



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Phys 214 – University Physics: Quantum Physics

Midterm Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb 11, 6-7:30pm Jay and Enrico Session 2: Feb 14, 1-2:30pm Alex and Greg

Can't make it to a session? Here's our schedule by course:

<https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/>

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into [Queue @ Illinois](#)
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!

1. **True or False:** The light intensity at any location on a screen is proportional to the probability that a photon arrives at that location, and that probability density is given by the square of the absolute value of the wavefunction.

True. The light intensity is correlated with the probability of detecting photons. That is, photons are more likely to be detected at those points where the wave intensity is high and less likely to be detected at those points where the wave intensity is low.

By definition, the probability density function is found by square the magnitude of the wavefunction. The integral of this function, over all space, must equal one.

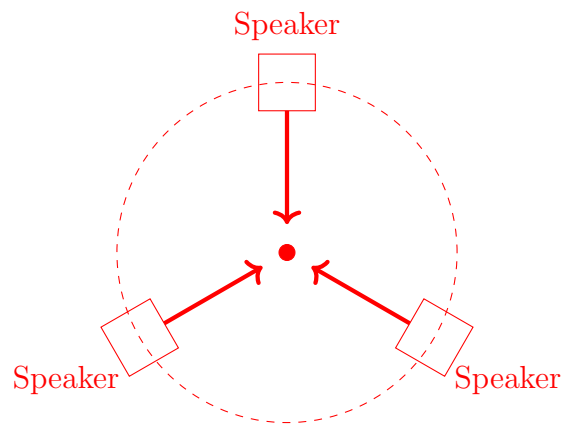
2. In a photoelectric effect demonstration, the intensity of the incident light is gradually increased, but no photocurrent is detected. Provide an explanation for this result.

It's light *frequency*, not intensity, that increases the kinetic energy of the photoelectrons. Since no current is detected, the energy of the incoming photons is less than the work function of the material, so no electrons escape. The frequency must be increased to change this.

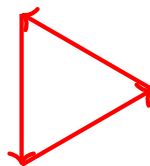
3. Three speakers lie on the perimeter of a circle. The sound intensity at each source is I_0 while the total intensity at the center of the circle is observed to be zero. Use phasors to determine the relative phase shift of each speaker such that this is possible.

All three speakers are equidistant from the circle's center. This means that any phase angle between waves is a result of the sources being out of phase, not the path difference. The phasor diagram of this system must be an equilateral triangle since the speakers have equal intensities, and the phase between each speaker can be deduced to be $(180 - 60) 120^\circ$.

Set-up:



Phasor Diagram:



4. A laser with time-varying frequency is directed at a barrier with a narrow slit followed by a screen. Assuming the laser intensity is constant, as the frequency increases, how does the number of photons per second arriving at the screen change?

The number of photons arriving at the screen will decrease. Using dimensional analysis:

$$\frac{\text{photons}}{\text{second}} * \frac{\text{joules}}{\text{photon}} = \frac{\text{joules}}{\text{second}}$$

The right hand side is proportional to the intensity (W/m^2), which is held constant. So if the frequency increases, the energy per photon increases, and the left hand side increases (J/photon). To keep intensity constant, the number of photons per second must decrease

5. The wave equation is a second order linear partial differential equation. Let f and g be any two solutions to the wave equations. Which of the following is also a solution?
Select all that apply.

- a) $y = fg$ c) $y = 2f + 3g$ e) $y = \left(\frac{df}{dt} + \frac{dg}{dt}\right)^2$
b) $y = (fg)^2$ d) $y = g - 4f$

Since the wave equation is linear, *any* linear combination of its solutions is also a solution

6. Simplify the following expression over the real numbers: $e^{ik\theta} + e^{-ik\theta}$

$$e^{ik\theta} + e^{-ik\theta} = \cos(k\theta) + i\sin(k\theta) + \cos(k\theta) - i\sin(k\theta) = 2\cos(k\theta)$$

7. A laser is directed at a barrier with a narrow slit followed by a screen. Applying a small angle approximation, if the slit width is halved while the wavelength is doubled, by what factor does the location of the first diffraction minimum change?

$$\lambda = a \sin(\theta) \rightarrow y = L \tan(\theta) \rightarrow y = L \tan\left(\arcsin\left(\frac{\lambda}{a}\right)\right) \approx \frac{L\lambda}{a}$$

So halving the slit width while doubling the wavelength results in the location of the first minimum increasing by (roughly) a factor of four

8. Given $\Psi(x) = N\sqrt{\omega ix}$ over some region of space, compute the probability density function associated with this wavefunction.

$$P(x) = \Psi\Psi^* = N\sqrt{\omega ix} * \sqrt{-\omega ix} = N^2\sqrt{(\omega x)^2} = N^2\omega x$$

9. A wave is traveling along the positive x-axis with speed 5 meters per second. Which equation could describe this wave?

a) $\frac{1}{2}e^{-(3x-4t)^2}$

A harmonic wave traveling in the $+\hat{x}$ direction has the general form

b) $\sin(12x + 2.6t)$

c) $3\cos(x - 0.3\pi t)$

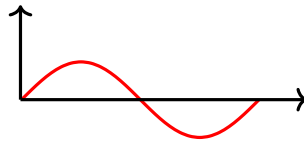
$$A\cos(kx - \omega t + \phi)$$

d) $\boxed{e^{-(x-5t)^4}}$

The velocity of such a wave is given by ω/k . Among the five candidate wave equations, the only choice that has this ratio equaling 5 is D

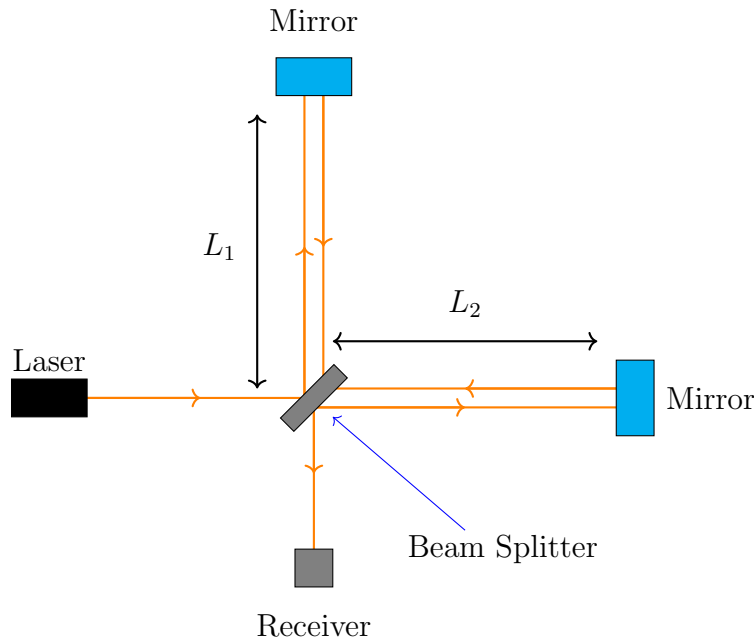
e) $\sin^2(x + t - \pi)$

To think about the direction of the wave, let's take a look at the general form (same as above).



Let's consider a test point, the max of the wave. If we press play and let our sine wave move in time, in order to stay at the maximum we need to keep the argument in the sine term constant. To achieve this effect we need to increase position. Thus, we can see that any wave with differently signed k and ω should be moving in the positive direction, while similarly signed k s and ω s move in the negative direction.

10. An interferometer has two arms of equal length ($L_1 = L_2$). A 200 W laser with wavelength 1064 nm is directed at the central beam splitter (shown below).



What is the minimum increase in L_2 required so that only 100 W goes to the detector? (Note that when the arms are of equal length, the detector receives 200 W).

- a) 266 nm
 b)
 c) 532 nm

Relevant equation:

$$I = 4I_0 \cos^2 \left(\frac{\phi}{2} \right)$$

where I_0 is the intensity from a single source. Since the beam goes into the splitter the intensity is halved so the value is first 100 W, this same thing is repeated when the beam enters the splitter the second time, thus the actual value of I_0 is 50W. Using this, combined with the knowledge that we need the final intensity to be 100 W we can solve for ϕ , the phase angle

The equation yields $\phi = \frac{\pi}{2}$. Plugging this into the phase angle equation

$$\phi = \frac{2\pi\delta}{\lambda}$$

we get that the minimum distance, δ , is 266 nm. Taking into account the fact that the beam bounces off the mirror, the distance it travels along that path is doubled. So we get that the minimum distance we need to move the mirror is 133 nm.

11. A wave propagating through the ocean is measured by a sensor and can be described by the equation $f(x, t) = \cos(0.4x - 2t)$. What is the wavelength, frequency and amplitude of the wave?

$$\lambda = 15.71m, f = 0.318Hz, A = 1$$

The general harmonic wave equation is

$$A \cos(kx - \omega t + \phi)$$

This can be used with $f = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$ to get the frequency and wavelength while the amplitude is the coefficient A

12. In which direction is the wave from the previous question traveling?

a) $-x$

b) $+x$

c) The direction is time dependent

As we increase time, the value of x must also increase for the argument of the wave equation to stay constant

13. A spacecraft is being pushed by a laser of wavelength 400 nm emitting photons at a rate of 10^{22} photons per second. Calculate the acceleration of the spacecraft given its mass is 4000 kg. Values are given in meters per second.

a) 3.25×10^{-6}

b) 1.53×10^{-3}

c) 4.14×10^{-9}

d) 1.7×10^{-5}

e) 5.5×10^{-4}

The photon's momentum can be found by

$$p = \frac{h}{\lambda}$$

which can be multiplied by the rate at which photons are emitted, R_γ to give us the recoiling force by Newton's second law:

$$F = \frac{dp}{dt} = pR_\gamma \equiv \frac{\text{momentum}}{\text{photon}} * \frac{\text{photons}}{\text{second}} = \frac{\text{momentum}}{\text{second}} = \text{Newtons}$$

This force, divided by the ship's mass giving us the acceleration

14. Compute the magnitude of the normalization constant for $\Psi(x) = Ne^{ikx}$ over the interval $0 \leq x \leq 3$. Assume the wavefunction equals zero for all other regions of space.
- 0.333
 - 0.577
 - 0.816

Taking the absolute square of this wavefunction will give N^2 as the probability density function

$$Ne^{ikx} * Ne^{-ikx} = N^2$$

If this is integrated from 0 to 3 and set equal to one (probabilities over all space must be equal to one), then we are left with $3N^2 = 1$, which when solved for N gives 0.577

15. Light with wavelength 100 nm is incident on a metal. The speed of the ejected photoelectrons is measured to be 10^6 meters per second. Find the work function of this metal.
- 1.99×10^{-18}
 - 1.53×10^{-18}
 - 4.55×10^{-18}

Through conservation of energy, we know that

$$hf = \frac{1}{2}mv^2 + \Phi$$

The frequency of this light can be found by dividing the speed of light by its wavelength

$$f = \frac{3 * 10^8}{100 * 10^{-9}} = 3 \times 10^{15} \text{ Hz}$$

Multiplying this with Planck's constant h gives us the total energy of the incoming photons:

$$1.988 \times 10^{-18} \text{ Joules}$$

The kinetic energy of the ejected electrons is

$$\frac{1}{2}mv^2 = 4.55 \times 10^{-19} \text{ Joules}$$

The difference in these values given the work function of this metal, using the above conservation equation.

$$\boxed{1.988 \times 10^{-18} - 4.55 \times 10^{-19} = 1.52 \times 10^{-18} \text{ J}}$$

16. An interferometer with equal arm lengths is sourced by a laser of wavelength $700 \mu\text{m}$. If the length of one arm is increased by 0.12 mm , by what amount are the waves out of phase?

$$\phi = \frac{2\pi\delta}{\lambda} \rightarrow \delta = 0.24 \times 10^{-3} \text{ m}$$

$$\phi = 2.15 \text{ radians}$$

17. Continuing from the previous question, assuming that the intensity received at the detector was 4 W/m^2 when the arm lengths were equal, what is the new intensity?

Values are given in W/m^2

- a) 2.95
- b) 0
- c) 0.898
- d) 1.21
- e) 4

Since the detector received 4 W/m^2 when the arm lengths were equal, we know that the source intensity is 4. We established the phase difference between these two waves in the previous question to be 2.15 radians. Using the equation

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

we can determine the new intensity in this interferometer. Remember that I_0 in this case is 1 because the beam is first split from 4 to 2, then 2 to 1 after the light passes the beam splitter a second time

18. The distance to the first minimum of a circular diffraction pattern is found to be 0.012 cm from the center. Assuming the distance to the screen is 10 mm and the diameter of the opening is 200 μm , what is the wavelength of the light used? Values are given in μm
- a) 1.97
- b) 2.28
- c) 0.94

For a circular diffraction pattern, the equation

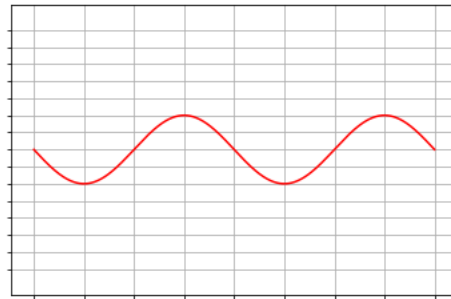
$$1.22\lambda = D \sin \left(\arctan \left(\frac{y}{L} \right) \right)$$

is used. However, a small angle approximation would be appropriate here since the angle is really small.

$$\lambda \approx \frac{Dy}{1.22L} = \frac{200 \times 10^{-6}(0.012 \times 10^{-2})}{1.22(10 \times 10^{-3})} = 1.97 \times 10^{-6} \text{ m}$$

Once converted to micrometers we get 1.97 μm

19. Below are two waves undergoing interference. Sketch the resulting waveform and determine the amplitude. What would the amplitude of this wave be if interference was constructive?



The resulting waveform will be the sum of the interfering waves. The amplitude is $4 - 2 = 2$. If we changed this so that the two were interfering constructively, the amplitude would be $4 + 2 = 6$.