UNIVERSIDADE DA BEIRA INTERIOR

CURSO: ....MESTRADO INTEGRADO EM ENGENHARIA AERONÁUTICA – 3º ANO
UNIDADE CURRICULAR:...................................... ESTRUTURAS AEROESPACIAIS II – 10373

FORMULÁRIO

Lei das Misturas

$$E_1 = V_f E_f + V_m E_m \ \ , \ \ \nu_{12} = V_f \nu_f + V_m \nu_m$$

Rotação de tensões

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} cos^2\,\theta & sin^2\,\theta & 2\,sin\,\theta\,cos\,\theta \\ sin^2\,\theta & cos^2\,\theta & -2\,sin\,\theta\,cos\,\theta \\ -\,sin\,\theta\,cos\,\theta & sin\,\theta\,cos\,\theta & cos^2\,\theta - sin^2\,\theta \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} cos^2\,\theta & sin^2\,\theta & -2\,sin\,\theta\,cos\,\theta \\ sin^2\,\theta & cos^2\,\theta & 2\,sin\,\theta\,cos\,\theta \\ sin\,\theta\,cos\,\theta & -\,sin\,\theta\,cos\,\theta & cos^2\,\theta - sin^2\,\theta \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix}$$

Rotação de extensões

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} cos^2\,\theta & sin^2\,\theta & sin\,\theta\,cos\,\theta \\ sin^2\,\theta & cos^2\,\theta & -\,sin\,\theta\,cos\,\theta \\ -2\,sin\,\theta\,cos\,\theta & 2\,sin\,\theta\,cos\,\theta & cos^2\,\theta - sin^2\,\theta \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} cos^2\,\theta & sin^2\,\theta & -\,sin\,\theta\,cos\,\theta \\ sin^2\,\theta & cos^2\,\theta & sin\,\theta\,cos\,\theta \\ 2\,sin\,\theta\,cos\,\theta & -2\,sin\,\theta\,cos\,\theta & cos^2\,\theta - sin^2\,\theta \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix}$$

Relações tensão-extensão

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{K}_{11} & \bar{K}_{12} & \bar{K}_{13} \\ \bar{K}_{21} & \bar{K}_{22} & \bar{K}_{23} \\ \bar{K}_{31} & \bar{K}_{32} & \bar{K}_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{K}_{11} \\ \bar{K}_{22} \\ \bar{K}_{33} \\ \bar{K}_{12} \\ \bar{K}_{13} \\ \bar{K}_{23} \end{Bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2)^2 \\ m^2n^2 & m^2n^2 & m^4+n^4 & -4m^2n^2 \\ m^3n & -mn^3 & mn^3-m^3n & 2(mn^3-m^3n) \\ mn^3 & -m^3n & m^3n-mn^3 & 2(m^3n-mn^3) \end{bmatrix} \begin{Bmatrix} K_{11} \\ K_{22} \\ K_{12} \\ K_{33} \end{Bmatrix} \ , m = cos\theta \ , n = sen\theta$$

$$K_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}} \quad ; \quad K_{12} = \frac{\upsilon_{21}E_1}{1-\upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{12}E_2}{1-\upsilon_{12}\upsilon_{21}} \quad ; \quad K_{22} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}} \quad ; \quad K_{33} = G_{12}$$

Relações extensão-tensão

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} \\ \bar{S}_{21} & \bar{S}_{22} & \bar{S}_{23} \\ \bar{S}_{31} & \bar{S}_{32} & \bar{S}_{33} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{S}_{11} \\ \bar{S}_{22} \\ \bar{S}_{33} \\ \bar{S}_{12} \\ \bar{S}_{13} \\ \bar{S}_{23} \end{Bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & m^2n^2 \\ 4m^2n^2 & 4m^2n^2 & -8m^2n^2 & (m^2-n^2)^2 \\ m^2n^2 & m^2n^2 & m^4+n^4 & -m^2n^2 \\ 2m^3n & -2mn^3 & 2(mn^3-m^3n) & mn^3-m^3n \\ 2mn^3 & -2m^3n & 2(m^3n-mn^3) & m^3n-mn^3 \end{bmatrix} \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{12} \\ S_{33} \end{Bmatrix}, m = cos\theta \; , n = sen\theta$$

$$S_{11} = \frac{1}{E_1} \quad ; \quad S_{12} = -\frac{\upsilon_{21}}{E_2} = -\frac{\upsilon_{12}}{E_1} \quad ; \quad S_{22} = \frac{1}{E_2} \quad ; \quad S_{33} = \frac{1}{G_{12}}$$

Critério de falha de Tsai-Hill

$$f = \left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 - \left(\frac{\sigma_1}{X}\right)\left(\frac{\sigma_2}{X}\right)$$

Análise de laminados

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} \; , \; A_{ij} = \sum_{p=1}^{N} t_p\left(\overline{K}_{ij}\right)_p \; , \; i = 1{,}3 \; , \; j = 1{,}3$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = t \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\sigma}_{xy} \end{Bmatrix}$$

$$a_{11} = (A_{22}A_{33} - A_{23}^2)/AA$$
$$a_{22} = (A_{11}A_{33} - A_{13}^2)/AA$$
$$a_{33} = (A_{11}A_{22} - A_{12}^2)/AA$$
$$a_{12} = (A_{13}A_{23} - A_{12}A_{33})/AA$$
$$a_{13} = (A_{12}A_{23} - A_{22}A_{13})/AA$$
$$a_{23} = (A_{12}A_{13} - A_{11}A_{23})/AA$$
$$AA = A_{11}A_{22}A_{33} + 2A_{12}A_{23}A_{13} - A_{22}A_{13}^2 - A_{33}A_{12}^2 - A_{11}A_{23}^2$$

$$t \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{m_x}{E_x} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{m_y}{E_y} \\ -\frac{m_x}{E_x} & -\frac{m_y}{E_y} & \frac{1}{G_{xy}} \end{bmatrix}$$

Análise de vigas de secção de paredes finas com diferentes materiais

Carga axial:

$$\sigma_Z = E_{Z,i}\varepsilon_z = E_{Z,i}\frac{P}{\sum_{j=1}^{n} A_j E_{Z,j}}$$

Flexão:

$$\sigma_Z = E_{Z,i}\left[\left(\frac{M_Y I'_{XX} - M_X I'_{XY}}{I'_{XX}I'_{YY} - {I'_{XY}}^2}\right)X + \left(\frac{M_X I'_{YY} - M_Y I'_{XY}}{I'_{XX}I'_{YY} - {I'_{XY}}^2}\right)Y\right]$$

Corte:

$$q_s = -E_{Z,i}\left[\left(\frac{S_X I'_{XX} - S_Y I'_{XY}}{I'_{XX}I'_{YY} - {I'_{XY}}^2}\right)\int_0^s t_i X ds + \left(\frac{S_Y I'_{YY} - S_X I'_{XY}}{I'_{XX}I'_{YY} - {I'_{XY}}^2}\right)\int_0^s t_i Y ds\right] + q_{s,0}$$

$$q_s = -E_{Z,i}\left[\left(\frac{S_X I'_{XX} - S_Y I'_{XY}}{I'_{XX}I'_{YY} - {I'_{XY}}^2}\right)\sum_{r=1}^{n} B_r X_r + \left(\frac{S_Y I'_{YY} - S_X I'_{XY}}{I'_{XX}I'_{YY} - {I'_{XY}}^2}\right)\sum_{r=1}^{n} B_r Y_r\right] + q_{s,0} \quad \text{para secção idealizada}$$

$$S_X\eta_0 - S_Y\xi_0 = \oint pq_b + 2Aq_{s,0}$$

Torção:

$$\frac{d\theta}{dZ} = \frac{T}{GJ}$$

$$GJ = \frac{4A^2}{\oint \frac{ds}{G_{XY,i}t_i}} \text{ para secções fechadas}$$

$$GJ = \frac{1}{3}\int_{sec} G_{XY,i}t_i^3 ds \text{ para secções abertas}$$

$$q = \frac{T}{2A} \text{ para secções fechadas}$$

$$\tau = 2G_{XY,i}n\frac{d\theta}{dZ} \text{ para secções abertas}$$

Área do "boom" associada a uma estrutura idealizada

$$B_r = A_r + \frac{1}{E_{Z,r}}\sum_{j=1}^{N} E_{Z,j}\frac{t_j b_j}{6}\left(2 + \frac{\sigma_{Z,j}}{\sigma_{Z,r}}\right)$$

Matriz de rigidez de uma treliça articulada

$$[K_{ij}] = \frac{AE}{L}\begin{bmatrix} \lambda^2 & \lambda\mu & -\lambda^2 & -\lambda\mu \\ \lambda\mu & \mu^2 & -\lambda\mu & -\mu^2 \\ -\lambda^2 & -\lambda\mu & \lambda^2 & \lambda\mu \\ -\lambda\mu & -\mu^2 & \lambda\mu & \mu^2 \end{bmatrix}, \lambda = cos\theta \,, \mu = sen\theta$$

Força interna:

$$S_{ij} = \frac{AE}{L}[\lambda \quad \mu]_{ij}\begin{Bmatrix} u_j - u_i \\ v_j - v_i \end{Bmatrix}$$

Matriz de rigidez de uma viga uniforme

$$[K_{ij}] = EI\begin{bmatrix} \frac{12\mu^2}{L^3} & -\frac{12\lambda\mu}{L^3} & \frac{6\mu}{L^2} & -\frac{12\mu^2}{L^3} & \frac{12\lambda\mu}{L^3} & \frac{6\mu}{L^2} \\ -\frac{12\lambda\mu}{L^3} & \frac{12\lambda^2}{L^3} & -\frac{6\lambda}{L^2} & \frac{12\lambda\mu}{L^3} & -\frac{12\lambda^2}{L^3} & -\frac{6\lambda}{L^2} \\ \frac{6\mu}{L^2} & -\frac{6\lambda}{L^2} & \frac{4}{L} & -\frac{6\mu}{L^2} & \frac{6\lambda}{L^2} & \frac{2}{L} \\ -\frac{12\mu^2}{L^3} & \frac{12\lambda\mu}{L^3} & -\frac{6\mu}{L^2} & \frac{12\mu^2}{L^3} & -\frac{12\lambda\mu}{L^3} & -\frac{6\mu}{L^2} \\ \frac{12\lambda\mu}{L^3} & -\frac{12\lambda^2}{L^3} & \frac{6\lambda}{L^2} & -\frac{12\lambda\mu}{L^3} & \frac{12\lambda^2}{L^3} & \frac{6\lambda}{L^2} \\ \frac{6\mu}{L^2} & -\frac{6\lambda}{L^2} & \frac{2}{L} & -\frac{6\mu}{L^2} & \frac{6\lambda}{L^2} & \frac{4}{L} \end{bmatrix}, \lambda = cos\theta \,, \mu = sen\theta$$

Força e momento internos:

$$\begin{Bmatrix} S_y \\ M \end{Bmatrix} = EI\begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{12}{L^3}x - \frac{6}{L^2} & -\frac{6}{L^2}x + \frac{4}{L} & \frac{12}{L^3}x + \frac{6}{L^2} & -\frac{6}{L^2}x + \frac{2}{L} \end{bmatrix}\begin{Bmatrix} \bar{v}_i \\ \theta_i \\ \bar{v}_j \\ \theta_j \end{Bmatrix}$$

Mecânica da fratura

$$\frac{da}{dN} = C(\Delta K)^m \;,\; N_r = \frac{1}{C(\Delta\sigma)^m}\int_{a_i}^{a_f}\frac{da}{(\Upsilon\sqrt{\pi a})^m}$$

Fator de intensidade de tensão:

$$K_I = XY\sigma\sqrt{\pi a}$$

Aeroelasticidade

Velocidade de divergência:

$$U_d = \sqrt{\frac{2k_\theta}{\rho S(x_{ea} - x_{aa})\frac{dC_l}{d\alpha}}}$$