LIO on Wheels

1. Assume odo's measurement is available in IMU frame:

that is. new measurement is available as $\begin{bmatrix} w_b \\ v_b \end{bmatrix}$, in which:

for diff. drive:

$$w_b = \begin{bmatrix} 0 \\ 0 \\ w_{b,z} \end{bmatrix} \qquad v_b = \begin{bmatrix} v_{b,x} \\ 0 \\ 0 \end{bmatrix}$$

2. So the relative pose constraint, imposed by Odo-Pre-Integration, can be stated as:

$$\dot{p}_{wb_i} = q_{wb_i} \cdot v_b \qquad = C_{wb_i} \cdot v_b$$

$$\dot{q}_{wb_i} = q_{wb_i} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} w_b \end{bmatrix}, \quad \dot{C}_{wb_i} = C_{wb_i} [w_b]^\times$$

$\Rightarrow$: $p_{wb_{k+1}} = p_{wb_k} + vT$, $v = \frac{1}{2}\left[C_k v_{b_k} + C_{k+1} v_{b_{k+1}}\right]$

$$C_{wb_{k+1}} = C_{wb_k} \exp(wT), \quad w = \frac{1}{2}\left[w_k + w_{k+1}\right]$$

3. For error propagation:

$$\because \quad \dot{p} = C v_b$$

$$\therefore \quad \dot{p} + \delta\dot{p} = C[I + \delta\theta^{\times}](v_b + n_v)$$

$$\therefore \quad \delta\dot{p} = C\delta\theta^{\times} v_b + C n_v$$

$$= -C v_b^{\times} \delta\theta + C n_v$$

$$\dot{C} = C w_b^{\times}$$

$$\therefore \dot{C} + \dot{C}\delta\theta^{\times} + C\delta\dot{\theta}^{\times} = C[I + \delta\theta^{\times}](w_b + n_w)^{\times}$$

$$= C w_b^{\times} + C\delta\theta^{\times} w_b^{\times} + C n_w^{\times}$$

$$\therefore C w_b^{\times}\delta\theta^{\times} + C\delta\dot{\theta}^{\times} = C\delta\theta^{\times} w_b^{\times} + C n_w^{\times}$$

$$\therefore \delta\dot{\theta}^{\times} = (\delta\theta^{\times} w_b^{\times} - w_b^{\times}\delta\theta^{\times}) + n_w^{\times}$$

$$\therefore \delta\dot{\theta} = \delta\theta^{\times} w_b + n_w$$

$$= -w_b^{\times}\delta\theta + n_w$$

$$\Rightarrow: \quad \delta\dot{p} = -C v_b^{\times}\delta\theta + C n_v$$

$$\delta\dot{\theta} = -w_b^{\times}\delta\theta + n_w$$

4. Error Propagation, Discretized:

$$\delta\theta_{k+1} = (I - \bar{w}^{\times} T)\delta\theta_k + \tfrac{1}{2} T \eta_{w,k} + \tfrac{1}{2} T \eta_{w,k+1}$$

$$\begin{aligned}\delta p_{k+1} = \delta p_k &- \tfrac{1}{2} T \left[C_k v_k^{\times} + C_k v_{k+1}^{\times} (I - \bar{w}^{\times} T) \right] \delta\theta_k \\ &+ \tfrac{1}{2} T C_k \eta_{v,k} + \tfrac{1}{2} T C_{k+1} \eta_{v,k+1} \\ &- \tfrac{1}{4} T^2 C_{k+1} v_{k+1}^{\times} (\eta_{w,k} + \eta_{w,k+1})\end{aligned}$$

5: Jacobian: NOT applicable to Odo pre-integration;

6: Residual:

$$r_\theta = \ln(C_{ij}^T C_{wi}^T C_{wj})$$

$$r_p = C_{wi}^T (p_j - p_i) - p_{ij}$$