Experiment 4: Pulses in Transmission Lines

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Abstract

In this experiment, we studied the time delay on electrical circuits. In previous experiments, we had ignored the non-zero time it takes a signal to propagate through a wire, but this was only just because this time scales used in our circuits were much, much larger than this difference. We studied the speed of propagation in different types of wires and determined that they were similar to our expected value of around two-thirds the speed of light. We looked at the incident, reflected, and transmitted waves of several different loads at the end of our circuits. For resistive loads, we saw a close agreement with our theory for the form of the reflected and transmitted waves. For loads with capacitors, we saw the general relation of our exponential decays being held but had some discrepancies between our expected values. For inductive loads, we obtained our general relation and had more luck getting values that were similar to our theory. Additionally, for an LC circuit, we saw our damped sine form and the values that we got for the frequency agreed with theoretical predictions. Finally, we used our detective skills to apply our analysis and determine that an unknown load was actually a resistor and capacitor in series. In the future, we would like to go in and obtain error bars on our values for our components to get a better idea of the range of the theory, as well as learning to better fit just the data we are looking at in Origin.

1 Purpose

The purpose of these experiments was to learn more about the time delay of circuits as well as the different responses that different components produce. To learn about the time delay of circuits, which we mentioned we usually neglect, we needed to first learn more about the speed of propagation in the wires we were using. In previous labs we had assumed that the propagation speed was nearly infinite and ignored; however, in the first part of the lab, we show that the speed of propagation is in fact finite. We find our results consistent with the manufacturing company says the waves propagate at. The second main part of this lab was to study the response of these time-delayed circuits. We first looked at circuits with resistive loads and determined the shape of the reflected (the wave bounced back at our oscilloscope) and transmitted (the wave detected at the load) follow fairly closely with the theory derived from Ohm's Law and Thevenin equivalence. Next, we repeated the experiments with a capacitor and then an inductor. For these two components, we used the transmitted wave to find the time constant of the charging and discharging on the elements. For both of them, we found a close relationship with the theoretical models, but for the capacitor, we struggled to find a fit that accurately matched the value for the capacitor. We performed a similar task on an LC circuit to determine the frequency of the damped oscillation of charge, this part also agreed fairly well with the theoretical models and we obtained values close to the actual values for our inductor and capacitor. Finally, we worked backward on a circuit: we were handed a black box with some unknown components and had to use the incident, reflected, and transmitted waves to figure out the components and their values. This was a direct application of the other parts of the lab as well as the Thevenin equivalence principle. Once we had figured out the components, their configuration, and their values we used software on the computers to check against what we obtained and found that our guesses were at least similar to what we should be expecting with our results.

2 Theory

In this lab, we spent most of our time looking at the incident, reflected, and transmitted waves of circuits with different loads attached to the end. The load in this case is going to be some combination of components that will have some impedance. If we were to assume that a wire has no loss across it we can describe our system using differential equations.

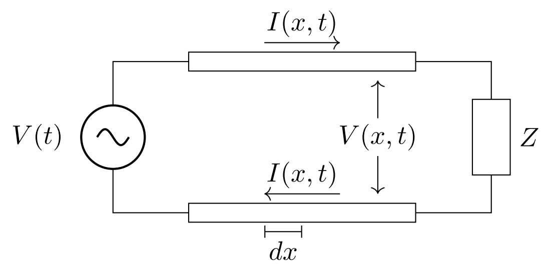

Figure 1: Example circuit with a lossless line having a load at the end

If we have the set-up as described in Figure 1 we could describe the system in terms of the capacitance and inductance per unit length.

$$C\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x} \tag{2.1}$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \tag{2.2}$$

Taking the time derivative of Eqn. 2.1 and the spatial derivative of Eqn. 2.2 and then combine them we obtain the following results.

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \qquad \frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} \tag{2.3}$$

Both results from Eqn. 2.3 look very similar to the wave equation, which indeed they are. Therefore we know that the speed of propagation should be $v = \frac{1}{\sqrt{LC}}$. We can also describe the characteristic impedance of a wire using the following equation.

$$Z_k \equiv \frac{V(x,t)}{I(x,t)} = \sqrt{\frac{L}{C}} \tag{2.4}$$

The capacitance per unit length, C, and inductance per unit length, L, both depend on the size of the wire as well as the dielectric constant, ϵ_r, and magnetic permeability μ_r. All of these factors contribute to determining the speed of propagation inside our wires.

We can describe the total voltage and current at any point on the wire through the following relations.

$$V = V_r + V_i \tag{2.5}$$

$$I = I_r + I_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k} \;\; \rightarrow \;\; V_i - V_r = Z_k I \tag{2.6}$$

Graphically, the speed of propagation of waves inside the different types of wires can be determined by looking at the same point on each incident and reflected waves and determining the time difference between those two events. Once we know that time difference, Δt, we already know the length of the wires, ℓ, therefore finding the speed of propagation is the usual relationship, taking into account that the wave has to travel twice the length of the wire.

$$v = \frac{2\ell}{\Delta t} \tag{2.7}$$

In general, for our resistive loads, we can set-up an equation to determine the voltage of the reflected wave by knowing the total resistance of the load and the impedance of the rest of the circuit.

For resistive loads, we can write Ohm's Law in terms of the resistance of the load.

$$R_L = \frac{V}{I} \tag{2.8}$$

The incident wave generates a reflected wave at the load with the following form.

$$\frac{R_L}{Z_k} = \frac{V}{Z_k I} = \frac{V_I + V_r}{V_I - V_r} \tag{2.9}$$

We can solve Eqn 2.9 for V_r to determine the form of the reflected wave.

$$V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i \tag{2.10}$$

We can look at a couple of key cases for Eqn. 2.10 to determine what our expected response for our resistive loads should be. The first case is where $0 \leq R_L < Z_k$. In this case, we can see that the numerator will be negative, therefore the reflected wave will have the opposite sign as the incident one. The next case we should look at is $R_L = Z_k$, at this point the numerator will go to zero and we will not have a reflected wave. In the third case, we must consider if $Z_k < R_L < \infty$, here we will have a reflected wave with the same sign of the incident just scaled down by some factor. Finally, our last case is $R_L = \infty$, this case is where we have an open line, in this limiting case $V_r = V_i$ and the reflected wave will look the same as the incident.

For the transmitted waves of loads, we need to use a principle called Thevenin equivalence. This equivalence principle states that any combination of impedances on a wire can be described as just the combination of all the elements as one single resistor.

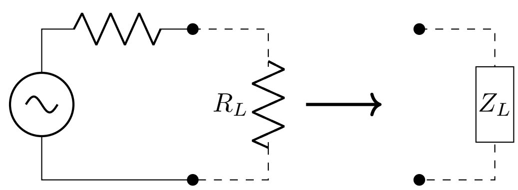

Figure 2: Example of Thevein Equivalent circuit, we can add any number of elements into the load and just treat it like one resistor

From Thevenin equivalence, we can learn more about the transmitted portion of our loads. For these situations, we can reference Eqns. 2.5 and 2.6 for the general form and solve for the current to obtain the following expression for the current at the load.

$$I = \frac{2V_I}{R_L + Z_k} \tag{2.11}$$

Plugging this into our usual $V = IR$ relation for Ohm's Law we obtain the following for the theoretical voltage across the load.

$$V_T = R_L \left(\frac{2V_I}{R_L + Z_k} \right) \tag{2.12}$$

This Thevenin equivalence principle also allows us to know more about our circuits that contain capacitors, inductors, and many combinations of elements without having to know specifically which elements are which.

For our experiments where we have an inductor or a capacitor at the end of the line, we can use our transmitted waves to measure the capacitance or inductance of the circuit. We can define the rise and fall times of these elements like the time to go from either 10% voltage to 90% voltage or 90% to 10%. This rise and fall time is particularly useful because it is related to the impedance out of the function generator and the value for our element.

Each of these elements at the end of the line has its own characteristic forms. A reflection of a capacitor's voltage undergoes the following voltage forms, the first for charging up and the second for discharging.

$$V_{\text{Charging}}(t) = V_i\left(1 - e^{-t/T}\right) \tag{2.13}$$

$$V_{\text{Discharging}}(t) = 2V_i\left(1 - e^{-T_1/\tau}\right)\left(1 - e^{-(t-T_1)/\tau}\right) \tag{2.14}$$

The reflection of the system with an Inductor at the end of the line has a voltage response that can be modeled with the following equation.

$$V(t) = \frac{(Z_k + Z_L)}{2}\left(1 - e^{-t/T}\right) \tag{2.15}$$

For a capacitor, the rise and fall times are modeled by an exponential decay that we can approximate using the Impedance and the Capacitance.

$$\tau_{\text{Rise/Fall},C} \approx 2.20 Z_k C_L \tag{2.16}$$

Meanwhile, for an inductor, we define the rise and fall time with this equation.

$$\tau_{\text{Rise/Fall},L} = \frac{L}{Z_k} \tag{2.17}$$

In addition to all of this we can, for LC circuits, define the frequency of oscillation in terms of the inductance and the capacitance, as shown below.

$$\omega = \sqrt{\frac{1}{LC}} \tag{2.18}$$

Our data analysis software, Origin, has a built-in damped sine function that it uses to evaluate our data for these LC circuits. Instead of graphing against ω, Origin fits against w. This parameter is defined as follows.

$$w = \frac{\pi}{\omega} \tag{2.19}$$

Finally, for Part F where we were tasked with finding the values of two unknown components in some mystery box. Spoiler Alert: it was a resistor and capacitor in series, but there is much more about this in Section 3.7. To actually calculate these values, we needed to solve some differential equations, which were slightly different on the leading edge and the trailing edge.

For the leading edge, we have the following differential equation and initial value.

$$(Z_k + R)Q'(t) + \frac{1}{C}Q(t) = 2V_0 \qquad Q(t=0) = 0 \tag{2.20}$$

Similarly, for the trailing edge we have this slightly different differential equation with a different initial value.

$$(Z_k + R)Q'(t) + \frac{1}{C}Q(t) = 0 \qquad Q(t=0) = 2V_0 C \tag{2.21}$$

This problem can be done for either differential equation and retrieve the same answer. Now that we know the differential equations we need to solve for $Q(t)$, doing this we obtain the following expression.

$$Q(t) = 2V_0 C + \exp\left(\frac{-\,t}{C(Z_k + R)}\right) \tag{2.22}$$

We can take our newly found expression for $Q(t)$ and use the following equation in order to solve for the total voltage across the series RC.

$$V_{\text{seriesRC}} = V_C(t) + V_R(t) = \frac{1}{C}Q(t) + RQ'(t) \tag{2.23}$$

Plugging and solving here we obtain an expression for the voltage across our box as a function of time.

$$V_{\text{series}} = 2V_0\left(1+\frac{R}{R+Z_k}\right)\exp\left(\frac{-t}{C(Z_k+R)}\right) \tag{2.24}$$

We can see the previous equation looks extremely similar to that of exponential decay with the form $A\exp(-t/\tau)$.

$$A = 2V_0\left(1+\frac{R}{R+Z_k}\right) \tag{2.25}$$

$$\tau = C(Z_k+R) \tag{2.26}$$

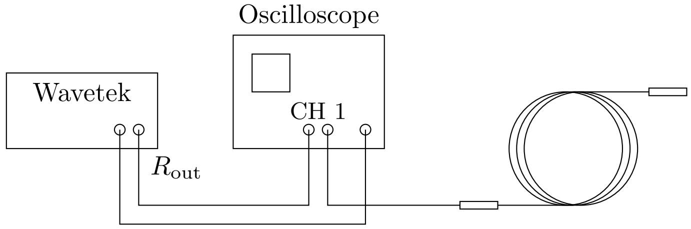

Figure 3: Basic set up for this experiment, we have our function generator that has some oscilloscope to read our pulses and the wire. At the end of the line we attach our components that we are experimenting with

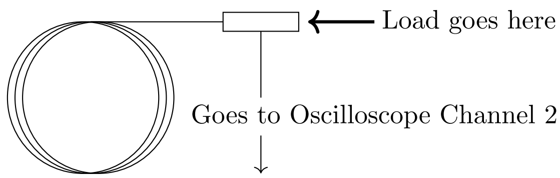

Figure 4: Close up view of our load with the added note that we have another wire that attaches to Channel 2 of our scope to measure the transmitted voltage

3 Experiment

3.1 Equipment

For this lab, we had a couple of different types of equipment. We had several resistors of 0, 25, 50, and 100 Ω as well as a capacitor of 2.79 nF and an inductor of 10 μH. We used these elements alone, with the one exception of setting up an LC circuit in Part E. Also, we were given a block with some unknown combination of elements (which is further discussed in Part F). To measure all of the transmitted waves we connected a BNC-T to our loads and scope.

For all the data taken, we used an oscilloscope to read off the voltages across both the oscilloscope and the 'end of the line'. In addition, we used a Wavetek function generator with an internal resistance of about 50 Ω. To interact with this function generator we were given a computer program that allowed us to set the duty factor, frequency, and amplitude. On this program, we could adjust those quantities as well as the type of wave that we were outputting, in our case a square wave. For the entirety of the lab we were looking at a square wave with an amplitude of 500 mV at 1 Mhz, for the beginning parts of the lab we had duty at 10% but starting in Part C we adjusted it to 40%.

Another key part of the lab that we looked at was the type of wire that we were using. We looked at two different types of wire, the RG8U, and the RG58U. The differences between the two were the thickness, the

RG8U was a thicker wire, and the length of the two wires, the RG8U was 15 meters while the RG58U was 30.48 meters.

Finally, to figure out which combination of elements we had for our mystery box, we used a program that simulates loads at the end of the line. We could choose whether we wanted a capacitor or inductor in series or parallel with our resistor. From there we could adjust our parameters and see how well our guess matches up with our data. As always we used Origin to do all of our plot making and data analysis.

3.2 Part A: Speed of Waves on RG8U and RG58U

In this part of the experiment, we wanted to learn about the velocity of propagation in each of these two types of wires. We did this by looking at the time delay between the incident pulse and the reflection coming back to the function generator. According to the lab write up each of the two wires has a speed of propagation $2/3c$.

Cable	Time of incident wave (μs)	Time of reflected wave (μs)	Difference (μs)
RG8U	1 ± 0.2	1.314 ± 0.2	$.314 \pm 0.1$
RG58U	1 ± 0.1	1.15 ± 0.1	$.15 \pm 0.05$

Table 1: Measurements of time of right edge of incident and reflected waves on oscilloscope

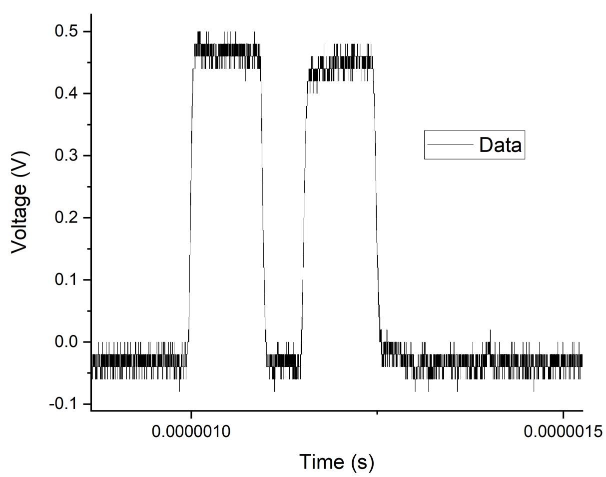

Figure 5: Incident and Reflected waves for open ended circuit with RG5U wire

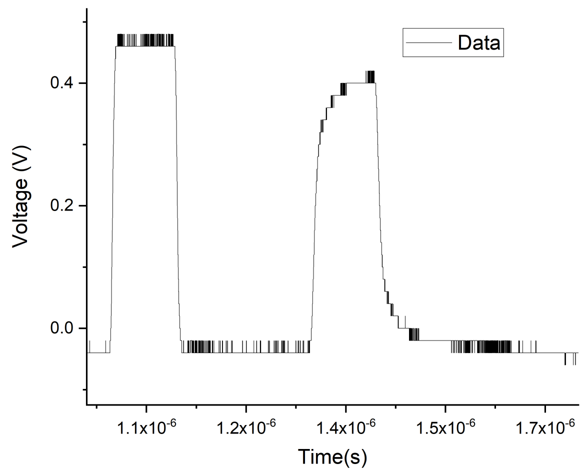

Figure 6: Incident and Reflected waves for open ended circuit with RG58U wire

As previously mentioned, the RG8U cable was 15 meters long and the RG58U cable was 30.48 meters long. We can employ Eqn. 2.7 to calculate the speed of propagation within the wires, we did so here in terms of the speed of light (which we took to be $3{\times}10^8$ m/s).

	RG8U	RG58U
In terms of c	$(0.667 \pm 0.222)c$	$(0.647 \pm 0.206)c$
In m/s	$2{\times}10^8 \pm 6.66{\times}10^7$	$1.941{\times}10^8 \pm 6.18{\times}10^7$

Table 2: Speeds of the two wires, in terms of c and in m/s

From Table 2, the two speeds for each wire are very similar and it would appear that the propagation speed inside the RG8U is slightly faster, but not significantly. Both of these values are fairly consistent with the speed given in the lab write-up.

One of the main sources of error in our calculation is in our measurement. When we measured the times for both the incident and reflected waves all we did was use our cursor and hover over the approximate value for the beginning of the peak. Another potential source of error is we did not measure the length of either wire with much precision, we only used integer values (15 m for RG8U and 100 ft for RG58U).

In the future for this part, we would like to go in and measure the length of each of the wires with more precision as well as looking more precisely at the times of the waves beginning.

3.3 Part B: Resistive termination and Thevenin Equivalent Circuit

For this part of the experiment (and the rest of the experiment), we used our RG8U cable and then varied the resistance at the end of the line. We used several different resistances for this part: open, 0, 25, 50, and 120 Ω.

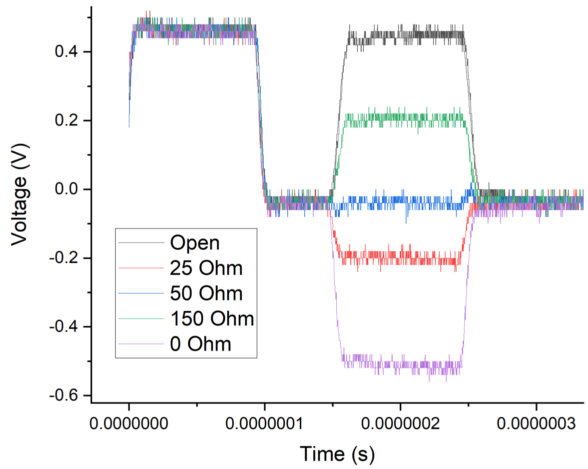

Figure 7: Incident and reflected waves for open, 0, 25, 50 and 150 Ω terminations

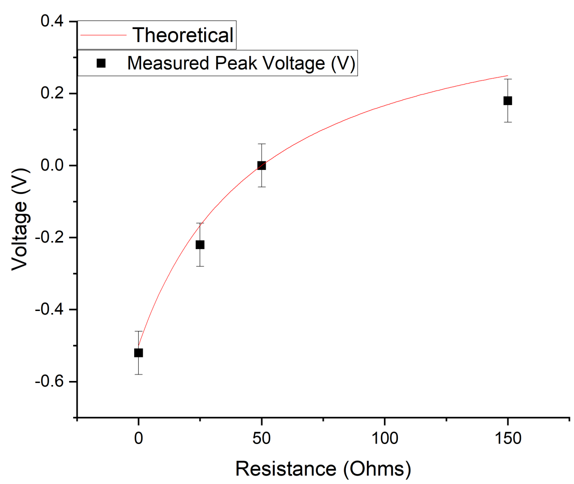

Figure 8: Magnitude of the reflected wave versus the resistance, with theoretical model (Eqn. 2.10)

As we can see the data follows our model fairly accurately. There are a couple of sources of error that could have arisen. One of the sources of error is that the way we obtained our values for each resistance, we looked at what we considered to be an "average" value on the peak of the reflected wave and used that. Although this method was fairly close there is certainly some uncertainty in our measurements: the error bars on this part of the measurement are around $\pm$ 0.02 V. A second, related, source of error comes from the fact that the voltage is not perfectly zeroed here. It oscillated around -0.04 V, so all of our values should be around 0.04 volts higher, which makes sense because they are all undershooting the theoretical model. These two contributions make the error bars on our measurements around $\pm$ 0.06 V.

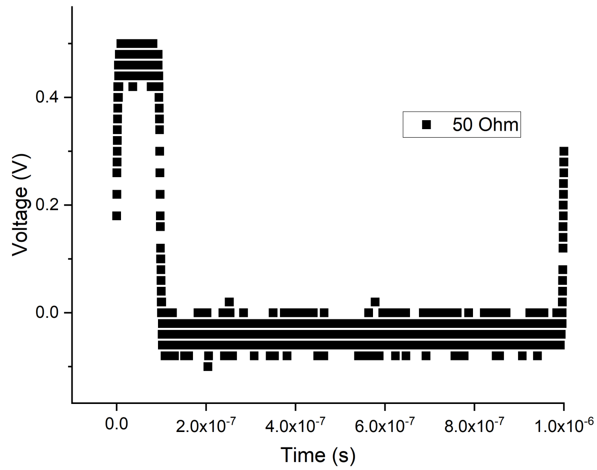

Figure 9: 50 Ohm termination resistance

Using Eqn. 2.10 and knowing that the impedance of the Wavetek is 50 Ω, we can solve for what value of the resistance is going to make the reflected voltage 0. The value this ends up being is, in fact, 50 Ω. We can see that this is also shown in Figure 9. Additionally looking at Figure 10 and the open termination, in combination with the fact that our V_0 is 500 mV, that the magnitude of the peak voltage is $2V_0$.

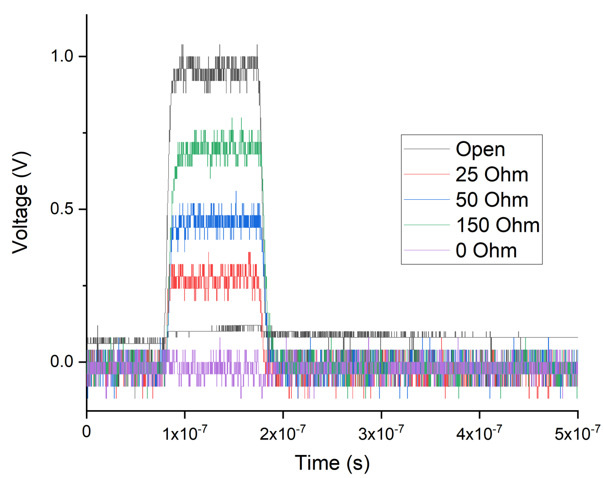

Figure 10: Transmitted wave for open, 0, 25, 50 and 150 Ω terminations

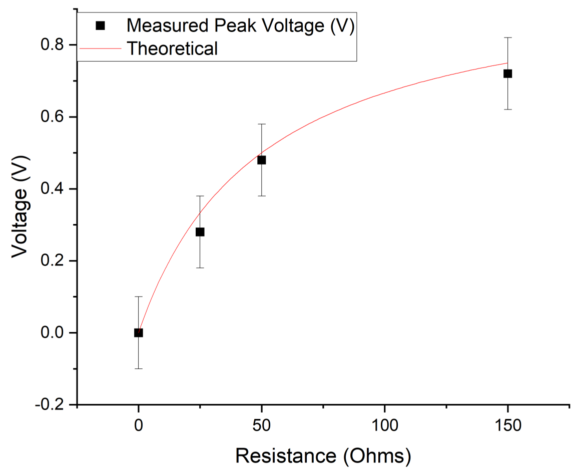

Figure 11: Magnitude of the transmitted wave versus the resistance, with theoretical model (Eqn. 2.12)

Similar to the reflected data, the data follows our model fairly accurately. Our error comes from very similar places as in the reflected part, too. The first potential source of error is that the way we obtained our values for each resistance, we looked at what we considered to be a roughly average value on the peak of the reflected wave and went with that. This was close but some uncertainty in our measurements remained: the error bars on this measurement were around $\pm$ 0.06 V. Another source of error comes from the fact that the voltage is not perfectly zeroed here. It oscillated around -0.04 V, so all of our values should be around 0.04 volts higher, which, again, makes sense because they are all undershooting the theoretical model. These two contributions make the total error bars of our measurements around $\pm$ 0.1 V.

In the future, we should be more careful with zeroing our oscilloscope. In addition, we could use Origin to obtain the average on the interval of the peak voltage, instead of picking out some arbitrary value on the interval.

3.4 Part C: Capacitor at the End of the Line

The set-up for this part of the experiment is very similar to that of Part B, only this time instead of resistors we are looking at a capacitor at the end of the line.

C (nF)	τ (s)
2.79	3.069×10^{-7}

Table 3: Value for our capacitor as well as the theoretical time constant (found using Eqn. 2.16)

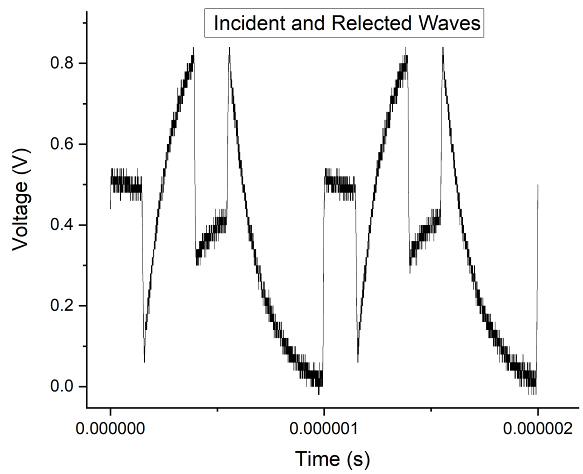

Figure 12: Incident and reflected waves for a capacitor at the end of the line

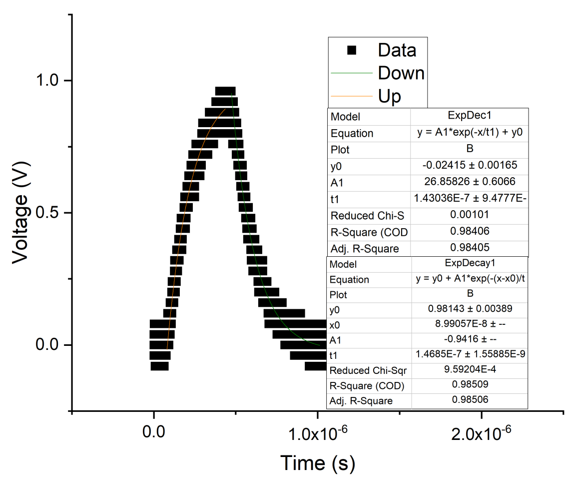

Figure 13: Transmitted wave for a capacitor at the end of the line, with fits for exponential decay, charging and discharing

τ_{up} (s)	τ_{down} (s)
$1.468\times10^{-7} \pm 9.478\times10^{-9}$	$1.430\times10^{-7} \pm 1.559\times10^{-9}$
C_{up} (nF)	C_{down} (nF)
$1.33 \pm .086$	1.30 ± 0.014

Table 4: Time constants from the fits on Figure 13. Also listed are capacitance values from Eqn. 2.16

Based on the values for τ in Tables 3 and 4 we can find the percentage error off the theoretical.

$$\%\text{error}_{\text{up}} = 100\left(\frac{|1.33 - 2.79|}{2.79}\right) = 52.17\%$$

$$\%\text{error}_{\text{down}} = 100\left(\frac{|1.30 - 2.79|}{2.79}\right) = 53.4\%$$

This is a pretty significant error in our values for the capacitance. One of the main sources of error comes from the fit that we gave to our data. There are a lot of values in our data that are very close to the beginning of the curve but not on it (i.e. before the wave hits for charging or while the system is still charging for the discharging case). These points can throw off our fit significantly and cause a lot of errors in our time constant. This compounded with the overall noise of our system gives us some of the error that we are seeing from theoretical. Additionally, we were not given error bars for the value of our capacitor so it is possible that our capacitor has some unaccounted for uncertainty and our time constant for the system is different. The general trend of the exponential follows it just seems that our value for the time constant is off.

In the future eliminating those values that do not belong in the exponential fit would help us a lot in measuring the time constant more accurately and well as having error bars on the value of the capacitor so we know a general range our time constant should be.

3.5 Part D: Inductor at the End of the Line

This part of the lab is nearly identical to Part C, only instead of a capacitor we now have an inductor.

L (μH)	τ (s)
10	$2{\times}10^{-7}$

Table 5: Value of inductor and theoretical time constant (found using Eqn. 2.26) for our circuit

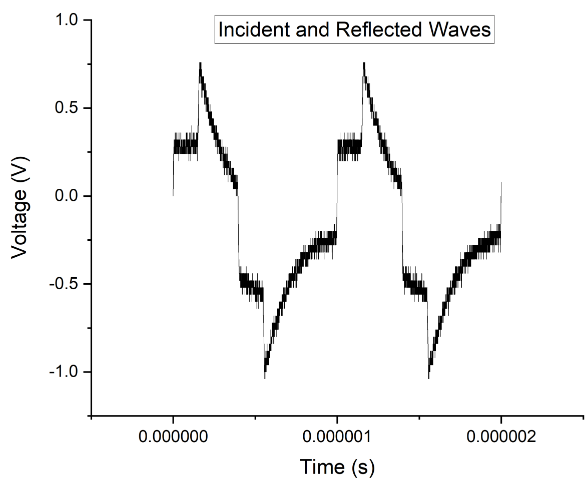

Figure 14: Incident and reflected waves for an inductor at the end of the line

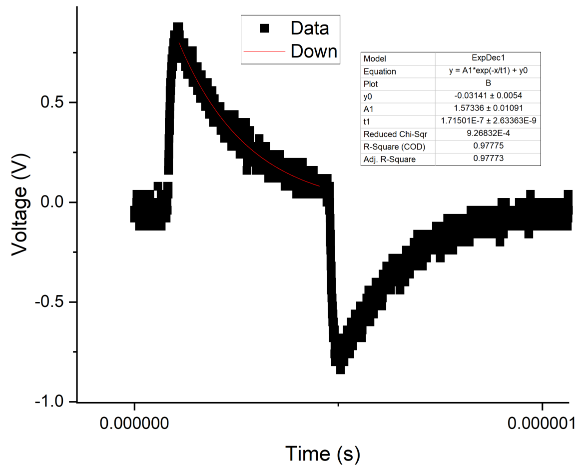

Figure 15: Transmission wave for an inductor with the time down part fitted

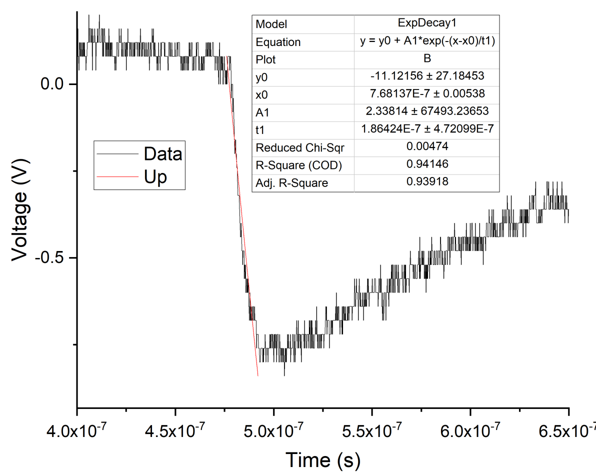

Figure 16: Transmission wave for an inductor with the time up part fitted, done separately from down for range requirements

τ_{up} (s)	τ_{down} (s)
$1.864\times10^{-7} \pm 4.72\times10^{-7}$	$1.715\times10^{-7} \pm 2.633\times10^{-9}$
L_{up} (μH)	L_{down} (μH)
9.32 ± 23.6	$8.575 \pm .132$

Table 6: Time constants from the fits on Figures 15 and 16. Also listed are attributed inductance values found from Eqn. 2.17

Based on the values for τ in Tables 5 and 6 we can find the percentage error of the theoretical

$$\%\text{error}_{\text{up}} = 100\left(\frac{|9.32 - 10|}{10}\right) = 6.8\%$$

$$\%\text{error}_{\text{down}} = 100\left(\frac{|8.575 - 10|}{10}\right) = 14.25\%$$

It can be clearly seen here that we obtained extremely close values for our inductance to our theory. It was not perfect, however. One of the main points of discussion in most of the fitting parts of this lab comes from values that should be excluded from the fit that we accidentally fit. This causes a lot of trouble in our parameters for our fits and it is an extremely easy mistake to make since all the charging and discharging happen so close to each other. As per usual the thermal noise of our system certainly played a role in the error bars on our estimates here, too. Both of these things combined to give us some error on the obtained values of our time constant.

As mentioned previously, in the future going in and really fine-tuned finding all the data points that belong in our fit and only fitting over those values would help us significantly in obtaining values even closer to theory.

3.6 Part E: LC Circuit at the End of the Line

This part of the experiment is a combination of Parts B and C. Now instead of looking at a capacitor or an inductor, we are looking at both of them at the same time.

C (nF)	L (μH)	ω (rad/s)
2.79	10	5.986×10^6

Table 7: Values for our circuit as well as our theoretical ω (found using Eqn. 2.18)

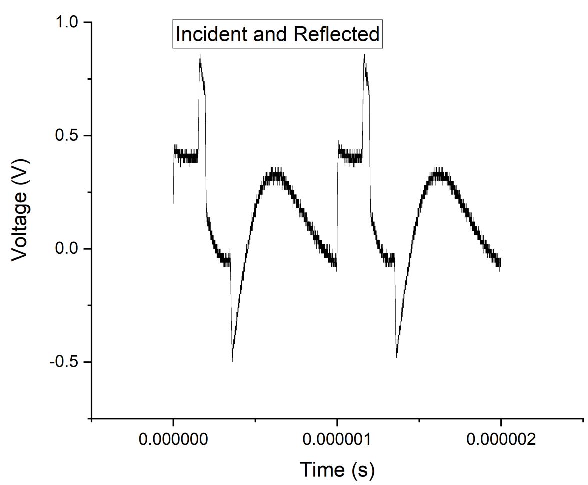

Figure 17: Incident and Reflected waves for our LC circuit

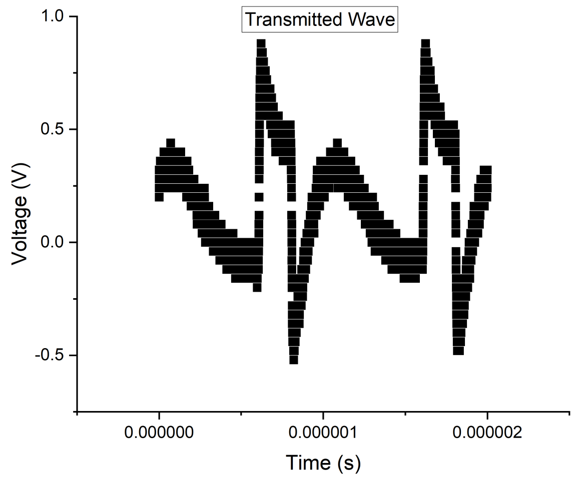

Figure 18: Transmitted wave for our LC circuit

In Figures 17 and 18, we can see the form of the incident, reflected and transmitted waves. We can see the same sort of behaviors as both the inductor and capacitor at the end of the line. We can see the same sort of charging up and down.

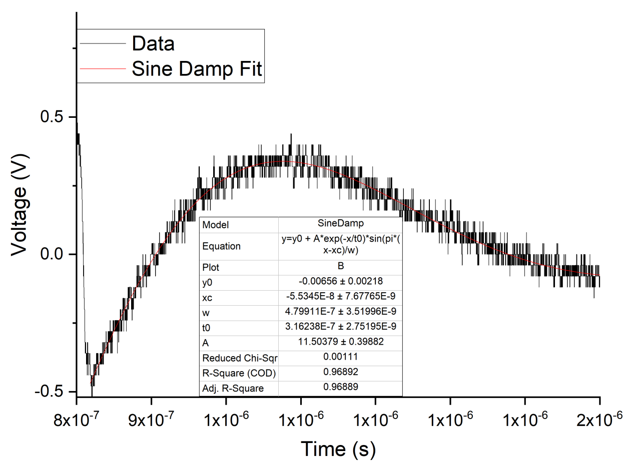

Figure 19: Damped sine fit over part of the transmitted wave

The reason we use a damped sine curve to obtain the fit for our LC circuit is that we have resistance in the wire, from the Wavetek, the elements, and the wires.

From Figure 19 we obtain the value for w. Using Eqn 2.19 we can convert this value to ω to compare against our theoretical.

w (s)	ω (rad/s)
$4.799\times10^{-7} \pm 3.520\times10^{-9}$	$6.546\times10^{6} \pm 4.801\times10^{4}$

Table 8: Origin fit parameter and physical parameter of circuit

$$\%\text{error} = 100\left(\frac{|6.546\times10^6 - 5.986\times10^6|}{5.986\times10^6}\right) = 9.35\%$$

As can be seen, we obtained a very similar value to our theoretical model. It was not exactly perfect, however. One of the potential sources of error on this is that we were not given error bars for our values for L and C, therefore the uncertainty for those values may attribute to this discrepancy. Another source of error could be the thermal noise of our setup. As can be seen in Figure 19, there are several points stacked very closely on top of each other, this comes from the noise. It is possible that this noise could contribute to the error we are seeing as well. One other place was this discrepancy could arise is in our fit for the data, as we can see in Figure 18 on the interval outside of where we took the fit it clearly does not follow a damped sine function. Therefore if we accidentally included some of those values in the data for our fit we could obtain a poor fit. Finally, part of our error could be from the unknown total resistance of the wire, we know the Waketek has some resistance of 50 Ω, but the rest of the wire (and the inductor itself) have some resistance to them so that will affect the fit we obtain.

As with other parts of the lab going back in the future and getting the exact intervals of the function that we want to study would help us obtain a better fit as well as going in and getting error bars on our circuit elements' values so that we can know a range for the potential ω's, instead of a particular value.

3.7 Part F: Unknown Element at the End of the Line

This section of the lab required some detective work. We were given a mystery box with some unknown impedance by our lab TA. We were told that there are two components inside the box and one of them was a resistor. We were tasked with finding out what other element was in the box, the configuration of the box, and the values for our elements. We were given Box #5.

In order to do this, we first looked at the reflected and incident waves to determine, roughly what each of them was the form of.

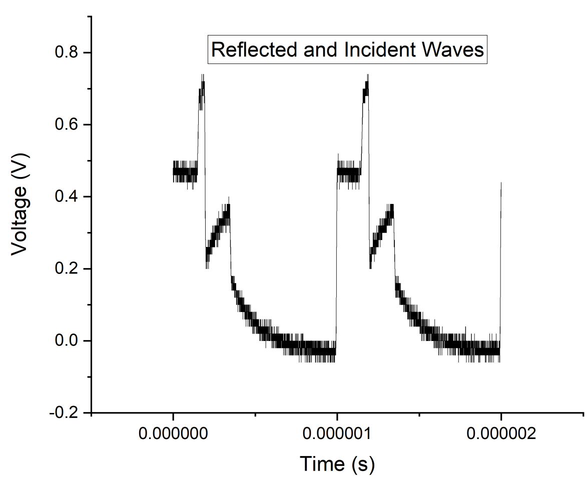

Figure 20: Reflected and Incident wave for our unknown box

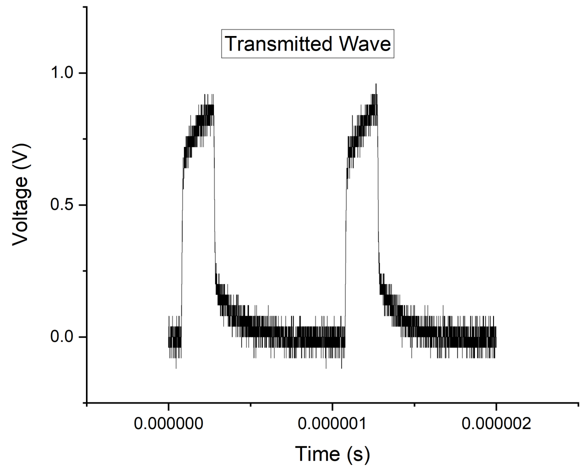

Figure 21: Transmitted wave for unknown box

As can be seen in Figures 20 and 21, there is a clear charging and discharging as we would see from a capacitor similar to the one in Figure 13. Thus, we know that our second component is a capacitor. Now we need to figure out if it is in series or parallel with our resistor. To do this we used the program we mentioned in Section 3.1.

We compared our incident and reflected pulses to that given by the program. We tried the two different configurations of series and parallel. Our data matched the model of a resistor and a capacitor in series. Now that we know the method of connection for our circuit we just need to determine values for our resistor and capacitor.

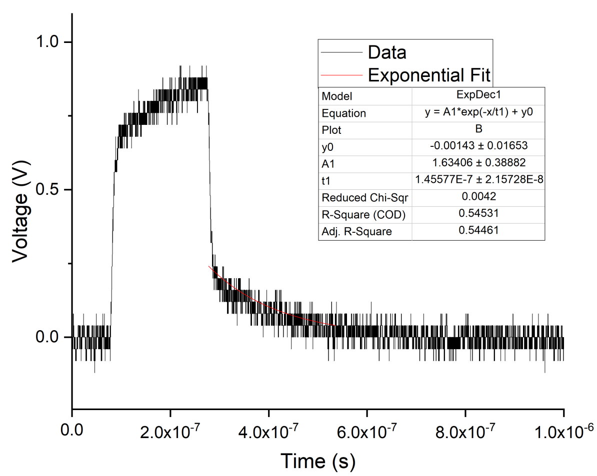

Figure 22: Transmitted wave with a fit for the unknown box, with parameters

A (V)	τ (s)
1.634 ± 0.388	$1.455\times10^{-7} \pm 2.157\times10^{-8}$

Table 9: Values for constants in Figure 22

As mentioned earlier in the Theory section the solution to our differential equation, used to find the total voltage across the circuit takes the form $A\exp(-t/\tau)$. Table 9 gives us the values for A and τ, therefore we can solve Eqns. 2.25 and 2.26 for R and C, respectively, to find approximate R and C values. We know that the value of Z_k for this circuit is 50 Ω.

R (Ω)	C (nF)
86.61 ± 144.82	1.07 ± .158

Table 10: Approximate values for R and C for unkownn circuit

As can clearly be seen in Table 10, there is a great deal of uncertainty for the value of the resistor. Part of this was because it was difficult to fit the exponential function to the data in Figure 22 without getting a value for A that would yield a negative resistance. Therefore, we had to play around with the fitting a lot before we got values that would give us a positive resistance. We can also say that we know the resistance should be positive so the error bar really should only extend to 86.61 on the bottom end. Another source of our error with this was how we confirmed our guess via the checking software. We looked at the values we got for R and C and compared to how the model on the software looked versus Figures 20 and 21. Because of this, we were biased towards certain values that looked close to our plots and fitted our data accordingly.

For this final part, we saw another situation where knowing the interval on which the function was charging or discharging would help us significantly in obtaining the values for our mystery components. Additionally not relying on the software to check our guesses as much would help us not be so biased in selecting the fits that matched our guesses better.

4 Conclusion

From all of this, we learned about time delayed circuits and the response of circuits with different loads to this time delay. During the first part of the experiment, we determined that for each of the wires we tested that their speeds of propagation were relatively similar. In terms of studying the response of the loads we, in general, saw a very close relation of our theory to our data. For resistive loads, we see that the reflected wave follows the form of Eqn. 2.10 rather closely while the transmitted wave follows the form of Eqn. 2.12 closely. Both the reflected and transmitted had a little deviation from theory due to our measurement procedures, mainly. For our load with a capacitor, we determined that the time constant followed the form of Eqn. 2.16, but we had some difficulties finding a fit that would yield the actual value of the capacitor we were using. Most of our error here (and in a lot of this part of the lab) comes from the way that we are fitting the data, including values outside our range gives us weird values for our parameters and unfortunately, the data is very sensitive to this. Similarly, for an inductor at the load, we found that the results for the time constant follow closely with Eqn. 2.17 and we obtained a fairly accurate value for the time constant and subsequent inductance. Again, a lot of this error was due to the sensitivity of our fit to the range we were fitting over. Additionally, for our LC circuit at the end of the line, we found that the frequency of oscillation of our damped sine wave followed the theoretical calculations of Eqn. 2.18 once again. This analysis suffered from the same issues as the other systems as well as an unknown resistance changing the damping parameters and possibly changing the value for our frequency in our fit. Finally, for our unknown black box, we used all of our previous knowledge on our systems to work backward and find that our box had a resistor and a capacitor in series, with values shown in Table 10. The sources of error in obtaining these values come, again, from our fit as well as our biases while guessing values in the software that outputted models based on our inputs for the values of the components.

We have mentioned it numerous times before in this lab but the main source of error in our measurements comes from the Origin fits. The fits are particularly sensitive to any data points that are included from outside the range that we should be fitting over. This sensitivity causes us to get fits that sometimes wildly vary based on only adding a couple of data points. Therefore, in the future, it would be in our best interest to look into being more precise in deciding over what intervals to be conducting our fits. In addition to the fits being fickle, none of the values for any of our components that were given to us contained any error bars. Error bars in this case would help us out because it would help us understand whether or not what we are getting from our experiments is a reasonable value or not. This would certainly help with the aforementioned fitting problem we had in this lab. Thus in the future going back and obtaining some sort of error bars for all of our components would help us out a great deal.