Brief Article

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Abstract

The purpose of this analysis: Given N pointings determine the placement of fibers that optimizes the determination of the asymptotic velocity of the rotation curve of a spiral galaxy. The procedure: Determine the combination of positions that minimize the uncertainty of the asymptotic velocity as estimated from Fisher Matrix calculations.

1 Data

The data are composed of N measurements. Each have a

- $\hat{\theta}$ - the angular coordinate where we think the fiber was positioned.
- $\hat{v}$ - velocity measurement.

There are a total of $2N$ measurements. Measurements are notated by the hat.

The velocity measurement uncertainties are given by σ_v. This uncertainty is taken to be the proportional to the inverse of the signal-to-noise of the flux measurement of the feature, that is

$$\sigma_v \propto \frac{\sqrt{S+B}}{S},$$

where S and B are the source and background fluxes respectively. For S we use the light of the spiral arm, which according to Wikipedia is well described by an exponential profile,

$$S \propto \exp\left(-b\frac{|\theta|}{R_e}\right), \tag{1}$$

where $b \approx 5/3$ and R_e is the half-light radius.

When the signal to noise gets poor enough a velocity measurement cannot even be obtained, meaning that there is a maximum angular separation from the core $\theta^{\max}$ with a velocity measurement that has the largest possible uncertainty $\sigma_v^{\max}$. Anticipating a Fisher matrix analysis, which does not accommodate the discrete dropout of data, we take

$$\sigma_v = \begin{cases} \sigma_v^{\max} \exp\left(b\frac{|\theta|-\theta^{\max}}{R_e}\right)\sqrt{\frac{\exp\left(-b\frac{|\theta|}{R_e}\right)+B'}{\exp\left(-b\frac{\theta^{\max}}{R_e}\right)+B'}} & |\theta| < \theta^{\max} \\ \sigma_v^{\max} \exp\left(c\frac{|\theta|-\theta^{\max}}{R_e}\right) & |\theta| \geq \theta^{\max} \end{cases}. \tag{2}$$

In this note, we take $c = 5 > b$. There are three parameters in the noise model of a galaxy, the maximum possible uncertainty $\sigma_v^{\max}$, the radius where that occurs $\theta^{\max}/R_e$, and the relative flux of the source and background B'.

2 Model

The true positions of the measurements are assumed to be

$$\theta \sim \mathcal{N}(\hat{\theta}, \sigma_\theta), \tag{3}$$

where σ_θ is the positioning error.

The velocity v along the semi-major axis is modeled as a hyperbolic tangent

$$f = v_\infty \tanh\left(\frac{\theta - \theta_0}{a}\right) + v_0, \tag{4}$$

where the model parameters are

- v_∞ - Asymptotic rotation velocity.
- θ - Coordinate of the measurement
- θ_0 - Coordinate of the galaxy centroid.
- a - Galaxy length along the semi-major axis.
- v_0 - the nominal redshift of the galaxy

There are a total of $4 + N$ parameters.

The elements of the Fisher Matrix for the above model is given in §A.

3 Results for DESI

DESI is interested in measuring Tully-Fisher distances of large spiral galaxies. Each of these galaxies will have at least 4 measurements, one of which is at the core of the galaxy. Of primary interest is the uncertainty in the asymptotic velocity $\sqrt{F^{-1}_{v_\infty v_\infty}}$. We therefore determine the placement of the three other fibers by minimizing this uncertainty.

Before moving forward, it is worth mentioning that the optimal solution for four free fibers does not have one of them at the core. In addition, we are ultimately interested in the peculiar velocity of the galaxy meaning that the uncertainty in v_0 is also relevant. However, the fiber placements that optimize the galaxy redshift and rotation velocities differ. As we proceed, we take that a core measurement is immutable and that the uncertainties in the rotation velocity dominate the peculiar velocity error budget.

The optimal fiber placement depends the specific details of the galaxy: its signal-to-noise as described by the maximum radius where a rotation velocity can be measured $\theta^{\max}$, the length-scale of its rotation curve a, and its size relative to the fiber positioning uncertainty σ_θ (all considered in units of half-light radius R_e).

The optimal fiber placements as a function of the angular extent where velocities can be measured $\theta^{\max}$ for fixed values of a, σ_θ, and $B' = 0$ (source noise-dominated) are shown in Figure 1. When rotation velocities can be only measured out to small radii, small $\theta^{\max}$, one of the fibers is best placed at the that extremum to provide as much leverage as possible in measuring the asymptote of the rotation curve. Above a certain value of $\theta^{\max}$ the fiber placements are the same, the optimal solutions settling on the fixed velocity signal-to-noises at those positions.

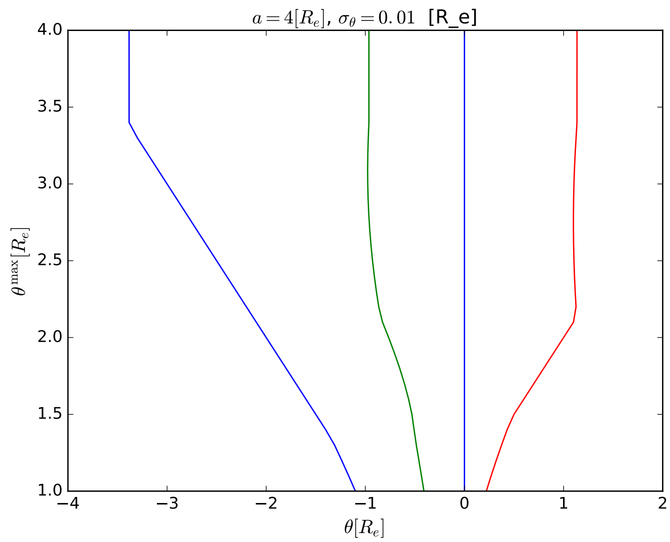

Figure 1: Optimal placements of three fibers, with one fixed at the core, as a function of $\theta^{\max}$ for fixed values of $a = 4$, $\sigma_\theta = 0.01$, and $B' = 0$. One of the four fibers is fixed at $\theta = 0$.

The optimal fiber placements as a function of the scale of the rotation curve a for fixed values of $\theta^{\max}$, σ_θ, and $B' = 0$ are shown in Figure 2. The value of $\theta^{\max}$ is large enough to give velocity measurements over most of the galaxy and so represents the asymptotic solution discussed in the previous example. A broader angular extent is preferred for broader rotation curves and, within the phase space shown, there is no asymptotic fixed-position solution. The value of a will not be known before the measurement is made, either an independent estimate of a or an effective value for the population could be used.

The optimal fiber placements as a function of the size of the galaxy in terms of the pointing error σ_θ for fixed values of $\theta^{\max}$, a, and $B' = 0$ are shown in Figure 3. There is a slight dependence on the galaxy size, with smaller galaxies preferring relatively higher radius fiber placement.

A Fisher Matrix Elements

The corresponding partial derivatives for the model prediction for measurement i are

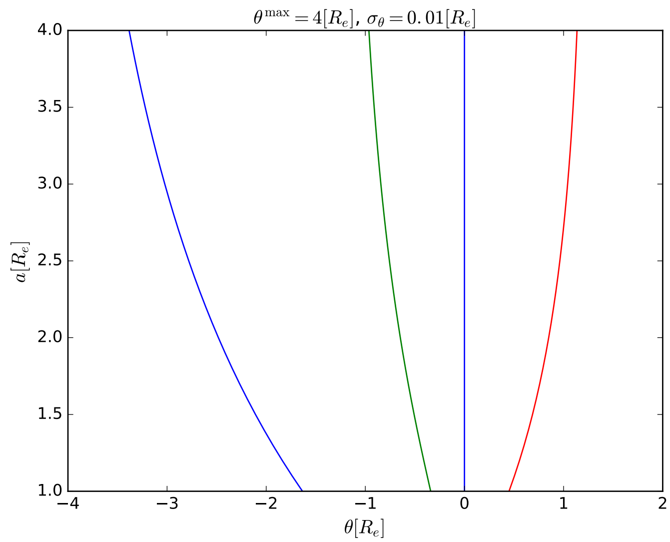

Figure 2: Optimal placements of three fibers, with one fixed at the core, as a function of a for fixed values of $\theta^{\max} = 4$, $\sigma_\theta = 0.01$, and $B' = 0$. One of the four fibers is fixed at $\theta = 0$.

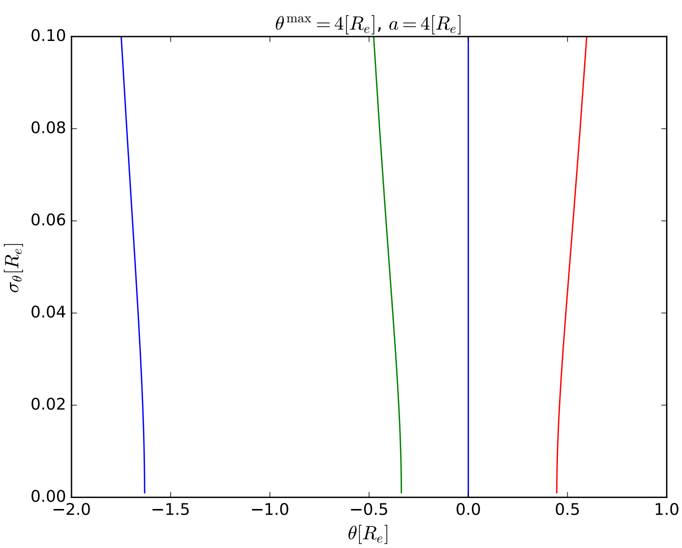

Figure 3: Optimal placements of three fibers, with one fixed at the core, as a function of $\sigma_\theta = 0.01$ for fixed values of $\theta^{\max} = 4$, $a = 4$, and $B' = 0$. One of the four fibers is fixed at $\theta = 0$.

- $\partial f/\partial v_\infty = \tanh\left(\frac{\theta_i-\theta_0}{a}\right)$
- $\partial f/\partial \theta_j = \frac{v_\infty}{a}\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)\delta^D_{ij}$
- $\partial f/\partial \theta_0 = -\frac{v_\infty}{a}\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $\partial f/\partial a = -\frac{v_\infty}{a^2}(\theta_i-\theta_0)\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $\partial f/\partial v_0 = 1$

The Fisher matrix elements are then

- $F_{v_\infty v_\infty} = \sum_i \sigma_{v_i}^{-2}\tanh^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{v_\infty \theta_i} = \frac{v_\infty}{a}\sigma_{v_i}^{-2}\tanh\left(\frac{\theta_i-\theta_0}{a}\right)\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{v_\infty \theta_0} = -\frac{v_\infty}{a}\sum_i \sigma_{v_i}^{-2}\tanh\left(\frac{\theta_i-\theta_0}{a}\right)\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{v_\infty a} = -\frac{v_\infty}{a^2}\sum_i(\theta_i-\theta_0)\sigma_{v_i}^{-2}\tanh\left(\frac{\theta_i-\theta_0}{a}\right)\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{v_\infty v_0} = \sum_i \sigma_{v_i}^{-2}\tanh\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{\theta_i\theta_j} = (\frac{v_\infty^2}{a^2}\sigma_{v_i}^{-2}\mathrm{sech}^4\left(\frac{\theta_i-\theta_0}{a}\right) + \sigma_\theta^{-2})\delta^D_{ij}$
- $F_{\theta_i\theta_0} = -\frac{v_\infty^2}{a^2}\sigma_{v_i}^{-2}\mathrm{sech}^4\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{\theta_i a} = -\frac{v_\infty^2}{a^3}(\theta_i-\theta_0)\sigma_{v_i}^{-2}\mathrm{sech}^4\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{\theta_i v_0} = \frac{v_\infty}{a}\sigma_{v_i}^{-2}\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{\theta_0\theta_0} = \frac{v_\infty^2}{a^2}\sum_i \sigma_{v_i}^{-2}\mathrm{sech}^4\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{\theta_0 a} = \frac{v_\infty^2}{a^3}\sum_i(\theta_i-\theta_0)\sigma_{v_i}^{-2}\mathrm{sech}^4\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{\theta_0 v_0} = -\frac{v_\infty}{a}\sum_i \sigma_{v_i}^{-2}\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{aa} = \frac{v_\infty^2}{a^4}\sum_i(\theta_i-\theta_0)^2\sigma_{v_i}^{-2}\mathrm{sech}^4\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{av_0} = -\frac{v_\infty}{a^2}\sum_i(\theta_i-\theta_0)\sigma_{v_i}^{-2}\mathrm{sech}^2\left(\frac{\theta_i-\theta_0}{a}\right)$
- $F_{v_0v_0} = \sum_i \sigma_{v_i}^{-2}$