On a Formal Model of Safe and Scalable Self-driving Cars

Shai Shalev-Shwartz, Shaked Shammah, Amnon Shashua

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Abstract

In recent years, car makers and tech companies have been racing towards self driving cars. It seems that the main parameter in this race is who will have the first car on the road. The goal of this paper is to add to the equation two additional crucial parameters. The first is standardization of safety assurance — what are the minimal requirements that every self-driving car must satisfy, and how can we verify these requirements. The second parameter is scalability — engineering solutions that lead to unleashed costs will not scale to millions of cars, which will push interest in this field into a niche academic corner, and drive the entire field into a "winter of autonomous driving". In the first part of the paper we propose a white-box, interpretable, mathematical model for safety assurance, which we call Responsibility-Sensitive Safety (RSS). In the second part we describe a design of a system that adheres to our safety assurance requirements and is scalable to millions of cars.

1 Introduction

The "Winter of AI" is commonly known as the decades long period of inactivity following the collapse of Artificial Intelligence research that over-reached its goals and hyped its promise until the inevitable fall during the early 80s. We believe that the development of Autonomous Vehicles (AV) is dangerously moving along a similar path that might end in great disappointment after which further progress will come to a halt for many years to come.

The challenges posed by most current approaches are centered around lack of safety guarantees, and lack of scalability. Consider the issue of guaranteeing a multi-agent safe driving ("Safety"). Given that society will unlikely tolerate road accident fatalities caused by machines, guarantee of Safety is paramount to the acceptance of autonomous vehicles. Ultimately, our desire is to guarantee zero accidents, but this is impossible since multiple agents are typically involved in an accident and one can easily envision situations where an accident occurs solely due to the blame of other agents (see Fig. 1 for illustration). In light of this, the typical response of practitioners of autonomous vehicle is to resort to a statistical data-driven approach where Safety validation becomes tighter as more mileage is collected.

To appreciate the problematic nature of a data-driven approach to Safety, consider first that the probability of a fatality caused by an accident per one hour of (human) driving is known to be 10^{-6}. It is reasonable to assume that for society to accept machines to replace humans in the task of driving, the fatality rate should be reduced by three orders of magnitude, namely a probability of 10^{-9} per hour[1]. In this regard, attempts to guarantee Safety using a data-driven statistical approach, claiming increasing superiority as more mileage is driven, are naive at best. The amount of data required to guarantee a probability of 10^{-9} fatality per hour of driving is proportional to its inverse, 10^9 hours of data (see details in the sequel), which is roughly in the order of thirty billion miles. Moreover, a multi-agent system interacts with its environment and thus cannot be validated offline[2], thus any change to the software of planning and control will require a new data collection of the same magnitude — clearly unwieldy. Finally, developing a system through data invariably suffers from lack of interpretability and explainability of the actions being taken — if an autonomous vehicle kills someone, we need to know the reason. Consequently, a model-based approach to Safety is required but the existing "functional safety" and ASIL requirements in the automotive industry are not designed to

[1] This estimate is inspired from the fatality rate of air bags and from aviation standards. In particular, 10^{-9} is the probability that a wing will spontaneously detach from the aircraft in mid air.

[2] unless a realistic simulator emulating real human driving with all its richness and complexities such as reckless driving is available, but the problem of validating the simulator is even harder than creating a Safe autonomous vehicle agent — see Section 2.2.

cope with multi-agent environments. Hence the need for a formal model of Safety which is one of the goals of this paper.

The second area of risk lies with lack of scalability. The difference between autonomous vehicles and other great science and technology achievements of the past is that as a "science project" the effort is not sustainable and will eventually lose steam. The premise underlying autonomous vehicles goes beyond "building a better world" and instead is based on the premise that mobility without a driver can be sustained at a lower cost than with a driver. This premise is invariably coupled with the notion of scalability — in the sense of supporting mass production of autonomous vehicles (in the millions) and more importantly of supporting a negligible incremental cost to enable driving in a new city. Therefore the cost of computing and sensing does matter, if autonomous vehicles are to be mass manufactured, the cost of validation and the ability to drive "everywhere" rather than in a select few cities is also a necessary requirement to sustain a business.

The issue with most current approaches is centered around a "brute force" state of mind along three axes: (i) the required "computing density", (ii) the way high-definition maps are defined and created, and (iii) the required specification from sensors. A brute-force approach goes against scalability and shifts the weight towards a future in which unlimited on-board computing is ubiquitous, somehow the cost of building and maintaining HD-maps becomes negligible and scalable, and that exotic super advanced sensors would be developed, productized to automotive grade, and at a negligible cost. A future for which any of the above holds is indeed plausible but having all of the above hold becomes a low-probability event. The combined issues of Safety and Scalability contain the risk of "Winter of autonomous vehicles". The goal of this paper is to provide a formal model of how Safety and Scalability are pieced together into an autonomous vehicles program that society can accept and is scalable in the sense of supporting millions of cars driving anywhere in the developed countries.

The contribution of this paper is twofold. On the Safety front we introduce a model called "Responsibility Sensitive Safety" (RSS) which formalizes the common sense of human judgement with regard to the notion of "who is responsible for causing an accident". RSS is interpretable, explainable, and incorporates a sense of "responsibility" into the actions of a robotic agent. The definition of RSS is agnostic to the manner in which it is implemented — which is a key feature to facilitate our goal of creating a convincing global safety model. RSS is motivated by the observation (as highlighted in Fig. 1) that agents play a non-symmetrical role in an accident where typically only one of the agents is responsible for the accident and therefore is to be blamed for it. The RSS model also includes a formal treatment of "cautious driving" under limited sensing conditions where not all agents are always visible (due to occlusions of kids behind a parking vehicle, for example). Our ultimate goal is to guarantee that an agent will never *cause* an accident, rather than to guarantee that an agent will never be involved in an accident (which, as mentioned previously, is impossible). It is important to note that RSS is not a formalism of blame according to the law but instead it is a formalism of the common sense of human judgement. For example, if some other car violated the law by entering an intersection while having the red light signal, while the robotic car had the green light, but had time to stop before crashing into the other car, then the common sense of human judgement is that the robotic car should brake in order to avoid the accident. In this case, the RSS model indeed requires the robotic car to brake in order not to cause an accident, and if the robotic car fails to do so, it shares responsibility for the accident.

Clearly, a model is useful only if it comes with an efficient Policy[3] that complies with RSS — in particular an action that looks innocent at the current moment might lead to a catastrophic event in the far future ("butterfly effect"). We prove that our definition of RSS is useful by constructing a set of local constraints on the short-term future that guarantees Safety for the entire future.

Our second contribution evolves around the introduction of a "semantic" language that consists of units, measurements, and action space, and specification as to how they are incorporated into Planning, Sensing and Actuation of the autonomous vehicles. To get a sense of what we mean by Semantics, consider how a human taking driving lessons is instructed to think about "driving policy". These instructions are not geometric — they do not take the form "drive 13.7 meters at the current speed and then accelerate at a rate of 0.8 m/s^2". Instead, the instructions are of a semantic nature — "follow the car in front of you" or "overtake that car on your left". The language of human driving policy is about longitudinal and lateral goals rather than through geometric units of acceleration vectors. We develop a formal Semantic language and show that the Semantic model is crucial on multiple fronts connected to the computational complexity of Planning that do not scale up exponentially with time and number of agents, to the manner in which

[3]a function that maps the "sensing state" to an action.

Safety and Comfort interact, to the way the computation of sensing is defined and the specification of sensor modalities and how they interact in a fusion methodology. We show how the resulting fusion methodology (based on the semantic language) guarantees the RSS model to the required 10^{-9} probability of fatality, per one hour of driving, while performing only offline validation over a dataset of the order of 10^5 hours of driving data.

Specifically, we show that in a reinforcement learning setting we can define the Q function[4] over actions defined over a semantic space in which the number of trajectories to be inspected at any given time is bounded by 10^4 regardless of the time horizon used for Planning. Moreover, the signal to noise ratio in this space is high, allowing for effective machine learning approaches to succeed in modeling the Q function. In the case of computation of sensing, Semantics allow to distinguish between mistakes that affect Safety versus those mistakes that affect the Comfort of driving. We define a PAC model[5] for sensing which is tied to the Q function and show how measurement mistakes are incorporated into Planning in a manner that complies with RSS yet allows to optimize the comfort of driving. The language of semantics is shown to be crucial for the success of this model as other standard measures of error, such as error with respect to a global coordinate system, do not comply with the PAC sensing model. In addition, the semantic language is also a critical enabler for defining HD-maps that can be constructed using low-bandwidth sensing data and thus be constructed through crowd-sourcing and support scalability.

To summarize, we propose a formal model that covers all the important ingredients of an autonomous vehicle: sense, plan and act. The model guarantees that from a Planning perspective there will be no accident of the autonomous vehicle's blame, and also through a PAC sensing model guarantees that, with sensing errors, a fusion methodology we present will require only offline data collection of a very reasonable magnitude to comply with our Safety model. Furthermore, the model ties together Safety and Scalability through the language of semantics, thereby providing a complete methodology for a safe and scalable autonomous vehicles. Finally, it is worth noting that developing an accepted safety model that would be adopted by the industry and regulatory bodies is a necessary condition for the success of autonomous vehicles — and it is better to do it earlier rather than later. An early adoption of a safety model will enable the industry to focus resources along a path that will lead to acceptance of autonomous vehicles. Our RSS model contains parameters whose values need to be determined through discussion with regulatory bodies and it would serve everyone if this discussion happens early in the process of developing autonomous vehicles solutions.

1.1 Outline

We follow the classic sense-plan-act robotic control methodology. The sensing system is responsible for understanding the present state of the environment. The planning part, which we call "driving policy", is responsible for figuring out what is the best next move (a "what will happen if" type of reasoning). The acting part is responsible for implementing the plan. The focus of the paper is on the sensing and planning parts (since the acting part is by and large well understood by control theory).

Mistakes that might lead to accidents can stem from sensing errors or planning errors. Planning is a multi-agent game, as there are other road users (humans and machines) that react to our actions. Section 2 underscores the problem with existing approaches to safety guarantees for the planning part, which we call multi-agent safety. We formally show that statistical estimation of the probability of planning errors must be done "online", namely, after every update of the software we must drive billions of miles with the new version. This is clearly infeasible. As an alternative, in Section 3 we propose a formal mathematical model for multi-agent safety which we call Responsibility Sensitive Safety (RSS). This model gives a 100% guarantee that the planning module will not make mistakes of the autonomous vehicle's responsibility (the notion of "responsibility" is formally defined). Such a model is useless without an efficient way to validate that a certain driving policy adheres to it. In Section 4 we accompany the RSS definitions with computationally efficient methods to validate them.

Mistakes of the sensing system are easier to validate, since sensing can be independent[6] of the vehicle actions, and therefore we can validate the probability of a severe sensing error using "offline" data. But, even collecting offline

[4]A function evaluating the long term quality of performing an action $a \in A$ when the agent is at state $s \in S$. Given such a Q-function, the natural choice of an action is to pick the one with highest quality, $\pi(s) = \operatorname{argmax}_a Q(s, a)$.

[5]Probably Approximate Correct (PAC), borrowing Valiant's PAC-learning terminology.

[6]Strictly speaking, the vehicle actions might change the distribution over the way we view the environment. However, this dependency can be circumvented by data augmentation techniques.

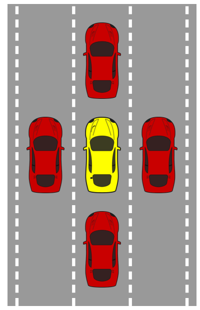


Figure 1: The central car can do nothing to ensure absolute safety.

data of more than 10^9 hours of driving is challenging. In Section 6.2, as part of a description of our sensing system, we present a fusion approach that can be validated using a significantly smaller amount of data.

The rest of the sections deal with Scalability. We outline a complete system that is safe and can scale to millions of cars. In Section 5 we describe our driving policy, starting from an explanation of why existing methods are so computationally demanding, and then showing how our semantic-based approach leads to a computationally efficient driving policy. In Section 6 we connect our semantic driving policy to semantic requirements from the sensing system, showing how it leads to sensing and mapping requirements that can scale to millions of cars in today's technology.

2 Multi-agent Safety

This section formalizes our arguments with regard to the necessity of a thorough safety definition, a minimal standard to which autonomous vehicle systems must abide.

2.1 Absolute Safety is Impossible

We begin with a naive attempt at defining a safe action-taking by a car, and immediately rule it out as infeasible. We say an action a taken by a car c is *absolutely safe* if no accident can follow it at some future time. It is easy to see that it is impossible to achieve absolute safety, by observing simple driving scenarios, for example, as depicted in Figure 1: from the central car's perspective, no action can ensure that none of the surrounding cars will crash into it, and no action can help it escape this potentially dangerous situation. We emphasize that solving this problem by forbidding the autonomous car from being in such situations is completely impossible — every highway with more than 2 lanes will lead to it and forbidding this scenario amounts to staying in the parking lot.

2.2 Deficiencies of the Statistical Approach

Since it is impossible to guarantee absolute safety, a popular approach is to propose a statistical guarantee, attempting to show that self-driving cars are statistically better than human drivers. There are several problems with this approach. First, as we formally prove below, validating this claim is infeasible. Second, statistical guarantees are not transparent. What will happen when a self-driving car will kill a little kid? Even if statistically self-driving cars will be involved in 50% less accidents than the average human driver, will the society be satisfied with this statistical argument? We believe that the statistical approach can be useful only if it leads to several orders of magnitude less accidents, and as shown in the next paragraph, this is infeasible to achieve.

Validating the statistical claim is infeasible: In the following technical lemma, we formally show why a statistical approach to validation of an autonomous vehicles system is infeasible, even for validating a simple claim such as "the system makes N accidents per hour".

Lemma 1 *Let X be a probability space, and A be an event for which $\Pr(A) = p_1 < 0.1$. Assume we sample $m = \frac{1}{p_1}$ i.i.d. samples from X, and let $Z = \sum_{i=1}^{m} \mathbf{1}_{[x \in A]}$. Then*

$$\Pr(Z = 0) \geq e^{-2}.$$

Proof We use the inequality $1 - x \geq e^{-2x}$ (proven for completeness in Appendix A.1), to get

$$\Pr(Z = 0) = (1 - p_1)^m \geq e^{-2p_1 m} = e^{-2}.$$

■

Corollary 1 *Assume an autonomous vehicle system AV_1 makes an accident with small yet insufficient probability p_1. Any deterministic validation procedure which is given $1/p_1$ samples, will, with constant probability, not distinguish between AV_1 and a different autonomous vehicle system AV_0 which never makes accidents.*

In order to gain perspective over the typical values for such probabilities, assume we desire an accident probability of 10^{-9} per hour, and a certain autonomous vehicle system provides only 10^{-8} probability. Even if we obtain 10^8 hours of driving, there is a constant probability that our validation process will not be able to tell us that the system is dangerous.

Finally, note that this difficulty is for invalidating a single, specific, dangerous autonomous vehicle system. A full *solution* cannot be viewed as a single system, as new versions, bug fixes, and updates will be necessary. Each change, even of a single line of code, generates a *new system* from a validator's perspective. Thus, a solution which is validated statistically, must do so online, over new samples after every small fix or change, to account for the shift in the distribution of states observed and arrived-at by the new system. Repeatedly and systematically obtaining such a huge number of samples (and even then, with constant probability, failing to validate the system), is infeasible.

On the problem of validating a simulator: As explained previously, multi-agent safety is hard to validate statistically as it should be done in an "online" manner. One may argue that by building a simulator of the driving environment, we can validate the driving policy in the "lab". The problem with this argument is that validating that the simulator faithfully represents reality is as hard as validating the policy itself. To see why this is true, suppose that the simulator has been validated in the sense that applying a driving policy π in the simulator leads to a probability of an accident of $\hat{p}$, and the probability of an accident of π in the real world is p, with $|p - \hat{p}| < \epsilon$. (Say that we need that ϵ will be smaller than 10^{-9}.) Now we replace the driving policy to be π'. Suppose that with probability of 10^{-8}, π' performs a weird action that confuses human drivers and leads to an accident. It is possible (and even rather likely) that this weird action is not modeled in the simulator, without contradicting its superb capabilities in estimating the performance of the original policy π. This proves that even if a simulator has been shown to reflect reality for a driving policy π, it is not guaranteed to reflect reality for another driving policy.

3 The Responsibility-Sensitive Safety (RSS) model for Multi-agent Safety

In the previous section we have shown that absolute safety is impossible. The implications might seem, at first glance, disappointing. Nothing is absolutely safe. However, we claim that this requirement is too harsh, as evident by the fact that humans do not get even close to following absolute safety. Instead, humans follow a safety notion that depends on responsibility. To be more precise, the crucial aspect missing from the absolute safety concept is the non-symmetry of most accidents - it is usually one of the drivers who is responsible for a crash, and is to be blamed. Clearly, in the example we consider in Figure 1, the central car is not to be blamed if the left car, for example, suddenly drives into it. We'd like to formalize the fact that considering its lack of responsibility, a behaviour of staying in its own lane can be considered safe. In order to do that, we develop a formal concept of "accident responsibility", which, we argue, captures the common sense behind human judgement of "who was driving safely and who was responsbile for the accident". The premise of RSS is that while self-driving cars might be involved in accidents, they will never *cause* an accident.

By and large, RSS is constructed by formalizing the following 4 "common sense" rules:

1. Keep a safe distance from the car in front of you, so that if it will brake abruptly you will be able to stop in time
2. Keep a safe distance from cars on your side, and when performing lateral manoeuvres and cutting-in to another car's trajectory, you must leave the other car enough space to respond
3. You should respect "right-of-way" rules, but "right-of-way" is given not taken
4. Be cautious of occluded areas, for example, a little kid might be occluded behind a parked car

The subsections below formalize these rules. The rules hold for both vehicles and pedestrians. For concreteness we start with vehicles, and explicitly explain the applicability to pedestrians in Section 3.4.

There are two aspects that a good formal model should satisfy:

- **Soundness**: when the model says that the self-driving car is not responsible for an accident, it should clearly match the "common sense" of human judgement. Note that we allow the model to assign responsibility on the self-driving car in fuzzy scenarios, possibly resulting in extra cautiousness, as long as the model is still useful.
- **Usefulness**: it is possible to efficiently create a policy that guarantees to never cause accidents while still maintaining normal flow of traffic.

Satisfying each of these requirements individually is trivial — a model that always assign responsibility to the self-driving car is sound but not useful, while a model that never assign responsibility to the self-driving car is useful but not sound. The proposed RSS model satisfies both soundness (which is the focus of this section) and usefulness (which is the focus of the next section).

3.1 Safe Distance

The first basic responsibility concept we formalize is "if someone hits you from behind it is not your fault". To gain some intuition, consider the simple case of two cars c_f, c_r, driving at the same speed, one behind the other, along a straight road, without performing any lateral manoeuvres. Assume c_f, the car at the front, suddenly brakes because of an obstacle appearing on the road, and manages to avoid it. Unfortunately, c_r did not keep enough of a distance from c_f, is not able to respond in time, and crashes into c_f's rear side. It is clear that the blame is on c_r; it is the responsibility of the rear car to keep safe distance from the front car, and to be ready for unexpected, yet reasonable, braking.

The following definition formalizes the concept of a "safe distance".

Definition 1 (Safe longitudinal distance — same direction) *A* longitudinal distance *between a car c_r that drives behind another car c_f, where both cars are driving at the same direction, is* safe *w.r.t. a response time ρ if for any braking of at most $a_{\max,\text{brake}}$, performed by c_f, if c_r will accelerate by at most $a_{\max,\text{accel}}$ during the response time, and from there on will brake by at least $a_{\min,\text{brake}}$ until a full stop then it won't collide with c_f.*

Remark 1

1. *The safe longitudinal distance depends on parameters: $\rho, a_{\max,\text{accel}}, a_{\max,\text{brake}}, a_{\min,\text{brake}}$. These parameters should be determined to some reasonable values by regulation.*
2. *The parameters can set differently for a robotic car and a human driven car. For example, the response time of a robotic car is usually smaller than that of a human driver and a robotic car can brake more effectively than a typical human driver, hence $a_{\min,\text{brake}}$ can set to be larger for a robotic car.*
3. *The parameters can also be set differently for different road conditions (wet road, ice, snow).*

Lemma 2 below calculates the safe distance as a function of the velocities of c_r, c_f and the parameters in the definition.

Lemma 2 *Let c_r be a vehicle which is behind c_f on the longitudinal axis. Let $\rho, a_{\max,\text{brake}}, a_{\max,\text{accel}}, a_{\min,\text{brake}}$ be as in Definition 1. Let v_r, v_f be the longitudinal velocities of the cars. Then, the minimal safe longitudinal distance between the front-most point of c_r and the rear-most point of c_f is:*

$$d_{\min} = \left[v_r\,\rho + \frac{1}{2}a_{\max,\text{accel}}\,\rho^2 + \frac{(v_r + \rho\,a_{\max,\text{accel}})^2}{2a_{\min,\text{brake}}} - \frac{v_f^2}{2a_{\max,\text{brake}}}\right]_+ .$$

Proof Let d_0 denote the initial distance between c_r and c_f. Denote $v_{\rho,\max} = v_r + \rho\,a_{\max,\text{accel}}$. The velocity of the front car decreases with t at a rate $a_{\max,\text{brake}}$ (until arriving to zero or that a collision happens), while the velocity of the rear car increases in the time interval $[0, \rho]$ (until reaching $v_{\rho,\max}$) and then decreases at a rate $a_{\min,\text{brake}} < a_{\max,\text{brake}}$ until arriving to zero or to a collision. It follows that if at some point in time the two cars have the same velocity, then from there on, the front car's velocity will be smaller, and the distance between them will be monotonically decreasing until both cars reach a full stop (where the "distance" can be negative if collision happens). From this it is easy to see that the worst-case distance will happen either at time zero or when the two cars reach a full stop. In the former case we should require $d_0 > 0$. In the latter case, the distances the front and rear car will pass until a full stop is $\frac{v_f^2}{2a_{\max,\text{brake}}}$ and $v_r\,\rho + \frac{1}{2}a_{\max,\text{accel}}\,\rho^2 + \frac{v_{\rho,\max}^2}{2a_{\min,\text{brake}}}$. At that point, the distance between them should be larger than zero,

$$d_0 + \frac{v_f^2}{2a_{\max,\text{brake}}} - \left(v_r\,\rho + \frac{1}{2}a_{\max,\text{accel}}\,\rho^2 + \frac{v_{\rho,\max}^2}{2a_{\min,\text{brake}}}\right) > 0 .$$

Rearranging terms, we conclude our proof. ■

The preceding definition of a safe distance is sound for the case that both the rear and front cars are driving at the same direction. Indeed, in this case, it is the responsibility of the rear car to keep a safe distance from the front car, and to be ready for unexpected, yet reasonable, braking. However, when the two cars are driving at opposite directions, we need to refine the definition. Consider for example a car c_r that is currently at a safe distance from a preceding car, c_f, that stands still. Suddenly, c_f is reversing into a parking spot and c_r hits it from behind. The common sense here is that the responsibility is not on the rear car, even though it hits c_f from behind. To formalize this common sense, we simply note that the definitions of "rear and front" do not apply to scenarios in which vehicles are moving toward each other (namely, the signs of their longitudinal velocities are opposite). In such cases we expect both cars to decrease the *absolute value* of their velocity in order to avoid a crash.

We could therefore define the safe distance between cars that drive in opposite directions to be the distance required so as if both cars will brake (after a response time) then there will be no crash. However, it makes sense that the car that drives at the opposite direction to the lane direction should brake harder than the one who drives at the correct direction. This leads to the following definition.

Definition 2 (Safe longitudinal distance — opposite directions) *Consider cars c_1, c_2 driving on a lane with longitudinal velocities v_1, v_2, where $v_2 < 0$ and $v_1 \geq 0$ (the sign of the longitudinal velocity is according to the allowed direction of driving on the lane). The longitudinal distance between the cars is safe w.r.t. a response time ρ, braking parameters $a_{\min,\text{brake}}, a_{\min,\text{brake,correct}}$, and an acceleration parameter $a_{\max,\text{accel}}$, if in case c_1, c_2 will increase the absolute value of their velocities at rate $a_{\max,\text{accel}}$ during the response time, and from there on will decrease the absolute value of their velocities at rate $a_{\min,\text{brake,correct}}, a_{\min,\text{brake}}$, respectively, until a full stop, then there will not be a collision.*

A calculation of the safe distance for the case of opposite directions is given in the lemma below (whose proof is straightforward, and hence omitted).

Lemma 3 *Consider the notation given in Definition 2. Define $v_{1,\rho} = v_1 + \rho\,a_{\max,\text{accel}}$ and $v_{2,\rho} = |v_2| + \rho\,a_{\max,\text{accel}}$. Then, the minimal safe longitudinal distance between c_1 and c_2 is:*

$$d_{\min} = \frac{v_1 + v_{1,\rho}}{2}\rho + \frac{v_{1,\rho}^2}{2a_{\min,\text{brake,correct}}} + \frac{|v_2| + v_{2,\rho}}{2}\rho + \frac{v_{2,\rho}^2}{2a_{\min,\text{brake}}} .$$

Before a collision between two cars, they first need to be at a non-safe distance. Intuitively, the idea of the safe distance definitions is that if both cars will respond "properly" to violations of the safe distance then there cannot be a collision. If one of them didn't respond "properly" then it is responsible for the accident. To formalize this, it is first important to know the moment just before the cars start to be at a non-safe distance.

Definition 3 (Dangerous Longitudinal Situation and Blame Time) *We say that time t is* dangerous *for cars c_1, c_2 if the distance between them at time t is non safe (according to Definition 1 or Definition 2). Given a dangerous time t, its* Blame Time*, denoted t_b, is the earliest non-dangerous time such that all the times in the interval $(t_b, t]$ are dangerous. In particular, an accident can only happen at time t if it is dangerous, and in that case we say that the blame time of the accident is the blame time of t.*

Next, we define what is a "proper response" to dangerous situations.

Definition 4 (Proper response to dangerous longitudinal situations) *Let t be a dangerous time for cars c_1, c_2 and let t_b be the corresponding blame time. The proper behaviour of the two cars is to comply with the following constraints on the longitudinal speed:*

1. *If at the blame time, the two cars were driving at the same direction, and say that c_1 is the rear car, then:*
   - *c_1 acceleration must be at most $a_{\max,\text{accel}}$ during the interval $[t_b, t_b + \rho)$ and at most $-a_{\min,\text{brake}}$ from time $t_b + \rho$ until reaching a full stop. After that, any non-positive acceleration is allowed.*
   - *c_2 acceleration must be at least $-a_{\max,\text{brake}}$ until reaching a full stop. After that, any non-negative acceleration is allowed.*
2. *If at the blame time the two cars were driving at opposite directions, and say that c_2 was driving at the wrong direction (negative velocity), then:*
   - *c_1 acceleration must be at most $a_{\max,\text{accel}}$ during the interval $[t_b, t_b + \rho)$ and at most $-a_{\min,\text{brake,correct}}$ from time $t_b + \rho$ until reaching a full stop. After that, it can apply any non-positive acceleration*
   - *c_2 acceleration must be at least $-a_{\max,\text{accel}}$ during the interval $[t_b, t_b + \rho)$ and at least $a_{\min,\text{brake}}$ from time $t_b + \rho$ until reaching a full stop. After that, any non-negative acceleration is allowed.*

As mentioned previously, collisions can only happen at dangerous times. It is easy to verify that if once the distance between cars become non-safe they both apply their "proper response to dangerous situations" from the corresponding blame time (until the distance between them becomes safe again) then collision cannot happen. This leads to assigning responsibility for accidents to the agent(s) that did not respond properly to the dangerous situation preceding the collision. The definition below is valid for scenarios where no lateral manoeuvres are being performed (that is, car keeps their lateral position in the lane all the time). In the next section we extend the definition to the more realistic case in which cars can perform lateral manoeuvres.

Definition 5 (Responsibility for an Accident — no lateral manoeuvres) *Consider an accident between c_1 and c_2 at time t and let t_b be the corresponding* Blame Time*. Car c_i is responsible for the accident if it did not follow the constraints defined by the "proper response to dangerous situations" (Definition 4) at some time in the interval $(t_b, t]$.*

Finally, note that it is possible that both c_1 and c_2 share the responsibility for an accident (if both of them did not comply with the proper response constraints).

3.2 Responsibility in Lateral Manoeuvres

We now move on to formally define responsibility when cars are performing lateral manoeuvres. We start with the case of a multi-lane road (typical highway situations or rural roads), where cars can change lanes, cut into other cars' paths and drive at different speeds. The cases of multiple geometries (merges, junctions, roundabouts, etc.), and unstructured roads, are discussed in the next subsection, where we introduce the concept of priority.

To simplify the following discussion, we assume a straight road on a planar surface, where the lateral, longitudinal axes are the x, y axes, respectively. This can be achieved, under mild conditions, by defining a bijection between the actual curved road and a straight road. See Appendix B. We refer to the longitudinal and lateral velocities as the derivatives of the longitudinal and lateral positions on the straight virtual road obtained by the bijection. Similarly, accelerations are second derivatives of positions.

Unlike longitudinal velocity, which can be kept to a value of 0 for a long time (the car is simply not moving), keeping lateral velocity at exact 0 is impossible as cars usually perform small lateral fluctuations. It is therefore required to introduce a robust notion of lateral velocity.

Definition 6 (μ-lateral-velocity) *Consider a point located at a lateral location l at time t. Its μ-lateral velocity at time t is defined as follows. Let $t_{out} > t$ be the earliest future time in which the point's lateral position, denoted l_{out}, is either $l - \mu/2$ or $l + \mu/2$ (if no such time exists we set $t_{out} = \infty$). If at some time $t' \in (t, t_{out})$ the point's lateral position is l, then the μ-lateral-velocity is 0. Otherwise, the μ-lateral-velocity is $(l_{out} - l)/(t_{out} - t)$.*

Roughly speaking, in order to have a collision between two vehicles, it is required that they will be close both longitudinally and laterally. For the longitudinal axis, we have already formalized the notion of "being close" using the safe distance. We will now do the same for lateral distance.

Definition 7 (Safe Lateral Distance) *The lateral distance between cars c_1, c_2 driving with lateral velocities v_1, v_2 is safe w.r.t. parameters $\rho, a^{\mathrm{lat}}_{\mathrm{min,brake}}, a^{\mathrm{lat}}_{\mathrm{max,accel}}, \mu$, if during the time interval $[0, \rho]$ the two cars will apply lateral acceleration of $a^{\mathrm{lat}}_{\mathrm{max,accel}}$ toward each other, and after that the two cars will apply lateral braking of $a^{\mathrm{lat}}_{\mathrm{min,brake}}$, until they reach zero lateral velocity, then the final lateral distance between them will be at least μ.*

A calculation of the lateral safe distance is given in the lemma below (whose proof is straightforward, and hence omitted).

Lemma 4 *Consider the notation given in Definition 7. W.l.o.g. assume that c_1 is to the left of c_2. Define $v_{1,\rho} = v_1 + \rho\, a^{\mathrm{lat}}_{\mathrm{max,accel}}$ and $v_{2,\rho} = v_2 - \rho\, a^{\mathrm{lat}}_{\mathrm{max,accel}}$. Then, the minimal safe lateral distance between the right side of c_1 and the left part of c_2 is:*

$$d_{\min} = \mu + \left[\frac{v_1 + v_{1,\rho}}{2}\rho + \frac{v_{1,\rho}^2}{2a^{\mathrm{lat}}_{\mathrm{min,brake}}} - \left(\frac{v_2 + v_{2,\rho}}{2}\rho - \frac{v_{2,\rho}^2}{2a^{\mathrm{lat}}_{\mathrm{min,brake}}} \right) \right]_+ .$$

As mentioned before, in order to have a collision between two cars, they must be both at a non-safe longitudinal distance and at a non-safe lateral distance. Intuitively, the idea of the safe distance definitions is that if both cars will respond "properly" to violations of safe distance then there cannot be a collision. If one of them didn't respond "properly" then it is responsible for the accident. To formalize this, we first refine the definitions of Dangerous Situation and Blame Time.

Definition 8 (Dangerous Situation and Blame Time) *We say that time t is* dangerous *for cars c_1, c_2 if both the longitudinal and lateral distances between them are non safe (according to Definition 1, Definition 2, and Definition 7). Given a dangerous time t, its* Blame Time*, denoted t_b, is the earliest non-dangerous time such that all the times in the interval $(t_b, t]$ are dangerous. In particular, an accident can only happen at time t if it is dangerous, and in that case we say that the blame time of the accident is the blame time of t.*

Next, we refine the definition of "proper response to dangerous situations".

Definition 9 (Proper response to dangerous situations) *Let t be a dangerous time for cars c_1, c_2 and let t_b be the corresponding blame time. The proper behaviour of the two cars is to comply with the following constraints on the lateral/longitudinal speed:*

1. *If before the blame time there was a safe longitudinal distance between c_1 and c_2 then the longitudinal speed is constrained according to Definition 4.*

2. *If before the blame time there was a safe lateral distance between c_1 and c_2, and w.l.o.g. assume that at that time c_1 was to the left of c_2:*

- *If $t \in [t_b, t_b + \rho)$ then both cars can do any lateral action as long as their lateral acceleration, a, satisfies $|a| \le a^{\mathrm{lat}}_{\mathrm{max,accel}}$.*
- *Else, if $t \ge t_b + \rho$:*
  - *Before reaching μ-lateral-velocity of 0, c_1 must apply lateral acceleration of at most $-a^{\mathrm{lat}}_{\mathrm{min,brake}}$ and c_2 must apply lateral acceleration of at least $a^{\mathrm{lat}}_{\mathrm{min,brake}}$*
  - *After reaching μ-lateral-velocity of 0, c_1 can have any non-positive μ-lateral-velocity and c_2 can have any non-negative μ-lateral-velocity*

3. Minimal evasive effort: *in addition to the rules above, we add the following constraints:*
   - *In case of item (1) above: Suppose that at the blame time the two cars are driving at the same direction and c_2 is the car on the front. Then, after time $t_b + \rho$ and until reaching a μ-lateral-velocity of 0, it should brake laterally by at least $a^{\mathrm{lat}}_{\mathrm{min,brake,evasive}}$. And, after that it should stay at μ-lateral-velocity of 0.*
   - *In case of item (2) above: Suppose that c_1 responds properly until reaching a μ-lateral-velocity of 0, and during that time c_2 did not respond properly and is now in front of c_1 (namely, all points of c_2 are longitudinally in front of all points of c_1, and some point of c_2 is at the same lateral position of some point of c_1). We call the first moment that this happens the* cut-in time*, denoted t_c. Then, if $t_b \le t_c \le t - \rho$ and currently c_1's velocity is positive, then it must brake longitudinally at a rate of at least $a_{\mathrm{min,brake,evasive}}$.*

In the above definition, all but the item titled "Minimal evasive effort" capture the essence of the assumptions in the definitions of safe distance, in the sense that if both cars respond properly then there will no collision. The item titled "Minimal evasive effort" deals with cases in which extra cautious is applied to prevent potential situations in which responsibility might be shared. The first case is when the rear car, c_1, should keep a safe distance from the front car, c_2, and c_2 is over cautious and avoid lateral manoeuvres so as to prevent collisions in case c_1 will not brake strong enough. The second case is when c_2 performs a cut-in, at a non-safe longitudinal distance, while c_1 was behaving properly. In this case, c_2 did not behave correctly. Nevertheless, we expect c_1 to make an effort to evase a potential collision, by applying braking of at least $a_{\mathrm{min,brake,evasive}}$. This ensures that dangerous situations cannot last long (unless both cars are at a zero longitudinal and lateral velocity).

Finally, the definition of RSS is straightforward.

Definition 10 (Responsibility for an Accident) *Consider an accident between c_1 and c_2 at time t and let t_b be the corresponding* Blame Time. *Car c_i is responsible for the accident if it did not respond properly according to Definition 9 at some time in $(t_b, t]$.*

Figure 2 illustrates the definitions.

Remark 2 *The parameters $a^{\mathrm{lat}}_{\mathrm{max,accel}}$, $a_{\mathrm{max,accel}}$, and $a_{\mathrm{max,brake}}$ do not necessarily reflect a physical limitation but instead they represent an upper bound on reasonable behavior we expect from road users. As the definitions imply, if a driver does not comply with these parameters at a dangerous time he immediately becomes responsible for the accident.*

Remark 3 *For simplicity, we assumed that cars can immediately switch from applying a "lateral braking" of $a^{\mathrm{lat}}_{\mathrm{min,brake}}$ to being at a μ-lateral-velocity of 0. This might not always be possible due to physical properties of the car. But, the important factor in the definition of proper response is that from time $t_b + \rho$ to the time the car reaches a μ-lateral-velocity of 0, the total lateral distance it will pass will not be larger than the one it would have passed had it applied a braking of $a^{\mathrm{lat}}_{\mathrm{min,brake}}$ until a full stop. Achieving this goal by a real vehicle is possible by first braking at a stronger rate and then decreasing lateral speed more gradually at the end. It is easy to see that this change will have no effect on the essence of RSS (as well as on the procedure for efficiently guaranteeing RSS safety that will be described in the next section).*

Remark 4 *The definitions hold for vehicles of arbitrary shapes, by taking the worst-case with respect to all points of each car. In particular, this covers semi-trailers or a car with an open door.*

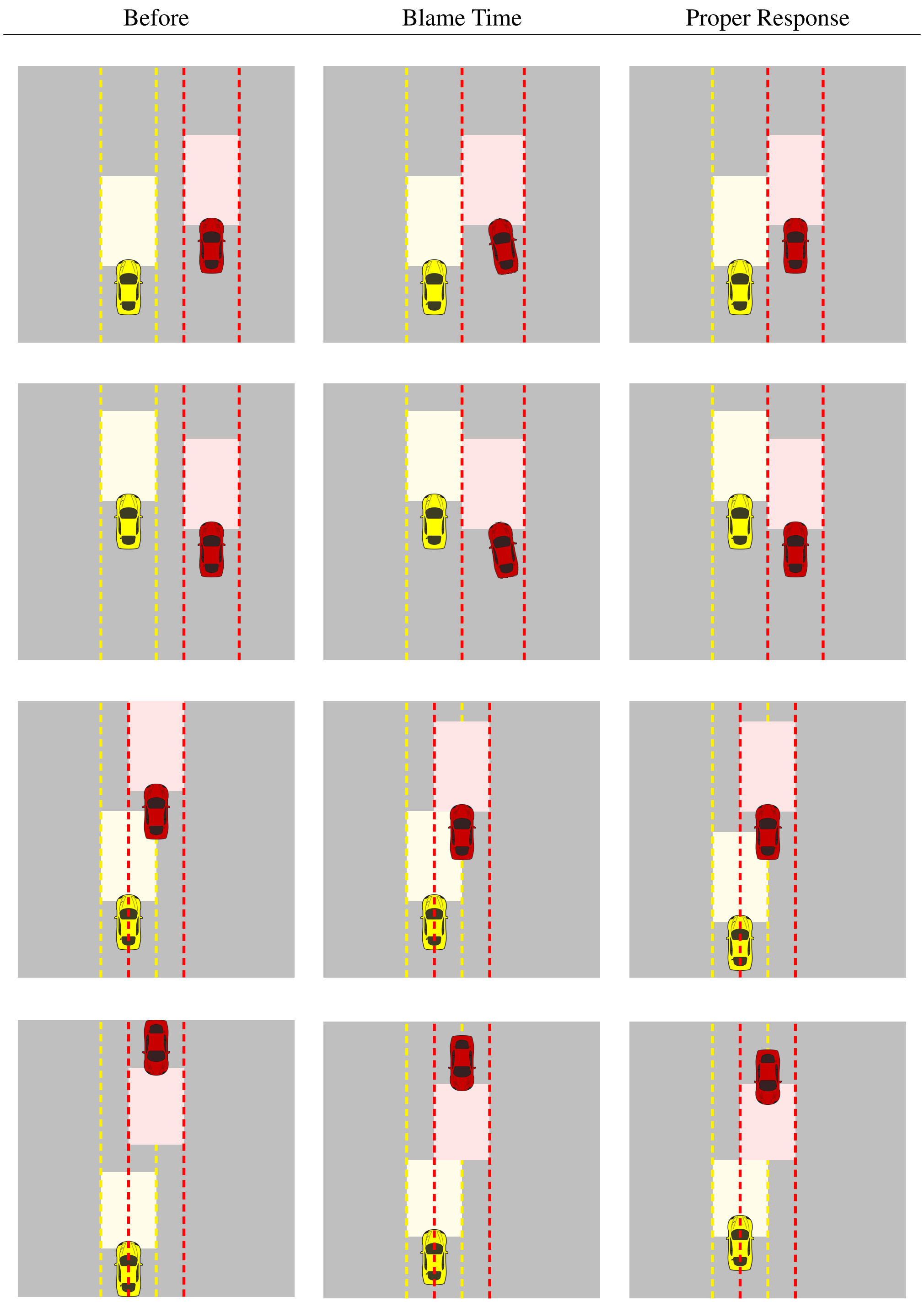
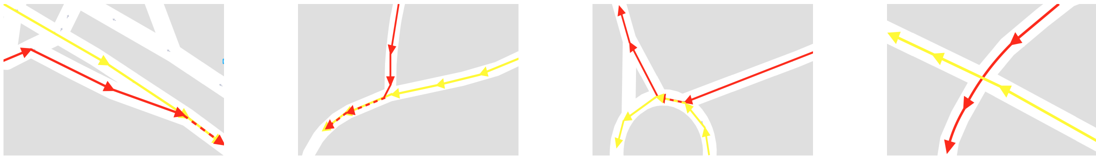


Figure 2: The vertical lines around each car show the possible lateral position of the car if it will accelerate laterally during the response time and then will brake laterally. Similarly, the rectangles show the possible longitudinal positions of the car (it it'll either brake by $a_{\max,\mathrm{brake}}$ or will accelerate during the response time and then will brake by $a_{\min,\mathrm{brake}}$). In the top two rows, before the blame time there was a safe lateral distance, hence the proper response is to brake laterally. The yellow car is already at μ-lateral-velocity of zero, hence only the red car brakes laterally. Third row: before the blame time there was a safe longitudinal distance, hence the proper response is for the car behind to brake longitudinally. Forth row: before the blame time there was a safe longitudinal distance, in an oncoming scenario, hence both car should brake longitudinally.



(a) (b) (c) (d)

Figure 3: Different examples for multiple routes scenarios. In yellow, the prioritized route. In red, the secondary route.

3.3 Multiple Geometry and Right-of-Way Rules

We next turn to deal with scenarios in which there are multiple different road geometries in one scene that overlap in a certain area. Examples include roundabouts, junctions, and merge into highways. See Figure 3 for illustration. In many such cases, one route has priority over others, and vehicles riding on it have the *right of way*.

In the previous section we could assume that the route is straight, by relying on Section B that shows how to construct a bijection between a general lane geometry and a straight road, with a coherent meaning for longitudinal and lateral axes. When facing scenarios of multiple route geometries, the definitions should be adjusted. To see this, consider for example the T-junction depicted on Figure 3b, and suppose that there is a stop sign for the red route. Suppose that c_1 is approaching the intersection on the yellow route and at the same time c_2 is approaching the intersection on the red route. According to the yellow route's coordinate system, c_2 has a very large lateral velocity, hence c_1 might deduce that c_2 is already at a non-safe lateral distance, which implies that c_1, driving on the prioritized route, must reduce speed in order to maintain a safe longitudinal distance to c_2. This means that c_2 should be very conservative w.r.t. traffic that coming from the red route. This is of course an unnatural behavior, as cars on the yellow route have the right-of-way in this case. Furthermore, even c_2, who doesn't have the priority, should be able to merge into the junction as long as c_1 can stop in time (this will be crucial in dense traffic). This example shows that when c_1 drives on r_1, it doesn't make sense to consider its position and velocity w.r.t the coordinate system of r_2. As a result, we need to generalize basic notions from the previous section such as "what does it mean that c_1 is in front of c_2", and what does it mean to be at a non safe distance.

Remark 5 *The definitions below assume that two cars, c_1, c_2 are driving on different routes, r_1, r_2. We emphasize that in some situations (for example, the T-junction given in Figure 3b), once there is exactly a single route r_1 such that both cars are assigned to it, and the time is not dangerous, then from that moment on, the definitions are solely w.r.t. r_1.*

We start with generalizing the definition of safe lateral distance. It is not hard to verify that applying the definition below to two routes of the same geometry indeed yields the same definition as in Definition 7. Throughout this section, we sometimes refer to a route as a subset of $\mathbb{R}^2$.

Definition 11 (Lateral Safe Distance for Two Routes of Different Geometry) *Consider vehicles c_1, c_2 driving on routes r_1, r_2 that intersect. For every $i \in \{1,2\}$, let $[x_{i,\min}, x_{i,\max}]$ be the minimal and maximal lateral positions in r_i that c_i can be in, if during the time interval $[0,\rho)$ it will apply a lateral acceleration (w.r.t. r_i) s.t. $|a^{\text{lat}}| \le a^{\text{lat}}_{\text{max,accel}}$, and after that it will apply a lateral braking of at least $a^{\text{lat}}_{\text{min,brake}}$ (again w.r.t. r_i), until reaching a zero lateral velocity (w.r.t. r_i). The lateral distance between c_1 and c_2 is safe if the restrictions[7] of r_1, r_2 to the lateral intervals $[x_{1,\min}, x_{1,\max}]$, $[x_{2,\min}, x_{2,\max}]$ are at a distance[8] of at least μ.*

Before we define longitudinal safe distance, we need to quantify ordering between cars when no common longitudinal axis exists.

Definition 12 (Longitudinal Ordering for Two Routes of Different Geometry) *Consider c_1, c_2 driving on routes r_1, r_2 that intersect. We say that c_1 is longitudinally in front of c_2 if either of the following holds:*

[7] The restriction of r_i to the lateral intervals $[x_{i,\min}, x_{i,\max}]$ is the subset of $\mathbb{R}^2$ obtained by all points $(x, y) \in r_i$ for which the semantic lateral position of (x, y) (as defined in Appendix B) is in the interval $[x_{i,\min}, x_{i,\max}]$.

[8] The distance between sets A, B is $\min\{\|a - b\| : a \in A, b \in B\}$.

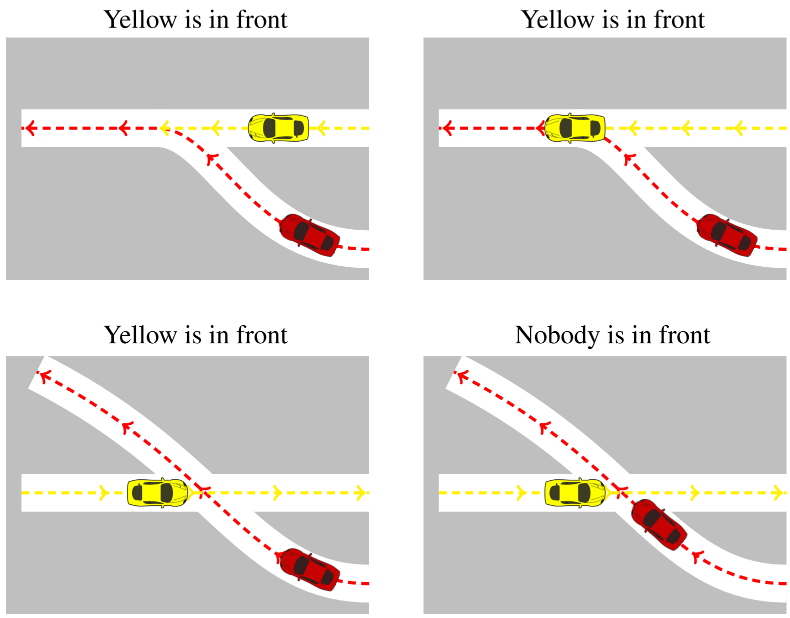


Figure 4: Illustration of Definition 12.

1. *For every i, if both vehicles are on r_i then c_1 is in front of c_2 according to r_i*

2. *c_1 is outside r_2 and c_2 is outside r_1, and the longitudinal distance from c_1 to the set $r_1 \cap r_2$, w.r.t. r_1, is larger than the longitudinal distance from c_2 to the set $r_1 \cap r_2$, w.r.t. r_2.*

Remark 6 *One may worry that the longitudinal ordering definition is not robust, for example, in item (2) of the definition, suppose that c_1, c_2 are at distances of $20, 20.1$ meters, respectively, from the intersection. This is not an issue as this definition is effectively being used only when there is a safe longitudinal distance between the two cars, and in that case the ordering between the cars will be obvious. Furthermore, this is exactly analogous to the non-robustness of ordering when two cars are driving side by side on a multi-lane highway road.*

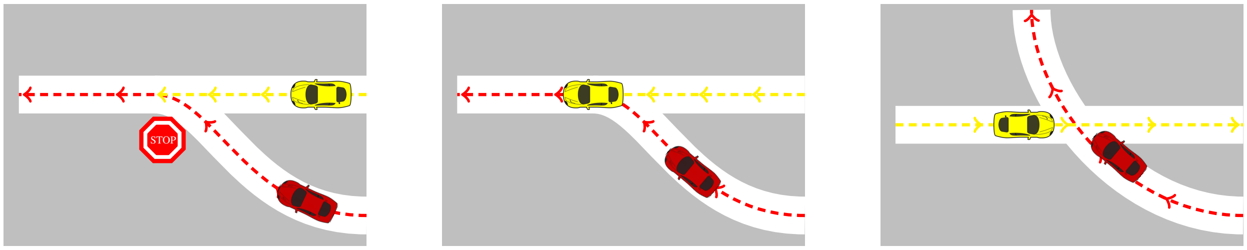
An illustration of the ordering definition is given in Figure 4.

Definition 13 (Longitudinal Safe Distance for Two Routes of Different Geometry) *Consider c_1, c_2 driving on routes r_1, r_2 that intersect. The longitudinal distance between c_1 and c_2 is safe if one of the following holds:*

1. *If for all $i \in \{1, 2\}$ s.t. r_i has no priority, if c_i will accelerate by $a_{\text{max,accel}}$ for ρ seconds, and will then brake by $a_{\text{min,brake}}$ until reaching zero longitudinal velocity (all w.r.t. r_i), then during this time c_i will remain outside of the other route.*

2. *Otherwise, if c_1 is in front of c_2 (according to Definition 12), then they are at a safe longitudinal distance if in case c_1 will brake by $a_{\text{max,brake}}$ until reaching a zero velocity (w.r.t. r_1), and c_2 will accelerate by at most $a_{\text{max,accel}}$ for ρ seconds and then will brake by at least $a_{\text{min,brake}}$ (w.r.t. r_2) until reaching a zero velocity, then c_1 will remain in front of c_2 (according to Definition 12).*

3. *Otherwise, consider a point $p \in r_1 \cap r_2$ s.t. for $i \in \{1, 2\}$, the lateral position of p w.r.t. r_i is in $[x_{i,\min}, x_{i,\max}]$ (as defined in Definition 11). Let $[t_{i,\min}, t_{i,\max}]$ be all times s.t. c_i can arrive to the longitudinal position of p w.r.t. r_i if it will apply longitudinal accelerations in the range $[-a_{\text{max,brake}}, a_{\text{max,accel}}]$ during the first ρ seconds, and then will apply longitudinal braking in the range $[a_{\text{min,brake}}, a_{\text{max,brake}}]$ until reaching a zero velocity. Then, the vehicles are at a safe longitudinal distance if for every such p we have that $[t_{1,\min}, t_{1,\max}]$ does not intersect $[t_{2,\min}, t_{2,\max}]$.*



(a) Safe because yellow has priority and red can stop before entering the intersection.

(b) Safe because yellow is in front of red, and if yellow will brake, red can brake as well and avoid a collision.

(c) If yellow is at a full stop and red is at a full lateral stop, safe by item (3) of Definition 13.

Figure 5: Illustration of safe longitudinal distance (Definition 13)

Illustrations of the definition is given in Figure 5.

Definition 14 (Dangerous & Blame Times, Proper Response, and Responsibility for Routes of Different Geometry) *Consider vehicles c_1, c_2 driving on routes r_1, r_2. Time t is dangerous if both the lateral and longitudinal distances are non-safe (according to Definition 11 and Definition 13). The corresponding blame time is the earliest non-dangerous time t_b s.t. all times in $(t_b, t]$ are dangerous. The proper response depends on the situation immediately before the blame time:*

- *If the lateral distance was safe, then both cars should respond according to the description of lateral safe distance in Definition 11.*
- *Else, if the longitudinal distance was safe according to item (1) in Definition 13, then if a vehicle is on the prioritized route it can drive normally, and otherwise it must brake by at least $a_{\mathrm{min,brake}}$ if $t - t_b \geq \rho$.*
- *Else, if the longitudinal distance was safe according to item (2) in Definition 13, then c_1 can drive normally and c_2 must brake by at least $a_{\mathrm{min,brake}}$ if $t - t_b \geq \rho$.*
- *Else, if the longitudinal distance was safe according to item (3) in Definition 13, then both cars can drive normally if $t - t_b < \rho$, and otherwise, both cars should brake laterally and longitudinally by at least $a^{\mathrm{lat}}_{\mathrm{min,brake}}, a_{\mathrm{min,brake}}$ (each one w.r.t. its own route).*

Finally, if a collision occur, then the responsibility is on the vehicle(s) that did not comply with the proper response.

Remark 7 *Note that there are cases where the route used by another agent is unknown: for example, see Figure 6. In such case, RSS is obtained by simply checking all possibilities.*

3.3.1 Traffic Lights

We next discuss intersections with traffic lights. One might think that the simple rule for traffic lights scenarios is “if one car’s route has the green light and the other car’s route has a red light, then the blame is on the one whose route has the red light”. However, this is not the correct rule. Consider for example the scenario depicted in Figure 7. Even if the yellow car’s route has a green light, we do not expect it to ignore the red car that is already in the intersection. The correct rule is that the route that has a green light have a priority over routes that have a red light. Therefore, we obtain a clear reduction from traffic lights to the route priority concept we have described previously. The above discussion is a formalism of the common sense rule of **right of way is given, not taken**.

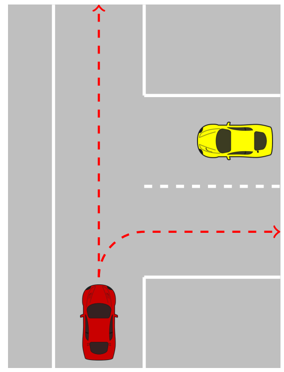


Figure 6: The yellow car cannot know for sure what is the route of the red one.

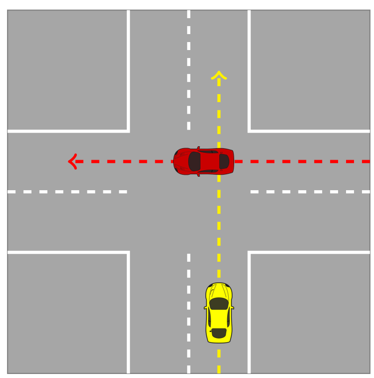
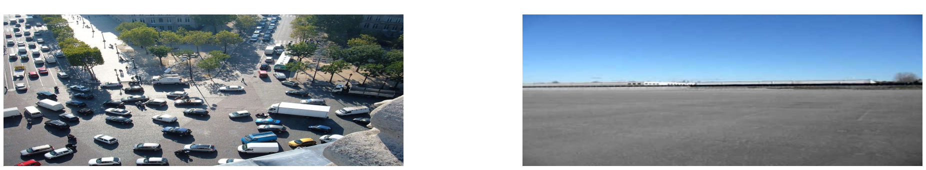


Figure 7: “Right of way is given, not taken”: The red car’s route has a red light and it is stuck in the intersection. Even though the yellow car’s route has a green light, since it has enough distance, it should brake so as to avoid an accident.



(a) (b)

Figure 8: Unstructured roads. (a) a wide roundabout around arc-de-triomphe. (b) a parking lot.

3.3.2 Unstructured Road

We next turn to consideration of unstructured roads, for example, see Figure 8. Consider first the scenario given in Figure 8a. Here, while the partition of the road area to lanes is not well defined, the partition of the road to multiple routes (with a clear geometry for every route) is well defined. Since our definitions of responsibility only depend on the route geometry, they apply as is to such scenarios.

Next, consider the scenario where there is no route geometry at all (e.g. the parking lot given in Figure 8b). Unlike the structured case, in which we separated the lateral and longitudinal directions, here we need two dimensional trajectories.

Definition 15 (Trajectories) *Consider a vehicle c riding on some road. A future trajectory of c is a function $\tau : \mathbb{R}_+ \to \mathbb{R}^2$, where $\tau(t)$ is the position of c in t seconds from the current time. The tangent vector to the trajectory at t, denoted $\tau'(t)$, is the Jacobian of τ at t. We denote $t_s(\tau) = \sup\{t : \forall t_1 \in [0, t),\ \|\tau'(t_1)\| > 0\}$, namely, t_s is the first time in which the vehicle will arrive to a full stop, where if no such t exists we set $t_s(\tau) = \infty$.*

Dangerous situations will depend on the possibility of a collision between two trajectories. This is formalized below.

Definition 16 (Trajectory Collision) *Let τ_1, τ_2 be two future trajectories of c_1, c_2, with corresponding stopping times $t_1 = t_s(\tau_1), t_2 = t_s(\tau_2)$. Given parameters ϵ, θ, we say that τ_1 and τ_2 do not collide, and denote it by $\tau_1 \cap \tau_2 = \emptyset$, if either of the following holds:*

1. *For every $t \in [0, \max(t_1, t_2)]$ we have that $\|\tau_1(t) - \tau_2(t)\| > \epsilon$.*
2. *For every $t \in [0, t_1]$ we have that $\|\tau_1(t) - \tau_2(t)\| > \epsilon$ and the absolute value of the angle between the vectors $(\tau_2(t_1) - \tau_1(t_1))$ and $\tau_2'(t_1)$ is at most θ.*

Given a set of trajectories for c_1, denoted $\mathcal{T}_1$, and a set of trajectories for c_2, denoted $\mathcal{T}_2$, we say that $\mathcal{T}_1 \cap \mathcal{T}_2 = \emptyset$ if for every $(\tau_1, \tau_2) \in \mathcal{T}_1 \times \mathcal{T}_2$ we have that $\tau_1 \cap \tau_2 = \emptyset$.

The first item states that both vehicles will be away from each other until they are both at a full stop. The second item states that the vehicles will be away from each other until the first one is at a full stop, and at that time, the velocity vector of the second one points away from the first vehicle.

Note that the collision operator we have defined is not commutative — think about two cars currently driving on a very large circle at the same direction, where c_1 is closely behind c_2, and consider τ_1 to be the trajectory in which c_1 brakes strongly and τ_2 is the trajectory in which c_2 continues at the same speed forever. Then, $\tau_1 \cap \tau_2 = \emptyset$ while $\tau_2 \cap \tau_1 \neq \emptyset$.

We continue with a generic approach, that relies on abstract notions of "braking" and "continue forward" behaviors. In the structured case the meanings of these behaviors were defined based on allowed intervals for lateral and longitudinal accelerations. We will later specify the meanings of these behaviors in the unstructured case, but for now we proceed with the definitions while relying on the abstract notions.

Definition 17 (Possible Trajectories due to Braking and Normal Driving) *Consider a vehicle c riding on some road. Given a set of constraints, C, on the behavior of the car, we denote by $\mathcal{T}(C, c)$ the set [9] of possible future trajectories of c if it will comply with the constraints given in C. Of particular interest are $\mathcal{T}(C_b, c), \mathcal{T}(C_f, c)$ representing the future trajectories due to constraints on braking behavior and constraints on continue forward behavior.*

We can now refine the notions of safe distance, dangerous situation, blame time, proper response, and responsibility.

Definition 18 (Safe Distance, Dangerous Situation, Blame Time, Proper Response, Responsibility) *The distance between c_0, c_1 driving on an unstructured road is safe if either of the following holds:*

1. *For some $i \in \{0, 1\}$ we have $\mathcal{T}(C_b, c_i) \cap \mathcal{T}(C_f, c_{1-i}) = \emptyset$ and $\mathcal{T}(C_b, c_{1-i}) \cap \mathcal{T}(C_f, c_i) \neq \emptyset$*

[9] A superset is also allowed. We will use supersets when it makes the calculation of the collision operator more easy.

2. $\mathcal{T}(C_b, c_0) \cap \mathcal{T}(C_b, c_1) = \emptyset$

We say that time t is dangerous w.r.t. c_0, c_1 if the distance between them is non safe. The corresponding blame time is the earliest non-dangerous time t_b s.t. during the entire time interval $(t_b, t]$ the situation was dangerous. The proper response of car c_j at a dangerous time t with corresponding blame time t_b is as follows:

- *If both cars were already at a full stop, then c_j can drive away from c_{1-j} (meaning that the absolute value of the angle between its velocity vector and the vector of the difference between c_j and c_{1-j} should be at most θ, where θ is as in Definition 16)*
- *Else, if t_b was safe due to item (1) above and $j = 1 - i$, then c_j should comply with the constraints of "continue forward" behavior, as in C_f, as long as c_{1-j} is not at a full stop, and after that it should behave as in the case that both cars are at a full stop.*
- *Otherwise, the proper response is to brake, namely, to comply with the constraints C_b.*

Finally, in case of a collision, the responsibility is on the vehicle(s) that did not respond properly.

Finally, to make this generic approach concrete, we need to specify the braking constraints, C_b, and the continue forward behavior constraints, C_f. Recall that there are two main things that a clear structure gives us. Firstly, vehicles can predict what other vehicles will do (other vehicles are supposed to drive on their route, and change lateral/longitudinal speed at a bounded rate). Secondly, when a vehicle is at a dangerous time, the proper response is defined w.r.t. the geometry of the route ("brake" laterally and longitudinally). It is very important that the proper response is not defined w.r.t. the other vehicle from which we are at a non-safe distance, because had this been the case, we could have conflicts when a vehicle were at a non-safe distance w.r.t. more than a single other vehicle. Therefore, when designing the definitions for unstructured scenarios, we must make sure that the aforementioned two properties will still hold.

The approach we take relies on a basic kinematic model of vehicles. For the speed, we take the same approach as we have taken for longitudinal velocity (bounding the range of allowed accelerations). For lateral movements, observe that when a vehicle maintains a constant angle of the steering wheel and a constant speed, it will move (approximately) on a circle. In other words, the heading angle of the car will change at a constant rate, which is called the yaw rate of the car. We denote the speed of the car by $v(t)$, the heading angle by $h(t)$, and the yaw rate by $h'(t)$ (as it is the derivative of the heading angle). When $h'(t)$ and $v(t)$ are constants, the car moves on a circle whose "radius" is $v(t)/h'(t)$ (where the sign of the "radius" determines clockwise or counter clockwise and the "radius" is ∞ if the car moves on a line, i.e. $h'(t) = 0$). We therefore denote $r(t) = v(t)/h'(t)$. We will make two constraints on normal driving. The first is that the inverse of the radius changes at a bounded rate The second is that $h'(t)$ is bounded as well. The expected braking behavior would be to change $h'(t)$ and $1/r(t)$ in a bounded manner during the response time, and from there on continue to drive on a circle (or at least be at a distance of at most $\epsilon/2$ from the circle). This behavior forms the analogue of accelerating by at most $a^{\text{lat}}_{\text{max,accel}}$ during the response time and then decelerating until reaching a lateral velocity of zero.

To make efficient calculations of the safe distance, we construct the superset $\mathcal{T}(C_b, c)$ as follows. W.l.o.g., lets call the blame time to be $t = 0$, and assume that at the blame time the heading of c is zero. By the constraint of $|h'(t)| \le h'_{\max}$ we know that $|h(\rho)| \le \rho\, h'_{\max}$. In addition, the inverse of the radius at time ρ must satisfy

$$\frac{1}{r(0)} - \rho\, r^{-1'}_{\max} \le \frac{1}{r(\rho)} \le \frac{1}{r(0)} + \rho\, r^{-1'}_{\max}\,, \tag{1}$$

where $r(0) = v(0)/h'(0)$. All in all, we define the superset $\mathcal{T}(C_b, c)$ to be all trajectories such that the initial heading (at time 0) is in the range $[-\rho\, h'_{\max}, \rho\, h'_{\max}]$, the trajectory is always on a circle whose inverse radius is according to (1), and the longitudinal velocity on the circle is as in the structured case. For the continue forward trajectories, we perform the same except that the allowed longitudinal acceleration even after the response time is in $[-a_{\text{max,brake}}, a_{\text{max,accel}}]$. An illustration of the extreme radiuses is given in Figure 9.

Finally, observe that these proper responses satisfy the aforementioned two properties of the proper response for structured scenarios: it is possible to bound the future positions of other vehicles in case an emergency will occur, and the same proper response can be applied even if we are at a dangerous situation w.r.t. more than a single other vehicle.

Figure 9: Illustration of the lateral behavior in unstructured scenes. The black line is the current trajectory. The blue and red lines are the extreme arcs.

3.4 Pedestrians

The rules for assigning responsibility for collisions involving pedestrians (or other road users) follow the same ideas described in previous subsections, except that we need to adjust the parameters in the definitions of safe distance and proper response, as well as to specify pedestrians' routes (possibly unstructured routes) and their priority w.r.t. vehicles' routes. In some cases, a pedestrian's route is well defined (e.g. a zebra crossing or a sidewalk on a fast road). In other cases, like a typical residential street, we follow the approach we have taken for unstructured roads except that unlike vehicles that typically ride on circles, for pedestrians we constrain the change of heading, $|h'(t)|$, and assume that at emergency, after the response time, the pedestrian will continue at a straight line. If the pedestrian is standing, we assign it to all possible lines originating from his current position. The priority is set according to the type of the road and possibly based on traffic lights. For example, in a typical residential street, a pedestrian has the priority over the vehicles, and it follows that vehicles must yield and be cautious with respect to pedestrians. In contrast, there are roads with a sidewalk where the common sense behavior is that vehicles should not be worried that a pedestrian on the sidewalk will suddenly start running into the road. There, cars have the priority. Another example is a zebra crossing with a traffic light, where the priority is set dynamically according to the light. Of course, priority is given not taken, hence even if pedestrians do not have priority, if they entered the road at a safe distance, cars must brake and let them pass.

Let us illustrate the idea by some examples. The first example is a pedestrian that stands on a residential road. The pedestrian is assigned to all routes obtained by rays originating from its current position. Her safe longitudinal distance w.r.t. each of these virtual routes is quite short. For example, setting a delay of 500 ms, and maximal acceleration[10] and braking of 2 m/s^2, yields that her part of the safe longitudinal distance is $50cm$. It follows that a vehicle must be in a kinematic state such that if it will apply a proper response (acceleration for ρ seconds and then braking) it will remain outside of a ball of radius $50cm$ around the pedestrian.

A second example is a pedestrian standing on the sidewalk right in front of a zebra crossing, the pedestrian has a red light, and a vehicle approaches the zebra crossing at a green light. Here, the vehicle route has the priority, hence the vehicle can assume that the pedestrian will stay on the sidewalk. If the pedestrian enters the road while the vehicle is at a safe longitudinal distance (w.r.t. the vehicle's route), then the vehicle must brake ("right of way is given, not taken"). However, if the pedestrian enters the road while the vehicle is not at a safe longitudinal distance, and as a result the vehicle hits the pedestrian, then the vehicle is not responsible. It follows that in this situation, the vehicle can drive at a normal speed, without worrying about the pedestrian.

The third example is a pedestrian that runs on a residential road at 10 km per hour (which is $\approx 2.7m/s$). The possible future trajectories of the pedestrian form an isosceles triangle shape. Using the same parmeters as in the first example, the height of this triangle is roughly $15m$. It follows that cars should not enter the pedestrian's route at a distance smaller than $15m$. But, if the car entered the pedestrian's route at a distance larger than $15m$, and the pedestrian didn't stop and crashed into the car, then the responsibility is of the pedestrian.

In the next subsection we deal with occlusions, and in particular with occluded pedestrians.

[10] As mentioned in [4], the estimated acceleration of Usain Bolt is 3.09 m/s^2.

3.5 Cautiousness with respect to Occlusion

A very common human response, when blamed for an accident, falls into the "but I couldn't see him" category. It is, many times, true. Human sensing capabilities are limited, sometimes because of an unaware decision to focus on a different part of the road, sometimes because of carelessness, and sometimes because of physical limitations - it is impossible to see a little kid hidden behind a parked car. While advanced automatic sensing systems are never careless, and have a $360°$ view of the road, they might still suffer from limited sensing due to physical occlusions or range of sensor detection. Few examples are:

- A kid that might be occluded behind a parked car
- Junctions in which a building or a fence occlude traffic that approach the junction from a different route
- When changing lanes on a highway, there is a limited view range for detecting cars that arrive from behind (especially when weather is bad)

Returning to our "but I couldn't see him" argument, a counter argument is often "well, you should've been more careful". Let us formalize what does it mean to be careful. To motivate the definition, consider the scenario depicted in Figure 10, where c_0 is trying to exit a parking lot, merging into a route, but cannot see whether there are cars approaching the merge point from the left side of the street. Let us assume that this is an urban, narrow street, with a speed limit of 30 km/h. A human driver's behaviour is to slowly merge onto the road, obtaining more and more field of view, until sensing limitations are eliminated. Observe that without making additional assumptions, this behaviour might lead to collision. Indeed, if c_1 is driving very fast, then at the moment that it will enter the view range of c_0, c_1 might reveal that the situation is already dangerous for more than ρ seconds and it didn't respond properly. What assumptions does the human driver make, which allows this manoeuvre?

Before we answer this question, another significant moment in time should be defined — the first time the occluded object is exposed to us; after its exposure, we can deal with it just like any other object we can sense.

Definition 19 (Exposure Time) *The* Exposure Time *of an object is the first time in which we see it.*

We can now formalize the "common sense" of human drivers when performing such manoeuvres by setting an upper bound on the reasonable speed of road users. In the previously described example, it is reasonable to assume that c_1 will not drive faster than, say, 60 km/h (twice the allowed speed). With this assumption in hand, c_0 can approach the merge point slow enough so as to make sure that it will not enter a dangerous situation w.r.t. any car whose speed is at most 60 km/h. This leads to the following definition.

Definition 20 (Responsibility due to Unreasonable Speed) *Consider an accident between c_0, c_1, where before the blame time there was a safe lateral distance, and where c_1's was driving unreasonably fast, namely, its average velocity from the exposure time until the blame time was larger than v_{limit}, where v_{limit} is a parameter associated with the position of c_1 on the map and possibly on other road conditions. Then, the responsibility is solely on c_1.*

This extension allows c_0 to exit the parking lot safely, in the same manner a human driver does. Based on the above definition, it suffices for c_0 to check the situation where there is a vehicle at the edge of the occluded area whose velocity is v_{limit}. Intuitively, this encourages c_0 to drive slower and further from the occluder, thus slowly increasing its field of view and later allowing for safe merging into the street.

This responsibility definition holds for a variety of cases by specifying **what** can be occluded (a potentially fast car cannot be occluded between two closely parked cars, but a kid can), and what is unreasonably fast (a kid's v_{limit} is obviously much smaller than that of a car).

3.5.1 Occluded pedestrians in a residential area

Or particular interest is the case of a kid running from behind parked car. If the road is at a residential area, the kid's route has priority, hence a vehicle should adjust speed such that if a kid will emerge behind a parked car then there will be no accident. By limiting the maximal speed of the kid, the vehicle can adjust speed according to the worst-case situation in which the kid will be exactly at the maximal speed.

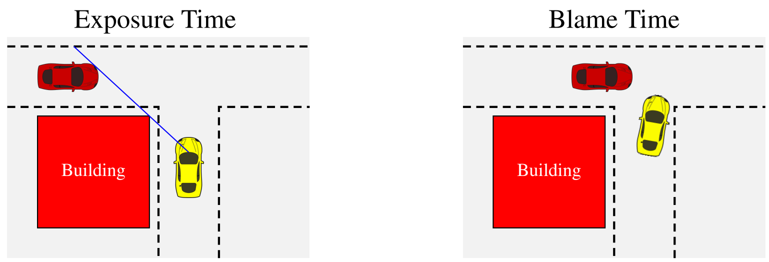


Figure 10: Illustration of the exposure time and the blame time.

However, this yields a too defensive behavior. Indeed, consider a typical scenario when a vehicle is driving next to a sequence of parking cars. A little kid, that runs into the road (e.g. chasing a ball) at a speed of $10km/h$, might be currently occluded by a parked car, where the vehicles' camera will sense it only when the longitudinal distance beween the vehicle and the pedestrian is smaller than $0.3m$. The longitudinal safe distance in the kid's route is around $15m$, which is typically more than the width of the residential road. Therefore, the vehicle is already at a non-safe longitudinal distance (w.r.t. the kid's route), and it therefore must be able to stop before entering the kid's route. However, the braking distance of a car driving at $1m/s$ (with reasonable setting of parameters) is more than $0.4m$. It follows that we should drive slower than $1m/s$ in this scenario, even if we drive at a lateral distance of, say, $5m$, from the parking cars. This is too defensive and does not reflect normal human behavior. What assumptions does the human driver make, which allows driving faster?

The first justification a human driver is making relies on the law — driving slower than the speed limit often feels safe. While autonomous vehicles will obviously follow the speed limit, we seek for stronger notions of safety. Below we propose a responsibility definition for this scenario, that leads to a safer driving while enabling reasonably fast driving.

Definition 21 (Responsibility for Collision with Occluded Pedestrian) *Consider a collision between a pedestrian and a vehicle at a residential road. The vehicle is not responsible for the accident if the following two conditions hold:*

- *Let t_e be the exposure time. The vehicle did not accelerate at the time interval $[t_e, t_e + \rho)$, and performed a longitudinal brake of at least $a_{\min,\text{brake}}$ from $t_e + \rho$ until the accident or until arriving to a full stop.*
- *The averaged velocity of the vehicle from the exposure time until the collision time was smaller than the averaged velocity of the pedestrian at the same time interval (where averaged velocity is the total distance an agent moved through a time interval divided by the length of the time interval).*

The first condition tells us that the velocity of the vehicle is monotonically non-increasing from the exposure time until the collision time, and the second condition tells us that the averaged velocity of the vehicle is smaller than that of the pedestrian. As implied by the proof of the lemma below, under some reasonable assumptions on the pedestrian maximal acceleration, the responsibility definition tells us that at the collision time, we expect that either the velocity of the car will be significantly smaller than that of the pedestrian, or that both the vehicle and the pedestrian will be at a very slow speed. In the latter case, the damage shouldn't be severe and in the former case, it complies with "common sense" that the pedestrian is to be blamed for the accident.

The lemma below shows the allowed driving speed that guarantees to never be responsible for a collision with an occluded pedestrian.

Lemma 5 *Suppose that a vehicle located at a lateral distance of x and longitudinal distance of y from an occluded* *spot. Then, if its lateral speed is zero and its longitudinal speed is smaller than*

$$v_{\max}(x,y) = a_{\min,\text{brake}} \left(-\rho + \sqrt{\rho^2 + \frac{x^2+y^2}{y\, a_{\min,\text{brake}}}} \right)$$

it can guarantee to not make an accident of the vehicle's responsibility. In particular, a constant driving speed of

$$v_{\max}(x) = \min_{y>0} v_{\max}(x,y) = a_{\min,\text{brake}} \left(-\rho + \sqrt{\rho^2 + \frac{2x}{a_{\min,\text{brake}}}} \right)$$

is possible.

Proof Suppose that at the exposure time, the pedestrian is located at position $(0,0)$ and the car is located at position $(x,-y)$. Suppose the collision is at the point (x, y_c) and let (x, y_s) be the point in which the car will come to a full stop (if there will be no collision). Define by t_c the collision time and by t_s the time the vehicle will reach a full stop. Note that if there is a collision of the vehicle's blame at point (x, y_c), then we must have

$$x^2 + y_c^2 \le (y_c + y)^2 \;\;\Rightarrow\;\; x^2 \le y^2 + 2y_c y \;\;\Rightarrow\;\; \frac{x^2 - y^2}{2y} \le y_c$$

But, since $y_s \ge y_c$, this also implies that

$$y_s \ge \frac{x^2-y^2}{2y} \;\;\Rightarrow\;\; x^2 + y_s^2 \le (y_s + y)^2 \;,$$

which implies that there can be a collision of the vehicle's blame at the braking time. So, to make sure that there is no collision of the vehicle's blame, we only need to verify that there is no such collision at the braking time, that is, that the following condition holds: $y_s < \frac{x^2-y^2}{2y}$. Since $y_s = -y + v\rho + 0.5\, v^2/a_{\min,\text{brake}}$, we obtain, after rearrangements,

$$\frac{v^2}{2a_{\min,\text{brake}}} + v\rho - \frac{x^2+y^2}{2y} < 0$$

This condition holds for the interval $v \in [0, v_{\max}(x,y)]$ where $v_{\max}(x,y)$ is the right root of the quadratic equation, namely,

$$v_{\max}(x,y) = a_{\min,\text{brake}} \left(-\rho + \sqrt{\rho^2 + \frac{x^2+y^2}{y\, a_{\min,\text{brake}}}} \right)$$

Since $f(y) = (x^2+y^2)/y$ is minimized at $y = x$ we obtain that the maximal constant speed we can ride at is

$$v_{\max}(x) = a_{\min,\text{brake}} \left(-\rho + \sqrt{\rho^2 + \frac{2x}{a_{\min,\text{brake}}}} \right)$$

■

3.6 Utopia is Possible

To wrap up the discussion, consider a utopic future, where all cars (and other road users) somehow are able to successfully verify that they will never be responsible for an accident. Since by definition, for every accident there is at least one car to blame, the meaning is that *there will never be any accidents*, leading us to a utopic future of absolute safety.

4 Efficiently Validated Conditions for Responsibility-Sensitive Safety

In Section 3, we have completely ignored the implementation aspect of RSS. Of course, those definitions will be useless if there's no efficient way of successfully verifying that we will never be responsbile for an accident. To appreciate the difficulty, an action that is performed now may have a butterfly effect, that will lead to a chain of events with an accident after 10 minutes of driving. A "brute-force" approach that will check all possible future outcomes is, of course, impossible. It is therefore crucial to accompany the responsibility-sensitive safety definitions with computationally efficient methods to validate them.

4.1 Computationally Feasible Safety Verification

The main mathematical tool for computationally feasible verification is "induction". To prove a claim by induction, one begins with proving the claim for simple cases, and then, each induction step extends the proof to more and more involved cases. Since we have constructed the responsibility based on the notion of proper response, we can easily make the inductive argument as follows. For simplicity, we ignore the case of occlusions as well as unstructured roads. The extension to these cases is tedious but straightforward.

Theorem 1 *Consider a vehicle c whose policy complies with the following constraints:*

- *The policy can react to the environment in less that ρ seconds*
- *The policy never accelerates longitudinally by more than $a_{\mathrm{max,accel}}$, never decelerates longitudinally by more than $a_{\mathrm{max,brake}}$, and never drives faster than v_{limit}.*
- *The policy never accelerates laterally by more than $a^{\mathrm{lat}}_{\mathrm{max,accel}}$ and in unstructured scenarios, it never changes the yaw rate by more than $h''_{\mathrm{max,accel}}$*
- *If the situation at time t is dangerous w.r.t. some other road user, then the policy adheres to the constraints of the proper response (as in Definition 9 or Definition 14).*

Then c is guaranteed to never be responsible for accidents.

Proof The proof is by induction. For the induction base we start with an initial state which is either not dangerous or when we are at a complete stop. The induction step is as follows. If the next time step is still a non-dangerous time then we are done. Otherwise, let t_b be the blame time. Since the situation at t_b was not dangerous, and since by the constraints on the policy we were performing the proper response during $[t_b, t]$, hence, either there is no collision, or it is not of our responsibility. Finally, observe that even if we are at a dangerous time w.r.t. several cars, the constraints on the policy can always be satisfied (by braking both laterally and longitudinally). This concludes our proof. ■

The main benefits of our inductive approach are that there is no need to check infinite future, and that it suffices to check the situation w.r.t. each car individually and take the intersection of all the resulting constraints.

4.2 Usefulness

We have shown that it is possible to efficiently comply with the RSS definitions. However, one may wonder if a policy that guarantees to never be responsible for an accident according to RSS will necessary lead to an extremely defensive driving. This is not the case. To see this, the reader can assign reasonable values to the parameters of RSS and see that normal traffic manoeuvres are far from violating the safe distance definitions. This can be further illustrated empirically by managing to drive in complex scenario in a natural way while adhering to RSS constraints. See for example the "double-merge" scenario as described in [6].

5 Driving Policy

A driving policy is a mapping from a sensing state (a description of the world around us) into a driving command (e.g., the command is lateral and longitudinal accelerations for the coming second, which determines where and at what speed should the car be in one second from now). The driving command is passed to a controller, that aims at actually moving the car to the desired position/speed.

In the previous sections we described a formal safety model and proposed constraints on the commands issued by the driving policy that guarantee safety. The constraints on safety are designed for extreme cases. Typically, we do not want to even need these constraints, and would like to construct a driving policy that leads to a comfortable ride. The focus of this section is on how to build an efficient driving policy, in particular, one that requires computational resources that can scale to millions of cars. For now, we ignore the issue of how to obtain the sensing state and assume an utopic sensing state, that faithfully represents the world around us without any limitations. In later sections we will discuss the effect of inaccuracies in the sensing state on the driving policy.

We can cast the problem of defining a driving policy in the language of Reinforcement Learning (RL). At each iteration of RL, an agent observes a state describing the world, denoted s_t, and should pick an action, denoted a_t, based on a policy function, π, that maps states into actions. As a result of its action and other factors out of its control (such as the actions of other agents), the state of the world is changed to s_{t+1}. We denote a (state,action) sequence by $\bar{s} = ((s_1, a_1), (s_2, a_2), \ldots, (s_{\text{len}(\bar{s})}, a_{\text{len}(\bar{s})}))$. Every policy induces a probability function over (state,action) sequences. This probability function is affected by the actions taken by the agent, but also depends on the environment (and in particular, on how other agents behave). We denote by P_π the probability over (state,action) sequences induced by π. The quality of a policy is defined to be $\mathbb{E}_{\bar{s}\sim P_\pi}[\rho(\bar{s})]$, where $\rho(\bar{s})$ is a reward function that measures how good the sequence $\bar{s}$ is. In most case, $\rho(\bar{s})$ takes the form $\rho(\bar{s}) = \sum_{t=1}^{\text{len}(\bar{s})} \rho(s_t, a_t)$, where $\rho(s, a)$ is an instantaneous reward function, that measures the immediate quality of being at state s and performing action a. For simplicity, we stick to this simpler case.

To cast the driving policy problem in the above RL language, let s_t be some representation of the road, and the positions, velocities, and accelerations, of the ego vehicle as well as other road users. Let a_t be a lateral and longitudinal acceleration command. The next state, s_{t+1}, depends on a_t as well as on how the other agents will behave. The instantaneous reward, $\rho(s_t, a_t)$, may depend on the relative position/velocities/acceleration to other cars, the difference between our speed and the desired speed, whether we follow the desired route, whether our acceleration is comfortable etc.

The main difficulty of deciding what action should the policy take at time t stems from the fact that one needs to estimate the long term effect of this action on the reward. For example, in the context of driving policy, an action that is taken at time t may seem a good action for the present (that is, the reward value $\rho(s_t, a_t)$ is good), but might lead to an accident after 5 seconds (that is, the reward value in 5 seconds would be catastrophic). We therefore need to estimate the long term quality of performing an action a when the agent is at state s. This is often called the Q function, namely, $Q(s, a)$ should reflect the long term quality of performing action a at state s. Given such a Q function, the natural choice of an action is to pick the one with highest quality, $\pi(s) = \text{argmax}_a Q(s, a)$.

The immediate questions are how to define Q and how to evaluate Q efficiently. Let us first make the (completely non-realistic) simplifying assumption that s_{t+1} is some deterministic function of (s_t, a_t), namely, $s_{t+1} = f(s_t, a_t)$. The reader familiar with Markov Decision Processes (MDPs), will quickly notice that this assumption is even stronger than the Markovian assumption of MDPs (i.e., that s_{t+1} is conditionally independent of the past given (s_t, a_t)). As noted in [6], even the Markovian assumption is not adequate for multi-agent scenarios, such as driving, and we will therefore later relax the assumption.

Under this simplifying assumption, given s_t, for every sequence of decisions for T steps, $(a_t, \ldots, a_{t+T})$, we can calculate exactly the future states $(s_{t+1}, \ldots, s_{t+T+1})$ as well as the reward values for times $t, \ldots, T$. Summarizing all these reward values into a single number, e.g. by taking their sum $\sum_{\tau=t}^{T} \rho(s_\tau, a_\tau)$, we can define $Q(s, a)$ as follows:

$$Q(s, a) = \max_{(a_t, \ldots, a_{t+T})} \sum_{\tau=t}^{T} \rho(s_\tau, a_\tau) \quad \text{s.t.} \quad s_t = s,\ a_t = a,\ \forall \tau,\ s_{\tau+1} = f(s_\tau, a_\tau)$$

That is, $Q(s, a)$ is the best future we can hope for, if we are currently at state s and immediately perform action a.

Let us discuss how to calculate Q. The first idea is to discretize the set of possible actions, A, into a finite set $\hat{A}$, and simply traverse all action sequences in the discretized set. Then, the runtime is dominated by the number of discrete action sequences, $|\hat{A}|^T$. If $\hat{A}$ represents 10 lateral accelerations and 10 longitudinal accelerations, we obtain 100^T possibilities, which becomes infeasible even for small values of T. While there are heuristics for speeding up the search (e.g. coarse-to-fine search), this brute-force approach requires tremendous computational power.

The parameter T is often called the "time horizon of planning", and it controls a natural tradeoff between computation time and quality of evaluation — the larger T is, the better our evaluation of the current action (since we explicitly examine its effect deeper into the future), but on the other hand, a larger T increases the computation time exponentially. To understand why we may need a large value of T, consider a scenario in which we are 200 meters before a highway exit and we should take it. When the time horizon is long enough, the cumulative reward will indicate if at some time τ between t and $t + T$ we have arrived to the exit lane. On the other hand, for a short time horizon, even if we perform the right immediate action we will not know if it will lead us eventually to the exit lane.

A different approach attempts to perform offline calculations in order to construct an approximation of Q, denoted $\hat{Q}$, and then during the online run of the policy, use $\hat{Q}$ as an approximation to Q, without explicitly rolling out the future. One way to construct such an approximation is to discretize both the action domain and the state domain. Denote by $\hat{A}, \hat{S}$ these discretized sets. We can perform an offline calculation for evaluating the value of $Q(s, a)$ for every $(s, a) \in \hat{S} \times \hat{A}$. Then, for every $a \in \hat{A}$ we define $\hat{Q}(s_t, a)$ to be $Q(s, a)$ for $s = \mathrm{argmin}_{s \in \hat{S}} \|s - s_t\|$. Furthermore, based on the pioneering work of Bellman [2, 3], we can calculate $Q(s, a)$ for every $(s, a) \in \hat{S} \times \hat{A}$, based on dynamic programming procedures (such as the Value Iteration algorithm), and under our assumptions, the total runtime is order of $T\,|\hat{A}|\,|\hat{S}|$. The main problem with this approach is that in any reasonable approximation, $\hat{S}$ is extremely large (due to the curse of dimensionality). Indeed, the sensing state should represent 6 parameters for every other relevant vehicle in the sense — the longitudinal and lateral position, velocity, and acceleration. Even if we discretize each dimension to only 10 values (a very crude discretization), since we have 6 dimensions, to describe a single car we need 10^6 states, and to describe k cars we need 10^{6k} states. This leads to unrealistic memory requirements for storing the values of Q for every (s, a) in $\hat{S} \times \hat{A}$.

A popular approach to deal with this curse of dimensionality is to restrict Q to come from a restricted class of functions (often called a hypothesis class), such as linear functions over manually determined features or deep neural networks. For example, [5] learned a deep neural network that approximates Q in the context of playing Atari games. This leads to a resource-efficient solution, provided that the class of functions that approximate Q can be evaluated efficiently. However, there are several disadvantages of this approach. First, it is not known if the chosen class of functions contain a good approximation to the desired Q function. Second, even if such function exists, it is not known if existing algorithms will manage to learn it efficiently. So far, there are not many success stories for learning a Q function for complicated multi-agent problems, such as the ones we are facing in driving. There are several theoretical reasons why this task is difficult. We have already mentioned that the Markovian assumption, underlying existing methods, is problematic. But, a more severe problem is that we are facing a very small signal-to-noise ratio due to the time resolution of decision making, as we explain below.

Consider a simple scenario in which we need to change lane in order to take a highway exit in 200 meters and the road is currently empty. The best decision is to start making the lane change. We are making decisions every 0.1 second, so at the current time t, the best value of $Q(s_t, a)$ should be for the action a corresponding to a small lateral acceleration to the right. Consider the action a' that corresponds to zero lateral acceleration. Since there is a very little difference between starting the change lane now, or in 0.1 seconds, the values of $Q(s_t, a)$ and $Q(s_t, a')$ are almost the same. In other words, there is very little *advantage* for picking a over a'. On the other hand, since we are using a function approximation for Q, and since there is noise in measuring the state s_t, it is likely that our approximation to the Q value is noisy. This yields a very small signal-to-noise ratio, which leads to an extremely slow learning, especially for stochastic learning algorithms which are heavily used for the neural networks approximation class. However, as noted in [1], this problem is not a property of any particular function approximation class, but rather, it is inherent in the definition of the Q function.

In summary, existing approaches can be roughly divided into two camps. The first one is the brute-force approach which includes searching over many sequences of actions or discretizing the sensing state domain and maintaining a huge table in memory. This approach can lead to a very accurate approximation of Q but requires unleashed resources, either in terms of computation time or in terms of memory. The second one is a resource efficient approach in which we either search for short sequences of actions or we apply a function approximation to Q. In both cases, we pay by having a less accurate approximation of Q, that might lead to poor decisions.

Our approach to constructing a Q function that is both resource-efficient and accurate is to depart from geometrical actions and to adapt a semantic action space, as described in the next subsection.

5.1 Semantics to the rescue

To motivate our semantic approach, consider a teenager that just got his driving license. His father seats next to him and gives him "driving policy" instructions. These instructions are not geometric — they do not take the form "drive 13.7 meters at the current speed and then accelerate at a rate of 0.8 m/s^2". Instead, the instructions are of semantic nature — "follow the car in front of you" or "quickly overtake that car on your left". We formalize a semantic language for such instructions, and use them as a semantic action space. We then define the Q function over the semantic action space. We show that a semantic action can have a very long time horizon, which allows us to estimate $Q(s,a)$ without planning for many future semantic actions. Yet, the total number of semantic actions is still small. This allows us to obtain an accurate estimation of the Q function while still being resource efficient. Furthermore, as we show later, we combine learning techniques for further improving the quality function, while not suffering from a small signal-to-noise ratio due to a significant difference between different semantic actions.

We now define our semantic action space. The main idea is to define lateral and longitudinal goals, as well as the aggressiveness level of achieving them. Lateral goals are desired positions in lane coordinate system (e.g., "my goal is to be in the center of lane number 2"). Longitudinal goals are of three types. The first is relative position and speed w.r.t. other vehicles (e.g., "my goal is to be behind car number 3, at its same speed, and at a distance of 2 seconds from it"). The second is a speed target (e.g., "drive at the allowed speed for this road times 110%"). The third is a speed constraint at a certain position (e.g., when approaching a junction, "speed of 0 at the stop line", or when passing a sharp curve, "speed of at most 60kmh at a certain position on the curve"). For the third option we can instead apply a "speed profile" (few discrete points on the route and the desired speed at each of them). A reasonable number of lateral goals is bounded by $16 = 4 \times 4$ (4 positions in at most 4 relevant lanes). A reasonable number of longitudinal goals of the first type is bounded by $8 \times 2 \times 3 = 48$ (8 relevant cars, whether to be in front or behind them, and 3 relevant distances). A reasonable number of absolute speed targets are 10, and a reasonable upper bound on the number of speed constraints is 2. To implement a given lateral or longitudinal goal, we need to apply acceleration and then deceleration (or the other way around). The aggressiveness of achieving the goal is a maximal (in absolute value) acceleration/deceleration to achieve the goal. With the goal and aggressivness defined, we have a closed form formula to implement the goal, using kinematic calculations. The only remaining part is to determine the combination between the lateral and longitudinal goals (e.g., "start with the lateral goal, and exactly at the middle of it, start to apply also the longitudinal goal"). A set of 5 mixing times and 3 aggressiveness levels seems more than enough. All in all, we have obtained a semantic action space whose size is $\approx 10^4$.

It is worth mentioning that the variable time required for fulfilling these semantic actions is not the same as the frequency of the decision making process. To be reactive to the dynamic world, we should make decisions at a high frequency — in our implementation, every 100ms. In contrast, each such decision is based on constructing a trajectory that fulfills some semantic action, which will have a much longer time horizon (say, 10 seconds). We use the longer time horizon since it helps us to better evaluate the short term prefix of the trajectory. In the next subsection we discuss the evaluation of semantic actions, but before that, we argue that semantic actions induce a sufficient search space.

Is this sufficient: We have seen that a semantic action space induces a subset of all possible geometrical curves, whose size is exponentially smaller (in T) than enumerating all possible geometrical curves. The first immediate question is whether the set of short term prefixes of this smaller search space contains all geometric commands that we will ever want to use. We argue that this is indeed sufficient in the following sense. If the road is free of other agents, then there is no reason to make changes except setting a lateral goal and/or absolute acceleration commands and/or speed constraints on certain positions. If the road contains other agents, we may want to negotiate the right of way with the other agents. In this case, it suffices to set longitudinal goals relatively to the other agents. The exact implementation of these goals in the long run may vary, but the short term prefixes will not change by much. Hence, we obtain a very good cover of the relevant short term geometrical commands.

5.2 Constructing an evaluation function for semantic actions

We have defined a semantic set of actions, denoted by A^s. Given that we are currently in state s, we need a way to choose the best $a^s \in A^s$. To tackle this problem, we follow a similar approach to the *options mechanism* of [7]. The basic idea is to think of a^s as a meta-action (or an option). For each choice of a meta-action, we construct a geometrical trajectory $(s_1, a_1), \ldots, (s_T, a_T)$ that represents an implementation of the meta-action, a^s. To do so we of course need to know how other agents will react to our actions, but for now we are still relying on (the non-realistic) assumption that $s_{t+1} = f(s_t, a_t)$ for some known deterministic function f. We can now use $\frac{1}{T}\sum_{t=1}^{T} \rho(s_t, a_t)$ as a good approximation of the quality of performing the semantic action a^s when we are at state s_1.

Most of the time, this simple approach yields a powerful driving policy. However, in some situations a more sophisticated quality function is required. For example, suppose that we are following a slow truck before an exit lane, where we need to take the exit lane. One semantic option is to keep driving slowly behind the truck. Another one is to overtake the truck, hoping that later we can get back to the exit lane and make the exit on time. The quality measure described previously does not consider what will happen after we will overtake the truck, and hence we will not choose the second semantic action even if there is enough time to make the overtake and return to the exit lane. Machine learning can help us to construct a better evaluation of semantic actions, that will take into account more than the immediate semantic actions. Previously, we have argued that learning a Q function over immediate geometric actions is problematic due to the low signal-to-noise ratio (the lack of advantage). This is not problematic when considering semantic actions, both because there is a large difference between performing the different semantic actions and because the semantic time horizon (how many semantic actions we take into account) is very small (probably less than three in most cases).

Another advantage of applying machine learning is for the sake of *generalization*: we can probably set an adequate evaluation function for *every* road, by a manual inspection of the properties of the road, and maybe some trial and error. But, can we automatically generalize to *any* road? Here, a machine learning approach can be trained on a large variety of road types so as to generalize to unseen roads as well.

To summarize, our semantic action space allows to enjoy the benefits of both worlds: semantic actions contain information on a long time horizon, hence we can obtain a very accurate evaluation of their quality while being resource efficient.

5.3 The dynamics of the other agents

So far, we have relied on the assumption that s_{t+1} is a deterministic function of s_t and a_t. As we have emphasized previously, this assumption is completely not realistic as our actions affect the behavior of other road users. While we do take into account some reactions of other agents to our actions (for example, we assume that if we will perform a safe cut-in, then the car behind us will adjust its speed so as not to hit us from behind), it is not realistic to assume that we model all of the dynamics of other agents.

The solution to this problem is to re-apply our decision making at a high frequency, and by doing this, we constantly adapt our policy to the parts of the environment that are beyond our modeling. In a sense, one can think of this as a Markovization of the world at every step. This is a common technique that tends to work very good in practice as long as the balance between modeling error and frequency of planning is adequate.

6 Sensing

In this section we describe the sensing state, which is a description of the relevant information of the scene, and forms the input to the driving policy module. By and large, the sensing state contains static and dynamic objects. The static objects are lanes, physical road delimiters, constraints on speed, constraints on the right of way, and information on occluders (e.g. a fence that occludes relevant part of a merging road). Dynamic objects are vehicles (bounding box, speed, acceleration), pedestrians (bounding box, speed, acceleration), traffic lights, dynamic road delimiters (e.g. cones at a construction area), temporary traffic signs and police activity, and other obstacles on the road (e.g. an animal, a mattress that fell from a truck, etc.).

In any reasonable sensor setting, we cannot expect to obtain the exact sensing state, s. Instead, we view raw sensor and mapping data, which we denote by $x \in X$, and there is a sensing system that takes x and produces an approximate sensing state. Formally,

Definition 22 (Sensing system) *Let S denote the domain of sensing state and let X be the domain of raw sensor and mapping data. A sensing system is a function $\hat{s} : X \to S$.*

It is important to understand when we should accept $\hat{s}(x)$ as a reasonable approximation to s. The ultimate way to answer this question is by examining the implications of this approximation on the performance of our driving policy in general, and on the safety in particular. Following our safety-comfort distinction, here again we distinguish between sensing mistakes that lead to non-safe behaviour and sensing mistakes that affect the comfort aspects of the ride.

Before we dive into the details, let us first describe the type of errors a sensing system might make:

- False negative: the sensing system misses an object
- False positive: the sensing system indicates a “ghost” object
- Inaccurate measurements: the sensing system correctly detects an object but incorrectly estimates its position or speed
- Inaccurate semantic: the sensing system correctly detects an object but misinterpret its semantic meaning, for example, the color of a traffic light

6.1 Comfort

Recall that for a semantic action a, we have used $Q(s, a)$ to denote our evaluation of a given that the current sensing state is s. Our policy picks the action $\pi(s) = \mathrm{argmax}_a Q(s, a)$. If we inject $\hat{s}(x)$ instead of s then the selected semantic action would be $\pi(\hat{s}(x)) = \mathrm{argmax}_a Q(\hat{s}(x), a)$. Clearly, if $\pi(\hat{s}(x)) = \pi(s)$ then $\hat{s}(x)$ should be accepted as a good approximation to s. But, it is also not bad at all to pick $\pi(\hat{s}(x))$ as long as the quality of $\pi(\hat{s}(x))$ w.r.t. the true state, s, is almost optimal, namely, $Q(s, \pi(\hat{s}(x))) \geq Q(s, \pi(s)) - \epsilon$, for some parameter ϵ. We say that $\hat{s}$ is ϵ-accurate w.r.t. Q in such case. Naturally, we cannot expect the sensing system to be ϵ-accurate all the time. We therefore also allow the sensing system to fail with some small probability δ. In such a case we say that $\hat{s}$ is Probably (w.p. of at least $1 - \delta$), Approximately (up to ϵ), Correct, or PAC for short (borrowing Valiant's PAC learning terminology [8]).

We may use several (ϵ, δ) pairs for evaluating different aspects of the system. For example, we can choose three thresholds, $\epsilon_1 < \epsilon_2 < \epsilon_3$ to represent mild, medium, and gross mistakes, and for each one of them set a different value of δ. This leads to the following definition.

Definition 23 (PAC sensing system) *Let $((\epsilon_1, \delta_1), \ldots, (\epsilon_k, \delta_k))$ be a set of (accuracy,confidence) pairs, let S be the sensing state domain, let X be the raw sensor and mapping data domain, and let D be a distribution over $X \times S$. Let A be an action space, $Q : S \times A \to \mathbb{R}$ be a quality function, and $\pi : S \to A$ be such that $\pi(s) \in \mathrm{argmax}_a Q(s, a)$. A sensing system, $\hat{s} : X \to S$, is Probably-Approximately-Correct (PAC) with respect to the above parameters if for every $i \in \{1, \ldots, k\}$ we have that $\mathbb{P}_{(x,s)\sim D}[Q(s, \pi(\hat{s}(x))) \geq Q(s, \pi(s)) - \epsilon_i] \geq 1 - \delta_i$.*

Few remarks are in order:

- The definition depends on a distribution D over $X \times S$. It is important to emphasize that we construct this distribution by recording data of many human drivers but not by following the particular policy of our autonomous vehicle. While the latter seems more adequate, it necessitates online validation, which makes the development of the sensing system impractical. Since the effect of any reasonable policy on D is minor, by applying simple data augmentation techniques we can construct an adequate distribution and then perform offline validation after every major update of the sensing system.
- The definition provides a sufficient, but not necessary, condition for comfort ride using $\hat{s}$. It is not necessary because it ignores the important fact that short term wrong decisions have little effect on the comfort of the ride. For example, suppose that there is a vehicle 100 meters in front of us, and it is slower than the host vehicle.

The best decision would be to start accelerating slightly now. If the sensing system misses this vehicle, but will detect it in the next time (after 100 mili-seconds), then the difference between the two rides will not be noticeable. To simplify the presentation, we have neglected this issue and required a stronger condition. The adaptation to a multi-frame PAC definition is conceptually straightforward, but involves more technicality and therefore we omit it.

We next derive design principles that follow from the above PAC definition. Recall that we have described several types of sensing mistakes. For mistakes of types false negative, false positive, and inaccurate semantic, either the mistakes will be on non-relevant objects (e.g., a traffic light for left turn when we are proceeding straight), or they will be captured by the δ part of the definition. We therefore focus on the "inaccurate measurements" type of errors, which happens frequently.

Somewhat surprisingly, we will show that the popular approach of measuring the accuracy of a sensing system via ego-accuracy (that is, by measuring the accuracy of position of every object with respect to the host vehicle) is not sufficient for ensuring PAC sensing system. We will then propose a different approach that ensures PAC sensing system, and will show how to obtain it efficiently. We start with some additional definitions.

For every object o in the scene, let $p(o), \hat{p}(o)$ be the positions of o in the coordinate system of the host vehicle according to $s, \hat{s}(x)$, respectively. Note that the distance between o and the host vehicle is $\|p\|$. The *additive* error of $\hat{p}$ is $\|p(o) - \hat{p}(o)\|$.The *relative* error of $\hat{p}(o)$, w.r.t. the distance between o and the host vehicle, is the additive error divided by $\|p(o)\|$, namely $\frac{\|p(o)-\hat{p}(o)\|}{\|p(o)\|}$.

We first argue that it is not realistic to require that the additive error is small for far away objects. Indeed, consider o to be a vehicle at a distance of 150 meters from the host vehicle, and let ϵ be of moderate size, say $\epsilon = 0.1$. For additive accuracy, it means that we should know the position of the vehicle up to 10cm of accuracy. This is not realistic for reasonably priced sensors. On the other hand, for relative accuracy we need to estimate the position up to 10%, which amounts to 15m of accuracy. This is feasible to achieve (as we will describe later).

We say that a sensing system, $\hat{s}$, positions a set of objects, O, in an ϵ-ego-accurate way, if for every $o \in O$, the (relative) error between $p(o)$ and $\hat{p}(o)$ is at most ϵ. The following example shows that an ϵ-ego-accurate sensing state does not guarantee PAC sensing system with respect to every reasonable Q. Indeed, consider a scenario in which the host vehicle drives at a speed of $30m/s$, and there is a stopped vehicle 150 meters in front of it. If this vehicle is in the ego lane, and there is no option to change lanes in time, we must start decelerating now at a rate of at least $3m/s^2$ (otherwise, we will either not stop in time or we will need to decelerate strongly later). On the other hand, if the vehicle is on the side of the road, we don't need to apply a strong deceleration. Suppose that $p(o)$ is one of these cases while $\hat{p}(o)$ is the other case, and there is a 5 meters difference between these two positions. Then, the relative error of $\hat{p}(o)$ is

$$\frac{\|\hat{p}(o) - p(o)\|}{\|p(o)\|} \;=\; \frac{5}{150} = \frac{1}{30} \leq 0.034 \;.$$

That is, our sensing system may be ϵ-ego-accurate for a rather small value of ϵ (less than 3.5% error), and yet, for any reasonable Q function, the values of Q are completely different since we are confusing between a situation in which we need to brake strongly and a situation in which we do not need to brake strongly.

The above example shows that ϵ-ego-accuracy does not guarantee that our sensing system is PAC. Is there another property that is sufficient for PAC sensing system? Naturally, the answer to this question depends on Q. We will describe a family of Q functions for which there is a simple property of the positioning that guarantees PAC sensing system. Intuitively, the problem of ϵ-ego-accuracy is that it might lead to semantic mistakes — in the aforementioned example, even though $\hat{s}$ was ϵ-ego-accurate with $\epsilon < 3.5\%$, it mis-assigned the vehicle to the correct lane. To solve this problem, we rely on *semantic units* for lateral position.

Definition 24 (semantic units) *A lane center is a simple natural curve, namely, it is a differentiable, injective, mapping $\ell : [a, b] \to \mathbb{R}^3$, where for every $a \leq t_1 < t_2 \leq b$ we have that the length* $\mathrm{Length}(t_1, t_2) := \int_{\tau=t_1}^{t_2} |\ell'(\tau)| d\tau$ *equals to $t_2 - t_1$. The width of the lane is a function $w : [a, b] \to \mathbb{R}_+$. The projection of a point $x \in \mathbb{R}^3$ onto the curve is the point on the curve closest to x, namely, the point $\ell(t_x)$ for $t_x = \mathrm{argmin}_{t \in [a,b]} \|\ell(t) - x\|$. The semantic longitudinal position of x w.r.t. the lane is t_x and the semantic lateral position of x w.r.t. the lane is $\|\ell(t_x) - x\|/w(t_x)$. Semantic speed and acceleration are defined as first and second derivatives of the above.*

Similarly to geometrical units, for semantic longitudinal distance we use relative error: if $\hat{s}$ induces a semantic longitudinal distance of $\hat{p}(o)$ for some object, while the true distance is $p(o)$, then the relative error is $\frac{|\hat{p}(o)-p(o)|}{\max\{p(o),1\}}$ (where the maximum in the denominator deals with cases in which the object has almost the same longitudinal distance (e.g., a car next to us on another lane). Since semantic lateral distances are small we can use additive error for them. This leads to the following defintion:

Definition 25 (error in semantic units) *Let ℓ be a lane and suppose that the semantic longitudinal distance of the host vehicle w.r.t. the lane is 0. Let $x \in \mathbb{R}^3$ be a point and let $p_{\mathrm{lat}}(x), p_{\mathrm{lon}}(x)$ be the semantic lateral and longitudinal distances to the point w.r.t. the lane. Let $\hat{p}_{\mathrm{lat}}(x), \hat{p}_{\mathrm{lon}}(x)$ be approximated measurements. The distance between $\hat{p}$ and p w.r.t. x is defined as*

$$d(\hat{p}, p; x) \;=\; \max\left\{ |\hat{p}_{\mathrm{lat}}(x) - p_{\mathrm{lat}}(x)| \;,\; \frac{|\hat{p}_{\mathrm{lon}}(x) - p_{\mathrm{lon}}(x)|}{\max\{p_{\mathrm{lon}}(x), 1\}} \right\}$$

The error of the lateral and longitudinal semantic velocities is defined analogously.

Equipped with the above definition, we are ready to define the property of Q and the corresponding sufficient condition for PAC sensing system.

Definition 26 (Semantically-Lipschitz Q) *A Q function is L-semantically-Lipschitz if for every a, $s, \hat{s}$, $|Q(s,a) - Q(\hat{s}(x), a)| \le L \max_o d(\hat{p}, p; o)$, where $\hat{p}, p$ are the measurements induced by $s, \hat{s}$ on an object o.*

As an immediate corollary we obtain:

Lemma 6 *If Q is L-semantically-Lipschitz and a sensing system $\hat{s}$ produces semantic measurements such that with probability of at least $1 - \delta$ we have $d(\hat{p}, p; o) \le \epsilon/L$, then $\hat{s}$ is a PAC sensing system with parameters ϵ, δ.*

6.2 Safety

We now discuss sensing mistakes that lead to non-safe behavior. As mentioned before, our policy is provably safe, in the sense that it won't lead to accidents of the autonomous vehicle's blame. Such accidents might still occur due to hardware failure (e.g., a break down of all the sensors or exploding tire on the highway), software failure (a significant bug in some of the modules), or a sensing mistake. Our ultimate goal is that the probability of such events will be extremely small — a probability of 10^{-9} for such an accident per hour. To appreciate this number, the average number of hours an american driver spends on the road is (as of 2016) less than 300. So, in expectation, one needs to live 3.3 million years to be in an accident.

Roughly speaking, there are two types of safety-critic sensing mistake. The first type is a *safety-critic miss*, meaning that a dangerous situation is considered non-dangerous according to our sensing system. The second type is a *safety-critic ghost*, meaning that a non-dangerous situation is considered dangerous according to our sensing system. Safety-critic misses are obviously dangerous as we will not know that we should respond properly to the danger. Safety-critic ghosts might be dangerous when our speed is high, we brake hard for no reason, and there is a car behind us.

Usually, a safety-critic miss is caused by a false negative while a safety-critic ghost is caused by a false positive. Such mistakes can also be caused from significantly incorrect measurements, but in most cases, our comfort objective ensures we are far away from the boundaries of non-safe distances, and therefore reasonable measurement errors are unlikely to lead to safety-critic mistakes.

How can we ensure that the probability of safety-critic mistakes will be very small, say, smaller than 10^{-9} per hour? As followed from Lemma 1, without making further assumptions we need to check our system on more than 10^9 hours of driving. This is unrealistic (or at least extremely challenging) — it amounts to recording the driving of 3.3 million cars over a year. Furthermore, building a system that achieves such a high accuracy is a great challenge. Our solution for both the system design and validation challenges is to rely on several sub-systems, each of which is engineered independently and depends on a different technology, and the systems are fused together in a way that ensures boosting of their individual accuracy.

Suppose we build 3 sub-systems, denoted, s_1, s_2, s_3 (the extension to more than 3 is straightforward). Each sub-system should decide if the current situation is dangerous or not. Situations which are non-dangerous according to the majority of the sub-systems (2 in our case) are considered safe.

Let us now analyze the performance of this fusion scheme. We rely on the following definition:

Definition 27 (One side c-approximate independent) *Two Bernoulli random variables r_1, r_2 are called one side c-approximate independent if*

$$\mathbb{P}[r_1 \wedge r_2] \le c\ \mathbb{P}[r_1]\ \mathbb{P}[r_2]\ .$$

For $i \in \{1, 2, 3\}$, denote by e_i^m, e_i^g the Bernoulli random variables that indicate if sub-system i has a safety-critic miss/ghost respectively. Similarly, e^m, e^g indicate a safety-critic miss/ghost of the fusion system. We rely on the assumption that for any pair $i \neq j$, the random variables e_i^m, e_j^m are one sided c-approximate independent, and the same holds for e_i^g, e_j^g. Before explaining why this assumption is reasonable, let us first analyze its implication. We can bound the probability of e^m by:

$$\begin{aligned}
\mathbb{P}[e^m] &= \mathbb{P}[e_1^m \wedge e_2^m \wedge e_3^m] + \sum_{j=1}^{3} \mathbb{P}[\neg e_j^m \wedge \wedge_{i \neq j} e_i^m] \\
&\le 3\,\mathbb{P}[e_1^m \wedge e_2^m \wedge e_3^m] + \sum_{j=1}^{3} \mathbb{P}[\neg e_j^m \wedge \wedge_{i \neq j} e_i^m] \\
&= \sum_{j=1}^{3} \mathbb{P}[\wedge_{i \neq j} e_i^m] \\
&\le c \sum_{j=1}^{3} \prod_{i \neq j} \mathbb{P}[e_i^m].
\end{aligned}$$

Therefore, if all sub-systems have $\mathbb{P}[e_i^m] \le p$ then $\mathbb{P}[e^m] \le 3\,c\,p^2$. The exact same derivation holds for the safety-critic ghost mistakes. By applying a union bound we therefore conclude:

Corollary 2 *Assume that for any pair $i \neq j$, the random variables e_i^m, e_j^m are one sided c-approximate independent, and the same holds for e_i^g, e_j^g. Assume also that for every i, $\mathbb{P}[e_i^m] \le p$ and $\mathbb{P}[e_i^g] \le p$. Then,*

$$\mathbb{P}[e^m \vee e^g] \le 6\,c\,p^2\ .$$

This corollary allows us to use significantly smaller data sets in order to validate the sensing system. For example, if we would like to achieve a safety-critic mistake probability of 10^{-9}, instead of taking order of 10^9 examples, it suffices to take order of 10^5 examples and test each system separately.

It is left to reason about the rational behind the one sided independence assumption. There are pairs of sensors that yield completely non-correlated errors. For example, radar works well in bad weather conditions but might fail due to non-relevant metallic objects, as opposed to camera that is affected by bad weather but is not likely to be affected by metallic objects. Seemingly, camera and lidar have common sources of mistakes — both are affected by foggy weather, heavy rain or snow. However, the type of mistake for camera and lidar would be different — camera might miss objects due to bad weather while lidar might detect a ghost due to reflections from particles in the air. Since we have distinguished between the two types of mistakes, the approximate independency is still likely to hold.

Remark 8 *Our definition of safety-critic ghost requires that the situation is dangerous by at least two sensors. We argue that even in difficult conditions (e.g. heavy fog), this is unlikely to happen. The reason is that in such situations, systems that are affected by the difficult conditions (e.g. the lidar), will dictate a very defensive driving to the policy, as they can declare that high velocity and lateral maneuvers would lead to a dangerous situation. As a result, we will drive slowly, and then even if we require an emergency stop, it is not dangerous due to the low speed of driving. Therefore, an adaptation of the driving style to the conditions of the road will follow from the definitions.*

6.3 Building a scalable sensing system

We have described the requirements from a sensing system, both in terms of comfort and safety. We now briefly suggest our approach for building a sensing system that meets these requirements while being scalable.

There are three main components of our sensing system. The first is long range, 360 degrees coverage, of the scene based on cameras. The three main advantages of cameras are: (1) high resolution, (2) texture, (3) price. The low price enables a scalable system. The texture enables to understand the semantics of the scene, including lane marks, traffic light, intentions of pedestrians, and more. The high resolution enables a long range of detection. Furthermore, detecting lane marks and objects in the same domain enables excellent semantic lateral accuracy. The two main disadvantages of cameras are: (1) the information is 2D and estimating longitudinal distance is difficult, (2) sensitivity to lighting conditions (low sun, bad weather). We overcome these difficulties using the next two components of our system.

The second component of our system is a semantic high-definition mapping technology, called Road Experience Management (REM). A common geometrical approach to map creation is to record a cloud of 3D points (obtained by a lidar) in the map creation process, and then, localization on the map is obtained by matching the existing lidar points to the ones in the map. There are several disadvantages of this approach. First, it requires a large memory per kilometer of mapping data, as we need to save many points. This necessitates an expensive communication infrastructure. Second, only few cars are equipped with lidar sensors, and therefore, the map is updated very infrequently. This is problematic as changes in the road can occur (construction zones, hazards), and the "time-to-reflect-reality" of lidar-based mapping solutions is large. In contrast, REM follows a semantic-based approach. The idea is to leverage the large number of vehicles that are equipped with cameras and with software that detects semantically meaningful objects in the scene (lane marks, curbs, poles, traffic lights, etc.). Nowadays, many new cars are equipped with ADAS systems which can be leveraged for crowd source based creation of the map. Since the processing is done on the vehicle side, only a small amount of semantic data should be communicated to the cloud. This allows a very frequent update of the map in a scalable way. In addition, the autonomous vehicles can receive the small sized mapping data over existing communication platforms (the cellular network). Finally, highly accurate localization on the map can be obtained based on cameras, without the need for expensive lidars.

REM is used for three purposes. First, it gives us a foresight on the static structure of the road (we can plan for a highway exit way in advance). Second, it gives us another source of accurate information of all of the static information, which together with the camera detections yields a robust view of the static part of the world. Third, it solves the problem of lifting the 2D information from the image plane into the 3D world as follows. The map describes all of the lanes as curves in the 3D world. Localization of the ego vehicle on the map enables to trivially lift every object on the road from the image plane to its 3D position. This yields a positioning system that adheres to the accuracy in semantic units described in Section 6.1.

The third component of our system is a complementary radar and lidar system. This system serves two purposes. First, they enable to yield an extremely high accuracy for the sake of safety (as described in Section 6.2). Second, they give direct measurements on speed and distances, which further improves the comfort of the ride.

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A Technical Lemmas

A.1 Technical Lemma 1

Lemma 7 *For all $x \in [0, 0.1]$, it holds that $1 - x \geq e^{-2x}$.*

Proof Let $f(x) = 1 - x - e^{-2x}$. Our goal is to show $f(x) \geq 0$ for $x \in [0, 0.1]$. Note that $f(0) = 0$, and it is therefore sufficient to have that $f'(x) \geq 0$ in the aforementioned range. Explicitly, $f'(x) = -1 + 2e^{-2x}$. Clearly, $f'(0) = 1$, and it is monotonically decreasing, hence it is sufficient to verify that $f'(0.1) > 0$, which is easy to do numerically, $f'(0.1) \approx 0.637$. ■

B The Lane-Based Coordinate System

The most basic simplifying assumption we made in the RSS definition was that the road is comprised by adjacent, straight lanes, of constant width. The distinction between lateral and longitudinal axes, along with an ordering of longitudinal position, play a significant role in RSS, and in common-sense driving. Moreover, the definition of those directions is clearly based on the lane shape. We propose a transformation from (global) positions on the plane, to a lane-based coordinate system, reducing the problem yet again to the original, "straight lane of constant width", case.

Assume that the lane's center is a smooth directed curve r on the plane, where all of its pieces, denoted $r^{(1)}, \ldots, r^{(k)}$, are either linear, or an arc. Note that smoothness of the curve implies that no pair of consecutive pieces can be linear. Formally, the curve maps a "longitudinal" parameter, $Y \in [Y_{\min}, Y_{\max}] \subset \mathbb{R}$, into the plane, namely, the curve is a function of the form $r : [Y_{\min}, Y_{\max}] \to \mathbb{R}^2$. We define a continuous lane-width function $w : [Y_{\min}, Y_{\max}] \to \mathbb{R}_+$, mapping the longitudinal position Y into a positive lane width value. For each Y, from smoothness of r, we can define the normal unit-vector to the curve at position Y, denoted $r^{\perp}(Y)$. We naturally define the subset of points on the plane which reside in the lane as follows:

$$R = \left\{ r(Y) + \alpha w(Y) r^{\perp}(Y) \;\mid\; Y \in [Y_{\min}, Y_{\max}],\ \alpha \in [\pm 1/2] \right\} .$$

Informally, our goal is to construct a transformation ϕ of R into $\mathbb{R}^2$,[11] such that for two cars which are on the lane, their "logical ordering" will be preserved:

- If c_r is "behind" c_f on the curve, then $\phi(c_r)_y < \phi(c_f)_y$.
- If c_l is "to the left of" c_r on the curve, then $\phi(c_l)_x < \phi(c_r)_x$.

In order to define ϕ, we rely on the assumption that for all i, if $r^{(i)}$ is an arc of radius ρ, then the width of the lane throughout $r^{(i)}$ is $\leq \rho/2$. Note that this assumption holds for any practical road. The assumption trivially implies that for all $(x', y') \in R$, there exists a unique pair $Y' \in [Y_{\min}, Y_{\max}]$, $\alpha' \in [\pm 1/2]$, s.t. $(x', y') = r(Y') + \alpha' w(Y') r^{\perp}(Y')$.

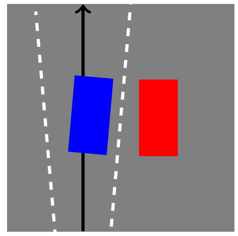

Figure 11: Changing lane width. Although the red car drives in parallel to the lane's center (black arrow), it clearly makes lateral movement towards the lane. The blue car, although getting further away from the lane's center, stays in the same position w.r.t. the lane boundary.

We can now define $\phi : R \to \mathbb{R}^2$ to be $\phi(x', y') = (Y', \alpha')$, where (Y', α') are the unique values that satisfy $(x', y') = r(Y') + \alpha' w(Y') r^{\perp}(Y')$.

This definition captures the notion of a "lateral manoeuvre" in lane's coordinate system. Consider, for example, a widening lane, with a car driving exactly on one of the lane's boundaries (see Figure 11 for an illustration). The widening of the lane means that the car is moving away from the center of the lane, and therefore has lateral velocity w.r.t. it. However, this doesn't mean it performs a lateral manoeuvre. Our definition of $\phi(x', y')_x = \alpha'$, namely, the lateral distance to the lane's center in $w(Y')$-units, implies that the lane boundaries have a fixed lateral position of $\pm 1/2$, hence, a car which sticks to one of the lane's boundaries is not considered to perform any lateral movement. Finally, it is easy to see that ϕ is a bijective embedding. We will use the term *lane-based coordinate system* when discussing $\phi(R) = [Y_{\min}, Y_{\max}] \times [\pm 1/2]$. We have thus obtained a reduction from a general lane geometry to a straight, longitudinal/lateral, coordinate system.

[11] Where, as in RSS, we will associate the y-axis with the "longitudinal" axis, and the x-axis with the "lateral".