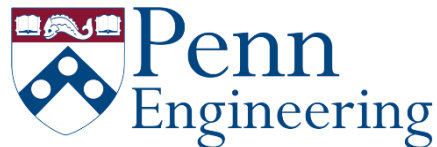


# Robotics

Estimation and Learning  
with Dan Lee

## Week 1. Gaussian Model Learning

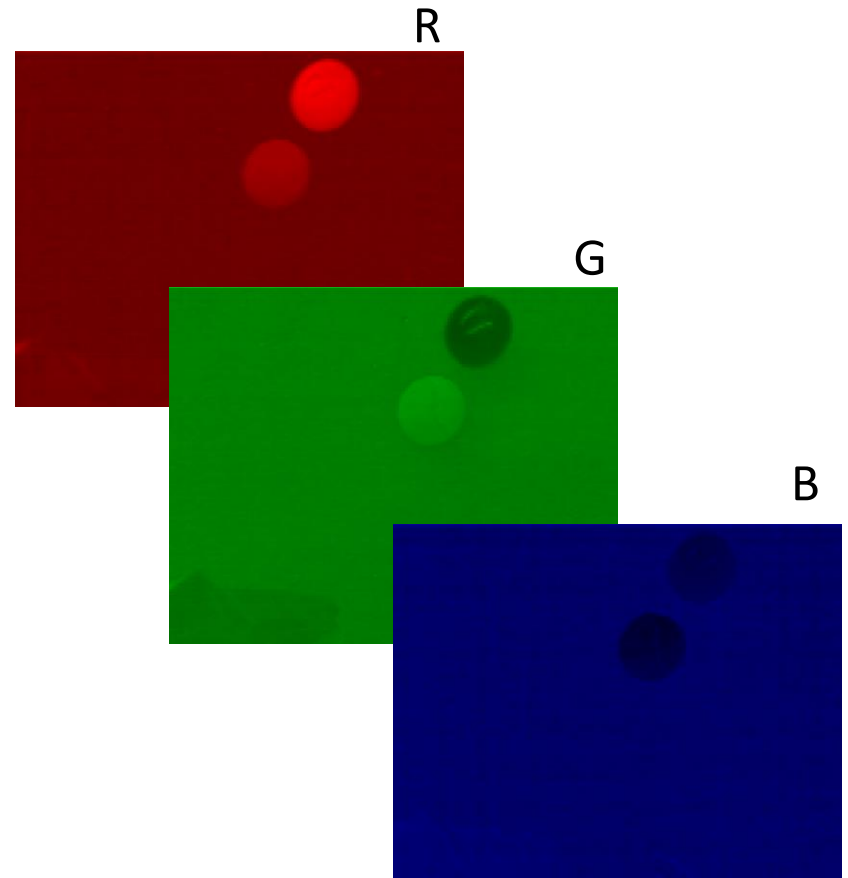
### 1.3.1 Multivariate Gaussian Distribution



# Multivariate Gaussian : Example

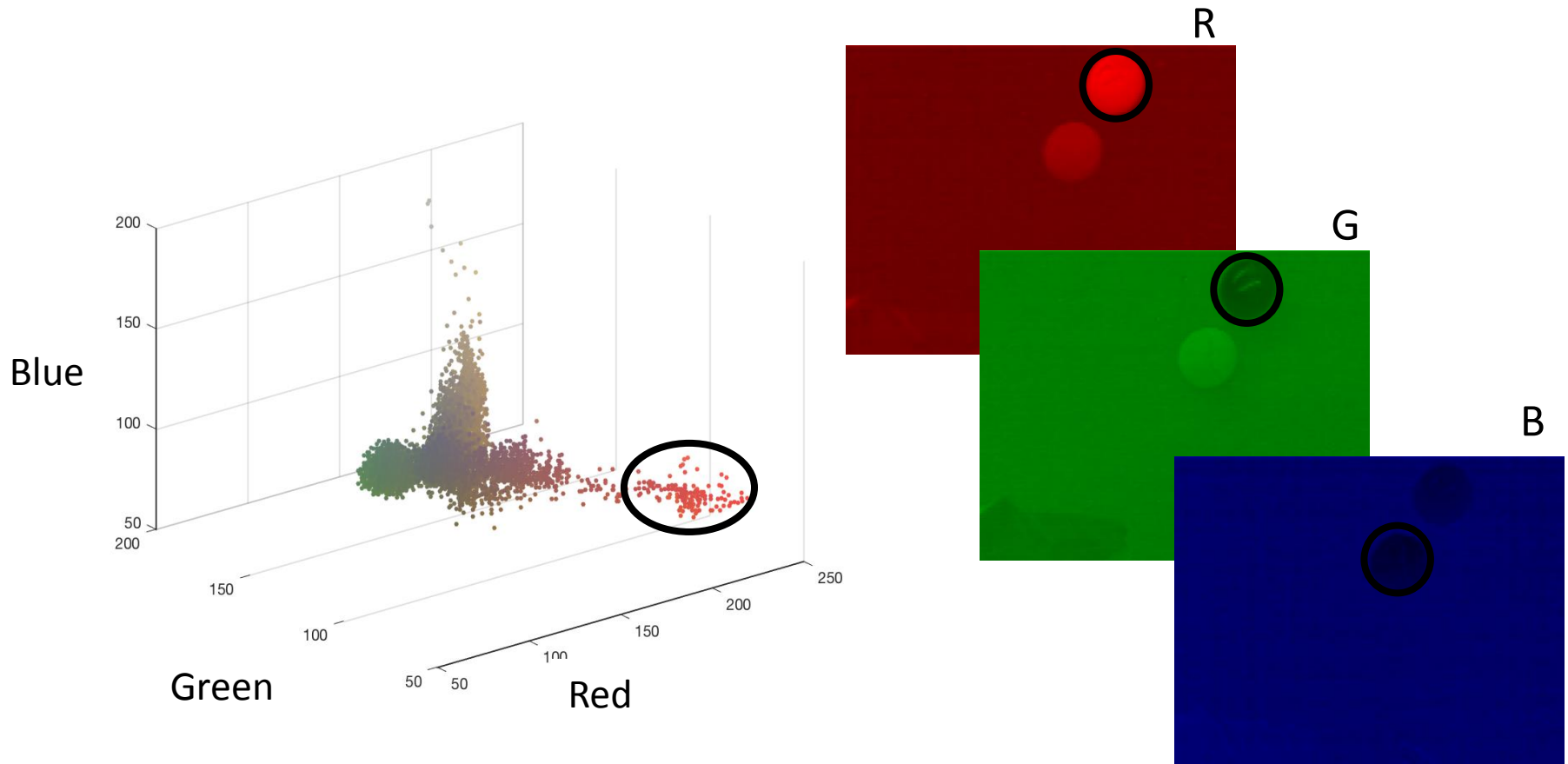
Ball color in multi-channels

RGB Image



# Multivariate Gaussian : Example

Ball color in multi-channels



# Multivariate Gaussian

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

$D$  Number of Dimensions

(Dimension = 1)

$\mathbf{x}$  Variable

$\boldsymbol{\mu}$  Mean *vector*

$\Sigma$  Covariance *matrix*

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}$$

# Multivariate Gaussian

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

$\Sigma$  Covariance *matrix*

- \* Diagonal terms: variance
- \* Off-diagonal terms: correlation

(Dimension = 2)

$$\Sigma = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1} \sigma_{x_2} \\ \sigma_{x_2} \sigma_{x_1} & \sigma_{x_2}^2 \end{bmatrix} \quad (\sigma_{x_1} \sigma_{x_2} = \sigma_{x_2} \sigma_{x_1})$$

# Multivariate Gaussian

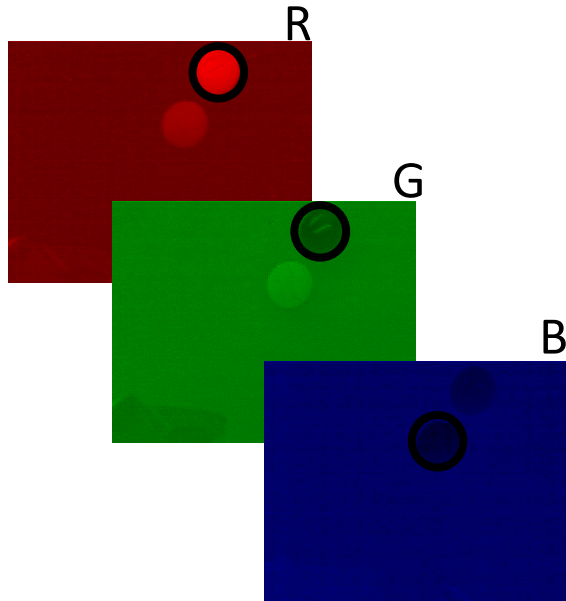
$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

Determinant of  $\Sigma$

# Multivariate Gaussian

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

Ball color in multi-channels



$$D = 3$$

$$\mathbf{x} = [x_R \quad x_G \quad x_B]$$

$$\boldsymbol{\mu} = [\mu_R \quad \mu_G \quad \mu_B]$$

$$\Sigma = \begin{bmatrix} \sigma_{x_R}^2 & \sigma_{x_R} \sigma_{x_G} & \sigma_{x_R} \sigma_{x_B} \\ \sigma_{x_R} \sigma_{x_G} & \sigma_{x_G}^2 & \sigma_{x_G} \sigma_{x_B} \\ \sigma_{x_R} \sigma_{x_B} & \sigma_{x_G} \sigma_{x_B} & \sigma_{x_B}^2 \end{bmatrix}$$

# Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{2\pi} \exp \left\{ -\frac{x^2 + y^2}{2} \right\}$$

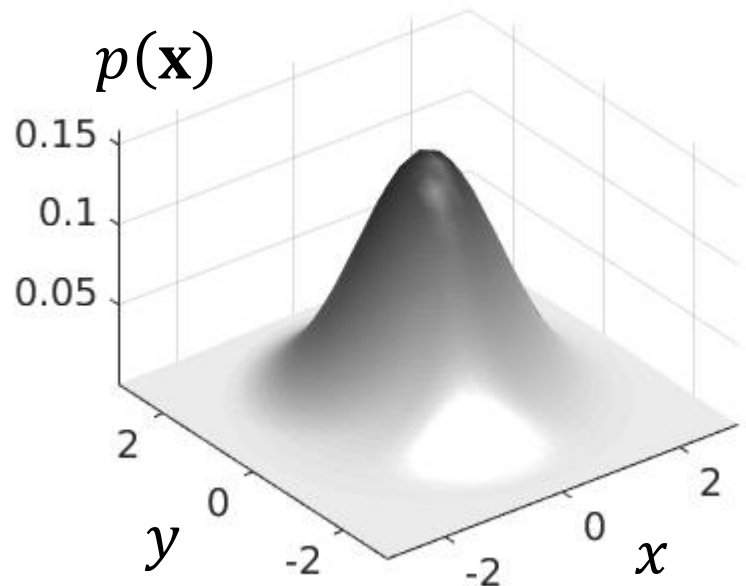
- 2D Zero-mean Spherical Case

$$D = 2$$

$$\mathbf{x} = [x \quad y]^T$$

$$\boldsymbol{\mu} = [0 \quad 0]^T$$

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$





# Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{2\pi} \exp \left\{ -\frac{x^2 + y^2}{2} \right\}$$

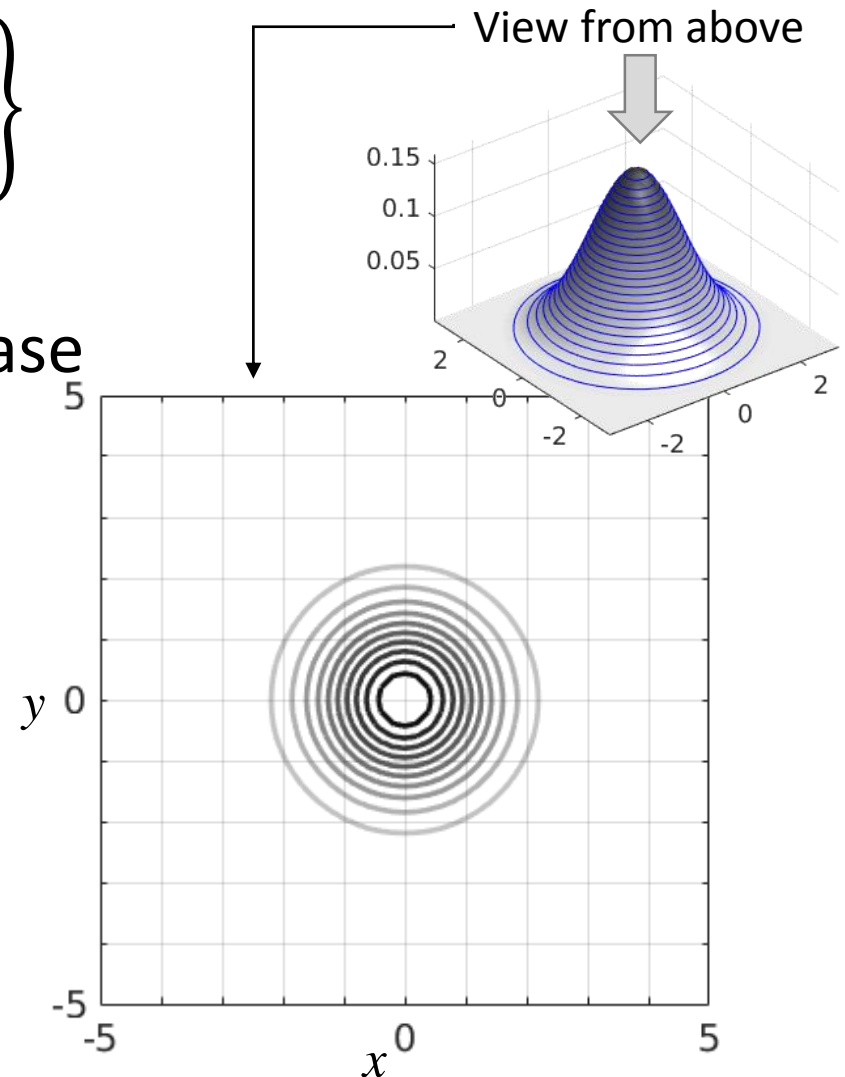
- 2D Zero-mean Spherical Case

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# Multivariate Gaussian: 2D

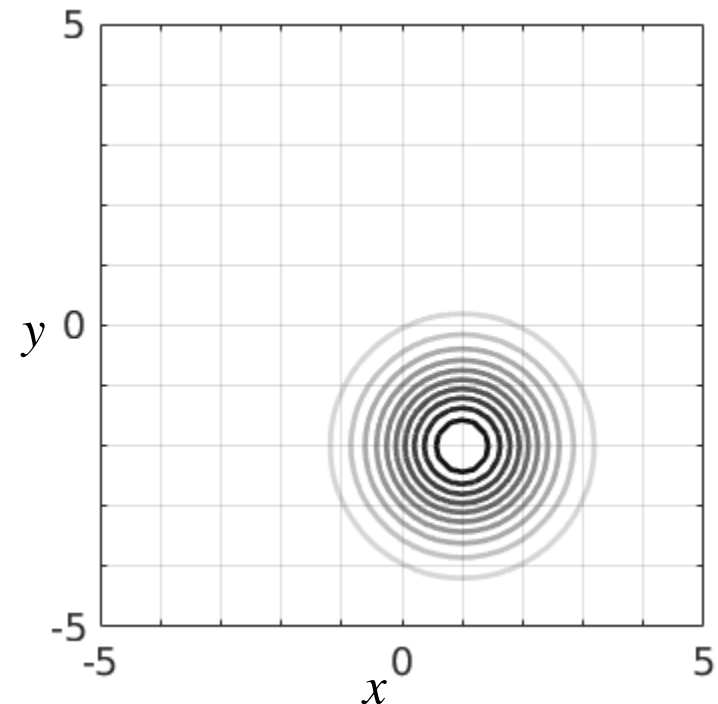
$$p(\mathbf{x}) = \frac{1}{2\pi} \exp \left\{ -\frac{(x - \mu_x)^2 + (y - \mu_y)^2}{2} \right\}$$

$$D = 2$$

$$\mathbf{x} = [x \quad y]^T$$

$$\boldsymbol{\mu} = [\mu_x \quad \mu_y]^T$$

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



# Multivariate Gaussian: 2D

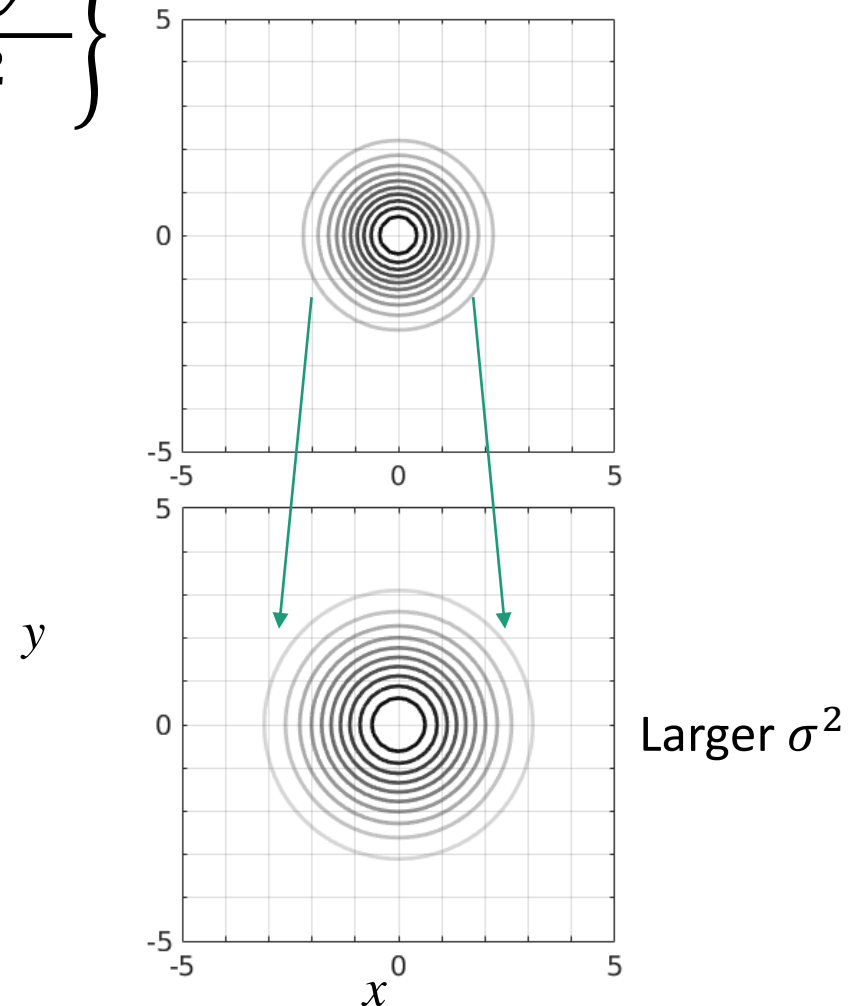
$$p(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{x^2 + y^2}{2\sigma^2}\right\}$$

$$D = 2$$

$$\mathbf{x} = [x \quad y]^T$$

$$\boldsymbol{\mu} = [0 \quad 0]^T$$

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$



# Multivariate Gaussian: 2D

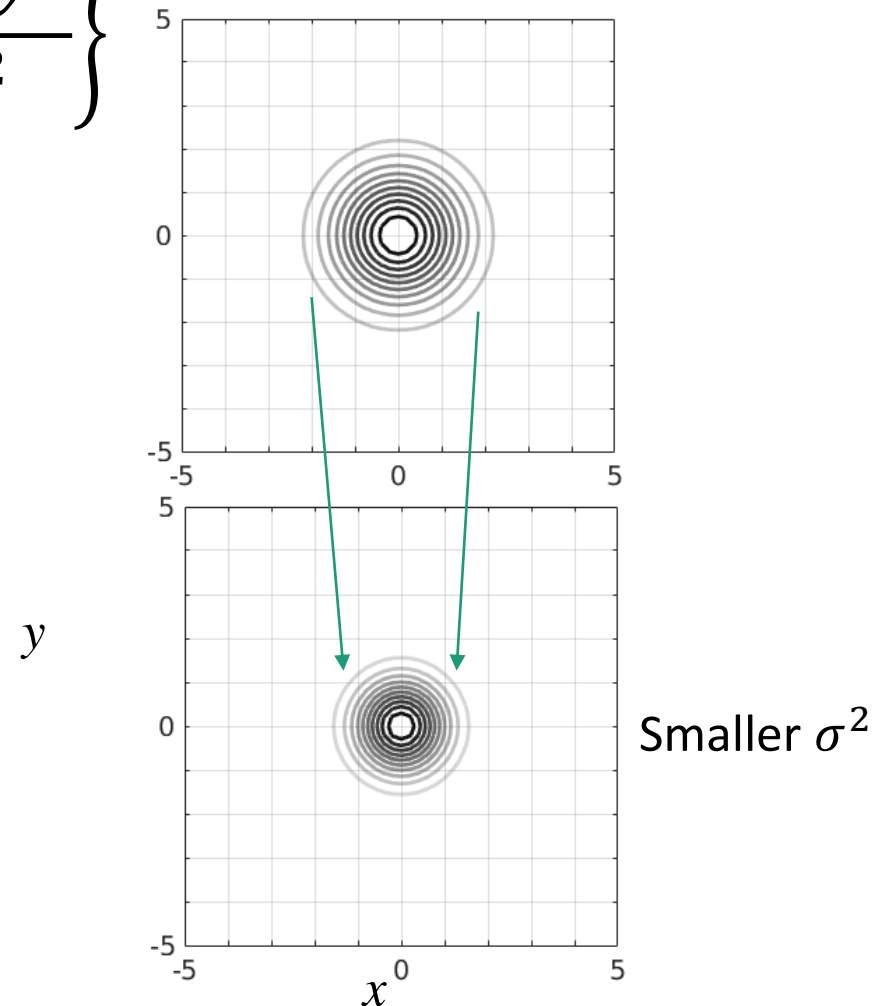
$$p(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{x^2 + y^2}{2\sigma^2}\right\}$$

$$D = 2$$

$$\mathbf{x} = [x \quad y]^T$$

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$$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

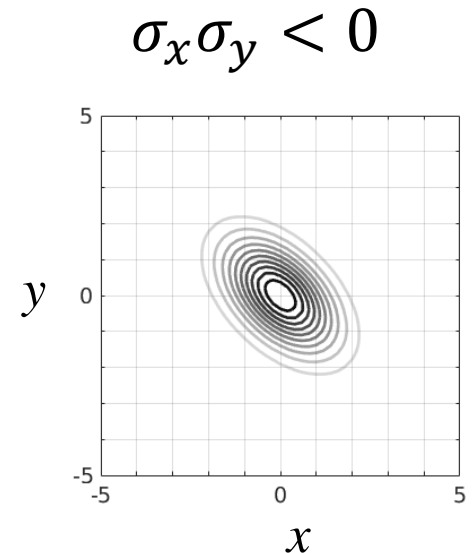
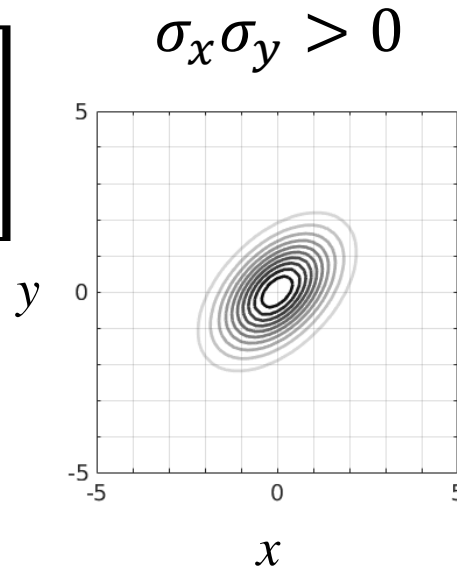


# Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

- 2D General Case

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_x \sigma_y \\ \sigma_x \sigma_y & \sigma_y^2 \end{bmatrix}$$



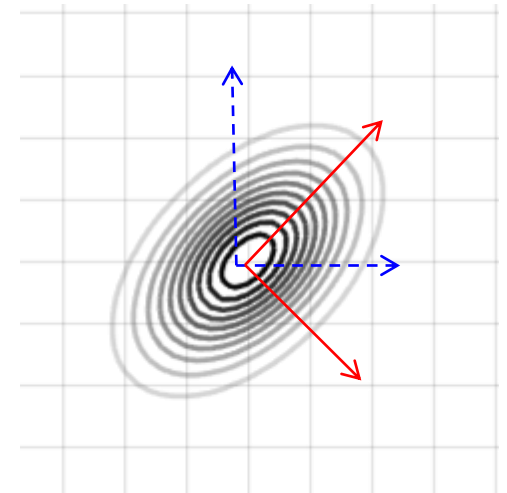
# Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

- Properties of Covariance Matrix  $\Sigma$

1)  $\Sigma$  is Symmetric and Positive Definite.

2) Diagonalization:  $\Sigma$  can be decomposed in the form of  $UDU^T$ .  
(D is a Diagonal matrix.)



# Multivariate Gaussian: 2D

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right\}$$

