

```
import pandas as pd
# Read data. This data represents the cumulative known cases to date (https://covidtracking.com/about-data/faq)
url = 'https://raw.githubusercontent.com/COVID19Tracking/covid-tracking-data/master/data/states_daily_4pm_et.csv'
df = pd.read_csv(url,index_col=0,parse_dates=[0])
```

```
df.head(5)
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	inIcuCurrently	inIcuCumulative	onVentilatorCur
date									
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	NaN	NaN	
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	335.0	
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	NaN	NaN	
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	NaN	
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	291.0	NaN	

Double-click (or enter) to edit

```
# Drop total, posNeg, and hospitalized columns as they are redundant
# Drop other columns that will not be used
#df_drop = df.drop(columns = [6, 7, 8, 9, 11, 12, 14, 15, 17, 18, 19, 20, 21, 22, 23])
df_drop = df.drop(columns = ['inIcuCurrently', 'inIcuCumulative',
                             'onVentilatorCurrently', 'onVentilatorCumulative',
                             'hash', 'dateChecked', 'hospitalized', 'total',
                             'posNeg', 'fips', 'deathIncrease',
                             'hospitalizedIncrease', 'negativeIncrease',
                             'positiveIncrease', 'totalTestResultsIncrease'])
df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults
date									
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0

```
# Create new features
# Divide positive by totalTestResults to get positive_percent
df_drop["percent_positive"] = ""
df_drop["percent_positive"] = 100*df_drop["positive"]/df_drop["totalTestResults"]
df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.7%
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.0%
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.5%
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.0%
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.7%

```
# Divide hospitalized by positive to get hospitalized_percent
import numpy as np
df_drop["hospitalized_percent"] = ""
df_drop["hospitalized_percent"] = np.nanmax(df_drop[['hospitalizedCurrently', 'hospitalizedCumulative']], axis=1)
df_drop["hospitalized_percent"] = 100*df_drop["hospitalized_percent"]/df_drop["positive"]
df_drop.head()
```

⏏ /usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:3: RuntimeWarning: All-NaN axis encountered
This is separate from the ipykernel package so we can avoid doing imports until

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.7%
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.0%
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.5%
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.0%
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.7%

```
# Divide recovered by positive to get recovered_percent
df_drop["recovered_percent"] = ""
df_drop["recovered_percent"] = 100*df_drop["recovered"]/df_drop["positive"]
df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.7%
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.0%
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.5%
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.0%
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.7%

```
# Divide death by positive to get death_percent
df_drop["death_percent"] = ""
df_drop["death_percent"] = 100*df_drop["death"]/df_drop["positive"]
df_drop.head()
```

⏏

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.70
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.06
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.55
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.00
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.72

```
# Fetch the latest state population data (nst-est2019-01.csv)
from google.colab import files
uploaded = files.upload()

Choose Files nst-est2019-01.csv
• nst-est2019-01.csv(application/vnd.ms-excel) - 676 bytes, last modified: 4/13/2020 - 100% done
Saving nst-est2019-01.csv to nst-est2019-01.csv
```

```
# Load latest state population data
import io
df_state_pop = pd.read_csv(io.StringIO(uploaded['nst-est2019-01.csv'].decode('utf-8')))
df_state_pop["Population"] = pd.to_numeric(df_state_pop["Population"])
df_state_pop.head()
```

	State	Population
0	AK	731545.0
1	AL	4903185.0
2	AR	3017804.0
3	AS	NaN
4	AZ	7278717.0

```
# Add column of state populations (population) to df_drop_total_posNeg
# Need to sort rows by state using index numbering from state_list

df_drop["population"] = ""

for i in range(len(df_drop)):
    for index in range(len(df_state_pop)):
        if df_drop.iloc[i, 0] == df_state_pop.iloc[index, 0]:
            df_drop.iloc[i, 13] = df_state_pop.iloc[index, 1]

df_drop[["population"]] = df_drop["population"].apply(pd.to_numeric)

df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.70
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.06
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.55
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.00
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.72

```
# Normalize positive to state population
df_drop["positive_norm"] = ""
df_drop["positive_norm"] = df_drop["positive"]/df_drop["population"]
df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_posi
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.70
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.00
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.50
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.00
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.70

```
# Normalize hospitalized to state population
df_drop["hospitalized_norm"] = ""
df_drop["hospitalized_norm"] = np.nanmax(df_drop[['hospitalizedCurrently','hospitalizedCumulative']], axis=1)
df_drop["hospitalized_norm"] = df_drop["hospitalized_norm"]/df_drop["population"]
df_drop.head()
```

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:2: RuntimeWarning: All-NaN axis encountered

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_posi
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.70
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.00
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.50
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.00
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.70

```
# Normalize recovered to state population
df_drop["recovered_norm"] = ""
df_drop["recovered_norm"] = df_drop["recovered"]/df_drop["population"]
df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_posi
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.70
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.00
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.50
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.00
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.70

```
# Normalize death to state population
df_drop["death_norm"] = ""
df_drop["death_norm"] = df_drop["death"]/df_drop["population"]
df_drop.head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.70
2020-05-02	AL	7434.0	84775.0	NaN	NaN	1023.0	NaN	288.0	92209.0	8.06
2020-05-02	AR	3372.0	48210.0	NaN	95.0	414.0	1987.0	73.0	51582.0	6.55
2020-05-02	AS	0.0	57.0	NaN	NaN	NaN	NaN	0.0	57.0	0.00
2020-05-02	AZ	8364.0	69633.0	NaN	718.0	1339.0	1565.0	348.0	77997.0	10.72

```
df_drop.info()
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 3265 entries, 2020-05-02 to 2020-01-22
Data columns (total 18 columns):
#   Column              Non-Null Count  Dtype
---  -
0   state                3265 non-null   object
1   positive              3250 non-null   float64
2   negative              3084 non-null   float64
3   pending               671 non-null    float64
4   hospitalizedCurrently  1152 non-null   float64
5   hospitalizedCumulative 1207 non-null   float64
6   recovered             997 non-null    float64
7   death                2538 non-null   float64
8   totalTestResults      3263 non-null   float64
9   percent_positive      3219 non-null   float64
10  hospitalized_percent    1822 non-null   float64
11  recovered_percent      997 non-null    float64
12  death_percent          2486 non-null   float64
13  population             3073 non-null   float64
14  positive_norm           3073 non-null   float64
15  hospitalized_norm       1783 non-null   float64
16  recovered_norm          913 non-null    float64
17  death_norm             2395 non-null   float64
dtypes: float64(17), object(1)
memory usage: 564.6+ KB
```

```
# Get the unique values of 'state' column
state_list = df.state.unique()
state_list
```

```
array(['AK', 'AL', 'AR', 'AS', 'AZ', 'CA', 'CO', 'CT', 'DC', 'DE', 'FL',
       'GA', 'GU', 'HI', 'IA', 'ID', 'IL', 'IN', 'KS', 'KY', 'LA', 'MA',
       'MD', 'ME', 'MI', 'MN', 'MO', 'MP', 'MS', 'MT', 'NC', 'ND', 'NE',
       'NH', 'NJ', 'NM', 'NV', 'NY', 'OH', 'OK', 'OR', 'PA', 'PR', 'RI',
       'SC', 'SD', 'TN', 'TX', 'UT', 'VA', 'VI', 'VT', 'WA', 'WI', 'WV',
       'WY'], dtype=object)
```

```
#create a data frame dictionary to store the state data frames
df_state_dict = {elem : pd.DataFrame for elem in state_list}
```

```
for key in df_state_dict.keys():
    df_state_dict[key] = df_drop[df_drop.state == key]
```

```
df_state_dict['AK'].head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos:
date										
2020-05-02	AK	365.0	21034.0	NaN	10.0	NaN	261.0	9.0	21399.0	1.7%
2020-05-01	AK	364.0	19961.0	NaN	25.0	NaN	254.0	9.0	20325.0	1.7%
2020-04-30	AK	355.0	18764.0	NaN	19.0	NaN	252.0	9.0	19119.0	1.8%
2020-04-29	AK	355.0	18764.0	NaN	14.0	NaN	240.0	9.0	19119.0	1.8%
2020-04-28	AK	351.0	16738.0	NaN	16.0	NaN	228.0	9.0	17089.0	2.0%

```
df_state_dict['CA'].head()
```

	state	positive	negative	pending	hospitalizedCurrently	hospitalizedCumulative	recovered	death	totalTestResults	percent_pos
date										
2020-05-02	CA	52197.0	634606.0	NaN	4722.0	NaN	NaN	2171.0	686803.0	7.5
2020-05-01	CA	50442.0	604543.0	NaN	4706.0	NaN	NaN	2073.0	654985.0	7.7
2020-04-30	CA	48917.0	576420.0	NaN	4981.0	NaN	NaN	1982.0	625337.0	7.8
2020-04-29	CA	46500.0	556639.0	NaN	5011.0	NaN	NaN	1887.0	603139.0	7.7
2020-04-28	CA	45031.0	532577.0	NaN	4983.0	NaN	NaN	1809.0	577608.0	7.7

```
from matplotlib import pyplot as plt
```

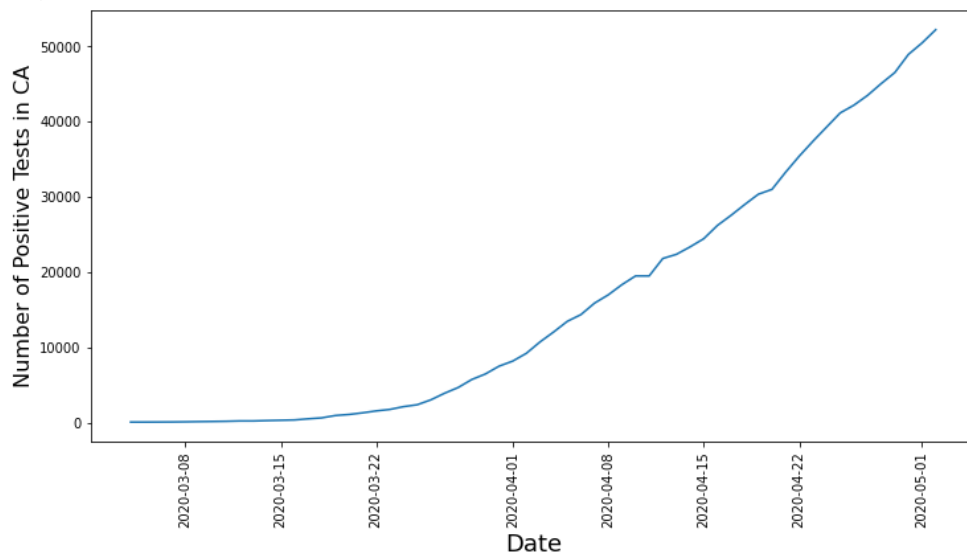
```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].positive)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Number of Positive Tests in CA', fontsize=16)
plt.show()
```

```
↳
```

No handles with labels found to put in legend.
<Figure size 432x288 with 0 Axes>

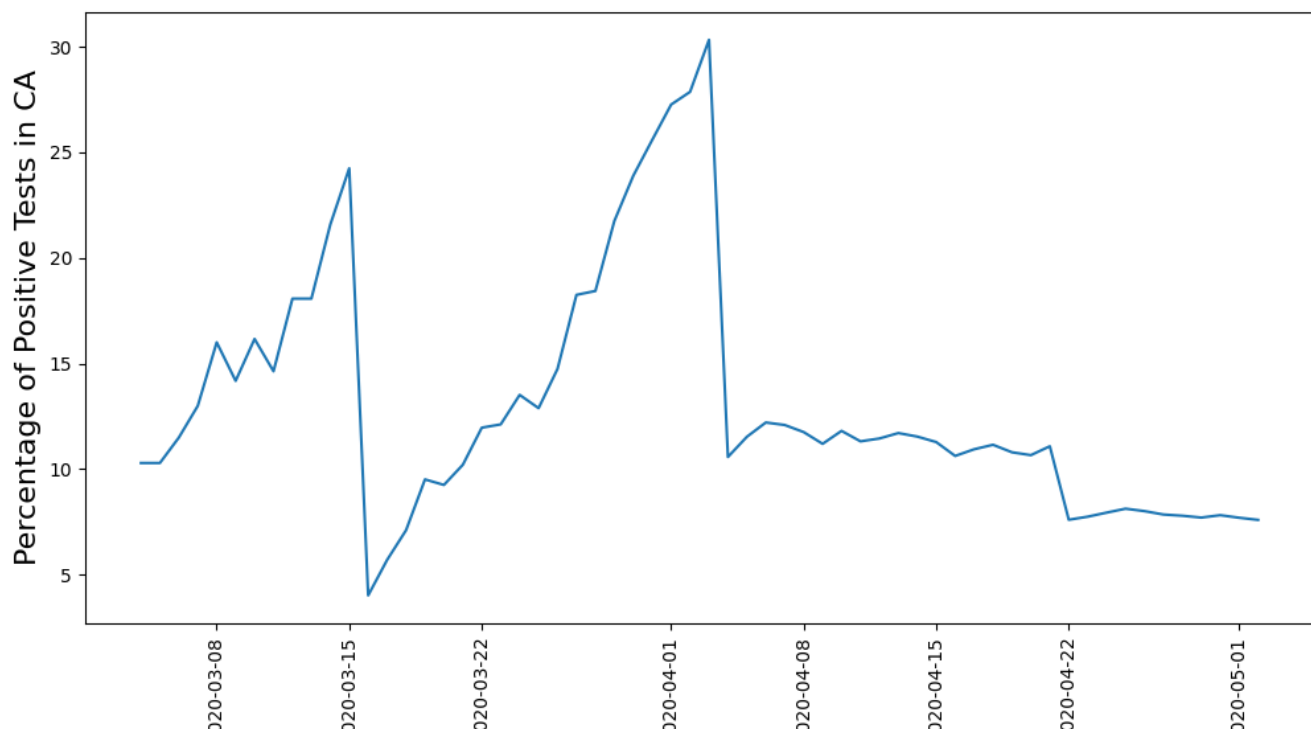


```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].percent_positive)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Percentage of Positive Tests in CA', fontsize=16)
plt.show()
```

⌵ No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>

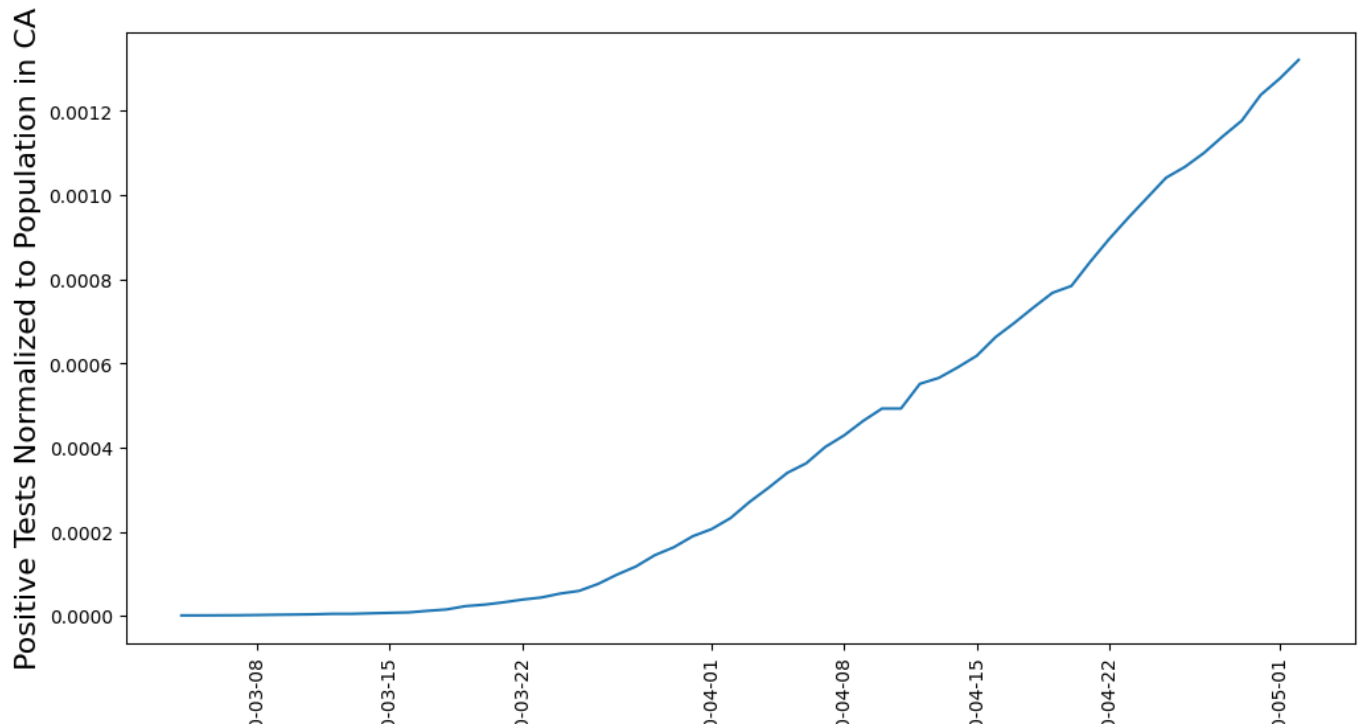


```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].positive_norm)
plt.xticks(rotation='vertical')
```

```
plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Positive Tests Normalized to Population in CA', fontsize=16)
plt.show()
```

⌵ No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>



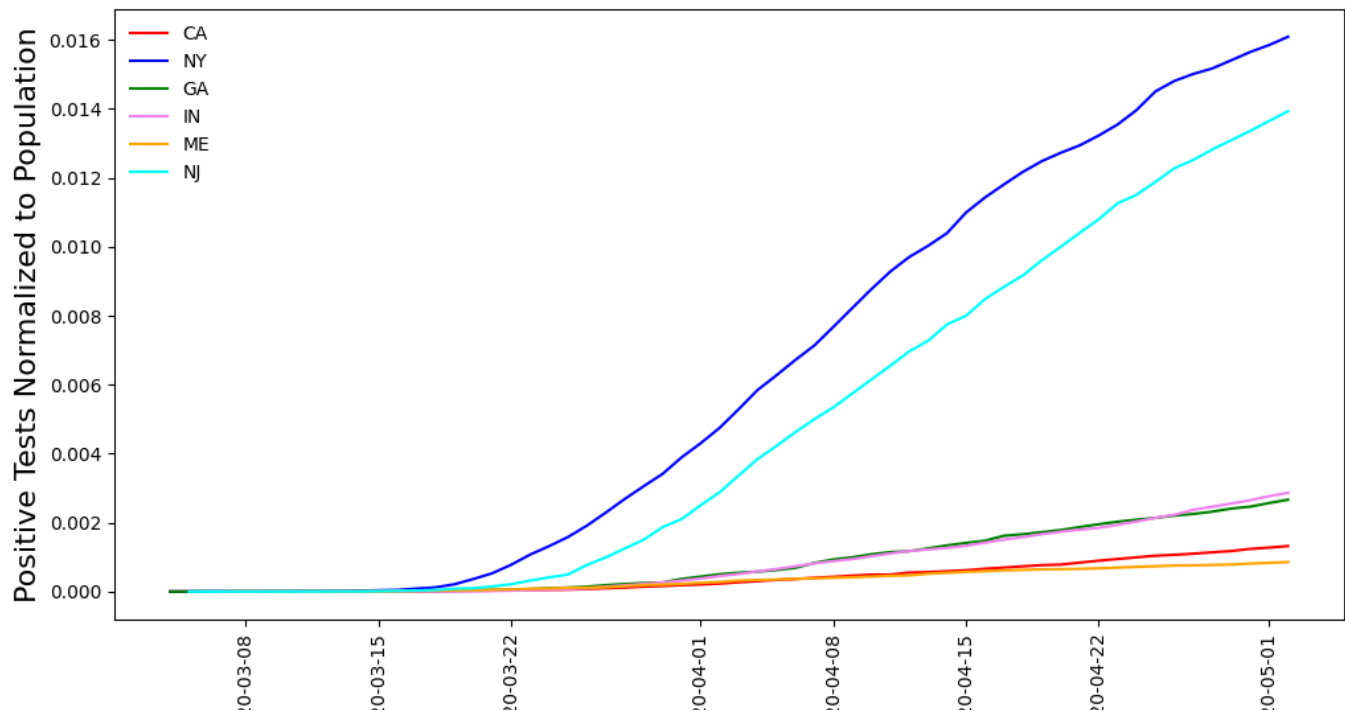
```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].positive_norm, color="red", label="CA")
plt.plot(df_state_dict['NY'].positive_norm, color="blue", label="NY")
plt.plot(df_state_dict['GA'].positive_norm, color="green", label="GA")
plt.plot(df_state_dict['IN'].positive_norm, color="violet", label="IN")
plt.plot(df_state_dict['ME'].positive_norm, color="orange", label="ME")
plt.plot(df_state_dict['NJ'].positive_norm, color="cyan", label="NJ")
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Positive Tests Normalized to Population', fontsize=16)
plt.show()
```

⌵

<Figure size 640x480 with 0 Axes>

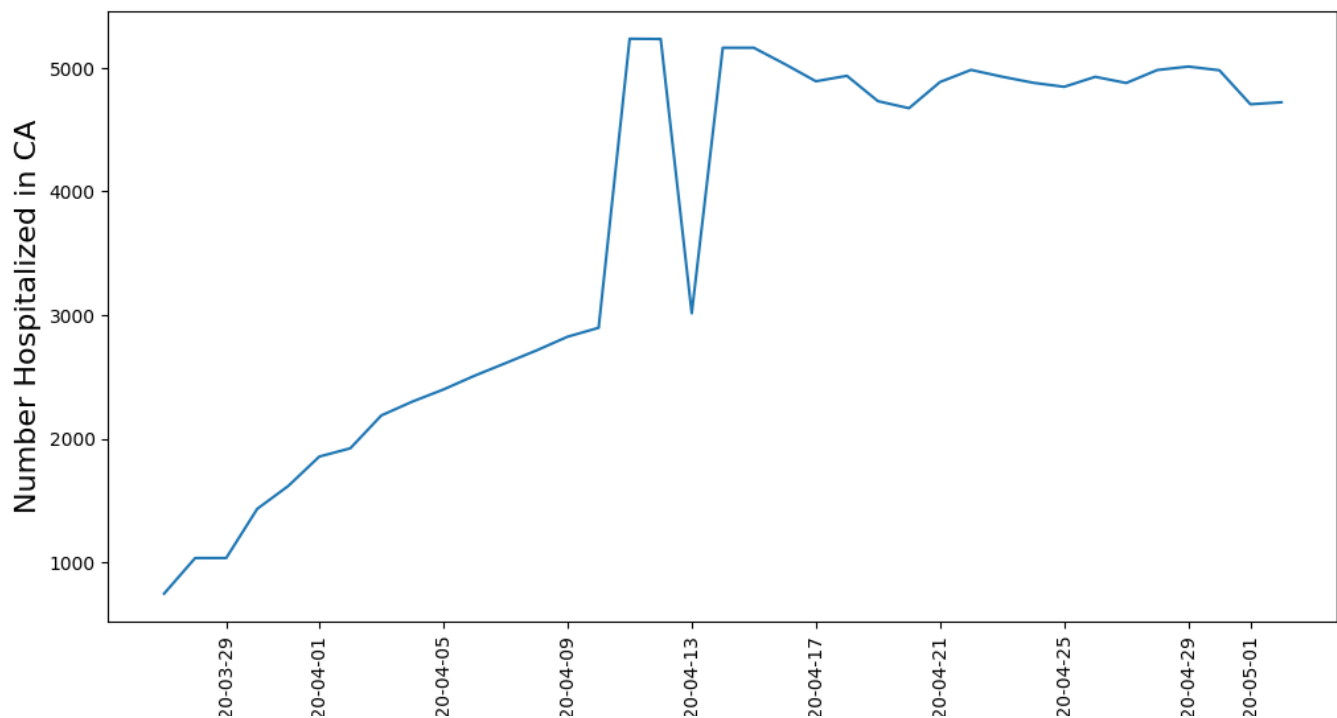


```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].hospitalizedCurrently)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Number Hospitalized in CA', fontsize=16)
plt.show()
```

⚠ No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>

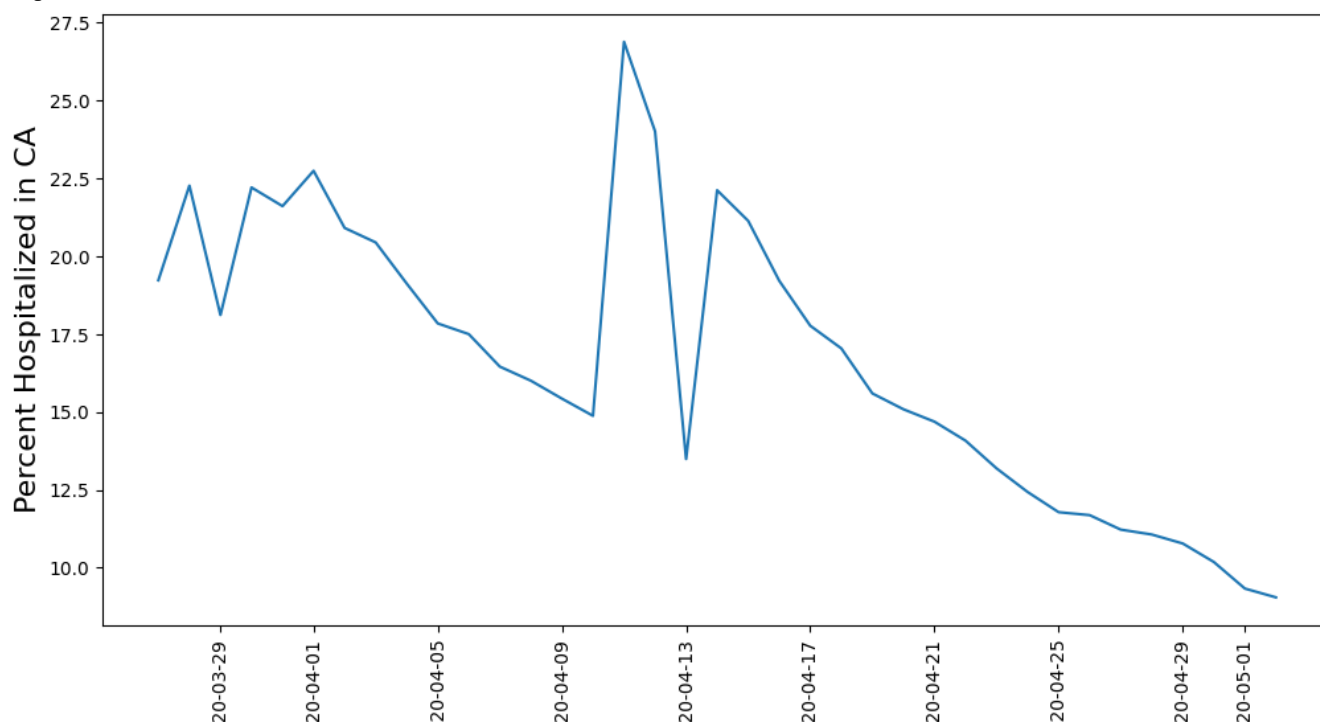


```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)
```

```
plt.plot(df_state_dict['CA'].hospitalized_percent)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Percent Hospitalized in CA', fontsize=16)
plt.show()
```

⌂ No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>



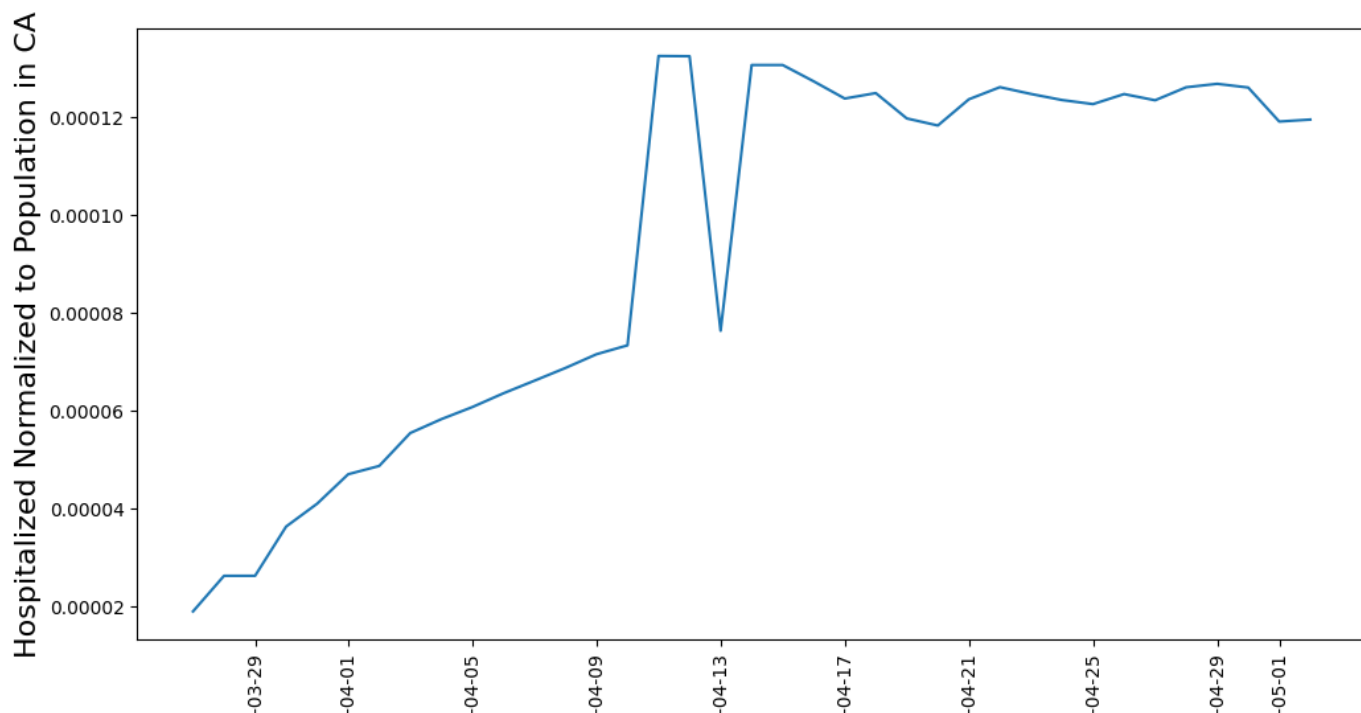
```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].hospitalized_norm)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Hospitalized Normalized to Population in CA', fontsize=16)
plt.show()
```

⌂

No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>



```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].hospitalized_norm, color="red", label="CA")
plt.plot(df_state_dict['NY'].hospitalized_norm, color="blue", label="NY")
plt.plot(df_state_dict['GA'].hospitalized_norm, color="green", label="GA")
plt.plot(df_state_dict['IN'].hospitalized_norm, color="violet", label="IN")
plt.plot(df_state_dict['ME'].hospitalized_norm, color="orange", label="ME")
plt.plot(df_state_dict['NJ'].hospitalized_norm, color="cyan", label="NJ")
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Hospitalized Normalized to Population', fontsize=16)
plt.show()
```



<Figure size 640x480 with 0 Axes>

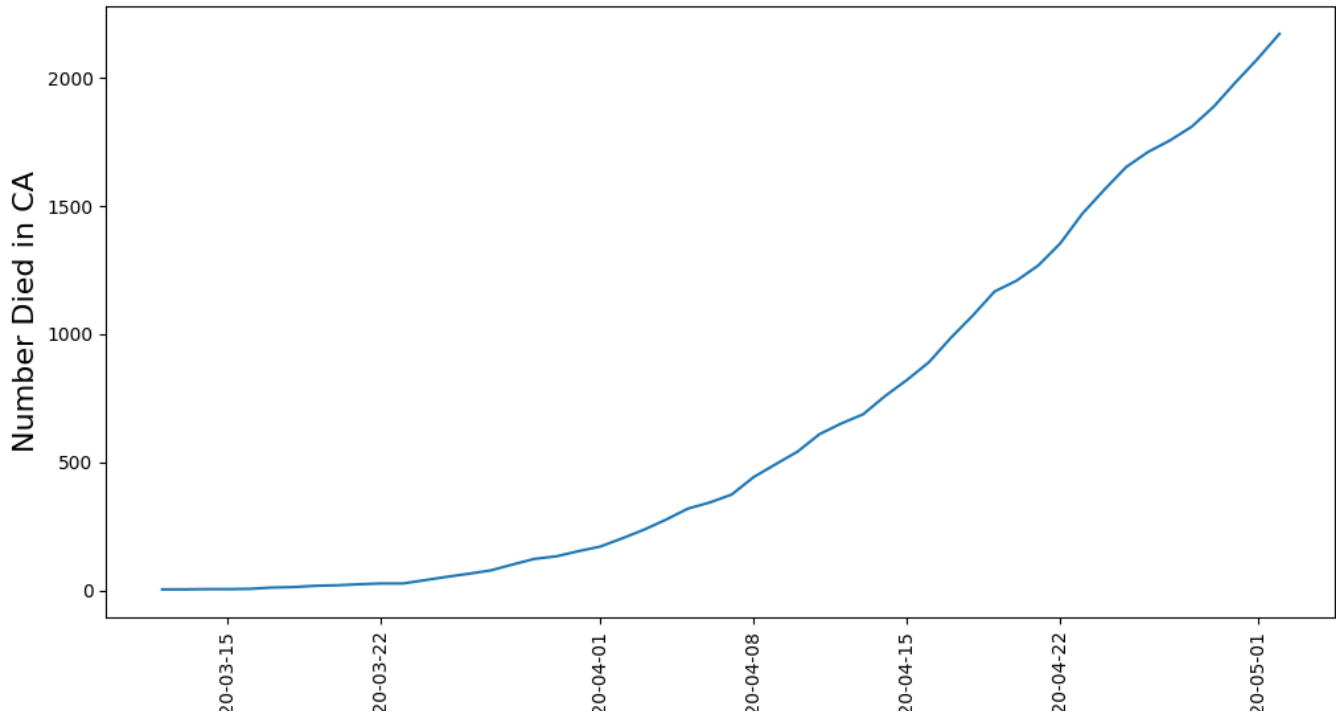
In several states, population normalized hospitalizations plateau, although population normalized death rate continues to grow.

```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].death)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Number Died in CA', fontsize=16)
plt.show()
```

⌘ No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>



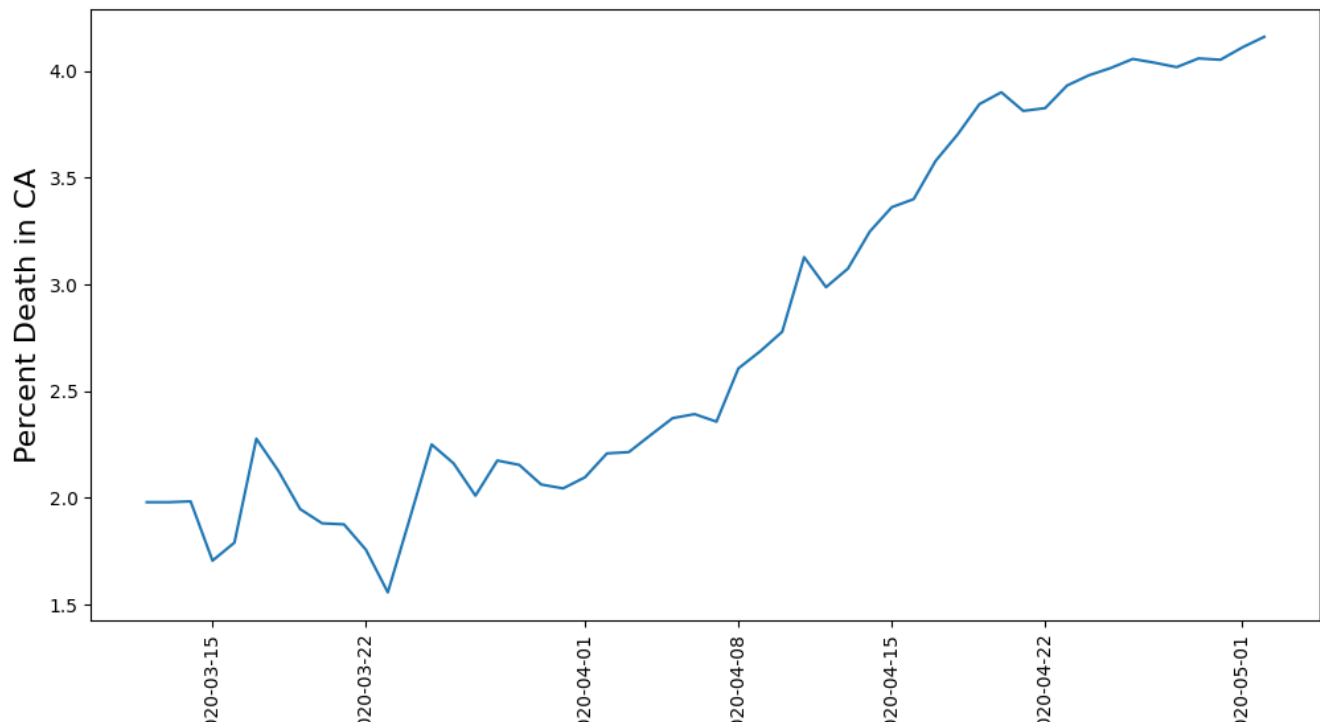
```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].death_percent)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Percent Death in CA', fontsize=16)
plt.show()
```

⌘

No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>

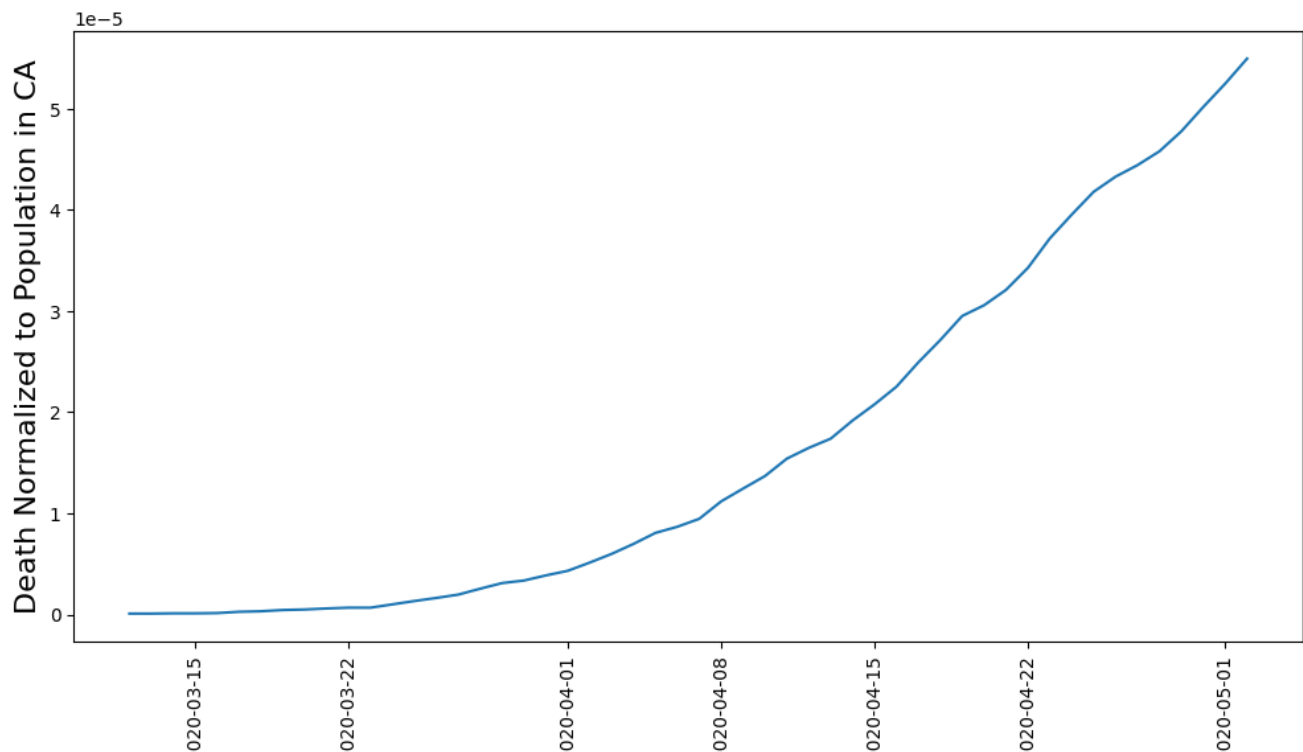


```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].death_norm)
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Death Normalized to Population in CA', fontsize=16)
plt.show()
```

✖ No handles with labels found to put in legend.
<Figure size 640x480 with 0 Axes>



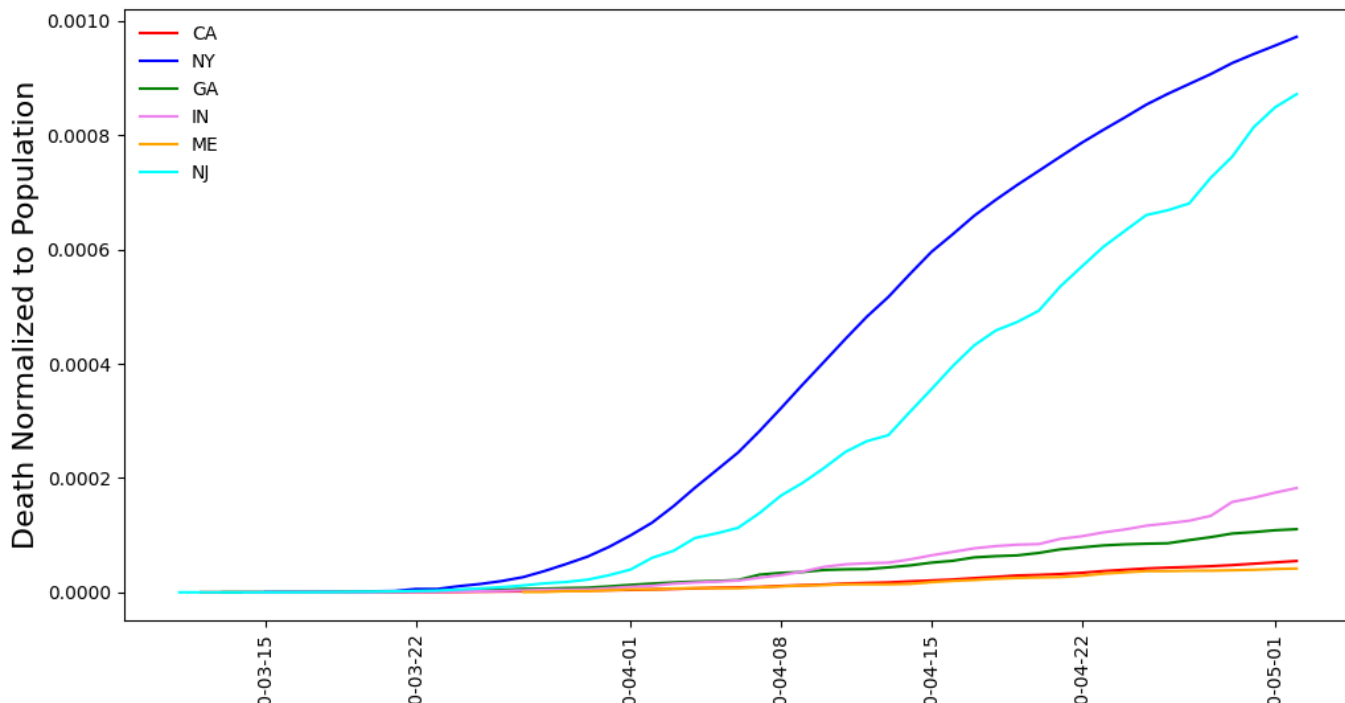
```
fig = plt.figure()
```

```
fig = plt.figure()
fig, ax = plt.subplots(figsize=(12, 6))
plt.rcParams.update(plt.rcParamsDefault)

plt.plot(df_state_dict['CA'].death_norm, color="red", label="CA")
plt.plot(df_state_dict['NY'].death_norm, color="blue", label="NY")
plt.plot(df_state_dict['GA'].death_norm, color="green", label="GA")
plt.plot(df_state_dict['IN'].death_norm, color="violet", label="IN")
plt.plot(df_state_dict['ME'].death_norm, color="orange", label="ME")
plt.plot(df_state_dict['NJ'].death_norm, color="cyan", label="NJ")
plt.xticks(rotation='vertical')

plt.legend(frameon=False)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Death Normalized to Population', fontsize=16)
plt.show()
```

Figure size 640x480 with 0 Axes



Note how the population normalized death curves relate closely to population normalized positive test curves

Curve fitting done at: <http://www.xuru.org/rt/NLR.asp#CopyPaste>

```
# Fetch the parameters for each state (CexpDx^1.csv) that fit to positive_norm = a*exp(b/x)
# where x is the number of days from March 4, 2020
from google.colab import files
uploaded = files.upload()
```

Choose Files CexpDx^1.csv

- CexpDx^1.csv(application/vnd.ms-excel) - 1680 bytes, last modified: 4/16/2020 - 100% done

Saving CexpDx^1.csv to CexpDx^1.csv

```
# Load the parameters for each state (CexpDx^1.csv) that fit to positive_norm = a*exp(b/x)
import io
df_state_params = pd.read_csv(io.StringIO(uploaded['CexpDx^1.csv'].decode('utf-8')))
df_state_params.head()
```

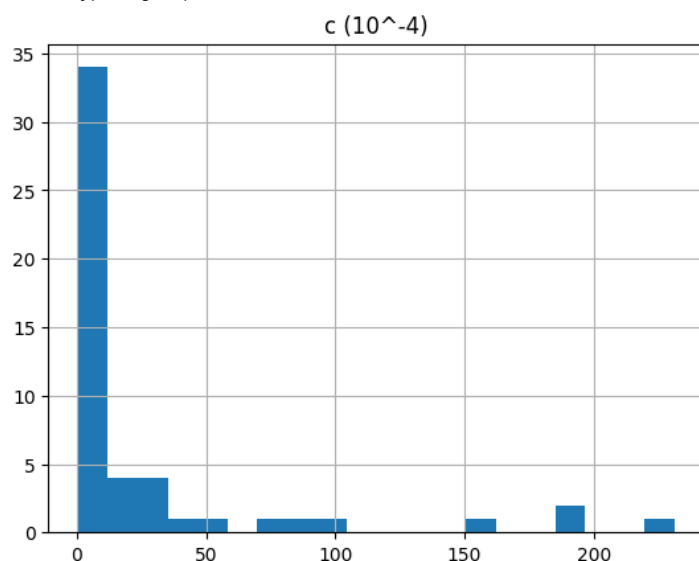
	State	c (10^-4)	d	fit rank
0	AK	1.331139	-95.882596	2.0
1	AL	8.124937	-145.096536	1.0
2	AR	1.444874	-108.708991	3.0
3	AS	NaN	NaN	NaN
4	AZ	4.374538	-129.204382	1.0

```
df_state_params.describe()
```

	c (10 ⁻⁴)	d	fit rank
count	51.000000	51.000000	51.000000
mean	28.922502	-142.879078	2.098039
std	53.235594	33.811201	2.156431
min	0.516899	-215.115296	1.000000
25%	3.745253	-165.040649	1.000000
50%	7.421743	-145.096536	1.000000
75%	20.958221	-123.240757	2.500000
max	231.216701	-47.945262	15.000000

```
df_state_params.hist(column='c (10-4)', bins=20)
```

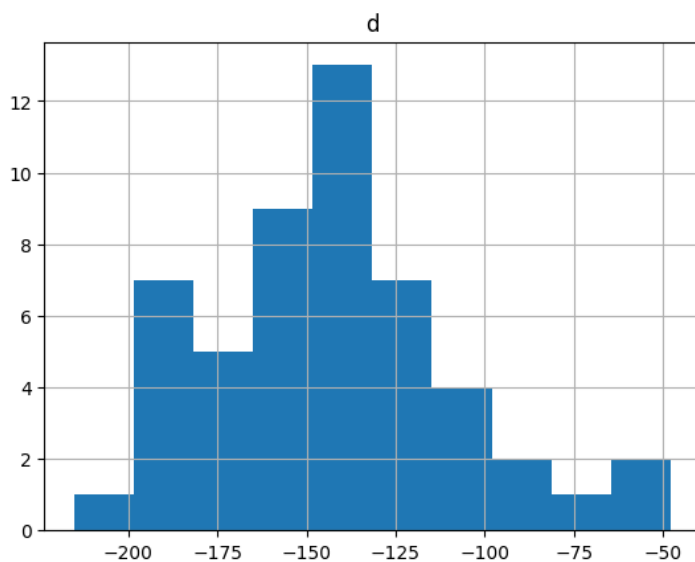
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f87007ee780>]],
      dtype=object)
```



High value outliers here are NJ (fit rank 1), NY, (fit rank 1), RI (fit rank 5), and SD (fit rank 4)

```
df_state_params.hist(column='d', bins=10)
```

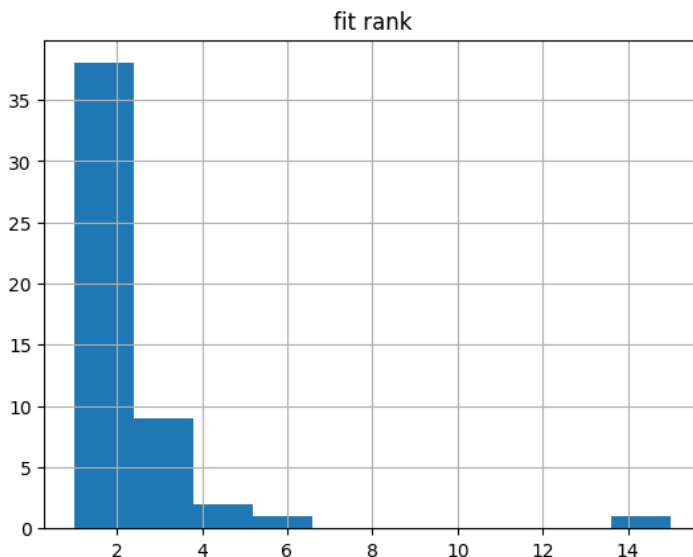
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f87007901d0>]],
      dtype=object)
```



Low value outliers here are RI (fit rank 5) and SD (fit rank 4).

```
df_state_params.hist(column='fit rank')
```

```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f8700762d68>]],
      dtype=object)
```



The $A \cdot \exp(B/x)$ functional form works extremely well for thirty of the 52 states (57.7%).

```
# Fetch static data for each state (CovidCompleteStateData.csv)
from google.colab import files
uploaded = files.upload()
```

```
Choose Files CovidCompl...teData.csv
• CovidCompleteStateData.csv(application/vnd.ms-excel) - 60510 bytes, last modified: 4/20/2020 - 100% done
Saving CovidCompleteStateData.csv to CovidCompleteStateData.csv
```

```
# Load static data for each state (CovidCurrentStateData.csv)
import io
df_state_data = pd.read_csv(io.StringIO(uploaded['CovidCompleteStateData.csv'].decode('utf-8')))
df_state_data.head()
```

```
State      Sum of      Sum of      Sum of      Sum of      Sum of      Sum of      Sum of
NUM_Medicare_BEN  NUM_BEN_Age_Less_65  NUM_BEN_Age_65_to_74  NUM_BEN_Age_75_to_84  NUM_BEN_Age_Greater_84  NUM_Female_BEN  NUM_

0  AK      1820384.0      270970.0      809516.0      468255.0      175296.0      1034762.0
1  AL      10804823.0      2065353.0      4386595.0      2980828.0      1190504.0      6237445.0
2  AR      15892716.0      2818665.0      6370265.0      4555468.0      1848506.0      9275039.0
3  AS              NaN              NaN              NaN              NaN              NaN              NaN
4  AZ      10786064.0      886596.0      4861035.0      3377040.0      1294375.0      5944519.0

5 rows x 116 columns
```

```
# Feature Engineering
# Land Area/Water Area
df_state_data['State Area Ratio'] = df_state_data['Land Area']/df_state_data['Water Area']
df_state_data['State Area Ratio'] = df_state_data['Land Area'].divide(df_state_data['Water Area'], fill_value=0)

# Elevation Ratio = Highest Elevation/Mean Elevation
df_state_data['Elevation Ratio'] = df_state_data['Highest Elevation']/df_state_data['Mean Elevation']
df_state_data['Elevation Ratio'] = df_state_data['Highest Elevation'].divide(df_state_data['Mean Elevation'], fill_v

# Capital Area Ratio = Capital Land Area/Capital Water Area
df_state_data['Capital Area Ratio'] = df_state_data['Capital Land Area']/df_state_data['Capital Water Area']
df_state_data['Capital Land Area'] = df_state_data['Capital Land Area'].astype(float)
df_state_data['Capital Area Ratio'] = df_state_data['Capital Land Area'].divide(df_state_data['Capital Water Area'],

# Boundaries = Number of boarding states + On Coast + Borders Another Country
df_state_data['Boundaries'] = df_state_data['Number of bordering states'] + df_state_data['On Coast'] + df_state_data['Number of bordering states']
```



```
# Latitude Difference to State Capital = Latitude - Capital Latitude
df_state_data['Latitude Difference to State Capital'] = df_state_data['Latitude'] - df_state_data['Capital Latitude']

# Longitude Difference to State Capital = Capital Longitude - Longitude
df_state_data['Longitude Difference to State Capital'] = df_state_data['Capital Longitude'] - df_state_data['Longitude']

# Latitude Difference to DC = Latitude - DC Latitude
df_state_data['Latitude Difference to DC'] = df_state_data['Latitude'] - 38.904722

# Longitude Difference to DC = DC Longitude - Longitude
df_state_data['Longitude Difference to DC'] = -77.016389 - df_state_data['Longitude']

# Latitude Difference to US Center = Latitude - Center Latitude
df_state_data['Latitude Difference to Center'] = df_state_data['Latitude'] - 39.833333

# Longitude Different to US Center = Center Longitude - Longitude
df_state_data['Longitude Difference to Center'] = -98.585522 - df_state_data['Longitude']
```

```
df_state_data.head()
```

↗

	State	Sum of NUM_Medicare_BEN	Sum of NUM_BEN_Age_Less_65	Sum of NUM_BEN_Age_65_to_74	Sum of NUM_BEN_Age_75_to_84	Sum of NUM_BEN_Age_Greater_84	Sum of NUM_Female_BEN	NUM_
0	AK	1820384.0	270970.0	809516.0	468255.0	175296.0	1034762.0	
1	AL	10804823.0	2065353.0	4386595.0	2980828.0	1190504.0	6237445.0	4
2	AR	15892716.0	2818665.0	6370265.0	4555468.0	1848506.0	9275039.0	6
3	AS	NaN	NaN	NaN	NaN	NaN	NaN	
4	AZ	10786064.0	886596.0	4861035.0	3377040.0	1294375.0	5944519.0	4

5 rows × 126 columns

```
df_state_data.shape
```

↗ (56, 126)

```
# Define variables for regression
df_temp1 = df_state_data.drop(df_state_data.index[[3, 12, 27, 42, 50, 55]])
X = df_temp1.drop('State', axis = 1)
df_temp2 = df_state_params.drop(df_state_data.index[[3, 12, 27, 42, 50, 55]])
y = df_temp2['d']
```

```
# Look at correlation coefficients
pd.set_option('display.max_columns', None)
pd.set_option('display.max_rows', 1000)
X.corr()
```

↗

	Sum of NUM_Medicare_BEN	Sum of NUM_BEN_Age_Less_65	Sum of NUM_BEN_Age_65_to_74	Sum of NUM_BEN_Age_75_to_84	Sum of NUM_BEN_Age_85_to_94
Sum of NUM_Medicare_BEN	1.000000	0.981244	0.998612	0.998085	0.998085
Sum of NUM_BEN_Age_Less_65	0.981244	1.000000	0.977935	0.969186	0.969186
Sum of NUM_BEN_Age_65_to_74	0.998612	0.977935	1.000000	0.996336	0.996336
Sum of NUM_BEN_Age_75_to_84	0.998085	0.969186	0.996336	1.000000	1.000000
Sum of NUM_BEN_Age_Greater_84	0.989852	0.960258	0.982527	0.992524	0.992524
Sum of NUM_Female_BEN	0.999917	0.982419	0.998360	0.997902	0.997902
Sum of NUM_Male_BEN	0.999896	0.979571	0.998622	0.998281	0.998281
Sum of NUM_Black_or_African_American_BEN	0.895536	0.925224	0.894585	0.882970	0.882970
Sum of NUM_Asian_Pacific_Islander_BEN	0.524429	0.473716	0.516336	0.528889	0.528889
Sum of NUM_Hispanic_BEN	0.894417	0.829126	0.903356	0.900554	0.900554
Sum of NUM_American_IndianAlaska_Native_BEN	0.077349	0.053905	0.086472	0.081806	0.081806
Sum of NUM_BEN_With_Race_Not_Elsewhere_Classified	0.821569	0.771437	0.801707	0.830466	0.830466
Sum of NUM_Non-Hispanic_White_BEN	0.996809	0.978655	0.994347	0.996101	0.996101
Sum of NUM_Minorities	0.958404	0.925675	0.961032	0.957614	0.957614
Sum of Average_Age_of_BEN	0.678752	0.726826	0.682844	0.659778	0.659778
Sum of NUM_BEN_Atrial_Fibrillation	0.990319	0.969220	0.985453	0.991337	0.991337
Sum of NUM_BEN_Asthma	0.995489	0.979353	0.991510	0.992852	0.992852
Sum of NUM_BEN_Cancer	0.994721	0.971958	0.992833	0.994822	0.994822
Sum of NUM_BEN_Heart_Failure	0.997108	0.985088	0.995323	0.993852	0.993852
Sum of NUM_BEN_Chronic_Kidney_Disease	0.997480	0.980181	0.997065	0.995383	0.995383
Sum of NUM_BEN_Chronic_Obstructive_Pulmonary_Disease	0.986081	0.980417	0.981434	0.983841	0.983841
Sum of NUM_BEN_Hyperlipidemia	0.996199	0.974138	0.994686	0.996386	0.996386
Sum of NUM_BEN_Diabetes	0.997736	0.981117	0.996508	0.995642	0.995642
Sum of NUM_BEN_Hypertension	0.998843	0.982162	0.998059	0.996914	0.996914
Sum of NUM_BEN_Ischemic_Heart_Disease	0.993954	0.974989	0.991463	0.994045	0.994045
Sum of NUM_BEN_Stroke	0.990470	0.971925	0.988713	0.989929	0.989929
Sum of PCT_MEDICARE	0.710503	0.759188	0.713882	0.692945	0.692945
% Urban Pop	0.239324	0.172542	0.233998	0.252295	0.252295
Density (P/mi2)	-0.099963	-0.110703	-0.100658	-0.096325	-0.096325
Children 0-18	0.884945	0.844648	0.874846	0.887257	0.887257
Adults 19-25	0.864191	0.823977	0.851022	0.867408	0.867408
Adults 26-34	0.846985	0.802138	0.833617	0.851409	0.851409
Adults 35-54	0.860076	0.817671	0.846322	0.864281	0.864281
Adults 55-64	0.838622	0.799478	0.819933	0.843902	0.843902
65+	0.840633	0.793344	0.820862	0.850354	0.850354
Latitude	-0.395637	-0.392189	-0.398492	-0.402613	-0.402613
Longitude	0.036162	0.081918	0.023777	0.029848	0.029848
Land Area	0.235431	0.200886	0.248419	0.236252	0.236252
Water Area	0.038411	0.051521	0.032297	0.034407	0.034407
Mean Elevation	-0.133770	-0.196098	-0.117766	-0.126100	-0.126100
Highest Elevation	-0.038246	-0.115800	-0.018904	-0.028611	-0.028611
Lowest elevation	-0.344113	-0.337087	-0.333651	-0.346722	-0.346722
Number of bordering states	0.092703	0.153356	0.090523	0.073651	0.073651
On Coast	0.464164	0.497887	0.435913	0.455132	0.455132
Bordering Another Country	0.254042	0.222222	0.257805	0.250755	0.250755

Borders Another Country	0.351913	0.303223	0.357825	0.350755
Capital Latitude	-0.386561	-0.391908	-0.392011	-0.390199
Capital Longitude	0.018177	0.067248	0.005968	0.010624
Capital Land Area	0.003972	-0.007988	0.013931	0.004629
Capital Water Area	-0.091118	-0.100314	-0.086948	-0.090518
Capital Mean Elevation	-0.166033	-0.186941	-0.154788	-0.163860
Capital is the Largest City	-0.154074	-0.128106	-0.149158	-0.156946
Largest City Latitude	-0.419120	-0.419459	-0.423088	-0.422919
Largest City Longitude	0.048321	0.092830	0.035728	0.041774
Number of Counties	0.659574	0.706073	0.666432	0.641478
Became a State	-0.126570	-0.186422	-0.115157	-0.112935
DaysSinceStayatHomeOrder	-0.021086	-0.020186	-0.030817	-0.027800
DaysSinceFirstPositive	0.357249	0.306142	0.355519	0.364255
DaysSinceTestStart	0.273593	0.219953	0.272942	0.282120
15-49yearsAllcauses	0.886884	0.854562	0.873498	0.888773
15-49yearsAsthma	0.822646	0.785134	0.805485	0.825296
15-49yearsChronickidneydisease	0.917925	0.892317	0.908566	0.917956
15-49yearsChronicobstructivepulmonarydisease	0.895564	0.876357	0.879172	0.896199
15-49yearsDiabetesmellitus	0.911319	0.879991	0.899800	0.913356
15-49yearsInterstitiallungdiseaseandpulmonarysarcoidosis	0.879916	0.862208	0.865322	0.878905
15-49yearsIschemicheartdisease	0.927678	0.926759	0.915842	0.922736
15-49yearsNeoplasms	0.886136	0.858150	0.871628	0.887471
15-49yearsOtherchronicrespiratorydiseases	0.905560	0.883613	0.891223	0.905653
15-49yearsRheumaticheartdisease	0.902424	0.891711	0.892262	0.897798
15-49yearsStroke	0.918867	0.897147	0.909310	0.918599
50-69yearsAllcauses	0.878744	0.853509	0.861522	0.880659
50-69yearsAsthma	0.799440	0.762340	0.778773	0.803715
50-69yearsChronickidneydisease	0.916387	0.896945	0.904561	0.915572
50-69yearsChronicobstructivepulmonarydisease	0.877906	0.870963	0.859255	0.877419
50-69yearsDiabetesmellitus	0.881134	0.855438	0.863901	0.883450
50-69yearsInterstitiallungdiseaseandpulmonarysarcoidosis	0.861583	0.838312	0.844421	0.862487
50-69yearsIschemicheartdisease	0.904978	0.899635	0.888882	0.901757
50-69yearsNeoplasms	0.871034	0.851227	0.852407	0.872097
50-69yearsOtherchronicrespiratorydiseases	0.883753	0.873315	0.866185	0.882303
50-69yearsRheumaticheartdisease	0.891423	0.888783	0.879360	0.885632
50-69yearsStroke	0.906978	0.890724	0.893997	0.906473
70+yearsAllcauses	0.847442	0.816751	0.826481	0.852488
70+yearsAsthma	0.789028	0.744699	0.766961	0.797072
70+yearsChronickidneydisease	0.875670	0.856224	0.857657	0.876360
70+yearsChronicobstructivepulmonarydisease	0.865156	0.840259	0.845077	0.869812
70+yearsDiabetesmellitus	0.843401	0.812744	0.821754	0.849108
70+yearsInterstitiallungdiseaseandpulmonarysarcoidosis	0.831802	0.797053	0.811884	0.837251
70+yearsIschemicheartdisease	0.839315	0.817188	0.816155	0.842376
70+yearsNeoplasms	0.835509	0.805555	0.813851	0.840697
70+yearsOtherchronicrespiratorydiseases	0.874566	0.857451	0.856689	0.874418
70+yearsRheumaticheartdisease	0.842665	0.837198	0.824793	0.837776
70+yearsStroke	0.870071	0.847350	0.852618	0.871917

AllAgesAllcauses	0.878588	0.849145	0.861845	0.881318
AllAgesAsthma	0.831304	0.792231	0.813720	0.835086
AllAgesChronicKidneydisease	0.904402	0.883840	0.890334	0.904351
AllAgesChronicobstructivepulmonarydisease	0.875803	0.858774	0.856544	0.878011
AllAgesDiabetesmellitus	0.878317	0.849967	0.860647	0.881652
AllAgesInterstitiallungdiseaseandpulmonarysarcoidosis	0.852165	0.823512	0.833849	0.855184
AllAgesIschemicheartdisease	0.882192	0.869062	0.862943	0.881839
AllAgesNeoplasms	0.863741	0.839097	0.844574	0.866307
AllAgesOtherchronicrespiratorydiseases	0.902524	0.884302	0.887253	0.902007
AllAgesRheumaticheartdisease	0.879079	0.873449	0.864765	0.873886
AllAgesStroke	0.894221	0.873914	0.879380	0.894925
AllAgesTotal	0.879105	0.851798	0.861916	0.881553
Airpollution	0.887961	0.886816	0.873716	0.881728
Highbody-massindex	0.892574	0.870891	0.875767	0.893521
Highfastingplasmaglucoese	0.885519	0.868208	0.867475	0.886276
HighLDLcholesterol	0.892016	0.880761	0.874040	0.890927
Highsystolicbloodpressure	0.896298	0.880918	0.879042	0.896085
Impairedkidneyfunction	0.888684	0.870825	0.871779	0.888904
Noaccesstohandwashingfacility	0.876183	0.855685	0.860915	0.875209
Smoking	0.880256	0.864750	0.861340	0.881441
Log10Pop	0.730625	0.738162	0.716041	0.724834
DaysSinceInfection	0.412821	0.360632	0.410278	0.422147
Children0-18	0.170467	0.184747	0.184614	0.162743
Allriskfactors	0.881460	0.858902	0.864027	0.883001
State Area Ratio	-0.128550	-0.166800	-0.113602	-0.122087
Elevation Ratio	0.006435	-0.008386	0.016149	0.010278
Capital Area Ratio	-0.107958	-0.139494	-0.098783	-0.101355
Boundaries	0.500872	0.558822	0.480645	0.479234
Latitude Difference to State Capital	-0.251296	-0.188306	-0.234552	-0.277897
Longitude Difference to State Capital	-0.132644	-0.120676	-0.128482	-0.139685
Latitude Difference to DC	-0.395637	-0.392189	-0.398492	-0.402613
Longitude Difference to DC	-0.036162	-0.081918	-0.023777	-0.029848
Latitude Difference to Center	-0.395637	-0.392189	-0.398492	-0.402613
Longitude Difference to Center	-0.036162	-0.081918	-0.023777	-0.029848

```
# Note that there are many highly correlated features which need to be dropped
# Create absolute value correlation matrix
corr_matrix = X.corr().abs()

# Select upper triangle of correlation matrix
upper = corr_matrix.where(np.triu(np.ones(corr_matrix.shape), k=1).astype(np.bool))

# Find index of feature columns with correlation greater than 0.95
to_drop = [column for column in upper.columns if any(upper[column] > 0.95)]

# Drop features by index which were identified as being highly correlated
X = X.drop(X[to_drop], axis=1)
```

```
X.head()
```



	Sum of NUM_Medicare_BEN	Sum of NUM_Black_or_African_American_BEN	Sum of NUM_Asian_Pacific_Islander_BEN	Sum of NUM_Hispanic_BEN	Sum of NUM_American_IndianAlaska_Native_BEN	!
0	1820384.0	62311.0	76773.0	46525.0		14
1	10804823.0	1549811.0	30624.0	65500.0		
2	15892716.0	1334245.0	19642.0	108428.0		6
4	10786064.0	221183.0	61840.0	689880.0		17
5	42579588.0	2072012.0	3276415.0	5674776.0		11

```
X.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 50 entries, 0 to 54
Data columns (total 38 columns):
#   Column                                     Non-Null Count  Dtype
---  -
0   Sum of NUM_Medicare_BEN                   50 non-null     float64
1   Sum of NUM_Black_or_African_American_BEN 50 non-null     float64
2   Sum of NUM_Asian_Pacific_Islander_BEN     50 non-null     float64
3   Sum of NUM_Hispanic_BEN                   50 non-null     float64
4   Sum of NUM_American_IndianAlaska_Native_BEN 50 non-null     float64
5   Sum of NUM_BEN_With_Race_Not_Elsewhere_Classified 50 non-null     float64
6   Sum of Average_Age_of_BEN                 50 non-null     float64
7   Sum of PCT_MEDICARE                       50 non-null     float64
8   % Urban Pop                               50 non-null     float64
9   Density (P/mi2)                           50 non-null     float64
10  Children 0-18                             50 non-null     float64
11  Latitude                                   50 non-null     float64
12  Longitude                                  50 non-null     float64
13  Land Area                                 50 non-null     float64
14  Water Area                               50 non-null     float64
15  Mean Elevation                            50 non-null     float64
16  Highest Elevation                         50 non-null     float64
17  Lowest elevation                          50 non-null     float64
18  Number of bordering states                50 non-null     float64
19  On Coast                                  50 non-null     float64
20  Borders Another Country                   50 non-null     float64
21  Captial Land Area                         50 non-null     float64
22  Capital Water Area                        50 non-null     float64
23  Capital Mean Elevation                    50 non-null     float64
24  Capital is the Largest City                50 non-null     float64
25  Became a State                            50 non-null     float64
26  DaysSinceStayatHomeOrder                  50 non-null     float64
27  DaysSinceFirstPositive                     50 non-null     float64
28  DaysSinceTestStart                        50 non-null     float64
29  Log10Pop                                  50 non-null     float64
30  DaysSinceInfection                         50 non-null     float64
31  Children0-18                              50 non-null     float64
32  State Area Ratio                          50 non-null     float64
33  Elevation Ratio                           50 non-null     float64
34  Capital Area Ratio                        50 non-null     float64
35  Boundaries                                50 non-null     float64
36  Latitude Difference to State Capital       50 non-null     float64
37  Longitude Difference to State Capital      50 non-null     float64
dtypes: float64(38)
memory usage: 15.2 KB
```

```
X.describe()
```

	Sum of NUM_Medicare_BEN	Sum of NUM_Black_or_African_American_BEN	Sum of NUM_Asian_Pacific_Islander_BEN	Sum of NUM_Hispanic_BEN	Sum of NUM_American_IndianAlaska_I
count	5.000000e+01	5.000000e+01	5.000000e+01	5.000000e+01	
mean	1.057661e+07	9.653450e+05	1.439833e+05	5.412557e+05	39
std	1.317051e+07	1.280319e+06	4.765951e+05	1.644850e+06	88
min	1.655870e+05	2.960000e+02	1.660000e+02	4.130000e+02	
25%	2.518838e+06	6.328700e+04	6.770500e+03	3.269350e+04	2
50%	6.848160e+06	3.978665e+05	2.777200e+04	1.050865e+05	7
75%	1.479523e+07	1.548688e+06	7.370350e+04	2.012818e+05	28
max	7.644909e+07	7.011107e+06	3.276415e+06	1.007620e+07	560

```
# Train/validate split: random 75/25% train/validate split.
from sklearn.model_selection import train_test_split

X_train, X_val, y_train, y_val = train_test_split(X, y, test_size = 0.25, random_state = 42)

X_train.shape, y_train.shape, X_val.shape, y_val.shape
```

```
((37, 38), (37,), (13, 38), (13,))
```

```
X_train.describe()
```

	Sum of NUM_Medicare_BEN	Sum of NUM_Black_or_African_American_BEN	Sum of NUM_Asian_Pacific_Islander_BEN	Sum of NUM_Hispanic_BEN	Sum of NUM_American_IndianAlaska_I
count	3.700000e+01	3.700000e+01	37.000000	3.700000e+01	
mean	1.157925e+07	1.130874e+06	98436.675676	5.365955e+05	41
std	1.384476e+07	1.398898e+06	171362.519286	1.696478e+06	97
min	1.655870e+05	2.960000e+02	166.000000	4.130000e+02	
25%	3.242760e+06	1.057920e+05	12709.000000	4.230000e+04	3
50%	8.517210e+06	5.217080e+05	30624.000000	1.112130e+05	7
75%	1.629170e+07	1.693845e+06	76800.000000	2.027260e+05	28
max	7.644909e+07	7.011107e+06	793067.000000	1.007620e+07	560

```
# Optimizing Hyperparameters
from sklearn.model_selection import GridSearchCV
from sklearn.ensemble import RandomForestRegressor

# Define classifier
forest = RandomForestRegressor(random_state = 1)

# Parameters to fit

max_depth = [2, 3, 4]
n_estimators = [28, 29, 30]
min_samples_split = [1.5, 2, 2.5]
min_samples_leaf = [3.5, 4, 4.5]
max_leaf_nodes = [None]
max_features = ['auto']
ccp_alpha = [0.0, 0.00625, 0.0125]
min_weight_fraction_leaf = [0.0, 0.00625, 0.0125]

hyperF = dict(n_estimators = n_estimators, max_depth = max_depth,
              min_samples_split = min_samples_split,
              min_samples_leaf = min_samples_leaf,
              max_leaf_nodes = max_leaf_nodes,
              max_features = max_features,
              ccp_alpha=ccp_alpha,
              min_weight_fraction_leaf=min_weight_fraction_leaf)
```

```

gridF = GridSearchCV(forest, hyperF, cv = 3, verbose = 10,
                     scoring='r2', return_train_score=True,
                     n_jobs = -1)
bestF = gridF.fit(X_train, y_train)

# Output best accuracy and best parameters
print('The score achieved with the best parameters = ', gridF.best_score_, '\n')
print('The parameters are:', gridF.best_params_)

```

```

↳ Fitting 3 folds for each of 729 candidates, totalling 2187 fits
[Parallel(n_jobs=-1)]: Using backend LokyBackend with 2 concurrent workers.
[Parallel(n_jobs=-1)]: Done 1 tasks      | elapsed: 1.2s
[Parallel(n_jobs=-1)]: Done 4 tasks      | elapsed: 1.4s
[Parallel(n_jobs=-1)]: Done 9 tasks      | elapsed: 1.5s
[Parallel(n_jobs=-1)]: Done 14 tasks     | elapsed: 1.6s
[Parallel(n_jobs=-1)]: Batch computation too fast (0.1862s.) Setting batch_size=2.
[Parallel(n_jobs=-1)]: Batch computation too fast (0.0837s.) Setting batch_size=4.
[Parallel(n_jobs=-1)]: Done 24 tasks     | elapsed: 1.7s
[Parallel(n_jobs=-1)]: Batch computation too fast (0.1513s.) Setting batch_size=8.
[Parallel(n_jobs=-1)]: Done 58 tasks     | elapsed: 2.2s
[Parallel(n_jobs=-1)]: Done 130 tasks    | elapsed: 3.9s
[Parallel(n_jobs=-1)]: Done 202 tasks    | elapsed: 4.7s
[Parallel(n_jobs=-1)]: Done 290 tasks    | elapsed: 6.0s
[Parallel(n_jobs=-1)]: Done 378 tasks    | elapsed: 7.7s
[Parallel(n_jobs=-1)]: Done 482 tasks    | elapsed: 9.1s
[Parallel(n_jobs=-1)]: Done 586 tasks    | elapsed: 10.5s
[Parallel(n_jobs=-1)]: Done 706 tasks    | elapsed: 12.8s
[Parallel(n_jobs=-1)]: Done 826 tasks    | elapsed: 14.4s
[Parallel(n_jobs=-1)]: Done 962 tasks    | elapsed: 16.8s
[Parallel(n_jobs=-1)]: Done 1098 tasks   | elapsed: 19.2s
[Parallel(n_jobs=-1)]: Done 1250 tasks   | elapsed: 21.4s
[Parallel(n_jobs=-1)]: Done 1402 tasks   | elapsed: 24.1s
[Parallel(n_jobs=-1)]: Done 1570 tasks   | elapsed: 26.5s
[Parallel(n_jobs=-1)]: Done 1738 tasks   | elapsed: 29.2s
[Parallel(n_jobs=-1)]: Done 1922 tasks   | elapsed: 32.4s
[Parallel(n_jobs=-1)]: Done 2106 tasks   | elapsed: 35.5s
The score achieved with the best parameters = -0.1309525586842498

The parameters are: {'ccp_alpha': 0.0, 'max_depth': 3, 'max_features': 'auto', 'max_leaf_nodes': None, 'min_samples_leaf': 4, 'min_sample
[Parallel(n_jobs=-1)]: Done 2187 out of 2187 | elapsed: 36.6s finished

```

```
!pip install category_encoders==2.0.0
```

```

↳ Collecting category_encoders==2.0.0
  Downloading https://files.pythonhosted.org/packages/6e/a1/f7a22f144f33be78afeb06bfa78478e8284a64263a3c09b1ef54e673841e/category\_encoder-2.0.0-py3-none-any.whl (92kB)
    92kB 2.9MB/s
Requirement already satisfied: statsmodels>=0.6.1 in /usr/local/lib/python3.6/dist-packages (from category_encoders==2.0.0) (0.10.2)
Requirement already satisfied: patsy>=0.4.1 in /usr/local/lib/python3.6/dist-packages (from category_encoders==2.0.0) (0.5.1)
Requirement already satisfied: pandas>=0.21.1 in /usr/local/lib/python3.6/dist-packages (from category_encoders==2.0.0) (1.0.3)
Requirement already satisfied: scikit-learn>=0.20.0 in /usr/local/lib/python3.6/dist-packages (from category_encoders==2.0.0) (0.22.2.post1)
Requirement already satisfied: numpy>=1.11.3 in /usr/local/lib/python3.6/dist-packages (from category_encoders==2.0.0) (1.18.3)
Requirement already satisfied: scipy>=0.19.0 in /usr/local/lib/python3.6/dist-packages (from category_encoders==2.0.0) (1.4.1)
Requirement already satisfied: six in /usr/local/lib/python3.6/dist-packages (from patsy>=0.4.1->category_encoders==2.0.0) (1.12.0)
Requirement already satisfied: pytz>=2017.2 in /usr/local/lib/python3.6/dist-packages (from pandas>=0.21.1->category_encoders==2.0.0) (2017.2)
Requirement already satisfied: python-dateutil>=2.6.1 in /usr/local/lib/python3.6/dist-packages (from pandas>=0.21.1->category_encoders==2.0.0) (2.6.1)
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.6/dist-packages (from scikit-learn>=0.20.0->category_encoders==2.0.0) (0.11.2)
Installing collected packages: category-encoders
Successfully installed category-encoders-2.0.0

```

```

from sklearn.ensemble import RandomForestRegressor
from sklearn.pipeline import make_pipeline
import category_encoders as ce
from sklearn.impute import SimpleImputer

pipeline1 = make_pipeline(
    ce.OneHotEncoder(use_cat_names=True),
    SimpleImputer(strategy='mean'),
    RandomForestRegressor(bootstrap=True, ccp_alpha=0.0, criterion='mse',
                          max_depth=3, max_features='auto', max_leaf_nodes=None,
                          max_samples=None, min_impurity_decrease=0.0,
                          min_impurity_split=None, min_samples_leaf=4,
                          min_samples_split=2, min_weight_fraction_leaf=0.0,
                          n_estimators=29, n_jobs=None, oob_score=False,
                          random_state=0, verbose=0, warm_start=False))

pipeline1.fit(X_train, y_train)

# Get the model's training accuracy

```

```
print("Training Accuracy: R^2 = ", pipeline1.score(X_train,y_train))
```

```
# Get the model's validation accuracy
```

```
print('Validation Accuracy: R^2 = ', pipeline1.score(X_val, y_val))
```

```
☞ Training Accuracy: R^2 = 0.6195488363328325
   Validation Accuracy: R^2 = 0.48283160859213214
```

```
print("Feature Importances =")
```

```
#print(RandomForestRegressor.feature_importances_)
```

```
print(pipeline1.steps[2][1].feature_importances_)
```

```
☞ Feature Importances =
[0. 0.00426752 0. 0. 0.07631863 0.00543221
 0.00740174 0. 0.0467188 0.14111398 0.03505111 0.14622923
 0.00726956 0.00351066 0. 0.01670479 0.07845557 0.
 0.01871196 0.00280924 0. 0.11643129 0.00982198 0.03083072
 0. 0.01720428 0.0157221 0.0044032 0.01288937 0.
 0. 0.03006819 0.00778861 0.00929704 0.08558693 0.02593157
 0.01157612 0.03245364]
```

```
# Plot of feature importances from pure Random Forest Regressor
```

```
%matplotlib inline
```

```
import matplotlib.pyplot as plt
```

```
# Get feature importances
```

```
encoder = pipeline1.named_steps['onehotencoder']
```

```
encoded = encoder.transform(X_train)
```

```
rf = pipeline1.named_steps['randomforestregressor']
```

```
importances1 = pd.Series(rf.feature_importances_, encoded.columns)
```

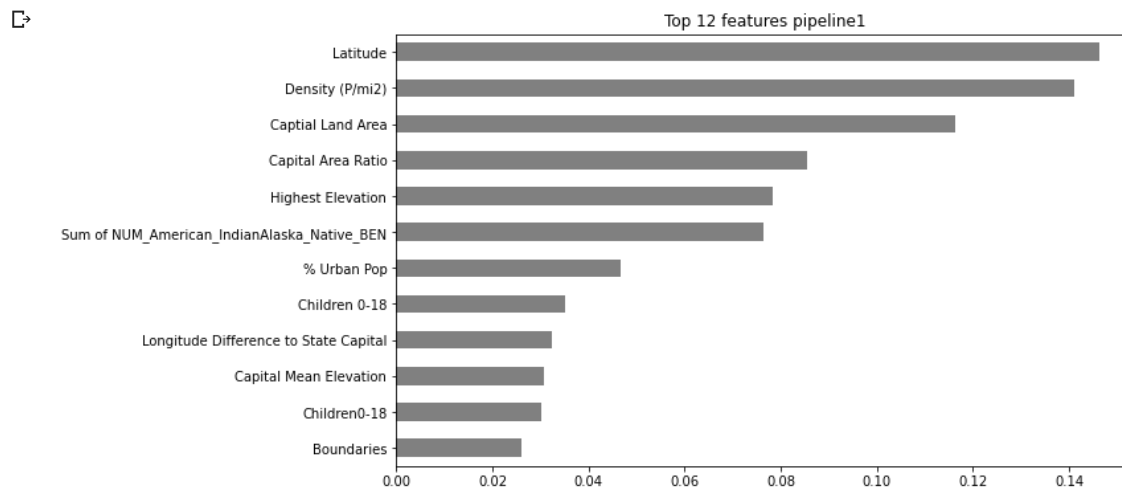
```
# Plot feature importances
```

```
n = 12
```

```
plt.figure(figsize=(10,n/2))
```

```
plt.title(f'Top {n} features pipeline1')
```

```
importances1.sort_values()[-n:].plot.barh(color='grey');
```



```
# Generate validation curves
```

```
%matplotlib inline
```

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
from sklearn.model_selection import validation_curve
```

```
pipeline2 = make_pipeline(
```

```
    ce.OrdinalEncoder(),
```

```
    SimpleImputer(),
```

```
    RandomForestRegressor()
```

```
)
```

```
depth = range(1, 10, 2)
```

```
train_scores, val_scores = validation_curve(
```

```
    pipeline2, X_train, y_train,
```

```
    param_name='randomforestregressor__max_depth',
```

```
    param_range=depth,
```

```
    cv=3,
```

```
    n_jobs=-1
```

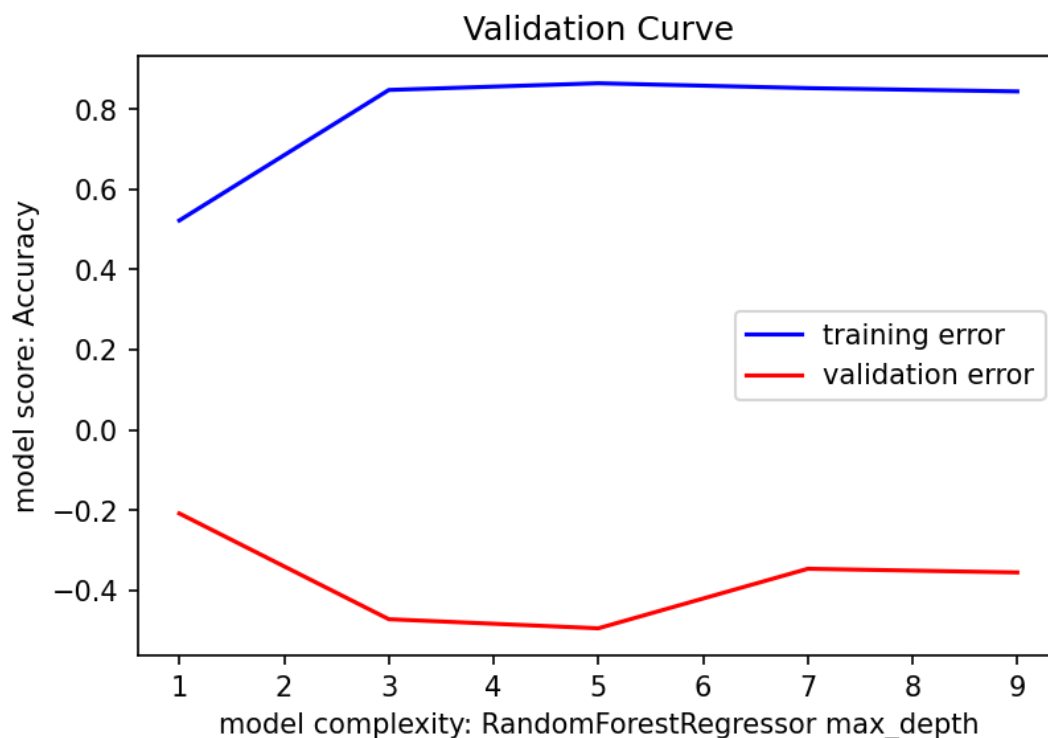
```
)
```

```
plt.figure(dpi=150)
```

```
plt.plot(depth, np.mean(train_scores, axis=1), color='blue', label='training error')
```



```
plt.plot(depth, np.mean(val_scores, axis=1), color='red', label='validation error')
plt.title('Validation Curve')
plt.xlabel('model complexity: RandomForestRegressor max_depth')
plt.ylabel('model score: Accuracy')
plt.legend();
```



```
# Get drop-column importances
column = 'Density (P/mi2)'

pipeline3 = make_pipeline(
    ce.OneHotEncoder(use_cat_names=True),
    SimpleImputer(strategy = 'most_frequent'),
    RandomForestRegressor(bootstrap=True, ccp_alpha=0.0, criterion='mse',
        max_depth=3, max_features='auto', max_leaf_nodes=None,
        max_samples=None, min_impurity_decrease=0.0,
        min_impurity_split=None, min_samples_leaf=4,
        min_samples_split=2, min_weight_fraction_leaf=0,
        n_estimators=29, n_jobs=None, oob_score=False,
        random_state=0, verbose=0, warm_start=False))

# Fit without column
pipeline3.fit(X_train.drop(columns=column), y_train)
score_without = pipeline3.score(X_val.drop(columns=column), y_val)
print(f'Validation Accuracy without {column}: {score_without}')

# Fit with column
pipeline3.fit(X_train, y_train)
score_with = pipeline3.score(X_val, y_val)
print(f'Validation Accuracy with {column}: {score_with}')

# Compare the error with & without column
print(f'Drop-Column Importance for {column}: {score_with - score_without}')
```



```
Validation Accuracy without Density (P/mi2): 0.5009375902676718
Validation Accuracy with Density (P/mi2): 0.48283160859213214
Drop-Column Importance for Density (P/mi2): -0.018105981675539673
```

```
# Using Eli5 library which does not work with pipelines
transformers = make_pipeline(
    ce.OneHotEncoder(use_cat_names=True),
    SimpleImputer(strategy='most_frequent')
)

X_train_transformed = transformers.fit_transform(X_train)
X_val_transformed = transformers.transform(X_val)
```

```

model1 = RandomForestRegressor(bootstrap=True, ccp_alpha=0.15, criterion='mse',
                                max_depth=3, max_features='auto', max_leaf_nodes=None,
                                max_samples=None, min_impurity_decrease=0.0,
                                min_impurity_split=None, min_samples_leaf=4,
                                min_samples_split=2, min_weight_fraction_leaf=0,
                                n_estimators=29, n_jobs=None, oob_score=False,
                                random_state=0, verbose=0, warm_start=False)

model1.fit(X_train_transformed, y_train)

```

```

↳ RandomForestRegressor(bootstrap=True, ccp_alpha=0.15, criterion='mse',
                          max_depth=3, max_features='auto', max_leaf_nodes=None,
                          max_samples=None, min_impurity_decrease=0.0,
                          min_impurity_split=None, min_samples_leaf=4,
                          min_samples_split=2, min_weight_fraction_leaf=0,
                          n_estimators=29, n_jobs=None, oob_score=False,
                          random_state=0, verbose=0, warm_start=False)

```

```

# Get permutation importances
! pip install eli5
from eli5.sklearn import PermutationImportance
import eli5

permuter = PermutationImportance(
    model1,
    scoring='r2',
    n_iter=2,
    random_state=42
)

permuter.fit(X_val_transformed, y_val)
feature_names = X_val.columns.tolist()

eli5.show_weights(
    permuter,
    top=None, # show permutation importances for all features
    feature_names=feature_names
)

```

```

↳

```

```
Collecting eli5
  Downloading https://files.pythonhosted.org/packages/97/2f/c85c7d8f8548e460829971785347e14e45fa5c6617da374711dec8cb38cc/eli5-0.10.1-py2.
  |████████████████████████████████████████| 112kB 2.8MB/s
Requirement already satisfied: numpy>=1.9.0 in /usr/local/lib/python3.6/dist-packages (from eli5) (1.18.3)
Requirement already satisfied: scipy in /usr/local/lib/python3.6/dist-packages (from eli5) (1.4.1)
Requirement already satisfied: scikit-learn>=0.18 in /usr/local/lib/python3.6/dist-packages (from eli5) (0.22.2.post1)
Requirement already satisfied: graphviz in /usr/local/lib/python3.6/dist-packages (from eli5) (0.10.1)
Requirement already satisfied: six in /usr/local/lib/python3.6/dist-packages (from eli5) (1.12.0)
Requirement already satisfied: Jinja2 in /usr/local/lib/python3.6/dist-packages (from eli5) (2.11.2)
Requirement already satisfied: tabulate>=0.7.7 in /usr/local/lib/python3.6/dist-packages (from eli5) (0.8.7)
Requirement already satisfied: attrs>16.0.0 in /usr/local/lib/python3.6/dist-packages (from eli5) (19.3.0)
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.6/dist-packages (from scikit-learn>=0.18->eli5) (0.14.1)
Requirement already satisfied: MarkupSafe>=0.23 in /usr/local/lib/python3.6/dist-packages (from Jinja2->eli5) (1.1.1)
Installing collected packages: eli5
Successfully installed eli5-0.10.1
/usr/local/lib/python3.6/dist-packages/sklearn/utils/deprecation.py:144: FutureWarning: The sklearn.metrics.scorer module is deprecated
  warnings.warn(message, FutureWarning)
/usr/local/lib/python3.6/dist-packages/sklearn/utils/deprecation.py:144: FutureWarning: The sklearn.feature_selection.base module is deprecated
  warnings.warn(message, FutureWarning)
Using TensorFlow backend.
```

Weight	Feature
0.1736 ± 0.1030	Capital Area Ratio
0.1545 ± 0.2241	Density (P/mi2)
0.0665 ± 0.0669	Capital Land Area
0.0546 ± 0.0414	Latitude
0.0284 ± 0.0096	% Urban Pop
0.0134 ± 0.0002	Longitude Difference to State Capital
0.0128 ± 0.0150	Became a State
0.0094 ± 0.0061	DaysSinceStayatHomeOrder
0.0072 ± 0.0004	Latitude Difference to State Capital
0.0068 ± 0.0193	Sum of NUM_American_IndianAlaska_Native_BEN
0.0060 ± 0.0121	Number of bordering states
0.0056 ± 0.0070	Boundaries
0.0046 ± 0.0216	Capital Mean Elevation
0.0034 ± 0.0172	Longitude
0.0027 ± 0.0078	Highest Elevation
0.0022 ± 0.0043	Land Area
0.0021 ± 0.0068	Sum of Average_Age_of_BEN
0.0020 ± 0.0000	Sum of NUM_Black_or_African_American_BEN
0.0013 ± 0.0088	DaysSinceTestStart
0.0013 ± 0.0031	State Area Ratio
0.0010 ± 0.0050	On Coast
0.0009 ± 0.0055	Capital Water Area
0 ± 0.0000	Sum of NUM_Asian_Pacific_Islander_BEN
0 ± 0.0000	Sum of NUM_Hispanic_BEN
0 ± 0.0000	Sum of PCT_MEDICARE
0 ± 0.0000	Sum of NUM_Medicare_BEN
0 ± 0.0000	Water Area
0 ± 0.0000	Lowest elevation
0 ± 0.0000	DaysSinceInfection
0 ± 0.0000	Borders Another Country
0 ± 0.0000	Log10Pop
0 ± 0.0000	Capital is the Largest City
-0.0004 ± 0.0009	DaysSinceFirstPositive
-0.0035 ± 0.0032	Elevation Ratio
-0.0060 ± 0.0175	Sum of NUM_BEN_With_Race_Not_Elsewhere_Classified
-0.0186 ± 0.0044	Children 0-18
-0.0193 ± 0.0076	Mean Elevation
-0.0236 ± 0.0123	Children0-18

```
from sklearn.metrics import mean_squared_error, r2_score

# Coefficient of determination r2 for the training set
pipeline_score = permutter.score(X_train_transformed,y_train)
print("Coefficient of determination r2 for the training set.: ", pipeline_score)

# Coefficient of determination r2 for the validation set
pipeline_score = permutter.score(X_val_transformed,y_val)
print("Coefficient of determination r2 for the validation set.: ", pipeline_score)

# The mean squared error
y_pred = permutter.predict(X_val_transformed)
print("Mean squared error: %.2f"% mean_squared_error(y_val, y_pred))

☐ Coefficient of determination r2 for the training set.: 0.6195488363328325
Coefficient of determination r2 for the validation set.: 0.48283160859213214
Mean squared error: 528.60
```

```
# Thus, Density remains important according to feature permutation than according to feature importance in the Random Forest
# Use importances for feature selection
print('Shape before removing features:', X_train.shape)
```

```
print('Shape before removing features:', X_train.shape)
```

```
↳ Shape before removing features: (37, 38)
```

```
# Remove features of 0 importance
zero_importance = 0.0
mask = permuted.feature_importances_ > zero_importance
features1 = X_train.columns[mask]
X_train = X_train[features1]
print('Shape after removing features:', X_train.shape)
```

```
↳ Shape after removing features: (37, 22)
```

```
# Random forest classifier with 22 features
X_val = X_val[features1]
pipeline4 = make_pipeline(
    ce.OneHotEncoder(use_cat_names=True),
    SimpleImputer(strategy='most_frequent'),
    RandomForestRegressor(bootstrap=True, ccp_alpha=0,
        max_depth=3, max_features='auto', max_leaf_nodes=None,
        max_samples=None, min_impurity_decrease=0.0,
        min_impurity_split=None, min_samples_leaf=4,
        min_samples_split=2, min_weight_fraction_leaf=0,
        n_estimators=29, n_jobs=None, oob_score=False,
        random_state=0, verbose=0, warm_start=False)
)

# Fit on train, score on val
pipeline4.fit(X_train, y_train);
```

```
# Coefficient of determination r2 for the training set
pipeline_score = pipeline4.score(X_train,y_train)
print("Coefficient of determination r2 for the training set.: ", pipeline_score)

# Coefficient of determination r2 for the validation set
pipeline_score = pipeline4.score(X_val,y_val)
print("Coefficient of determination r2 for the validation set.: ", pipeline_score)

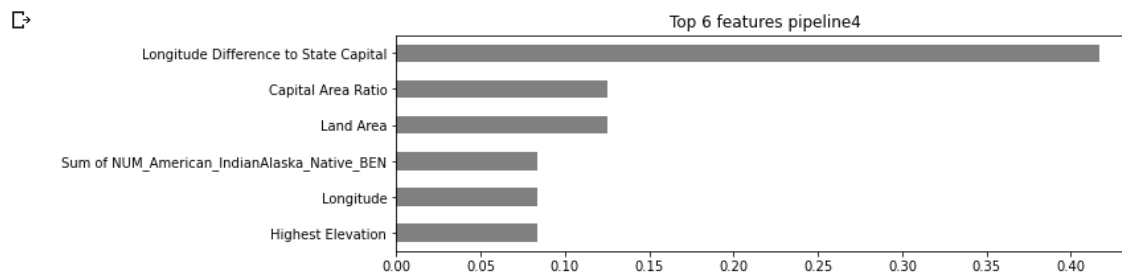
# The mean squared error
y_pred = pipeline4.predict(X_val)
print("Mean squared error: %.2f"% mean_squared_error(y_val, y_pred))
```

```
↳ Coefficient of determination r2 for the training set.: 0.6266222750237471
Coefficient of determination r2 for the validation set.: 0.4897612195667084
Mean squared error: 521.52
```

```
pipeline4.fit(X_val, y_val)
# Plot of features
%matplotlib inline
import matplotlib.pyplot as plt

# Get feature importances
encoder = pipeline4.named_steps['onehotencoder']
encoded = encoder.transform(X_val)
rf = pipeline4.named_steps['randomforestregressor']
importances2 = pd.Series(rf.feature_importances_, encoded.columns)

# Plot feature importances
n = 6
plt.figure(figsize=(10,n/2))
plt.title(f'Top {n} features pipeline4')
importances2.sort_values()[-n:].plot.barh(color='grey');
```



```
# Gradient boosting using XGboost with 15 estimators
from xgboost import XGBRegressor
```

```

pipeline5 = make_pipeline(
    ce.OrdinalEncoder(),
    XGBRegressor(n_estimators=15,
                  max_depth=1,
                  learning_rate=0.41, # try a higher learning rate
                  random_state=42,
                  n_jobs=-1)
)
pipeline5.fit(X_train, y_train);

```

⌘ [19:45:31] WARNING: /workspace/src/objective/regression_obj.cu:152: reg:linear is now deprecated in favor of reg:squarederror.

```

# Coefficient of determination r2 for the training set
pipeline_score = pipeline5.score(X_train,y_train)
print("Coefficient of determination r2 for the training set.: ", pipeline_score)

# Coefficient of determination r2 for the validation set
pipeline_score = pipeline5.score(X_val,y_val)
print("Coefficient of determination r2 for the validation set.: ", pipeline_score)

# The mean squared error
y_pred = pipeline5.predict(X_val)
print("Mean squared error: %.2f"% mean_squared_error(y_val, y_pred))

```

⌘ Coefficient of determination r2 for the training set.: 0.7838756216629406
 Coefficient of determination r2 for the validation set.: 0.605786244166149
 Mean squared error: 402.93

The best validation score (0.605786) and lowest MSE (402.93) comes from using Gradient Boosting with 13 estimators.

```

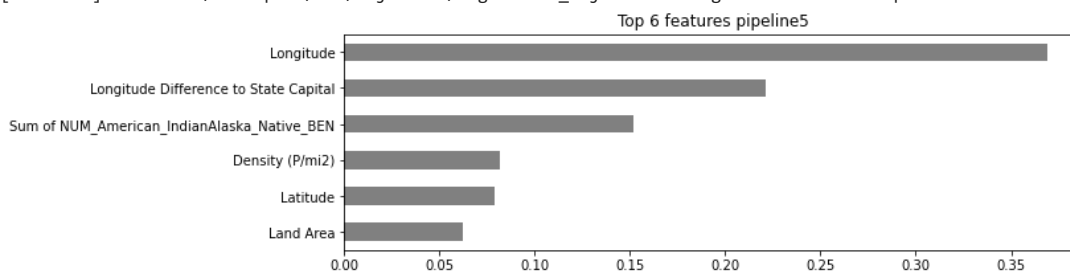
pipeline5.fit(X_val, y_val)
# Plot of features
%matplotlib inline
import matplotlib.pyplot as plt

# Get feature importances
encoder = pipeline5.named_steps['ordinalencoder']
encoded = encoder.transform(X_val)
rf = pipeline5.named_steps['xgbregressor']
importances3 = pd.Series(rf.feature_importances_, encoded.columns)

# Plot feature importances
n = 6
plt.figure(figsize=(10,n/2))
plt.title(f'Top {n} features pipeline5')
importances3.sort_values()[-n:].plot.barh(color='grey');

```

⌘ [19:45:31] WARNING: /workspace/src/objective/regression_obj.cu:152: reg:linear is now deprecated in favor of reg:squarederror.



```

# Gradient boosting using XGboost with 1000 estimators
encoder = ce.OrdinalEncoder()
X_train_encoded = encoder.fit_transform(X_train)
X_val_encoded = encoder.transform(X_val)
X_train.shape, X_val.shape, X_train_encoded.shape, X_val_encoded.shape

```

⌘ ((37, 22), (13, 22), (37, 22), (13, 22))

```

eval_set = [(X_train_encoded, y_train),
            (X_val_encoded, y_val)]

model2 = XGBRegressor(
    n_estimators=1000, # <= 1000 trees, depends on early stopping
    max_depth=1,

```

```

learning_rate=0.41, # try higher learning rate
n_jobs=-1)

model2.fit(X_train_encoded, y_train, eval_set=eval_set, eval_metric='rmse',
          early_stopping_rounds=50)

```

⚠ [19:45:31] WARNING: /workspace/src/objective/regression_obj.cu:152: reg:linear is now deprecated in favor of reg:squarederror.
 [0] validation_0-rmse:91.9687 validation_1-rmse:92.6396
 Multiple eval metrics have been passed: 'validation_1-rmse' will be used for early stopping.

Will train until validation_1-rmse hasn't improved in 50 rounds.

[1]	validation_0-rmse:59.8496	validation_1-rmse:59.9746
[2]	validation_0-rmse:41.644	validation_1-rmse:42.4277
[3]	validation_0-rmse:31.6015	validation_1-rmse:32.9433
[4]	validation_0-rmse:26.6825	validation_1-rmse:27.7771
[5]	validation_0-rmse:24.0516	validation_1-rmse:26.4663
[6]	validation_0-rmse:22.2411	validation_1-rmse:24.8219
[7]	validation_0-rmse:20.9869	validation_1-rmse:23.9266
[8]	validation_0-rmse:19.944	validation_1-rmse:22.0353
[9]	validation_0-rmse:19.1823	validation_1-rmse:20.8479
[10]	validation_0-rmse:18.435	validation_1-rmse:19.5501
[11]	validation_0-rmse:17.6743	validation_1-rmse:20.4497
[12]	validation_0-rmse:17.0584	validation_1-rmse:21.0673
[13]	validation_0-rmse:16.5325	validation_1-rmse:20.8832
[14]	validation_0-rmse:15.9991	validation_1-rmse:20.0731
[15]	validation_0-rmse:15.4335	validation_1-rmse:20.1635
[16]	validation_0-rmse:15.0028	validation_1-rmse:20.8857
[17]	validation_0-rmse:14.5576	validation_1-rmse:20.6639
[18]	validation_0-rmse:14.2005	validation_1-rmse:20.4452
[19]	validation_0-rmse:13.8148	validation_1-rmse:20.5736
[20]	validation_0-rmse:13.4191	validation_1-rmse:20.8303
[21]	validation_0-rmse:13.098	validation_1-rmse:20.5082
[22]	validation_0-rmse:12.765	validation_1-rmse:20.4495
[23]	validation_0-rmse:12.4537	validation_1-rmse:21.0325
[24]	validation_0-rmse:12.1802	validation_1-rmse:20.8216
[25]	validation_0-rmse:11.8973	validation_1-rmse:20.5977
[26]	validation_0-rmse:11.6352	validation_1-rmse:20.8435
[27]	validation_0-rmse:11.3495	validation_1-rmse:20.7061
[28]	validation_0-rmse:11.085	validation_1-rmse:20.8546
[29]	validation_0-rmse:10.8404	validation_1-rmse:20.8113
[30]	validation_0-rmse:10.6297	validation_1-rmse:21.0464
[31]	validation_0-rmse:10.3895	validation_1-rmse:21.3184
[32]	validation_0-rmse:10.1956	validation_1-rmse:21.5471
[33]	validation_0-rmse:9.95444	validation_1-rmse:21.5849
[34]	validation_0-rmse:9.76568	validation_1-rmse:21.4027
[35]	validation_0-rmse:9.58377	validation_1-rmse:21.4196
[36]	validation_0-rmse:9.37852	validation_1-rmse:21.3164
[37]	validation_0-rmse:9.17668	validation_1-rmse:21.3223
[38]	validation_0-rmse:8.99263	validation_1-rmse:21.0383
[39]	validation_0-rmse:8.84106	validation_1-rmse:21.0943
[40]	validation_0-rmse:8.64903	validation_1-rmse:21.1314
[41]	validation_0-rmse:8.47994	validation_1-rmse:21.4191
[42]	validation_0-rmse:8.29076	validation_1-rmse:21.471
[43]	validation_0-rmse:8.1178	validation_1-rmse:21.5766
[44]	validation_0-rmse:7.96713	validation_1-rmse:21.5596
[45]	validation_0-rmse:7.82337	validation_1-rmse:21.6025
[46]	validation_0-rmse:7.67152	validation_1-rmse:21.8777
[47]	validation_0-rmse:7.51868	validation_1-rmse:21.9137
[48]	validation_0-rmse:7.36816	validation_1-rmse:21.7134
[49]	validation_0-rmse:7.23374	validation_1-rmse:21.6397
[50]	validation_0-rmse:7.09412	validation_1-rmse:21.474
[51]	validation_0-rmse:6.95826	validation_1-rmse:21.6141
[52]	validation_0-rmse:6.83248	validation_1-rmse:21.7338
[53]	validation_0-rmse:6.68974	validation_1-rmse:21.6532
[54]	validation_0-rmse:6.55768	validation_1-rmse:21.6003
[55]	validation_0-rmse:6.44008	validation_1-rmse:21.682
[56]	validation_0-rmse:6.32668	validation_1-rmse:21.5601
[57]	validation_0-rmse:6.19023	validation_1-rmse:21.7015
[58]	validation_0-rmse:6.06951	validation_1-rmse:21.7155
[59]	validation_0-rmse:5.95723	validation_1-rmse:21.6837
[60]	validation_0-rmse:5.83285	validation_1-rmse:21.6631

Stopping. Best iteration:

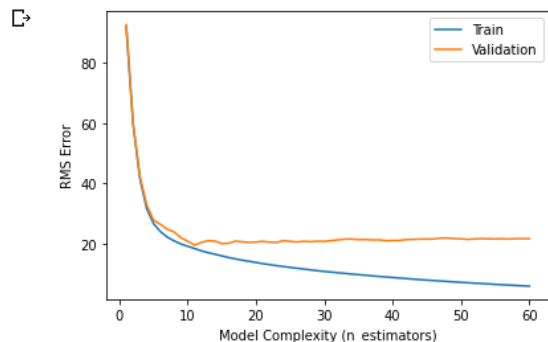
[10]	validation_0-rmse:18.435	validation_1-rmse:19.5501
------	--------------------------	---------------------------

```

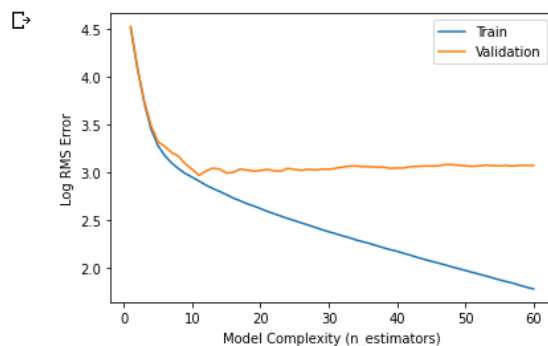
XGBRegressor(base_score=0.5, booster='gbtree', colsample_bylevel=1,
              colsample_bynode=1, colsample_bytree=1, gamma=0,
              importance_type='gain', learning_rate=0.41, max_delta_step=0,
              max_depth=1, min_child_weight=1, missing=None, n_estimators=1000,
              n_jobs=-1, nthread=None, objective='reg:linear', random_state=0,
              reg_alpha=0, reg_lambda=1, scale_pos_weight=1, seed=None,
              silent=None, subsample=1, verbosity=1)

```

```
# Plot the results
results = model2.evals_result()
train_error = results['validation_0']['rmse']
val_error = results['validation_1']['rmse']
epoch = range(1, len(train_error)+1)
plt.plot(epoch, train_error, label='Train')
plt.plot(epoch, val_error, label='Validation')
plt.ylabel('RMS Error')
plt.xlabel('Model Complexity (n_estimators)')
# plt.ylim((0.18, 0.22)) # Zoom in
plt.legend();
```



```
# Plot log classification error versus model complexity
import numpy as np
results = model2.evals_result()
log_train_error = np.log(results['validation_0']['rmse'])
log_val_error = np.log(results['validation_1']['rmse'])
epoch = range(1, len(train_error)+1)
plt.plot(epoch, log_train_error, label='Train')
plt.plot(epoch, log_val_error, label='Validation')
plt.ylabel('Log RMS Error')
plt.xlabel('Model Complexity (n_estimators)')
# plt.ylim((0.18, 0.22)) # Zoom in
plt.legend();
```



#Gradient Boosting R^2

```
gb = make_pipeline(
    ce.OrdinalEncoder(),
    # XGBRegressor(n_estimators=45, objective='reg:squarederror', n_jobs=-1)
    XGBRegressor(n_estimators=15,
                  objective='reg:squarederror',
                  max_depth=1, # try deeper trees because of high cardinality categoricals
                  learning_rate=0.41, # try a higher learning rate
                  random_state=42,
                  n_jobs=-1)
)
gb.fit(X_train, y_train)

#y_pred = gb.predict(X_val)
#print('Gradient Boosting  $R^2$ ', r2_score(y_val, y_pred))
```



```
Pipeline(memory=None,
          steps=[('ordinalencoder',
                  OrdinalEncoder(cols=[], drop_invariant=False,
                                handle_missing='value', handle_unknown='value',
                                mapping=[], return_df=True, verbose=0)),
                 ('xgbregressor',
                  XGBRegressor(base_score=0.5, booster='gbtree',
                                colsample_bylevel=1, colsample_bynode=1,
                                colsample_bytree=1, gamma=0,
                                importance_type='gain', learning_rate=0.41,
                                max_delta_step=0, max_depth=1, min_child_weight=1,
                                missing=None, n_estimators=15, n_jobs=-1,
                                nthread=None, objective='reg:squarederror',
                                random_state=42, reg_alpha=0, reg_lambda=1,
                                scale_pos_weight=1, seed=None, silent=None,
                                subsample=1, verbosity=1))),
          verbose=False)
```

```
# Coefficient of determination r2 for the training set
#pipeline_score = gb.score(X_train,y_train)
y_train_pred = gb.predict(X_train)
pipeline_score = r2_score(y_train, y_train_pred)
print("Coefficient of determination r2 for the training set.: ", pipeline_score)

# Coefficient of determination r2 for the validation set
#pipeline_score = gb.score(X_val,y_val)
y_val_pred = gb.predict(X_val)
pipeline_score = r2_score(y_val, y_val_pred)
print("Coefficient of determination r2 for the validation set.: ", pipeline_score)

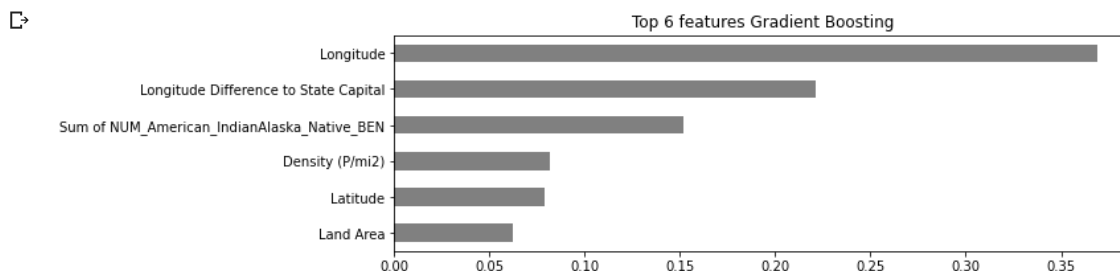
# The mean squared error
#y_pred = gb.predict(X_val)
#print("Mean squared error: %.2f"% mean_squared_error(y_val, y_pred))
print("Mean squared error: %.2f"% mean_squared_error(y_val, y_val_pred))
```

```
↗ Coefficient of determination r2 for the training set.: 0.7838756216629406
Coefficient of determination r2 for the validation set.: 0.605786244166149
Mean squared error: 402.93
```

```
#pipeline5.fit(X_val, y_val)
gb.fit(X_val, y_val)
# Plot of features
%matplotlib inline
import matplotlib.pyplot as plt

# Get feature importances
encoder = gb.named_steps['ordinalencoder']
encoded = encoder.transform(X_val)
rf = gb.named_steps['xgbregressor']
importances4 = pd.Series(rf.feature_importances_, encoded.columns)

# Plot feature importances
n = 6
plt.figure(figsize=(10,n/2))
#plt.title(f'Top {n} features pipeline5')
plt.title(f'Top {n} features Gradient Boosting')
importances4.sort_values()[-n:].plot.barh(color='grey');
```



```
!pip install pdpbox
```

↗

Collecting pdpbox

Downloading <https://files.pythonhosted.org/packages/87/23/ac7da5ba1c6c03a87c412e7e7b6e91a10d6ecf4474906c3e736f93940d49/PDPbox-0.2.0.tar>

57.7MB 62kB/s

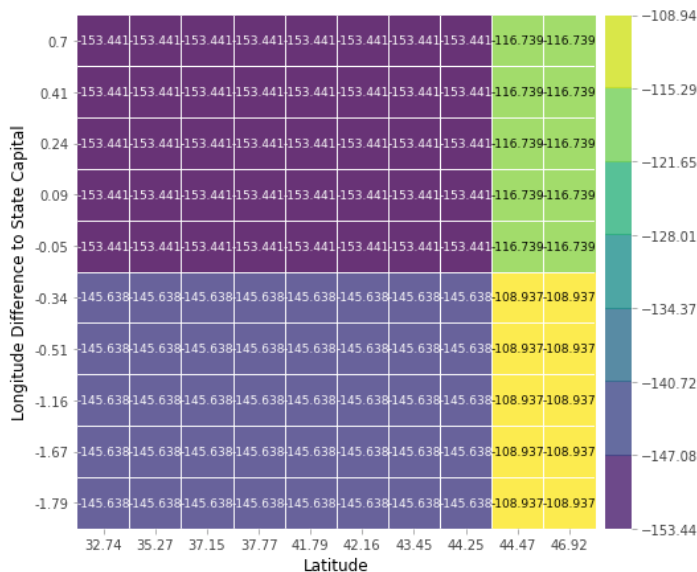
Requirement already satisfied: pandas in /usr/local/lib/python3.6/dist-packages (from pdpbox) (1.0.3)
 Requirement already satisfied: numpy in /usr/local/lib/python3.6/dist-packages (from pdpbox) (1.18.3)
 Requirement already satisfied: scipy in /usr/local/lib/python3.6/dist-packages (from pdpbox) (1.4.1)
 Requirement already satisfied: matplotlib>=2.1.2 in /usr/local/lib/python3.6/dist-packages (from pdpbox) (3.2.1)
 Requirement already satisfied: joblib in /usr/local/lib/python3.6/dist-packages (from pdpbox) (0.14.1)
 Requirement already satisfied: psutil in /usr/local/lib/python3.6/dist-packages (from pdpbox) (5.4.8)
 Requirement already satisfied: scikit-learn in /usr/local/lib/python3.6/dist-packages (from pdpbox) (0.22.2.post1)
 Requirement already satisfied: pytz>=2017.2 in /usr/local/lib/python3.6/dist-packages (from pandas->pdpbox) (2018.9)
 Requirement already satisfied: python-dateutil>=2.6.1 in /usr/local/lib/python3.6/dist-packages (from pandas->pdpbox) (2.8.1)
 Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in /usr/local/lib/python3.6/dist-packages (from matplotlib>=2.1.2->pdpbox) (0.10.0)
 Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.6/dist-packages (from matplotlib>=2.1.2->pdpbox) (0.10.0)
 Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.6/dist-packages (from matplotlib>=2.1.2->pdpbox) (1.2.0)
 Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.6/dist-packages (from python-dateutil>=2.6.1->pandas->pdpbox) (1.12.0)
 Building wheels for collected packages: pdpbox
 Building wheel for pdpbox (setup.py) ... done
 Created wheel for pdpbox: filename=PDPbox-0.2.0-cp36-none-any.whl size=57690722 sha256=9178ce1471a64bb075e7143d79a73f38354c202a78f9fb0f
 Stored in directory: /root/.cache/pip/wheels/7d/08/51/63fd122b04a2c87d780464eeffb94867c75bd96a64d500a3fe
 Successfully built pdpbox
 Installing collected packages: pdpbox
 Successfully installed pdpbox-0.2.0

```
# Partial Dependence Plots with 2 features
from pdpbox.pdp import pdp_interact, pdp_interact_plot
features2 = ['Latitude', 'Longitude Difference to State Capital']
interaction = pdp_interact(
    model=gb,
    dataset=X_val,
    model_features=X_val.columns,
    features=features2
)
pdp_interact_plot(interaction, plot_type='grid', feature_names=features2);
```

```
⚠ findfont: Font family ['Arial'] not found. Falling back to DejaVu Sans.
findfont: Font family ['Arial'] not found. Falling back to DejaVu Sans.
findfont: Font family ['Arial'] not found. Falling back to DejaVu Sans.
findfont: Font family ['Arial'] not found. Falling back to DejaVu Sans.
```

PDP interact for "Latitude" and "Longitude Difference to State Capital"

Number of unique grid points: (Latitude: 10, Longitude Difference to State Capital: 10)



```
# A two feature partial dependence plot in 3D
pdp = interaction.pdp.pivot_table(
    values='preds',
    columns=features2[0],
    index=features2[1]
)[::-1] # Slice notation to reverse index order so y axis is ascending

import plotly.graph_objs as go
```

```

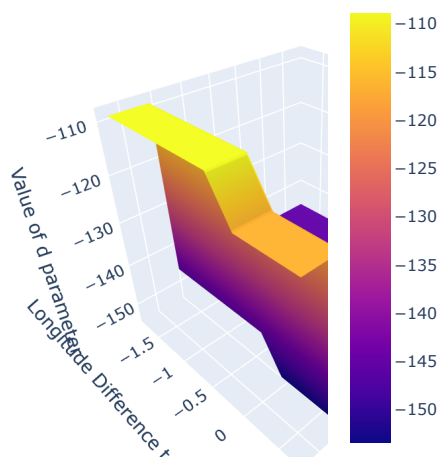
target = 'Value of d parameter'

surface = go.Surface(x=pdp.columns,
                    y=pdp.index,
                    z=pdp.values)

layout = go.Layout(
    scene=dict(
        xaxis=dict(title=features2[0]),
        yaxis=dict(title=features2[1]),
        zaxis=dict(title=target)
    )
)

fig = go.Figure(surface, layout)
fig.show()

```



In order to establish feature importances, Shapley Force Plots are used. SHAP is both consistent and accurate as a way to allocate feature importances. The details are in a recent paper by Lundberg and Lee (papers.nips.cc/paper/7062-a-unified-approach-to-interpreting-model-predictions.pdf)

```

! pip install shap==0.23.0
! pip install -I shap

```



```
Collecting shap==0.23.0
  Downloading https://files.pythonhosted.org/packages/60/0d/8bd076821f7230edb2892ad982ea91ca25f2f925466563272e61eae891c6/shap-0.23.0.tar.gz
    184kB 2.6MB/s
Requirement already satisfied: numpy in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (1.18.3)
Requirement already satisfied: scipy in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (1.4.1)
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (0.22.2.post1)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (3.2.1)
Requirement already satisfied: pandas in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (1.0.3)
Requirement already satisfied: tqdm in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (4.38.0)
Requirement already satisfied: ipython in /usr/local/lib/python3.6/dist-packages (from shap==0.23.0) (5.5.0)
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.6/dist-packages (from scikit-learn->shap==0.23.0) (0.14.1)
Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.6/dist-packages (from matplotlib->shap==0.23.0) (1.2.0)
Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in /usr/local/lib/python3.6/dist-packages (from matplotlib->shap==0.23.0) (2.8.1)
Requirement already satisfied: python-dateutil>=2.1 in /usr/local/lib/python3.6/dist-packages (from matplotlib->shap==0.23.0) (2.8.1)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.6/dist-packages (from matplotlib->shap==0.23.0) (0.10.0)
Requirement already satisfied: pytz>=2017.2 in /usr/local/lib/python3.6/dist-packages (from pandas->shap==0.23.0) (2018.9)
Requirement already satisfied: pygments in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (2.1.3)
Requirement already satisfied: decorator in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (4.4.2)
Requirement already satisfied: pickleshare in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (0.7.5)
Requirement already satisfied: traitlets>=4.2 in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (4.3.3)
Requirement already satisfied: simplegeneric>=0.8 in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (0.8.1)
Requirement already satisfied: pexpect; sys_platform != "win32" in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (4.2.1)
Requirement already satisfied: setuptools>=18.5 in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (46.1.3)
Requirement already satisfied: prompt-toolkit<2.0.0,>=1.0.4 in /usr/local/lib/python3.6/dist-packages (from ipython->shap==0.23.0) (1.0.1)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.6/dist-packages (from python-dateutil->shap==0.23.0) (1.14.0)
Requirement already satisfied: ipython-genutils in /usr/local/lib/python3.6/dist-packages (from traitlets->shap==0.23.0) (0.2.0)
Requirement already satisfied: ptyprocess>=0.5 in /usr/local/lib/python3.6/dist-packages (from pexpect; sys_platform != "win32"->ipython->shap==0.23.0) (0.6.0)
Requirement already satisfied: wcwidth in /usr/local/lib/python3.6/dist-packages (from prompt-toolkit<2.0.0,>=1.0.4->ipython->shap==0.23.0) (0.1.9)
Building wheels for collected packages: shap
  Building wheel for shap (setup.py) ... done
  Created wheel for shap: filename=shap-0.23.0-cp36-cp36m-linux_x86_64.whl size=235681 sha256=5603ae310a497a67d8b08f126db57be6be8b61f724c
  Stored in directory: /root/.cache/pip/wheels/c1/2c/aa/10d1782fe066536fcd564a2f8adea4dd05f57768236038855b
Successfully built shap
Installing collected packages: shap
Successfully installed shap-0.23.0
Collecting shap
  Downloading https://files.pythonhosted.org/packages/a8/77/b504e43e21a2ba543a1ac4696718beb500cfa708af2fb57cb54ce299045c/shap-0.35.0.tar.gz
    276kB 2.8MB/s
Collecting numpy
  Downloading https://files.pythonhosted.org/packages/03/27/e35e7c6e6a52fab9fcc64fc2b20c6b516eba930bb02b10ace3b3820d3ab/numpy-1.18.4-cp36-cp36m-linux_x86_64.whl
    20.2MB 63.7MB/s
Collecting scipy
  Downloading https://files.pythonhosted.org/packages/dc/29/162476fd44203116e7980cfbd9352eef9db37c49445d1fec35509022f6aa/scipy-1.4.1-cp36-cp36m-linux_x86_64.whl
    26.1MB 1.5MB/s
Collecting scikit-learn
  Downloading https://files.pythonhosted.org/packages/5e/d8/312e03adf4c78663e17d802fe244007237fee46cada1404f1727ed77a32/scikit_learn-0.22.2.post1-cp36-cp36m-linux_x86_64.whl
    7.1MB 41.5MB/s
Collecting pandas
  Downloading https://files.pythonhosted.org/packages/bb/71/8f53dbdbcb67c912b888b40def255767e475402e9df64050019149b1a943/pandas-1.0.3-cp36-cp36m-linux_x86_64.whl
    10.0MB 175KB/s
Collecting tqdm>4.25.0
  Downloading https://files.pythonhosted.org/packages/c9/40/058b12e8ba10e35f89c9b1fd4c2d4c7f8c05947df2d5eb3c7b258019fda0/tqdm-4.46.0-py2.py3-none-any.whl
    71kB 10.3MB/s
Collecting joblib>=0.11
  Downloading https://files.pythonhosted.org/packages/28/5c/cf6a2b65a321c4a209efc64c2689efae2cb62661f8f6f4bb28547cf1bf/joblib-0.14.1-py2.py3-none-any.whl
    296kB 50.3MB/s
Collecting pytz>=2017.2
  Downloading https://files.pythonhosted.org/packages/4f/a4/879454d49688e2fad93e59d7d4efda580b783c745fd2ec2a3adf87b0808d/pytz-2020.1-py2.py3-none-any.whl
    512kB 48.3MB/s
Collecting python-dateutil>=2.6.1
  Downloading https://files.pythonhosted.org/packages/d4/70/d60450c3dd48ef87586924207ae8907090de0b306af2bce5d134d78615cb/python_dateutil-2.8.1-py2.py3-none-any.whl
    235kB 49.7MB/s
Collecting six>=1.5
  Downloading https://files.pythonhosted.org/packages/65/eb/1f97cb97bfc2390a276969c6fae16075da282f5058082d4cb10c6c5c1dba/six-1.14.0-py2.py3-none-any.whl
    4.9kB 10.3MB/s
Building wheels for collected packages: shap
  Building wheel for shap (setup.py) ... done
  Created wheel for shap: filename=shap-0.35.0-cp36-cp36m-linux_x86_64.whl size=394118 sha256=0a0aa13f591a373298b5792d516d10896c3c2c014b4
  Stored in directory: /root/.cache/pip/wheels/e7/f7/0f/b57055080cf8894906b3bd3616d2fc2bfd0b12d5161bcb24ac
Successfully built shap
ERROR: google-colab 1.0.0 has requirement six<=1.12.0, but you'll have six 1.14.0 which is incompatible.
ERROR: datascience 0.10.6 has requirement folium==0.2.1, but you'll have folium 0.8.3 which is incompatible.
ERROR: convertdate 2.2.0 has requirement pytz<2020,>=2014.10, but you'll have pytz 2020.1 which is incompatible.
ERROR: alumentations 0.1.12 has requirement imgaug<0.2.7,>=0.2.5, but you'll have imgaug 0.2.9 which is incompatible.
Installing collected packages: numpy, scipy, joblib, scikit-learn, pytz, six, python-dateutil, pandas, tqdm, shap
Successfully installed joblib-0.14.1 numpy-1.18.4 pandas-1.0.3 python-dateutil-2.8.1 pytz-2020.1 scikit-learn-0.22.2.post1 scipy-1.4.1 shap-0.35.0
WARNING: The following packages were previously imported in this runtime:
[dateutil,joblib,numpy,pandas,pytz,scipy,six,sklearn,tqdm]
You must restart the runtime in order to use newly installed versions.
```

RESTART RUNTIME

```
# Local Interpretation using SHAP (for prediction at State # = 4, row 32)
import shap

model_shap = XGBRegressor(n_estimators=15,
                           objective='reg:squarederror',
                           max_depth=1, # try deeper trees because of high cardinality categoricals
                           learning_rate=0.41, # try a higher learning rate
                           random_state=42,
                           n_jobs=-1)

#encoder = ce.OrdinalEncoder()
#X_train_shap_encoded = encoder.fit_transform(X_train)
#X_val_shap_encoded = encoder.transform(X_val)

#eval_set = [(X_train_shap_encoded, y_train),
#            (X_val_shap_encoded, y_val)]
eval_set = [(X_train, y_train),
            (X_val, y_val)]

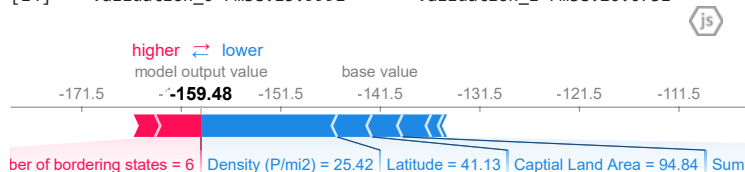
#model_shap.fit(X_train_shap_encoded,
model_shap.fit(X_train,
               y_train,
               eval_set=eval_set,
               eval_metric='rmse',
               early_stopping_rounds=50)

shap.initjs()
#explainer = shap.TreeExplainer(model2)
explainer = shap.TreeExplainer(model_shap)
#shap_values = explainer.shap_values(X_train_shap_encoded)
shap_values = explainer.shap_values(X_train)
i = 32
shap.force_plot(explainer.expected_value,
                shap_values[i],
                features=X_train_shap_encoded.loc[i],
                features=X_train.loc[i],
                feature_names=X_train_shap_encoded.columns)
                feature_names=X_train.columns)
```

```
[0] validation_0-rmse:91.9687 validation_1-rmse:92.6396
Multiple eval metrics have been passed: 'validation_1-rmse' will be used for early stopping.
```

Will train until validation_1-rmse hasn't improved in 50 rounds.

[1]	validation_0-rmse:59.8496	validation_1-rmse:59.9746
[2]	validation_0-rmse:41.644	validation_1-rmse:42.4277
[3]	validation_0-rmse:31.6015	validation_1-rmse:32.9433
[4]	validation_0-rmse:26.6825	validation_1-rmse:27.7771
[5]	validation_0-rmse:24.0516	validation_1-rmse:26.4663
[6]	validation_0-rmse:22.2411	validation_1-rmse:24.8219
[7]	validation_0-rmse:20.9869	validation_1-rmse:23.9266
[8]	validation_0-rmse:19.944	validation_1-rmse:22.0353
[9]	validation_0-rmse:19.1823	validation_1-rmse:20.8479
[10]	validation_0-rmse:18.435	validation_1-rmse:19.5501
[11]	validation_0-rmse:17.6743	validation_1-rmse:20.4497
[12]	validation_0-rmse:17.0584	validation_1-rmse:21.0673
[13]	validation_0-rmse:16.5325	validation_1-rmse:20.8832
[14]	validation_0-rmse:15.9991	validation_1-rmse:20.0731



```
# Find Shapley Forces across the training sample i (i = 0 - 37)
processor = make_pipeline(
    ce.OrdinalEncoder(),
    SimpleImputer(strategy='median')
)

X_train_processed = processor.fit_transform(X_train)
column_names = X_train.columns
shap_values_array = pd.DataFrame(columns = column_names)

for i in range(len(y_train)):
    row = X_train.iloc[[i]]
```

```

explainer = shap.TreeExplainer(model_shap)
row_processed = processor.transform(row)
shap_values_input = explainer.shap_values(row_processed)
shap_values_array = np.concatenate((shap_values_array, shap_values_input), axis=0)

```

```

# Create a 3D plot of force as a function of state curve displacement from mean curve and features for validation set
# A two feature partial dependence plot in 3D

```

```

import plotly.graph_objs as go
surface = go.Surface(x=column_names,
                    y=y_train,
                    z=shap_values_array)

layout = go.Layout(
    scene=dict(
        xaxis=dict(title= ''),
        yaxis=dict(title= 'Value of d for state'),
        zaxis=dict(title= 'Shapley Force')
    )
)
fig = go.Figure(surface, layout)
fig.show()

```

