Experiment 8: AC Measurement of Magnetic Susceptibility

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Abstract

In this lab, the properties of ferromagnetic materials are explored. Ferromagnets are materials with a very high magnetic susceptibility and are commercial magnets used in everyday life. In the first experiment, we examine the permeability as a function of the applied magnetizing field, magnetic field as a function of the magnetizing field, and power losses of four different ferromagnetic samples. For each sample, the magnetic field versus magnetizing field represents the hysteresis or the effect of history on the material. From this, we found the coercivity and 'hardness' of each ferromagnet. In the second experiment, we sought to determine the temperature dependence of the susceptibility of the sample. We found that at a certain, critical temperature the sample undergoes a phase transition and the susceptibility drops linearly. In the future of this experiment, an improvement that could be made for the first experiment was to make sure the sample is not being tampered with during the data taking process. For the second part, running the correct DC sweep through the circuit as well as creating hysteresis curves at lower temperatures would help us better understand the temperature dependence.

1 Purpose

The purpose of this lab is to better understand ferromagnetic materials. Ferromagnets are typical magnets found in the household, they have some built-in magnetization and are especially useful for things like refridgerator magnets. In the first experiment, we run a program to strip the material of its polarization and then run a DC sweep to apply some magnetizing field to the sample. This magnetizing field induces a magnetic field, a combination of the built-in polarization and magnetizing field, on the sample. A plot of this magnetic field versus the magnetizing field is called a hysteresis loop. Studying this loop we can learn about the "hardness" of the ferromagnet, which translates to a measure of how easily a ferromagnetic material can be magnetized. Additionally, since the permeability is complex, the system has some energy losses. The energy losses can be quantified by the volume of the sample times the area subtended by the hysteresis curve.

For the second part of this lab, the objective is to explore the temperature dependence of the susceptibility. We submerged the sample in corn oil and attached a temperature ramp, to increase the temperature of the sample. The first thing we did, as the sample heated up, was to create hysteresis curves at a couple of different temperatures. As the sample was heated to near the critical temperature, the diminishing magnetic field becomes more apparent. This is due to a phase transition causing the material to act more like a paramagnet than a ferromagnet. After reaching the final temperature, we turn off the temperature ramp and allowed the system to cool back down. During this time we took a temperature scan and examined the susceptibility response over all the temperatures as it cools. We used the Weiss-Curie law to determine the critical temperature from an Origin fit.

2 Theory

The magnetic field can be thought of in two different quantities, B and H. The magnetic induction or magnetic flux density, B, represents the forces on free moving charges via the Lorentz force law. The magnetic field intensity or magnetizing field, H, represents the created only by free moving charges. In general, we can write an expression for B in terms of the H-field and another factor, known as the magnetic polarization, M.

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) \tag{2.1}$$

Some scientists refer to this B-field as the magnetic field, while others call the H-field the magnetic field. In general, we will refer to the B-field as the magnetic field. The magnetic polarization and magnetizing field can be utilized to define the magnetic susceptibility, χ. For most materials, the response is approximately linear

$$\vec{M} = \chi \vec{H} \tag{2.2}$$

We can define a relative permeability of a medium using this magnetic susceptibility, which in turn can be defined with a derivative of the magnetic field.

$$\mu_r = 1 + \chi = \frac{1}{\mu_0} \frac{\partial B}{\partial H} \tag{2.3}$$

From this definition of relative permeability, we can define general permeability. This is useful because we can express in simply in terms of a derivative of the magnetic field.

$$\mu = \mu_r \mu_0 = \frac{\partial B}{\partial H} \tag{2.4}$$

Thus, we can rewrite the equation for the magnetic field in terms of only the susceptibility or permeability.

$$\vec{B} = (1 + \chi)\vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H} \tag{2.5}$$

We can classify all materials into three different types based on their magnetic susceptibility.

Diamagnetic	$\chi < 0$	$\mu_r < 1$	Weakly Repelled
Paramagnetic	$\chi > 0$	$\mu_r > 1$	Weakly Attracted
Ferromagnetic	$\chi \gg 0$	$\mu_r \gg 1$	Strongly Attracted

Table 1: Three different types of magnetic materials

In general, we should note that the magnetic susceptibility is a function of the magnetizing field, is a 2^{nd}-rank tensor, is complex and may depend on its history (*hysteresis*)

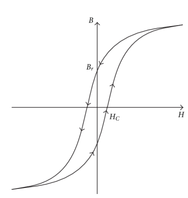


Figure 1: Sample hysteresis curve

Remanence is the remaining magnetic field left on the material when there no magnetizing field is being applied. This can be thought of as the magnetization term in Eqn. 2.1, this magnetization can be applied or taken away by running a current through our system. We utilize this several times in our experiments. From Figure 1, the remanence is the height along the B axis where H is 0.

Coercivity describes the ability of a ferromagnetic material to resist an applied magnetizing field without becoming demagnetized. In Figure 1, the coercivity is the width of the loop. A ‘hard’ ferromagnet has a wide hysteresis loop while a ‘soft’ ferromagnet has a narrower loop.

In terms of the actual circuit, seen in Figure 2, there are several useful being measured that we can describe. What makes these quantities useful is their ability to help us transform our measurements of susceptibility in our data acquisition software into a hysteresis loop.

The first useful quantity that we use comes from the lock-in amplifier. The lock-in amplifier measures the EMF imposed on the pickup coil.

$$V_{\text{lock-in}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\vec{B}\cdot\vec{S}\right) \tag{2.6}$$

We also know that the AC generated by the Wavetek across the inductor L_2 can be described as follows.

$$I_p = \frac{V_0 \sin(\omega t)}{R_2} \tag{2.7}$$

This primary coil is a toroid with N_p turns carrying a current I_p that creates a magnetizing field H and thus adds to the following flux $d\Phi$ to the toroid.

$$d\Phi = \mu \int \vec{H}\cdot d\vec{a} \tag{2.8}$$

Thus, the only flux detected by the pickup coil is the following.

$$\Phi = N_{\text{pickup}} d\Phi = \frac{\mu N_{\text{pickup}} I_p N_p t}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \tag{2.9}$$

The inductance of the toroid can be described by the ratio of the flux and current. This ratio then simplifies to a product of the permeability and some constant of the geometric inductance.

$$L = \frac{\Phi}{I} = \mu_r L_0 = (\mu' - i\mu'')L_0 \tag{2.10}$$

Where the geometric inductance constant is defined as follows.

$$L_0 = \frac{\mu_0 N_{pickup} N_p t}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \tag{2.11}$$

Thus, we can rewrite Eqn. 2.6 in terms of the permeability, the geometric inductance constant, and a derivative of the current.

$$V_{\text{lock-in}} = \mu_r L_0 \frac{dI_p}{dt} \tag{2.12}$$

Where the derivative of the current can be expressed in the following manner.

$$\frac{dI_p}{dt} = \omega \frac{V_0}{R_2} \cos(\omega t) \tag{2.13}$$

Putting this all together, we obtain the expression for the lock-in voltage as the following.

$$V_{\text{lockin}} = \mu_r L_0 \frac{\omega V_0}{R_2} \cos(\omega t) \tag{2.14}$$

In general, the permeability will be a complex function (the imaginary term corresponds to energy loss in the work required to create the hysteresis loop). Combined with our knowledge that a lockin amplifier outputs real (in-phase) and imaginary (quadrature) voltages, we can express the two components of V_{lockin} with the two components of μ_r. Note: one prime represents real, two represents imaginary.

$$V'_{\text{lockin}} = \mu'_r L_0 \frac{\omega V_0}{R_2} \cos(\omega t) \tag{2.15}$$

$$V''_{\text{lockin}} = -\mu''_r L_0 \frac{\omega V_0}{R_2} \cos(\omega t) \tag{2.16}$$

The next step to going from the permeability to the hysteresis loops is to integrate. The permeability, $\mu_r(H)$, is a local derivative, so integrating it will give us $B(H)$.

$$\mu(H_0) = \mu_0 \mu_r(H_0) = \left.\frac{dB}{dH}\right|_{H_0} \tag{2.17}$$

$$B(H) = \mu_0 \int \mu_r(H) dH \tag{2.18}$$

Now, to complete our hysteresis loop, we need to be able to convert from the applied current to the magnetizing field, H. In general, we can write H as the sum of two terms, H_0 and H_1.

$$H = H_0 + H_1 \cos(\omega t) \tag{2.19}$$

Each of these terms comes from the current being applied, H_0 from the DC, and H_1 from the AC. In this experiment, we will say that $H_0 \gg H_1$ and therefore will neglect the H_1 term. We can write the DC term of the H field in the following way.

$$H_0 = \frac{N_{DC} I_{DC}}{2\pi r} \tag{2.20}$$

To find the power losses of the process, we need to look at the work done throughout the entire process. We can obtain the work done on the system by integrating over the hysteresis loop.

$$W = V \oint \vec{H} d\vec{B} = V A_{\text{loop}} \tag{2.21}$$

Where V is the volume of the sample. Finding the volume of a toroid is found by calculating the area of the square section and then multiplying it by the radius of revolution, with some coefficients.

$$V_{\text{toroid}} = 2\pi r A \tag{2.22}$$

To convert our work into power, we simply need to multiply the work, found in Eqn. 2.21, by the frequency that we are driving the system.

$$P_{\text{loss}} = W f \tag{2.23}$$

The second part of this lab is to examine the temperature dependence of permeability. An important feature of ferromagnets is their phase change from paramagnets at a certain temperature. In the paramagnetic phase, the sample is free to point along the direction of the applied field, meanwhile, in the ferromagnetic phase, the sample begins to point in the opposite direction. The temperature at which approximately half of the sample has undergone this switch is called the critical temperature. This effect can be achieved in either direction, heating up or cooling down.

We can write the temperature dependence of the permeability with the following relation.

$$\mu(T) = \mu_0 \mu_r(T) = \frac{dB}{dH}(T) \tag{2.24}$$

The susceptibility will have the following relation as a function of temperature (Note: the critical temperature, T_C, is the temperature at which the permeability severely drops). This is known as the Curie-Weiss Law, after Pierre Curie and Pierre Weiss.

$$\chi(T) = \frac{C}{T - T_C} \tag{2.25}$$

For practical reasons when looking at the data, examining the inverse of χ is useful. In this case, the inverse can be written as the following.

$$\frac{1}{\chi} = \frac{T - T_C}{C} \tag{2.26}$$

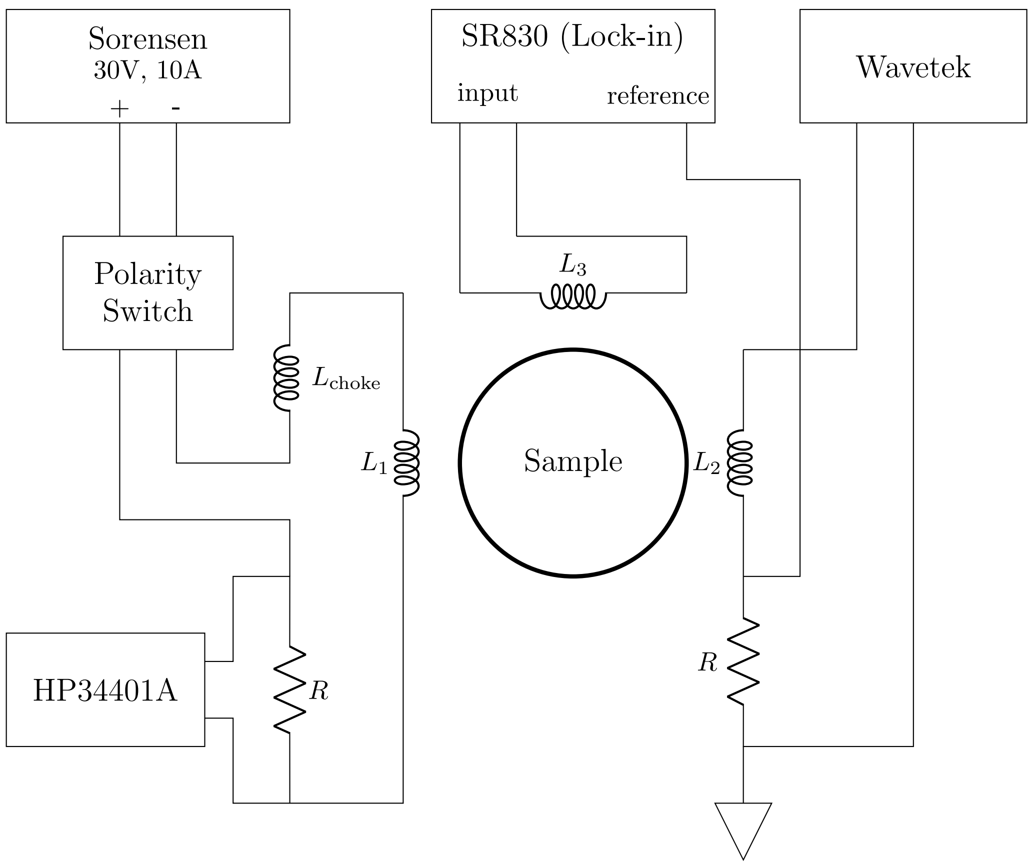


Figure 2: Circuit set-up for the first parts of the experiment

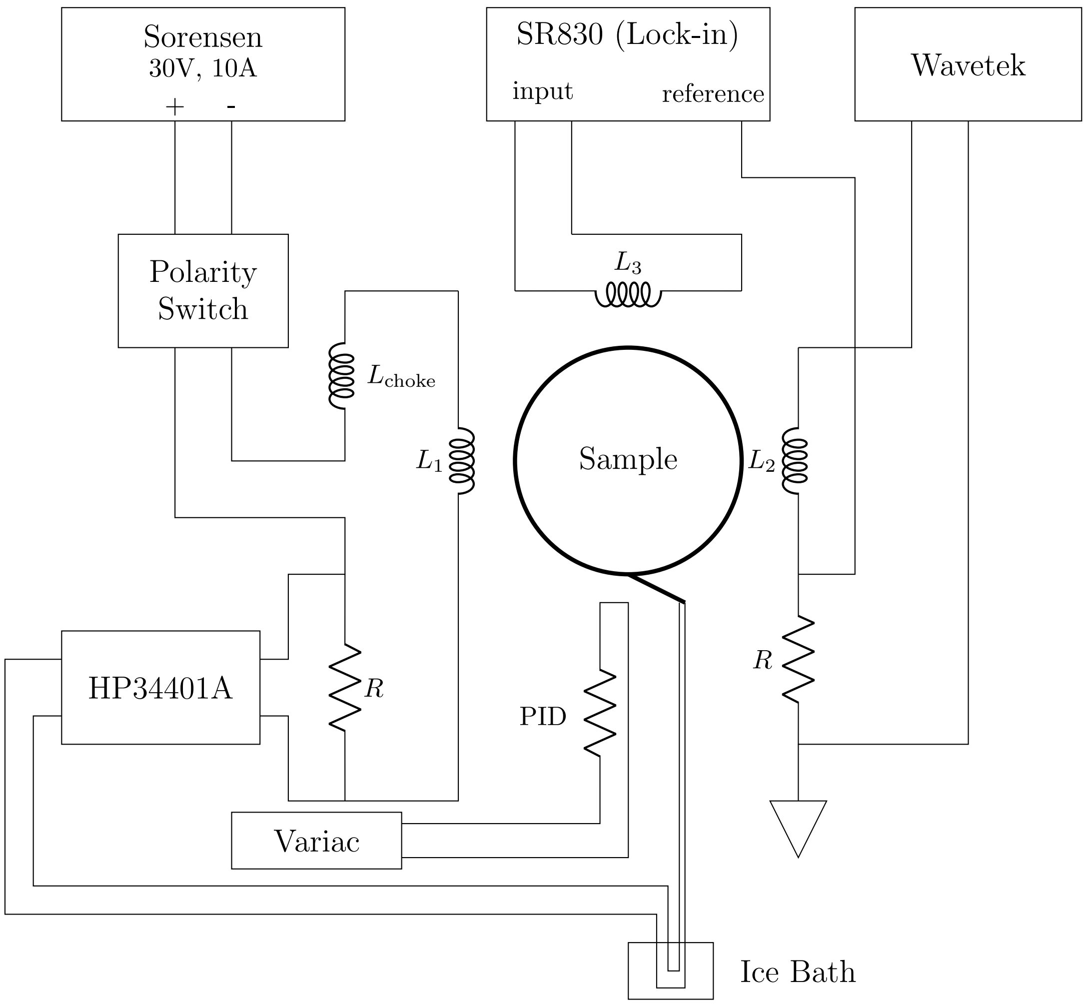


Figure 3: Circuit set-up with the ice bath and Thermocouple Reference for the second portion of the lab

3 Experiments

3.1 Equipment

There was a lot of equipment used in this lab. The modules we used (seen as rectangles in Figures 2 and 3) are a Sorensen, a Digital Multimeter (HP34401A), an SR830 lock-in amplifier, and a Wavetek function generator. The Sorensen and Wavetek are the DC and AC power supplies, respectively. Also, we use a polarity switch to flip the polarity of the DC exiting the Sorensen. We use the DMM to read the voltage across the sample. The lock-in amplifier helps differentiate the real and complex parts of the permeability.

For the first experiments, we used four different samples: Ferroxcube 4C65 #27, Ferroxcube 4S2, Magnetics ZP44715TC and Ferroxcube 4A20. For the second part of the lab, we used Ferroxcube 3E12. All of the parameters of these coils are described in the appropriate section.

For the second part of the lab, we add a Variac, and Omega PID temperature controller, an ice bath, and a thermocouple. In addition, the whole sample is submerged in a fluid. The temperature controller (PID stands for Proportional, Integral, and Differential which are the three standard linear operations it performs) controls the fluid by heating it. The Variac displays the temperature within the fluid and outputs it on our computer through lab software which allows us to control the PID. The thermocouple (shown as the stem off the bottom of the Sample in Figure 3) is a T-type Thermocouple.

There are a couple of circuit elements without our set-up. We have a couple of different inductors (L_1, L_2, and L_3) that are interwoven on the toroid. We have a couple of resistors that help deliver the current to the inductors in a predictable manner. Finally, there is a choke inductor (L_{choke}) which serves to minimize any AC that may be induced via mutual inductance from L_2, while leaving the DC unaffected.

For the first experiment, we ran the system at an RMS voltage of 3.535 V with a frequency of 1 kHz. For the first part of the second experiment, the same parameters were using. Then for the back half of the second, the RMS voltage was set to 4 V with a frequency of 10 kHz.

As always, our data analysis and plot creation were done in Origin Pro. The data taking method came from a program installed on the computer that had a couple of different modes, two of which were 'B versus H' and 'Temperature Scan'. These two different methods ran sweeps of different currents and measured voltage from the lock-in.

3.2 Mapping the Hysteresis Loop

In this part of the lab, we are taking the toroid samples, demagnetizing them, and then measuring the magnetic field (B) response to an applied magnetizing field (H). From this information, we created the hysteresis loops. We used four different samples for this part of the lab: a Ferroxcube 4C65 #27, a Ferroxcube 4S2 a Magnetics ZP44715TC, and a Ferroxcube 4A20. The specific parameters for each of these samples is shown in Table 2. A mock-up of the set-up is in Figure 2.

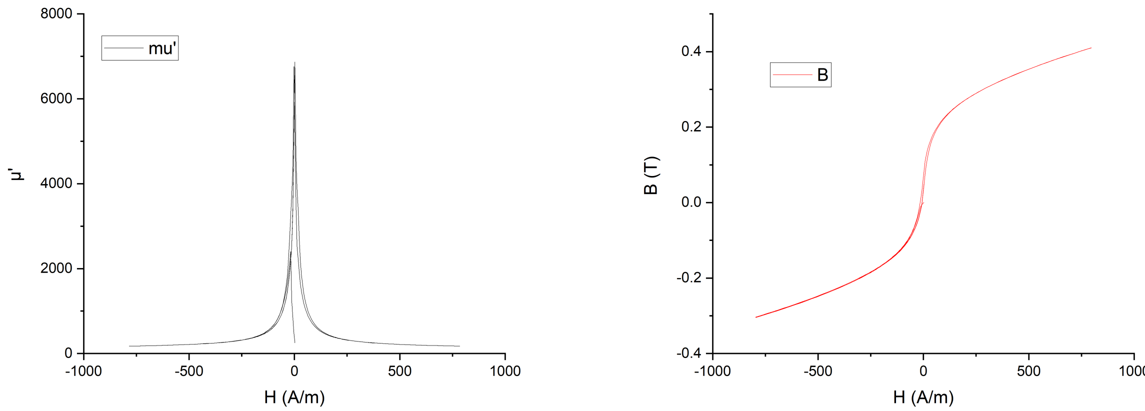
Name	DC Primary Turns (N_{DC})	AC Primary Turns (N_{AC})	Pickup Turns (N_{pickup})
Ferroxcube 4C65 #27	30	10	10
Ferroxcube 4S2	60	40	40
Magnetics ZP44715TC	15	15	10
Ferroxcube 4A20	15	15	10
	Outer Diameter (mm)	Inner Diameter (mm)	Height (mm)
Ferroxcube 4C65 #27	22.42	13.4	6.72
Ferroxcube 4S2	22.1	13.2	8.5
Magnetics ZP44715TC	46.9	27	15
Ferroxcube 4A20	22.4	13.2	8.7

Table 2: Parameters for each of the four coils. All lengths have an error bar of ± 0.1 mm

Before each measurement was made we demagnetized the toroid by running it through the demagnetization protocol on the computer program. This applied a current through the sample in a decreasing periodic function to eliminate the built-in polarization, M. This is known as removing the magnet's remanence. Zero-ing the built-in polarization allows us to start the hysteresis loop at roughly the origin.

After the sample has been demagnetized, we ran the B versus H routine on the program. This gives us data with the current on the horizontal axis and the magnitude of the lock-in voltage on the vertical axis. To obtain the desired μ_{real} versus H curve to integrate into our hysteresis loop, we need to employ Eqns. 2.15 and 2.20. All of the constants are provided in Table 2, with the approximation of r in Eqn. 2.20 being the average of the inner and outer radii.

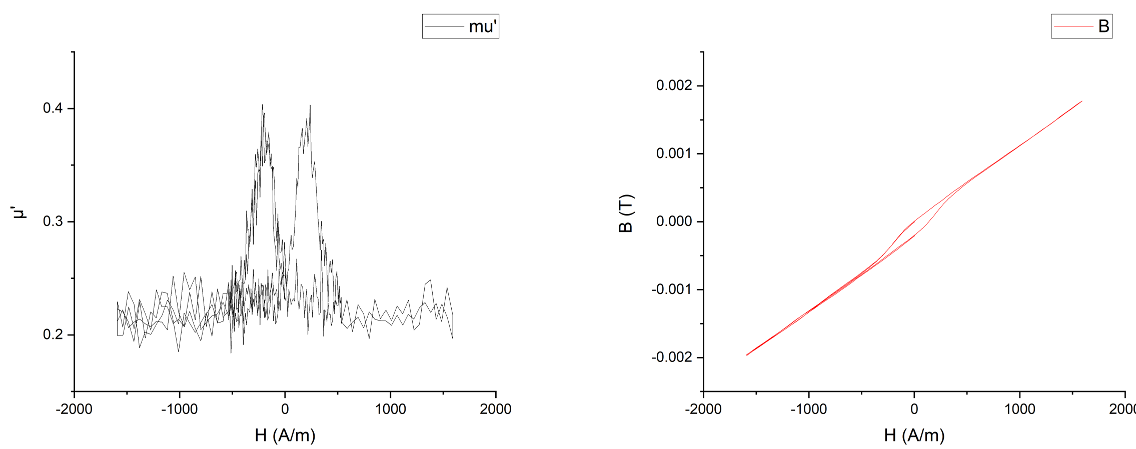
Converting the axes to the correct units and integrating, we obtain the hysteresis loop for the given sample, except for one small detail. Recalling Eqn. 2.18, we need to multiply the integrated vertical axis by μ_0 to get that axis into the magnetic field, B.



A) B)

Figure 4: μ' versus H, A) and hysteresis loop, B) for the first sample, Ferroxcube 4C65 #27

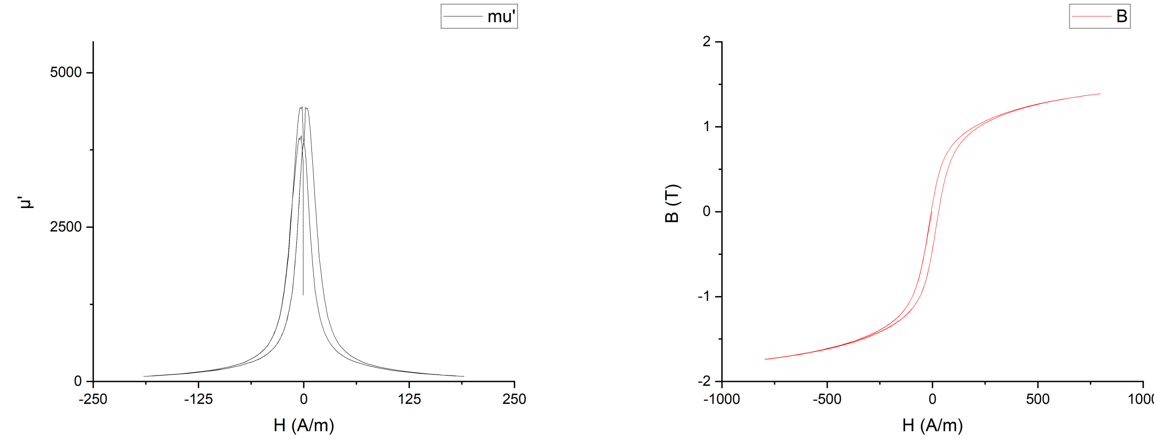
From Figure 4B) we can see that this particular sample has very low coercivity and thus our magnetic sample, Ferroxcube 4C65 #27, appears to be a rather "soft" ferromagnet.



A) B)

Figure 5: μ' versus H, A) and hysteresis loop, B) for the second sample, Ferroxcube 4S2

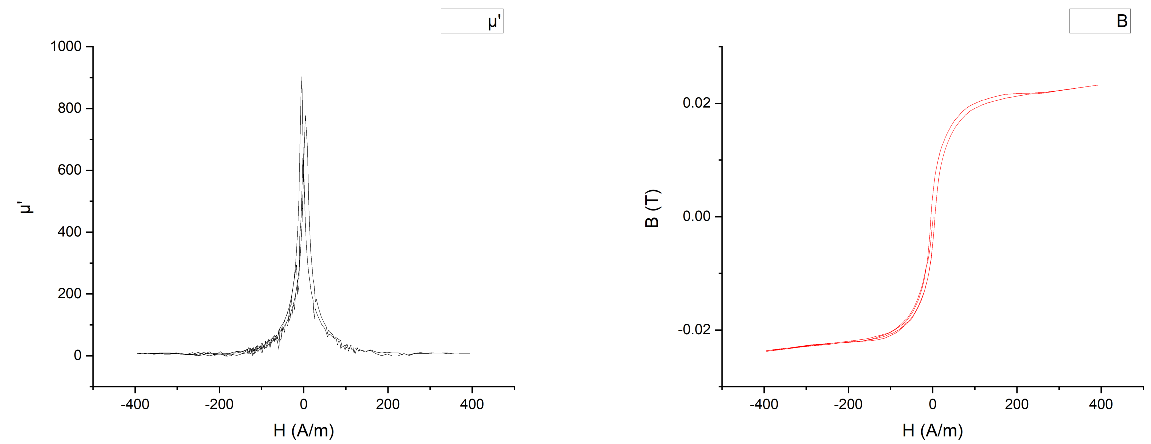
As can clearly be seen by Figure 5A) this data is very noisy. This noise is most likely caused by interference with the setup as the data was being taken. During this first section of the lab, the TA, Jordan, most likely was moving around the sample as we were collecting data, inadvertently, causing a lot of noise. While the data does have a lot of noise, we still obtain a hysteresis curve from it. Based on the signs that Figure 5B) point to, we could say that it appears this sample has a higher coercivity and thus is a relatively "hard" ferromagnet, although we can say none of this with any real certainty.



A) B)

Figure 6: μ' versus H, A), and hysteresis loop, B), for the third sample, Magnetics ZP44715TC

From Figure 6B) we can see that this particular sample has a higher coercivity than the first sample (note the difference in horizontal scale). Thus, this sample, Magnetics ZP44715TC, appears to be a "hard" ferromagnet.



A) B)

Figure 7: μ' versus H, A), and hysteresis loop, B), for the fourth sample, Ferroxcube 4A20

From Figure 7B) we can see that this particular sample has a low coercivity, similar to the first sample. Thus, we can say that this fourth sample, Ferroxcube 4A20, is a "soft" ferromagnet.

In addition to creating the hysteresis loops, this part of the lab was about finding the power losses. The energy loss in the hysteresis curve is described in Eqn. 2.21. We have to integrate the curve and then multiply it by the volume of the sample.

	Ferroxcube 4C65 #27	Ferroxcube 4S2	Magnetics ZP44715TC	Ferroxcube 4A20
Volume (m^3)	3.43×10^{-6}	4.23×10^{-6}	3.73×10^{-5}	4.62×10^{-6}
Area ($kgs^{-2}m^{-1}$)	-2.686	-0.186	-131.80	-0.68
Work (J)	-9.21×10^{-6}	-7.867×10^{-7}	-4.92×10^{-3}	-3.14×10^{-6}

Table 3: Values associated with each sample's volume, area of hysteresis loop and energy loss

As mentioned in the equipment section, we are running the Wavetek at 1 kHz for each of these samples. Therefore, by Eqn. 2.23, to find the power lost we just need to multiply the energy loss by the frequency.

	Ferroxcube 4C65 #27	Ferroxcube 4S2	Magnetics ZP44715TC	Ferroxcube 4A20
Power Loss (W)	-9.21×10^{-3}	-7.867×10^{-4}	-4.92	-3.14×10^{-3}

Table 4: Power loss for each of the samples

From Table 4, it seems that the "soft" ferromagnets have larger energy losses than the "hard" ones.

In the future of this part of the experiment, making sure that the data does not have so much noise during the lab would help us out significantly. Additionally, demagnetizing the sample for a long time initially would help us have the hysteresis curves more centered around the origin.

3.3 Permeability as a function of temperature

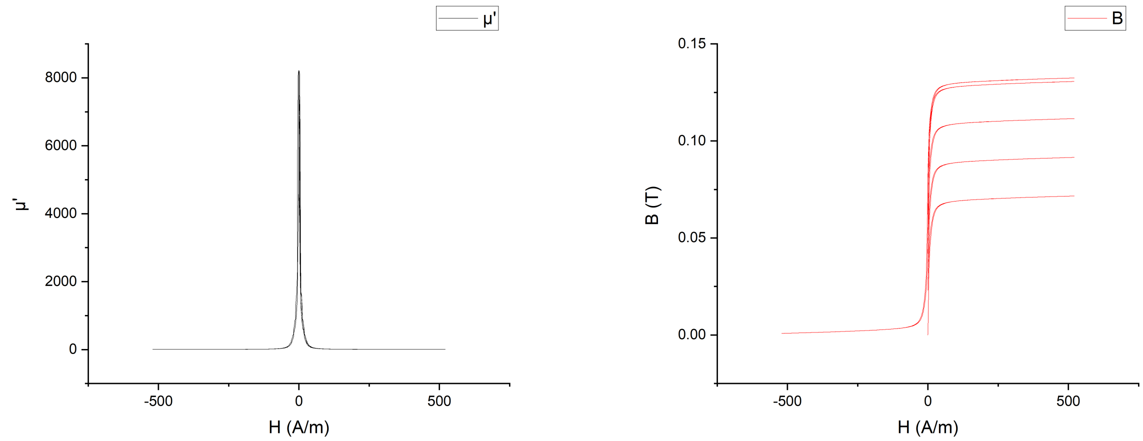
The second part of the lab centers around learning more about the temperature dependence of susceptibility in ferromagnetic materials. As described in the Theory section, as the temperature is increased (or decreased from a high temperature) there is a phase transition where the magnet begins to change from a ferromagnet to a paramagnet (or visa versa). The objective is to examine the transition in both increasing and decreasing temperatures, as well as find the critical temperature of the sample.

For this section of the lab, the toroid is submerged in a can of corn oil, and a few elements are added to the set-up. The set-up for this section of the lab can be seen in Figure 3. The temperature ramp was used to increase the temperature of the system and measurements were taken at various increments (60, 94.5, 97.4, 104, and 135 °C, all with some error bar of ± 1 °C due to thermal inertia) for the heating data. After the sample had undergone the phase transition, the temperature ramp was turned off and measurements were taken using the 'Temperature Scan' module on the computer program for the cooling data.

DC Primary Tuns	AC Primary Turns	Pickup Turns
20	15	20
Outer Diamater (mm)	Inner Diameter (mm)	Height (mm)
22.3	13.6	8.0

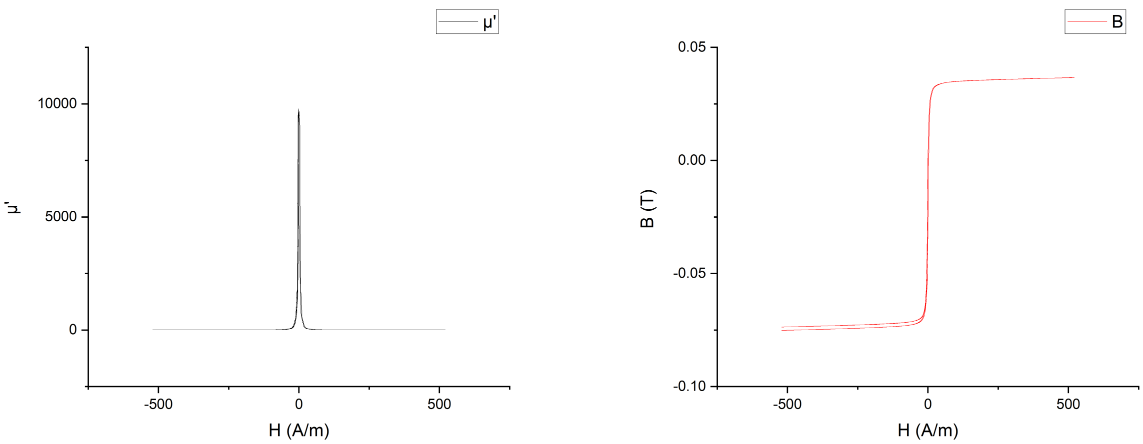
Table 5: Parameters for Ferroxcube 3E12 used in the second portion of the lab. All lengths have an error bar of ± 0.1 mm

The first objective of the second part of this lab is to create hysteresis curves for the sample at different temperatures. Similarly to the last section, we run the computer program in B versus H mode, which output the lock-in voltage components and DC according to some set sweep. To convert the vertical axis to permeability we used Eqn. 2.14, for the horizontal axis we used Eqn. 2.20. Using Eqn. 2.18 we obtain a hysteresis loop. Below are the results for the μ_r' versus H and hysteresis loops for the sample at each temperature.



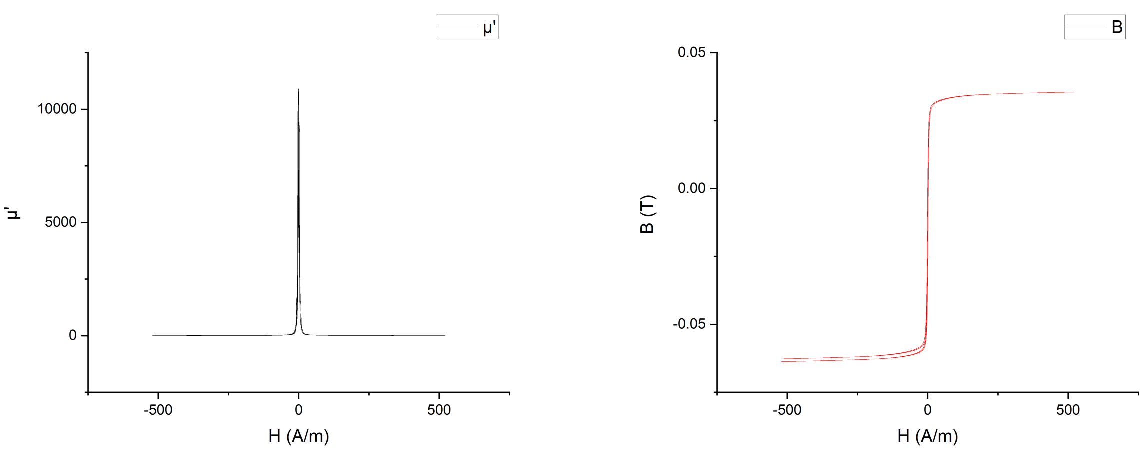
A) B)

Figure 8: μ' versus H, A), and hysteresis loop, B), for our sample at 60 °C



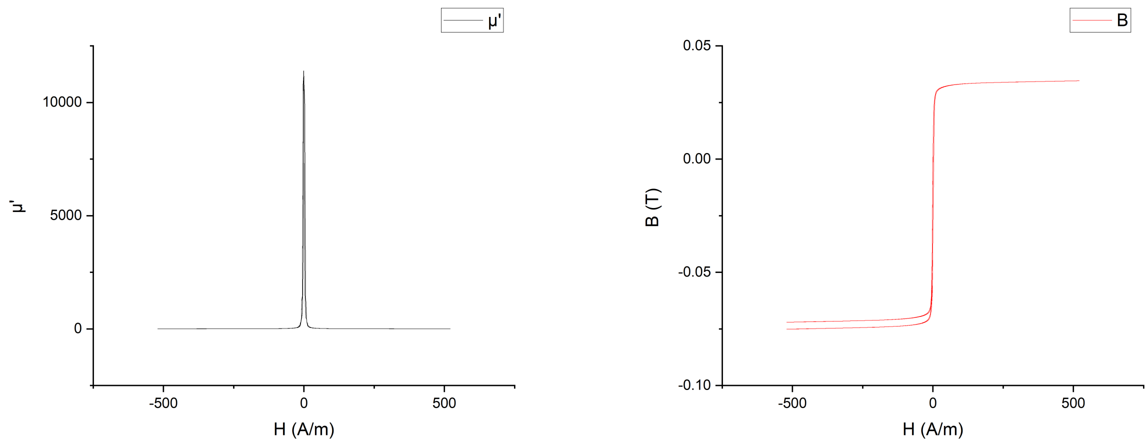
A) B)

Figure 9: μ' versus H, A), and hysteresis loop, B), for our sample at 94.5 °C



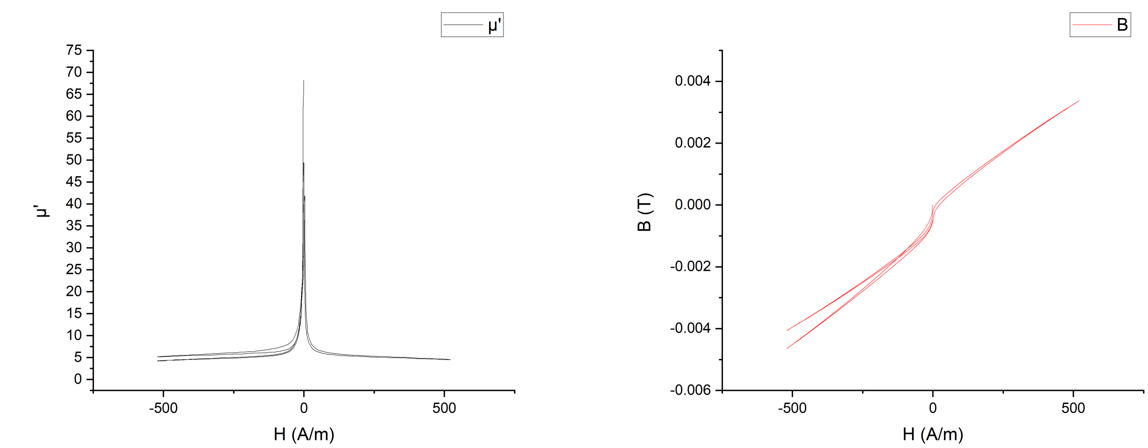
A) B)

Figure 10: μ' versus H, A), and hysteresis loop, B), for our sample at 97.4 °C



A) B)

Figure 11: μ' versus H, A), and hysteresis loop, B), for our sample at 104 °C



A) B)

Figure 12: μ' versus H, A), and hysteresis loop, B), for our sample at 135 °C

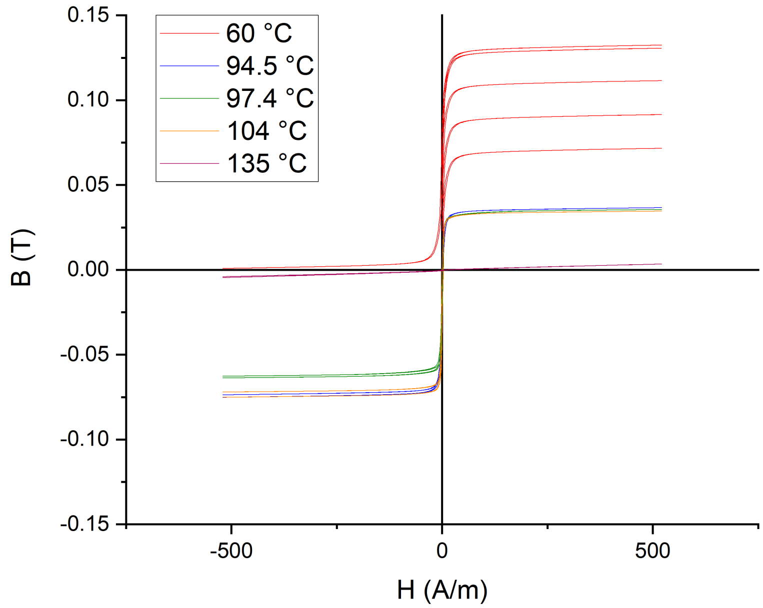
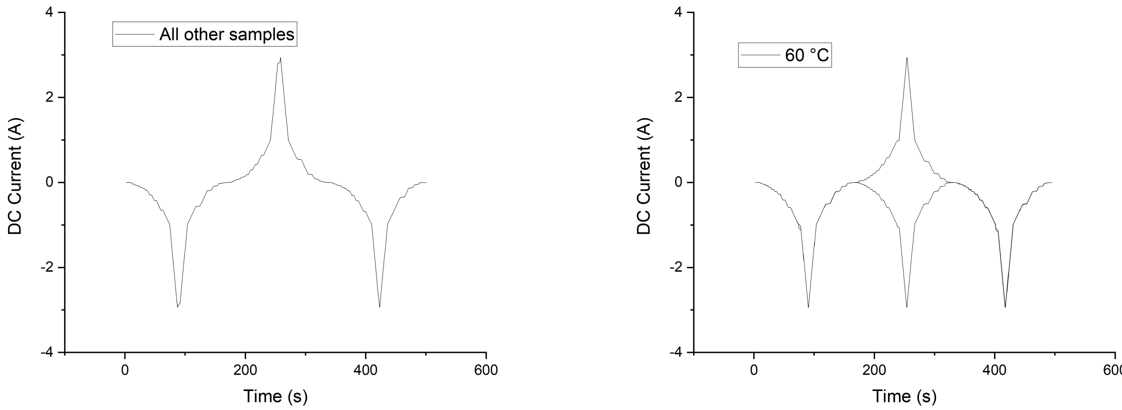


Figure 13: All of the hysteresis loops overplotted on the same figure

As can clearly be seen from the general trend of Figure 13, the amount of the produced B-field by the induced H-field dramatically decreases with temperature, as expected. This is the effect of the ferromagnet undergoing the phase transition into a paramagnet.

There are a great number of things that went awry in this part of the second experiment. From being transferred to four different lab tables to not collecting temperature samples at room temperature. This section of the lab is the reason the report has the subtitle "An Exercise in Patience". As mentioned, the 60 °C hysteresis loop appears very much different than the rest of the loops, this is because we inadvertently ran the wrong current program for this measurement. Below are the currents as a function of time for the 60 °C loop and the one used for the rest of the loops.

Another thing in the future that we would like to change about this part of this experiment is to take pick lower temperatures to sample at. While our lowest temperature that we measured at was 60 °C, the room the experiment was done in was around 25 °C. Having more temperatures measured would help us understand the trend a little better.



A) B)

Figure 14: A) The DC sweep used in the loops at all temperatures except 60 °C. B) The DC current sweep used in the 60 °C sample.

While the first part of this experiment was to heat the sample, the second is to examine the temperature dependence of the sample while cooling. As described earlier, we changed the program into “Temperature Scan” mode and let the sample cool for a long while. During this time the voltage and temperature were recorded. As before, the lock-in voltage can be converted to the permeability using Eqn. 2.14, then to convert the permeability to susceptibility, χ, the first part of Eqn. 2.3 is used. The discussed in the theory section, the temperature dependence of the susceptibility comes from Eqn. 2.25. To see the linear relationship more clearly, we plot the inverse of the susceptibility versus temperature (Eqn. 2.26).

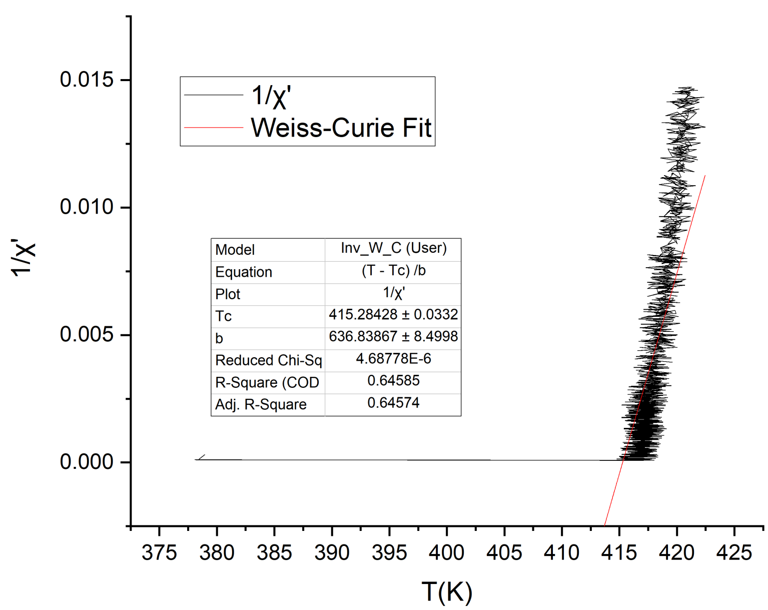


Figure 15: Temperature scan of sample while it cools

From this fit, we obtain a critical temperature of 415.28 ± 0.03 K (142.13 ± 0.03 ° C).

In the future of the second part of this experiment, one thing that can be improved upon is finding a good way to reduce the deviation of the susceptibility around similar temperatures. As can be seen in Figure 15, each value of the susceptibility has a couple of different temperatures associated with it. This thermal deviation in either direction causes the fit to be skewed a little in either direction. Obtaining a method to better eliminate noise would help with this effect a lot.

4 Conclusion

In conclusion, from this lab, we learned a great deal about ferromagnets. For the first part of the experiment, we found that of the four ferromagnetic materials that we sampled, two were 'hard' ferromagnets and two were 'soft' ferromagnets based on the coercivity of the hysteresis loops. In addition, we found that it seems like softer ferromagnets have larger energy losses than the harder ones. In the future, for this part of the lab, making sure that our apparatus is not being tampered with as we measure should prove essential in getting quality hysteresis loops. Additionally, allowing more time for our sample to demagnetize would make the hysteresis loops centered around the origin and more symmetrical around the axes. This would help us compute the power loss more accurately because the area of the would not be skewed either more positively or negatively.

In the second part of the lab, we examined the temperature dependence of the susceptibility. After submerging the sample in corn oil and increasing the temperature, we created hysteresis loops at several different temperatures. These loops allowed us to see that, in fact, the magnetic field response decreases drastically with temperature. Additionally, while the sample was cooling down, we took a temperature scan of the susceptibility and found a quite good agreement to the Weiss-Curie law (Eqn. 2.25) with a critical temperature of 415.28 ± 0.03 K (142.14 ± 0.03 °C). For this section, one improvement that we could make on creating the B versus H hysteresis loops would be to sample at lower temperatures. Our first sample was at 60 °C, but the room sat at around 25 °C. Getting more samples at different temperatures would help better map the different responses. For the second part of this experiment, in the future, we can improve the data by finding a better routine for measuring the susceptibility at a single temperature. Figure 15 shows that for a given susceptibility there are a couple of different temperatures associated with it. If we had a method of getting less noise then perhaps we could obtain a better fit and a more accurate estimate of the critical temperature.