Sensor Fusion - 07 Sliding Window - YanGe FAD

1. Jacobians for tightly coupled LOAM

$\because$ for lio-mapping

$$\bar{X}^{L}_{(k+1,i)} = T_{wk}^{-1} T_{wk+1} X^{L}_{(k+1,i)}$$
$$= R_k^T (R_{k+1} X^{L}_{(k+1,i)} + t_{k+1} - t_k)$$

$$\therefore \frac{\partial \bar{X}^{L}_{(k+1,i)}}{\partial R_k} = \bar{X}^{L\wedge}_{(k+1,i)}$$

$$\therefore \frac{\partial \bar{X}^{L}_{(k+1,i)}}{\partial t_k} = -R_k^T$$

$$\therefore \frac{\partial \bar{X}^{L}_{(k+1,i)}}{\partial R_{k+1}} = R_k^T \frac{\partial R_{k+1} X^{L}_{(k+1,i)}}{\partial R_{k+1}}$$
$$= -R_k^T R_{k+1} X^{L\wedge}_{(k+1,i)}$$

$$\therefore \frac{\partial \bar{X}^{L}_{(k+1,i)}}{\partial t_{k+1}} = R_k^T$$

$\because$ for point-line association:

$$\frac{\partial d_{pl}}{\partial \bar{X}^{L}_{(k+1,i)}} = \alpha^T$$

$$\alpha = \frac{v_{il} \times v_{im} \times v_{lm}}{\| v_{il} \times v_{im} \times v_{lm} \|_2}$$

in which:
$$\begin{cases} v_{il} = \bar{X}^{L}_{(k+1,i)} - \bar{X}^{L}_{(k,l)} \\ v_{im} = \bar{X}^{L}_{(k+1,i)} - \bar{X}^{L}_{(k,m)} \\ v_{lm} = \bar{X}^{L}_{(k,m)} - \bar{X}^{L}_{(k,l)} \end{cases}$$

$\because$ for point-plane association:

$$\frac{\partial d_{pp}}{\partial \bar{X}^{L}_{(k+1,i)}} = \beta^T$$

$$\beta = \mathrm{sgn}(c) \cdot n$$

in which:
$$n = \frac{(\bar{X}^{L}_{(k,j)} - \bar{X}^{L}_{(k,l)}) \times (\bar{X}^{L}_{(k,l)} - \bar{X}^{L}_{(k,m)})}{\|(\bar{X}^{L}_{(k,j)} - \bar{X}^{L}_{(k,l)}) \times (\bar{X}^{L}_{(k,j)} - \bar{X}^{L}_{(k,m)})\|_2}$$

$$c = (\bar{X}^{L}_{(k+1,i)} - \bar{X}^{L}_{(k,j)})^T n$$

$$\mathrm{sgn}(\sigma) = \begin{cases} 1 & \sigma \geq 0 \\ -1 & \sigma < 0 \end{cases}$$

$\because$ Jacobian for tightly-coupled LOAM are

a. point-line:

$$\text{pose } k\,(i) \begin{cases} \dfrac{\partial d_{pL}}{\partial R_k} = \alpha^T \bar{X}^{L}_{(k+1,i)}{}^{\wedge} \\ \dfrac{\partial d_{pl}}{\partial t_k} = -\alpha^T R_k^T \end{cases}$$

$$\text{pose } k+1\,(j) \begin{cases} \dfrac{\partial d_{pl}}{\partial R_{k+1}} = -\alpha^T R_k^T R_{k+1} X^{L}_{(k+1,i)}{}^{\wedge} \\ \dfrac{\partial d_{pl}}{\partial t_{k+1}} = \alpha^T R_k^T \end{cases}$$

b. point-plane

$$\text{pose } k\,(i) \begin{cases} \dfrac{\partial d_{pp}}{\partial R_k} = \beta^T \bar{X}^{L}_{(k+1,i)}{}^{\wedge} \\ \dfrac{\partial d_{pp}}{\partial t_k} = -\beta^T R_k^T \end{cases}$$

$$\text{pose } k+1\,(j) \begin{cases} \dfrac{\partial d_{pp}}{\partial R_{k+1}} = -\beta^T R_k^T R_{k+1} X^{L}_{(k+1,i)}{}^{\wedge} \\ \dfrac{\partial d_{pp}}{\partial t_{k+1}} = \beta^T R_k^T \end{cases}$$