Derivation

In [1]:
```
# set up session
from sympy import *
```

In [2]:
```
# variables:
x, y, z = symbols('x y z')
```

In [3]:
```
# parameters:
f, k0, k1 = symbols('f k0 k1')
```

In [4]:
```
# normalized plane:
x_normalized = x / z
y_normalized = y / z
```

In [5]:
```
# distortion coefficient:
r_2 = x_normalized*x_normalized + y_normalized*y_normalized
d = 1 + (k0 + k1*r_2)*r_2
```

In [6]:
```
# image plane:
u = f*d*x_normalized
v = f*d*y_normalized
```

result

u

In [7]:
```
u.diff(x)
```

Out[7]:
$$\frac{fx\left(\frac{2k_1x\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)}{z^2}+\frac{2x\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)}{z^2}\right)}{z}+\frac{f\left(\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)+1\right)}{z}$$

In [8]:
```
# for code implementation:
print(u.diff(x))
```

```
f*x*(2*k1*x*(x**2/z**2 + y**2/z**2)/z**2 + 2*x*(k0 + k1*(x**2/z**2 + y**2/z**
2))/z**2)/z + f*((k0 + k1*(x**2/z**2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) +
1)/z
```

In [9]:
```
u.diff(y)
```

Out[9]:
$$\frac{fx\left(\frac{2k_1y\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)}{z^2}+\frac{2y\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)}{z^2}\right)}{z}$$

In [10]:
```
# for code implementation:
print(u.diff(y))
```

```
f*x*(2*k1*y*(x**2/z**2 + y**2/z**2)/z**2 + 2*y*(k0 + k1*(x**2/z**2 + y**2/z**
2))/z**2)/z
```

In [11]:
```
u.diff(z)
```

Out[11]:
$$\frac{fx\left(k_1\left(-\frac{2x^2}{z^3}-\frac{2y^2}{z^3}\right)\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)+\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)\left(-\frac{2x^2}{z^3}-\frac{2y^2}{z^3}\right)\right)}{z} - \frac{fx\left(\left(k_0 -\right.\right.}{}$$

In [12]:
```
# for code implementation:
print(u.diff(z))
```

```
f*x*(k1*(-2*x**2/z**3 - 2*y**2/z**3)*(x**2/z**2 + y**2/z**2) + (k0 + k1*(x**
2/z**2 + y**2/z**2))*(-2*x**2/z**3 - 2*y**2/z**3))/z - f*x*((k0 + k1*(x**2/z*
*2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) + 1)/z**2
```

V

In [13]:
```
v.diff(x)
```

Out[13]:
$$\frac{fy\left(\frac{2k_1x\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)}{z^2}+\frac{2x\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)}{z^2}\right)}{z}$$

In [14]:
```
# for code implementation:
print(v.diff(x))
```

```
f*y*(2*k1*x*(x**2/z**2 + y**2/z**2)/z**2 + 2*x*(k0 + k1*(x**2/z**2 + y**2/z**
2))/z**2)/z
```

In [15]:
```
v.diff(y)
```

Out[15]:
$$\frac{fy\left(\frac{2k_1y\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)}{z^2}+\frac{2y\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)}{z^2}\right)}{z}+\frac{f\left(\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)+1\right)}{z}$$

In [16]:
```
# for code implementation:
print(v.diff(y))
```

```
f*y*(2*k1*y*(x**2/z**2 + y**2/z**2)/z**2 + 2*y*(k0 + k1*(x**2/z**2 + y**2/z**
2))/z**2)/z + f*((k0 + k1*(x**2/z**2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) +
1)/z
```

In [17]:
```
v.diff(z)
```

Out[17]:

$$\frac{fy\left(k_1\left(-\frac{2x^2}{z^3}-\frac{2y^2}{z^3}\right)\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)+\left(k_0+k_1\left(\frac{x^2}{z^2}+\frac{y^2}{z^2}\right)\right)\left(-\frac{2x^2}{z^3}-\frac{2y^2}{z^3}\right)\right)}{z} - \frac{fy\left(\left(k_0+\right.\right.}{}$$

In [18]:
```
# for code implementation:
print(v.diff(z))
```

```
f*y*(k1*(-2*x**2/z**3 - 2*y**2/z**3)*(x**2/z**2 + y**2/z**2) + (k0 + k1*(x**
2/z**2 + y**2/z**2))*(-2*x**2/z**3 - 2*y**2/z**3))/z - f*y*((k0 + k1*(x**2/z*
*2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) + 1)/z**2
```