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Derivation

```
In [1]:
          # set up session
          from sympy import *
In [2]:
          # variables:
          x, y, z = symbols('x y z')
In [3]:
          # parameters:
          f, k0, k1 = symbols('f k0 k1')
In [4]:
          # normalized plane:
          x normalized = x / z
          y \text{ normalized} = y / z
In [5]:
          # distortion coefficient:
          r 2 = x normalized*x normalized + y normalized*y normalized
          d = 1 + (k0 + k1*r 2)*r 2
In [6]:
          # image plane:
          u = f*d*x normalized
          v = f*d*y normalized
        result
        u
In [7]:
          u.diff(x)
Out[7]:
In [8]:
          # for code implementation:
          print(u.diff(x))
         f*x*(2*k1*x*(x**2/z**2 + v**2/z**2)/z**2 + 2*x*(k0 + k1*(x**2/z**2 + v**2/z**2)
         (k_0)/z^{2}/z + f^{2}(k_0 + k_1^{2}(x^{2}/z^{2}) + y^{2}/z^{2}))^{2}(x^{2}/z^{2} + y^{2}/z^{2}) + (k_0 + k_1^{2}/z^{2})^{2}
         1)/z
In [9]:
          u.diff(y)
Out[9]:
```

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```
In [10]:
                                                                                   # for code implementation:
                                                                                  print(u.diff(y))
                                                                             f^*x^*(2^*k1^*y^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2})
                                                                             2))/z**2)/z
In [11]:
                                                                                 u.diff(z)
                                                                                                                                                                                                                                                                                                                              +\left(k_0+k_1\left(rac{x^2}{z^2}+rac{y^2}{z^2}
ight)
ight)\left(
Out[11]:
In [12]:
                                                                                   # for code implementation:
                                                                                  print(u.diff(z))
                                                                             f*x*(k1*(-2*x**2/z**3 - 2*y**2/z**3)*(x**2/z**2 + y**2/z**2) + (k0 + k1*(x**2/z**2 + y**2/z**2))*(-2*x**2/z**3 - 2*y**2/z**3))/z - f*x*((k0 + k1*(x**2/z**2))*(-2*x**2/z**3))/z - f*x*((k0 + k1*(x**2/z**3))/z - f*x*((k0 + k1*(x**2/z**3)))/z - f*x*((k0 + k1*(x**2/z**3))/z - f*x*((k0 + k1*(x**2/z**3))/z - f*x*
                                                                            *2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) + 1)/z**2
                                                                       V
In [13]:
                                                                                  v.diff(x)
Out[13]:
                                                                                                                                                                                                                           z
In [14]:
                                                                                  # for code implementation:
                                                                                  print(v.diff(x))
                                                                             f^*y^*(2^*k1^*x^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2} + 2^*x^*(k0 + k1^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2})/z^{**2} + 2^*x^*(k0 + k1^*/z^{**2}/z^{**2})/z^{**2} + 2^*x^*(k0 + k1^*/z^{**2}/z^{**2})/z^{**2})/z^{**2} + 2^*x^*(k0 + k1^*/z^{**2}/z^{**2})/z^{**2} + 2^*x^{**2}/z^{**2}/z^{**2}
                                                                             2))/z**2)/z
In [15]:
                                                                                  v.diff(y)
Out[15]:
                                                                                                                                                                                                                                                                                                                                                                                                          f\left(\left(k_0+k_1\left(rac{x^2}{z^2}+rac{y^2}{z^2}
ight)
ight)
                                                                                                                                                                                                                          z
In [16]:
                                                                                   # for code implementation:
                                                                                  print(v.diff(y))
                                                                             f^*y^*(2^*k1^*y^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*(x^{**2}/z^{**2} + y^{**2}/z^{**2})/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2})/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{**2}/z^{**2}/z^{**2})/z^{**2} + 2^*y^*(k0 + k1^*/z^{
                                                                             2))/z**2)/z + f*((k0 + k1*(x**2/z**2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) +
                                                                             1)/z
In [17]:
                                                                                 v.diff(z)
Out[17]:
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 $rac{fy\left(k_{1}\left(-rac{2x^{2}}{z^{3}}-rac{2y^{2}}{z^{3}}
ight)\left(rac{x^{2}}{z^{2}}+rac{y^{2}}{z^{2}}
ight)+\left(k_{0}+k_{1}\left(rac{x^{2}}{z^{2}}+rac{y^{2}}{z^{2}}
ight)
ight)\left(-rac{2x^{2}}{z^{3}}-rac{2y^{2}}{z^{3}}
ight)
ight)}{z}-rac{fy\left(\left(k_{0}+k_{1}\left(rac{x^{2}}{z^{2}}+rac{y^{2}}{z^{2}}
ight)
ight)\left(-rac{2x^{2}}{z^{3}}-rac{2y^{2}}{z^{3}}
ight)
ight)}{z}$

In [18]:

for code implementation:
print(v.diff(z))

f*y*(k1*(-2*x**2/z**3 - 2*y**2/z**3)*(x**2/z**2 + y**2/z**2) + (k0 + k1*(x**2/z**2 + y**2/z**2))*(-2*x**2/z**3 - 2*y**2/z**3))/z - f*y*((k0 + k1*(x**2/z**2 + y**2/z**2))*(x**2/z**2 + y**2/z**2) + 1)/z**2