Software Language Engineering Semantics: Interpreters

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Recap

- Grammar -> Parser -> Parse Tree -> AST
- Name resolution: recover referential structure
- Checking: find errors not captured by syntax
- Today:
  - semantics
  - interpreters

Formal semantics

- Axiomatic semantics
- Operational semantics
  - small-step
  - big-step (aka “Natural semantics”)
- Denotational semantics

Formal semantics

- Prove things (e.g., determinism, type soundness)
- Simulate & explore (e.g. using Redex)
- Generate interpreter

Big-step

$$\frac{\langle b, \sigma \rangle \Downarrow \mathbf{true} \quad \langle c_0, \sigma \rangle \Downarrow \sigma'}{\langle \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, \sigma \rangle \Downarrow \sigma'} \quad \text{(conditionals)}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \mathbf{false} \quad \langle c_1, \sigma \rangle \Downarrow \sigma'}{\langle \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, \sigma \rangle \Downarrow \sigma'}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \mathbf{false}}{\langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma \rangle \Downarrow \sigma} \quad \text{(while loops)}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \mathbf{true} \quad \langle c, \sigma \rangle \Downarrow \sigma'' \quad \langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma'' \rangle \Downarrow \sigma'}{\langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma \rangle \Downarrow \sigma'}$$

Small-step

$$[\text{LEFT}] \quad \frac{E_0 \longrightarrow E_0'}{E_0 \times E_1 \longrightarrow E_0' \times E_1}$$

$$[\text{MUL}] \quad \frac{}{z_0 \times z_1 \longrightarrow z} \quad z = z_0 z_1$$

$$[\text{RIGHT}] \quad \frac{E_1 \longrightarrow E_1'}{E_0 \times E_1 \longrightarrow E_0 \times E_1'}$$

$$[\text{COND}] \quad \frac{E_0 \longrightarrow E_0'}{E_0 \,?\, E_1 : E_2 \longrightarrow E_0' \,?\, E_1 : E_2}$$

$$[\text{LEFT}_0] \quad \frac{}{0 \times E_1 \longrightarrow 0}$$

$$[\text{COND}_\text{Z}] \quad \frac{}{z \,?\, E_1 : E_2 \longrightarrow E_2} \quad z = 0$$

$$[\text{RIGHT}_0] \quad \frac{}{E_0 \times 0 \longrightarrow 0}$$

$$[\text{COND}_\text{NZ}] \quad \frac{}{z \,?\, E_1 : E_2 \longrightarrow E_1} \quad z \neq 0$$

In code…

- Big-step: eval(Exp, Env, Store) -> <Value, Store>
- Small-step: step(Exp, Env, Store) -> <Exp, Store>
- Denotational: map(Exp) -> (Env x Store -> Store)

```
    evalquote[fn;x] = apply[fn;x;NIL]
```

where

```
    apply[fn;x;a] =
          [atom[fn] → [eq[fn;CAR] → caar[x];
                        eq[fn;CDR] → cdar[x];
                        eq[fn;CONS] → cons[car[x];cadr[x]];
                        eq[fn;ATOM] → atom[car[x]];
                        eq[fn;EQ] → eq[car[x];cadr[x]];
                        T → apply[eval[fn;a];x;a]];
          eq[car[fn];LAMBDA] → eval[caddr[fn];pairlis[cadr[fn];x;a]];
          eq[car[fn];LABEL] → apply[caddr[fn];x;cons[cons[cadr[fn];
                                                   caddr[fn]];a]]]
    eval[e;a] = [atom[e] → cdr[assoc[e;a]];
          atom[car[e]] →
                    [eq[car[e],QUOTE] → cadr[e];
                    eq[car[e];COND] → evcon[cdr[e];a];
                    T → apply[car[e];evlis[cdr[e];a];a]];
          T → apply[car[e];evlis[cdr[e];a];a]]
```

pairlis and assoc have been previously defined.

```
    evcon[c;a] = [eval[caar[c];a] → eval[cadar[c];a];
                  T → evcon[cdr[c];a]]
```

and

```
    evlis[m;a] = [null[m] → NIL;
                  T → cons[eval[car[m];a];evlis[cdr[m];a]]]
```

Recursion over structure of expressions

```
public int eval(Exp exp) {
  switch (exp) {
    case nat(int nat): return nat;

    case mul(Exp lhs, Exp rhs): return eval(lhs) * eval(rh

    case div(Exp lhs, Exp rhs): return eval(lhs) / eval(rhs);

    case add(Exp lhs, Exp rhs): return eval(lhs) + eval(rhs);

    case min(Exp lhs, Exp rhs): return eval(lhs) - eval(rhs);

    case gt(Exp lhs, Exp rhs):  return eval(lhs) > eval(rhs) ? 1 : 0;

    case lt(Exp lhs, Exp rhs):  return eval(lhs) < eval(rhs) ? 1 : 0;

    case geq(Exp lhs, Exp rhs): return eval(lhs) >= eval(rhs) ? 1 : 0;

    case leq(Exp lhs, Exp rhs): return eval(lhs) <= eval(rhs) ? 1 : 0;

    case cond(Exp cond, Exp then, Exp otherwise):
      return eval(cond) != 0 ?
          eval(then) : eval(otherwise);
  }
}
```

Example

Formal parameter

Actual parameter

```
fact(n) =
  if n <= 1 then
    1
  else
    n * fact(n-1)
  end
```

Idea: substitute formals

Call stack

```
fact(4)
 4 * fact(3)
  3 * fact(2)
   2 * fact(1)
        1
```

```
if 4 <= 1 then
  1
else
  4 * fact(3)
end
```

```
if 3 <= 1 then
  1
else
  3 * fact(2)
end
```

```
if 2 <= 1 then
  1
else
  2 * fact(1)
end
```

```
if 1 <= 1 then
  1
else
  1 * fact(0)
end
```

Lookup table for functions

```
alias PEnv = map[str, Func];
```

a procedure environment

maps names to functions

```
data Prog = prog(list[Func] funcs);
data Func = func(str name, list[str] formals, Exp body);
```

```
public int eval(Exp exp) {
  switch (exp) {
    case nat(int nat): return nat;

    case mul(Exp lhs, Exp rhs): return eval(lhs) * eval(rhs);

    case div(Exp lhs, Exp rhs): return eval(lhs) / eval(rhs);

    case add(Exp lhs, Exp rhs): return eval(lhs) + eval(rhs);

    case min(Exp lhs, Exp rhs): return eval(lhs) - eval(rhs);

    case gt(Exp lhs, Exp rhs):  return eval(lhs) > eval(rhs) ? 1 : 0;

    case lt(Exp lhs, Exp rhs):  return eval(lhs) < eval(rhs) ? 1 : 0;

    case geq(Exp lhs, Exp rhs): return eval(lhs) >= eval(rhs) ? 1 : 0;

    case leq(Exp lhs, Exp rhs): return eval(lhs) <= eval(rhs) ? 1 : 0;

    case cond(Exp cond, Exp then, Exp otherwise):
      return eval(cond) != 0 ?
          eval(then) : eval(otherwise);
  }
}
```

```
public int eval(Exp exp, PEnv penv) {
  switch (exp) {
    case nat(int nat): return nat;

    case mul(Exp lhs, Exp rhs): return eval(lhs, penv) * eval(rhs, penv);

    case div(Exp lhs, Exp rhs): return eval(lhs, penv) / eval(rhs, penv);

    case add(Exp lhs, Exp rhs): return eval(lhs, penv) + eval(rhs, penv);

    case min(Exp lhs, Exp rhs): return eval(lhs, penv) - eval(rhs, penv);

    case gt(Exp lhs, Exp rhs):  return eval(lhs, penv) > eval(rhs, penv) ? 1 : 0;

    case lt(Exp lhs, Exp rhs):  return eval(lhs, penv) < eval(rhs, penv) ? 1 : 0;

    case geq(Exp lhs, Exp rhs): return eval(lhs, penv) >= eval(rhs, penv) ? 1 : 0;

    case leq(Exp lhs, Exp rhs): return eval(lhs, penv) <= eval(rhs, penv) ? 1 : 0;

    case cond(Exp cond, Exp then, Exp otherwise):
      return eval(cond, penv) != 0 ?
          eval(then, penv) : eval(otherwise, penv);
  }
}
```

Evaluating calls

lookup the called function in the env

```
...
case call(str name, list[Exp] args): {
        f = penv[name];
        b = subst(f.body, f.formals, [ eval(a, penv) | a <- args])
        return eval(b, penv);
}
...
```

eval body where all variables have been substituted for values

eval actual args to perform substitution

Let bindings

```
(define let
  (macro (bindings . body)
    (define (named-let name bindings body)
      `(let ((,name #f))
         (set! ,name (lambda ,(map first bindings) . ,body))
         (,name . ,(map second bindings))))
  (if (symbol? bindings)
      (named-let bindings (first body) (rest body))
      `((lambda ,(map first bindings) . ,body) .
       ,(map second bindings)))))
```

```
(define-syntax let
    (syntax-rules ()
      ((let ((name val) ...) body1 body2 ...)
       ((lambda (name ...) body1 body2 ...)
        val ...))
      ((let tag ((name val) ...) body1 body2 ...)
       ((letrec ((tag (lambda (name ...)
                        body1 body2 ...)))

          tag)
        val ...))))
```

Func1 = Func0 + {let}

local variables

scope of bindings

```
syntax Exp = let: "let" {Binding ","}* "in" Exp "end";
syntax Binding = binding: Ident "=" Exp;
```

```
data Exp = let(list[Binding] bindings, Exp exp);
data Binding = binding(str var, Exp exp);
```

Example

```
fact(n) =
 let
   x = n
  in
   if x <= 1 then
     x
   else
     x * fact(x-1)
   end
  end
```

Shadowing

```
fact(n) =
 let
   n = n + 1
 in
   if n <= 1 then
     n
   else
     n * fact(n-1)
   end
 end
```

NB: not an assignment!

formal param *n* is shadowed by let-bound *n*

Substitution?

```
fact(4) =
 let
    n = 4 + 1
  in
    if 4 <= 1 then
      4
    else
      4 * fact(4-1)
    end
  end
```

Wrong

Different approach: environments

```
alias Env = map[str, int];
```

variable

values

```
public int eval(Exp exp, PEnv penv) {
  switch (exp) {
    case nat(int nat): return nat;

    case mul(Exp lhs, Exp rhs): return eval(lhs, penv) * eval(rhs, penv);

    case div(Exp lhs, Exp rhs): return eval(lhs, penv) / eval(rhs, penv);

    case add(Exp lhs, Exp rhs): return eval(lhs, penv) + eval(rhs, penv);

    case min(Exp lhs, Exp rhs): return eval(lhs, penv) - eval(rhs, penv);

    case gt(Exp lhs, Exp rhs):  return eval(lhs, penv) > eval(rhs, penv) ? 1 : 0;

    case lt(Exp lhs, Exp rhs):  return eval(lhs, penv) < eval(rhs, penv) ? 1 : 0;

    case geq(Exp lhs, Exp rhs): return eval(lhs, penv) >= eval(rhs, penv) ? 1 : 0;

    case leq(Exp lhs, Exp rhs): return eval(lhs, penv) <= eval(rhs, penv) ? 1 : 0;

    case cond(Exp cond, Exp then, Exp otherwise):
      return eval(cond, penv) != 0 ?
          eval(then, penv) : eval(otherwise, penv);
  }
}
```

```
public int eval(Exp exp, Env env, PEnv penv) {
  switch (exp) {
    case nat(int nat): return nat;

    case mul(Exp lhs, Exp rhs): return eval(lhs, env, penv) * eval(rhs, env, penv);

    case div(Exp lhs, Exp rhs): return eval(lhs, env, penv) / eval(rhs, env, penv);

    case add(Exp lhs, Exp rhs): return eval(lhs, env, penv) + eval(rhs, env, penv);

    case min(Exp lhs, Exp rhs): return eval(lhs, env, penv) - eval(rhs, env, penv);

    case gt(Exp lhs, Exp rhs):  return eval(lhs, env, penv) > eval(rhs, env, penv) ? 1 : 0;

    case lt(Exp lhs, Exp rhs):  return eval(lhs, env, penv) < eval(rhs, env, penv) ? 1 : 0;

    case geq(Exp lhs, Exp rhs): return eval(lhs, env, penv) >= eval(rhs, env, penv) ? 1 : 0;

    case leq(Exp lhs, Exp rhs): return eval(lhs, env, penv) <= eval(rhs, env, penv) ? 1 : 0;

    case cond(Exp cond, Exp then, Exp otherwise):
      return eval(cond, env, penv) != 0 ?
          eval(then, env, penv) : eval(otherwise, env, penv);
  }
}
```

Evaluating variables

```
case var(str name):
   return env[name];
```

lookup *name*
in *env*

Evaluating calls

```
...
case call(str name, list[Exp] args): {
  f = penv[name];
  env = bind(f.formals, [ eval(a, env, penv) | a <- args ]);
  return eval(f.body, env, penv);
}
...
```

create a **new** environment by binding actuals to formals

evaluate the body of *f* in the new env

Evaluating let

```
...
case let(list[Binding] bindings, Exp exp): {
   env += ( b.var : eval(b.exp, env, penv) | b <- bindings );
   return eval(exp, env, penv);
}
...
```

update the current environment

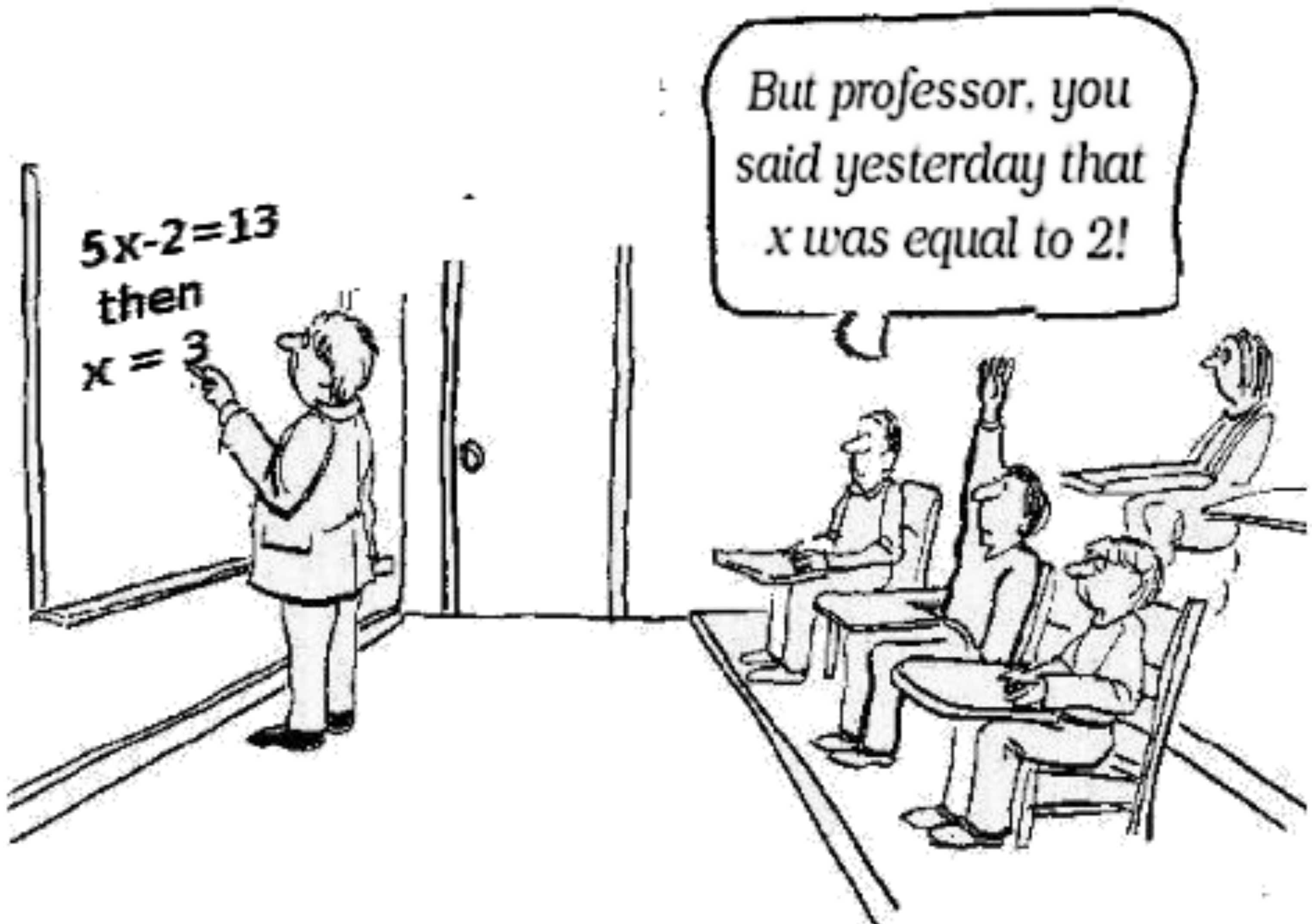
eval exp in the updated env

Evaluating with *env*

Call stack	**Output**	**Env**
fact(4):		
4*fact(3)	= 24	(“n”: 4)
3*fact(2)	= 6	(“n”: 3)
2*fact(1)	= 2	(“n”: 2)
1	= 1	(“n”: 1)

```
fact(n) =
  if n <= 1 then
    1
  else
    n * fact(n-1)
  end
```

Imperative features



Func2 = Func1 + {:=, ;}

```
syntax Exp =
           ...
           >
           right assign: Exp ":=" Exp
           >
           right seq: Exp ";" Exp;
```

sequencing is a right assoc binary operator

```
data Exp = seq(Exp lhs, Exp rhs)
         | assign(Exp exp, Exp exp);
```

Example

```
fact(n) =
    if n <= 1 then
      n := 1
    else
      n := n * fact(n-1)
    end;
    n
```

Returning the environment

- Previously, *env* was only passed *down* the expression tree
- For `S1; S2`, if `S1` performs assignments, they are lost!
- => the environment must be passed through evaluation

Result

```
alias Result = tuple[Env, int];
```

the (updated) environment

the computed result

```
public int eval(Exp exp, Env env, PEnv penv) {
  switch (exp) {
    case nat(int nat): return nat;

    case mul(Exp lhs, Exp rhs): return eval(lhs, env, penv) * eval(rhs, env, penv);

    case div(Exp lhs, Exp rhs): return eval(lhs, env, penv) / eval(rhs, env, penv);

    case add(Exp lhs, Exp rhs): return eval(lhs, env, penv) + eval(rhs, env, penv);

    case min(Exp lhs, Exp rhs): return eval(lhs, env, penv) - eval(rhs, env, penv);

    case gt(Exp lhs, Exp rhs):  return eval(lhs, env, penv) > eval(rhs, env, penv) ? 1 : 0;

    case lt(Exp lhs, Exp rhs):  return eval(lhs, env, penv) < eval(rhs, env, penv) ? 1 : 0;

    case geq(Exp lhs, Exp rhs): return eval(lhs, env, penv) >= eval(rhs, env, penv) ? 1 : 0;

    case leq(Exp lhs, Exp rhs): return eval(lhs, env, penv) <= eval(rhs, env, penv) ? 1 : 0;

    case cond(Exp cond, Exp then, Exp otherwise):
      return eval(cond, env, penv) != 0 ?
          eval(then, env, penv) : eval(otherwise, env, penv);
  }
}
```

```
public Result eval(Exp exp, Env env, PEnv penv) {
  switch (exp) {
    case nat(int nat):
       return <env, nat>;

    case var(str name):
       return <env, env[name]>;

    case mul(Exp lhs, Exp rhs): {
      <env, x> = eval(lhs, env, penv);
      <env, y> = eval(rhs, env, penv);
      return <env, x * y>;
    }

    case div(Exp lhs, Exp rhs): {
      <env, x> = eval(lhs, env, penv);
      <env, y> = eval(rhs, env, penv);
      return <env, x / y>;
    }

    case add(Exp lhs, Exp rhs): {
      <env, x> = eval(lhs, env, penv);
      <env, y> = eval(rhs, env, penv);
      return <env, x + y>;
    }

    case min(Exp lhs, Exp rhs): {
      <env, x> = eval(lhs, env, penv);
      <env, y> = eval(rhs, env, penv);
      return <env, x - y>;
    }

    case gt(Exp lhs, Exp rhs): {
      <env, x> = eval(lhs, env, penv);
      <env, y> = eval(rhs, env, penv);
      return <env, x > y ? 1 : 0>;
    }

    case lt(Exp lhs, Exp rhs): {
      <env, x> = eval(lhs, env, penv);
      <env, y> = eval(rhs, env, penv);
      return <env, x < y ? 1 : 0>;
    }
    ...
    case cond(Exp cond, Exp then, Exp otherwise
      <env, c> = eval(cond, env, penv);
      return c != 0 ?
       eval(then, env, penv) :
       eval(otherwise, env, penv);
    }
    case let(list[Binding] bindings, Exp exp):
       for (b <- bindings) {
         <env, x> = eval(b.exp, env, penv);
         env[b.var] = x;
       }
       return eval(exp, env, penv);
    }
```

Spot the difference

```
case mul(Exp lhs, Exp rhs): {
   <env, x> = eval(lhs, env, penv);
   <env, y> = eval(rhs, env, penv);
   return <env, x * y>;
 }
```

side-effects +
order matters

```
case mul(Exp lhs, Exp rhs): {
   <_, x> = eval(lhs, env, penv);
   <_, y> = eval(rhs, env, penv);
   return <env, x * y>;
 }
```

no side-effects
order irrelevant

Evaluating sequences

```
case seq(Exp lhs, Exp rhs): {
   <env, _> = eval(lhs, env, penv);
   return eval(rhs, env, penv);
}
...
```

ignore the value returned by *lhs*

pass the env of *lhs* to *rhs*

Evaluating assignment

only allow assigning variables (for now)

```
case assign(var(str name), Exp exp): {
  <env, v> = eval(exp, env, penv);
  env[name] = v;
  return <env, v>;
}
```

update the env with the result of eval'ing *exp*

Eval with side-effects

Call stack	**Output**	**Env**
fact(4):		
4*fact(3)	= <("n": 24), 24>	("n": 4)
3*fact(2)	= <("n": 6), 6>	("n": 3)
2*fact(1)	= <("n": 2), 2>	("n": 2)
1	= <("n": 1), 1>	("n": 1)

```
fact(n) =
  if n <= 1 then
    n := 1
  else
    n := n * fact(n-1)
  end;
  n
```

Summary

- Semantics:
  - Operational: big step & small step
  - Denotational: compositional mapping to semantic domain
- Interpreters: executable, operational semantics
  - Context information: environment, stores, output etc.

Next up

- State machines! ;)
- Outlook to QL exercise