ILLINOIS

Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Phys 100 − Thinking About Physics

Midterm Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Nov 1, 4-6pm Alex and Cindy   Session 2: Nov 3, 4-6pm Jason and Yvonne

Can't make it to a session? Here's our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into Queue @ Illinois

2. Click "New Question"

3. Add your NetID and Name

4. Press "Add to Queue"

5. Join the zoom link in the staff message

   Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!

1. The following two parts refer to their respective plots and are mutually exclusive.

(a) Below is a graph of position versus time and an equation of an object traveling along some path
described by: $s(t) = \begin{cases} 3 & t < 3 \\ t & t > 3 \end{cases}$
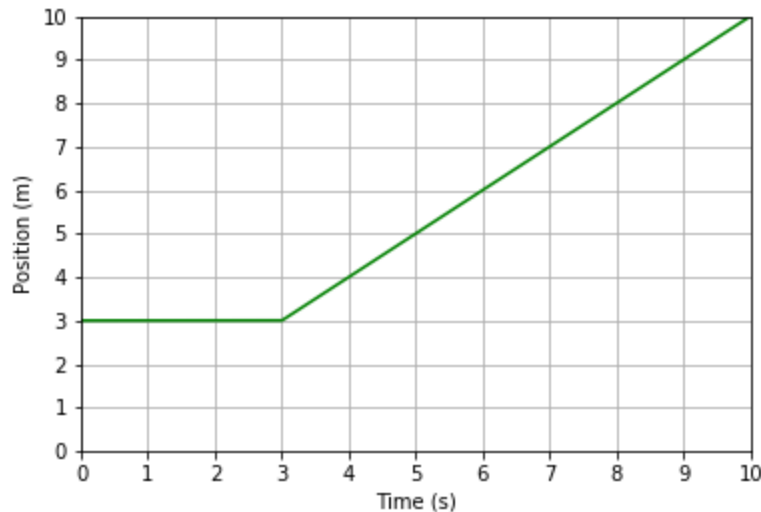
i) What is the object's velocity at time $t = 2$ s?

Velocity at $t = 2$ s is 0 m/s since slope is 0

ii) What is the object's velocity at time $t = 5$ s?

Velocity at $t = 5$ s is 1 m/s since slope is 1

iii) When is the object accelerating?

The object has zero acceleration on all intervals where the velocity is constant.
Aside: At $t = 3$ s since the velocity changes there is some instantaneous acceleration, but since
there is a discontinuity in the velocity function we ignore it.

(b) Below is a graph of velocity versus time and an equation of an object traveling along some path

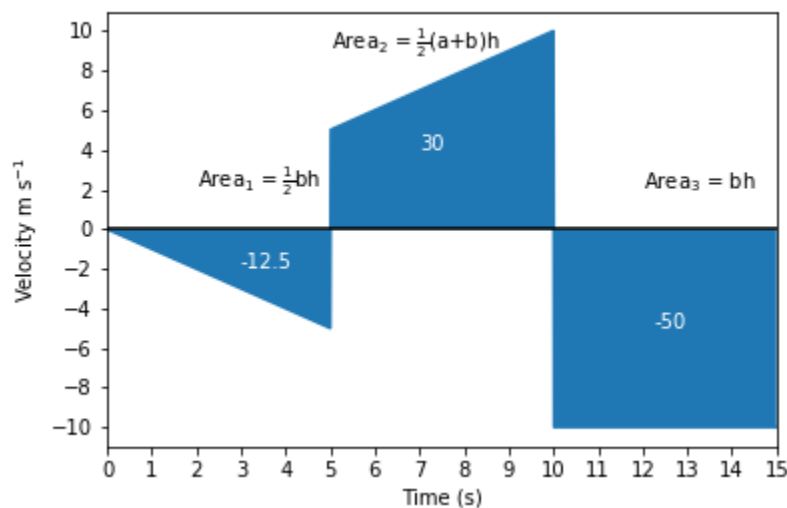described by: $v(t) = \begin{cases} -t & 0 < t \leq 5 \\ t & 5 < t \leq 10 \\ -10 & 10 < t \leq 15 \end{cases}$
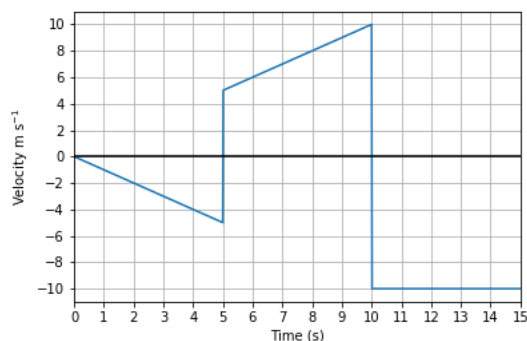
i) If the object started at initial position $x = 15$ m, where does it end up after the whole 15 s?

Change in position on a velocity versus time graph is the area under the curve

$$x_0 = 15, \ \Delta x_1 = -12.5, \ \Delta x_2 = +30, \ \Delta x_3 = -50$$

Thus x_f = 15 - 12.5 + 30 - 50 = $\boxed{-17.5 \text{ m}}$

ii) On which intervals of time does the object have positive acceleration?

The object is accelerating on the interval t ∈ (5,10]

iii) On which intervals of time in the object decelerating?

The object is accelerating on the interval t ∈ (0,5]

2. A blue ball is thrown upward with an initial speed of $v_b = 20$ m/s, from a height of $h_1 = 0.6$ meters above the ground. $t_d = 2.4$ seconds after the blue ball is thrown, a red ball is thrown down with an initial speed of $v_r = 8.7$ m/s from a height of $h_2 = 22.8$ meters above the ground. The force of gravity due to the earth results in the balls each having a constant downward acceleration of $g = 9.81$ m/s^2
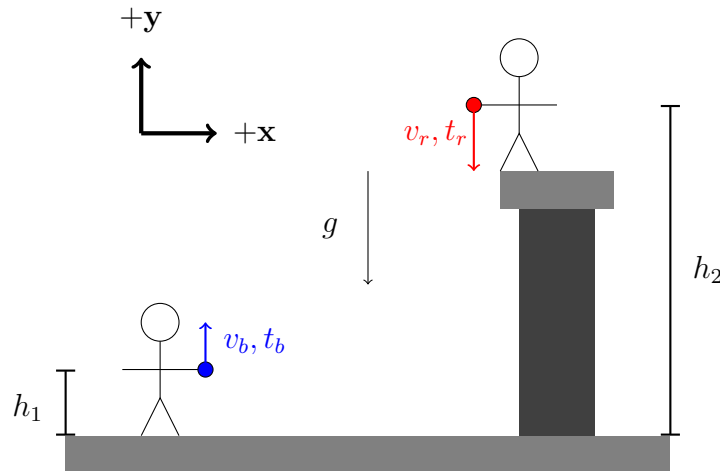
(a) What is the speed of the blue ball when it reaches its maximum height?

The speed of the blue ball at its max height is 0 m/s

(b) How long does it take the blue ball to reach its maximum height?

$$v_i = v_f + at$$
$$v_b = 20, \ v_f = 0, \ g = 9.81$$
$$\boxed{t = 2.038 \text{ s}}$$

(c) What is the maximum height of the blue ball?

$$x_{f,b} = x_0 + v_0 t_b + \frac{1}{2} a t_b^2$$
$$h_1 = 0.6, \ v_b = 20, \ t = 2.038, \ g = -9.81$$
$$\boxed{x_f = 20.822 \text{ m}}$$

(d) Write the position of the red ball as a function of time, keeping in mind it is released 2.4 seconds after the blue ball

$$x_{f,r} = x_0 + v_0 t_r + \frac{1}{2} a t_r^2 \qquad \text{where } t_r \equiv t_b - 2.4$$

(e) When are the red ball and the blue ball at the same height?

Set $x_{f,r} = x_{f,b}$ and solve:

t_d defined as the time difference in the dropped balls (2.4s)

$$h_1 + v_b t_b + \frac{1}{2}g t_b^2 = h_2 + v_r(t_b - t_d) + \frac{1}{2}g(t_b - t_d)^2$$

$$(h_1 - h_2) + v_b t_b + \frac{1}{2}g t_b^2 = v_r t_b - v_r t_d + \frac{1}{2}g\left[t_b^2 - 2t_b t_d + t_d^2\right]$$

$$(h_1 - h_2 + v_r t_d - \frac{1}{2}g t_d^2) = (v_r - v_b - g t_d)t_b$$

$$t_b = \frac{(h_1 - h_2 + v_r t_d - \frac{1}{2}g t_d^2)}{(v_r - v_b - g t_d)}$$

$$\boxed{t_b = 2.875 \text{ s}}$$

3. A quarterback throws a football toward a receiver who catches it t seconds later D meters away. Assume the ball is thrown and caught at the same height above the horizontal field and that you can ignore air resistance.
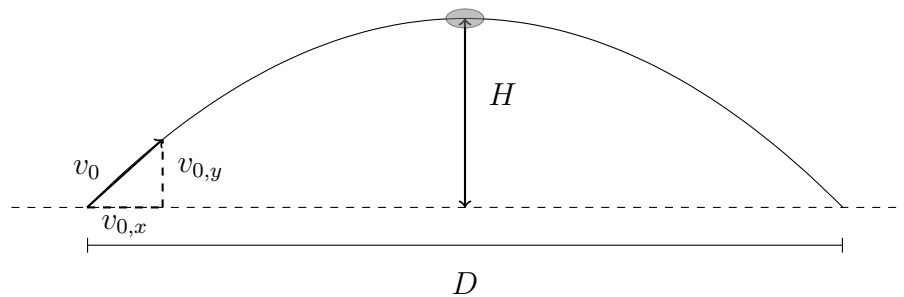
(a) If $t = 3$ seconds and $D = 60$ m, what is the horizontal component $v_{0,x}$ of the initial velocity of the ball?

Since there is no force in the x direction, the velocity, v_x, is constant.

$$v_{0,x} = \frac{D}{t}$$

$$\boxed{v_{0,x} = 20 \text{ m/s}}$$

(b) If $t = 3$ seconds and $D = 60$ m, what is the maximum height H reached by the ball (above its initial position)?

$$v_{f,y} = v_{i,y} + gt_{top}, v_{f,y} = 0$$
$$v_{i,y} = -gt_{top} = 14.715 m/s$$

$$y_f = y_i + v_{0,y}t_{top} + \frac{1}{2}at_{top}^2$$
$$H = v_{0,y}t_{top} + \frac{1}{2}at_{top}^2$$

$$\boxed{H = 11.036 \text{ m}}$$

4. A boat is traveling directly across a river (as seen by an observer standing on the shore) that flows at a uniform rate of $v_{r,g} = 10$ ft/s, as shown in the figure. To compensate for the flow of the river, the boat must head upstream as it travels. The speed of the boat is 18 ft/s with respect to the water
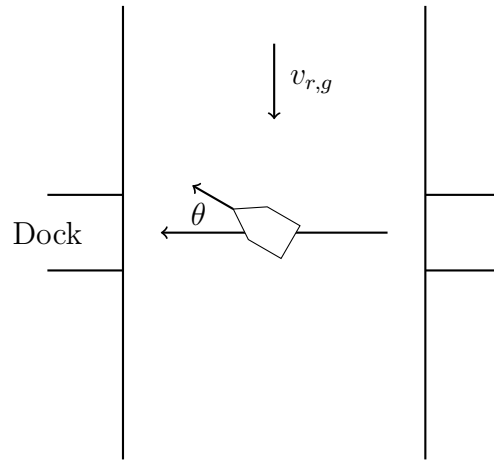
(a) What is the angle between the direction the boat points and the direction it is traveling with respect to the shore?

These problems are often tricky because there's usually some caution that's needed when converting the words into vectors. Nonetheless, it's a relative motion problem, so it's best to start with the relation for relative motion:

$$\mathbf{v}_{b,s} = \mathbf{v}_{b,w} + \mathbf{v}_{w,s}$$

where the subscripts are boat, shore, and water respectively. Let's also say that $+\hat{\mathbf{x}}$ is leftward (in the direction of motion) and $+\hat{\mathbf{y}}$ is upward.

We see from the diagram that the water flows in the $-\hat{\mathbf{y}}$ direction at a speed of 10 ft/s. In vectors, that is $\mathbf{v}_{w,s} = -10\hat{\mathbf{y}}$. We also know the *speed* of the boat relative to the water, but not its direction. So in vectors, that is

$$\mathbf{v}_{b,w} = 18\cos(\theta)\hat{\mathbf{x}} + 18\sin(\theta)\hat{\mathbf{y}}.$$

Now here's a tricky part: the problem tells us that the boat moves directly across the river as viewed from the shore. This means that the y-component is zero, and $\mathbf{v}_{b,s} = v_{b,s_x}\hat{\mathbf{x}} + 0\hat{\mathbf{y}}$. So we only know one of the components. But we can solve for the angle without knowing the other one fortunately because all the y-components are known.

$$\mathbf{v}_{b,s} = \mathbf{v}_{b,w} + \mathbf{v}_{w,s}$$
$$v_{b,s_x}\hat{\mathbf{x}} + 0\hat{\mathbf{y}} = 18\cos(\theta)\hat{\mathbf{x}} + 18\sin(\theta)\hat{\mathbf{y}} - 10\hat{\mathbf{y}}$$

Taking the y-components as one equation, we can solve

$$0 = 18\sin(\theta) - 10$$
$$\theta = \arcsin\left(\frac{10}{18}\right) \approx \boxed{33.7^\circ}$$

(b) If the river has width, $W = 500$ ft, and the angle is $\theta = 50°$:

Breaking up the components of the vectors we can figure out the velocity of the boat wrt the ground $(v_{b,g})$
(Use this equation from your formula sheet:)

$$v_{b,g} = v_{b,r} + v_{r,g}$$

<div align="center">

X: Y:

</div>

$$v_{(b,g)_x} = 18\cos(\theta) + 0 \qquad\qquad v_{(b,g)_y} = 18\sin(\theta) - 10$$
$$v_{(b,g)_x} = 11.57 \text{ ft / s} \qquad\qquad v_{(b,g)_y} = 3.788 \text{ ft / s}$$

i) How long does it take the boat to reach the other side

Here we only need to look at the x-component

$$\boxed{t = \frac{d}{v} = 43.2 \text{ s}}$$

ii) How far upstream does the boat end up?

Here we only need to look at the y-component

$$\boxed{d = vt = 164.64 \text{ ft}}$$

5. Three blocks are placed in contact on a horizontal frictionless surface. A constant force of magnitude $F = 30$ N is applied to the box of mass $M_1 = 8$kg. There is friction between the surfaces of blocks $M_2 = 2M_1$ and $M_3 = 3M_1$ ($\mu_s = 0.5$, $\mu_k = 0.3$) so the three blocks accelerated together to the right.
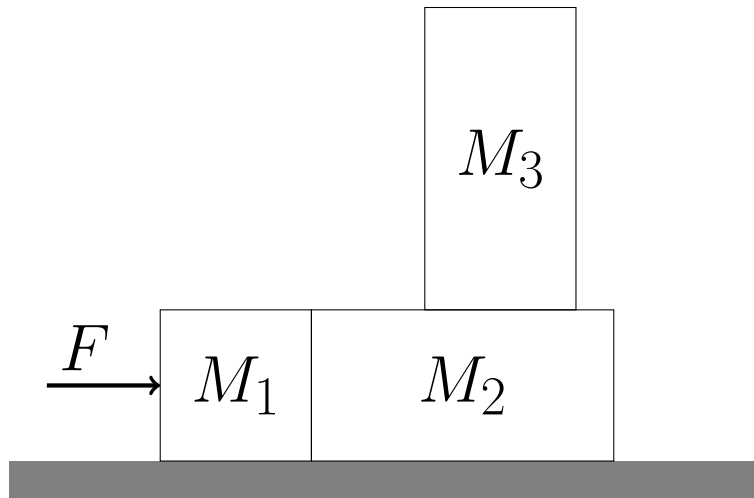

(a) What is the acceleration of M_3? Is it different than the acceleration of mass M?

The blocks are accelerating together, so since F = ma, $\boxed{a = 0.625 \text{ ms}^{-2}}$
Which is the same as the rest for the blocks

(b) What is the maximum force F that can be applied, before the M_3 block slides off? (Hint: draw force diagrams for all three of the blocks)

In the previous problem, we treated the three blocks as a single system, but this time, we will write a Newton's second law equation for each block. Since we are working in magnitudes, the following facts hold: $f_s \leq mg\mu_s$ and $F_{1,2} = F_{2,1}$.

Block 1

$F - F_{2 \text{ on } 1} = Ma$

Block 2

$F_{1 \text{ on } 2} - f_s = 2Ma$

Block 3

$f_s = 3Ma$
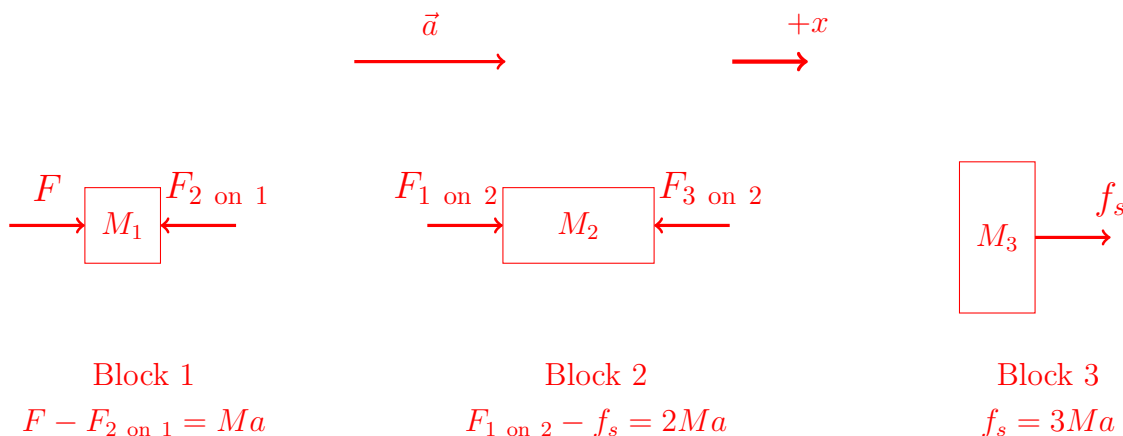
Now, using these three equations, we essentially want to find a function for F in terms of f_s and maximize it. We also want to eliminate the term a.

Combining the first two equations gives $F = 3Ma + f_s$. Taking this and combining it with the third equation, we get $F = 2f_s$.

It's easy to see that the maximum value of F occurs at the maximum value of f_s since they're directly related. Therefore, we can say (using the inequality of static friction) that

$$F_{max} = 2f_{s,max} = 6Mg\mu_s$$

And the maximum force occurs at $F_{max} = 6Mg\mu_s$ Plugging in the given values gives the final answer of

$$\boxed{F = 235.2 \text{ N}}$$

6. Consider the following vertical spring system before and after a mass is attached:
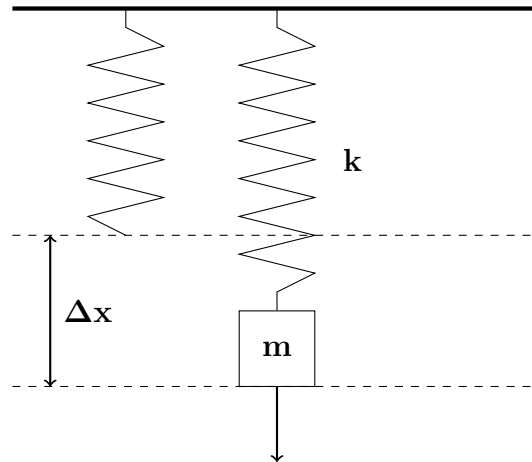
After the mass, m, is attached to the spring with spring constant k, the spring stretches by some Δx. Find an expression for Δx in terms of k and m

Draw the free body diagram for the mass on the spring:

There are two forces acting on the system, the force of gravity $(-\hat{y})$ and the restoring spring force $(+\hat{y})$
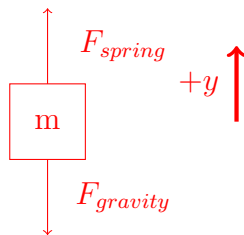
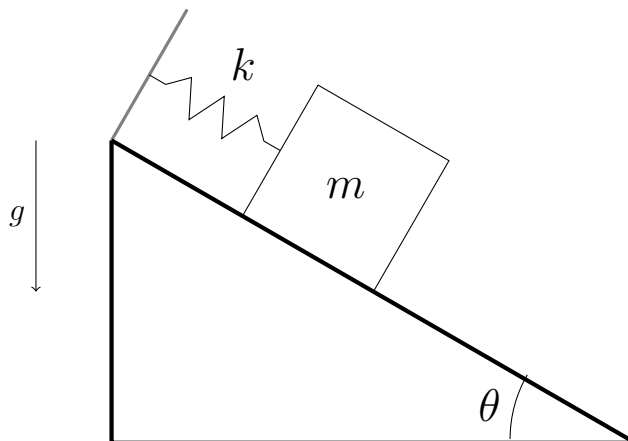
$$\Sigma F = ma$$

$$\Sigma F = 0$$

Since the system is at rest in equilibrium

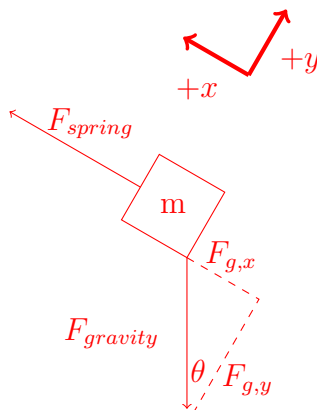$$0 = F_{spring} - F_{gravity}$$
$$0 = k\Delta x - mg$$

Thus our equation is $\boxed{\Delta x = \dfrac{mg}{k}}$

7. A block of mass m is kept in place on a smooth ramp by a spring, as shown. The ramp makes an angle θ with respect to the horizontal, and the spring constant is k. Neglect friction.

What is Δx_{min}, the minimum amount the spring must be stretched in order for the block to be at rest?

Draw the Free Body Diagram for this situation

Writing out the sum of the forces (we are working in magnitudes):

x-direction	y-direction
$mg\sin(\theta) = k\Delta x$	$N - mg\cos(\theta) = 0$
$mg\sin(\theta) - k\Delta x = 0$	$N = mg\cos(\theta)$

$$\Delta x_{min} = \frac{mgsin(\theta)}{k}$$

Be careful to note that $\sin(\theta)$ is the x-component here because of the trig in the situation.