5.4.3 (m_{pump})
Feed system	• Section 5.4.2 (m_{fs})
Structural supports	• $m_{ss} = 0.1\,[m_{mc} + m_{noz} + m_{tvc} + m_{inj} + m_{tank} + m_{press} + m_{pump} + m_{fs}]$ • 10% of the other inert masses

Propellant choices

Choose Propellants

The mission-level analysis (Chap. 10) suggests that RP-1 and LOx propellants are a good choice to meet the requirements. Review of Appendix B indicates that the performance of these propellants is very similar to that of HTPB / LOx, so we choose them for our preliminary design. The optimal *O/F* is 2.1, based on the

frozen-flow thermochemical analysis presented in Appendix B. Remember that the *O/F* shifts over the burn duration. We want to restrict the *O/F* shift to the smallest range possible. Thus, we choose an initial *O/F* of 1.2.

<u>For our design</u>

We chose to use Paraffin for our propellent.

In their case there mission analysis indicated RP-1 and LOx as good propellants.

Step one Preliminary Design

7.7.2 Make Preliminary Design Decisions

Basic Decisions

As mentioned above, we need to make several decisions that allow us to design the system. These decisions generate a solution but are not necessarily optimal. To find an optimal solution, we would iterate on these decisions to improve the result.

- Number of ports = 8 (wagon wheel with no center hole burning)
- Initial $O/F = 1.2$
- Initial thrust-to-weight ratio = 1.5
- Initial chamber pressure = 1.4 MPa
- Helium gas pressurant stored in a titanium pressure vessel
- Conventional metallic structure for thrust chamber and oxidizer tank

Summary

Define parameters to later iterate on.

For our design

…Need to determine…

Number of ports refers to the entry cap to the fuel grain which results in different burn patterns and therefore varying performance. Various ports can be seen below

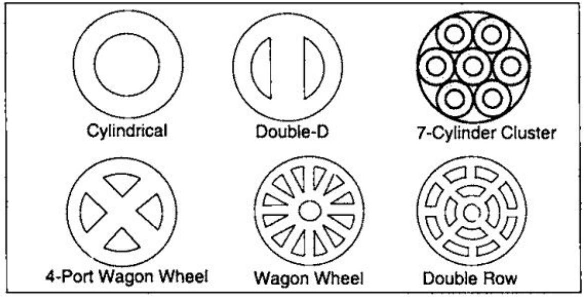


Fig. 7.18. Typical Port Configurations for Hybrid Rockets. The trick is to choose a configuration that optimizes volumetric loading efficiency of the solid fuel. Such a configuration requires the fuel web to be constant around the perimeter of each port.

Feed levels

Determine Engine and Feed System Pressure Levels

We start by looking at how chamber pressure affects system performance. Assuming the optimal *O/F* of 2.1 as an analysis point, we can determine the thermochemical parameters for combustion from Appendix B (assume a frozen-flow combustion efficiency of 1.0):

- Combustion temperature = 3593 K
- $\gamma = 1.231$
- Molecular mass = 22.84 kg/kmol
- $c^* = 1747.4$ m/s [Eq. (7.71)]

Using the process outlined in Table 5.5, we can calculate the variation in vacuum specific impulse as a function of nozzle expansion ratio. Fig 7.25 shows this relationship, assuming a nozzle efficiency of 0.98. A reasonable expansion ratio is 70, using the argument of diminishing returns. This assumption gives us a specific impulse of 330 s.

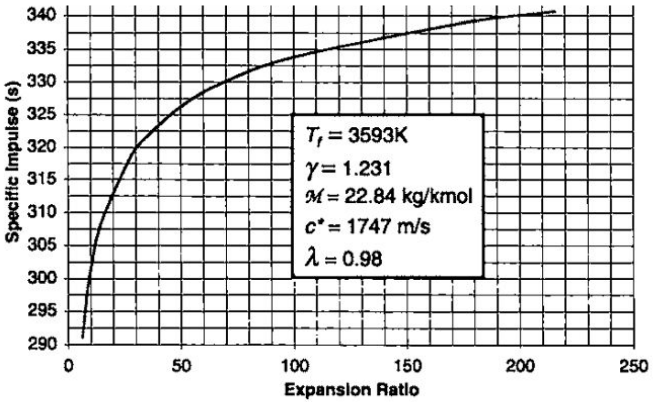


Fig. 7.25. Performance of Vacuum Specific Impulse for Polybutadiene and Liquid Oxygen. The chart shows the performance at an *O/F* of 2.1 and a nozzle efficiency of 98%.

Summary

Assuming optimal O/F you can graph specific impulse and expansion ratio and define the parameters of combustion for your chamber.

Characterizing our Nozzle

Finding Area of the throat

7.7.4 Size and Configure Components

Now that we are fairly confident our system meets the basic performance requirements, we must configure the various systems. From the ideal-rocket analysis, we know

- Total mass = 12,000 kg
- Payload mass = 4914 kg
- Propellant mass = 5399 kg
- Inert mass of the stage = 12,000 – 5399 – 4914 = 1687 kg
- Inert-mass fraction = $\dfrac{1687}{12,000 - 4914} = 0.238$

The sum of the individual component masses must be less than the inert mass number by some reasonable margin.

Nozzle—We start the component analysis at the nozzle and work back through the system. We need to design our nozzle so we get the correct initial mass flow rate (to get the specified initial thrust level). From the characteristic velocity equation, we can determine the throat area (A_t). We can then determine the nozzle exit area (A_e) from the expansion ratio (ε). We choose the initial condition to specify the thrust level [Eq. (3.133)].

$$A_t = \frac{\dot{m}c^*}{p_c} = \frac{64.2\,(1545)}{1,400,000} = 0.0708\ \mathrm{m}^2$$

Summary

Finding Area of throat based on parameters of the project.

Characterising our Nozzle 2

Finding Area of the exit and nozzle mass

$$A_e = A_t\varepsilon = 0.0708\,(70) = 4.959 \text{ m}^2$$

These translate into:

- Throat diameter = 30.03 cm
- Exit diameter = 251.3 cm

If we were to use a conical nozzle with a 15° half angle, the length of this nozzle would be:

$$\frac{2.513 - 0.300}{2\tan 15} = 4.128 \text{ m} \qquad \text{[Eq. (5.41)]}$$

Using the bell nozzle figures in Chap. 5 (Fig. 5.25) and requiring a 98% nozzle efficiency, we can determine the bell nozzle's length:

- Bell-nozzle fraction = 0.675
- Nozzle length = 0.675 × 4.128 = 2.787 m

We then use the empirical mass equation for phenolic nozzles [Eq. (7.103)] to determine the nozzle mass:

$$125\left(\frac{5399}{5400}\right)^{\frac{2}{3}}\left(\frac{70}{10}\right)^{\frac{1}{4}} = 203 \text{ kg}$$

Summary

Finding area of exit and nozzle mass based on calculated values.

Combustion Chamber

Combustion chamber—The next step is to evaluate the thrust chamber. The port length is 3.493 meters, but we must allow for an aft mixing section and an injector section. The typical rule of thumb for the aft mixing section is 0.5–1.0 grain diameter. We choose the total grain diameter:

- Grain length = 3.493 m
- Fore and aft mixing section length = $2 \times 0.4675 = 0.9350$ m
- Chamber length = $3.493 + 0.9350 = 4.4280$ m

The injector section uses a hemispherical dome with a radius equal to the case radius (0.4675 m). Figure 7.30 shows the engine configuration.

The process for estimating the thrust-chamber mass is similar to that for the cases of solid rocket motors (Sec. 6.3.3). To be conservative, we assume an aluminum case material with material properties taken from Table 6.4:

- Ultimate allowable stress (F_{tu}) = 414,000,000 Pa
- Density = 2800 kg/m^3

We assume that burst pressure is twice the maximum expected operating pressure:

- Burst pressure = $2 \times 1{,}400{,}000 = 2{,}800{,}000$ Pa

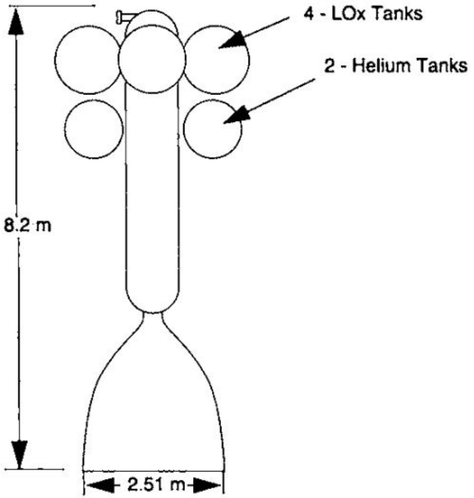


Fig. 7.30. **Configuration of the Thrust Chamber.** This is the design-point configuration showing the main dimensions.

__Summary__

We can characterize the thrust chamber using the previously solved values …

Combustion chamber 2

From Eq. (6.11), we know the required wall thickness is (assuming a thickness factor of 1.5 to allow for stress concentrations):

$$t_{cs} = \frac{2\,(1.5)\ (2,800,000)\ (0.4675)}{414,000,000} = 0.0048 \text{ m}$$

The approximate material volume (surface area × thickness) is 0.0754 m^3, giving a

- Mass of the combustion chamber case = 209 kg

If we estimate the interior volume of the tank assuming a cylinder capped by two hemispherical ends, we calculate a volume of 3.47 m^3. Multiplying this by the chamber pressure and dividing by the case weight, we end up with a structural mass factor of 2366 m. This result agrees with Chap. 5 for typical numbers (2500 m).

We estimate the injector mass by calculating the mass of a 2.5-cm thick aluminum plate spanning the chamber radius:

- Injector mass = $2800 \times \pi(0.4675)^2(0.025) = 48$ kg

The total thrust-chamber mass is:

- Thrust-chamber mass = 209 + 48 + 203 = 460 kg

Summary

And from those values we can solve the wall thickness, injector mass and thrust chamber mass.

Extras pieces

Oxidizer tank—The required volume of oxidizer is 3.295 m^3 (allowing for 10% ullage), and the pressure is 1,787,100 Pa from Sec. 7.7.2. Assuming a structural tank factor of 2500 m (see Sec. 5.4.4) for an aluminum tank and assuming a 2.0 burst-pressure factor, we can determine the oxidizer tank's mass:

- Oxidizer tank's mass = 528 kg.

Pressurant system—Using the process discussed in Sec. 5.4.5 for designing a regulated pressure system for isentropic blowdown with a helium pressurant, we need

- Pressurant volume = 1.132 m^3
- Pressurant mass = 42 kg (assuming 21,000,000-Pa initial pressure)
- Tank mass = 191 kg (assuming 12,700 as the pressurant tank factor)

Support structure and ancillary parts—As in the other sections, we allow for mass of the support structure by assuming it is 10% of the inert mass:

- Mass of the support structure = $0.1 \times (460 + 528 + 191) = 118$ kg

Summary—Adding up the masses of all components gives

- Total inert mass = 1339 kg
- Inert mass margin = $1687 - 1339 = 348$ kg

These values give us a 27% margin in inert mass. Figure 7.30 shows the layout of the overall system.

Summary

We can also determine the oxidizer tank mass, pressurant mass and inert mass and other values.

Iteration

7.7.5 Iterate

The analysis of this case study in the previous sections gives us a single point in the "design space." Varying the decision parameters (Sec. 7.7.2) could allow us to improve our design somewhat. For example, decreasing the initial thrust-to-weight ratio or increasing the chamber pressure decreases the nozzle size. Changing the number of ports changes the volumetric efficiency, which in turn changes the size of the thrust chamber.

Notice that we have violated the length constraint. Long, narrow, thrust chambers are typical for hybrid rockets. It is unlikely that we can design a practical hybrid rocket system to meet the length requirement. Reducing the nozzle expansion ratio, increasing the chamber pressure, and using a segmented and deployable nozzle skirt can substantially decrease the nozzle size. Decreasing the fore and aft mixing area and increasing the number of ports can decrease the combustion-chamber length. However, extreme changes in these design parameters compromise the utility of this system, making the liquid system much more attractive.

Summary

By changing initial parameters we can iterate our design.

Hybrid rocket interior ballistics model

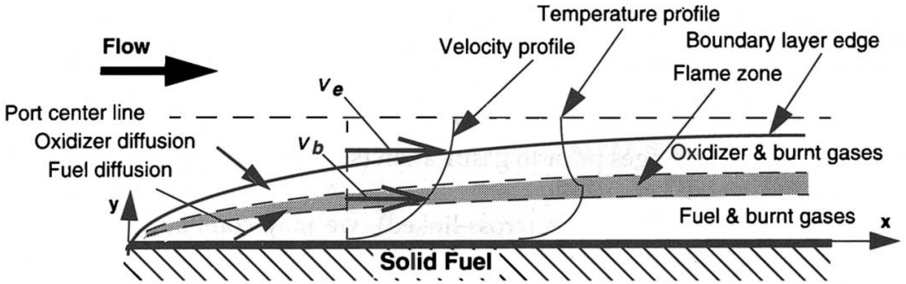


Fig. 7.4. **Schematic of Combustion Zone Above Hybrid Fuel.** This basic model is used to develop the theoretical expressions for regression rate.

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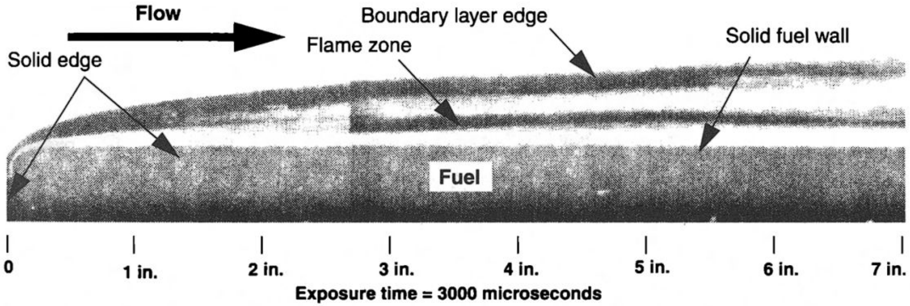

Fig. 7.5. **Schlieren Photograph of a Boundary Layer for Hybrid Combustion.** This photograph shows that our basic model (Fig. 7.4) is based on empirical observation. The "flame zone" and boundary layer features are shown [Muzzy, 1963].

Additional information to look into