Experiment 5: Measurement of the Electronic Charge by the Millikan Oil Drop Model

Alex Garcia
Partners: Alex Huynh
TA: Jordan Sickle
Data taken October 6, 2020

Original Report: October 13, 2020
Resubmitted: December 13, 2020

Abstract

In this experiment we replicated the Millikan Oil Drop experiment in order to find the charge of an electron and to prove that charge is quantized. First we looked at our own lab section's small subset of our entire class' data and ran the process through just to get a feel for it. For this partial set we saw, for the most part, fairly decent values for the charge of an electron as well as demonstration of the quantization of charge. We then performed the same process for the full class' data and ran the same statistical examinations. Again we saw pretty good agreement with the the value of the charge of an electron. As for quantized charge we saw slightly worse agreement in the full class data-set, but the general trend held. In the future we would really like to get in the lab and actually perform this experiment. Some experimental errors that we came across could have come from an imperfect set-up that we just did not know about because we were only given the data-set. Additionally, some of the analysis done was biased in the sense that we arbitrarily selected values around what we already knew our result should be. Therefore, in the future finding a different way to analyze the peaks would help us obtain more accurate, unbiased statistics.

1 Purpose

Starting in the 1700's with Benjamin Franklin's studies of electricity there has been a great quest to find out more information on elementary particles. Things really picked up at the end of the end of the 19$^{\text{th}}$ century with Helmholtz and subsequently Thompson describing electricity using tiny packets, or 'atoms of electricity' as Helmholtz called them. Finally, in 1909 Robert Millikan along with his graduate student Harvey Fletcher developed an experimental set-up to determine the charge of this packet. We aim to recreate this same set-up that Millikan used over 100 years ago. With this experiment Millikan proved not only the charge of an electron but that charge is quantized, which is to say can only be integer multiples of the charge of an electron. The first data-set that was provided for us contained information from a small number of tests measuring the rise and fall times of oil droplets in our capacitor. Between the large and small data-sets we saw good agreement between our calculation of charge and the theoretical value for an electron. In terms of quantization of charge, surprisingly, the small data-set shows better evidence than the full class data-set.

2 Theory

During this lab we attempted to find the charge of an electron in the same way that Millikan and his graduate students did. The main set-up for this experiment is shown in Figure 1. In effect we are subjecting the charged oil droplet to some number of forces and peering at it through a microscope in order to watch how it moves. The specific things we are looking to measure are the rise and fall times of the particle.

From the rise and fall times, along with several constants of the environment we can formulate a process for finding the charge of an electron, as well as dividing our data by our theoretical value for an electron to find the quantization of charge. We will get more into this formulation after an overview of our set-up.

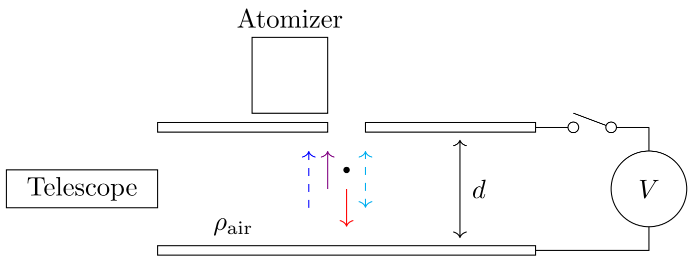


Figure 1: Model of experimental set-up with the forces and some physical parameters shown.

Looking at the Figure 1 the colored arrows all represent the forces that the oil droplets are subject to during their fall. This one is the drag force. This one is gravity. This is the buoyant force. This is the electric force.

$$\vec{F} = m\frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E \tag{2.1}$$

In Eqn. 2.1 we lump the gravitational and buoyant forces into one force F_g.

$$\vec{F}_g = -mg\hat{z} \tag{2.2}$$

$$\vec{F}_{\text{drag}} = -6\pi\eta a\vec{v} \tag{2.3}$$

$$\vec{F}_E = Q\vec{E} \tag{2.4}$$

Even though we have several forces acting in different directions on our oil droplet terminal velocity is achieved within around 10 μs of being subject to all these forces. This means that during the rise and fall times of the oil droplet we can treat it as traveling at constant velocity.

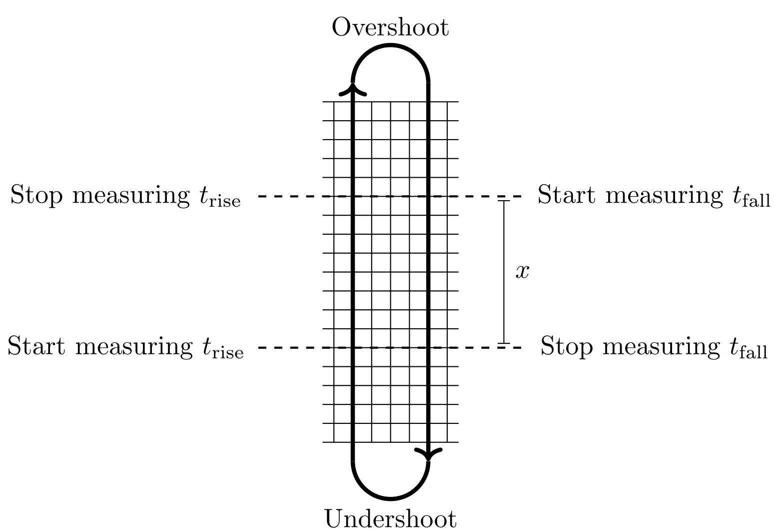


Figure 2: Set-up for measuring t_{rise} and t_{fall} for the oil drop falling in our system.

Figure 2 represents what we would see if we were to look into the telescope at the end of the capacitor. The grid is superposed on the background with a light shined on it so we can see the droplet. One student looks through the scope to monitor the time it takes each droplet to go from the top dashed line to the bottom and visa versa. Once the object passes the dashed line the electric field is turned off and the particle slows down. At this point we then flip the electric field and watch the droplet rise up to the top. This process is done several times for each droplet and for several droplets.

There are number of parameters that go into finding the charge and consequent number of elemental charges. We need to worry about the following:

Drop	Apparatus	Measured
a (radius)	E (E-field)	t_g (fall time)
ρ (net density)	V (Voltage)	t_{rise} (rise time)
v (velocity)	η (viscosity)	
Q (charge)	g (gravity)	
	x (drift distnace)	

Table 1: List of all the parameters and constants of our experiment

Each of these parameters and constants impact the formulation of the charge of the droplet. Finding the total charge on the droplet involves a number of different values based on these parameters that in the end will be multiplied together. We expect that the total charge on the droplet will be some integer multiple of e (the charge of an electron), therefore we can write the charge in this way, too.

$$Q = ne = FST \tag{2.5}$$

There are several steps in deriving this formula. The first one we will explore is the modification of Stokes' Law. It was discovered that for particles of radius less than approximately 15 μm, Stokes' Law must have a correction factor called the Cunningham factor, this factor is added to Eqn. 2.3.

$$\vec{F}_{\text{drag}} = -6\pi\eta\frac{a}{f_c}\vec{v} \tag{2.6}$$

This Cunningham factor has a derivation from Newton's Laws and complex equation but it simplifies down to the following approximation.

$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \tag{2.7}$$

$$r_c = \frac{6.18 \times 10^{-5}m}{p[mm\ Hg]} \tag{2.8}$$

Knowing r_c (Eqn. 2.8) we can describe another parameter of the system called τ_g. This parameter is sort of an intermediary step in calculating F.

$$\tau_g = \frac{2\eta x}{\rho g r_c^2} \tag{2.9}$$

From τ_g we can describe the aforementioned Cunningham correction to some power, which we will call F, in terms of our τ.

$$F = f_c^{-2/3} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}} \tag{2.10}$$

S in Eqn. 2.5 is known as the Stokes' law force balance and can be described as follows.

$$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}} \tag{2.11}$$

T from Eqn. 2.5 can be described by the timing measurements that we take while looking through the telescope.

$$T = \sqrt{\frac{1}{t_g}}\left[\frac{1}{t_g} + \frac{1}{t_{\text{rise}}}\right] \tag{2.12}$$

Now that we have explain what each part is we can rewrite Eqn. 2.5 in it's full form.

$$Q = ne = \frac{1}{f_c^{2/3}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g}} \left[\frac{1}{t_g} + \frac{1}{t_{\text{rise}}}\right] \tag{2.13}$$

In addition to all of this η is actually a function of temperature that we can approximate rather simply. We calculate the value of $\eta(T)$ around 25° C using the following relation.

$$\eta(T) = \eta(25) - \frac{d\eta}{dT}(T - 25) \tag{2.14}$$

In addition, we know from any science course that the charge of an electron, e, is roughly 1.602×10^{-19} C.

Finally, we can define the percent error that we will use to calculate how close we were to our theoretical values as the following.

$$\%\ \text{error} = (100)\frac{|\text{theoretical} - \text{experimental}|}{\text{theoretical}} \tag{2.15}$$

Quantity	**Value**
V	500 V
η	1.8478×10^{-5} kg/m s (25°C)
$\frac{d\eta}{dT}$	4.8×10^{-8} kg/m s/°C
ρ_{oil}	886 kg/m^3
ρ_{air}	1.29 kg/m^3
$\rho_{\text{oil}} - \rho_{\text{air}}$	884.71 kg/m^3
g	9.801 m/s^2

Table 2: Table of constants

The temperature and pressure also matters for each of these systems. For the partial data set, the temperature ranged from 22 °C to 26 °C and the pressures ranged from 761 mm Hg to 769 mm Hg. For the full data set, the temperature ranged from 20.5 °C to 28.5 °C and the pressure ranged from 760 mm Hg to 769 mm Hg. All of these temperatures and pressures just depended on the time and place we measured the oil drops at.

3 Experiment

3.1 Equipment

For this lab we used a number of different pieces of equipment in our analysis of the oil drop experiment. The bulk of the equipment is shown in Figure 3.

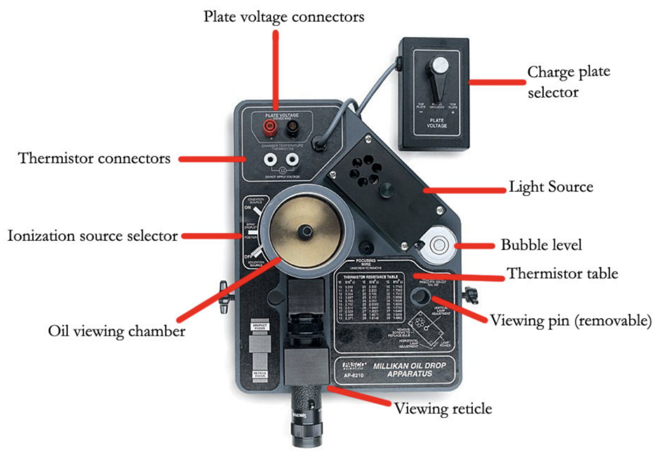


Figure 3: All the Equipment used in lab

The Thermistor is a resistor that changes resistance with temperature so we can accurately read off the temperature of the environment, the table showing that conversion is also used. There is also the charge selector plate which allows us to turn off and on the electric field within the conductor to let the charge rise and fall. We used a microscope in order to look at the droplet rising and falling. Additionally, we have the viewing reticle which is the grid that we see in Figure 2 as well as the light source which illuminates the droplet so we can see it. The oil viewing chamber is where we conduct our experiment and watch the oil. The plate voltage connectors connect to the voltage source. The last thing on this apparatus is the bubble level which we use to make sure that the whole system is level and the gravity vector points directly down.

For our data analysis we used Origin for the plots and Python for calculation of the charges and quantization levels given rise and fall times. One issue that was noticed when using Python versus inputting the data into the Origin template provided was that the values were off by a small amount. It was not enough to raise alarm but it was non-zero.

3.2 Small Data-set

First, during the lab section we were given a small sample of the larger data set in order to understand the process of analyzing the data. In addition we were limited by time in our lab section so analyzing the full data-set was set to be outside of lab.

3.2.1 Charge of electron

The first thing we did was take the measurements of t_g and t_{rise} and use Eqn. 2.5 to find the total charge in Coulombs. We looked at histogram of charges we obtained to obtain the charge of an electron.

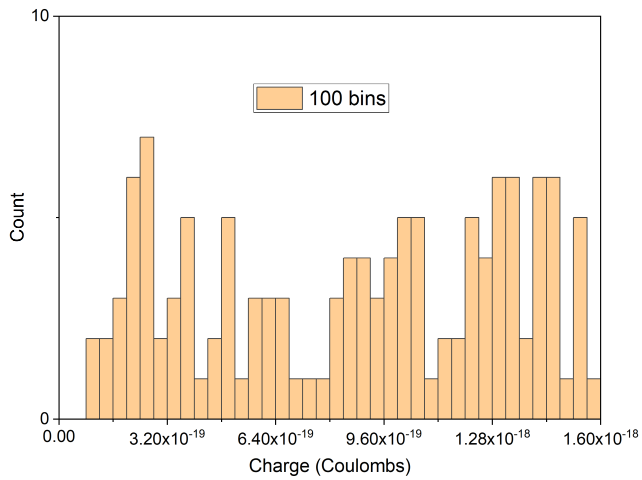
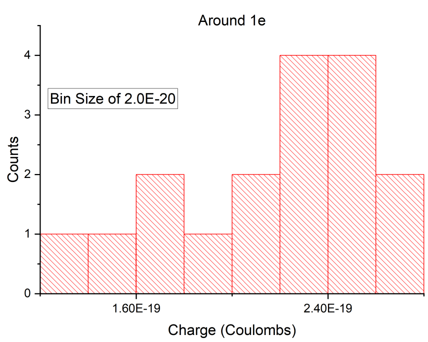
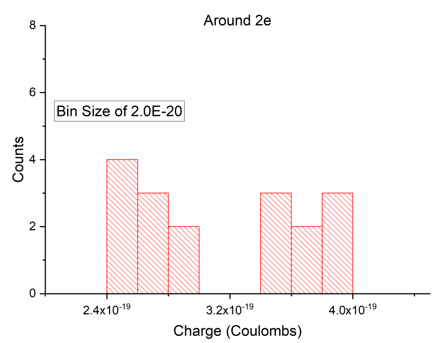


Figure 4: Histogram of all charges less than $10e$ for our partial data-set, in terms of Coulombs

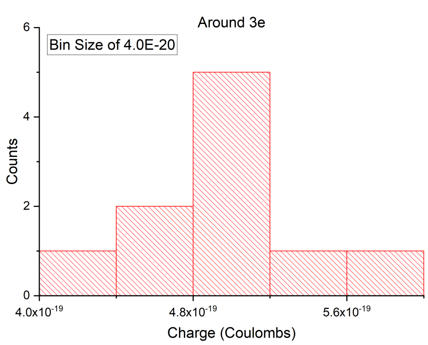
As you can see from Figure 4 there are some peaks but none of them are particularly clearly Gaussian. There are a number of reasons that we do not see a nice distribution, chief among them should be that we simply do not have enough data to get this well-behaved curve. With more data it is more likely that we will observe values closer to our theoretical value and thus our histograms will become 'more normal'. Regardless, we performed statistical analysis on the values around our theoretical ne and make histograms of those values then recorded their mean and standard deviation.



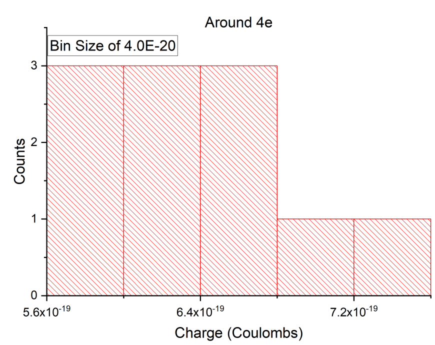
A)



B)



C)



D)

Figure 5: Histograms of values for charges around ne in terms of coulombs. A) $n = 1$, B) $n = 2$, C) $n = 3$, D) $n = 4$

n	N data points	Mean	Standard Deviation
1	17	$2.15\times10^{-19} \pm 1.04\times10^{-20}$	4.27×10^{-20}
2	17	$3.26\times10^{-19} \pm 1.45\times10^{-20}$	5.96×10^{-20}
3	10	$4.94\times10^{-19} \pm 1.44\times10^{-20}$	4.56×10^{-20}
4	11	$6.39\times10^{-19} \pm 1.67\times10^{-20}$	5.53×10^{-20}

Table 3: Table for mean and standard deviation of charge values for partial data-set

n	1	2	3	4
% error	34.2	1.7	2.9	0.15

Table 4: Percentage Error of charge

As you can see there is already fairly strong evidence, barring the n =1 data, that the charge of an electron is around our theoretical value, even in the small data set. Most of the error on these values come from the way we were measuring the t_g and t_{rise}. When we look into the microscope and time the droplet falling or rising through the grid there exists a chance that the time we record is just a little off. This error could propagate through our calculation of Eqn. 2.5 and lead to the small error bars we're seeing. In terms of why specifically the $n = 1$ data is so far off the theoretical value, we can see that Figure 5A) that there is a significantly larger amount of charges above e than below e. This can be explained through the small number of charges that we are sampling. For example, if we look at 5B) we do not even see any charges measured in our theoretical value, so it's not unreasonable to measure charges above and below e. In the case of $n = 1$ we just happen to measure more charges greater than e than less than e, hence our large value for the mean.

3.2.2 Quantization of charge

This second part of data analysis we took our value for the charge we obtained in 3.2.1 and divided it by our theoretical value for e. We plot a histogram of these values to show that there is spacing between integer values in order to prove the quantization of charge.

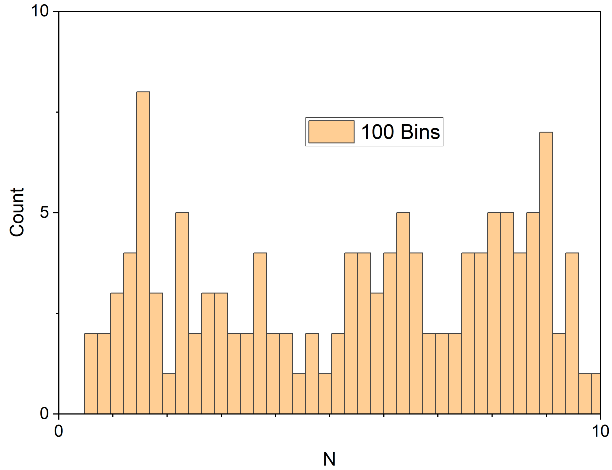


Figure 6: Histogram of all charges less than $10e$ for our partial data-set, in terms of elementary charge Q/e

Similar to the charge histogram we don't really see the Gaussian behavior we would expect. We again attribute this to the small number of datum in our small data-set. We performed the same process as above for the charges divided by our theoretical value of the charge of an electron.

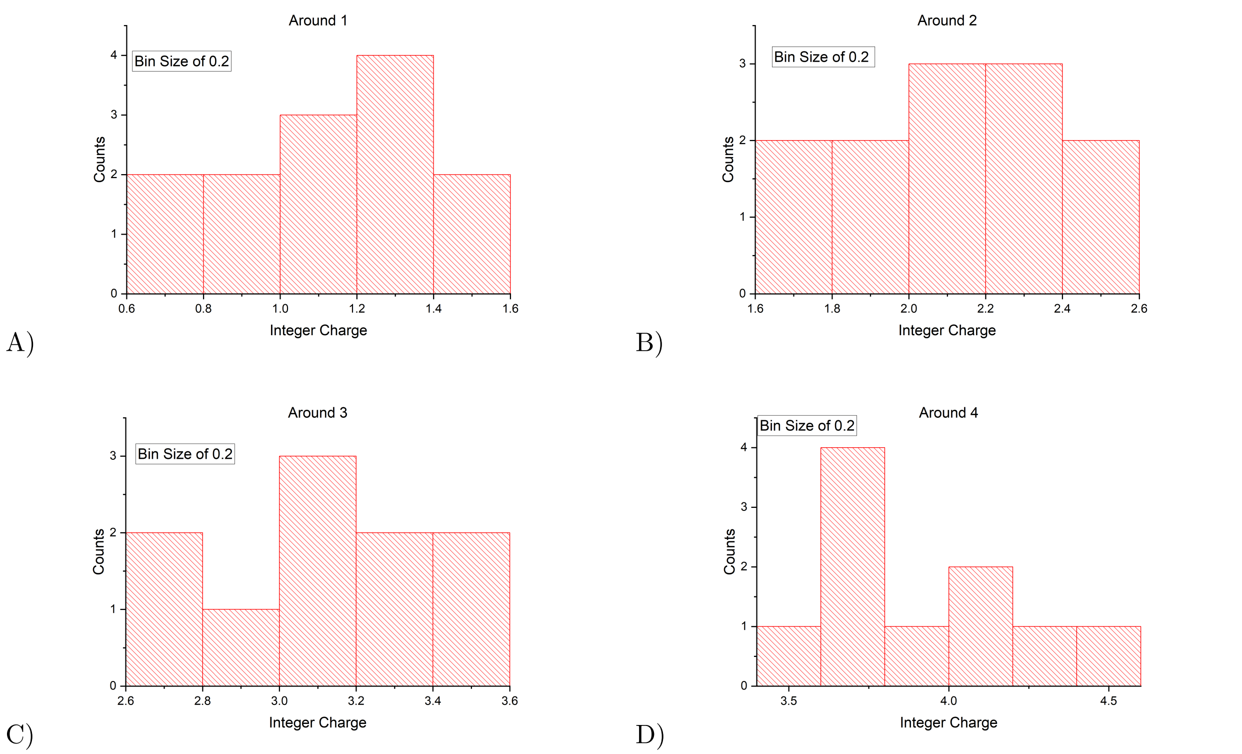


Figure 7: Histograms of integer values of elementary charges around n. A) $n = 1$, B) $n = 2$, C) $n = 3$, D) $n = 4$

n	N data points	Mean	Standard Deviation
1	13	1.12 $\pm$ 0.080	0.290
2	12	2.13 $\pm$ 0.082	0.283
3	10	3.08 $\pm$ 0.090	0.284
4	10	3.92 $\pm$ 0.091	0.289

Table 5: Table for mean and standard deviation of elementary particles for partial data-set

From Table 5 we can start to see this idea of quantization of charge. We know that *if* we have a Gaussian distribution that approximately 68% of measurements should be within one standard deviation. Looking at this we see that most of the charges that we measure are approximately within 0.3 of an integer value of charge. We can see in this case that there is no evidence for no half charges.

n	1	2	3	4
% error	12.0	6.5	2.7	2.0

Table 6: Percentage Error for integer number of elementary charges for the partial data-set

Looking at Table 6 we can see that there is pretty close agreement, again. Again, some of the errors that we could create here come from measurement, most likely the measurement of t_g and t_{rise}. Similarly one issue that could cause some discrepancy in our values comes from the heat generated by us and the light bulb. Presumably, we measure the Thermistor at the start of the experiment while the apparatus was sitting alone for a while. Once we measure we are in close contact with the system and we turn on a light very near to the apparatus. Since the calculation is temperature dependent these changes in temperature cause inconsistencies in our values.

When we examine why this $n = 1$ result has much more success matching the model compared to the charge statistics, we notice that we sampled less points this time around. This helped us find a closer value to the theoretical because we do not have as many of those high values we referenced before. This brings us to one of the main issues with the way we are describing the data. When we go to make the histograms around our specified value we are manually selecting the range of values we look at. Evidently, this choice can make a significant difference in our agreement with the theoretical models.

3.3 Class Data-set

Now, outside of the lab section, we were given the full set of over 1000 measurements to analyze. The idea behind more observations goes hand-in-hand with what we were discussing in Section 3.2, with more measurements we should see more normal data with small deviations and a closer mean.

3.3.1 Charge of electron

We ran the same formulations as in Section 3.2.1 with our larger data-set.

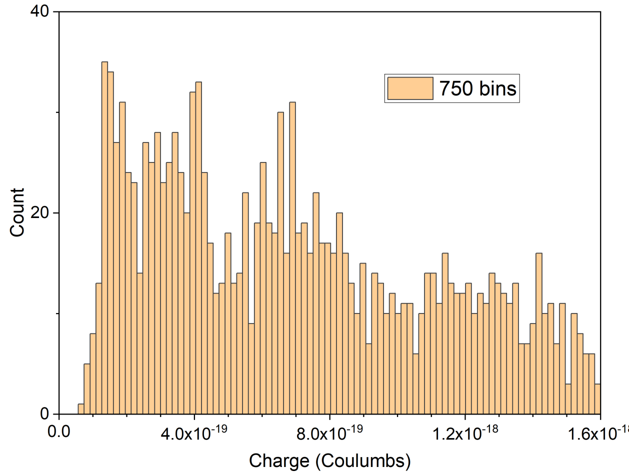


Figure 8: Histogram of all charges less than $10e$ for the class data-set, in terms of Coulombs

The peaks are just a little bit clearer than from the partial set, but for the most part it still appears fairly difficult to see each individual peak of charge. To help with this process we took values around the theoretical and made histograms.

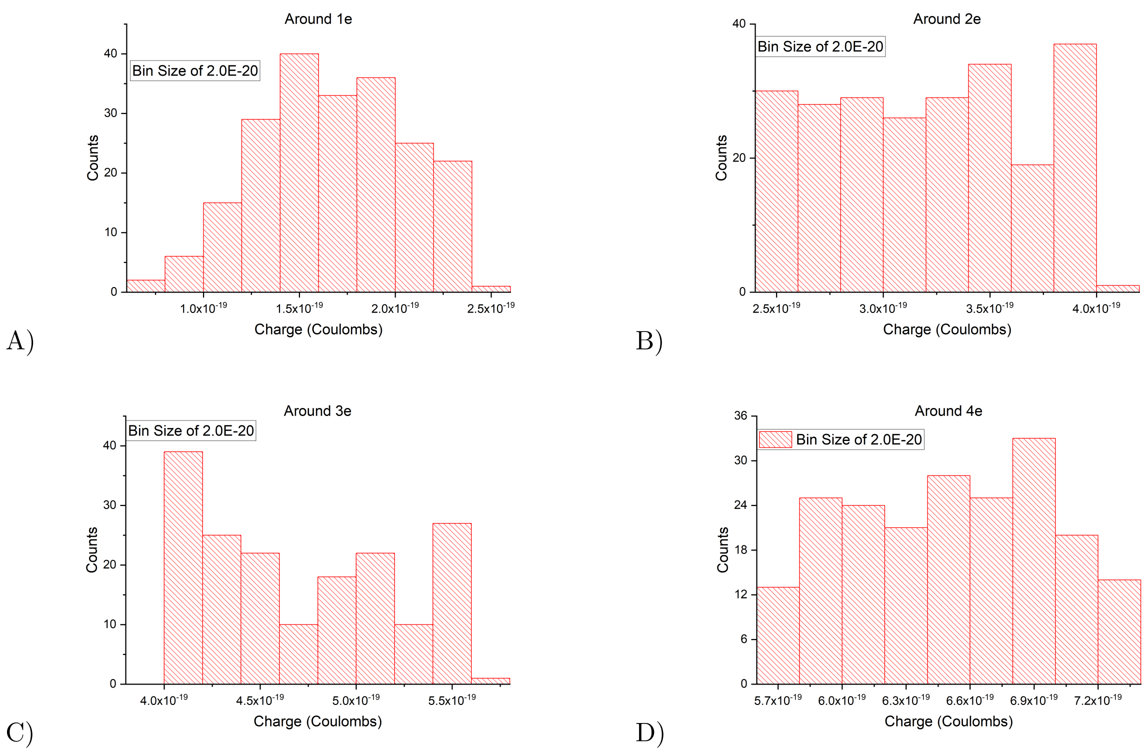


Figure 9: Histograms of values for charges around ne in terms of coulombs. A)$n = 1$, B)$n = 2$, C)$n = 3$, D)$n = 4$

From Figure 9 we can see that plots A) and D) follow more closely the Gaussian distribution than any other histogram from Section 3.2.

n	N data points	Mean (C)	Standard Deviation (C)
1	209	$1.67\times10^{-19} \pm 2.65\times10^{-21}$	3.824×10^{-20}
2	233	$3.21\times10^{-19} \pm 3.07\times10^{-21}$	4.682×10^{-20}
3	174	$4.72\times10^{-19} \pm 3.89\times10^{-21}$	5.135×10^{-20}
4	203	$6.51\times10^{-19} \pm 3.24\times10^{-21}$	4.611×10^{-20}

Table 7: Table for mean and standard deviation of charge values for class data-set

n	1	2	3	4
% error	4.2	0.3	1.7	1.7

Table 8: Percentage Error of charge

As demonstrated in Table 8 the data follows very closely with the theoretical model. Some of these small errors come from our experimental set-up. Without knowing in too great of detail of how the exact experiment was conducted, our best guess for some of the experimental error comes from our apparatus. As mentioned before, the thermal variations from warm bodies could provide some deviation from theory. In addition, the apparatus could not have been completely level, this could mean that our grid would be slightly skewed from the droplet falling, so there would be some component of gravity that we are not accounting for. This would cause us to measure the t_g and t_{rise} inaccurately. These errors could cause the calculated values to be slightly off.

3.3.2 Quantization of charge

We performed the same analysis as from Section 3.2.2 on our larger data-set now.

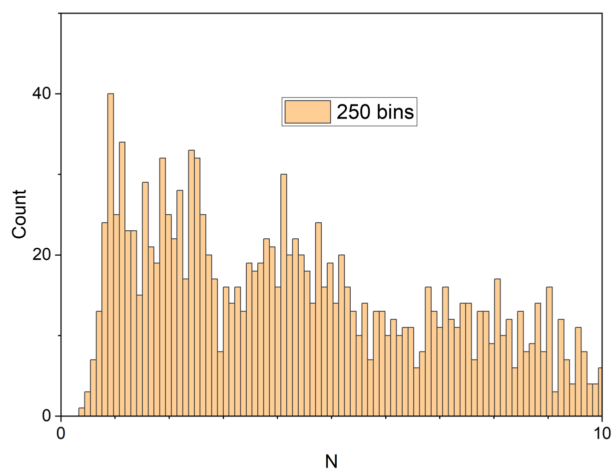


Figure 10: Histogram of all charges less than $10e$ for the class data-set, in terms of elementary charge Q/e

If we look closely we can sort of see the the peaks at each integer value a little more clearly, which is good in proving the quantization of charge. To help analyze the peaks to better see these relations.

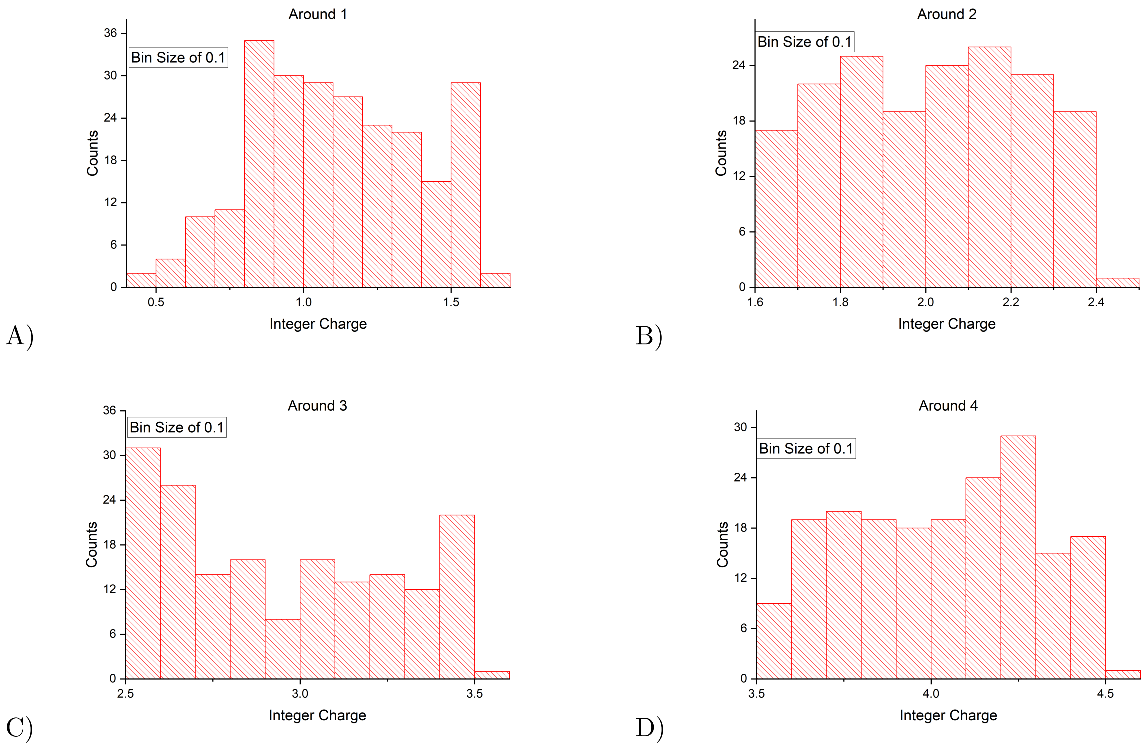


Figure 11: Histograms of integer values of elementary charges around n. A)n = 1, B)n =2, C)n = 3, D)n = 4

n	N data points	Mean	Standard Deviation
1	239	1.11 ± 0.02	0.283
2	176	2.01 ± 0.02	0.221
3	173	2.95 ± 0.02	0.320
4	190	4.03 ± 0.02	0.270

Table 9: Table for mean and standard deviation of elementary particles for class data-set

Interestingly, we notice that the average standard deviation of each of these integer values is slightly higher than those of the smaller data set. This means that there is actually less evidence for the quantization of charge in this larger data-set. Of course, the difference is not really that significant and 68% of the charges fall within approximately 0.3 of an integer. We can still say that charge is quantized, but we would expect the larger data set to have a smaller standard deviation and more concretely show this.

n	1	2	3	4
% error	11.0	0.5	1.7	0.75

Table 10: Percentage Error for integer number of elementary charges for the partial data-set

All of the experimental errors discussed before can contribute to the error that we see on each of these values. Thermal heating of the apparatus by outside sources increases the temperature and deviates our calculation from theory. Similarly, a poorly leveled apparatus leads to some difficult geometry measuring the oil droplet and getting us bad data for the rise and fall times. As well as worrying about inconsistencies is measuring t_g and t_{rise}, the reaction time required to accurately measure the droplet is faster than a human possesses, especially 1000 times. All of these things can easily explain away the $\tilde{1}$% on n = 1,2,3.

For $n = 1$, we see a similar situation to what we saw with the percent error on the charge in the partial data-set. In this case it's not just the number of samples because we have a good number of samples. In this case it is how we made our histograms and goes into the bias we had previously discussed. It appears that we just made a bad biased decision for searching for values around $n = 1$. If we were to take a different sample of less data we would obtain a value with a lower percent error (this can be best seen with the large spike in Figure 11A).

4 Conclusion

In conclusion we can see that the charge of an electron is roughly equal to our predicted value of 1.602×10^{-19} C and we also obtain that charge is quantized. From our small data-set we see larger deviation from our actual value for the mean value. We also have a fairly difficult time discerning that charge is quantized as the standard deviations are rather large. Meanwhile from the larger, class data-set we obtain a closer value to our theoretical value. Somewhat surprisingly, the quantization of charge is much more obvious as the standard deviation is a little tighter around the integer values for the partial data-set.

Overall, a lot of the experimental error is difficult for us to accurately explain as we did not perform the experiment. With this being said we can certainly use our judgement to figure out reasonable experimental errors. Certainly one error that could have been made was in the measurement of t_g or t_{rise}. Pressing the button in order to time the rise and fall requires a certain reaction time that could vary from person to person so there could be discrepancies from the actual time it took to the measured time. Additionally, there is a chance that the apparatus was not perfectly level meaning there was some x-component of gravity that we did not account for and thus our calculations are a little off. Another point is that since charge is quantized, why are we seeing anything but integer numbers of charge? This can be explained by possible changes in the environment as we take data. Perhaps the pressure increased as a storm came into the area or the temperature increased as we turned on the light to view the oil drop. All of these experimental errors can compound and give us the error bars that we see in our analysis. In terms of analytical errors, one thing that is mentioned earlier is that our Python algorithm to give us the charge and elementary number was *close* to what the sample Origin template outputted. The difference came in around the 3^{rd} or 4^{th} decimal place, so it was not a huge error, but still existed. Finally, as mentioned earlier our method for looking at values about the theoretical is biased as we just arbitrarily select some max and min to look over the interval on based on the correct answer that we already know. In the future a method to analyze the peaks and determine the average and standard deviation would help us get more believable statistics.