Foundation of Computer Engineering

Lesson 02 - Boolean Switching Functions Continued

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1. Use truth tables to show whether the following Boolean equations are for equivalent functions

a. xyz + x’y’ + z’ = xyz’ + x’y’z’ + x’y’z + xy + x’yz’ + xy’z’

Below is the required truth table.

x	y	z	xyz + x’y’ + z’	xyz’ + x’y’z’ + x’y’z + xy + x’yz’ + xy’z’
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

Below are the SOP form of the two expressions:

```
Left hand side = xyz + x'y'z + x'y'z' + x'yz' + xy'z' + xyz' = sigma(0, 1, 2, 4, 6, 7)

Right hand side = xyz + xyz' + xy'z' + x'yz' + x'y'z + x'y'z' = sigma(0, 1, 2, 4, 6, 7)
```

Which agrees exactly with the above truth table.

b. xy + z = xyz’ + z

Below is the required truth table.

x	y	z	xy + z	xyz’ + z
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Since

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Left hand side = (xyz + xyz') + z = xyz' + (xyz + z) = xyz' + (xy + 1)z = xyz' + z = Right
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This also confirms the conclusion from the above truth table.

c. (x+y)(x+z) = x + yz + xyz

Below is the required truth table.

x	y	z	(x+y)(x+z)	x + yz + xyz
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1

x	y	z	(x+y)(x+z)	x + yz + xyz
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Since

```
Left hand side = (x'y' + x'z')' = (sigma(0, 1, 2))' = sigma(3, 4, 5, 6, 7)

Right hand size = x + xyz + x'yz + xyz = x + xyz + x'yz = x(1 + yz) + x'yz = x + x'yz = si
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This agrees very well with the above truth table.

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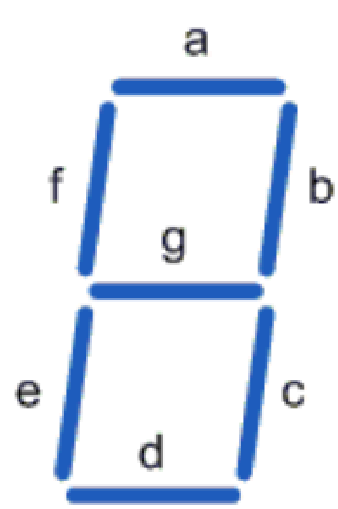
2. Consider the following encoding for binary coded decimal (BCD) numbers (left) and associated 7-segment display (right). Using truth tables, don't care conditions and K-maps, create minimized SOP equations for the following segments of the 7-segment display (b, c, g). For example, segments b and c would be illuminated for the BCD code 0001. All segments would be illuminated for the BCD code 1000.

Decimal digit	BCD w x y z
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1

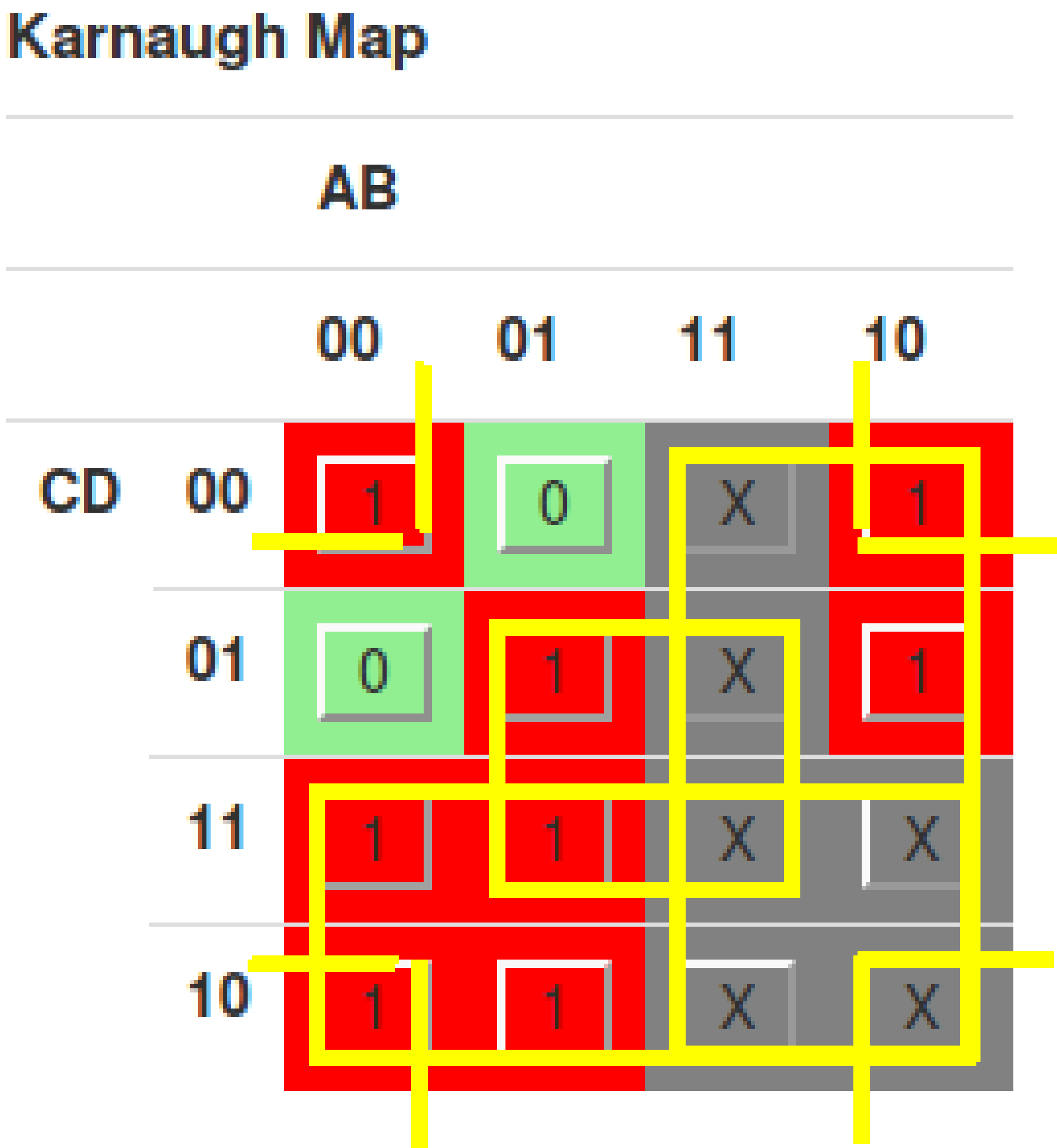


Below is the required truth table

w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0

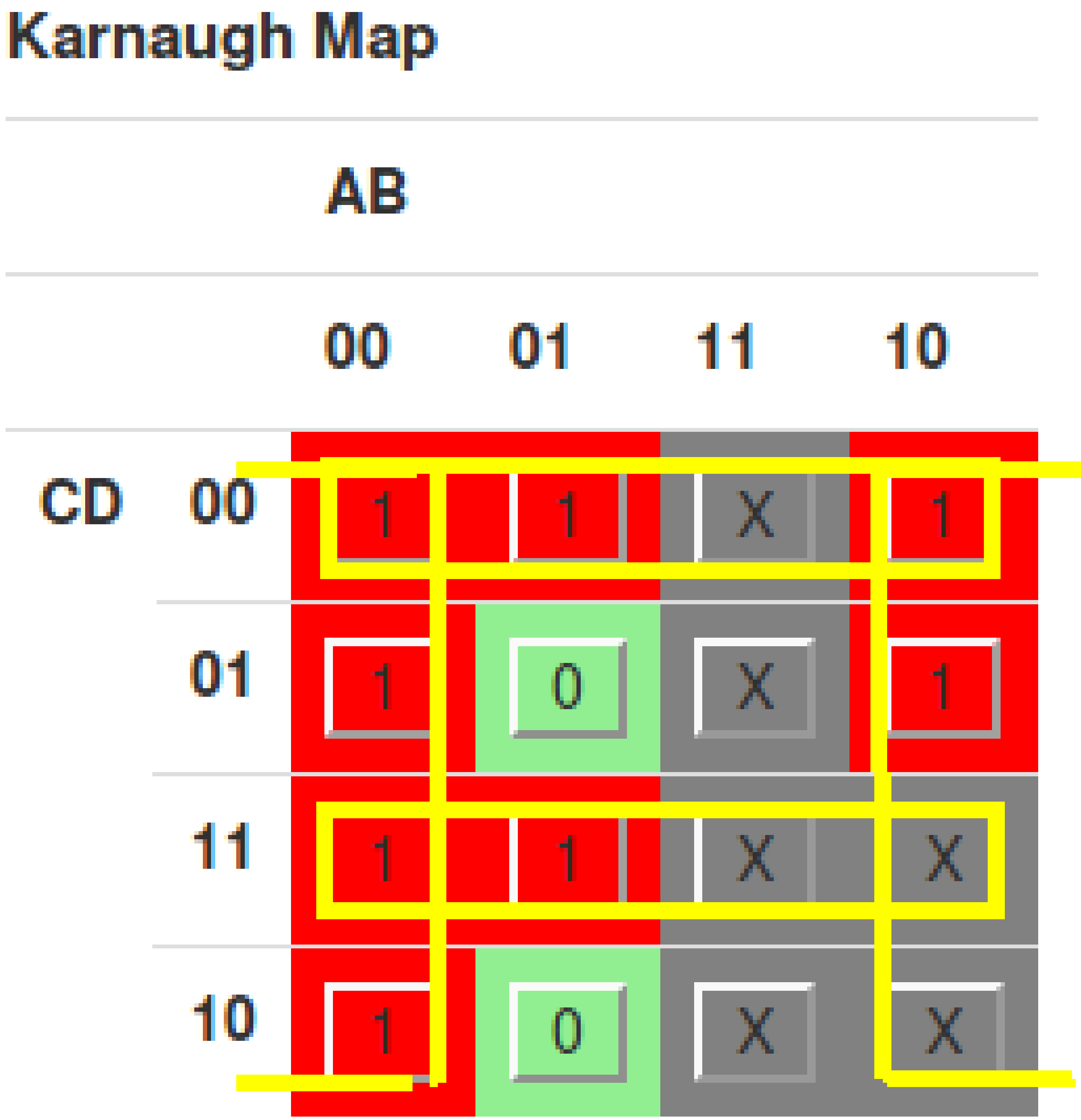
w	x	y	z	a	b	c	d	e	f	g
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1

a. K-map for output channel a (ABCD := wxyz)



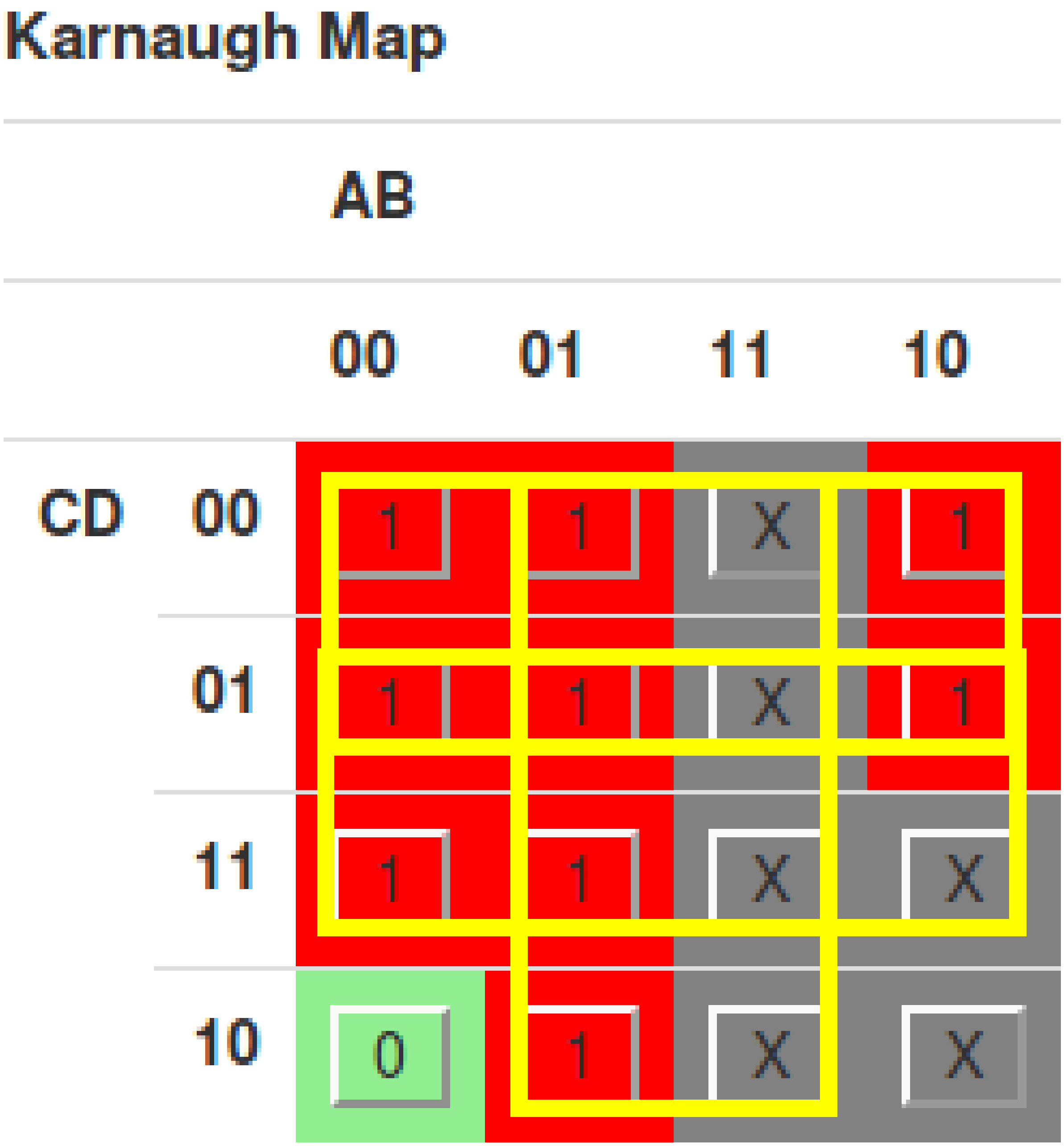
So **a = w + y + xz + x'z'**

b. K-map for output channel b (ABCD := wxyz)



So **b = x' + y'z' + yz**

c. K-map for output channel c (ABCD := wxyz)



So **c = x + y' + z**

d. K-map for output channel d (ABCD := wxyz)

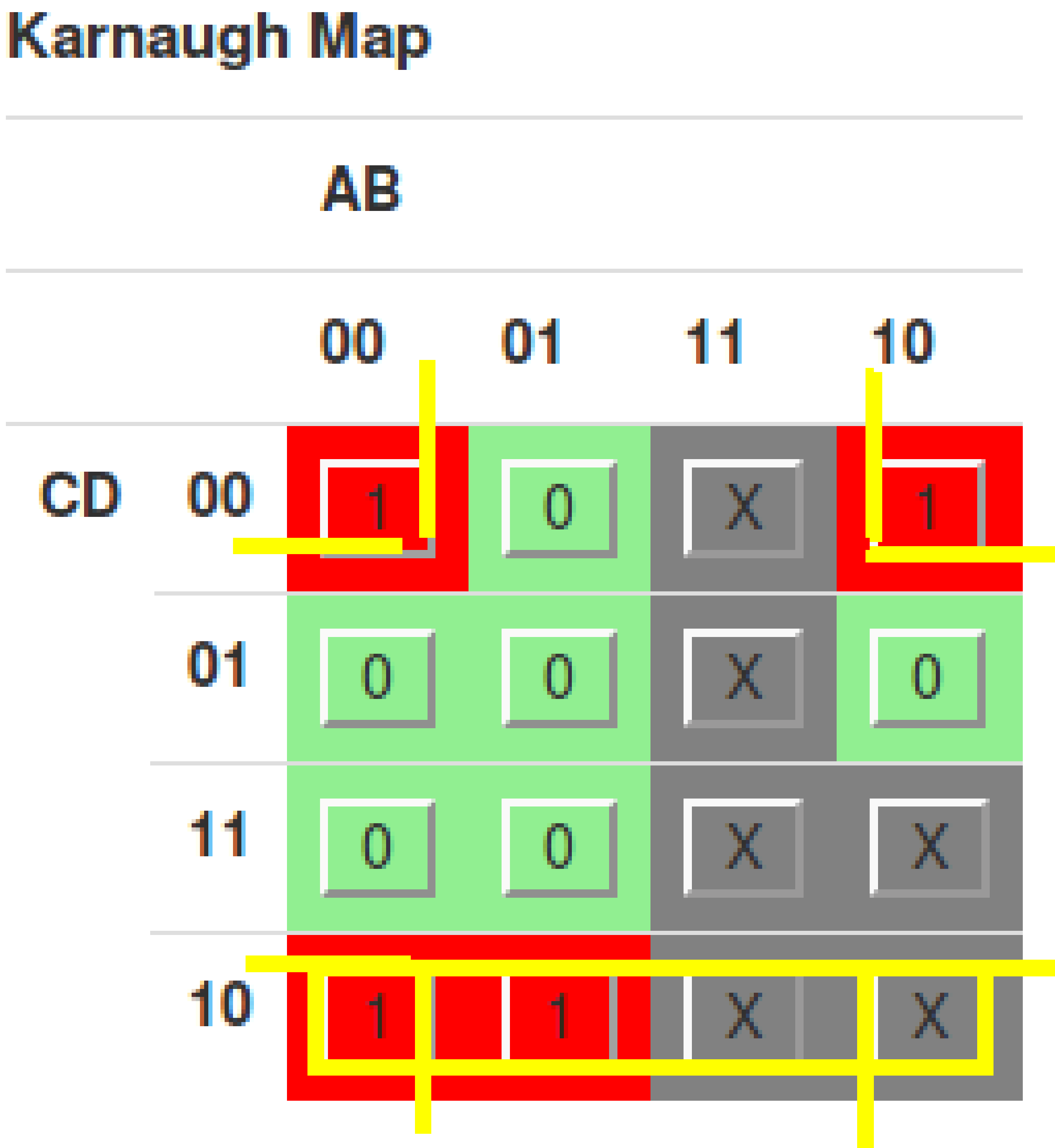
Karnaugh Map

AB

CD	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	0	X	X
10	1	1	X	X

So **d = w + x'z' + x'y + yz' + xy'z**

e. K-map for output channel e (ABCD := wxyz)



So **e = x'z' + yz'**

f. K-map for output channel f (ABCD := wxyz)

So **f = w + y'z' + xz' + xy'**

g. K-map for output channel g (ABCD := wxyz)

So **g = w + xy' + x'y + yz'**

Output Summary

a = w + y + xz + x'z'

b = x' + y'z' + yz

c = x + y' + z

d = w + x'z' + x'y + yz' + xy'z

e = x'z' + yz'

f = w + y'z' + xz' + xy'

g = w + xy' + x'y + yz'

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3. Problem 7.1 in the Kohavi and Jha text

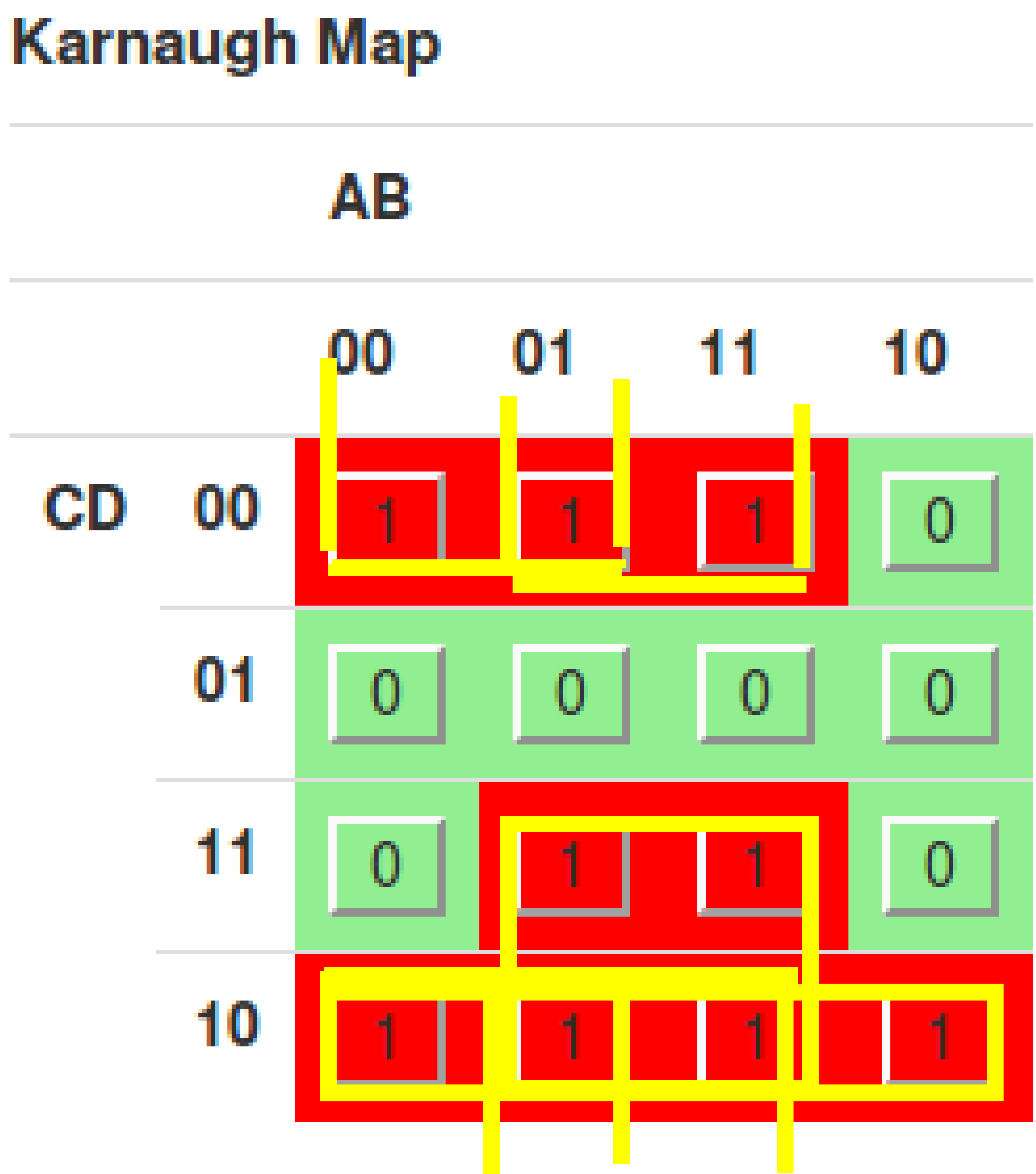
a. Question A

First is the truth table for this unit:

x1	x2	x3	x4	w1	w2	w3	w4	z=sigma(xi*wi)	T	f
0	0	0	0	-1	2	2	-3	0	-1/2	1
0	0	0	1	-1	2	2	-3	-3	-1/2	0
0	0	1	0	-1	2	2	-3	2	-1/2	1
0	0	1	1	-1	2	2	-3	-1	-1/2	0
0	1	0	0	-1	2	2	-3	2	-1/2	1
0	1	0	1	-1	2	2	-3	-1	-1/2	0
0	1	1	0	-1	2	2	-3	4	-1/2	1
0	1	1	1	-1	2	2	-3	1	-1/2	1
1	0	0	0	-1	2	2	-3	-1	-1/2	0
1	0	0	1	-1	2	2	-3	-4	-1/2	0
1	0	1	0	-1	2	2	-3	1	-1/2	1

x1	x2	x3	x4	w1	w2	w3	w4	z=sigma(xi*wi)	T	f
1	0	1	1	-1	2	2	-3	-2	-1/2	0
1	1	0	0	-1	2	2	-3	1	-1/2	1
1	1	0	1	-1	2	2	-3	-2	-1/2	0
1	1	1	0	-1	2	2	-3	3	-1/2	1
1	1	1	1	-1	2	2	-3	0	-1/2	1

Below is the corresponding K-map:



The final minimal SOP expression is:

f = w'z' + xz' + yz' + xy

b. Question B

First is the truth table for this unit:

x1	x2	x3	x4	gw1	gw2	gw3	gw4	zg=sigma(xi*gwi)	T	g	fw1	fw2
0	0	0	0	-2	-2	1	1	0	-1/2	1	2	2
0	0	0	1	-2	-2	1	1	1	-1/2	1	2	2
0	0	1	0	-2	-2	1	1	1	-1/2	1	2	2
0	0	1	1	-2	-2	1	1	2	-1/2	1	2	2
0	1	0	0	-2	-2	1	1	-2	-1/2	0	2	2
0	1	0	1	-2	-2	1	1	-1	-1/2	0	2	2
0	1	1	0	-2	-2	1	1	-1	-1/2	0	2	2
0	1	1	1	-2	-2	1	1	0	-1/2	1	2	2
1	0	0	0	-2	-2	1	1	-2	-1/2	0	2	2
1	0	0	1	-2	-2	1	1	-1	-1/2	0	2	2
1	0	1	0	-2	-2	1	1	-1	-1/2	0	2	2
1	0	1	1	-2	-2	1	1	0	-1/2	1	2	2
1	1	0	0	-2	-2	1	1	-4	-1/2	0	2	2

x1	x2	x3	x4	gw1	gw2	gw3	gw4	zg=sigma(xi*gwi)	T	g	fw1	fw2
1	1	0	1	-2	-2	1	1	-3	-1/2	0	2	2
1	1	1	0	-2	-2	1	1	-3	-1/2	0	2	2
1	1	1	1	-2	-2	1	1	-2	-1/2	0	2	2

Below is the corresponding K-map:

Karnaugh Map

		AB			
		00	01	11	10
CD	00	1	0	1	0
	01	1	0	1	0
	11	1	1	1	1
	10	1	0	1	0

The final minimal SOP expression is:

f = w'x' + wx + yz

4. Problem 4.1, parts a and b in the Kohavi and Jha text

a. f = sigma(0, 1, 2, 3, 4, 6, 8, 9, 10, 11) (ABCD := wxyz)

Karnaugh Map

		AB			
		00	01	11	10
CD	00	1	1	0	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	1	0	1

So **f = x' + w'z'**

b. f = sigma(0, 1, 5, 7, 8, 10, 14, 15) (ABCD := wxyz)

Karnaugh Map

		AB			
		00	01	11	10
CD	00	1	0	0	1
	01	1	1	0	0
	11	0	1	1	0
	10	0	0	1	1

So **f = x'y'z' + w'y'z + xyz + wyz'**

5. Problem 4.2, part b in the Kohavi and Jha text

Karnaugh Map

		AB			
		00	01	11	10
CD	00	1	1	1	0
	01	X	X	0	1
	11	0	X	1	0
	10	1	0	0	X

So **f = w'y' + xy'z' + x'y'z + xyz + x'yz'**

6. Problem 4.14 in the Kohavi and Jha text

Below is the original map:

Karnaugh Map

		AB			
		00	01	11	10
CD	00	0	1	0	1
	01	1	1	0	1
	11	1	1	1	1
	10	0	0	1	0

a. Use the map to obtain the set of all prime implicants and indicate specifically the essential ones

First is the map with essential prime implicants marked out:

Karnaugh Map

		AB			
		00	01	11	10
CD	00	0	1	0	1
	01	1	1	0	1
	11	1	1	1	1
	10	0	0	1	0

They are:

1. w'xy'
2. wx'y'
3. wxy

Then are the three non-essential prime implicants:

Karnaugh Map

		AB			
		00	01	11	10
CD	00	0	1	0	1
	01	1	1	0	1
	11	1	1	1	1
	10	0	0	1	0

They are:

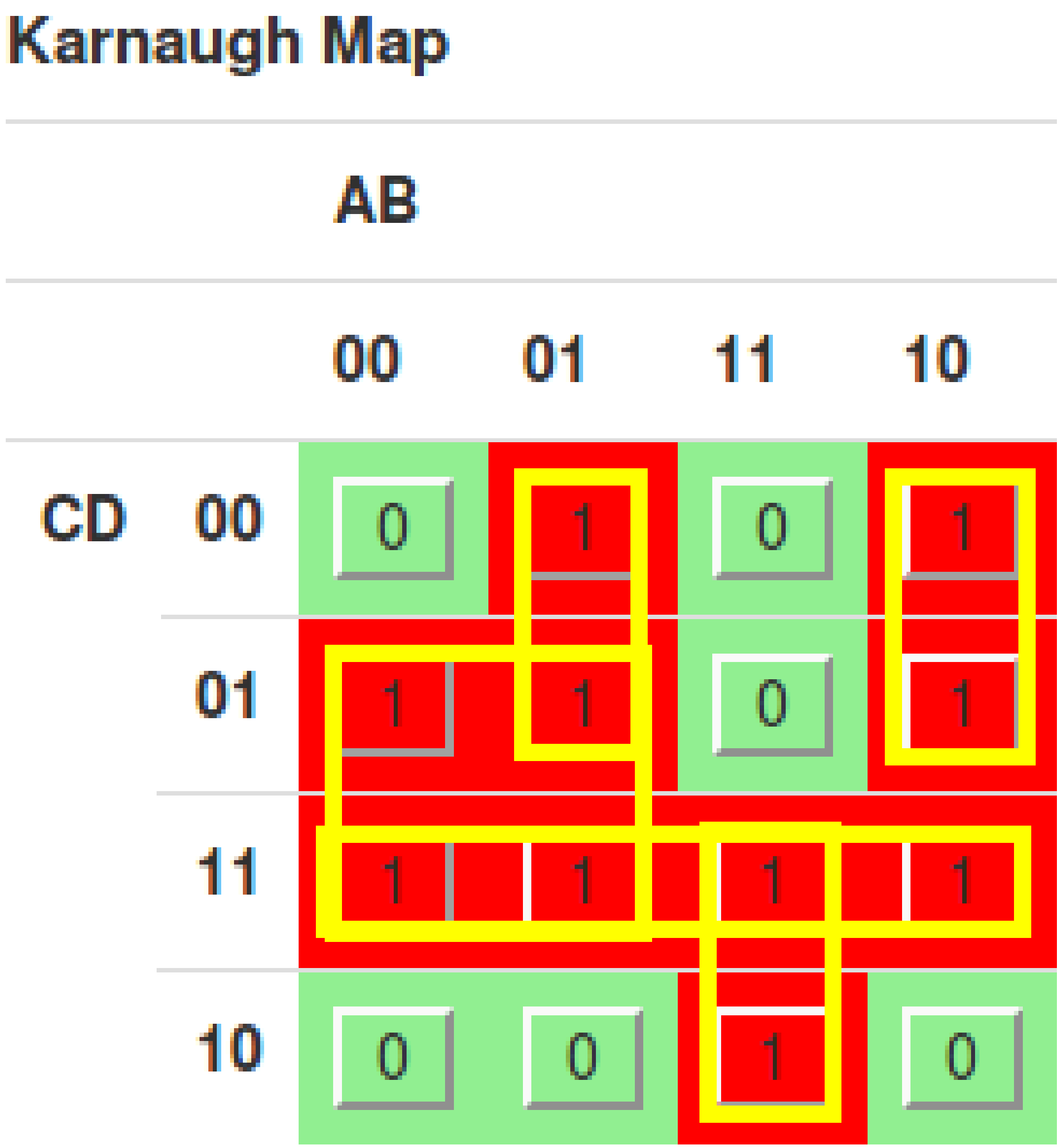
1. w'z
2. yz
3. x'z

So the final set of all prime implicants is:

P = {w'xy', wx'y', wxy, w'z, yz, x'z}

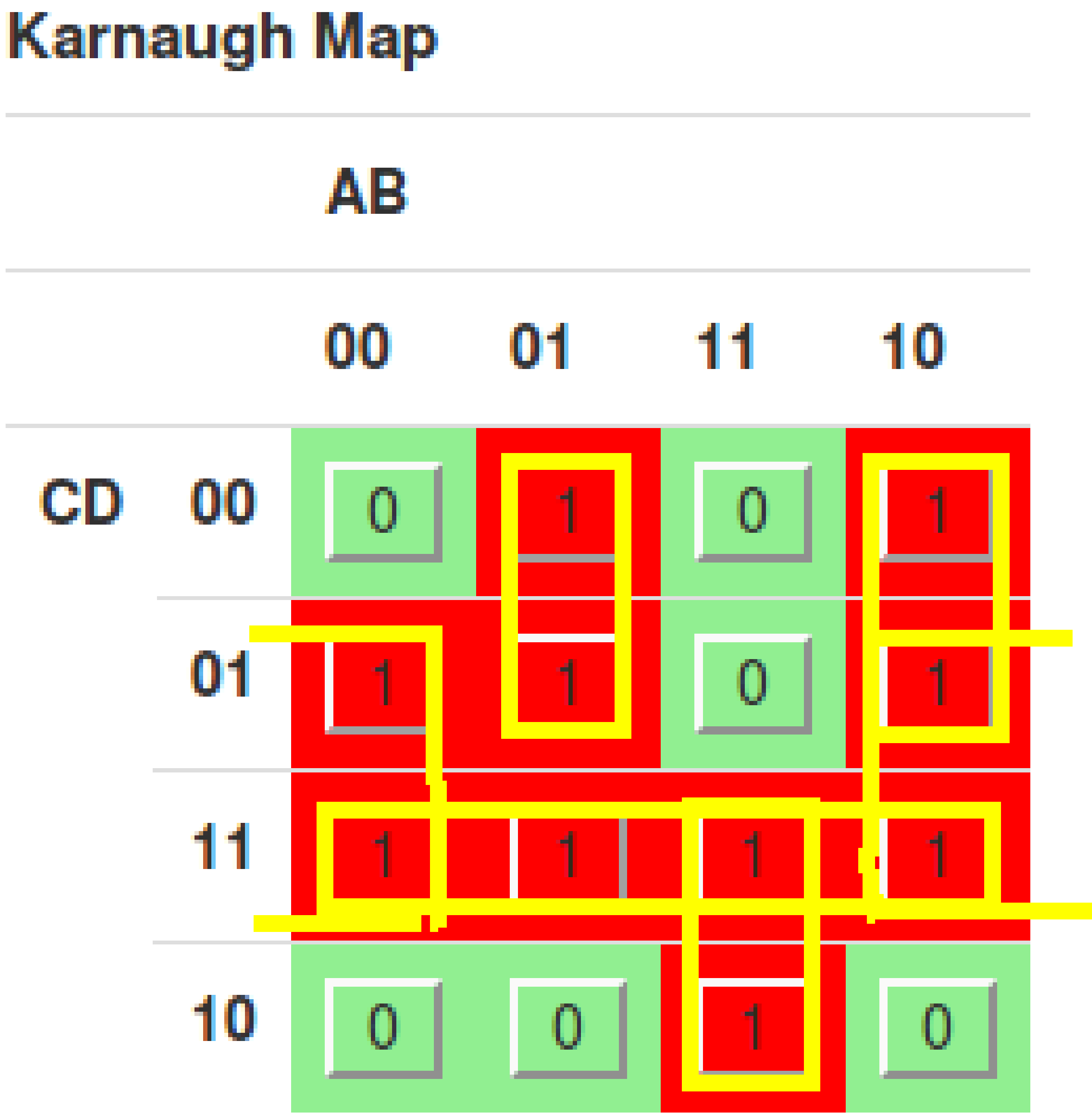
b. Find three distinct minimal expression for T

1. First (ABCD := wxyz)



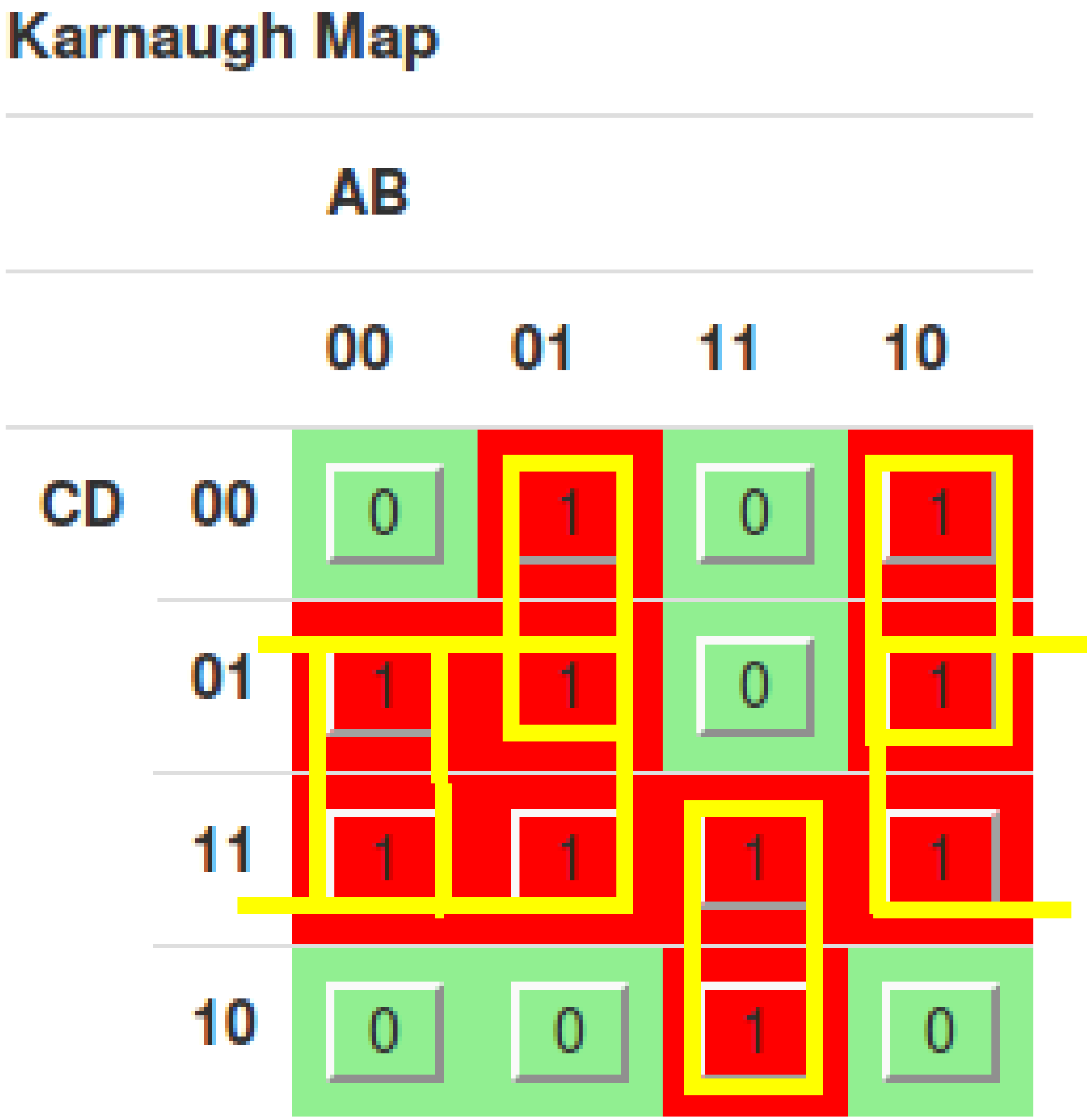
T = w'z + yz + w'xy' + wx'y' + wxy

2. Second (ABCD := wxyz)



T = x'z + yz + w'xy' + wx'y' + wxy

3. Third (ABCD := wxyz)



T = x'z + w'z + w'xy' + wx'y' + wxy

c. Find the complement T' directly from the map

Below is the map for the complement T'

Karnaugh Map

		AB			
		00	01	11	10
CD	00	1	0	1	0
	01	0	0	1	0
	11	0	0	0	0
	10	1	1	0	1

Karnaugh Map

		AB			
		00	01	11	10
CD	00	1	0	1	0
	01	0	0	1	0
	11	0	0	0	0
	10	1	1	0	1

T' = w'x'z' + wxy' + w'yz' + x'yz'

d. Assume that only unprimed variables are available and construct a circuit that realizes T and requires no more than 13 gate inputs and two NOT gates

Sorry up to now I have no clue. Still working on it.

7. Problem 4.18 in the Kohavi and Jha text

a. Let f(x1, x2, ..., xn) be equal to 1 if and only if exactly k of the variables equal 1. How many prime implicants does this function have?

Sorry up to now I have no clue. Still working on it.

b. Repeat (a) for the case where f assumes the value 1 if and only if k or more of the variables are equal to 1

Sorry up to now I have no clue. Still working on it.

8. Problem 4.19 in the Kohavi and Jha text

a. Let T(A, B, C, D) = A'BC + B'C'D. Prove that any expression for T must contain at least one instance of the literal D or of the literal D'

Sorry up to now I have no clue. Still working on it.

b. If in a minimal sum-of-products expression, each variable appears either in a primed form or in an unprimed form but not in both then the function is said to be unate. Prove that the minimal sum-of-products form of a unate function is unique.

Sorry up to now I have no clue. Still working on it.

c. Is the converse true, i.e., if the minimal sum-of-products expression is unique then the function is unate?

Sorry up to now I have no clue. Still working on it.