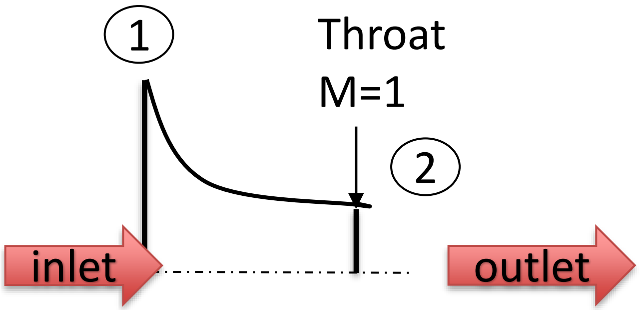
Nozzle Design



From the conservation of mass

$$\dot{m}_1 = \dot{m}_2 = Av\rho$$

From the perfect gas law

$$pV = RT$$

Isentropic flow process

$$\frac{T_1}{T_2} = \frac{p_1}{p_2}^{\frac{k-1}{k}}$$

$$k = \frac{c_p}{c_v}$$

$$c_p - c_v = R$$

$$c_p = R\frac{k}{k-1}$$

Derivation from last time

$$h_o = h + v^2$$

$$c_{p0}T_0 = c_pT + v^2$$

$$\frac{T_0}{T} = 1 + \frac{v^2}{2c_pT}$$

$$\frac{p_1}{p_2}^{\frac{k-1}{k}} = 1 + \frac{v^2}{2c_pT}$$

$$v_2 = \sqrt{\frac{2k}{k-1}RT_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]}$$

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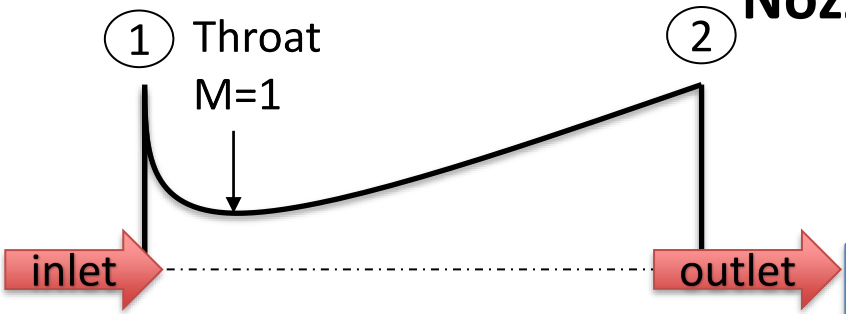

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$$k = \frac{c_p}{c_v} \qquad c_p - c_v = R$$

Total "stagnation" enthalpy

$$h_o = h + v^2$$

$$c_{p0} T_0 = c_p T + v^2$$

$$\frac{p_0}{p} = [1 + \frac{v^2}{2 c_p T}]^{\frac{k-1}{k}}$$

Speed of sound

$$a = \sqrt{kRT}$$

Mach number

$$M = \frac{v}{a} = v/\sqrt{kRT}$$

$$T_0 = T(1 + \frac{1}{2}(k-1)M^2)$$

$$T_c = T_t(1 + \frac{1}{2}(k-1))$$

$$T_t = T_c(\frac{2}{k+1})$$

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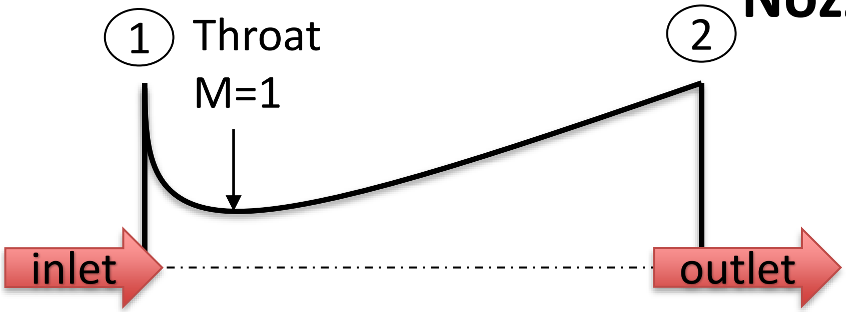

$$\frac{A_t}{A_e} = \frac{v_e \rho_e}{v_t \rho_t} = \frac{\sqrt{\frac{2k}{k-1} R T_c \left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}}{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{k R T_t}} \frac{\rho_e}{\rho_c}$$

$$\frac{A_t}{A_e} = \frac{v_e \rho_e}{v_t \rho_t} = \frac{\sqrt{\frac{2k}{k-1} R T_c \left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}}{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{k R T_t}} \frac{\rho_e}{\frac{p_c}{R T_c}}$$

$$\frac{A_t}{A_e} = \frac{\sqrt{\frac{2}{k-1} \left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}}{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \frac{p_c}{p_e} \frac{T_e}{T_c} \sqrt{\frac{2}{k+1}}}$$

$$\boxed{\frac{A_t}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{p_e}{p_c}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1}\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}}$$

$$\epsilon = \frac{A_e}{A_t}$$