Review: Quantum Gates ($|0\rangle$, $|1\rangle$)

X: $X|0\rangle \to |1\rangle$ — Bit Flip Gate

$X|1\rangle \to |0\rangle$

Z: $Z|0\rangle \to |0\rangle$ — Phase Flip Gate

$Z|1\rangle \to -|1\rangle$

H: $H|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|0\rangle \to |\rightarrow\rangle$

$H|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$|1\rangle \to |\uparrow\rangle$

Acts linearly on superposition states.

$$X\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = \frac{X|0\rangle + X|1\rangle}{\sqrt{2}}$$

$$= \frac{|1\rangle + |0\rangle}{\sqrt{2}}$$

Requirements:

Quantum State

↳ $|\alpha|^2 + |\beta|^2 = 1$

↳ $|0\rangle$, $|1\rangle$ need to be orthogonal

Linearity

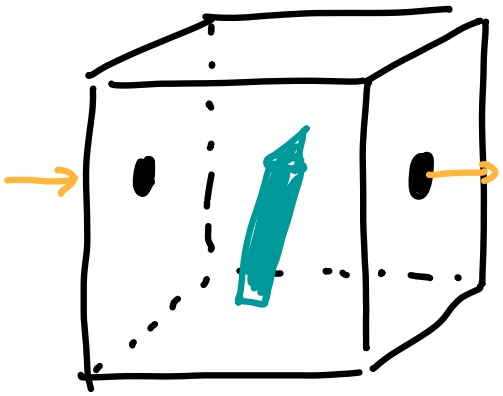
↳ $L(\bar{x} + \bar{y}) = L\bar{x} + L\bar{y}$

Distributivity on Addition

↳ $L(\alpha \bar{x}) = \alpha L\bar{x}$

Scalar Multiplication

Puzzle: Firework in a Box



Q: Is there a firework in the box?

case 1: No firework

↳ light shines thru

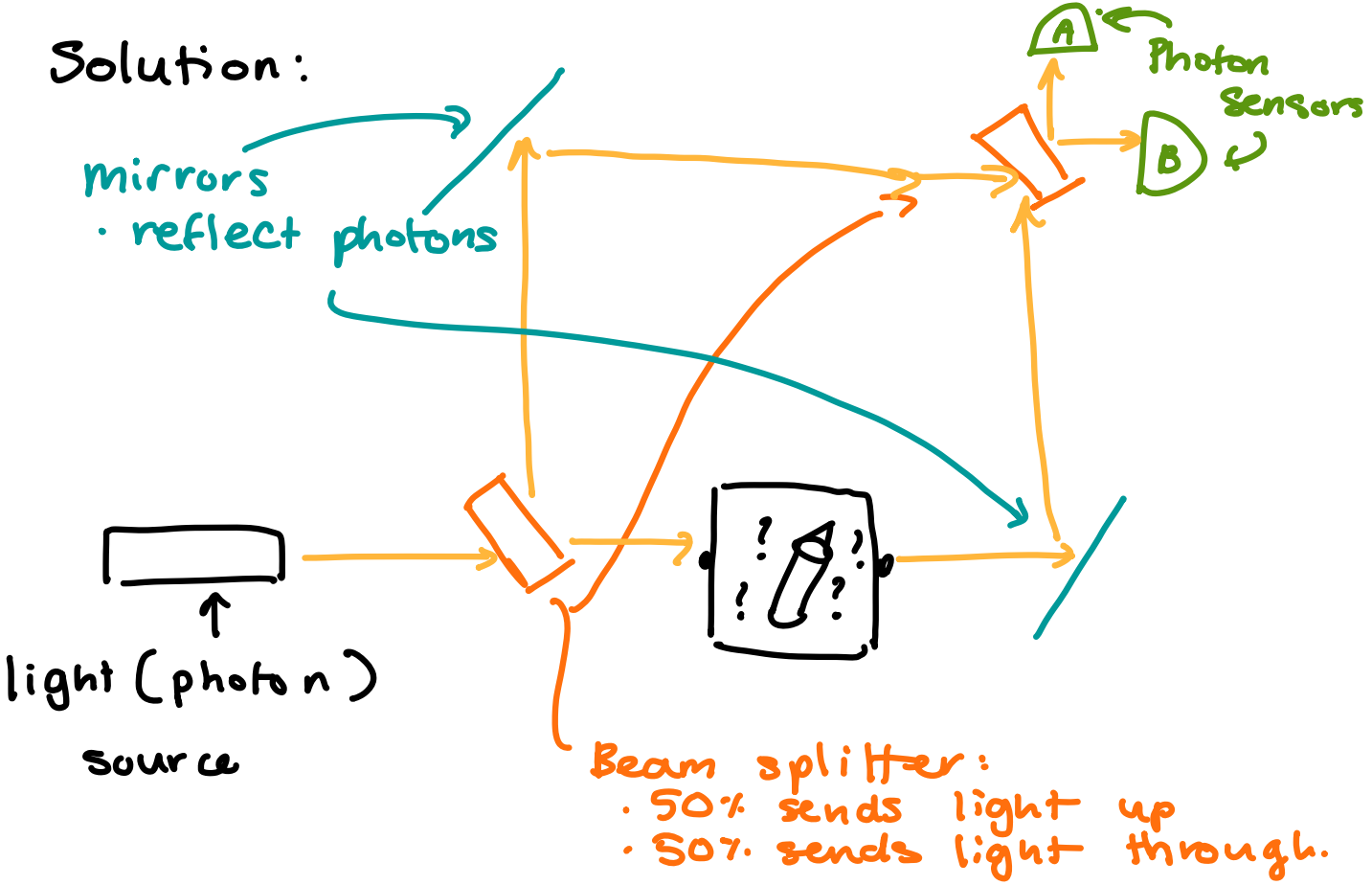
Case 2: firework

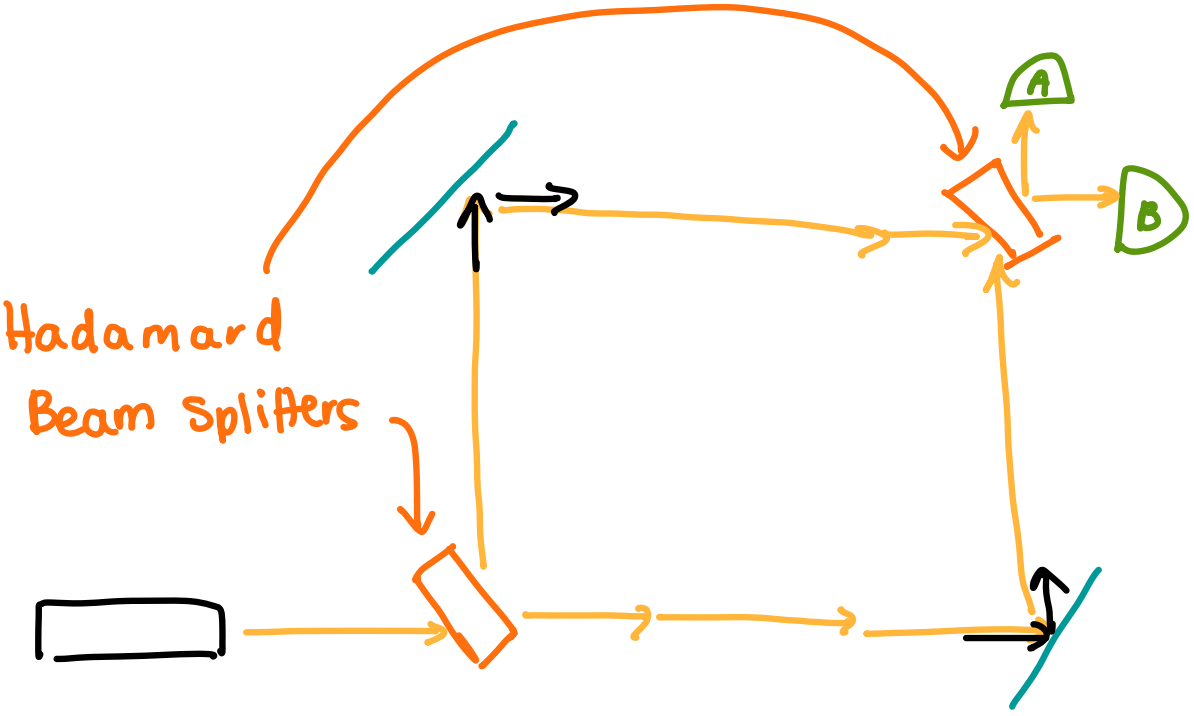
↳ firework explodes at light.

can we find out if a firework is in the box without letting it explode?

↳ No.

Solution:





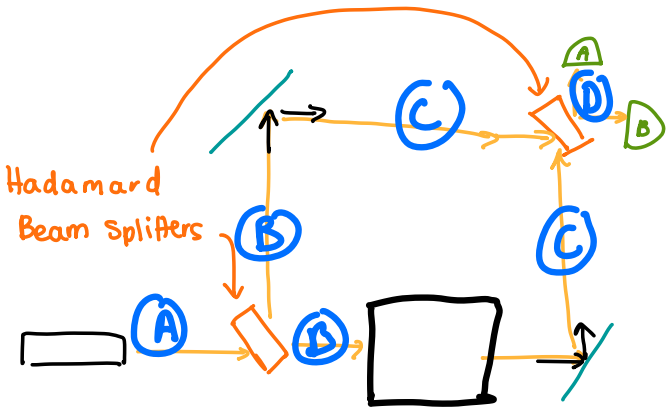
what percentage photons hit:

↳ sensor A?

↳ sensor B?

Quantum Circuit	Our Setup
Qubit	Photon.
$\|0\rangle$	$\|\rightarrow\rangle$
$\|1\rangle$	$\|\uparrow\rangle$
Hadamard (H)	Beam Splitter
Not (X) gate	Mirror.
Measurement Devices	 , Box

Case 1: No Firework.



$$|\psi_A\rangle = |\rightarrow\rangle$$

$$|\psi_B\rangle = H|\psi_A\rangle = \frac{|\rightarrow\rangle + |\uparrow\rangle}{\sqrt{2}}$$

$$|\psi_C\rangle = X|\psi_B\rangle = X\left(\frac{|\rightarrow\rangle + |\uparrow\rangle}{\sqrt{2}}\right) = \frac{X|\rightarrow\rangle + X|\uparrow\rangle}{\sqrt{2}} = \frac{|\uparrow\rangle + |\rightarrow\rangle}{\sqrt{2}}$$

$|1\rangle$ $|0\rangle$

$$|\psi_D\rangle = H|\psi_C\rangle = \frac{1}{\sqrt{2}}\left(H|\uparrow\rangle + H|\rightarrow\rangle\right)$$

$|0\rangle - |1\rangle$ $|0\rangle + |1\rangle$

$$= \frac{1}{\sqrt{2}}\left(\frac{|\rightarrow\rangle - |\uparrow\rangle}{\sqrt{2}} + \frac{|\rightarrow\rangle + |\uparrow\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left(|\rightarrow\rangle - |\uparrow\rangle + |\rightarrow\rangle + |\uparrow\rangle\right)$$

$$= \frac{1}{2}\left(2|\rightarrow\rangle\right) = |\rightarrow\rangle$$

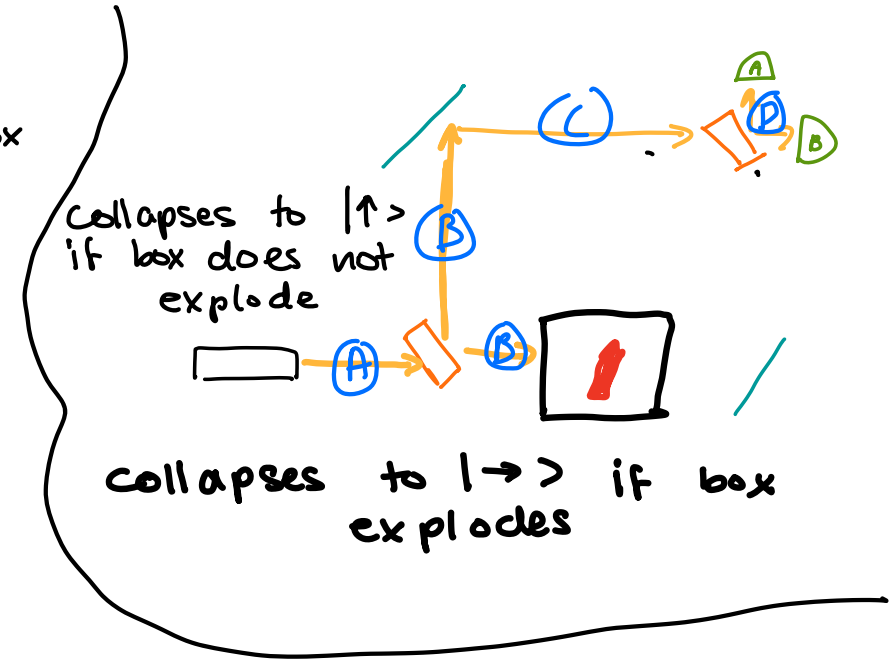
Photon would only be detected at B

Case 2: Firework

↳ Case 2a: Photon sent to box

0% of being measured at A or B

↳ case 2b: Photon sent to mirror



$$|\psi_B\rangle = |\uparrow\rangle$$

$$|\psi_C\rangle = X|\psi_B\rangle$$
$$= X|\uparrow\rangle$$
$$= |\rightarrow\rangle$$

$$|\psi_D\rangle = H|\psi_C\rangle$$
$$= H|\rightarrow\rangle$$
$$= \frac{|\uparrow\rangle + |\rightarrow\rangle}{\sqrt{2}}$$

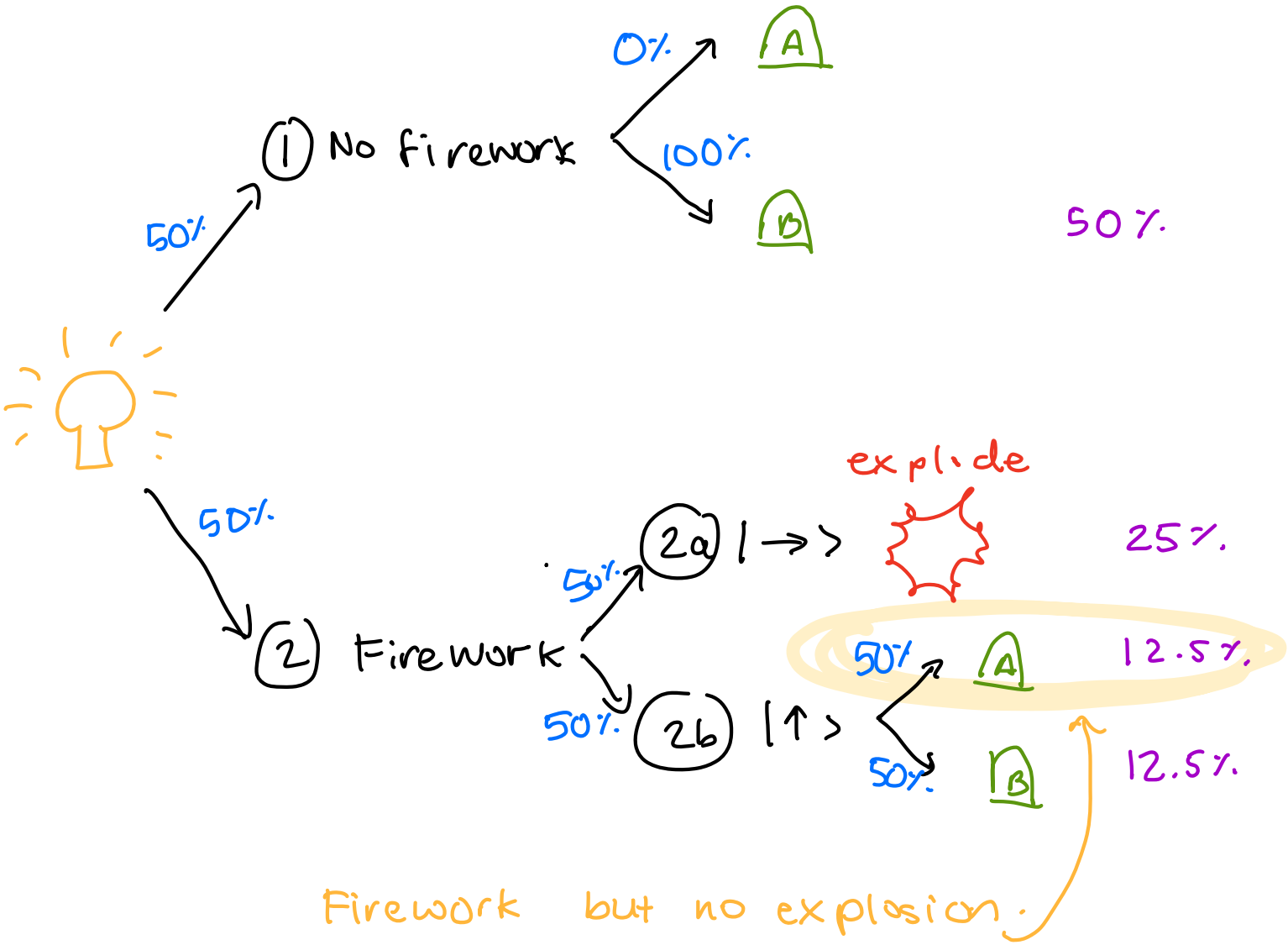
Case 2 summary:

↳ 50% : photon is not measured since firework explode (case 2a)

↳ 25% : photon measured at A

↳ 25% : photon measured at D

Summary


0%
A
1 No firework
100%
B
50%
50%
50%
explode
2a |→>
25%
50%
2 Firework
50%
A
12.5%
50%
2b |↑>
50%
B
12.5%
Firework but no explosion.