peer-graded-assignment

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1 Vertex Cover on 4-Colorable Graph

1.1 Q1. Give a relation between the value val(X) and the cardinality of the two sets

A1.

$$val(X) = |V^1| + \frac{1}{2}|V^{\frac{1}{2}}|$$

1.2 Q2. Give a tight lower bound on the cardinality between the two sets

A2. Because

$$|V^{\frac{1}{2}}| = \sum\nolimits_{c \in C} |V_c^{\frac{1}{2}}| \leqslant 4|V_3^{\frac{1}{2}}|$$

So we have

$$\frac{1}{4}|V^{\frac{1}{2}}| \leqslant |V_3^{\frac{1}{2}}|$$

1.3 Q3. Deduce an upper bound for the given expression

A3. From the derivation of Q2 we have

$$-|V_3^{\frac{1}{2}}| \leqslant -\frac{1}{4}|V^{\frac{1}{2}}|$$

Since

$$|V_0^{\frac{1}{2}}| + |V_1^{\frac{1}{2}}| + |V_2^{\frac{1}{2}}| = |V^{\frac{1}{2}}| - |V_3^{\frac{1}{2}}|$$

So we have

$$|V_0^{\frac{1}{2}}| + |V_1^{\frac{1}{2}}| + |V_2^{\frac{1}{2}}| \leqslant |V^{\frac{1}{2}}| - \frac{1}{4}|V^{\frac{1}{2}}| = \frac{3}{4}|V^{\frac{1}{2}}|$$

1.4 Q4. Give an upper bound on the number of vertices in S based on val(X)

A4. Since

$$|S| = |V^1| + |V_0^{\frac{1}{2}}| + |V_1^{\frac{1}{2}}| + |V_2^{\frac{1}{2}}|$$

So we have

$$|S| \leqslant |V^1| + \frac{3}{4}|V^{\frac{1}{2}}| = val(X) + \frac{1}{4}|V^{\frac{1}{2}}|$$

1.5 Q5. Conclude on the approximation ratio of the rounding procedure

A5. The ratio is

$$\frac{3}{2}$$

and the ratio is tight. This can be illustrated using the following example.
Consider the graph G = (V, E) in which
V = {A, B, C, D}
E = {(A, B), (A,C), (B,D), (C,D)}
The color of the vertex is specified as the following Python dict.
V = {A:0, B:1, C:2, D:3}
The fractional solution given by LP is

$$x_A^* = x_B^* = x_C^* = x_d^* = \frac{1}{2}$$

The integer solution given by the rounding procedure is

$$z_A = z_B = z_C = 1, z_D = 0$$

So we have

$$\frac{3}{2} val(X) = S$$

So the ratio of the rounding procedure is tight.

1.6 Q6. Show the contradiction under the given condition

A6. Since X is the solution of LP, from the constraints of LP we know that

$$x_u + x_v \geqslant 1$$

So we have

$$x_v = 1$$

1.7 Q7. Deduce from the previous question the belonging of the two vertices

A7. The derived result of A6 is equivalent to

$$v \in S$$

which contradicts with the given condition

1.8 Q8. Show the contradiction under the given condition

A8. They must both belong to

$$V_3^{\frac{1}{2}}$$

1.9 Q9. Recall that C is a 4-colouring. Explain the contradiction

A9. The derived result of Q8 shows that

$$C(u) = C(v) = 3$$

which contradicts with the given condition

1.10 Q5. Give an example of a well-known class of graphs that is 4-colourable.

A10. Every planar graph is 4-colorable. A well-known instance is the world map.