Ergodicity Economics in Property & Casualty Insurance: A Simulation Framework for Understanding Risk Appetite

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Abstract

This white paper introduces a new simulation framework based on ergodicity economics principles for analyzing insurance risk appetite in Property & Casualty markets. Traditional ensemble-based risk models may not adequately capture the temporal dynamics of insurance company decision-making. This introductory paper demonstrates that insurance appetite varies significantly with company size, measured by revenue.

Keywords: ergodicity economics, insurance appetite, risk modeling, simulation framework

Contents

1 Executive Summary

This section should provide a concise overview of the entire paper, typically 1-2 pages covering:

- The problem addressed
- Your key methodology (simulation framework)
- Primary finding about company size and insurance appetite
- Practical implications for the industry

2 Introduction

2.1 Current Challenges in Insurance Appetite Modeling

Traditional approaches to modeling insurance appetite in P&C markets face several limitations:

- Reliance on ensemble averages rather than temporal sequences
- Inadequate consideration of path-dependent effects
- Limited incorporation of company-specific factors

2.2 The Promise of Ergodicity Economics

Ergodicity economics, developed by Peters and Adamou (2025), offers a new perspective on decision-making under uncertainty. Unlike traditional expected utility theory, which focuses on ensemble averages, ergodicity economics emphasizes the importance of time averages and growth rates.

3 Theoretical Foundation

3.1 Ergodicity vs. Non-Ergodicity in Financial Systems

The distinction between time averages and ensemble averages lies at the heart of ergodic economics, a framework pioneered by Peters (2019) that challenges fundamental assumptions in economic theory. A stochastic process is ergodic if its time average equals its ensemble average:

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f(X_t)dt = \mathbb{E}[f(X)] \tag{1}$$

For ergodic systems, what happens to an individual over time matches the average across many individuals at a single point in time. However, most economic processes violate this equality, creating a fundamental divergence between individual experience and statistical expectations.

The ensemble average represents the expected value across many parallel scenarios:

$$\langle W \rangle = \mathbb{E}[W_t] = \frac{1}{N} \sum_{i=1}^{N} W_i(t) \tag{2}$$

where $W_i(t)$ represents the wealth of entity i at time t. This is the traditional expected value used in most economic and actuarial analysis.

In contrast, the time average captures what a single entity experiences over its lifetime:

$$g_{\text{time}} = \lim_{T\to\infty} \frac{1}{T} \ln\left(\frac{W_T}{W_0}\right) \tag{3}$$

For multiplicative processes with uncertainty, Jensen's inequality ensures these two averages diverge systematically, with profound implications for insurance and risk management (Peters and Gell-Mann, 2016). Jensen's inequality states that for a random variable W and a convex function f:

$$f(\mathbb{E}[W]) \leq \mathbb{E}[f(W)] \tag{4}$$

In the context of multiplicative wealth dynamics, this means that the expected growth rate (ensemble average) can be misleadingly optimistic compared to the actual growth experienced over time (time average).

3.2 Insurance Markets as Non-Ergodic Systems

Insurance markets exhibit fundamentally non-ergodic properties that invalidate traditional expected value analysis. The non-ergodicity arises from several structural features:

Multiplicative Wealth Dynamics: Corporate wealth evolves through multiplicative processes where returns compound on existing capital:

$$W_{t+1} = W_t \cdot (1 + r_t - l_t) \tag{5}$$

where r_t represents returns and l_t represents proportional losses. This multiplicative structure means that the order and magnitude of losses matter critically. For example, a 50% loss followed by a 50% gain leaves wealth at 75% of its original value, not 100%.

Absorbing Bankruptcy Barrier: Once a company reaches zero equity (bankruptcy), it cannot recover:

$$W_t = 0 \Rightarrow W_{t+k} = 0 \quad \forall k > 0 \tag{6}$$

This absorbing barrier creates fundamental asymmetry: while gains are unbounded, losses are limited to -100%. The existence of this barrier means that time averages systematically underperform ensemble averages.

Path-Dependent Capital Requirements: Regulatory and market constraints depend on the entire history of losses and capital positions, not just current values. A company that experienced large losses faces higher capital costs and restricted growth opportunities, even after recovering to its original capital level.

Finite Decision Horizons: While ensemble averages assume infinite repeated trials, real companies operate with finite time horizons constrained by management tenure, regulatory cycles, and strategic planning periods. Over finite horizons, the divergence between time and ensemble averages can be substantial.

These non-ergodic features explain why insurance appears valuable even when premiums substantially exceed expected losses. The relevant metric is not the ensemble average (expected value) but the time average (growth rate) (Peters, 2011).

3.3 Mathematical Framework

Consider a company with wealth W_t evolving under stochastic dynamics. In the absence of insurance, wealth follows:

$$W_{t+1} = W_t \cdot (1 + \mu - \sigma Z_t - L_t) \tag{7}$$

where:

- μ is the drift (expected return from operations)
- σZ_t represents operational volatility with $Z_t \sim \mathcal{N}(0, 1)$
- L_t represents catastrophic losses occurring with frequency λ and severity distribution F_L

The time-average growth rate without insurance is:

$$g_{\text{uninsured}} = \mu - \frac{\sigma^2}{2} - \lambda \cdot \mathbb{E}\left[\ln\left(1 - \frac{L}{W}\right)\right] \tag{8}$$

The first two terms represent the familiar geometric Brownian motion result, while the third term captures the impact of jump losses. Note the appearance of $\mathbb{E}[\ln(1 - L/W)]$ rather than $\mathbb{E}[L/W]$—this logarithmic transformation is crucial and explains why losses have a disproportionate impact on growth.

With insurance featuring deductible D and premium rate π, the growth rate becomes:

$$g_{\text{insured}} = \mu - \frac{\sigma^2}{2} - \pi - \lambda \cdot \mathbb{E}\left[\ln\left(1 - \frac{\min(L, D)}{W}\right)\right] \tag{9}$$

The insurance creates value when:

$$g_{\text{insured}} > g_{\text{uninsured}} \tag{10}$$

This inequality can hold even when $\pi > \lambda \cdot \mathbb{E}[L]$ (premium exceeds expected losses), particularly for companies with:

- Lower capital levels (higher L/W ratios)
- Higher operational leverage (larger μ potential)
- Greater loss severity risk (heavy-tailed F_L)

The optimal insurance strategy maximizes time-average growth, not expected wealth. This leads naturally to the Kelly criterion (Kelly Jr, 1956), which in the insurance context becomes:

$$D^* = \arg\max_D \mathbb{E}\left[\ln\left(\frac{W_{t+1}}{W_t}\right)\right] \tag{11}$$

This optimization framework explains the simulation results showing that smaller companies optimally choose lower deductibles despite higher relative premium costs: they face greater proportional impact from losses and thus benefit more from variance reduction.

Special Case: Deterministic Operations. The framework simplifies elegantly when operational volatility is zero ($\sigma = 0$), as in this simulation. The growth rates become:

$$g_{\text{uninsured}} = \mu - \lambda \cdot \mathbb{E}\left[\ln\left(1 - \frac{L}{W}\right)\right] \tag{12}$$

$$g_{\text{insured}} = \mu - \pi - \lambda \cdot \mathbb{E}\left[\ln\left(1 - \frac{\min(L, D)}{W}\right)\right] \tag{13}$$

Even without operational volatility, insurance creates value because losses themselves introduce multiplicative volatility. The insurance value condition becomes:

$$\lambda \cdot \mathbb{E}\left[\ln\left(\frac{1-\min(L,D)/W}{1-L/W}\right)\right] > \pi \tag{14}$$

This expression is positive whenever losses exceed the deductible, demonstrating that insurance enhances growth by capping catastrophic losses—converting unpredictable large losses into predictable premium costs. The absence of operational volatility actually clarifies the pure insurance value proposition: reducing the multiplicative impact of tail risks on long-term growth rates.

4 Simulation Framework Overview

4.1 Architecture and Key Components

The simulation framework implements a comprehensive ergodic analysis system that bridges corporate finance, insurance economics, and stochastic modeling. Rather than treating insurance as a simple expected value calculation, the framework captures the fundamental non-ergodicity of business growth under uncertainty.

4.1.1 Framework Design Philosophy

The architecture centers on a critical distinction from traditional actuarial models: it compares time-average growth rates (what a single company experiences over its lifetime) with ensemble-average growth rates (statistical expectations across many parallel scenarios). This ergodic economics approach, pioneered by Peters (2019), reveals that insurance can enhance growth even when premiums substantially exceed expected losses, a result invisible to traditional expected value analysis.

The framework employs Monte Carlo simulation to generate thousands of independent business trajectories, each experiencing unique sequences of loss events. By tracking individual path dynamics, the system quantifies the lift in time-average growth rate offered by insurance in certain business scenarios.

4.1.2 Corporate Business Model

At the framework's core lies an illustrative manufacturing company with realistic financial dynamics. The model tracks complete corporate accounting relationships while maintaining computational efficiency for large-scale simulations.

Revenue generation follows an asset turnover model where annual revenues equal total assets multiplied by an asset turnover ratio (typically 0.8 to 1.2). This approach naturally scales business activity with company size while allowing exploration of capital efficiency effects. Operating income derives from revenues through configurable operating margins, capturing the fundamental profitability before considering loss events.

The income statement dynamics incorporate operating revenues, margins, claim losses, insurance premiums, and corporate taxes. Critically, revenues serve as the exposure base for loss generation, creating a direct link between business scale and risk exposure. Premiums are calibrated at the outset, assuming accurate matching to the expected loss distribution, and loaded for costs to a realistic target loss ratio. Premiums are subsequently scaled with revenues, in line with the underlying loss exposure base. After-tax earnings flow to retained earnings, driving balance sheet growth absent external capital raises.

Balance sheet evolution tracks assets, liabilities, and equity through time, maintaining the fundamental accounting equation. When losses are incurred, they follow a 10-year payment schedule, reflecting typical commercial insurance claim settlement patterns. The company's portion (deductible amount) requires immediate collateral via letter of credit. The collateral accrues at market rates and is gradually released as claim payments are made over the years. Insurance recoveries are tracked as receivables, but have no practical effect on the company as insurer defaults are not modeled.

4.1.3 Loss Generation Module

The framework implements a three-tier loss structure that captures the full spectrum of business risks:

Attritional losses represent high-frequency, low-severity events such as minor operational disruptions, small liability claims, or routine equipment failures. These follow a compound Poisson process with lognormal severities, creating a steady baseline loss burden.

Large losses capture moderate-frequency events with more substantial impact, such as significant liability claims, major equipment breakdowns, or supply chain disruptions. These also follow a compound Poisson-lognormal structure but with parameters shifted toward less frequent, more severe events.

Catastrophic losses model low-probability, high-impact events that can threaten business continuity, such as natural disasters, major litigation, or systemic failures. These events, while rare, fundamentally alter growth trajectories, erode capital, and drive much of the insurance value proposition. Poisson-Pareto distributions capture the heavy-tailed nature of these risks.

Each loss type draws from distinct frequency and severity distributions, with all claim counts scaled by revenue exposure. Claim correlations are not modeled.

4.1.4 Insurance Mechanism

The insurance module focuses on deductible optimization, the primary decision variable for most corporate insurance programs. While the framework supports complex insurance towers with multiple attachment points, coverage types and limits, for this first experiment, the configuration was simplified with virtually unlimited coverage. These high coverage limits effectively eliminate ruin risk, isolating the deductible's impact on growth dynamics.

For each loss event, the model determines payment allocation between the company (up to the deductible) and insurers (excess of deductible). Premium calculations incorporate expected losses, expense loadings, and risk charges, with pricing that reflects realistic market conditions rather than actuarially fair rates. Again, for this first experiment, the pricing was simplified to a single target loss ratio, with no risk loadings or insurance cycle dynamics. Future work will explore more sophisticated pricing models.

4.1.5 Ergodic Analysis Engine

The framework's analytical core computes and compares growth metrics across insured and uninsured scenarios, revealing the ergodic effects of insurance.

The time-average growth rate for each trajectory is calculated as:

$$g_{\text{time}} = \frac{1}{T} \ln\left(\frac{W_T}{W_0}\right) \tag{15}$$

where W_T represents terminal wealth (equity) and W_0 initial wealth. This logarithmic formulation captures the multiplicative nature of business growth, where percentage changes compound over time.

Growth lift quantifies insurance value as the difference between insured and uninsured time-average growth rates:

$$\text{Growth Lift} = g_{\text{time}}^{\text{insured}} - g_{\text{time}}^{\text{uninsured}} \tag{16}$$

Positive growth lift indicates that insurance enhances long-term wealth accumulation despite premium costs.

The framework distinguishes between survival optimization (avoiding ruin) and growth optimization (maximizing wealth accumulation). While high coverage limits virtually eliminate bankruptcy risk in the modeled scenarios, different deductible levels create a trade-off between premium costs and retained volatility that affects growth rates. The optimal deductible maximizes time-average growth rather than merely ensuring survival.

4.1.6 Parameter Interactions

The simulation explores a multidimensional parameter space that captures key drivers of insurance appetite:

Capitalization levels ($5M, $10M, $25M) represent different company sizes, with smaller firms facing proportionally larger impacts from individual losses. This size effect proves central to understanding why optimal deductibles vary across companies. At retentions beyond $25M and with the loss distribution unaltered, full retention becomes optimal (i.e., no insurance) as loss volatility relative to capital diminishes, so these higher capitalization levels were removed from consideration.

Asset turnover ratios (0.8 to 1.2) capture capital efficiency, with higher ratios generating more revenue (and thus more loss exposure) per dollar of assets.

Operating margins determine profitability buffers against losses. Higher-margin businesses can absorb losses more easily, potentially supporting higher deductibles, while low-margin operations may require more insurance protection to maintain growth.

Loss ratios scale the overall loss burden, allowing sensitivity analysis across different risk environments. As loss ratios increase, and thus insurance costs decrease, the divergence between time and ensemble averages typically widens, amplifying insurance benefits.

These parameters interact in complex ways. For instance, a small company with high asset turnover and low margins faces compounded vulnerabilities that may make lower deductibles optimal despite higher premium costs. The framework systematically explores these interactions to identify shifts in optimal insurance strategies.

4.2 Input Parameters and Data Requirements

Simulation Hyperparameters:

- Number of simulation paths (100,000)
- Projection horizon (50 years)
- Pricing simulation runs (500,000)
    - These runs are used to estimate the starting expected loss for pricing purposes

Key Input Parameters:

- Initial capital levels (\$5M, \$10M, \$25M)
- Asset turnover ratios (0.8, 0.9, 1.0, 1.1, 1.2)
- Operating margins (10%, 12.5%, 15%)
- Target loss ratios (60%, 70%, 80%)
- Loss distribution parameters
  - Attritional losses:
    - Frequency: Poisson with mean=2.85 at \$10M revenue
    - Severity: Lognormal with mean=\$40K and CV=0.8
  - Large losses:
    - Frequency: Poisson with mean=0.20 at \$10M revenue
    - Severity: Lognormal with mean=\$500K and CV=1.5
  - Catastrophic losses:
    - Frequency: Poisson with mean=0.02 at \$10M revenue
    - Severity: Pareto with minimum value=\$5M and α=1.5
- Deductible levels (\$0, \$50K, \$100K, \$250K, \$500K, and no insurance)

Simplifying Assumptions:

- Working capital is set to 0% of revenue to maximize cash generation
- Steady growth rate parameter is set to 0%; all growth derives from retained earnings
- No stochastic revenue or margin fluctuations; operating margins are deterministic
- Loss events are independent across time and types
- No correlation between losses and business performance (other than through revenue exposure)
- Taxes are applied at a flat rate of 25%
- PP&E ratio is set to 0%; no property, plant, equipment or depreciation expense
- No external capital raises
- Retention ratio is set at 70% of net income (30% dividend payout)
- Insurance premiums are fixed at inception using the underlying loss distribution and subsequently scale with revenue
- Insurance pricing uses a fixed target loss ratio (60%, 70%, or 80%) without risk loadings or market cycle dynamics
- Insurance recoveries above the deductible have no balance sheet impact; insurer credit risk is not modeled
- No inflation, discounting, or time value of money considerations

- No regulatory capital requirements
- No investment income
- Timing resolution is annual (not monthly)
- Letter of Credit (LoC) mechanism for claim collateral:
  - When losses occur, the company's deductible portion requires immediate collateral via LoC, which has a 1.5% annual interest rate charge on outstanding total collateral
  - Collateral is posted immediately upon claim occurrence
  - Collateral is released gradually as claims are paid according to the payment pattern
  - Creates restricted assets on the balance sheet that cannot be used for operations
- Claim payment patterns follow a fixed 10-year schedule:

Year	1	2	3	4	5	6	7	8	9	10
Payment %	10%	15%	20%	15%	10%	10%	8%	6%	4%	2%

4.3 Model Validation and Calibration

The parameters were calibrated to achieve realistic loss burdens: a company with $10 million initial capital, 1.0 asset turnover ratio, and 10% operating margin experiences approximately 5% EBITDA after a $100,000 deductible. This is substantial enough to matter strategically but not so punitive as to overwhelm business fundamentals. Importantly, the calibration targets reasonableness rather than industry-specific accuracy, maintaining generality for cross-sector insights. The framework allows tailoring to specific industries and company adjustments as needed, although other industries may require additional features to be built out. In particular, stochastic revenue or complex risk management in investment firms and banks will require further development.

4.4 Computational Considerations

The simulation employs Monte Carlo methods with:

- 10,000 simulation paths per scenario
- 50-year projection horizons
- Annual decision points

Parallel processing and efficient data structures ensure tractability despite the high dimensionality of the parameter space. Each full scenario run takes approximately 30-60 minutes on a standard workstation, allowing exploration of multiple parameter combinations within reasonable timeframes. The experiment described in this paper was performed in a cloud computing environment (Google Colab) in a few hours and at a cost of approximately $10, so it can be easily replicated by anybody looking to validate or extend the findings.

Static parameters, such as tailoring the model to a specific industry or company, or incorporating discounting, will not significantly affect run times. However, adding stochastic revenue or margin fluctuations, more complex insurance structures, or regulatory capital requirements will increase computational demands and may require further optimization.

5 Case Study: Company Size and Insurance Appetite

5.1 Methodology and Assumptions

Our primary hypothesis is that insurance appetite, defined as the willingness to underwrite additional risk, varies systematically with company size.

Key assumptions:

- Companies maximize long-term growth rates rather than expected utility
- Risk appetite is measured by the Kelly criterion adaptation for insurance
- Company size is proxied by annual revenue

5.2 Data Sources and Parameterization

5.3 Results Presentation and Interpretation

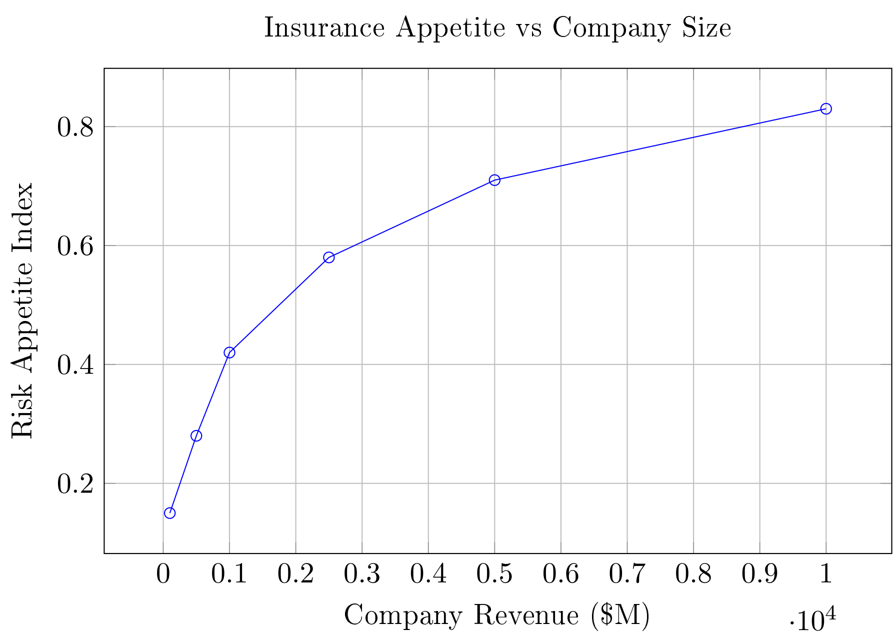

Figure 1: Risk appetite increases with company size (simulated results)

Figure 1 demonstrates the positive correlation between company size and risk appetite, consistent with ergodicity economics predictions.

5.4 Sensitivity Analysis

6 Industry Implications

6.1 Applications for Underwriting and Pricing

The simulation framework provides several practical applications:

- Dynamic pricing models that account for company size effects

- Portfolio optimization considering temporal risk dynamics
- Competitive positioning analysis

6.2 Portfolio Management Considerations

6.3 Regulatory and Capital Allocation Insights

7 Future Research Directions

7.1 Framework Extensions and Enhancements

Potential areas for future development include:

- Integration with catastrophe modeling
- Multi-line portfolio effects
- Reinsurance optimization

7.2 Additional Applications in P&C Insurance

7.3 Industry Collaboration Opportunities

8 Conclusion

This white paper has introduced a novel simulation framework based on ergodicity economics principles for analyzing insurance risk appetite. Our key findings demonstrate that company size significantly influences risk appetite, with larger insurers exhibiting greater willingness to underwrite risk. This finding has important implications for competitive dynamics and market structure in P&C insurance.

The simulation framework provides a powerful tool for:

- Understanding temporal risk dynamics
- Optimizing portfolio decisions
- Informing strategic planning

I encourage industry practitioners to explore these concepts further and welcome collaboration on extending this research.

The framework will provide new insights for companies making insurance decisions and is intended to answer questions such as "what is the ROI of our insurance program?", "how much insurance do we need?" and considerations of optimal insurance deductibles and limits.

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A Technical Appendix

A.1 Simulation Algorithm Details

A.2 Parameter Estimation Methods

A.3 Code Repository

The simulation framework code is available at:
https://github.com/AlexFiliakov/Ergodic-Insurance-Limits