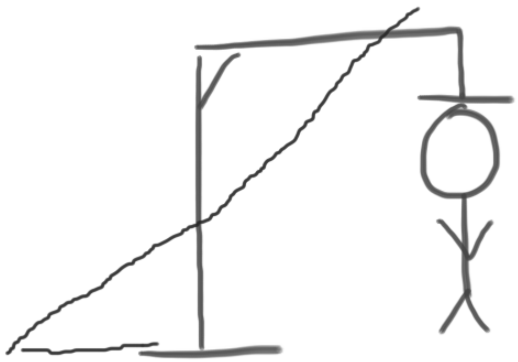
CONGRADS
AY!
YAY!
Yee haw!
rocket
Joana



s c h o o l

c a i u k r

guess

$$\left.\begin{array}{l}\boxed{f(0)}\\[1em] \boxed{f(1)}\end{array}\right\} \begin{array}{l} \xrightarrow{\text{match}} \text{constant} \\[1em] \xrightarrow{\text{differ}} \text{balanced} \end{array}$$

a	b	c	$(a \oplus b) \oplus c$	$a \oplus (b \oplus c)$
0	0	0	$0 \oplus 0 = 0$	$0 \oplus 0 = 0$
0	0	1	$0 \oplus 1 = 1$	$0 \oplus 1 = 1$
0	1	0	$1 \oplus 0 = 1$	$0 \oplus 1 = 1$
0	1	1	$1 \oplus 1 = 0$	$0 \oplus 0 = 0$
1	0	0	$1 \oplus 0 = 1$	$1 \oplus 0 = 1$
1	0	1	$1 \oplus 1 = 0$	$1 \oplus 1 = 0$
1	1	0	$0 \oplus 0 = 0$	$1 \oplus 1 = 0$
1	1	1	$0 \oplus 1 = 1$	$1 \oplus 0 = 1$

$$|\psi\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}}|0\rangle$$

$$U_f|\psi\rangle = U_f\left(\frac{|00\rangle + |10\rangle}{\sqrt{2}}\right)$$

$$= \frac{U_f|0\rangle|0\rangle + U_f|1\rangle|0\rangle}{\sqrt{2}}$$

$$= \frac{|0\rangle|\oplus f(0)\rangle + |1\rangle|\oplus f(1)\rangle}{\sqrt{2}}$$

$$|\psi\rangle = |x\rangle|-\rangle$$

$$= |x\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$= \frac{|x\rangle|0\rangle - |x\rangle|1\rangle}{\sqrt{2}}$$

$$U_f|\psi\rangle = \frac{U_f|x\rangle|0\rangle - U_f|x\rangle|1\rangle}{\sqrt{2}}$$

$$= \frac{|x\rangle|f(x)\rangle - |x\rangle|1 \oplus f(x)\rangle}{\sqrt{2}}$$

$$= \frac{|x\rangle|f(x)\rangle - |x\rangle|1 \oplus f(x)\rangle}{\sqrt{2}} = \frac{|x\rangle}{\sqrt{2}}\left(|f(x)\rangle - |1 \oplus f(x)\rangle\right)$$

$$\left.\begin{array}{ll} f(x) = 0, & |x\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \\ f(x) = 1, & |x\rangle\left(\frac{|1\rangle - |0\rangle}{\sqrt{2}}\right) \end{array}\right\} = (-1)^{f(x)}|x\rangle|-\rangle$$

$$\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$\boxed{|\psi\rangle = U_f|x\rangle|-\rangle = (-1)^{f(x)}|x\rangle|-\rangle}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left((-1)^{f(0)} |0\rangle + (-1)^{f(1)} |1\rangle \right)$$

$$= \frac{\boxed{(-1)^{f(0)}}}{\sqrt{2}}\left(|0\rangle + (-1)^{f(0)} (-1)^{f(1)} |1\rangle \right)$$

$$(-1)^{f(0)} (-1)^{f(0)} = 1$$

$$(-1)^{1} (-1)^{1} = 1$$

$$(-1)^{0} (-1)^{0} = 1$$

$$(-1)^{f(0)+f(1)} = (-1)^{f(0) \oplus f(1)}$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^{\boxed{f(0) \oplus f(1)}} |1\rangle \right)$$

1 if $f(x)$ balanced

0 if $f(x)$ constant

i) $|0\rangle|0\rangle$

$I X\,|0\rangle|0\rangle = |0\rangle|1\rangle$

$HH\,|0\rangle|1\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$

$= |+\rangle\,|-\rangle$

$U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$

$|-\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}} = |y\rangle$

$|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}} = |x\rangle$

$\boxed{|\psi\rangle = U_f|x\rangle|-\rangle = (-1)^{f(x)}|x\rangle|-\rangle}$

ii) $U_f\,\overset{x}{|+\rangle}\,\overset{y}{|-\rangle} = U_f\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|-\rangle$

$= \frac{U_f|0\rangle|-\rangle + U_f|1\rangle|-\rangle}{\sqrt{2}}$

$= \frac{(-1)^{f(0)}|0\rangle|-\rangle + (-1)^{f(1)}|1\rangle|-\rangle}{\sqrt{2}}$

$$= \frac{(-1)^{f(0)}|0\rangle|-\rangle + (-1)^{f(1)}|1\rangle|-\rangle}{\sqrt{2}}$$

$$= \boxed{\frac{\left((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right)}{\sqrt{2}}}|-\rangle$$

$$L = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right)$$

$$= \frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{f(0)}(-1)^{f(1)}|1\rangle\right)$$

$$(-1)^{f(0)+f(1)} = ($$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^{\boxed{f(0)\oplus f(1)}}|1\rangle\right)$$

if $f(0)\oplus f(1) = 0$, $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$

" $f(0)\oplus f(1) = 1$, $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$

$H|+\rangle = |0\rangle$

$H|-\rangle = |1\rangle$

iii) apply H, measure

$|0\rangle \rightarrow$ constant

$|1\rangle \rightarrow$ balanced