Lab 11: You Can Weigh Jupiter

Purpose

How can we measure the masses of such distant bodies? The answer is that we really have only one method of "weighing" astronomical bodies; we must observe the gravitational effect with some other body. When we weigh ourselves on a bathroom scale, we are actually measuring how hard the scale has to push on our body to hold it in place against the Earth's gravity!

The objective of this computerized experiment is to acquaint you with the way in which astronomers determine the masses of remote bodies such as planets and stars.

Introduction

Let's consider a planet orbiting the Sun. The orbit is stable only because the the Sun's gravity redirects the object from otherwise moving in a straight line (Newton's first law of motion). If the planet were to stop, it would immediately fall toward the Sun! If the Sun were more massive or if the planet were closer to the Sun, the planet would have to orbit faster to resist the increased force of gravity. This implies a relationship between the mass of the Sun, the size of the planet's orbit, and the planet's speed in that orbit.

Orbital Motion

Johannes Kepler was the first to give a mathematical form to this relationship. Kepler came up with his three laws of orbital motion. His third law, formulated in 1618, related the radius of a planet's orbit around the Sun, r,[a] to the time it takes to complete one orbit, P (also known as the orbital period). Kepler found the following relation where P is expressed in years and r is expressed in astronomical units (AU – defined by the Earth's average distance to the Sun):

$$P^2_{\text{years}} = r^3_{\text{AU}}$$

Kepler's third law was purely empirical - that is, it was based on fiddling with the parameters until something worked out. A half-century later, the great Issac Newton used his laws of motion and gravitation to derive a far more general version of Kepler's harmonic law that was now based on physical understanding of the problem. Because Newton's version included the mass, M, of the central body, the law could now be applied not only to planets orbiting the Sun, but also to satellites orbiting the planets, and even stars orbiting each other. Let's go through a quick sketch of a Newtonian argument for Kepler's Third Law:

[a] According to Kepler's First Law, orbits are elliptical meaning they are elongated and not necessarily circular. The eccentricity, a measure of how oblong the orbit actually is, is low for all of these moons; therefore, assuming a circular orbit will do for this lab!

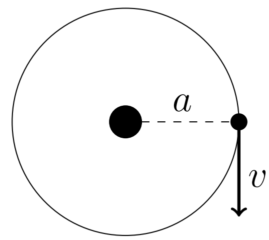


Figure 11.1: Circular orbit

The distance that an object traveling in a circle (sketched in Figure 11.1) moves is just the circumference of the circle, given by $C = 2\pi r$ (where r is the radius), therefore we know that the velocity, v, which is distance over time, can be expressed as the total distance for one orbit divided by the period of the orbit:

$$v = \frac{2\pi r}{P}$$

Additionally, from any introductory physics course, we know that the centrifugal force exactly balances the force of gravity in circular motion:

$$F_{\text{grav}} = \frac{GMm}{r^2} = F_{\text{c}} = \frac{mv^2}{r}$$

We can combine these two equations by seeing that the distance between the two objects in the force equations is just r in our diagram. Then, we can substitute in the velocity:

$$\frac{GMm}{r^2} = \frac{m(2\pi r)^2}{P^2 r}$$

Solving for P we obtain our desired expression:

$$P^2 = \frac{4\pi^2}{GM} r^3$$

As a matter of brevity, Newton's Law is often written as the following, with M typically being expressed in unit of solar masses:

$$P^2 = \frac{r^3}{M}$$

Newton himself used this to estimate the masses of Jupiter, Saturn and the Earth, the only planets known at the time to have satellites. Due to the poor quality of the available measurements, his results were not very accurate. Hopefully, we can do better.

We will use Newton's equation to measure the mass of the giant planet Jupiter, a mass so large that it greatly exceeds the masses of all the other solar system planets combined. The key to our experiment lies in the fact that Jupiter has four moons that are so bright they can be glimpsed with a good pair of binoculars. These satellites were discovered by the Italian scientist Galileo in January of 1610, soon after he turn the first astronomical telescope on the heavens.

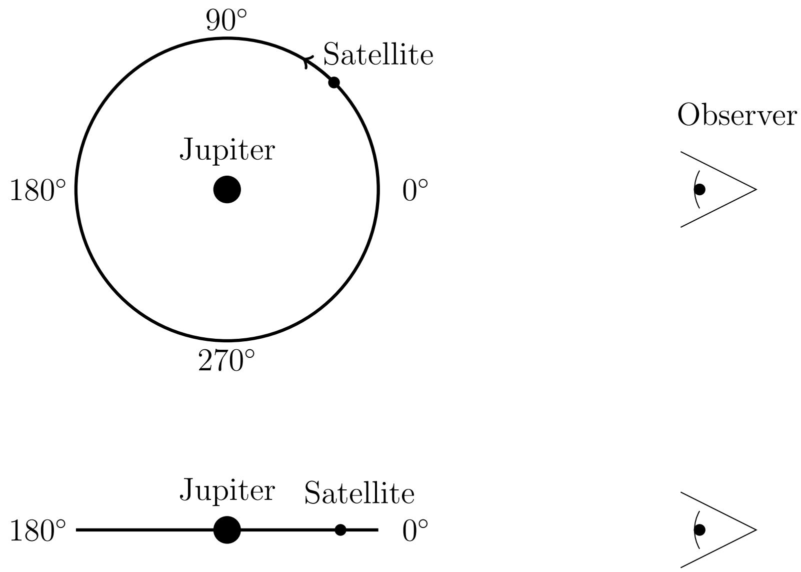


Figure 11.2: Two visualizations of the orbit of a moon of Jupiter. The top is seen from above, and the bottom is an edge-on orientation.

Sinusoidal Motion

We, as observers, are nearly in the plane of the satellite orbits, we see each moon oscillating from one side of Jupiter to the other, almost in a straight line (see Figure 11.2 for two different perspectives of the single orbit). The apparent distance, r, of the moon from the center of Jupiter follows a sinusoidal curve as illustrated in Figure 11.3. ("Sinusoidal" is from the Latin *sinus*, meaning a bend or semicircular fold.) When the moon is in front of Jupiter (0°), $a = 0$. As the moon moves from 0° to 90°, r increases steadily from 0 to 1. Then r decreases smoothly as the moon moves from 90° to 180° where $r = 0$ again. Between 180° and 360° the curve simply repeats itself, but in the opposite direction. Many natural phenomena follow sinusoidal curves: sound waves, vibrating musical strings, radio waves, electrical currents and voltages, oscillating springs, etc. An example sine curve is show in Figure 11.3 with the inner regions of the orbit not filled in as the moon passes behind Jupiter during this time.

Laboratory Procedure

Please DO NOT modify, create, or delete any files on the computers unless EXPLICITLY told to do so by your instructor

In this section, we will detail how the steps necessary in completing this lab. Your instructor will also provide you with guidance.

1. Navigate to the ALEX Labs "You Can Weigh Jupiter" link.
2. You should see a page with the orientation that the planets will be in during our observation window. This orientation is known as opposition, where the two planets are on the same side

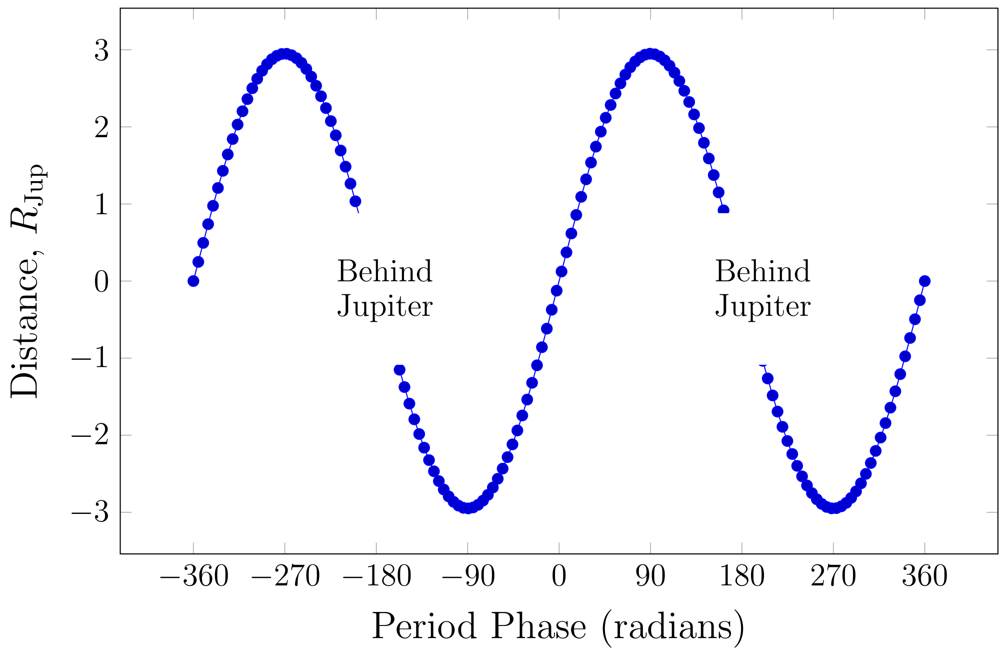


Figure 11.3: Example fine-resolution curve of what an observer would see for an edge-on inclination of the orbit of a moon around Jupiter. The reason there are no data points between -1 and 1 is because the moon is behind the planet during this phase of its orbit.

of the sun. Once you've figured out how far apart the two planets are, click on the `Start!` button.

3. You are now on a page with information regarding all four of the Galilean moons of Jupiter. Your instructor will assign you moon(s) to examine during this activity. As you read about the moons, note the colors as they will appear in the simulation.

   - Io is orange
   - Europa is red
   - Ganymede is green
   - Callisto is blue

4. Navigate to the bottom of the webpage and select `Start!`

5. We're now almost ready to take data. Based on Table 11.1, use the top slider to set the time-step of the observations

Moon	**Recommended Time-step** (hours)
Io	3
Europa	5
Ganymede	10
Callisto	22

Table 11.1: Recommended time-steps for the four Galilean moons. **Note:** You may deviate slightly from these if you so choose, but doing so lead to not obtaining a full period in your data or fine enough resolution!

6. Using the second slider, you can zoom into the picture for more accurate data-taking.

7. You are now ready to collect data! To get the positions of the moon, all you need to do is click on it in the picture and it should populate the **bottom of the screen** with the X-position on the picture in arcseconds, with respect to the center of the image.

   - The positions from Jupiter have either positive or negative. The sign simply denotes which side of Jupiter the moon is on, left being negative.

8. Once you've recorded the first data point into Data Table 11.0, you can advance the simulation to the next time-step by simply pressing the `Next Timestep` button. Continue to do this until you run out of spaces in your table. If you miss a position, simply press the `Back One Timestep` button to go back.

9. Once Data Table 11.0 is completely filled in, create the sine-curve graph corresponding to the moon(s) you observed. From this, and some simple unit conversions, you can estimate the mass of Jupiter.

You Can Weigh Jupiter Worksheet

Name: ______________________________ **Date:** ________________ **Section #:** ________

This worksheet should be filled out as you work through the experiments. Your instructor will either collect it or ask you to upload it to Canvas. Please read the accompanying lab and instructions carefully. Show your calculations and write in complete sentences when appropriate.

Introductory Questions

1. When Jupiter and Earth are in opposition (as they are for data collection in this lab) how far away are they in AU? What about in meters?

2. Given this distance and the fact that the radius of Jupiter is 7.149×10^7 m, what is the full *angular* size of Jupiter as seen from Earth, in arcseconds? (Hint 1: drawing a right triangle may help you recall which trigonometric relation to use) (Hint 2: be careful with units)

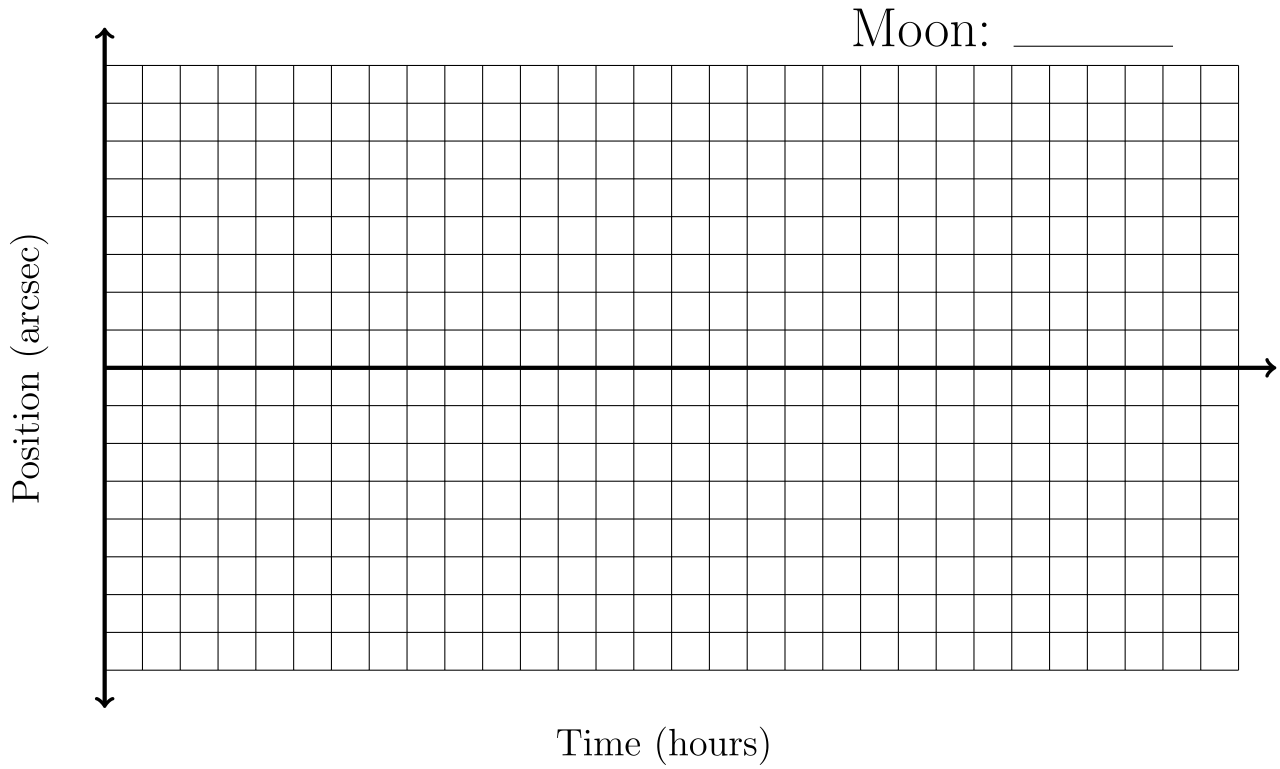
3. If an object has an angular size of $50''$ and is 10 AU away from us, what is the *physical* size of the object in AU? What about in meters?

Data Collection

Data Table 11.0

(1) Time	Moon: ____________

(2) Time	Moon: ____________

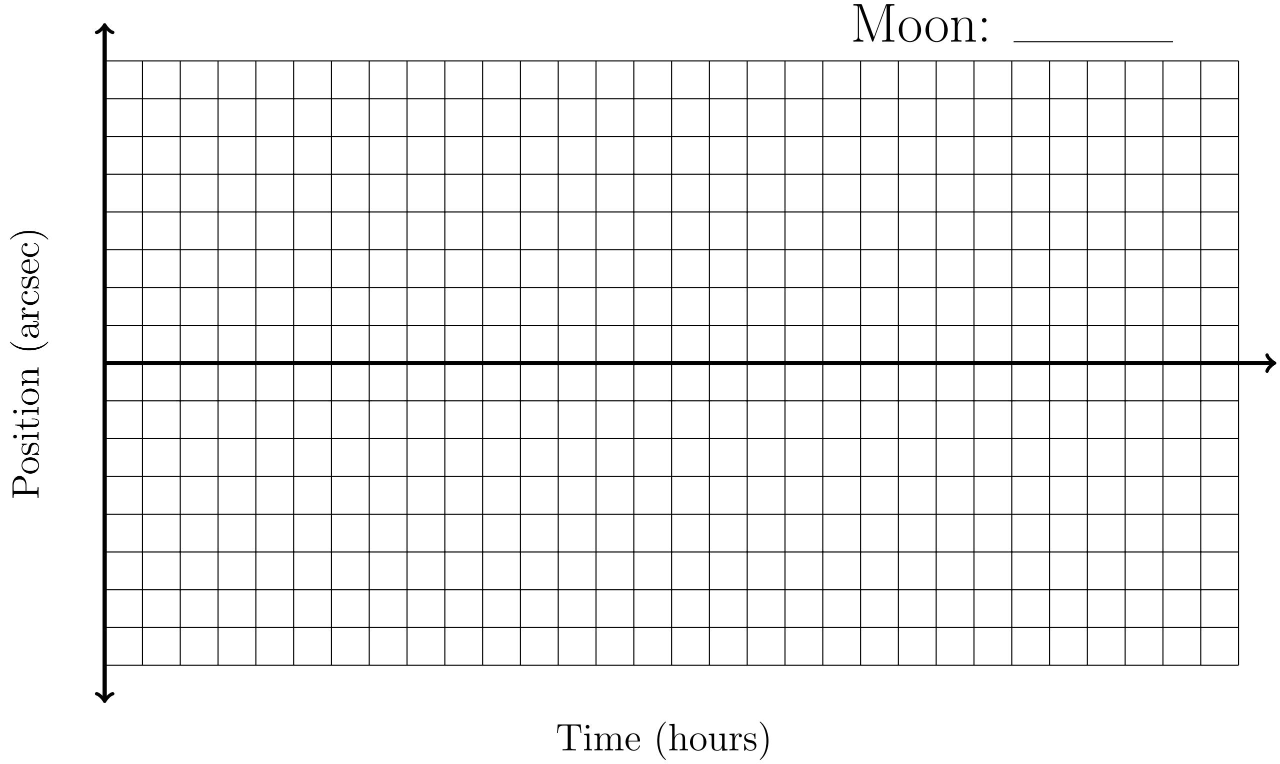


Graph 11.1

Period (hours)		Semi-major Axis (arcsec)	
Period (years)			

Data Table 11.1

Moon: ________



Graph 11.2

Period (hours)		Semi-major Axis (arcsec)	
Period (years)			

Data Table 11.2

Analysis

4. What are the semi-major axes of your moons' orbits in meters? What are they in AU? Use the distance you found in Question 1 and use Question 3 as a model.

5. Use Kepler's Third Law to compute the mass of Jupiter, in units of solar masses, for each of the moons you observed

$$P_{\text{years}}^2 = \frac{a_{\text{AU}}^3}{M}$$

6. Compute the mass of Jupiter in grams (1 $M_\odot$, solar mass, is equivalent to 1.992×10^{33} grams) for all the moons you observed. If you observed multiple, are your answers consistent?

7. Calculate the percentage error of your calculated value for the mass of Jupiter, given that the accepted value for the mass of Jupiter is 1.9897×10^{30} grams. What are some sources of error for this calculation?

8. The mass of the Earth is 5.975×10^{27} grams. Using your prior knowledge of Jupiter and Earth, does your answer to question 6 seem reasonable? Why or why not?

9. Density is defined as the mass within a fixed volume. The volume of a sphere is given by the following, where r is the radius of the sphere:

$$V = \frac{4}{3}\pi r^3$$

Given that the radius of Jupiter is 7.149×10^9 cm, use the following equation to compute the average density of Jupiter in g/cm^3 (use your measured mass of Jupiter)

$$\rho\ [\text{grams/cm}^3] = \frac{m\ [\text{grams}]}{V\ [\text{cm}^3]}$$

10. How does the mean density of Jupiter compare with the mean density of Earth ($\sim$5 g/cm^3)?

11. Some typical densities for different materials are shown below:

Material	ρ [g/cm^3]
Water	1
Ice	$\sim$0.9
Rock	$\sim$3
Metal	$\sim$5

From spectroscopy, we can determine that Jupiter is about 90% Hydrogen and 10% Helium. Given this information and your result from the previous problem, what would you infer about the composition and structure of Jupiter?