Experiment 7: Hall Probe Measurements of Magnetic Fields

Alex Garcia
Partners: Alex Huynh
TA: Jordan Sickle
Data taken November 10, 2020

November 24, 2020

Abstract

This lab focuses on the measurement of magnetic fields. We measured the magnetic field of Helmholtz coils as well as a series of Halbach arrays. During the first experiment with the Helmholtz coils, we determined that the coils, when separated by a distance of the coil's radius, create a region of the constant magnetic field in all directions. This result follows closely with the theoretical models derived from Biot-Savart. For the second experiment, our theoretical models from MATLAB followed somewhat closely to our measurements of these arrays. The arrays examined were uniform, quadrupole, and field gradient. In the future, testing the Helmholtz coil with fewer turns may prove useful to eliminate the small effect of the wires having a non-infinitesimal length. Similarly, for the Halbach arrays finding an effective method for measure the entire array would help us better determine how well the data matches the model.

1 Purpose

The purpose of this lab is to become more familiar with the measurement of magnetic fields. Magnetic fields are different from electric fields in the sense that they have easy, practical measurement techniques. The measurement technique of choice for this lab is the Hall effect. The Hall probe uses this effect to measure both the axial and transverse components of the magnetic field. In the first experiment with the Helmholtz coils, we verify that the Hall probe measures the axial component of the magnetic field by varying the angle of the probe head at the center of the Helmholtz coil. Next, we confirm that the magnetic field has some constant region inside the two coils in the z-direction by moving the coil through the coils. The next step is very similar, only this time in the x-direction. Finally, we determine the radius of the sphere of a constant magnetic field inside the coils. For the second experiment, we employ the Hall probe to measure the magnetic fields of the different types of Halbach arrays.

2 Theory

This lab centers around the idea of studying magnetic field distributions created by various systems. The two systems that are examined in this lab are Helmholtz Coils and Magnetic Halbach Arrays.

2.1 Helmholtz Coils

The most fundamental thing that needs to understand about magnetic fields is how to calculate them. To this, we defer to the Biot-Savart Law which can be expressed as follows.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3} \tag{2.}$$

Where μ_0 is the permeability of free space and is equal to $4\pi{\times}10^{-7}$ N/A^2. This equation allows us to calculate the magnetic field anywhere around a current-carrying wire.

If we consider a loop of current, as with Helmholtz coils, we can employ this Biot-Savart Law to compute the field at some point along the central axis of the loop. The geometry of this situation can be seen in Figure 1.

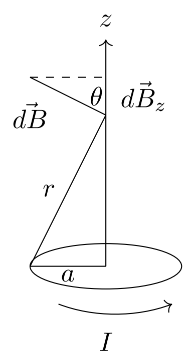


Figure 1: Caption

If we just look at B-fields along the z axis we can say the following.

$$dB_z = dB\cos(\theta) = dB\frac{a}{r} \tag{2.}$$

Integrating around each infinitesimal length around the loop we obtain the following for the length term of Eqn 2.

$$d\vec{l} = ad\vec{\varphi} \tag{2.}$$

Using the Biot-Savart law to solve for the differential B-field elements we obtain the following expression.

$$dB_z = \frac{\mu_0}{4\pi}\frac{Ia^2 d\varphi}{r^3} \tag{2.}$$

Taking the integral on both sides and evaluating from 0 to B_z for the B-field, from 0 to 2π for the angular component φ and making the substitution that $r = \sqrt{a^2 + z^2}$ we obtain the general form of the B-field along the z axis for a current-carrying loop.

$$B_z = \frac{\mu_0 I}{2}\frac{a^2}{(a^2 + z^2)^{3/2}} \tag{2.}$$

This general form can be extended to a loop with multiple current-carrying wires by multiplying by the number of wires in the loop.

$$B_z = \frac{\mu_0 NI}{2}\frac{a^2}{(a^2 + z^2)^{3/2}} \tag{2.}$$

This brings us to the experimental set-up for the first half of the lab, Helmholtz coils. Each of these coils is just a multiple-wired loop with current running through it. These coils have the same radii and current running in the same direction. The set-up can be seen in the figure below.

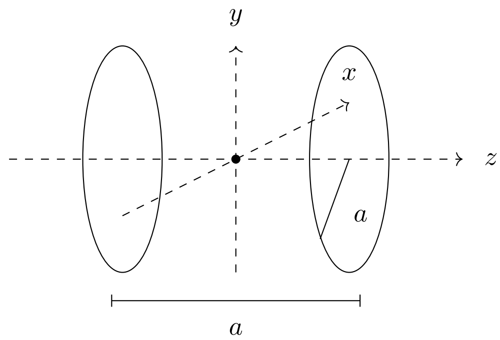


Figure 2: Experimental set-up, two Helmholtz coils placed coaxially with several wires wrapped around around them (Diagram not to scale).

Since these coils are coaxial, their B-field contributions will add in between them. This addition will, generally, consist of two peaks on either side of the origin and a valley at the origin. Moving the two coils closer to each other shifts the peaks together as well. This effect, when the coils are a distance of one radius apart, will create a constant magnetic field at the origin. This is what makes Helmholtz coils so useful, they provide an easy, straightforward way of generating a uniform magnetic field.

The B-field generated at the origin when the coils are separated by a distance of one radius is well described by Biot-Savart and the conclusion derived in Eqn. 2.. If we take the positions of the two coils to be $\left(\frac{a}{2}, 0\right)$ and $\left(-\frac{a}{2}, 0\right)$ we can express the B-field as the following.

$$B = \frac{\mu_0 N I a^2}{2} \left\{ \frac{1}{\left[\left(z + \frac{a}{2}\right)^2 + a^2\right]^{\frac{3}{2}}} + \frac{1}{\left[\left(z - \frac{a}{2}\right)^2 + a^2\right]^{\frac{3}{2}}} \right\} \hat{z} \tag{2.}$$

The following diagram visualizes the magnetic field

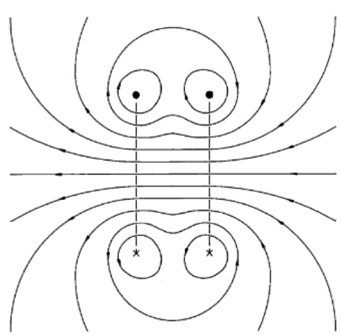
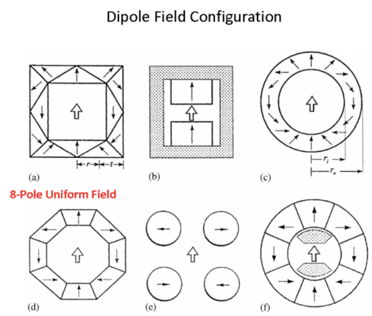


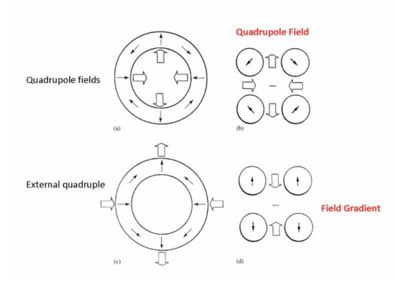
Figure 3: Field Lines on Helmholtz coils

2.2 Halbach Arrays

A Halbach array is a particular configuration of permanent magnets that concentrates the flux on one side of the array while cancelling it on the other side.



A)



B)

Figure 4: Examples of Halbach Arrays, large arrow indicates direction of B-field. A) Dipole and uniform field Halbach magnetic configurations and B) Quadrupole and field gradient configurations. (Courtesy of lab manual)

These arrays all have individual elements that add up to give the desired magnetic field either uniformly, in a dipole or a quadrupole.

3 Experiments

3.1 Equipment

In the first part of this lab, our apparatus consists of two Helmholtz coils, a digital multimeter to measure the current, and a battery to supply the current. The second apparatus consisted of a series of permanent magnets lined up in a certain configuration that we needed to analyze.

The measurements of the magnetic fields were taken with an AlphaLab Hall DC magnetometer. This magnetometer, known as a gaussmeter, has two probes, one axial and the other transverse. The axial probe measures the parallel component to the cylindrical axis of the B-field, while the transverse probe measures the perpendicular component of the B-field.

Our data analysis was performed in Origin as well as some MATLAB scripts written by our TA, Jordan. To run the experiments we used a program on the computer that controlled the position of the Hall probe, which was attached on a track that we could manipulate.

3.2 Measurement of the Helmholtz Coil Field

In this part of the lab, we are experimenting with the Helmholtz coils' magnetic fields in several different configurations. The set-up for this part of the lab can be seen in Figure 2. We used a digital multi-meter to measure the current and set it near 3.0 A.

Inner Diameter (cm)	Outer Diameter (cm)	Turns	Separation (cm)	Current (A)
19.7 ± 0.1	21.5 ± 0.1	125	10.4 ± 0.1	2.98 ± 0.02

Table 1: Specifications of Helmholtz coils and to measured current

The first part of this experiment was to prove that the field at the center of a Helmholtz coil is axial. To do this we took the probe and measured the magnetic field at a series of angles with the transverse probe. The measurements were made over a series of angles, from 0° to 180° in increments of 10°.

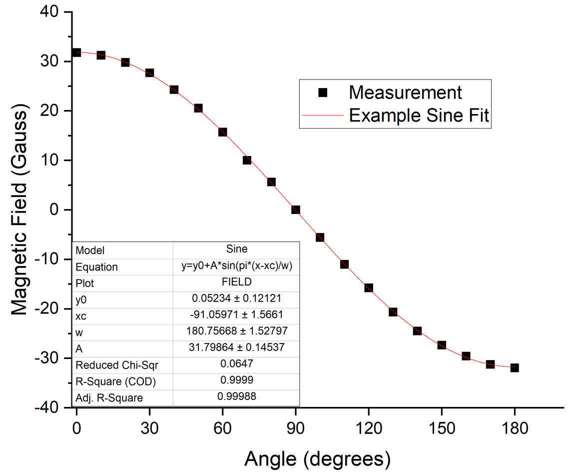


Figure 5: Plot of magnetic field versus angle measured at

We found that the relationship between the magnetic field and the angle we measure it with the transverse probe is non-linear. In fact, it follows extremely closely with a sinusoidal curve.

Some of the error in this sinusoidal relationship comes from the method in which we measured the angle. Our TA, Jordan, used a protractor to manually set the angle of the probe each time. This process has the element of human error, setting each angle exactly is rather difficult with a protractor. The angle will be relatively close, but not exact. Thus, we have a small amount of error on the sinusoidal fit.

The second part of this experiment was centered around mapping the axial component of the magnetic field (B_z) as a function of distance in the z-direction (definition unit vectors can be seen in Figure 2).

We took the Hall probe in axial mode along the perpendicular axis of the coils and varied its position along that axis. We used distances of -20 to 20 cm in increments of 0.5 cm.

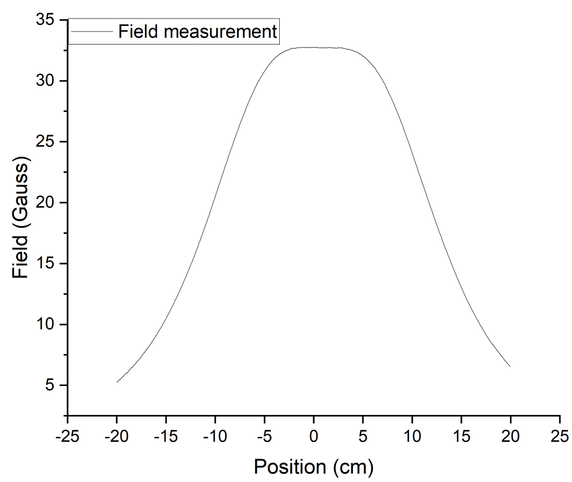


Figure 6: Plot of axial magnetic field as a function of position in the z-direction

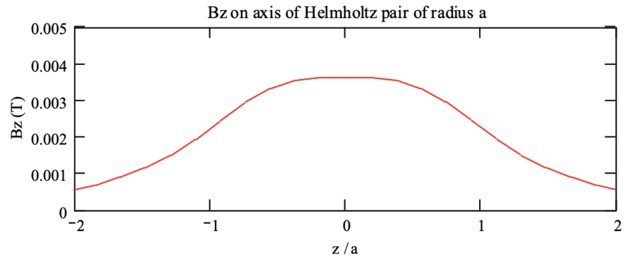


Figure 7: Theoretical Model (Eqn. 2.) of the field measurements as a function of position (Courtesy of lecture slides)

Comparing Figures 6 and 7 we can see that our experimental data follows closely to the general trend of the theoretical model. The peak value of the theoretical curve, based on plugging $z = 0$ into Eqn. 2., is 32.0 $\pm$ 0.1 Gauss. The peak value of the experimental curve, based on Figure 6, is 32.5 $\pm$ 0.1 Gauss. These two values are extremely close, with approximately 1.6% error. Some of this error stems from the fact that the coils have some of their own lengths. The coils having their own length means that there are some contributions from every wire that are not exactly stacked on top of each other. This effect will have a small impact on the final value of the peak magnetic field.

The third and final part of this experiment is very similar to the second part. We again measured the magnetic field as a function of position, except this time instead of moving axially, we moved the probe away in the x-direction.

We mapped the field on the median plane using the transverse probe. The transverse probe is used in this case to avoid running into the coils during the measurement process. The distances used in this case were -20 to 20 cm in increments of 0.5 cm.

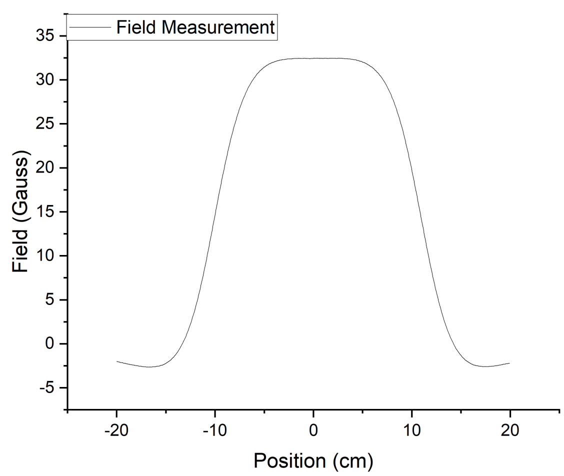


Figure 8: Plot of magnetic field measurements as a function of position in the x-direction

Similarly to the second experiment a region of a constant magnetic field is easily seen, as expected. This region extends much further than in the z-direction, this matches with the visualization in Figure 3. Thus, this result is consistent with our theoretical models.

The final part of the experiment was to determine the radius of a spherical volume centered within the Helmholtz coils that has a constant field within 5%.

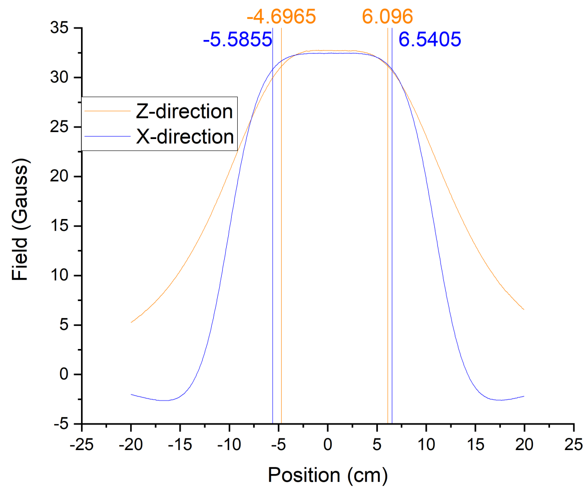


Figure 9: Both x- and z-directions magnetic fields along with their corresponding 5% deviation from uniform

	Peak Value (Gauss)	5% deviation value (Gauss)	Corresponding distances (cm)
z	32.50 $\pm$ 0.1	30.88 $\pm$ 0.1	-4.697 $\pm$ 0.3 & 6.096 $\pm$ 0.3
x	32.45 $\pm$ 0.1	30.84 $\pm$ 0.1	-5.590 $\pm$ 0.3 & 6.541 $\pm$ 0.3

Table 2: Measurements of peak values and the corresponding 5% decrease from peaks and distances

As can be seen from Figure 9 and Table 2, the sphere does not have any specific radius naturally. It appears that on one positive side the radius could be around 6.3 cm, while on the negative side the radius could be around 5.2 cm. Since there is much variation in these distances, we decided to take the mean of the magnitude of these values, which yields a value of 5.731 $\pm$ 0.3 cm.

3.3 Measurement of Halbach Array

In this section of the lab, we explored three different Halbach arrays. The three different arrays that we experimented with were quadrupole, uniform, and field gradient configurations. These Halbach arrays can be seen in Figure 4 (each of the individual elements labeled in red).

To measure the magnetic field we used the Halbach probes, axial and transverse, from the last experiment and now moved them over a square 10x10 cm area. The actual area of the array was 14x14 cm, but we looked within that range to ensure we got all the points. Once we had our data, we compared it to a MATLAB simulation. Our discussion in this section will center around the comparison of these plots.

For all of these figures the image on the left is the MATLAB simulation, while on the right is our data

First, we examined the field gradient array.

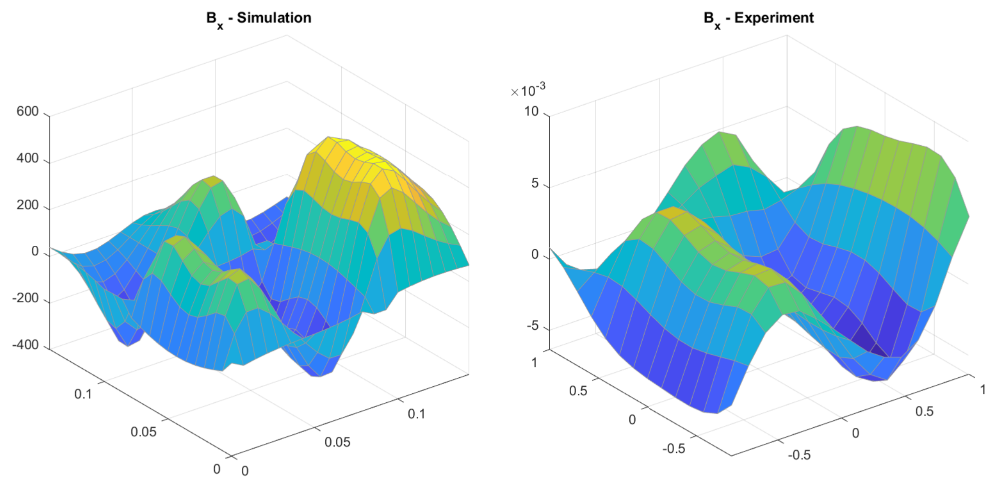


Figure 10: Field gradient array measured with the transverse probe

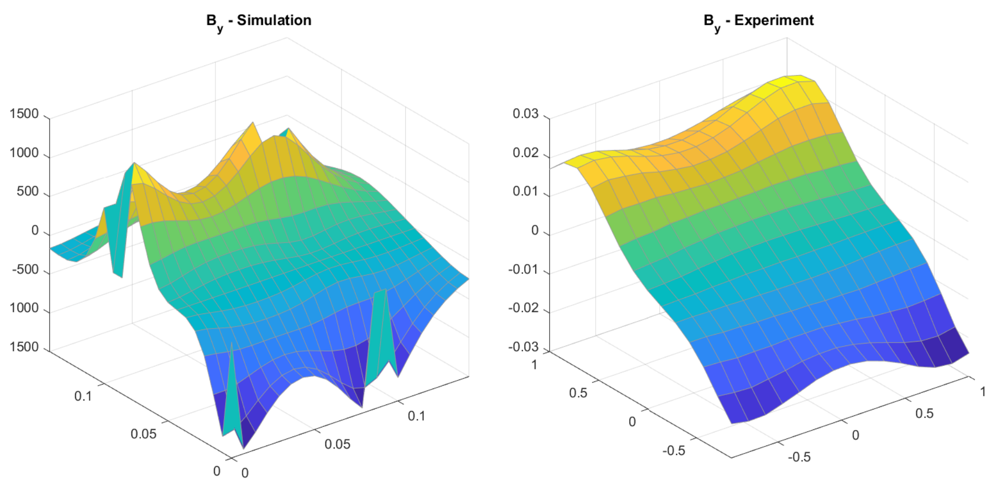


Figure 11: Field gradient array measured with the axial probe

Both of Figures 10 and 11 show that our observations match fairly closely with our models. Based on the agreement that we observe we can say that both the axial and transverse models are fairly accurate for the field gradient Halbach array.

While both of these plots demonstrate a good relation between simulation and experiment, there is some deviation. We can see that the experimental data follows the trend, but differs in the peaks and valleys. This deviation could stem from the fact that we only took the 10x10 cm square of the array and our MATLAB simulation might run the entire 14x14 cm Halbach array. Therefore our experimental data is just a small subset of the simulation that MATLAB ran.

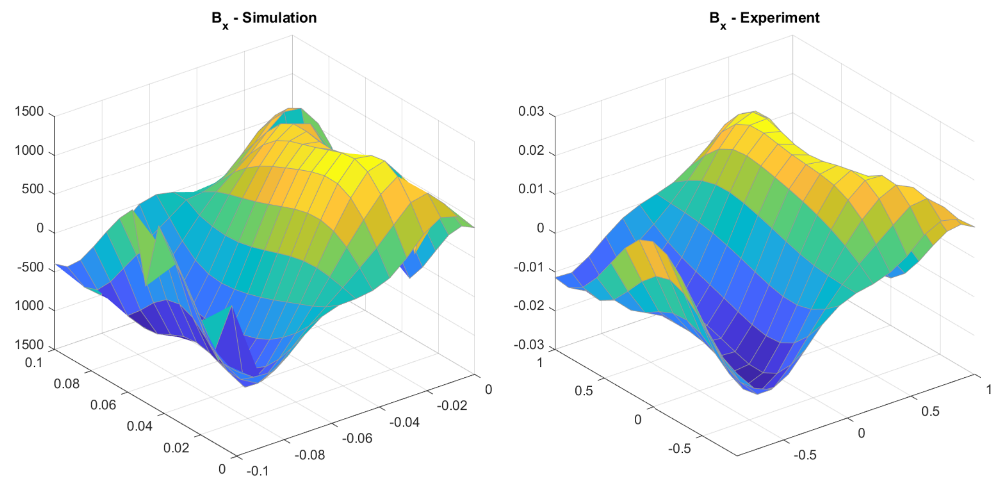


Figure 12: Quadrupole array measured with transverse probe

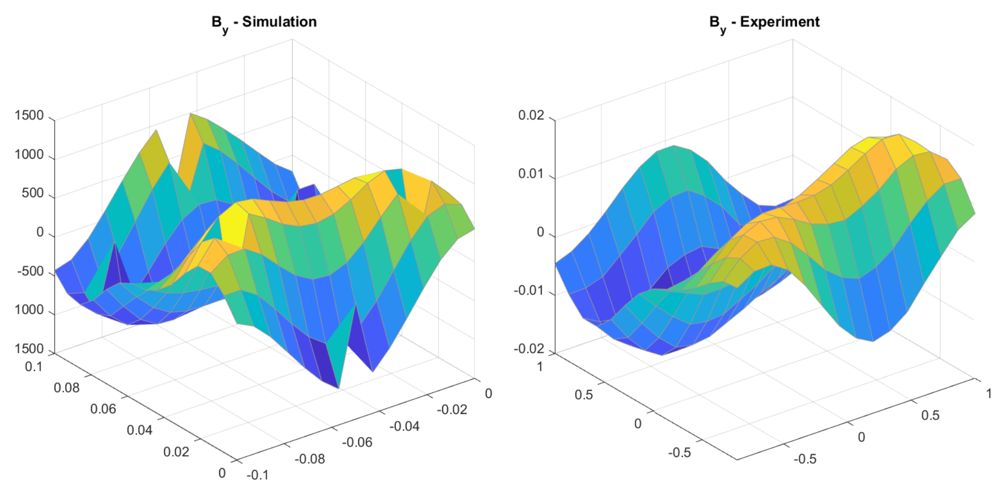


Figure 13: Quadrupole array measured with axial probe

Similar to the field gradient array, the quadrupole magnetic field theoretical model matches quite closely with the experimental values. The quadrupole array seems to have a clearer trend between experimental and theoretical for both probes than the field gradient array. Based on this agreement, we can say that the transverse and axial models are reasonably accurate for the quadrupole array.

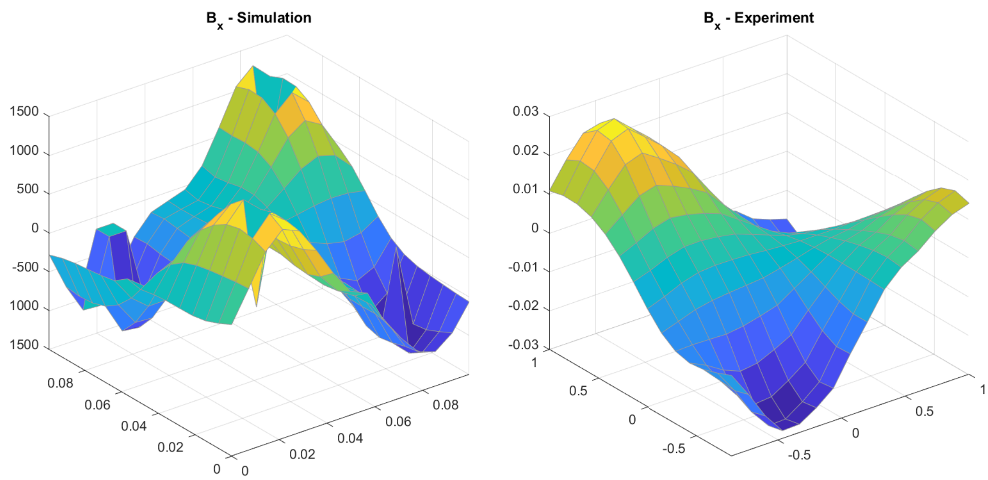


Figure 14: Uniform array measured with transverse probe

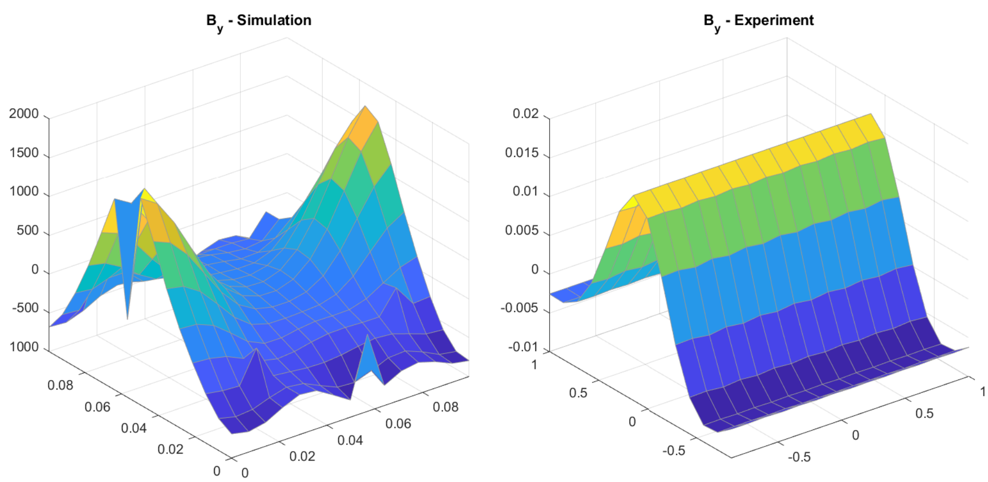


Figure 15: Uniform array measured with axial probe

For the uniform array, we can see that the agreement between the simulation and experimental is much less than in either of the other two arrays. While the transverse measurements seem similar under a 90° rotation of the figure, the axial probe measurements are quite different. The transverse measurements being rotated might be due to an experimental error of orienting the array in the wrong way or a computational error in not rotating the simulated data in the correct direction. As for the axial measurements, since this array is designated as 'uniform' the experimental data seems to make logical sense with the unimodal distribution. Since neither the transverse, nor axial, simulated fields are uniform on the array, we assert that there may be some miscalculation in the MATLAB simulation. It almost seems like the simulation depicts a dipole measurement, though we are not certain of this.

4 Conclusion

In conclusion, we can see that utilizing the Hall effect in the Hall probe to measure the magnetic fields is incredibly effective. For the Helmholtz coils, we verified that the relationship between the magnetic field and angle of measurement is non-linear. In fact, we derived that it follows a sine curve. This implies that the Hall probe's measuring device, while in axial mode, truly only measures along the axial direction of the coils. Additionally, we found that the coils create a region of a constant magnetic field in two directions (the z and x) when their separation is the radius of the coils. Using these results, we obtained a radius for the sphere of a constant magnetic field. Some of the error in our measurements in this section came from the wires coiled around the loop have some non-zero width. This effect causes the magnetic field to be slightly different than expected. In the future, perhaps trying a smaller number of loops or thinner loops would help eliminate this effect. For the Halbach arrays, we saw a fairly good agreement between the MATLAB simulations for the field gradient and quadrupole arrays. Some of the errors in these experiments came from the fact that we were measuring a 10x10 cm square on the 14x14 cm array. In the future, finding a way to accurately measure the magnetic field on more of the array would help decrease this error. The uniform array, especially the axial measurements, had some more significant deviation from the simulations. One of the errors on the transverse measurements for this uniform array came from rotation, the experimental and theoretical plots seemed similar, but only under a 90° rotation. As for the axial probe, it seemed like the theoretical model was not modeling a uniform array, but rather a dipole, which caused significant deviation from the experimental values.