IMU TK LM Derivation Yao Ge FAD.

$\because h(a^s, \theta^{acc}) = TK(a^s - B)$

in which:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ S_{ayx} & 1 & 0 \\ -S_{azx} & S_{azy} & 1 \end{bmatrix} \quad K = \begin{bmatrix} K_{ax} & 0 & 0 \\ 0 & K_{ay} & 0 \\ 0 & 0 & K_{az} \end{bmatrix} \quad B = \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \end{bmatrix}$$

$$\therefore h(a^s, \theta^{acc} + \Delta\theta) \doteq h(a^s, \theta^{acc}) + \frac{\partial h}{\partial \theta}\Big|_{a^s, \theta^{acc}} \Delta\theta$$

Define $a^{s(1)} := K(a^s - B)$

$\because a^0 = T a^{s(1)}$

$$= \begin{bmatrix} a_x^{s(1)} \\ S_{ayx} a_x^{s(1)} + a_y^{s(1)} \\ -S_{azx} a_x^{s(1)} + S_{azy} a_y^{s(1)} + a_z^{s(1)} \end{bmatrix}$$

$$\therefore \frac{\partial h}{\partial [S_{ayx}, S_{azx}, S_{azy}]} = \begin{bmatrix} 0 & 0 & 0 \\ a_x^{s(1)} & 0 & 0 \\ 0 & -a_x^{s(1)} & a_y^{s(1)} \end{bmatrix}$$

Define $a^{s(2)} = a^s - B$

$$\because a^0 = T \cdot K \cdot a^{s(2)} = T \cdot \mathrm{diag}(a^{s(2)}) \cdot \begin{bmatrix} K_{ax} \\ K_{ay} \\ K_{az} \end{bmatrix}$$

$$\therefore \frac{\partial h}{\partial [K_{ax}, K_{ay}, K_{az}]} = T \cdot \mathrm{diag}(a^{s(2)})$$

$$\therefore \frac{\partial h}{\partial \theta}\Big|_{a^s, \theta^{acc}} = \left[\begin{bmatrix} 0 & 0 & 0 \\ a_x^{s(1)} & 0 & 0 \\ 0 & -a_x^{s(1)} & a_y^{s(1)} \end{bmatrix}, T \cdot \mathrm{diag}(a^{s(2)}), -TK \right] \in R^{3 \times p}$$

$\therefore$ Define

$$L_k = \|g\|^2 - \|h(a_k^s, \theta^{acc})\|^2 = \|g\|^2 - h_k^T h_k$$

$$\therefore \frac{\partial L_k}{\partial h_k} = -2h_k^T$$

$\therefore$ Define:

$$e_k = \|g\|^2 - \|h(a_k^s, \theta^{acc}\|^2$$

$$J_k = \frac{\partial L_k}{\partial h_k} \cdot \frac{\partial h_k}{\partial \theta}$$

$$= -2h_k^T \left[\begin{matrix} 0 & 0 & 0 \\ a_x^{s(1)} & 0 & 0 \\ 0 & -a_x^{s(1)} & a_y^{s(1)} \end{matrix} , T\,\mathrm{diag}(a^{s(2)}), -TK \right]$$

$$\therefore L(\theta^{acc} + \Delta\theta) \doteq \sum_{k=1}^{M} (e_k + J_k\Delta\theta)^T (e_k + J_k\Delta\theta)$$

$$\therefore H = \sum_{k=1}^{M} J_k^T J_k + \mu I$$

$$\therefore b = -\sum_{k=1}^{M} J_k^T e_k$$

$\therefore$ Q.E.D.