MATH-1564, K1, TA: Sam, Instructor: Nitzan, Sigal Shahaf
HW4 ; Alexander Guo

1. (a) $AB = \begin{pmatrix} 5 & 2 & 5 \\ 0 & -1 & 5 \end{pmatrix}$
   (b) $BA =$ Undefined, inner dims don't match. $(2 \times \underline{3})(\underline{2} \times 2)$
   (c) $D^2 = \begin{pmatrix} 7 & -3 & 6 \\ -2 & 3 & -1 \\ 3 & -2 & 9 \end{pmatrix}$
   (d) $B^2 =$ Undefined, inner dims don't match. $(2 \times \underline{3})(\underline{2} \times 3)$
   (e) $DC = \begin{pmatrix} 7 & -1 \\ 0 & 5 \\ 1 & 2 \end{pmatrix}$
   (f) $CB = \begin{pmatrix} 5 & 2 & 5 \\ 5 & 1 & 10 \\ 0 & -1 & 5 \end{pmatrix}$
   (g) $BC = \begin{pmatrix} 7 & -1 \\ 7 & 4 \end{pmatrix}$
   (h) $FE = \begin{pmatrix} 2 \end{pmatrix}$
   (i) $EF = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & -3 \end{pmatrix}$
   (j) $CE =$ Undefined, inner dims don't match. $(3 \times \underline{2})(\underline{3} \times 1)$
   (k) $EC =$ Undefined, inner dims don't match. $(3 \times \underline{1})(\underline{3} \times 2)$

2. (a) i. A is invertible. $A^{-1} = \begin{pmatrix} 0.2 & 0.4 \\ 0.4 & -0.2 \end{pmatrix}$
   ii. B is not invertible because its rows and columns do not match. Its REF cannot be an identity matrix.
   iii. F is not invertible because its rows and columns do not match. Its REF cannot be an identity matrix.
   iv. C is not invertible because its rows and columns do not match. Its REF cannot be an identity matrix.
   v. D is invertible. $D^{-1} = \begin{pmatrix} 0.1 & 0.3 & 0.3 \\ 0.5 & 0.5 & -0.5 \\ 0.3 & -0.1 & -0.1 \end{pmatrix}$

vi. E is not invertible because its rows and columns do not match. Its REF cannot be an identity matrix.

(b) $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \to A^{-1}A\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ -2 \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.2 & 0.4 \\ 0.4 & -0.2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -0.6 \\ 0.8 \end{pmatrix}$

(c) You cannot find x, y which solve this equation because $B\begin{pmatrix} x \\ y \end{pmatrix}$ is undefined since their dimensions don't match.

(d) $DG = E \to D^{-1}DG = D^{-1}E \to G = \begin{pmatrix} 0.1 & 0.3 & 0.3 \\ 0.5 & 0.5 & -0.5 \\ 0.3 & -0.1 & -0.1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 2 \\ 2 & -1 & -2 \\ -1 & 2 & 6 \end{pmatrix} \to$

$G = \begin{pmatrix} 0.6 & 0.3 & 1.4 \\ 3 & -1.5 & -3 \\ 0.8 & -0.1 & 0.2 \end{pmatrix}$

3. (a) **False**. If $A \in M_n(\mathbb{R})$, fix $n = 2$, let us say that $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. $A^2 = 0$, but $A \neq 0$, thus the statement is false.

(b) **True**. Let $A, B \in M_n(\mathbb{R})$. Rewrite $AB^2 = B^2A$ as $A(BB) = (BB)A$. Apply associative property on matrices: $(AB)B = B(BA)$. Since we are given that $AB = BA$, substitute in equation: $(BA)B = B(AB)$. Apply associative property again and get $B(AB) = B(AB)$. We therefore conclude that the terms are equal and that $AB^2 = B^2A$ as long as $AB = BA$.

(c) **False**. If $A, B, C \in M_n(\mathbb{R})$, fix $n = 2$, let us set $A = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$. $AB = \begin{pmatrix} 0 & 0 \\ 2 & 2 \end{pmatrix}$, and $CB = \begin{pmatrix} 0 & 0 \\ 2 & 2 \end{pmatrix}$. However, $A \neq C$, so clearly the statement is false.

4. (a)