6 Trajectory Planning

6.1 Constant Velocity Trajectory

$$\dot{x}(t) = C$$
$$\rightarrow \int_0^T \dot{x}(t)dt = \int_0^T C dt$$
$$x(T) - x(0) = C \cdot [T - 0]$$
$$\rightarrow C = \frac{x(T) - x(0)}{T}$$
$$\rightarrow x(t) = \frac{x(T) - x(0)}{T} \cdot t + x(0)$$
$$\implies [x, \dot{x}, \ddot{x}] = \left[\frac{x(T) - x(0)}{T} \cdot t + x(0), \frac{x(T) - x(0)}{T}, 0\right]$$

6.2 Trapezoidal Velocity Trajectory

There are 3 domains to the piece-wise velocity function:

$$\left[(0 < t < \frac{T}{6}), (\frac{T}{6} < t < \frac{5T}{6}), (\frac{5T}{6} < t < T)\right]$$

Domain 1 ($0 < t < \frac{T}{6}$):

$$\ddot{x}_1 = a$$
$$\dot{x}_1 = at + C_1 \rightarrow \dot{x}_1 = 0 \rightarrow C_1 = 0$$
$$\rightarrow x_1 = \frac{1}{2}at^2 + C_2 \rightarrow x_1(t = 0) = x(0) = C_2$$
$$\rightarrow x_1 = \frac{1}{2}at^2 + x(0)$$

Domain 2 ($\frac{T}{6} < t < \frac{5T}{6}$):

$$\ddot{x}_2 = 0$$
$$\dot{x}_2 = C_1 \rightarrow \dot{x}_2(t = \frac{T}{6}) = a \cdot \frac{T}{6} = C_1$$
$$x_2 = a\frac{T}{6} \cdot t + C_2 \rightarrow x_2(t = \frac{T}{6}) = \frac{1}{2}a\frac{T^2}{36} + x(0) = a\frac{T^2}{72} + C_2 \rightarrow C_2 = x(0) - a\frac{T^2}{72}$$
$$\rightarrow x_2 = a\frac{T}{6} \cdot t - a\frac{T^2}{72} + x(0)$$

Domain 3 ($\frac{5T}{6} < t < T$):

$$\ddot{x}_3 = -a$$
$$\rightarrow \dot{x}_3 = -at + C_1 \rightarrow \dot{x}_3(t = T) = 0 \rightarrow C_1 = a \cdot T$$
$$\rightarrow \dot{x}_3 = -at + a \cdot T$$
$$\rightarrow x_3 = -\frac{1}{2}a \cdot t^2 + aT \cdot t + C_2$$
$$\rightarrow x(t = \frac{5T}{6}) = \frac{1}{8}aT^2 + x(0) = -\frac{1}{2}a \cdot \frac{5T}{6}^2 + aT \cdot \frac{5T}{6} + C_2$$
$$\rightarrow C_2 = x(0) - \frac{13}{36}aT^2$$
$$\rightarrow x_3 = -\frac{1}{2}a \cdot t^2 + aT \cdot t - \frac{13}{36}aT^2 + x(0)$$

Condition on acceleration:

$$x_3(t = T) = \frac{5}{36}aT^2 + x(0) = x(T)$$
$$\implies a = (x(T) - x(0)) \cdot \frac{36}{5T^2}$$

$$\Longrightarrow [x, \dot{x}, \ddot{x}]_{domain1} = \left[\frac{1}{2}at^2 + x(0), at, a\right]$$

$$\Longrightarrow [x, \dot{x}, \ddot{x}]_{domain2} = \left[a\frac{T}{6} \cdot t - a\frac{T^2}{72} + x(0), a \cdot \frac{T}{6}, 0\right]$$

$$\Longrightarrow [x, \dot{x}, \ddot{x}]_{domain3} = \left[-\frac{1}{2}a \cdot t^2 + aT \cdot t - \frac{13}{36}aT^2 + x(0), -at + a \cdot T, -a\right]$$

6.3 Polynomial Velocity

Constraints:

$$x(t=0) = x(0), x(t=T) = x(T)$$

$$\dot{x}(t=0) = 0, \dot{x}(t=T) = 0$$

$$\ddot{x}(t=0) = 0, \ddot{x}(t=T) = 0$$

6 constraints $\Longrightarrow 5_{th}$ order polynomial for the trajectory:

$$x(t) = a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0$$

$$\dot{x}(t) = 5a_5t^4 + 4a_4t^3 + 3a_3t^2 + 2a_2t + a_1$$

$$\ddot{x}(t) = 20a_4t^4 + 12a_4t^2 + 6a_3t + 2a_2$$

from plugging in the constraints, we discover $a_0, a_1, a_2 = 0$, and are left with the following system of equations:

$$\begin{bmatrix} 20T^2 & 12T & 6 \\ 5T^2 & 4T & 3 \\ T^5 & T^4 & T^3 \end{bmatrix} \cdot \begin{bmatrix} a_5 \\ a_4 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ x(T) - x(0) \end{bmatrix}$$

Using symbolic MATLAB, we find:

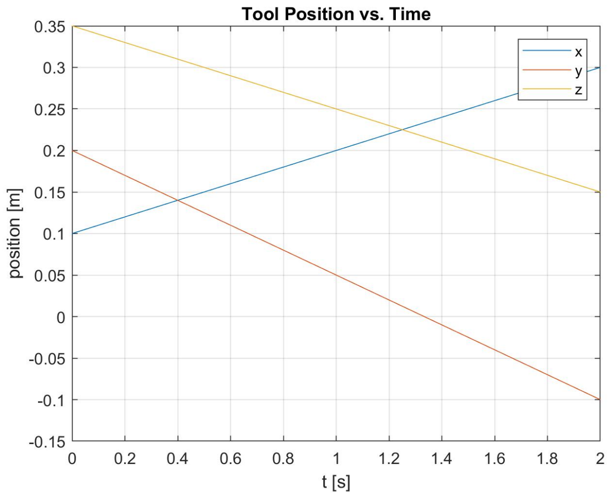
$$\begin{bmatrix} a_5 \\ a_4 \\ a_3 \end{bmatrix} = \begin{bmatrix} 6\frac{x(T)-x(0)}{T^5} \\ -15\frac{x(T)-x(0)}{T^4} \\ 10\frac{x(T)-x(0)}{T^3} \end{bmatrix}$$

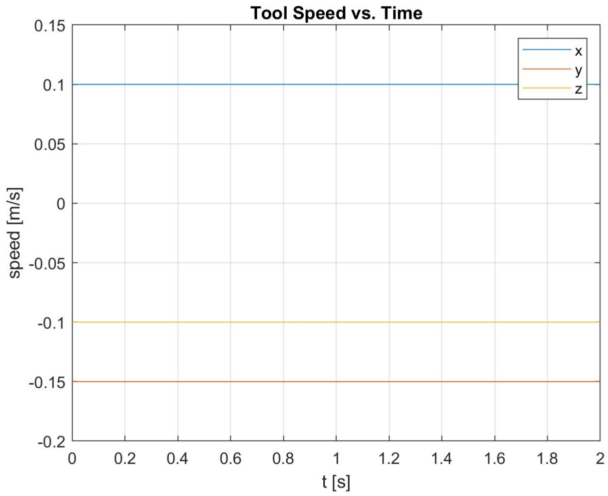
$$\Longrightarrow [x, \dot{x}, \ddot{x}] = \left[a_5t^5 + a_4t^4 + a_3t^3, 5a_5t^4 + 4a_4t^3 + 3a_3t^2, 20a_4t^4 + 12a_4t^2 + 6a_3t\right]$$

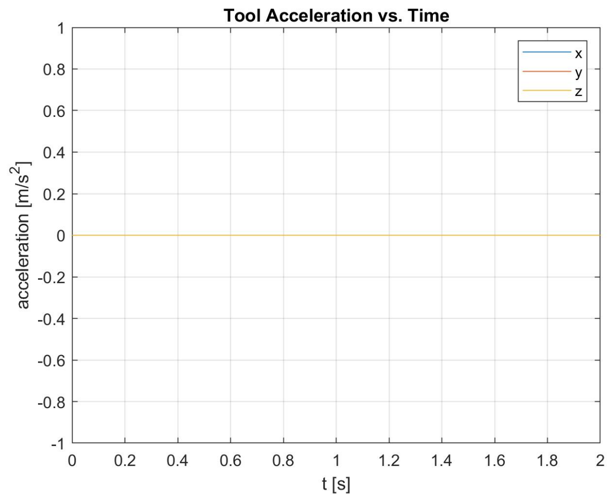
6.4 Tool Trajectory Results

The following graphs display tool position, velocity, and acceleration relative to the world frame.

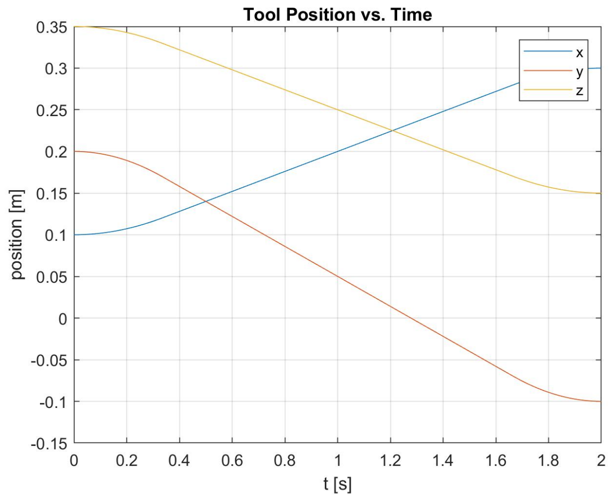
6.4.1 Constant Velocity Profile

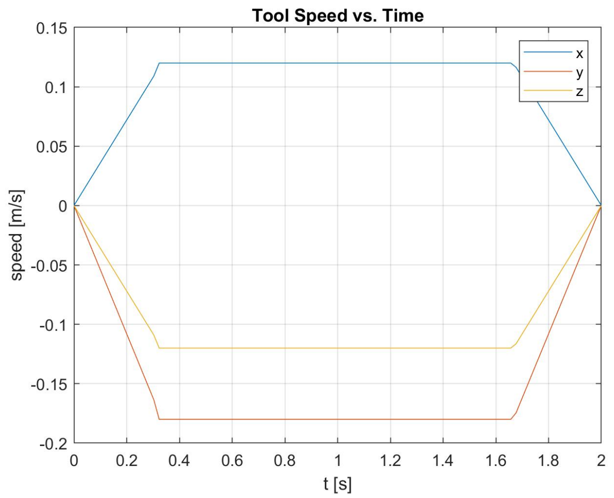


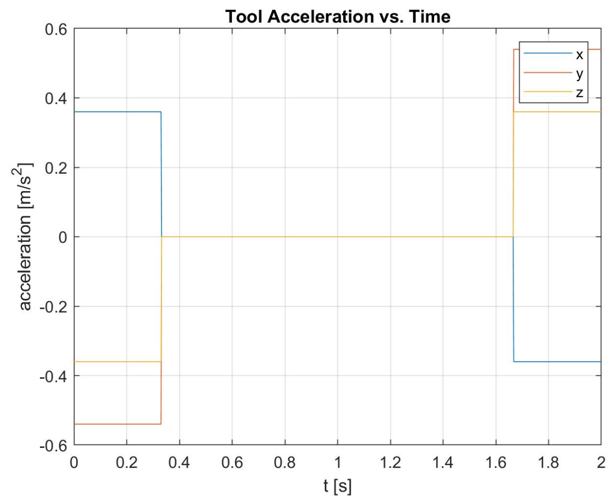




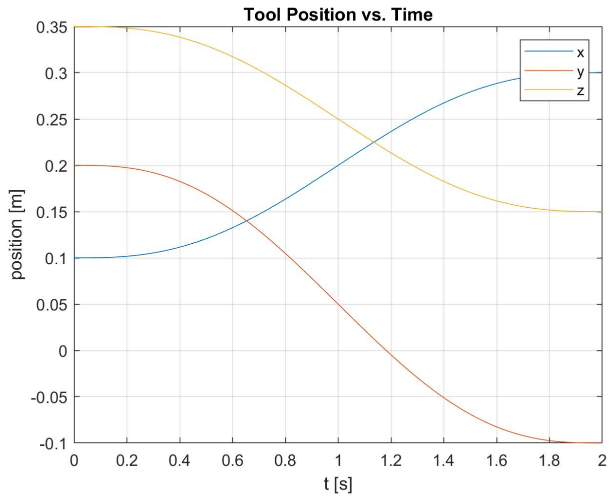
6.4.2 Trapezoidal Velocity Profile

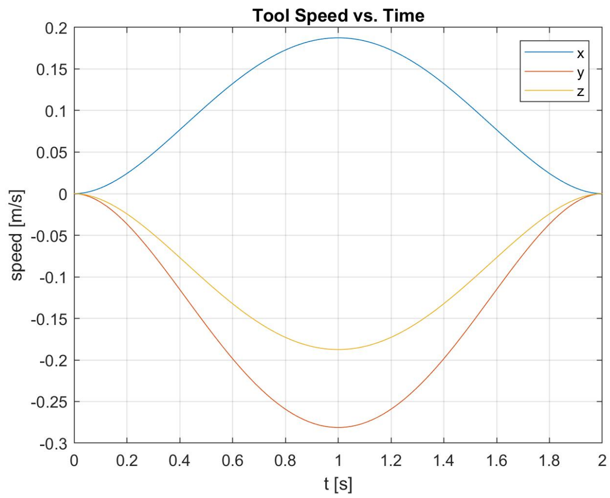


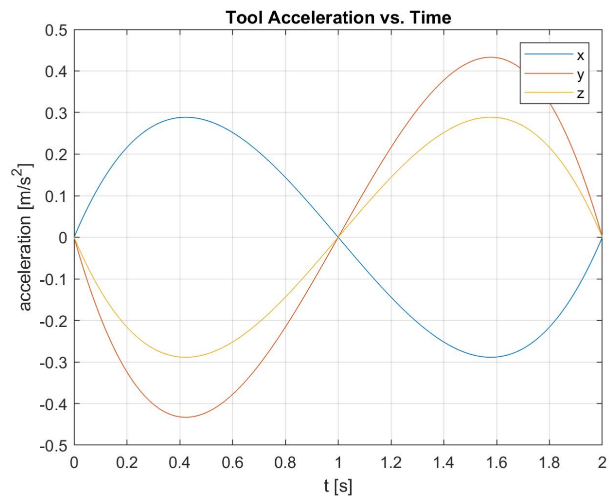




6.4.3 Polynomial Velocity Profile



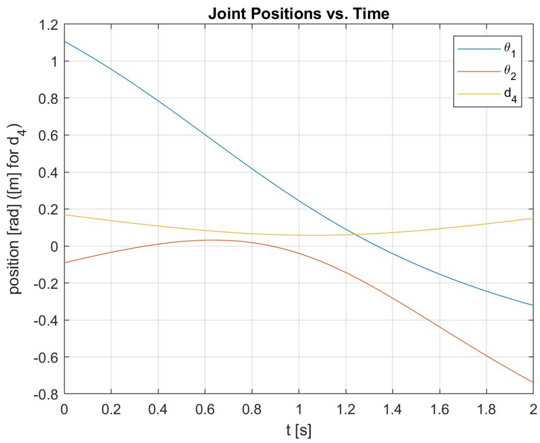


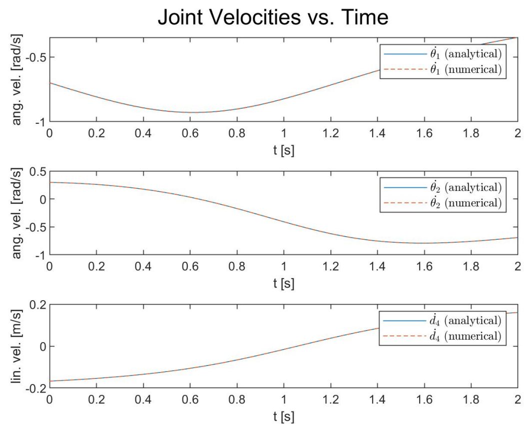


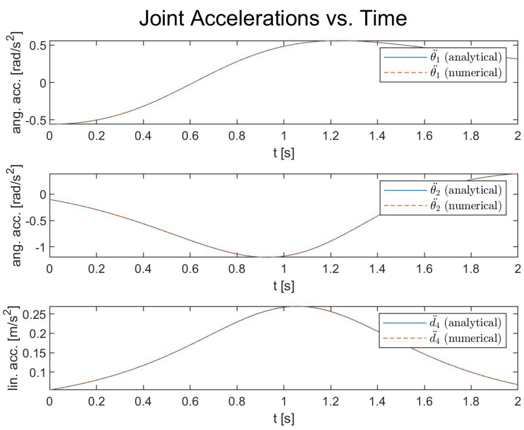
6.5 Joint Trajectory Results

The following graphs of joint position prove that we remain within the mechanical limitations, and therefore this specific trajectory is achievable. Furthermore, we see that the analytical and numerical solutions for joint velocity and acceleration are almost identical, except for a few points of discontinuity which occur in the numerically derived joint velocity and acceleration curves of the Trapezoidal trajectory. This is expected since the gradient function makes use of 2 points next to each other in an array, and therefore if there was a piece-wise jump in the graph, the function will not return an accurate value of the instantaneous derivative.

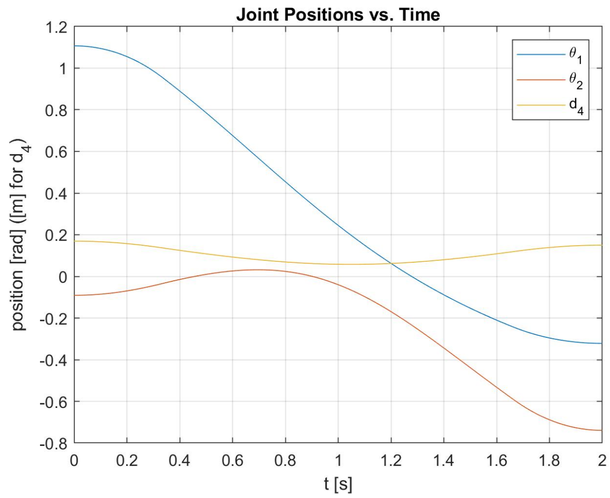
6.5.1 Constant Velocity Profile

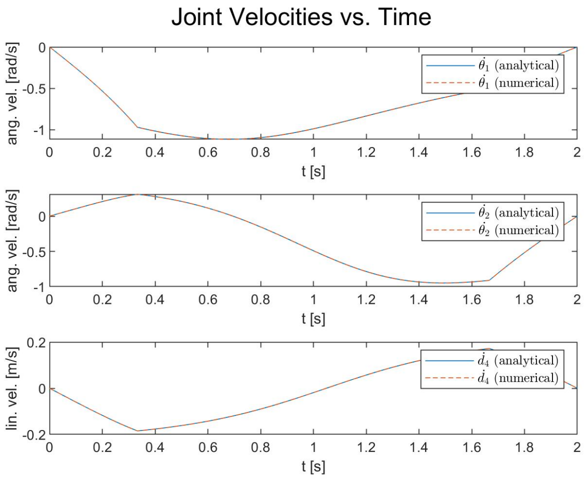


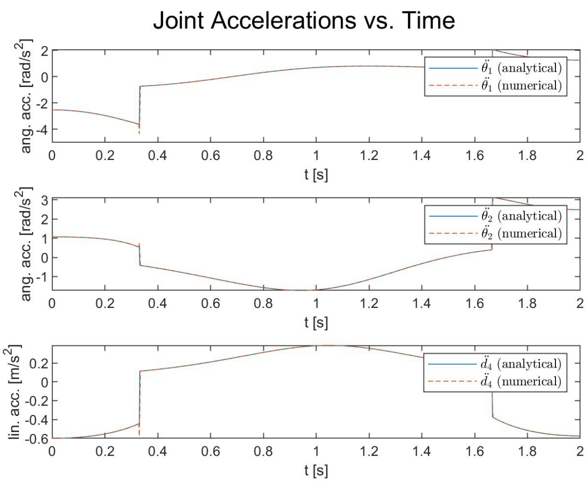




6.5.2 Trapezoidal Velocity Profile







6.5.3 Polynomial Velocity Profile