

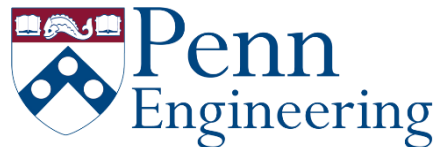
Robotics

Estimation and Learning
with Dan Lee

Week 1.

Gaussian Model Learning

1.2.2 Maximum Likelihood Estimate



Maximum Likelihood Estimate of Gaussian Model Parameters

- Objective

Estimate the mean and the variance given observed data

Likelihood:

$$p(\{x_i\}|\mu, \sigma)$$



Observed data



Unknown parameters

Maximum Likelihood Estimate of Gaussian Model Parameters

- Objective

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} p(\{x_i\} | \mu, \sigma)$$

Maximum Likelihood Estimate of Gaussian Model Parameters

- Objective

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} p(\{x_i\} | \mu, \sigma)$$

Assuming independence of observations,

$$p(\{x_i\} | \mu, \sigma) = \prod_{i=1}^N p(x_i | \mu, \sigma)$$

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma)$$

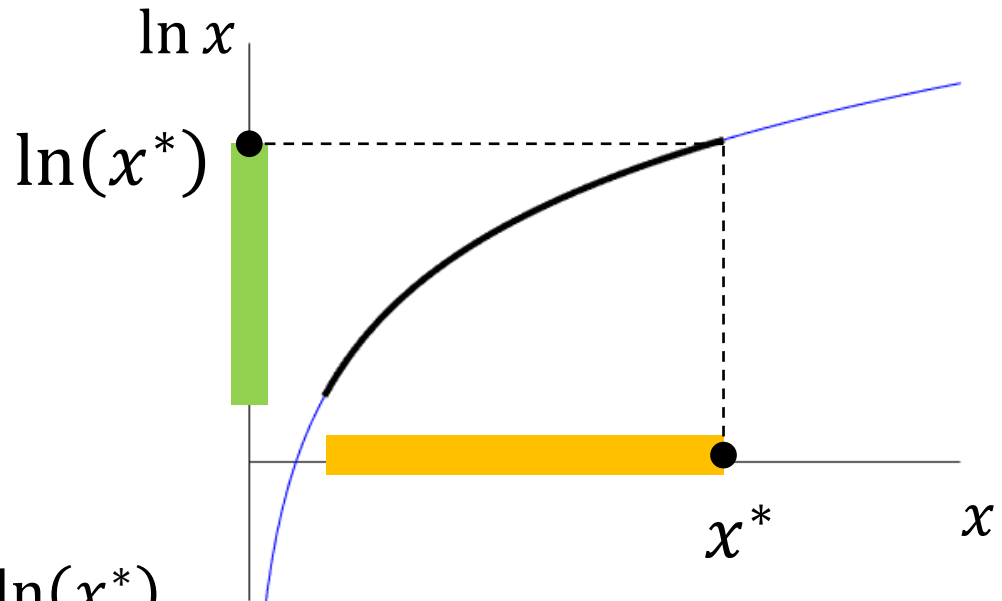
Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma)$$

(1) Take the log!

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma)$$



NOTE 1:

$$x \leq x^* \Leftrightarrow \ln(x) \leq \ln(x^*)$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma)$$

$$(1) \quad \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma) = \arg \max_{\mu, \sigma} \ln \left\{ \prod_{i=1}^N p(x_i | \mu, \sigma) \right\}$$

NOTE 1:

$$x \leq x^* \Leftrightarrow \ln(x) \leq \ln(x^*)$$

Maximum Likelihood Estimate of Gaussian Model Parameters


$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma)$$

$$\begin{aligned} (1) \quad \arg \max_{\mu, \sigma} \prod_{i=1}^N p(x_i | \mu, \sigma) &= \arg \max_{\mu, \sigma} \ln \left\{ \prod_{i=1}^N p(x_i | \mu, \sigma) \right\} \\ &= \arg \max_{\mu, \sigma} \sum_{i=1}^N \ln p(x_i | \mu, \sigma) \end{aligned}$$

NOTE 2:

$$\log(x_1 \times x_2 \times \cdots \times x_k) = \log(x_1) + \log(x_2) + \cdots + \log(x_k)$$


Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \sum_{i=1}^N \ln \underbrace{p(x_i | \mu, \sigma)}$$


(2) Gaussian!

$$\frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(x_i - \mu)^2}{2\sigma^2} \right\}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \sum_{i=1}^N \ln p(x_i | \mu, \sigma)$$

$$\ln \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(x_i - \mu)^2}{2\sigma^2} \right\}$$
$$= \left\{ -\frac{(x_i - \mu)^2}{2\sigma^2} - \ln \sigma - \ln \sqrt{2\pi} \right\}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \max_{\mu, \sigma} \sum_{i=1}^N \left\{ -\frac{(x_i - \mu)^2}{2\sigma^2} - \ln \sigma \right\}$$



$$\hat{\mu}, \hat{\sigma} = \arg \min_{\mu, \sigma} \sum_{i=1}^N \left\{ \frac{(x_i - \mu)^2}{2\sigma^2} + \ln \sigma \right\}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg \min_{\mu, \sigma} \underbrace{\sum_{i=1}^N \left\{ \frac{(x_i - \mu)^2}{2\sigma^2} + \ln \sigma \right\}}_{J(\mu, \sigma)}$$

- At optimum,

$$\frac{\partial J}{\partial \mu} = 0 \longrightarrow \hat{\mu}$$

$$\frac{\partial J(\hat{\mu}, \sigma)}{\partial \sigma} = 0 \longrightarrow \hat{\sigma}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

- The MLE Solution:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \hat{\mu})^2$$

MLE Estimate: Example

Ball color distribution

Segmented Ball Image



$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \hat{\mu})^2$$

